UNIVERSITAT DE BARCELONA

Business Innovation: measurement, treatment and decision making in uncertain environments

Víctor G. Alfaro-García

PhD in Business

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PhD student:
Víctor G. Alfaro - García

Advisor:
Anna M. Gil - Lafuente

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A mis padres, fuente inagotable de amor, apoyo y fortaleza.
A mi hermana, que día a día me demuestra que no existen imposibles.
Especialmente a ti Cecilia, ejemplo de vida, amor y entrega.
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Chapter 1. Introduction

"If you can't measure it, you can't improve it."
Peter Drucker.

Presentation

It is generally accepted that innovation is a source of competitiveness for companies and nations. The impact of innovation activities on business performance ranges from the effect on sales and market share to the improvement of productivity and operational efficiency. At a national level, the main effects of innovative activities are characterized by the raise of the sectorial competition, this increases the yield of the productive industry, along to the systematical permeation of know-how to the linked business networks.

The first definition of the concept of innovation was established by Schumpeter (1934), indicating that innovation is what we call in a non-scientific way, *economic progress*. The UK Department of Trade and Industry (DTI, 1998) mentions that innovation is the successful exploitation of new ideas. The Oslo Manual (OECD, 2006) indicates that an innovation is the implementation of a new or significantly improved product (good or service), or process, a new marketing method, or a new organizational method in business practices, workplace organization or external relations.

It is noteworthy that the concept of innovation involves elements such as economic progress, business success, successful exploitation, etc. But, how does this happens exactly? Identifying the implications of innovative activities in business performance is highly relevant, as it opens a path for assisting a key issue in innovation research: systematically determine which resources, from those intended to manage continuous change within companies are justified, moreover, categorize which of them have had a major impact on the organization's goals and objectives.

The formula to address the previous question is somehow logical and has been widely studied by several authors. It consists on collecting information of the determinants of success or failure of companies in specialized studies on innovation management, then generating a list with the most relevant and frequent characteristics and elements, finally applying a scoring system to the selected best practices (Tidd and Bessant, 2013). However, that method faces complex challenges in the innovation environment: the heterogeneous terminology, dissimilar views and definitions of the concept, among other elements obstruct a standard measurement framework. Thus a correct evaluation, quantification and comparison of the innovative competencies of the organizations is highly challenging, since there is no single or main tendency to evaluate the measure of innovation.

It is conventionally accepted that innovation is a concept demarcated by uncertainty. Thus, one of the most recurring problems is to treat innovation information with traditional methodologies, "the uncertainty that the innovation process entails is manifested by the fact that only one third of the new products introduced in the market end up being successful, a proportion that seems not to have been overcome in the last decades" (Velasco et al., 2008). Despite innovation is, by its nature, governed by random processes, in the business reality innovation must be the result of a deliberate process, guided by human intuition, intelligence and foresight (Cotec, 1999). It is therefore necessary to understand and manage the innovation process, so that there is little room for chance (Tidd; Bessant and Pavitt, 2005).

Motivated on shedding light to the scientific gap found in the literature, we present our proposal, which is mainly focused in the application of methodologies and techniques for the treatment of information under uncertainty towards innovation management measurement and its interlaced complex decision-making process.

Preliminaries

The research on business innovation decision-making, specifically the one based on the novel frameworks for innovation management measurement is especially relevant for emerging economies, such as the Mexican market. Since the 1980s México has been transiting from a closed to an open market economy. Such external and internal economic liberalization has affected all the companies within the territory at some level. The free market strategies, along to the free trade treaties subscribed by the country have increased the competitive environment where Mexican companies operate. In this scenario, the productivity and quality of the manufactured products need to be continually improved in order to maintain market share (Chauca, 1999).

The present thesis focuses much of its analysis in the city of Morelia, Mexico. The main justification relies in the role that science contributes to society. It is generally accepted that rigorous scientific studies detonate several benefits to the communities in which it is applied. This is the main reason why the core of the study, the data collection and most of the conclusions are bound to this particular city. Morelia is also an example of the role that the research on emerging cities could represent for the profit of nations.

The case of Morelia is of particular importance as it is the political capital of the State of Michoacán de Ocampo. Its political, social and economic influence extends throughout the region. Information from the National Institute of Statistics and Geography shows that in 2010 Morelia had a total population of 729.279 people, 16% of all the population in the entity. Around 3.900 economic units were dedicated to manufacturing activities, representing 14,2% of the state's total. In the same year 816.710 people visited the municipality, attracting 28,6% of the total tourism of the state. The average educational level of the population is 10 years, compared to the state's average of 7,4 years. Literacy rates reach 99% of the population aged 15 or older. The same institute catalogued Morelia as "very low" in the marginalization index. Trade and tourism are the core of the cities' economic activity. The primary sector (agriculture, livestock, hunting and fishing) occupies 6,64% of the total economic activity. The secondary sector (manufacturing industry, construction, and electricity) represents 25,91%. The tertiary sector (trade, tourism and services) represents 63,67%. Despite the recent city's considerable demographic growth, the industrial activity has not had a favorable development compared to other cities in the region. This fact is observable in the ecological industrial park, in which, from the 354 total hectares only 180 companies operate, offering around 9 thousand jobs. However less than 30% of those companies are dedicated to manufacturing activities.

This Doctoral Thesis seeks to generate new models for business decision-making processes based on innovation as source of competitiveness. The target are small and medium enterprises (SMEs) located in the city of Morelia, Mexico. The objective aims to provide tools for the continuity and sustainability of the SMEs' operations over time.

It is well known that SMEs play a key role in the economic development of cities, which is in part, because generally a large part of jobs creation occurs in these kind of businesses. The labor-intensive production processes developed by SMEs generate innovations in goods and in the same organization (Huber, 1991). For example, in Mexico, according to data from the National Institute of Statistics and Geography (INEGI), in 1998 there were around 2 million 500 thousand commercial establishments, of which 99,7% are SMEs, together they generate 42% of Gross domestic product and 64% of the employment in the country. In that same year, the institute established that in the State of Michoacán, there were around 135 thousand establishments, of which 20,465 were manufacturing enterprises, 66,420 of them developed commercial operations, 49,652 were service enterprises and 708 worked in other sectors. From the total of manufacturing units, 98% were classified as micro enterprises, 1,7% were small enterprises, 0,2% were medium enterprises and only 1% were large enterprises.

Finally, it is commonly accepted that innovation involves uncertainty and subjectivity (Roberts, 1998, Rese and Baier, 2011, Tidd and Bessant, 2013, Bowers and Khorakian, 2014). Studies with a Fuzzy standpoint have been increasing since the last century and have demonstrated efficacy when dealing with subjective or incomplete information, the treatment of complex phenomena and information in uncertain environments (Ribeiro, 1996). As stated by Bellman and Zadeh (1970), much of the real-world decision-making takes place in an environment in which the objectives, constraints and consequences of possible actions are not precisely known. That is why we have chosen tools derived from Fuzzy Logic research as the main methodological resource for the development of this study.

Please note that the Fuzzy methodological tools selected and presented in this Thesis not only provide mathematical robustness when treating phenomena under highly uncertain environments, but also, it allows an intuitive, logical and more human representation to the general modelling process.

Objectives

The goal of this Doctoral Thesis is to develop new models and tools for business decision-making based on innovation as main driver for competitiveness. In order to reach this goal, a line of work is proposed with the following specific objectives:

Objective I. State of the art.

Description: to propose a structured review of the relationship between the concepts: Fuzzy Logic, Innovation, Competitiveness and Innovation Measurement. The goal is to review, evaluate and consolidate the specific theoretical foundation that will support the rest of our research.

Objective II. Business environment analysis.

Description: To study the endogenous and exogenous elements that compose the industrial environment of Morelia, México. The in-depth knowledge of the manufacturing framework of Morelian companies will allow us to adjust the subsequent scientific developments of the Doctoral Thesis.

Milestone:

Journal Citation Report indexed publication: "A Fuzzy Approach to a Municipality Grouping Model towards Creation of Synergies". Computational and Mathematical Organization Theory (CMOT) journal. JCR Thomson Reuters Impact Factor: 0.370.

Objetive III: Development of a Fuzzy Logic oriented model designed to quantify the measure of innovation.

Description: Propose a new methodology to measure SMEs' innovation capabilities. The new technique includes tools derived from Fuzzy Logic and aggregation operators to obtain a representative and general value of the opinions of decision-makers in the manufacturing environment of the city of Morelia, Mexico.

Milestone:

Journal Citation Report indexed publication: "A fuzzy methodology for innovation management measurement". Published in Kybernetes Journal, Vol. 46 Issue: 1, pp.50-66. JCR Thomson Reuters Impact Factor: 0.637.

Objective IV. New tools for decision- making in innovation management

Description: To develop new models for business decision-making. Apply new techniques of Fuzzy Logic tools for the exploitation of the past objectives and on the other hand, the generation of new mathematical aggregation operators based on the Ordered Weighted Average (OWA) operator.

Methodology

Our research is based on the Uncertainty Theory, specifically in Fuzzy Logic theory, which since the publication of "Fuzzy Sets. Information and Control" in 1965, by Professor Lofti Zadeh, has been used to generate significant contributions in various fields of knowledge, especially in the treatment of information and decision-making under uncertain conditions.

Since the moment of its appearance, the Fuzzy Logic theory has been widely discussed. Over the years, scientists from various branches have agreed that Fuzzy Logic seeks to formalize or mechanize two remarkable human capabilities, in the one hand, the ability to converse, reason and make rational decisions in an environment of imprecision, uncertainty, lack of information, conflicting information, biases of truth and biases of possibility, i.e. phenomena in an environment where information is imperfect. On the other hand, the human capacity of developing a variety of physical and mental tasks without the need of measurements or calculations (Zadeh, 2008).

Fuzzy Logic tools and methodologies as background for research

The complex reality in which we are submerged requires the development and application of tools that help to study phenomena both qualitatively and quantitatively. In this section we briefly address some of the tools that support our research.

Linguistic variables

Zadeh and Bellmann (1970) propose and define the linguistic variable as the one which is not represented by numbers but by words or phrases in a natural or synthetic language. When working on any problem with linguistic variables we may be able to present their meanings. At that time we can assess and measure the different conditions using fuzzy numbers and linguistic variables. The linguistic variables represent the relative importance as well as the properties of each valuation method that simultaneously considers the metric distance and a proposed average fuzzy value. The distance between the ideal solution and the mean value of the fuzzy number is a usual criterion for the evaluation of the fuzzy numbers.

Fuzzy sets

Fuzzy sets are those that contain elements with membership degrees. The Fuzzy sets were introduced simultaneously by Zadeh and Klaua in 1965, as an extension of the classical notion of sets. In classical theory, the membership of elements is bounded to binary terms i.e. to bivalent conditions (an element belongs or not to the set in question). In contrast, the theory of Fuzzy sets allows the gradual belonging of an elements in a set. This gradual belonging is described by a membership function evaluated in the real interval [0,1], this allows a broad description of the relationship between variables.

Hamming distance

In 1950 Richard W. Hamming presents the Hamming distance in his work "Detection of error and error correction codes". The use of the Hamming distance has resulted very effective when ordering Fuzzy sets. The result of the procedure shows the item which is “closests” to an ideal, this is done by measuring the importance of certain characteristics or qualities between items. In any case, the “best” element is the one having the smallesst distance from the ideal. The application of the Hamming Distance to any business phenomenon is a novel approach and has different and very attractive results. Additionaly, the model facilitates decision making by considering subjective elements and measurements in the process.

Expertons method

The Expertons method represents an important extension to the fuzzy subset model developed by Professor Kaufmann. According to Gil-Lafuente et al., (2007) the Expertons method allows the aggregation of diverse expert opinions along to the treatment of Fuzzy numbers.

The result of the Expertons method is the aggregated and representative view of all the subjective opinions that can be given by a group of experts. The Expertons method facilitates decision making by treating the qualitative data collected from the dialogue that is maintained with various interest groups that surround certain phenomena. In general, the approach allows a useful and innovative tool in the process of information aggregation, which also corresponds to a very good technique that allows us to unify different points of view and expectations of diverse groups with divergent interests. Moreover, the model allows us to visualize the distribution levels of the aggregated values of the characteristic membership function.

The Ordered Weighted Averaging (OWA) operator

The Ordered weighted averaging (OWA) operator is a well-known method that has been extensively studied for information aggregation. The operator provides a family of operators that include special criteria and cases e.g. the minimum, maximum and the mean.

In 1988 Ronald Yager introduced the so-called OWA aggregation operator, generalizing 4 decision-making models into one: the optimistic criterion, the Wald's pessimistic criterion, the Hurwicz's criterion and the Laplace's criterion. The OWA operator can be defined as:

An OWA operator is a mapping $OWA: R^n \rightarrow R$, with a n associated weighting vector, characterized by having the properties $w_j \in [0,1]$, and $\sum_{j=1}^{n} w_j = 1$. According to the formula:

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j b_j,$$

where b_j is the jth largest of the arguments a_i.

The OWA operator and the extensions that have been extensively developed in recent years provide great flexibility to model a wide variety of problems.

Forgotten effects theory

The Forgotten effects theory was developed by Kaufman and Gil Aluja in 1988. This theory allows to obtain all the direct and indirect relations between diverse variables. According to the authors all the phenomena around us is part of a system or subsystem, this implies that almost any activity related to any problem is a result of "causes" and "effects."

When dealing with a problem, despite having a good control system, some causal relationships can be voluntarily or involuntarily left aside, this happens especially when causal relationships are not explicit, obvious or visible. If this happens, it is common that the deviations result in greater miscalculations when analyzing the results.

It is in this scenario, where the theory of forgotten effects results in handy. The method provides an innovative way of minimizing the risk of lost information by taking into account all the relationships of the processes analyzed, providing information of each one of them.

Moreover, the theory takes into account all the direct and indirect causal relations. In the development of the application, this model results of great support as it gives us the weight of the direct and indirect causal relation between several elements. The model is properly addressed in the fifth chapter of this work.

Structure and content

The present Doctoral Thesis has a total of six chapters. It has been designed to follow a single research line routed to accomplish the main goal of the study. Please note that the results, findings, discussions and conclusions of one chapter follows the next one.

The general structure consists on presenting a brief overview of the corresponding chapter followed by contributions published in peer-reviewed journals and international congresses that treat the specific topic.

The approach pursues two objectives, on the one hand to present the scientific contributions that have been developed throughout the Doctoral study, and on the other, to guide the reader from the broader aspects of the scientific research on innovation, to the most novel research of mathematical aggregation operators designed for strategic decision-making in innovation management measurement.

The first chapter contains the general introduction to our research. We propose a brief presentation, the justification of the location, the main focus of the general work and a concise description of the followed methodologies.

The second chapter proposes a general review of the state of the art of the study, i.e. Fuzzy Logic tools applied to the research on innovation management measurement. This chapter is aimed to review, evaluate and consolidate the specific theoretical bases that will support the rest of our research. This includes the relationship between innovation and business competitiveness and the challenges of innovation measurement in highly uncertain environments.

The third chapter studies the business environment of Morelia, México. This chapter contains two publications, one of them published in the Computational and Mathematical Organizational Theory Journal (indexed in the WOS database). It presents an exploratory analysis of the business context and the manufacturing tissue of the city. The combined applications of Fuzzy Logic tools with Moore families and the Galois Lattice provide a robust method for exploring both the business composition and the possible synergies for collaboration and strategic alliances that would ultimately produce innovations in different orders of the selected companies.

The fourth chapter contains the core of this Doctoral Thesis. The research article published in the Kybernetes Journal (indexed in the WOS database) proposes a new methodology for innovation management measurement applying the method of the Expertons in combination with aggregation operators.

The fifth chapter presents the most technical and novel part of the whole study. Three articles are presented, one of them already published, the rest of them are currently being revised by first quartile journals indexed in the WOS database. The three contributions propose new aggregation operators, they aim to support business decision-making in highly uncertain environments.

Finally, the last chapter compiles the general conclusion of the thesis.

Chapter 2. State of the Art

Innovation is a concept linked to competitiveness, to economic progress of companies and nations, however, there are important challenges that need to be addressed in order to properly analyze and measure it.

The present chapter collects diverse perspectives through which researchers around the world have tried to model the concept of innovation. The study presents their current visions, dichotomies, processes and models that have been developed to abstract the complex reality that encompasses the phenomenon.

The result is an exploratory analysis that converges on the current need of continuing the research on innovation, however, the preceding challenges require a novel standpoint, an alternative holistic perspective that could shed some light treating the uncertainty that surrounds the phenomenon.

A fuzzy logic approach towards innovation measurement[1]

Abstract

Innovation is a convened critical factor for firm success in today's economic environment. As academics and practitioners acquire knowledge on innovation, tendencies, points of view and practices arise. Yet measurement approaches meant to help decision makers to evaluate their current innovative position do not follow a main stream, moreover much of the information needed for an accurate evaluation tends to be qualitative or subjective. The objective of the present investigation is to review how Fuzzy Logic is currently dealing with subjective complex data in innovation management approaches, results will turn as implications for further applications in innovation measurement. An examination of new methodologies towards innovation measurement is presented and linked to a systematic review on Fuzzy Logic applications to innovation management. Results convey that there is no ultimate model to address innovation measurement in firms, yet a set of innovation measurement key issues are described in novel frameworks. Fuzzy Logic stands as a viable way to adopt decision-making due to its capacity of dealing with uncertain and subjective conditions. According to results, the use of Fuzzy Logic to evaluate qualitative and subjective factors in innovation measurement is encouraged.

Keywords: innovation measurement, fuzzy logic, uncertainty, decision making

[1] Published in Global Journal of Business Research, v. 9 (3) p. 53-71. The Journal is distributed in print and through EBOSCOHost, ProQuest ABI/Inform, SSRN and RePEc. The Journal is listed in Cabell's directories. The Journal is also indexed in the American Economic Association's *Econlit*, *e-JEL* and *JEL* on CD, Ulrich's Periodicals Directory and Colciencias.

Introduction

Research on the concept of innovation has been evolving since the last decades, currently there is no manager or decision maker that could affirm that innovation does not carry competitiveness, it is in some way a given fact. As Porter (1990) states *Companies gain advantage against the world's best competitors because of generating innovations*. The results of innovative activities in firms and organizations can ~~go~~ range from effects on sales and market share up to the improvement of productivity and efficiency. The significant impacts in the sector of activity are the evolution of international competitiveness and the total productivity of the factors; the knowledge spillovers of innovations produced by enterprises and the growth in the volume of knowledge that flows over the network.

Since there is a convened positive impact regarding innovation activities, scholars from diverse expertise address the topic. Gopalakrishnan and Damanpour (1997) identify three main groups of researchers: Economists, whose perspectives centers on growth at industry level and evaluate the impact of radical product and process innovation. Technologists, whose studies center ~~in~~ around the process of generating and improving new technology~~,~~—with a focus on radical and incremental product process innovations. Sociologists, whose studies mainly focus on the organizational features and the adoption of innovations within firms, and who study technical and administrative product and process innovations. As an effect of such widespread research, there are diverse approaches around the concept of innovation, in some way leading to inconsistencies. Several studies assess this gap by reviewing the evolution of the research on innovation (García and Calantone, 2002; Hansen and Wakonen, 1997; Landry et al., 2002).

Due to the broad range of ways that the concept of innovation can be addressed, there is no definition that covers all the aspects of innovation. The earliest definition of innovation was established by Schumpeter (1934) stating *innovation is what we call in a non-scientific way "economic progress", which means in essence the use of productive resources in ways not tested yet in practice, and the retirement of the uses that have had so far*. In a market oriented standpoint Drucker (1987) has pointed out that *innovation is the tool in which innovative entrepreneur's exploit the change as an opportunity for new*, and Kanter (1983) claims that *innovation refers to the process of establishing any new idea, which resolves a problem*. A broader definition of innovation is established by the UK Department of Trade and Industry's (DTI, 1998) implicating that innovation is *the successful exploitation of new ideas.* The Oslo Manual (OECD, 2006), mentions that *innovation is the implementation of a new or significantly improved product (good or service), or process, a new marketing method, or a new organizational method in business practices, workplace organization or external relations.*

Topics within the definition of innovation compile concepts such as: progress, success, solution of problems, etc. Hence, knowing how innovative activities affect companies' performance is also needed to be discussed, as it opens the path to attend an important challenge, which is to determine in a systematic way whether innovative efforts within firms are justified, objectives are being reached and the further generation of incentives (Cordero, 1990). The formula is somehow logic, we use information acknowledged in large studies concerning success and failure around innovation, then generate a checklist with the most relevant features and apply a scorecard with the best practices (Tidd and Bessant, 2013). However such logic faces complicated challenges as conceptions, terminology, and standpoints around innovation differ from authors and studies. Cooper (1979) centered attention on the evaluation of 77 success/failure *key activities* on product innovation. De Brentani (1991) expands the discussion introducing the study of service firms. Rothwell (1992) in a large-scale empirical study evaluated successful industrial innovations. Atuahene-Gima (1995) addressed market and project performance with 2 success dimensions. Benedetto (1999) took a product launch performance orientation, and Gerwin and Moffat (1997) addressed the challenge of evaluating 3 successful dimensions in order to establish relations between companies' cultural activities and innovation. The mentioned works do not pretend to be an exhaustive list, instead show how authors focus on different types of innovation and measurements to evaluate firm's performance often dealing with subjective, incomplete or vague information.

Through the last years uncertainty, understood as imprecision and imperfect or vague information in innovation management has been acquiring attention, see e.g. Macdonald et al. (1994); Gales & Mansour-Cole (1995); Hansen et al. (1998); Tidd (2001); Lane & Maxfield (2005); Hidalgo et al. (2008); Buddelmeyer et al., (2009); O'Connor & Rice (2013); among others. As stated by Tidd and Bessant (2013) "by its nature innovation is about the unknown, about possibilities and opportunities associated with doing something new and so the process involves dealing with uncertainty".

A correct evaluation, quantification and comparison of the innovative competences of contemporary organizations is difficult since there is no single or main trend to assess innovation measurement (Frenkel et al., 2000). Yet a generalized measurement framework would provide a useful basis for managers to monitor and evaluate their innovation processes and create incentives around them (Cebon and Newton 1999). The objective of the present investigation is to address this gap by revising the main trends on practical frameworks of innovation measurement, identifying common critical elements and utilizing Fuzzy Logic to deal with subjective complex data.

We have structured the paper as follows. The first section presents a literary review, containing preliminaries of the study, initial theories, framework to address innovation measurement and the main challenges of the topic. The second section comprehends the methodology that we used for the systematic review on the updated roll that Fuzzy Logic has on innovation management. The third section, results and discussion, presents the main findings of our study and analyze how Fuzzy Logic could aid decision makers in innovation management measurement. Finally we present our concluding comments.

Literature review

Preliminaries

When trying to assess performance measurement of innovation, we must establish some ground classifications. Neely et al. (1995) emphasize the need for a proper performance measurement of systems. They define performance *measurement as the process of quantifying the efficiency and effectiveness of action.* A performance measure can be defined as *a metric used to quantify the efficiency and/or effectiveness of an action* and a performance measurement system can be defined as *the set of metrics used to quantify both the efficiency and effectiveness of actions.*

However positive managerial implications of a correct innovation performance measurement system have been (Simons 1990; Gimbert et al., 2010), scholars have not yet reached consensus on a definite approach

(Nilsson et al., 2012). The widespread vision on innovation, its definitions and related inconsistencies addressed in the introductory section tend to be resilient challenges (Gopalakrishnan and Damanpour, 1997).

Even though there is no definite consensus around the measurement of innovation, significant advances have flourished over the last years. Approaches like Adams et al. (2006) whose work focuses on the description of a holistic framework retrieving successful critical factors over the years of innovation measurement. Crossan and Apaydin (2010), whose work reveals extensive research on innovation, consolidating fundamental theories around innovation academic approaches; Edison et al., (2013) whose empirical studies describe a specific (yet scalable) industry framework; and the evaluation of dichotomies and established practices (Nilsson et al., 2012; Jensen and Webster, 2009).

Initial Theories

In order to understand the latest advances on innovation measurement we first must address the basic concepts on innovation, its principles and theories. Crossan and Apaydin (2010) compile peer-reviewed scientific academic research done over 27 years (1981-2008). The analysis includes a systematic review of 367 highly cited (minimum 5 citations per year using 2009 as base year) articles and organized them by level (individual, organization, macro, multilevel). Table 2.1 presents the quantity of researched articles found.

Learning, knowledge management, adaptation and change theories prevailed as the most concurred with 17 articles, followed up by 8 articles with foundations on Network theories and also 8 articles with roots on economics and evolution. In spite of these findings, authors conclude on not finding a strong underlying theory, and the theoretical perspectives that were employed tended to be quite disparate and generally operating at a single level.

Table 2.1. Innovation Measurement Approaches

	Multilevel	Macro	Organization	Micro
Institutional	1	3	2	
Economics and evolution	2	3	3	
Network	2	4	2	
Resource-based view and dynamic capabilities			4	
Learning, knowledge management, adaptation, change	2	2	11	2
Other theories	1	1	1	5

Source: Adapted from Crossan and Apaydin (2010)

Conclusions on a lack of coherent and explicit theoretical base pair with those of Hobday (2005); Rothwell (1994); Tidd et al. (2006); Velasco & Zamanillo (2008), works that focus on reviewing the evolution of innovation models, finding again dissimilar approaches, e.g. Static and Dynamic Innovation (Afuah, 1998); Organizational Model of Technological Innovation (Kelly and Kranzberg, 1978); Model of Innovation of Schumpeter, Conversion Models, Technology – Push and Market – Pull Models, Marquis Model (Myers and Marquis, 1969); Strategic Option model (Freeman and Soete, 1997), Abernathy and Clark (1985) Model, Tushman and Anderson (1986) model. The S Curve model Foster (1986), Abernathy and Utterback (1978) model, Tushman and Rosenkopf Model (1992).

A framework to address innovation measurement

Since there is no ultimate model or theory to address innovation measurement, authors have chosen to develop frameworks that represent the main focus areas to consider. One of the latest approaches has been developed by Adams et al., (2006), work based on a review of six innovation models (Cooper and Kleinschmidt, 1995; Chiesa et al. 1996; Goffin and Pfeiffer, 1999; Cormican and Sullivan, 2004; Burgelman et al. 2004; Verhaeghet and Kfir, 2002) that proposes a seven-factor framework of categories specified in terms of the necessary structural capabilities in a firm to make and manage change. This holistic framework takes into account multiple perspectives e.g. Cooper and Kleinschmidt (1995) whose work focuses on the generation of five techno-centric factors for new product performance, yet overlooking the non-technical context of innovation; Chiesa et al. (1996) whose technical innovation audit tool explores a wide variety of indicators that are meant to evaluate the performance of systems and tools that managers hold in order to enable “hard innovations”. Corrnican and Sullivan (2004) explore the continuous and cross-functional connections needed inside an organization to produce effective product innovations. Table 2.2 shows the holistic framework proposed by Adams et al., (2006).

Table 2.2. Innovation Measurement Framework Areas

Framework category	**Measurement areas**
Inputs	People
	Physical and financial resources
	Tools
Knowledge management	Idea generation
	Knowledge repository
	Information flows
Innovation strategy	Strategic orientation
	Strategic leadership
Organization and culture	Culture
	Structure
Portfolio management	Risk/return balance
	Optimization tool use
Project management	Project efficiency
	Tools
	Communications
	Collaboration
Commercialization	Market research
	Market testing
	Marketing and sales

Source: retrieved from Adams et al., (2006)

Rather than giving specific measurement proxies of each category described in their framework, Adams et al., (2006) focus their attention on shedding light on useful implications around innovation measurement topics.

Concluding remarks in the category inputs, reflect a need for the literature to balance, not only raw financial R&D and NDP key measurements, but also process and business model innovations. Tacit and softer skills that deal with knowledge and creativity require more attention on further input measurement.

Knowledge management has to deal with explicit and implicit knowledge held by the organization in idea generation, knowledge reposition and information flows. Putting special emphasis on the correct measurement of codified information such as patents.

Innovation strategy is found to follow two main orientations; the first is the measurement of whether the firm has an established innovation strategy and identifiable roles for new products and services. The second trend measures whether innovation strategy is *a defined instrument that shapes and guides innovation in the organization*. Special attention has to be brought to the measurement of the leaders or innovation "champions" as they have proven to be a driver towards innovation and strategic performance.

Organization and culture must be measured in line with both structural and psychological standpoints, as work environment stands as a known variable on the level of innovation in organizations. In this section the authors emphasize that much literature has been focusing in culture and organization, yet little is known about structural shift and flexibility in organizations.

Portfolio management is a relatively new key topic in the literature, reflectors turn into this area as it deals with the allocation of scarce resources of the enterprise (money, time, people, machinery, etc.) on potential projects under uncertain conditions. Performance can be measured in this subject from different angles. Evaluation on quantity, quality, organizational capability, correct alignment to business objectives and balance in both risk and timespan seem to be some conductive approaches to measure portfolio management.

Project management is one on the most challengeable topics, as it needs to measure the capability of an enterprise to create marketable innovations through specific inputs. The plethora of dissimilar business activities makes it almost impossible to have a valid measurement layout, however, the evaluation of efficiency, tools, communication and collaboration on how a firm generates outputs is commonly addressed. Other factors such as internal collaboration, synergy and transparency had been named to be important, however not yet tested.

The author’s catalogue commercialization as the least attended topic in innovation management studies. Assessing the measure of marketing, sales, distribution and joint ventures, commercialization is one of the most important activities as it is the final step of the chain, and the real test for ideas to become successful innovations. Products launched per period, market analysis and monitoring tend to be the recurrent measures, although launch proficiency and post – launch reviews are new trend topics.

Challenges on Innovation Measurement

Encompassed with the early-discussed discrepancies, extent visions and differences in terminology, innovation measurement holds implicit challenges for decision makers. Nilsson et al. (2012) present a frame in which challenges assessed as dichotomies represent the main problems when attending radical and incremental innovation measurement in firms. Topics as uncertainty, defined not only as the inherent risk of an innovative project, but also as the possibility of different outcomes in a given situation (Loch et al., 2008), complexity and unfamiliar relations (Bordia et al., 2004) and lack of information (McLain, 2009). Time, distinguished as the management of different perspectives on timespan by a radical or an incremental innovation project. The flexibility of the companies’ processes, to launch a radical or an incremental innovation, while the structure and allocation of resources are some of the main constraints in pairing both perspectives (Adams et al., 2006). Control understood as the way firms manage the culture and working environment to pull up both incremental and radical innovations at the same time. Table 2.3 gathers the main dichotomies in innovation management found by Nilsson et al., (2012).

The dichotomies presented show the importance of a holistic framework, taking into account several perspectives in order to perform a measurement system that actually adds value to the company (Kaplan and Norton 1992; Micheli et al. 2010).

Since our main objective is to identify main approaches on innovation measurement, the theories and models presented are addressed in a general and illustrative perspective; a robust theoretical review would need extensive depth, an issue that overreaches our present work scheme.

Table 2.3. Dichotomies in innovation management.

Dimension in Dichotomies	Issue in Measurement
Uncertainty	Radical innovation:
Technical	Requires a higher number of market and external environmental measures than incremental
Market	Need to be measured on sales growth rather than profitability in the commercialization stage in contrast to incremental innovation
Project scope	Requires high amount of data from different sources compared to incremental
Strategy	Need to not be measured using strategic, operational and business model fit as a requirement why the opposite is needed for incremental
Resources	
	Prototypes or probes may replace traditional project management measures in the development of radical innovation
Time	Valuation and selection of idea and projects require different measures: ex. ROI, net present value (for incremental) vs. Opportunity cost (for radical).
Long and short (Length)	
Discontinuous and continuous (Rhythm)	Radical need to be supported by measures that trace rapid and unexpected events and incremental measures that traces alignment to a predefined path.
Rapid and slow (Pace)	

Flexibility (vs. stability)	Incremental innovation benefit from using the same measures for a long period of time
Process	More measures for external communication and for measuring relations needed for radical innovation.
Structure	Radical innovation:
Strategy	Requires a broad number of quantitative and qualitative measures that can easily be exchanged
	Requires measures to support strategy development i.e. what works and what does not why measures that control the alignment to goals and strategies are sufficient for incremental innovation.
Control (vs. freedom)	Measure identification and implementation for radical innovation require both audit (bottom up) and need driven procedures (top down) why incremental innovation is supported by a need driven procedure alone.
Roles	
Leadership	Measurements need to be aligned to and support both radical and incremental recognition and reward systems

Source: retrieved from Nilsson et al., 2012.

Data and methodology

In order to appreciate how Fuzzy Logic could be useful to the resilient challenges that innovation measurement drags, we must first know which advances of the Fuzzy Logic theory had reached the scope of innovation management. In order to do so, we propose a systematic literature review (Denyer and Neely, 2004), this clear and reproducible procedure has shown increasing interest among scholars (Adams et al., 2006) and has proven efficiency in dealing with large amounts of information, establishing main paths: development of clear objectives, pre-plan auditable methods, quality execution of the search and synthesis of impartial results using clear frameworks.

In our case our main objective is to utilize peer-reviewed journals in order to explore the quantity and quality of articles that have a Fuzzy Logic methodology to address innovation. We concentrated the search in Thomson Reuters database ISI Web of Science, since it compiles one of the foremost influential pools of peer-reviewed articles (Crossan and Apaydin, 2010). Articles within the timespan of January 1986 until September 2014 were included in the search.

In order to reach inclusion of a relevant set of articles, the selection criteria utilized was defined first by the Keywords: "Fuzzy Logic" *and* Innovation*. Document type: Article and Review. No further restriction selections were made. A total of 66 articles were retrieved, this initial set was fixed for further analyses. However, 19 papers were selected for deep analyses due to our criterion of including papers that have Fuzzy Logic as a methodological foundation.

Table 2.4 presents how articles are scattered around different journals, from environmental and pollution to artificial intelligence topics. The diverse fields that Fuzzy Logic techniques covers describe the flexibility of the methodologies to address different problems of various scientific topics.

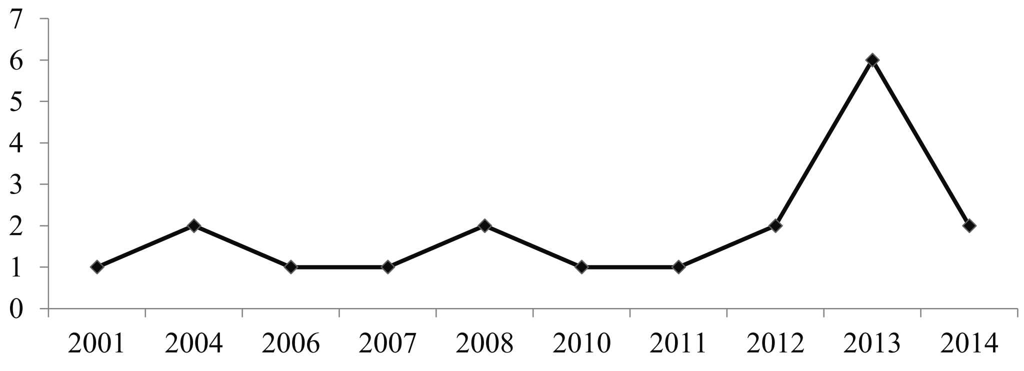
Table 2.4. Most Cited Journals

Journal	Times Cited
Proceedings of the IEEE	61
International Journal of Production Economics	53
Technovation	33
Engineering Applications of Artificial Intelligence	17
International Journal of Environment and Pollution	17
Production Planning and Control	10
Journal of Intelligent Manufacturing	7
Renewable Energy	5
Scientific World Journal	2
International Journal of Computers Communications & Control	1

Source: retrieved from Web of Science 2014.

Figure 2.1 denotes an increasing trend of publications addressing innovation management with Fuzzy Logic techniques; also it shows the novelty of these kinds of studies in the formal sciences. The increment of publications shows the rising interest from scholars to adopt diverse perspectives to address innovation management.

Figure 2.1. Growth of articles assessing innovation through a Fuzzy Logic approach



Source: retrieved from Web of Science 2014.

Our methodology has certain limitations e.g. the utilization of the ISI Web of Science narrows the scope of search. A depth analysis discriminated

47 papers out of the original 66 due to diverse issues; the main one is the fact that authors catalogue their work as “innovative”; the introduction of that keyword misleads the search and results obtained.

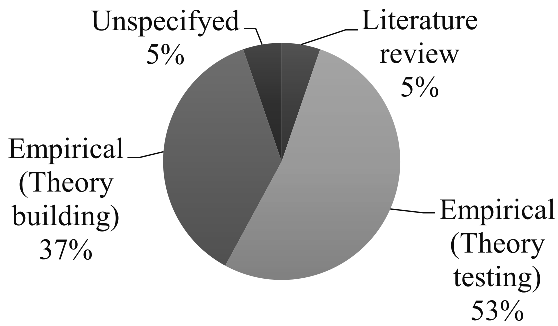
Results and discussion

In this section we present the main findings of the systematic review. We firstly present aggregated results, a specific quantification of article type, followed up by an analysis of the innovation areas that the publications address. Secondly, we present a classification of the main approaches of the chosen articles.

Aggregated results

From the 19 articles chosen for deep revision, the majority, 10 articles, present an empirical theoretical testing structure, putting into practice diverse Fuzzy Sets theories, being the most recurred theories the use of linguistic variables and fuzzy triangular numbers, thus dealing with imprecision or vagueness in information. A total of 7 articles propose the construction of theoretic frameworks, new approaches to deal with innovation management challenges with emphasis on uncertainty management and expert support systems. From the pool of articles only 1 describes a literature review, mainly focusing on soft computing industrial applications. Figure 2.2 shows the aggregated results by paper type of the 19 articles chosen for deep examination.

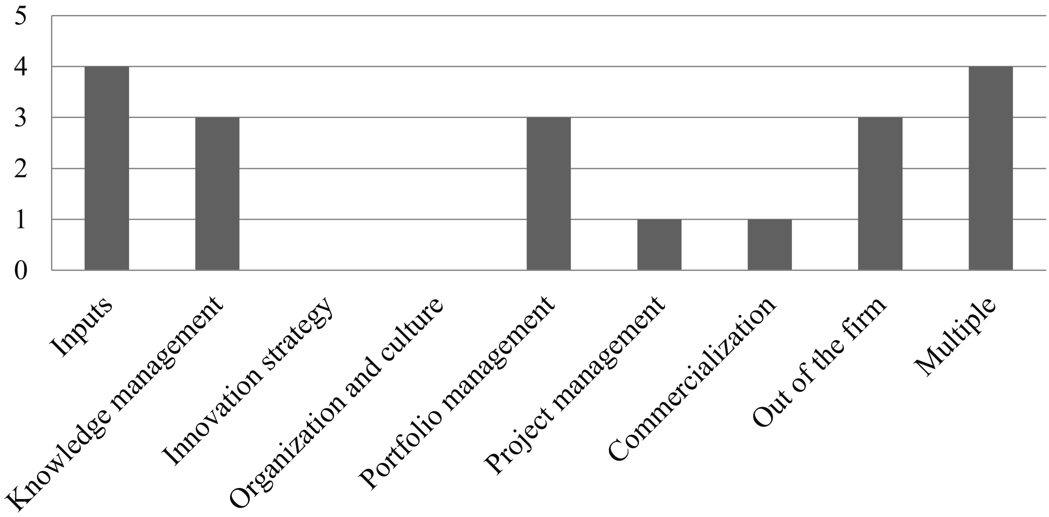
Figure 2.2. Aggregated results by paper type



Quantification of results of the selected papers differentiated by type: empiric, literature review or theoretic. Source: Self-elaborated.

Continuing with the results, the selected papers were catalogued by the innovation area they address (Adams et al., 2006). The majority of the papers reach the scope of innovation inputs and multiple stages; the first oriented to new product development tools and the second addressing several areas of innovation within firms, no main trend of innovation management approach was identified. Knowledge management, portfolio management and actions outside of the firm were recurrent categories; surprisingly we did not find many publications specifically addressing innovation strategy nor organization and culture. It is encouraged to fulfill such gaps in the near future because of its importance in the survival of firms. Figure 2.3 shows the aggregated results of the 19 papers selected for deep examination by innovation area as proposed by (Adams et al., 2006).

Figure 2.3. Aggregated results by innovation área



Number of articles differentiated by innovation area that selected papers attend. Source: Self-elaborated.

Categorization of articles

A deep categorization of empiric and methodologically robust articles was performed. 14 out of the initial 19 articles were classified first by area of specialization, i.e. the main focus of their research paper. Secondly, the approach adopted towards innovation management. Thirdly the specific

Fuzzy Logic methodology applied to address the area of specialization. Lastly, the articles were catalogued by the approach they carried out towards the definition of uncertainty. Table 2.5 presents the main findings over the deep examination of the selected papers.

A deep analysis shows how authors tend to create decision making support models based on Fuzzy Logic to face the inherent characteristics of innovation (Segev et al., 2013). Moreover, they tend to focus models to a specific domain, e.g. Büyüközkan and Feyzıoğlu, (2004); Achiche et al., (2013) for product development, Kong et al., 2008 for technological innovation capability. Zouggari and Benyoucef (2012) for partner selection models. Innovation is a complex activity, diversified, with a high amount of components that interact with each other creating new sources of ideas and it is difficult to discover the consequences that new events can develop, Escorsa & Valls (2003), in that issue, authors on Fuzzy Logic have combined rough quantitative indicators mixed with expert qualitative information (Kaklauskas and Zavadskas, 2007) to create a robust set of tools to assess innovation e.g. Taşkin et al., (2004); Serrano and Robledo (2013) for the evaluation of technological innovation capabilities for firms and institutions. The different factors such as competition, rapid markets, highly changing trends and advanced technology have to meet the shifting interests of the firm's stakeholders, a correct visualization of the transversal innovation capabilities is needed, for that matter Maravelakis et al., (2006); Lin et al., (2011) propose Fuzzy Logic based holistic models for the evaluation of innovative capabilities, the first focusing on SME's and the second on service sectors. Even with the diverse approaches to assess innovation, there is a common thing between the articles; the utilization of Fuzzy Logic techniques to address uncertainty in innovation management, whether faced as subjective judgments, partial truths (Ross, 2009), or approximation and characterization of phenomena that are too complex or ill-defined (Zadeh, 1965).

Table 2.5. Categorization of articles

Author	Area of specialization	Approach	Methodology	Uncertainty Approach
Büyüközkan and Feyzıoğlu, (2004)	New product development	Fuzzy logic decision making support system	Pseudo-order fuzzy preference model (Roy and Vincke, 1984; Wang, 1997), the fuzzy weighted average (FWA) method (Vanegas and Labib, 2001), fuzzy analytic hierarchy process (FAHP) (Triantaphyllou, 2000)	Information defect (Spender, 1993)
Taşkin et al., (2004)	Technological intelligence as competitive advantages	Technological survey analysis	Fuzzy expert system (Frantti and Mähören, 2001; Ordoobadi and Mulvaney, 2001)	Fuzzy logic to encapsulate partial truths (Ross, 1995)
Maravelakis et al., (2006)	Innovation benchmark for SME's	Three-dimensional fuzzy logic approach for measuring innovation	Fuzzy sets (Zadeh 1965)	Fuzzy Logic qualitative, subjective nature and linguistically expressed values (Yager and Zadeh 1992)
Kaklauskas and Zavadskas, (2007)	Pollution minimization and mitigation	Combined expert and decision support systems	Fuzzy relation model (Zhou et al., 2004)	Environmental uncertainty

Wang et al., (2008)	Technology innovation capability	Quantitative and qualitative multi-criteria analytical approach.	Triangular fuzzy numbers. Fuzzy averaging technique and defuzzying method (Chen and Klein (1997)	Technological innovation uncertainties (Afuah 1998)
Kong et al., (2008)	Evaluation of technological innovation capability	Fuzzy decision support models	Triangular fuzzy sets; Fuzzy Vikor Algorithm (Opricovic, 1998)	Uncertainty in the subjective judgments.
Lin et al., (2011)	Tourists service management	Fuzzy model for the evaluation of performance in the service sector	Fuzzy Quality Function Deployment (Hisdal, 1988)	Uncertainties in the tourism service design process (Chien & Tsai, 2000)
Zouggari and Benyoucef, (2012)	Supplier selection based on innovative characteristics	Fuzzy logic decision making support system	Fuzzy Analytic Hierarchy Process (Chang 1996, Wang et al., 2008b); Fuzzy Technique for Order Performance by Similarity to Ideal Solution (Hwang andYoon,1981)	Uncertainty as imprecision associated with information (Zadeh 1965)
Echeverri et al., (2012)	Group product development	Fuzzy model for the evaluation of group contributions	Fuzzy sets (Zadeh 1965)	Uncertainty in the subjective judgments.
Hsueh and Yan, (2013)	Facilitating Green Innovation	Fuzzy logic inference system	Fuzzy sets (Zadeh 1965; 1976; 1996); Triangular functions, bell shaped functions (Yu and Skibniewski 1999)	Complexity, and tolerance for imprecision used in natural language

Segev et al., (2013)	Multilingual knowledge innovation in Patents	Fuzzy Logic reasoning and decision making process	Fuzzy Logic Knowledge Interface (Aliev & Aliev, 2001)	Vagueness in linguistics can be captured mathematically by applying Fuzzy Sets (Lin & Lee, 1996).
Serrano and Robledo, (2013)	Evaluating Innovation Capabilities at University Institutions	Combination between a fuzzy logic system and the experience or knowledge of experts	Fuzzy inference system (Medina, 2006; Kosko, 1994; Mizutani and Sun, 1997)	Multi-value logic that allows reasoning about a world of objects as relational entities (Pedrycz and Gomide, 1998).
Achiche et al., (2013)	New product development	Fuzzy decision support models	Triangular fuzzy sets (Achiche et al. 2006; Duda 2001), Genetically generated Fuzzy Models (Achiche et al., 2004)	Approximate characterization of phenomena that are too complex or illdefined (Zadeh 1975)
Sorayaei et al., (2014)	Marketing strategy	Fuzzy logic decision making support system	Fuzzy Analytic Hierarchy Process (Saaty, 2000; Chang 1996)	Uncertainty in the subjective judgments.

Selected papers categorized by specialization, main approaches, methodological structure, and treatment of uncertainty. Source: Self-elaborated.

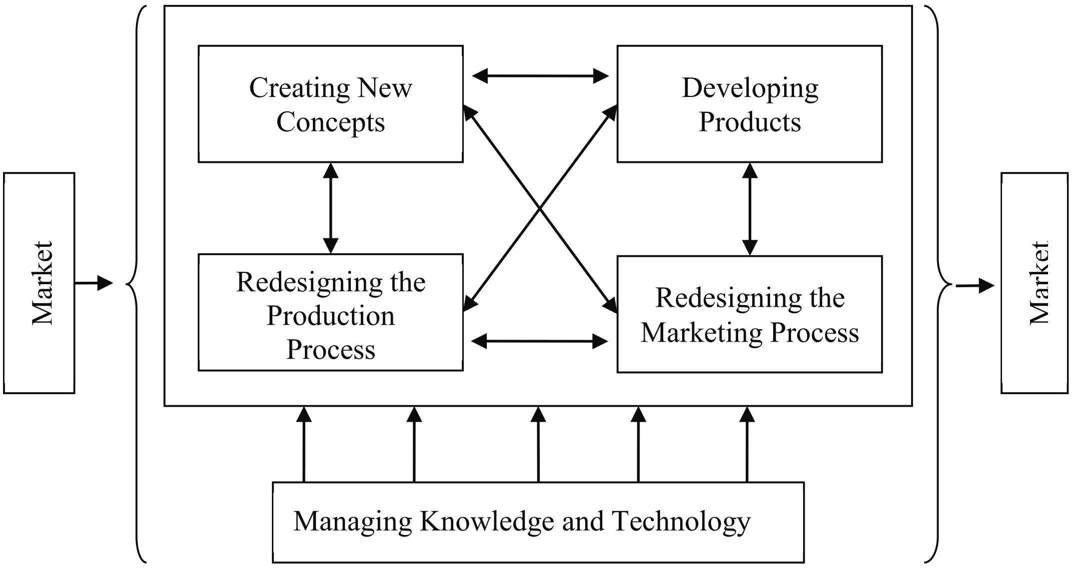
Uncertainty and innovation

Uncertainty is an attribute of information, Zadeh (2006). Not simply the absence of information but inadequate, inexact, unreliable and border with ignorance, Funtowicz & Ravetz (1990). Information is the key of the concept because it can increase or decrease the level of uncertainty phenomena express, more knowledge illuminates that our understanding is more limited or that the processes are more complex than thought before, Van der Sluis (1997). For detailed studies about uncertainty, its evolution and diverse perspectives see Walker et al. (2003); Perminova et al. (2008). In the present study we will follow the idea in which uncertainty is present whenever an outcome of a process is not known due to the attributes on the information that surrounds the phenomena. In that sense, what distinguishes innovation management from gambling? Both involve committing resources to something which (unless the game is rigged) have an uncertain outcome (Tidd and Bessant, 2013).

It is widely accepted that the concept of Innovation involves uncertainty, imprecision and imperfect or vague information. The challenges faced then must be addressed by overrunning that level of uncertainty and providing useful tools in the terms of administration models for the analysis and treatment of variables, taking into account endogenous and exogenous elements, qualitative and quantitative information, among other components.

When conceptualized as a process, the concept of uncertainty in innovation can be more visual, *the importance of an understanding of innovation as a process is that it shapes the way in which we try and manage it* Tidd (2001). The term innovation means a process as well as its result, Drejer (2002). I Ohme (2002) show an example of all the components involved within an organization. Figure 2.4 shows CIDEM innovation process model (i Ohme, 2002) which is a highly cited process innovation approaches and it was thought to evaluate and measure the intensity in which a firm conducts its innovative actions.

Figure 2.4. CIDEM Innovation Process Model



Source: i Ohme (2002).

It is an accepted convention that external factors of the *market* involve uncertainty, Roberts (1998); Rese & Baier (2011); Tidd and Bessant, 2013; Bowers & Khorakian (2014). *The facts of nature are uncertain; the economic, social, financial sphere of business change without ceasing; the acts of man – because he is free and provided with imagination – like relationships between mankind – because these are no robots – are all the fundamental causes of uncertainty*, Gil-Aluja (2004). Economic environment, competitors, suppliers, available workforce, users, highly changing trends, technology, politics and R&D facilities are some of the elements that make the environment uncertain. These elements envelope the scope in which a firm will develop and manage innovation. The procurement and awareness that a firm upholds on external information is base for the success upcoming projects.

At an internal level of a firm, uncertainty plays a key role in diverse aspects: *Creating New Concepts, Developing Products, Redesigning the Production Process and Redesigning of the Marketing Process, Managing Knowledge and Technology* all need complex interactions and fast connections in order to generate effective outcome. "Economic life, in all of its possibilities, is submerged in this context and decisions have to be taken within its realm are even more complex as a consequence of the uncertainty in the outcomes of future events”, Gil-Aluja (2001). Table 2.6 shows the diverse sub-processes, which have been matched to elements that involve uncertainty in CIDEM’s Innovation Process Model.

Fuzzy logic models and innovation

Studies with a fuzzy-oriented standpoint have been increasing since the last century and have proven efficacy while dealing with complex phenomena. As stated by Bellman & Zadeh (1970) “much of the decision making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely”. The theory of decision under uncertainty initializes with the appearance of the article *Fuzzy sets. Information and Control,* Zadeh (1965), and has proven efficiency handling incomplete and uncertain knowledge information see Ribeiro (1996). The theory of Fuzzy Sets has been applied in the field of the formal sciences; nonetheless in the past 44 years researchers from all over the world have been publishing diverse research studies with applications in varied fields of knowledge.

As stated by Zadeh (2008) major implications about using Fuzzy Logic into innovation management could be in the machinery of linguistic variables and fuzzy if–then rules, which is unique to fuzzy logic, the concepts of precisiation and cointension that play important roles in nontraditional view of fuzzy logic, the use of Natural Language Computation that opens the door to a wide-ranging enlargement of the role of natural languages in scientific theories, enabling the Possibility theory, which may be viewed as a formalization of perception of possibility a direct relevance to knowledge representation, semantics of natural languages, decision analysis and computation with imprecise probabilities, and Fuzzy logic as a modeling language, which is natural when the objects of modeling are not well defined, e.g., data compression, information compression and summarization. The result of imprecisiation is an object of modeling which is not precisely defined. A fuzzy modeling language comes into play at this point. This is the key idea, which underlies the fuzzy logic gambit. The fuzzy logic gambit is widely used in the design of consumer products – a realm in which cost is an important consideration.

Table 2.6. Elements of Uncertainty in an Innovation Process Model

Process	Sub-Process	Elements on Uncertainty
1. Creating New Concepts	Generating new product concepts Product innovation planning Innovativeness and creativity Exploiting innovation	Evaluating market needs Screening new concept ideas Selection of new or enhanced products Planning product innovation Favoring creativity and inventiveness Evaluating alternatives for developing new business opportunities Choosing appropriate people for critical innovative roles
2. Developing Products	Product development process Teamwork Transfer to manufacturing and distribution Teamwork and organization Industrial design	Managing product development projects Facilitating communication among different groups Establishing role and priority projects Defining states of project managers in the organization Integrating customer needs in product development Establishment of cross-functional teams
3. Redesigning the Production Process	Formulating a manufacturing strategy Implementation of new processes Continuous improvement	Matching process capabilities to the requirements of the marketplace Linking process innovation to process innovation

		Allocating resources for developing new process technologies Identifying opportunities for improvement in process Benchmark production process performance
4. Redesigning of the Marketing Process	Formulating marketing strategy Product introduction Product promotion Product placement	Grouping affine products for effective market introduction Assigning the best placement of products Link market preferences to image Relating customers characteristics to products or services
5. Managing knowledge and Technology	Managing intellectual property Human Resources Climate for innovation Systems Formulating technology strategy Selection generation and sourcing of technology	Choosing sources of technologies (In house), R&D, licensing, partnering, etc. Identifying emerging technologies Relating technology to business objectives and strategies Use methods to evaluate to evaluate R&D projects Identifying key issues in R&D

Source: Adapted from Chiesa et al. (1996)]

Other applications that have successfully conducted the application of Fuzzy Logic in the fields of social sciences can be found in the aggrupation of municipalities under uncertain conditions towards the creation of synergies, Alfaro et al. (2012), aggrupation of stakeholders for a better administration of enterprises see Gil-Lafuente and Barcellos de Paula (2013), a personnel selection model see e.g. Keropyan and Gil-Lafuente (2013).

In our research, the adoption and further application of Fuzzy Logic methodologies has multiple significances, at a first instance it introduces the possibility of addressing uncertainty at a different standpoint than traditional methods, also it allows to group, assign, link and relate different variables whether endogenous or exogenous that are present in the process of innovation in certain circumstances of a firm.

Concluding comments

The purpose of this study is to review how Fuzzy Logic is currently dealing with subjective complex data in innovation management; the objective is to show the relevance of such methodologies and techniques in innovation measurement approaches. A systematic review within the timespan of January 1986 until September 2014 is proposed; papers from Thomson Reuters database ISI Web of Science were utilized. Results show an increasing interest for assessing innovation management theories under a Fuzzy Logic approach. Decision support making models for innovation management were found to be the most numerous articles in the systematic review. The treatment of information under uncertain conditions with a high level of confidence is considered to be one of the main benefits of utilizing fuzzy logic techniques around innovation management. Although there is no ultimate path for measuring innovation in firms, new frameworks lead the discussion towards a set of key activities that must be covered for a firm to continuously revise their innovative capabilities in order to achieve competitive advantages, however much of the information needed to support those key activities tend to be qualitative or subjective. There are several recognized limitations to our study, firstly, we focused on gathering research and categorizing it, such classification may have omitted relevant topics. Secondly, our review uses only one database; such database may have omitted relevant research. Thirdly, the timespan and filtering methods may have also omitted relevant papers. Further research needs to be conducted, firstly to apply the Fuzzy methodologies on specific conditions, and secondary to keep reconnoitering additional Fuzzy Logic models, which could support decision making under undefined environments. Results motivate the use of Fuzzy Logic methodologies in social studies as key for the development of effective innovative strategies in enterprises towards the creation of competitive advantage.

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Chapter 3. Environment

Think global, act local. It is necessary to study the exogenous and endogenous variables that permeate the industrial fabric of a region in order to adapt, coherent and effectively, new models for business decision-making.

In the present chapter two applications are presented, they seek the same objective, the creation of synergies based on the productive capacities of the studied companies. The foundation is the robust mathematical models that aid the identification of the factors that could be shared among related groups, achieving benefits in several orders, from economies of scale, to the generation of strategic alliances.

The result is on the one hand to propose a methodology for the generation of clusters that at first glance do not show a clear relation, and on the other, to study the environment in which the manufacturing companies of the city of Morelia, Mexico are immersed.

A fuzzy approach to competitive clusters using Moore families[2]

Abstract.

Our investigation applies a fuzzy grouping model in order to identify the characteristics of the manufacturing network of enterprises in a specific city. The aim is to create clusters towards the construction of competitive advantages, cost reduction and economies of scale. We utilize tools of Fuzzy Sets Theory, evaluating productive capacities of local enterprises under Moore Family model. Results conclude in 18 different clusters formed by 2, 3, 4 and 5 firms located in 6 different zones of a specific city. This work seeks to shed light in the conformation of groups under uncertain conditions, and the deep examination of the manufacturing activities in a specific territory for decision and policy making.

Keywords: Fuzzy clustering; Multiple criteria evaluation; Group decision-making; Moore family model

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Introduction

In the last decades the economic openness and globalization has simplified trade barriers making economies more dependent on each other and so affecting the competitiveness of organizations and regions. In this context, governments have been reflecting about competitiveness and sustainable growth, focusing their efforts on the promotion of economic policy as dominating key element to boost the regional development within a worldwide economic context. Competitiveness as center of development strategy has focused the development of cluster initiatives and programs [1] in order to develop competitive environments and create competitive advantages for the existing industrial and business tissues. In this sense, the cluster empowerment initiatives are directed to promote economic development, improve microeconomic business environment, increment productivity and stimulate entrepreneurship and the entry of new business [2, 3]. Also, location plays a major role highlighting the conditions of the region as geographic area forming an integrated economic space, which offers distinctive qualities to boost growth.

In this regard, one of the points on which the decision-making process is based on, resides on knowing the competitive advantages of each zone, which is subject of study as well as their peculiarities, and specifics who can lead to fruitful cooperation, strategic alliances and added value activities. In this order of ideas, the primary target of this work consists on finding the possible affinities of enterprises in each zone of a specific city by analyzing their manufacturing activities, which can help firms to identify potential clusters allowing positive synergies and for policy maker's infrastructure optimal utilization and correct development.

Theoretical Framework

Competitive Advantages and Clusters

Globalization, economic openness, technological development and information management contribute to simplify trade barriers. This changes affected economic activity significantly leading to the development of more efficient and competitive processes to enable competitiveness. These dynamics in economic activities leads to a better use of the available resources highlighting the importance of the area as available resource platform for enterprises. In this regard location plays a significant role in enterprise development acting as operations center in which enterprise and surroundings interact. These interactions allow the generation of economic activities to improve competitiveness. In this context nations focused their efforts towards finding competitiveness theories to support economic growth and prosperity. These efforts are reflected in the promotion of integral economic policy as key factor to boost regional and enterprise development in a global economic context. Location plays a fundamental role by highlighting the qualities of the region as place for economic development and promotion of competitiveness.

Many authors have contributed to understand clusters' characteristics; see e.g. Hill et al., [4]; Porter [3]; Bell [5]. Porter [6] defines an industry cluster as "a set of industries related through buyer-supplier relationships or common technologies, common buyers, distribution channels or common workplace." The author describes two types of clusters: vertical and horizontal clusters. The firsts are made of industries linked through relationships between buyers and sellers, while horizontal groupings are the result of the existence of interconnections between companies of the same level, in order to share resources.

In recent definitions geographic concentration has been included as an important characteristic of a cluster. Authors like Jacobs & De Man [7] propose three definitions related to the concept of a cluster. The regional cluster, composed of spatially concentrated industries, the sectorial cluster, composed of sectors or groups of sectors and the network clusters, integrated by value chains.

Porter [8] redefines the concept of cluster incorporating geographic concentration as an important element, as "geographic concentrations of interconnected companies, specialized suppliers, service providers, related industries firms and associated institutions; competing in particular fields but also cooperating". Such definition will be adopted for the purposes of the present research.

The OECD [9] has defined the cluster as a grouping, or local productive system (LPS) in a same region that operate in similar business lines and whose relations foster the development of multiple interdependencies between them, strengthening their competitiveness in a wide variety of areas, such as: training, financial resources, technological development, product design, marketing, distribution and tourism.

Studied City

Morelia is a city located in the center of the Mexican Republic; it is the capital of the state of Michoacán de Ocampo. It is immersed in the Mega – region called "Greater Mexico City", where approximately 45 million people live and generates $290,000 million in LRP, more than half of the whole nation, Florida [10].

In economic aspects, the city has an overall gross domestic product of 7,774.5 dollars per capita, when the mean in the republic is 9,980 dollars. The city raised its gross domestic product from the 2003 to 2008 by 15%. The main economic activities of the city are tourism, education and commerce. The city reaches 1,606,399 economically active citizens which 1,554,720 are employed.

In terms of tourism, the city of Morelia is one of the first touristic destinations in the country due to its architectonical, cultural and historic legacy. The city also connects with a series of natural destinations, which increase the affluence of tourists. The city has over 110 lodging establishments and only in 2010 Morelia attracted 2,449,805 national tourists and 269,179 international tourists.

In terms of education, the city of Morelia offers 882 educational facilities, 7,744 classrooms, 81 libraries, 103 workshops and 165 laboratories. An Institute of Technology resides in the city; around 4,650 students specialize in technological fields of the knowledge. In general terms there are 9 public institutes of advance studies and 15 private ones.

Despite the laudable touristic, educational and cultural indicators, the development of the industrial tissue is incipient. In terms of industry, the city has over 16 mining economical units, where around 100 people work, 3143 manufacturing enterprises where 14,606 people work. In 2008 only 16 licenses for industrial land use were petitioned. The city has an industrial park where 180 enterprises offer around 9,000 employments, most of the enterprises established have only distributing activity and the manufacturing enterprises are small or medium size companies.

In the present study we will take into count all the manufacturing enterprises from the studied city with 51 or more employees. A total of 41 firms were retrieved from the National Statistical Directory of Economic Units [11]. By generating clusters of medium and large enterprises we endorse economies of scale, identification of weak and strong industries and in time, incentive policies.

Table 3.1. Total Manufacturing Enterprises with 50 Employees or More

A1	GS Alimentos de México	C2	Arpillas de Exportación
A2	Rafias Industriales	C3	Sacos de Propileno Especializados
A3	Sigma Alimentos de Occidente	C4	Quimic
A4	Estación la Antigua Valladolid	C5	Fundidora Morelia
A5	Infasa	D1	Harinera Michoacana
A6	Alstom Mexicana	D2	Matec
A7	Andritz Hidro	D3	El Pino
A8	De Acero	D4	Polystrech Ortiz
A9	Molinos Morelia	D5	Trico Casa Grande
A10	Tron Hermanos	E1	Bebidas Purificadas de Michoacán
A11	La Universal Impresora	E2	Congeladora y Empacadora Nacional
A12	Michoacana de Plásticos	E3	Fabrica de Trenzados Marinos
A13	Placoyt	E4	Internacional de Sacos y Arpillas
A14	Agro Metálica Michoacana	E5	Grupo Papelero Scribe
A15	Comprovet	F1	Kimberly y Clark de México
A16	Fabricación de Escobas. Cepillos y Similares	F2	La Voz de Michoacán
B1	Afrima de Michoacán	F3	Resinas Sintéticas
B2	Favetex	F4	Cartonera de Morelia
B3	Industrias Jafher	F5	Fabricas la Central
C1	Aarhuskarlshamn México	F6	ULTM de Morelia
		F7	Industrias Oken

Source: DENUE [11].

Methodology

When conducting traditional grouping of elements, trial and error methods are employed, therefore the confidence level in which the inherent characteristics of the studied variables relate to each other tends to be scarce. In order to optimize the grouping process, we propose the use of the Affinities Theory, Moore Families and Galois Group Theory, which allows us to know exactly the role of the interacting variables with a significant level of confidence. In this sense, is it emphasized that the concept of affinity serves as foundation on three aspects: homogeneity, relation and structure.

Based on these concepts, the methodological procedure consists of three phases: the first starts from the original matrix to describe the fuzzy subset [12] and transform it to a Boolean matrix [13] with an threshold $\alpha = n$; in the second phase we proceed to propose an algebraic approach, in which the relation between affinities in Families of Moore Model is founded [14]. We proceed to define each of the mathematical procedures.

Fuzzy Subset

Starting from the principal matrix of fuzzy relations$[\tilde{R}]$, a limit for each considered criteria is given, such limit represents the level in which the groupings of elements possess the required criteria. We define a fuzzy subset of elements [15], as:

$$[\tilde{U}] = \begin{matrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_n \end{matrix} \begin{matrix} \boxed{u_1} \\ \boxed{u_2} \\ \boxed{u_3} \\ \boxed{\vdots} \\ \boxed{u_n} \end{matrix}$$

$$u_1 \in [0,1], i = 1,2, \dots, n$$

This delimited Fuzzy Subset allows a fuzzy relation $[\tilde{R}]$, to be transformed into a Boolean Matrix if it demonstrated that:

If: $r_{ij} \geq u_i$ then $b_{ij} = 1$; $r_{ij} < u_i$ then $b_{ij} = 0$,

$$j = 1,2, \dots, m$$

$$i = 1,2, \dots, n$$

where b_{ij} represents the elements of the Boolean Matrix $[B]$.

Moore Families

Starting from the concept of "power set" [15], the limited set E_1 as power set, $\prod E_1$, generated by all the possible combinations of elements 1 in 1, 2 in 2, … m in m, if it is the corresponding cardinal. We obtain the next set:

$$E_1 = \{a, b, c, \dots, m\}, \tag{1}$$

the set of all its parts or "power set" is:

$$\prod(E_1) = \{\emptyset, a, b, c, \dots, m, ab, ac, bc, \dots, mm, E_1\}. \tag{2}$$

Let a family of Moore of $\prod(E_1)$, $F(E_1)$, such that, $F(E_1) \subset \prod(E_1)$, if $F(E_1)$: (1) $E_1 \subset F(E_1)$; 2. The intersection of the number of parts of $\prod(E_1)$ which belongs to $F(E_1)$, belongs also $F(E_1)$, it is written:

$$\big(A \in F(E_1), B \in F(E_1)\big) \Rightarrow \big(A \cap B \in F(E_1)\big), \tag{3}$$

then $F(E_1)$ is considered a Moore's family.

Starting from a family of Moore, Moore closing can be constructed. The Moore closing is a functional application in which all the components from the subset $A \subset E_1$ are made to correspond a MA, such that:

$$MA = \bigcap_{F \in F_A(E_1)} F, \tag{4}$$

where $F_A(E_1)$ represents the subset of the elements of $F(E_1)$ that contains A and F all elements of $F_A(E_1)$.

Notice that in order to make a correct mathematical Moore closing the properties of: Extensitivity: $\forall A \in \Pi(E_1): A \subset MA$; Idempotence: $\forall A \in$

$\Pi(E_1): M(MA) = MA$; Isotony: $\forall A \in \Pi(E_1): A \subset B \Longrightarrow (MA \subset MB)$, must be satisfied.

Studied variables

In the present study we propose seventeen variables that will be used to create clusters of enterprises in different zones within the studied city. Each variable represents the main economic activities that can be developed by a constituted manufacturing firm in the city of Morelia [16]. These variables are evaluated for every firm as they represent a unified classification system in the Mexican territory, thus results could be repeated and compared with other cities. Table 3.2 concentrates the main manufacturing activities. Table 3.3 presents the intensity in which a firm develops a certain manufacturing activity. Table 3.4 presents the different zones in which the firms could be located within the city.

Table 3.2. Manufacturing Activities

V1	Apparel manufacturing
V2	Machinery and equipment manufacturing
V3	Beverage and tobacco industries
V4	Chemical industry
V5	Food industry
V6	Petroleum and coal products manufacturing
V7	Plastic and rubber industry
V8	Electric appliances, accessories and electric power generation equipment manufacturing
V9	Basic metal industry
V10	Metal products manufacturing
V11	Other manufacturing industries
V12	Nonmetallic mineral products manufacturing
V13	Paper industry
V14	Printing and related industries
V15	Furniture, mattresses and blinds manufacturing
V16	Textile inputs manufacturing, and textiles finishing
V17	Textile products manufacturing, except apparel

Source: North American Industry Classification System, [16]

Table 3.4. Manufacturing Firms by Zones of the Studied City

A. Industrial Park	A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12}, A_{13}, A_{14}, A_{15}, A_{16}
B. Technologic Institute	B_1, B_2, B_3
C. Pedregal	C_1, C_2, C_3, C_4, C_5
D. Industrial Colony	D_1, D_2, D_3, D_4, D_5
E. State University	E_1, E_2, E_3, E_4, E_5
F. South West	F_1, F_2, F_3, F_4, F_5, F_6, F_7

Source: Self elaborated

In our case each firm is evaluated according to the intensity in which they generate a specific manufacturing activity. Table 3.5 presents the evaluation for each corresponding linguistic tag.

Table 3.5. Evaluation of variables

1	Excellent performance
0.9	Great performance
0.8	Really good performance
0.7	Good performance
0.6	Rather a better tan a poor performance
0.5	Nor a good or poor performance
0.4	Rather a poor tan a good performance
0.3	Poor performance
0.2	Really poor performance
0.1	Worst performance
0	Disastrous performance

Source: Self elaborated.

Table 3.3. Manufacturing Activities Evaluated by Firm

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}	V_{14}	V_{15}	V_{16}	V_{17}
A_1			0.5	0.6	1												
A_2											0.4				0.2	0.8	1
A_3			0.4	0.6	1												
A_4						1	0.4										
A_5		0.3		1							0.2						
A_6	0.5							1	0.7		0.7						
A_7	0.3							1	0.5		0.5						
A_8									0.6	1	0.6						
A_9				0.3	1												
A_{10}				0.5	1						0.6						
A_{11}													0.3				
A_{12}				0.6			1										
A_{13}				0.3			1										
A_{14}	0.6	1									0.2						
A_{15}	0								0.6	1	0.2						
A_{16}	0.5										1						
B_1				0.2	1												
B_2	1	0.7															
B_3															1	0.4	0.7
C_1			0.4	0.6	1												

C_2															0.3	0.6	1
C_3							0.5									0.6	1
C_4			0.6	1	0.7												
C_5									1	0.7							
D_1				0.4	1												
D_2															0.3	1	0.6
D_3				1													
D_4				0.4			1										
D_5					1												
E_1			1	0.5	0.6												
E_2			0.6	0.4	1												
E_3											0.3				0.3	0.6	1
E_4															0.2	0.7	1
E_5													1				
F_1													1				
F_2														1			
F_3				1			0.6										
F_4													1				
F_5				1													
F_6	0.3										0.5	1					
F_7	0.7	1									0.6						

Source: Self elaborated

Results

Boolean matrices with fuzzy subsets of thresholds are generated according to the models stated in the methodology section. The threshold level is set at $\propto \geq 0,4$, i.e. the enterprises that develop a specific manufacturing activity with a minimum intensity of 0,4 are included in the model. This threshold is taken firstly with an intention to build clusters capable of generating synergies at different levels, stressing the main intention of building economies of scale, therefore cost reduction.

Table 3.6. Boolean Matrix A. Industrial Park

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V14	V16	V17
A1			1	1	1									
A2													1	1
A3				1	1									
A4						1								
A5				1										
A6	1							1	1		1			
A7								1	1		1			
A8									1	1	1			
A9					1									
A1				1	1						1			
A11												1		
A12				1			1							
A13							1							
A14	1	1												
A15									1	1				
A16	1										1			

Table 3.7. Boolean Matrix B. Tecnologic Institute

	V_1	V_2	V_5	V_{15}	V_{17}
B_1			1		
B_2	1	1			
B_3				1	1

Table 3.8. Boolean Matrix C. Pedregal

	V3	V4	V5	V7	V9	V10	V16	V17
C1		1	1					
C2							1	1
C3				1			1	1
C4	1	1	1					
C5					1	1		

Table 3.9. Boolean Matrix D. Industrial Colony

	V_4	V_5	V_7	V_{16}	V_{17}
D_1		1			
D_2				1	1
D_3	1				
D_4			1		
D_5		1			

Table 3.10. Boolean Matrix E. State University

	V_3	V_4	V_5	V_{13}	V_{16}	V_{17}
E_1	1	1	1			
E_2	1		1			
E_3					1	1
E_4					1	1
E_5				1		

Table 3.11. Boolean Matrix F. South West

	V_1	V_2	V_4	V_7	V_{11}	V_{12}	V_{13}	V_{14}
F_1							1	
F_2								1
F_3			1	1				
F_4							1	
F_5			1					
F_6					1	1		
F_7	1	1			1			

Tables 3.6 – 3.11 source: self elaborated.

Continuing with the procedure, Families of Moore are obtained by setting a threshold of $\propto \geq 0,4$, following the criteria defined in previous sections:

Table 3.12. Families of Moore

A. Industrial Park	C. Pedregal	F. South West
E_1, Ø	E_1, Ø	E_1, Ø
$V_1V_8V_9V_{11}$, A_6	$V_7V_{16}V_{17}$, C_3	$V_1V_2V_{11}$, F_7
$V_4V_5V_{11}$, A_{10}	V_9V_{10}, C_5	$V_{11}V_{12}$, F_6
$V_9V_{10}V_{11}$, A_8	$V_{16}V_{17}$, C_2	V_4V_7, F_3
$V_8V_9V_{11}$, A_7	V_4V_5, C_1	V_{14}, F_2
$V_3V_4V_5$, A_1	V_{17}, C_2C_3	V_{13}, F_1F_4
V_1V_{11}, A_{16}	V_{16}, C_2C_3	V_{11}, F_6F_7
V_9V_{10}, A_{15}	V_5, C_1C_4	V_4, F_3F_5
V_1V_2, A_{14}	V_4, C_1C_4	E_2, Ø
V_4V_7, A_{12}	V_3, C_4	
V_4V_5, A_3	E_2, Ø	
$V_{16}V_{17}$, A_2		
V_{14}, A_{11}		
V_{11}, $A_6A_7A_8A_{10}A_{16}$		
V_{10}, A_8A_{15}		
V_9, $A_6A_7A_8A_{15}$		
V_8, A_6A_7		
V_7, $A_{12}A_{13}$		
V_6, A_4		
V_5, $A_1A_3A_9A_{10}$		
V_4, $A_1A_3A_5A_{10}A_{12}$		
V_1, $A_6A_{14}A_{16}$		
E_2, Ø		

E. State University	D. Industrial Colony	B. Technologic Institute
E_1, Ø	E_1, Ø	E_1, Ø
$V_{16}V_{17}$, E_4	$V_{16}V_{17}$, D_2	$V_{15}V_{17}$, B_3
$V_3V_4V_5$, E_1	V_4, D_3	V_1V_2, B_2
V_3V_5, E_2	V_5, D_1D_5	V_5, B_1
V_{17}, E_3E_4	V_7, D_4	E_2, Ø
V_{16}, E_3E_4	E_2, Ø	
V_{13}, E_5		
V_3, E_1E_2		
E_2, Ø		

Source: self elaborated.

The computing procedures finalize when we obtain the Families of Moore. At this stage of the process we observe that zone A presents the bigger amount of enterprises, therefore a bigger concentration of groups. Despite the different locations of Zones C, F, E within the city, a similar amount of groups with diverse manufacturing activities is observed. The least amount of groups is given in zones D and B, where the quantity of firms tends to be scarce.

Analysis of Results

The totality of the manufacturing firms with 50 employees or more of the studied city were introduced to the mathematical model. Results for each zone show similar behavior, grouping the majority of the enterprises in the second level, i.e. one firm that develops one or more manufacturing activity, however not linked with any other firm. The rest of the levels present a larger amount of firms related to each other. Only Zone A. presents the capacity of conforming clusters with 3 or more enterprises. See Table 3.13.

Zone A. being the zone with the largest attraction of manufacturing firms, has achieved 6 levels of groups. The main manufacturing activities displayed are V_1, V_4, V_5, V_9 and V_{11}, showing a preponderancy of metal and chemical industry in the zone. The largest groups of enterprises that could create clusters because of their characteristic manufacturing activities are groups of 5 enterprises: A_1, A_3, A_5, A_{10}, A_{12} and A_6, A_7, A_8, A_{10}, A_{16}.

Zone B. is the zone with the least amount of enterprises, no group was created because of the dissimilar manufacturing characteristics that firms develop in that specific zone of the city.

Zone C. Presents 2 levels of groups. Main manufacturing activities are V_4, V_5 and V_{16}, V_{17}, i.e. Textile, chemical and food manufacturing activities, 4 different groups of 2 related firms were assembled: C_1, C_4 and C_2, C_3.

Zone D. Exhibits 2 levels of groups. The preponderant manufacturing activity is V_5 (food industry). Only one cluster was defined: D_1, D_5.

Zone E. Shows 2 levels of groups. Main manufacturing activities are Textile and Beverage and tobacco industries. 2 Clusters were found to be possible: E_3, E_4 and E_1, E_2.

Zone F. Presents 2 levels of groups, the main manufacturing activities in the zone are Paper Industry and other manufacturing activities. From these preponderant activities 2 clusters were generated: F_1, F_4 and F_6, F_7.

Table 3.13. Aggregated Results

	A. Industrial Park			C. Pedregal			F. South West		
	G	F	MA	G	F	MA	G	F	MA
Level 1			E_1			E_1			E_1
Level 2	13	1	1, 2, 3 and 4	5	1	1, 2 and 3	4	1	1, 2 and 3
Level 3	3	2	1	4	2	1	3	2	1
Level 4	1	3	1			E_2			E_2
Level 5	2	4	1						
Level 6	2	5	1						
Level 7			E_2						

	E. State University			D. Industrial Colony			B. Technologic Institute		
	G	F	MA	G	F	MA	G	F	MA
Level 1			E_1			E_1			E_1
Level 2	4	1	1, 2 and 3	3	1	2	3	1	1 and 2
Level 3	2	2	1	1	2	1			E_2
Level 4			E_2			E_2			

G: Groups, F: Firms, MA: Manufacturing Activities. Source: self elaborated.

Results have a variety of implications; at first instance the preponderant manufacturing activities that firms within the studied city are concentrated in a few categories. This fact narrows the possibility of large clusters of enterprises.

The industrial park, as the zone with the largest amount of groups has been effective at attracting enterprises that share common manufacturing activities; however other zones present potential clusters, further research needs to be conducted in order to evaluate the incentives needed to promote the growth of the industries.

Discussion and Further Research

Continuing with the initial statement and the objectives pursed in our study, one of the foremost applications, which allows the constructive visualization and further exploitation of the results of the model is precisely the grouping of firms towards the creation of synergies that could foster the economic development of the city and therefore the region.

When conducting traditional groupings, trial and error methods are employed; therefore the confidence level in which the inherent characteristics of studied cities relate to each other tends to be scarce. In order to optimize the economic zone grouping process, we propose as further research the use of Galois Group Theory, which allows us to know exactly which variables are strong at determined zones with a significant level of confidence, thus allowing the creation of synergies at allocation of resources; centralized management and knowledge management (see [18]).

Galois group theory has been proven efficient in different fields because it provides "order- or structure-preserving passage between two worlds of our imagination - and thus are inherent in human thinking wherever logical or mathematical reasoning about certain hierarchical structures is involved" Denecke et al. [19].

Other applications that have conducted with success the application of Galois Lattices can be found in the grouping of stakeholders for a better administration of enterprises e.g. Gil Lafuente & Barcellos de Paula [20], and in human resources, with a personnel selection model e.g. Keropyan, A., & Gil-Lafuente, A. M. [21].

Conclusion

Results show clusters of enterprises conformed by affinity relationships. Localization within the studied city and related manufacturing activities were the variables chosen to generate groups with a homogeneity level of 60%. A total of 18 different clusters were obtained in 6 different zones of the city.

We aim to group firms in a specific city in order to identify possible positive synergies between them. Our model is founded on the basic principles of Moore Families Model, the processes allow us to group different variables with a certain level of significance. The present work tries to shed a light in the academic world by offering a group based model in which subjective and relative factors are intrinsic for the decision making process. Also this analysis tries to aid decision makers so they can create common policies due to the results of the grouping processes. Further research needs to be conducted, firstly to study the nature of the variables stated to know whether they need to be weighted, and if this weight plays a significant role on the results obtained and also apply the model in specific conditions; secondly to generate structured graphs of the results over Galois Group theory in order to achieve a better visualization of the results implications. The model we present can be applied to different locations and it may allow optimizing the process of grouping firms under subjective and uncertain conditions.

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A Fuzzy Approach to a Municipality Grouping Model towards Creation of Synergies[3]

Abstract

In recent years, a trend for accelerating the economic, social and environmental development of cities through associations, organization and creation of synergies has been identified. Our investigation applies a grouping model in order to identify municipalities that could create optimal synergies towards the construction of competitive advantages. In order to achieve this task, we use tools of Fuzzy Logic to evaluate subjective and qualitative characteristic elements of different municipalities under Galois' group theory. Results conclude on 32 different groups ordered in 7 different levels, relating 12 municipalities of a specific region according to 8 competitive variables. This work seeks to shed light in the conformation of groups under uncertain conditions and the deep examination of the characteristic competitive elements in a specific region for further policy and decision-making processes.

Keywords: Fuzzy clustering, Decision analysis, Multiple criteria evaluation, Group decision-making.

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Introduction

In the last decades the economic openness and globalization have simplified trade barriers making economies more dependent on each other and so affecting the competitiveness of organizations and regions. In this context, governments have been reflecting about competitiveness and sustainable growth, focusing their efforts on the promotion of economic policy as dominating key element to boost the regional development within a worldwide economic context. Competitiveness as center of development strategy has focused the development of cluster initiatives and programs (Ketels 2013) in order to develop competitive environments and create competitive advantages for the existing industrial and business tissues. In this sense, the cluster empowerment initiatives are directed to promote economic development, improve microeconomic business environment, increment productivity and stimulate entrepreneurship and the entry of new business (Porter 1996; Porter 2000). Also, location plays a major role highlighting the conditions of the region as geographic area forming an integrated economic space, which offers distinctive qualities to boost growth.

In this regard, one of the points on which the decision and policy making process is based on, resides on knowing the competitive advantages of zones and regions, their peculiarities and specificities in order to lead to fruitful synergies, strategic alliances and added value activities of their enterprise network. In this order of ideas, the primary target of this work consists on finding the possible affinities of diverse municipalities by analyzing their competitive characteristics under uncertain conditions.

The objective pursued in this work is especially relevant to emergent economies such as the Mexican market, which since the 1980's has been transiting from a closed economy to an open market strategy. Such external and internal economic liberation has affected in a certain degree to all the enterprises within the territory. The increasing commercial openness and free market treaties signed by the nation have increased the competitive environment in a way that productivity and higher quality of manufactured goods needs to be continuously pursed in order to retain market share.

It is widely accepted that the decision-making process involves uncertainty, imprecision and imperfect or vague information. "Much of the decision making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely" (Bellman and Zadeh 1970). The theory of decision under uncertainty initializes with the appearance of the article Fuzzy sets. Information and Control (Zadeh 1965), and has proven efficiency handling incomplete and uncertain knowledge information (Ribeiro 1996). The theory of Fuzzy Sets has been applied in the field of the formal sciences; nonetheless, in the past 44 years, researchers from all over the world have published diverse research studies with applications in varied fields of knowledge.

The relation that is established between the competitive variables of cities as criteria for aggrupation and creation of synergies starts from the proposal of (Kaufmann and Gil-Aluja 1988). The purpose of this work is to classify and group different municipalities that could by association, organization and creation of synergies, increase their competitiveness as a whole. The method to classify and group these municipalities has as foundation Galois' group theory e.g. see (Keropyan and Gil-Lafuente 2013) and the theory of fuzzy sets e.g. see (Gil-Aluja et al 2011). These approximations allow us to construct a generalized model adapted to the conditions of expectancy and uncertainty.

Theoretical Framework

Implementing a model based on group-processes to know in which measure different municipalities can benefit from the creation of synergies has both decision makers and academics' interest. From the first point of view, decision makers can (by the results obtained in the process of aggrupation) know the cities whose inherent characteristics make them the most suitable to adopt common public policies. From the academics point of view, the model offers an intuitive and robust grouping methodology in which variables with high subjective values can be treated by utilizing semantic scales "allowing us to transform the language descriptions to numeric values easier when it comes to dealing with real life problems" (Jiang et al 2008; Keropyan and Gil-Lafuente 2013).

Competitive Municipalities

The foundations of the present work rely on the positive impact that light shedding studies regarding economic competitiveness of cities and regions exhibit, see e.g. (Rogerson 1999; Porter 2003; Budd and Hirmis 2004; Turok 2004; Gardiner et al 2006; Malecki 2007; Caragliu et al 2011; Hansel and Wither 2011). Our study aims to group different municipalities based on their unique characteristics, hence cities' competitive variables in order to generate synergies. The enhanced efforts made by the decision and policy makers towards the group of municipalities should exert benefits in the economic, social and environmental aspect of cities producing in combined actions, greater effects than the sum of individual efforts (Alfaro and Gil-Lafuente 2012).

An initial approach towards the study that different variables could act as enhancers of cities is addressed in Begg, 1999. The variables proposed intend the attraction of firms, which ultimately favors the creation of wealth in the region. The development of one city, the level of wealth creation and attraction of enterprises are represented in the combination of the following input elements: sectorial trends, characteristics of the local firms, business environment, innovation and learning. Urban planners should create policies that help "equip cities to adapt and to foster a dynamic economic environment" (Begg 1999).

The study of variables concerning the measurement of competitiveness of a city is addressed in Parkison et al, 2004. The well-known difficulty of defining the concept of urban competitiveness is revised on three conceptual drivers: Critical drivers: innovation in firms and organizations, skilled workforce, connectivity internal and external, economic diversity, strategic decision-making capacity. Important drivers: an inclusive and diverse society. Ambiguous drivers: exhibition facilities, a distinctive city center, cultural facilities, quality housing, fiscal incentives to cities, national policies, a reputation for environmental excellence and responsibility.

Alliances and synergies need to be measured as well, as they offer opportunities to increase productivity in a region. In that sense, the three-stage model proposed in (Florida 2008) including economic, social and technological elements is suitable. The main objects of study are focused firstly on finding a feasible approach to measure human capital and secondly identifying the factors that lead the distribution of human capital around different regions. Positive effects appear as the interrelations between human capital, the effect of technologic capacities and economic variables (salaries and wages) are connected. Main results show that "human capital and the creative class play different but complementary roles in regional development. The creative class—or occupational skill—operates through the channel of wages and exerts its effect on regional labor productivity. Human capital—or education—operates by increasing regional income and wealth" Florida (2008). Additionally, the development of cities through synergies and alliances has been analyzed in numerous European researches; see e.g. (Creative Metropoles 2010;

Chapain et al 2010; Compete Network 2010). These European Community financed studies establish the relations and results that different cities in the EU have applied towards competitiveness and growth. Cities' investment in competitive industries, competitive cities theories, interactions and synergies foster the development of regions.

Methodology

The proposed model is an intuitive and structurally improved way of establishing groups to create synergies in a regional level. The optimal grouping can lead to share capacity resources and in general terms, profit structures to increment the progress, development and quality of life in the region.

In the model, we propose a transition from verbal semantics to the corresponding numerical semantics in order to measure the most adequate fundamental characteristics of the city according to the match between the variables obtained from the literature and the level of performance that each city displays.

Fuzzy Approach

Most of the concepts addressed in our analysis can be subjective and cannot be assessed in a precise quantitative form. It is in this kind of scenario where the use of the fuzzy sets tools is recommended (see e.g. Zadeh 1997; Delgado et al 1998; Chen 2001; Gil-Aluja 2009). In the present research we use the linguistic approach technique detailed in Herrera and Herrera-Viedma (2010). The linguistic approach "is beneficial because it introduces a more flexible framework which allows us to represent the information in a more direct and adequate way when we are unable to express it precisely" (Herrera and Herrera-Viedma 2000).

For example, here we assume that there are seven different municipalities, which have ten different levels of performance on competitive variables:

$$M = \{M_1; M_2; M_3; M_4; M_5; M_6; M_7\},$$

where:

$$M_1 = \{a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1\}$$
$$M_2 = \{a_2, b_2, c_2, d_2, e_2, f_2, g_2, h_2, i_2, j_2\}$$
$$M_3 = \{a_3, b_3, c_3, d_3, e_3, f_3, g_3, h_3, i_3, j_3\}$$
$$\vdots$$
$$M_7 = \{a_{10}, b_{10}, c_{10}, d_{10}, e_{10}, f_{10}, g_{10}, h_{10}, i_{10}, j_{10}\}.$$

Here $\{a_i, b_i, c_i, d_i, e_i, f_i, g_i, h_i, i_i, j_i = 0{,}0; 0{,}1; 0{,}2; \dots 1{,}0\}$ values represent how the municipalities' inherent characteristics overlay each variable, e.g. if the first municipality has a very high integration of population level then $a_1 = E = 1{,}0$. But if the education level shows a really poor performance, then $a_6 = RP = 0{,}2$. Once the analysis is made for all the municipalities, a matrix which links the performance of each variable and the cities addressed can be created.

The adequacy of the model is important in terms of correctly measuring the characteristics of the city and determining if these characteristics can be matched with other cities' to build strong synergies. The proposed model allows introducing subjective characteristics for certain special cases where measurement is possible. Although some objective characteristics may exist, we have to accept the fact that the transition from verbal semantics to numerical semantics is subjective for those special cases that could have been measured (Gil-Lafuente 2002).

Moore's closure and Moore's closed sets

In order to build a coherent and structured lattice we apply the concepts of Moore's closure and Moore's closed sets (Gil-Aluja et al 2009), such concepts adapt well to the necessities of our case and allows the construction of the proposed model. We start with a relation $[R]$ between the elements of a referential $B = \{B_i / i = 1, 2 \dots n\}$, that is to say the fuzzy subsets that describe the municipalities and the elements of a referential set E:

$$[R] = \{(B_i, x_j) \subset B \times E\}.$$

Each element of the matrix $[R]$ takes value one if it has the corresponding attribute or zero if it does not have it. Now the predecessors of x_j and the successors of B_i are defined as:

$$\left.\begin{array}{c} x_j \text{ is a sucessor of } B_i \\ B_i \text{ is a predecessor of } x_j \end{array}\right\} \quad if\ :\ (B_i, x_j) \in [R].$$

Thus, the successors of each municipality, B_i, will be the elements of the attributes set that it has. Taking in count the definitions stated above and the notion of a "power set" (Gil-Aluja et al 2009), i.e. all the possible groupings of the attributes x_j, we can find the connection to the right R^+A_k and to the left R^-F_t.

$$R^+A_k = \{x_j \in E/(B_i, x_j) \in [R], \forall\, B_i \in A_k\}, \qquad with\ R^+\emptyset = E.$$
$$R^-F_t = \{B_i \in B/(B_i, x_j) \in [R], \forall\, B_i \in A_k\}, \qquad with\ R^-\emptyset = B.$$

The right connection R^+A_k and the left one R^-F_t, can be found directly by reading of the matrix, or if its preferred, from the relation $[R]$, in the following way:

$$\forall B_i \subset A_k \in C(B): \quad \begin{array}{l} R^+A_k = \displaystyle\bigcap_{B_i \subset A_k} R^+\{B_i\}, \\ R^-F_t = \displaystyle\bigcap_{x_i \subset F_t} R^-\{x_j\}. \end{array}$$

Two Moore's closures corresponding to the relation $[R]$ are obtained by the max-min convolution R^- with $R^+$and R^+ with R^-. It is written as:

$$M^{(1)} = R^- \circ R^+,$$
$$M^{(2)} = R^+ \circ R^-,$$

where $M^{(1)}$ and $M^{(2)}$ are Moore's closures.

Two sets of Moore's closed sets are formed by the elements that comply with the following conditions:

$$\forall A_k \in C(B): A_k = MA_k.$$

$$\forall F_t \in D(B): F_t = MF_t.$$

If we determine a subset of closed sets of $D(E)$ corresponding to the Moore's closure $M^{(2)}$ as $T\big(E, M^{(2)}\big)$. As R^-F_t is a closed of $D(E)$ for $M^{(2)}$, it can be written as follows:

$$T\big(E, M^{(2)}\big) = \bigcup_{F_t \in D(E)} R^-F_t,$$

and, as R^+A_k is a closed for $M^{(1)}$, and we determine the subset of closed sets of $C(B)$ corresponding to Moore's closure $M^{(1)}$ as $T\big(B, M^{(1)}\big)$, we get:

$$T\big(B, M^{(1)}\big) = \bigcup_{A_k \in C(B)} R^+A_k.$$

Both families $T\big(B, M^{(1)}\big)$ and $T\big(E, M^{(2)}\big)$ are isomorphic and dual for each other. They are also antitone. Therefore, it can be written:

$$A_k \in T\big(B, M^{(1)}\big) \Rightarrow \big(F_t = R^-A_k \in T\big(E, M^{(2)}\big)\ and\ R^+F_t = A_k\big),$$
$$F_t \in T\big(E, M^{(2)}\big) \Rightarrow \big(A_k = R^+F_t \in T\big(B, M^{(1)}\big)\ and\ R^-A_k = F_t\big).$$

These families of closed sets can be associated with each other and each of the families forms a finite lattice. When superposing two lattices we obtain a single lattice that collects in each of its vertices the relation of the groupings of elements of the set B with the set E. When this happens it is said that there is affinity between them, which is the basis of the maximum correspondence algorithm (Gil-Aluja 1999).

Maximum Inverse Correspondence Algorithm

Based on the concepts described previously (Gil-Aluja et al 2009), the maximum inverse correspondence algorithm is defined as follows:

1. It is started out from a Boolean relation $[R]$.
2. The set with the lowest number of elements is chosen between two sets B and E.
3. We create a set $C(B)$ where B is the set with the lowest number of elements selected as established in the previous step.

4. “Right connection” $[R]$ is obtained, that is to say, for all $A_k \in C(B), R^+A_k$ compiles the successors of all the elements that belong to A_k.
5. For each non-empty set of R^+A_k we select the set corresponding to A_k which has the greatest number of elements of the referential B.

The relations obtained form a lattice. If we add the empty-referential and the referential-empty vertices, we obtain a Galois Lattice. The generalization that implies adopting a rectangular relation $[R] \subset B \times E$, as an initial element, made it possible to construct the algorithm.

Galois group theory and Galois Lattices

Galois theory is a connection between the field theory and the group theory. Certain problems in field theory can be reduced to group theory using Galois theory. This allows us to understand the problems easier and to solve them in a simpler way. To begin with, Galois used permutation groups to explain how the various roots of a given polynomial equation were related to each other (Edwards and Tardieu, 1984).

Galois theory is based on a remarkable correspondence between subgroups of the Galois group of an extension E/F and intermediate fields between E and F.

If G = Gal (E/F) is considered to be the Galois group of the extension E/F. If H is a subgroup of G, the fixed field of H is the set of elements fixed by every automorphism in H, that is:

$$F(H) = \{x \in E : \sigma(x) = x \; for \; every \; \sigma \in H\}$$

If *K* is an intermediate field, that is, $F \leq K \leq E$ define:

$$G(K) = Gal(E/K) = \{\sigma \in G : \sigma(x) = x \; for \; every \; x \in K\}.$$

In other words, fixing group of K for G (K), since G (K) is the group of automorphisms of E that leave K fixed. Galois Theory is about the relation between fixed fields and fixing groups (Edwards and Tardieu 1984; Artin 1998).

Definitions of the theory

Definition 1. A lattice is a partially ordered set in which any two elements have a least upper bound (LUB) and a greatest lower bound (GLB). A complete lattice is a lattice where any set has a LUB and a GLB.

Definition 2. A context K is a triple (O, F, ζ) where O is a set of objects; F is a set of attributes and ζ is a mapping from $O \times F$ into $\{0, 1\}$.

Definition 3. Given a context $K = (O, F, \zeta)$ let us define two mappings from $P(O)$ into $P(F)$ and from $P(F)$ into $P(O)$ using the same notation 0 by the formula:

$$\forall A \subset O, A' = \{f \in F | \forall o \in A, \zeta(o, f) = 1\}$$
$$\forall B \subset F, B' = \{o \in O | \forall o \in B, \zeta(o, f) = 1\},$$

A' is called the dual of A, similarly B' is called the dual of B.

Definition 4. Given a context $K = (O, F, \zeta)$, the pair $C = (A, B)$ is called a concept of K if and only if $A' = B$ and $B' = A$.

Definition 5. A is called the extent of the concept C and B is called its intent.

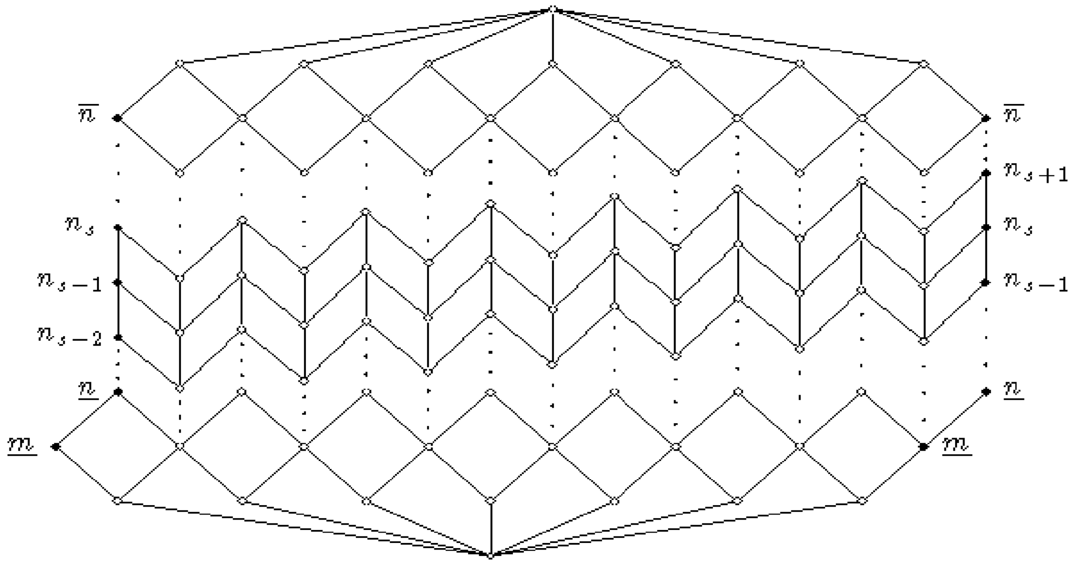
This is denoted by $A = extent\ (C)$ and $B = intent\ (C)$.

Considering an order relationship defined through inclusion of intents, one may define a Galois lattice or concept lattice.

Definition 6. The complete lattice $L(K)$ of concepts of the context K is called (general) Galois lattice or concept lattice.

According to Figure 1, the lattice is infinite and indirectly irreducible, but the variety it generates contains only finitely many other varieties, refuting an old conjecture. If this lattice is imagined as wrapped around a cylinder, then the elements at the sides with the same labels can be identified. The middle part of this diagrams moves up when going from left to right so that when it is wrapped around the cylinder, it is helical. Although the lattice is infinite in Figure 1, we can limit it to the variables that we are going to analyze and apply it to our model (Nation 1985).

Figure 4.1. J. B. Nation's counter example to the finite height conjecture



Source: Retrieved from Nation (1985)

Application of the Model

Proposed variables

In the present study, we use eight variables that according to the literature (Florida 2008; Porter 2003; Chapain et al 2010; Compete Network 2010) define each municipality addressed in terms of competitiveness. Table 4.1 concentrates the economic, social, cultural, political and environmental variables used in the model. Such variables were selected in order to present a general overview of the city and its main drivers (for a detailed compound of the variables see Alfaro et al 2014).

Table 4.1. Competitive variables

A	Integration *(of the Population)*
B	Infrastructure
C	Public Services
D	Tourism
E	Education
F	Marginality
G	Technology
H	Human development index

Source: Self-elaborated. Retrieved from Alfaro et al 2014

It is important to mention that the variables are not exhaustive and have been treated with the same level of importance. Further research needs to be conducted to study the nature of the variables and the relative importance they imply, thus introduce weights in order to improve the model.

Proposed municipalities

Table 4.2 presents the municipalities introduced in the model. The selected 12 of the whole 113 municipalities that comprehend the region were chosen because of the quantity of inhabitants and the economic relevance they display in the region (INAFED 2010). It is important to mention that this study can be expanded, adding municipalities in order to advance in the knowledge of synergies that could be applied in the whole region.

Table 4.2. Studied municipalities

a	Morelia
b	Uruapan
c	Zamora
d	Lázaro Cárdenas
e	Zitácuaro
f	Apatzingán
g	Hidalgo
h	La Piedad
i	Pátzcuaro
j	Maravatío
k	Tarímbaro
l	Zacapu

Source: Self-elaborated. Retrieved from INAFED (2010)

Grouping by affinities

The first step of the process of grouping is the generation of fuzzy subsets. The objective is to evaluate the characteristics, singularities and peculiarities of each municipality in the next way:

$$\underset{\sim}{i} = \begin{array}{cccc} A & B & & N \\ \boxed{\mu_A{}^{(i)}} & \boxed{\mu_B{}^{(i)}} & \cdots & \boxed{\mu_N{}^{(i)}} \end{array}$$

$$i = a, b, c, \ldots, m$$

$$\mu_j{}^{(i)} \in [0,1], j = A, B, \ldots, N$$

With this information we generate a fuzzy matrix comprehended by:

$$\left[\underset{\sim}{R}\right] = \begin{array}{c|cccc} & A & B & & N \\ a & \boxed{\mu_A{}^{(a)}} & \boxed{\mu_B{}^{(a)}} & \cdots & \boxed{\mu_N{}^{(a)}} \\ b & \boxed{\mu_A{}^{(b)}} & \boxed{\mu_A{}^{(b)}} & \cdots & \boxed{\mu_N{}^{(b)}} \\ & & \ldots & \ldots & \ldots \\ m & \boxed{\mu_A{}^{(m)}} & \boxed{\mu_A{}^{(m)}} & \ldots & \boxed{\mu_N{}^{(m)}} \end{array}$$

Table 4.3 shows the set of ten semantic scales used to valuate each competitive variable of the chosen municipalities. The valuation is set from E (*Excellent performance)* to D (*Disastrous performance).*

Table 4.3. Linguistic valuations

E	Excellent performance	(1,0)
GG	Great performance	(0,9)
RG	Really good performance	(0,8)
G	Good performance	(0,7)
RG	Rather good than a poor performance	(0,6)
NG	Neither a good nor poor performance	(0,5)
RPG	Rather a poor than a good performance	(0,4)
P	Poor performance	(0,3)
RP	Really poor performance	(0,2)
W	Worst performance	(0,1)
D	Disastrous performance	(0,0)

Source: Self-elaborated

Table 4.4 shows the valuation of each municipality according to their adequacy for each of the variables stated. The valuation was performed treating information retrieved from the National Institute of Statistics and Geography of Mexico (INEGI 2014), the National Institute for Federalism and Municipal Development (INAFED 2010) and the United Nations Development Programme (UNPD 2014).

Table 4.4. Evaluation Matrix

$\left[\underset{\sim}{R}\right] =$

	A	B	C	D	E	F	G	H
a	0,8	0,9	0,8	1	0,9	1	0,8	1
b	0,8	0,7	0,6	0,8	0,8	1	0,3	0,7
c	0,4	0,4	0,7	0,4	0,5	0,8	0,1	0,7
d	0,7	0,6	0,4	0,5	0,6	0,8	0,9	0,8
e	0,8	0,4	0,2	0,2	0,6	0,1	0,8	0,3
f	0,4	0,3	0,8	0,3	0,2	0,3	0,1	0,3
g	0,4	0,3	0,3	0,4	0,3	0,1	0,1	0,3
h	0,3	0,3	0,9	0,4	0,4	0,8	0,8	0,8
i	0,4	0,3	0,3	0,8	0,4	0,3	0,3	0,3
j	0,3	0,3	0,2	0,3	0,2	0,1	0,3	0,2
k	0,1	0,3	0,8	0,1	0,2	0,1	0,1	0,3
l	0,4	0,3	0,8	0,4	0,3	0,8	0,8	0,8

Source: Self-elaborated. Calculated using (INAFED 2010; INEGI 2014; UNPD 2014)

Once this information has been established and accepted, a desired level of homogeneity for the groups of municipalities must be chosen. The level of homogeneity has to represent the level in which the groups hold specific characteristics, qualities and peculiarities. For each competitive variable we establish:

$$0 \leq \theta_j \leq 1 \, , j = A, B, \ldots, N.$$

In our case we define θ as:

$$\theta_A = 0{,}4, \ \theta_B = 0{,}6, \theta_C = 0{,}7, \theta_D = 0{,}5, \theta_E = 0{,}6, \theta_F = 0{,}5, \theta_G = 0{,}7, \theta_H = 0{,}5$$

The values of θ_j are based on (Alfaro and Gil-Lafuente 2012; Alfaro et al 2014). It is manifest that a high level of $\theta_j \geq 6$ has been established to variables *Infrastructure, Public Services, Education* and *Technology* pursuing the generation of technology consolidated groups.

Continuing with the procedure, values of θ_j and the valuations of each column of competitive variables will be compared. If the valuation given to the specific characteristic is equal or superior to the desired level of homogeneity then the valuation is substituted with a 1, in the contrary 0. Specifically:

$$\mu_j{}^{(i)} \geq \theta_j \;,\; \beta_j{}^{(i)} = 1$$
$$\mu_j{}^{(i)} < \theta_j \;,\; \beta_j{}^{(i)} = 0.$$
$$i = a, b, \dots, m$$
$$j = A, B, \dots, N$$

Table 4.5 shows the homogeneity matrix for the municipalities' case. Notice slots display only 0 or 1.

Table 4.5. Homogeneity Matrix

$[B] =$

	A	B	C	D	E	F	G	H
a	1	1	1	1	1	1	1	1
b	1	1	0	1	1	1	0	1
c	1	0	1	0	0	1	0	1
d	1	1	0	1	1	1	1	1
e	1	0	0	0	1	0	1	0
f	1	0	1	0	0	0	0	0
g	1	0	0	0	0	0	0	0
h	0	0	1	0	0	1	1	1
i	1	0	0	1	0	0	0	0
j	0	0	0	0	0	0	0	0
k	0	0	1	0	0	0	0	0
l	1	0	1	0	0	1	1	1

Source: Self-elaborated

Maximum inverse correspondence algorithm

In order to group the most affine municipalities, we will use the theory of affinities, specifically the maximum inverse correspondence algorithm (Gil-Aluja 1999). Studies and applications of this algorithm in economic and business sectors have led to applicable results while dealing with uncertain conditions (Gil-Lafuente 2002).

From the homogeneity matrix choose the one conjunct that presents the fewer elements. In our case:

$$\{A, B, C, D, E, F, G, H\},$$

Create the *power set*, which represents all the possible combinations of the conjunct with the fewer elements. In our case:

{∅, A, B, C, D, E, F, G, H, AB, AC, AD, AE, AF, AG, AH, BC, BD, BE, BF, BG, BH, CD, CE, CF, CG, CH, DE, DF, DG, DH, EF, EG, EH, FG, FH, GH, ABC, ABD, ABE, ABF, ABG, ABH, ACD, ACE, ACF, ACG, ACH, ADE, ADF, ADG, ADH, AEF, AEG, AEH, AFG, AFH, AGH, BCD, BCE, BCF, BCG, BCH, BDE, BDF, BDG, BDH, BEF, BEG, BEH, BFG, BFH, BGH, CDE, CDF, CDG, CDH, CEF, CEG, CEH, CFG, CFH, CGH, DEF, DEG, DEH, DFG, DFH, DGH, EFG, EFH, EGH, FGH, ABCD, ABCE, ABCF, ABCG, ABCH, ABDE, ABDF, ABDG, ABDH, ABEF, ABEG, ABEH, ABFG, ABFH, ABGH, BCDE, BCDF, BCDG, BCDH, BCEF, BCEG, BCEH, BCFG, BCFH, BCGH, CDEF, CDEG, CDEH, CDFG, CDFH, CDGH, DEFG, DEFH, DEGH, EFGH, ABCDE, ABCDF, ABCDG, ABCDH, ABCEF, ABCEG, ABCEH, ABCFG, ABCFH, ABCGH, BCDEF, BCDEG, BCDEH, BCDFG, BCDFH, BCDGH, CDEFG, CDEFH, CDEGH, DEFGH, ABCDEF, ABCDFG, ABCDGH, ABCEFG, ABCEGH, ABCFGH, BCDEFG, BCDEGH, BCDFGH, CDEFGH, ABCDEFG, ABCDEGH, ABCDFGH, BCDEFGH, ABCDEFGH}

For each element of the *power set* include the corresponding elements of the conjunct that has not been chosen for having a greater number of elements. Table 4.6 presents the so-called *connection to the right*

.

Table 4.6. Connection to the right

A	abcdefgil	FG	adhl	BEF	abd	ABDG	ad	ABCEG	a
B	abd	FH	abcdhl	BEG	ad	ABDH	abd	ABCEH	a
C	acfhkl	GH	adhl	BEH	abd	ABEF	abd	ABCFG	a
D	abdi	ABC	a	BFG	ad	ABEG	ad	ABCFH	a
E	abde	ABD	abd	BFH	abd	ABEH	abd	ABCGH	a
F	abcdhl	ABE	abd	BGH	ad	ABFG	ad	BCDEF	a
G	adehl	ABF	abd	CDE	a	ABFH	abd	BCDEG	a
H	abcdhl	ABG	ad	CDF	a	ABGH	ad	BCDEH	a
AB	abd	ABH	abd	CDG	a	BCDE	a	BCDFG	a
AC	acfl	ACD	a	CDH	a	BCDF	a	BCDFH	a
AD	abdi	ACE	a	CEF	a	BCDG	a	BCDGH	a
AE	abde	ACF	acl	CEG	a	BCDH	a	CDEFG	a
AF	abcdl	ACG	al	CEH	a	BCEF	a	CDEFH	a
AG	adel	ACH	acl	CFG	ahl	BCEG	a	CDEGH	a
AH	abcdl	ADE	abd	CFH	achl	BCEH	a	DEFGH	ad
BC	a	ADF	abd	CGH	ahl	BCFG	a	ABCDEF	a
BD	abd	ADG	ad	DEF	abd	BCFH	a	ABCDFG	a
BE	abd	ADH	abd	DEG	ad	BCGH	a	ABCDGH	a
BF	abd	AEF	abd	DEH	abd	CDEF	a	ABCEFG	a
BG	ad	AEG	ade	DFG	ad	CDEG	a	ABCEGH	a

BH	abd	AEH	abd	DFH	abd	CDEH	a	ABCFGH	a
CD	a	AFG	adl	DGH	ad	CDFG	a	BCDEFG	a
CE	a	AFH	abcdl	EFG	ad	CDFH	a	BCDEGH	a
CF	achl	AGH	adl	EFH	abd	CDGH	a	BCDFGH	a
CG	ahl	BCD	a	EGH	ad	DEFG	ad	CDEFGH	a
CH	achl	BCE	a	FGH	adhl	DEFH	abd	ABCDEFG	a
DE	abd	BCF	a	ABCD	a	DEGH	ad	ABCDEGH	a
DF	abd	BCG	a	ABCE	a	EFGH	ad	ABCDFGH	a
DG	ad	BCH	a	ABCF	a	ABCDE	a	BCDEFGH	a
DH	abd	BDE	abd	ABCG	a	ABCDF	a	ABCDEFGH	a
EF	abd	BDF	abd	ABCH	a	ABCDG	a		
EG	ade	BDG	ad	ABDE	abd	ABCDH	a		
EH	abd	BDH	abd	ABDF	abd	ABCEF	a		

Source: Self-elaborated

We choose, from every non-void conjunct of the *connection to the right* the corresponding conjunct of the *power set*, which possess the greater number of elements. Table 4.7 exhibits the corresponding elements for the municipalities' case.

Table 4.7. Maximum inverse correspondence matrix

A	abcdefgil	AEG	ade	ABDH	abd
C	acfhkl	AFG	adl	ABEF	abd
G	adehl	AFH	abcdl	ABEG	ad
AC	acfl	AGH	adl	ABEH	abd
AD	abdi	CFG	ahl	ABFG	ad
AE	abde	CFH	achl	ABFH	abd
AG	adel	CGH	ahl	ABGH	ad
FH	abcdhl	FGH	adhl	DEFH	abd
ACF	acl	ABDE	abd	CDEGH	ad
ACG	al	ABDF	abd	ABCDEFGH	a
ACH	acl	ABDG	ad		

Source: Self-elaborated

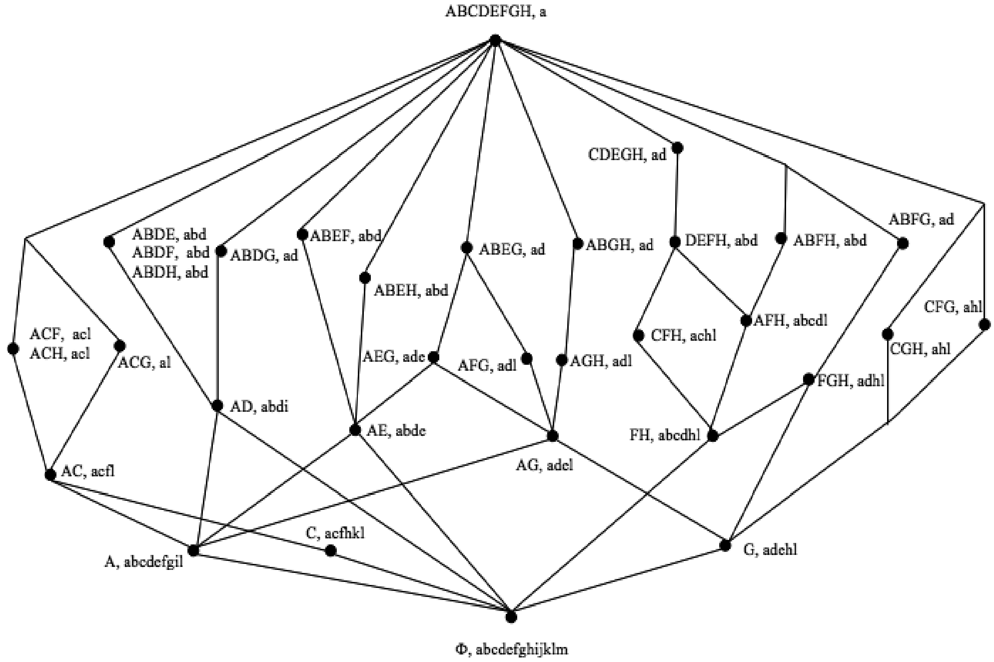
At this point we have found the maximum number of relations, named *affinities*. The algorithm applied allowed in an unambiguous method to create the biggest amount of groups, under to the chosen homogeneity level.

The relations between both conjuncts create themselves a Galois lattice. Such representation allows demonstrating in an ordered way the homogeneous groups as well as a proposed structure of the elements. Figure 4.2 represents the proposed Galois lattice for our case.

Results and Discussion

The generated Galois lattice allows a systematic and ordered perspective of the affinities between the selected municipalities. The lattice shows the characteristics that each of the homogeneous groups of municipalities holds and interrelates them thorough a coherent structure.

Figure 4.2. Galois Lattice for the Municipalities Grouping Case



Source: Self-elaborated

The lattice presents 7 different levels. Each level groups the municipalities that share characteristic elements. Starting form the lower vertex, when ascending in Figure 4.2, the amount of characteristics increase and the amount of grouped municipalities decrease. The first level shows there is no characteristic element that groups the total set of municipalities. However, the second level shows 3 homogeneous groups that share a single characteristic, element A groups municipalities *a, b, c, d, e, f, g, i* and *l*; element C groups *a, c, f, h, k* and *l*; and element G groups a, d, e, h and l. If we aim to build groups that share two characteristics, we will find 5 possibilities: AC, AD, AE, AG and FH. It is noticeable that we loose certain municipalities when we try to build groups with more characteristics e.g. AC groups a, c, f and l, in this case losing municipalities *b, d, e, g, i, h* and k in the process. In the fourth level we find 11 groups that share 3 characteristics in common, the largest amount of grouped municipalities in this level is found in AFH grouping municipalities *a, b, c, d* and l. In the fifth level we find 11 groups that share 4 common characteristics, the largest amount of grouped municipalities is shared by ABDE, ABDF, ABDH, ABEF, ABEH, ABFH and DEFH, comprising municipalities *a, b* and *d*. These municipalities tend to be the most affine in the upper levels of the lattice. The sixth level holds one single group sharing characteristics CDEGH, the municipalities grouped are *a* and *d*. The seventh and last level shows *a* as the municipality that groups all the characteristics.

The generated lattice presents multiple significances for the decision making process. In the one hand it helps to visualize the amount of characteristics that each municipality holds, this allows to rapidly know which characteristics are robust in certain municipalities, in the other hand, the fact that certain municipalities do not figure in the upper levels of the lattice represent the major challenges that need to be addressed in order to strengthen their position in the region.

When conducting traditional grouping of municipalities, trial and error methods are employed, therefore, the confidence level in which the inherent characteristics of studied cities relate to each other tends to be scarce. In order to optimize the process of grouping municipalities, we propose the use of Galois Group Theory, which allows us to know exactly which municipalities are robust at determined characteristics with a significant level of confidence, thus allowing the creation of synergies at allocation of resources; centralized management and knowledge management (Porter 1998).

In our research, the adoption and application of the Galois group theory to the measured variables has multiple significances:

Firstly, it allows us to establish different levels of synergies that could be created as of the inherent characteristics of the municipalities analyzed.

Secondly, once the level of synergies has been detected and established, the model allows us to know precisely which the specific characteristics that foster the optimization of the regional groups are.

Thirdly, a selection on which of the characteristics operated are the ones that we want to prioritize in a specific environment of regional development.

Galois group theory has been proven efficient in different fields because of the "order- or structure-preserving passage between two worlds of our imagination - and thus are inherent in human thinking wherever logical or mathematical reasoning about certain hierarchical structures is involved" Denecke et al (2004), they provide.

Other applications that have conducted the application of Galois lattices with success can be found in the aggrupation of stakeholders for a better administration of enterprises see e.g. Gil-Lafuente and Paula (2013), and in human resources areas, with a personnel selection model, see e.g. (Keropyan and Gil-Lafuente 2013).

Conclusion

We propose a group-based model that relies on the comparison between determined variables due to the inherent characteristics of the studied municipalities in order to create positive synergies between them. Our model is founded on the basic principles of Galois group theory; this process allows us to group different municipalities with a certain level of significance. The present work tries to shed a light on the academic world by offering an intuitive group based model in which subjective and relative factors are intrinsic for the decision making process. Also, this analysis tries to aid decision makers so they can create common policies according to the results of the grouping processes. Further research needs to be conducted. Firstly, a study of the nature of the competitive variables selected to know whether they need to be weighted, and if this weight would play a significant role in the results obtained, secondly, the application of the model in diverse scenarios to expand the visualization of results. The model we present can be applied to different locations and it may allow optimizing the process of grouping of municipalities under flexible, subjective and uncertain conditions.

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Chapter 4. Innovation measurement under uncertainty

Since the first definition of innovation in 1934, different points of view, methodologies and models have been published trying to explain the phenomenon.

Today, we find seven organizational areas that show a tendency to define the intensity of innovation within a company. However, the uncertainty connected to the concept difficult its correct measurement.

Our proposal is a methodology to quantify the measure of innovation capabilities in the organization combining Fuzzy Logic tools and aggregation operators.

The result is the added representative value of the opinions of more than 90 decision-makers addressing the before mentioned seven organizational study areas in their own businesses.

With this approach, it is possible to quantify the measure of innovation in an industrial sector, encompassing perspectives, opinions and subjective conditions through an intuitive, agile and highly reproducible tool for comparison and benchmark among companies of the sector.

A Fuzzy Methodology for Innovation Management Measurement[4]

Abstract

Innovation has been recognized as one of the main sources of competitive advantage for organizations and nations. The purpose of this study is to present an innovation management measurement approach applying fuzzy techniques to small and medium manufacturing enterprises. This study presents a survey focusing on seven innovation measurement areas: innovation strategy, knowledge management, project management, portfolio management, internal enablers, organization and structure and external enablers. A total of 91 small and medium enterprises (SMEs) located in the city of Morelia, Mexico, participated in the study. Furthermore, the data collected were analyzed under a multi-criteria decision-making approach using the theory of expertons and the induced generalized ordered weighted averaging (IGOWA) operator. The results show that the most valued areas are innovation strategy, organization and structure, knowledge management and project management. Furthermore, portfolio management, external drivers and internal drivers are the areas with the lowest valuations. This paper presents an original methodological structure based on an expertise process designed to achieve well-founded results from uncertain and subjective opinions directly from the managers of the surveyed SMEs.
Keywords: Innovation measurement; Fuzzy Analysis; Innovation Management; Theory of expertons; Aggregation operators; Small and medium sized firms.

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Introduction

J. Schumpeter (1934) proposed the first study of innovation as a dynamic concept for economic development and economic progress. Since then, a plethora of definitions for innovation have appeared, e.g., the department of trade and industry of the UK defines that innovation is the successful exploitation of new ideas (Branson, 1988), while the Oslo Manual (OECD, 2005) states that innovation is the implementation of a new or significantly improved product (good or service), process, marketing method, or organizational method in business practices, workplace organization or external relations.

Over the past six decades, the literature concerning innovation management has experienced constant evolution (Drejer, 2002; Keupp et al., 2012; Tidd, 2001). Much of the interest centers on the idea that innovation is a driver for competitiveness (see, e.g., Porter, 1990), and some of its positive effects include increases in sales and market share, productivity improvement, and efficiency in operations.

The late 1970s saw the emergence of a focused interest in the study of innovation management in small and medium sized enterprises (SMEs). Increasing attention received by SMEs has been primarily triggered by the dynamism they generate in the countries' economies (Wolff and Pett, 2006), increase in employment rates (Birch, 1989), and especially in their ability to perform remarkable innovations (Fritz, 1989; Keizer et al., 2002). The sustainable introduction of innovations by SMEs strengthens the position of industry and generates profits above the average in the market (Porter, 1980).

An interesting issue in innovation management is its measurement (Gimbert et al., 2010; Simons, 1990). Approximately 40 years ago, only input indicators were analyzed, e.g., spending on research and development (R&D), number of acquired patents and the amount of highly qualified human resources. Currently, the concept has advanced toward consideration of innovation measurement as a system, e.g., the approach of the holistic framework of innovation management measurement suggested by Adams et al. (2006), the consolidation of theories around innovation and its measurement by Crossan and Apaydin (2010), and the empirical study undertaken by Edison et al., (2013).

The main objective of innovation measurement is to systematically determine whether the resources committed to innovation activities within companies are justified. This allows the revision of a firm's innovation goals and objectives and the most adequate set of incentives for its promotion and management (Cordero, 1990). However, this measurement is complex because there is no single or main trend to assess the extent of innovation, plus the information regarding the concept is highly subjective and uncertain (Alfaro-García et al., 2015; Frenkel et al., 2000).

The aim of this paper is to present a methodological structure for the measurement of innovation management in manufacturing SMEs under a fuzzy approach. This is an original methodology that first presents a survey designed to evaluate the opinions of SME managers around the seven key innovation measurement areas proposed by Adams et al. (2006): innovation strategy, knowledge management, project management, portfolio management, internal drivers, organization and structure, and external drivers. The process used to analyze the subjective information retrieved from the survey is a multi-criteria decision-making problem employing two models. The first is the theory of expertons (Kaufmann and Gil-Aluja, 1993; Kaufmann, 1988); the main advantage of this tool is the complete consideration of group information, including all of the individual opinions, and the production of a final single result. The second is the application of the induced generalized ordered weighted averaging (IGOWA) operator (Yager, 2004) to aggregate the expertons results. The objective of the IGOWA operator is the aggregation of information considering different degrees of importance to the arguments included in the problem. It has been observed that aggregation operators are useful when there is a need to assess information in a more efficient way than can be accomplished through traditional averaging (Emrouznejad and Marra, 2014).

The article is structured as follows. Section 1 presents the preliminaries of the study, considering the innovation management measurement literature, the theory of expertons and the IGOWA operator. Section 2 discusses the methodology and the results of the survey. Section 3 presents the application of the theory of expertons and IGOWA analysis to our specific problem. Finally, Section 5 summarizes the main conclusions of the article.

Preliminaries

In this section, we briefly review some basic concepts regarding the innovation management measurement, the theory of expertons and the induced generalized ordered weighted aggregation operator.

Innovation management measurement

This study takes the proposal developed by Adams et al. (2006) as the primary innovation measurement reference. The framework developed by Adams et al. (2006) is based on a review of six models and frameworks of innovation measurement (Burgelman et al., 2009; Chiesa et al., 1996; Cooper and Kleinschmidt, 1995; Cormican and O'Sullivan, 2004; Goffin and Pfeiffer, 1999; Verhaeghe and Kfir, 2002). It should be noted that from the seven main areas outlined by the authors, a framework for measuring innovation has been adapted. This approach, which is based on the theory of dynamic capabilities (Teece et al., 1997; Pavlov and El Sawy, 2011), has been widely adopted to measure, model and describe innovation management performance, e.g., a theoretically based framework for innovation capabilities measurement in SMEs (Saunila, 2016); empirical research of process innovation activities and mechanisms in manufacturing and service firm performance (Piening and Salge, 2015); a four staged multi-dimensional process-based innovation performance measurement scheme (Dewangan and Godse, 2014); and an innovation index to benchmark innovation capabilities in SMEs (Galvez et al., 2013).

The first innovation measurement area (IMA) is innovation strategy in firms. Studies have suggested that inefficiencies in processes can be reduced by maintaining an innovation strategy integrated to the culture, behavior and actions of an organization (O'Brien, 2003). Therefore, when assessing the area of innovation strategy in a firm, it is relevant to analyze factors such as long-term administrative commitment and the direct location of resources toward innovation efforts (Cooper et al., 2004), the link between key business objectives and leadership created by a shared and robust vision along the organizational structure (Pinto and Prescott, 1988), the risk aversion level of the board, and the pro-activity of the direction as well as their persistence and commitment to innovation (Saleh and Wang, 1993). Moreover, empirical studies have demonstrated that appropriate formulation, implementation and monitoring of an innovation strategy for the firm can assure the development of strategic capabilities (Vicente et al., 2015).

The second IMA is knowledge management. This includes the management of explicit and implicit knowledge within organizations (Nonaka, 1991), as well as the process of collection and use of such information. It is therefore worth determining the level of absorptive capacity (AC), understood as the ability to recognize the value of new knowledge from ideas generated within the company (Chiesa et al., 1996; Valentim et al., 2015) or obtained from external connections with other companies or information resources (Atuahene-Gima, 1995; Tipping et al., 1995; Chang et al., 2013), as well as the assimilation and application of knowledge to commercial activities (Cohen and Levinthal, 1990).

The third IMA considers project management. The literature agrees on the importance of the relationship between innovation and efficiency along with its impact on business performance when managing projects (Spieth and Lerch, 2014). Commonly, such a relationship is measured in terms of cost, duration and return on investment (Adams et al., 2006; Chiesa et al., 1996; Lichtenthaler, 2016). Additionally, it is interesting to know the extent of the internal communication of the implicated areas when developing a new product (Damanpour, 1991), as well as collaboration with suppliers (Bessant, 2003) and customers (Von Hippel, 1986) given their identification as sources that contribute to the innovation process.

The fourth IMA is new product portfolio management. Given the fast changing environment in which SMEs govern their production activities, the effectiveness in which an organization manages its new products portfolio is often a key determinant of competitive advantage (Bard et al., 1988). It is important to know the extent to which firms base their operations on systematized processes that are guided by clear criteria, as doing so facilitates the optimal use of limited resources and improves an organization's competitive position (Hall and Nauda, 1990). Furthermore, highly competitive companies use formal tools applied consistently to all developing projects of a given portfolio (Cooper et al., 1999) and the selection of the product to be developed must be a rational and objective choice based on a systematized path of multi-criteria dimensions (Jugend and Da Silva, 2013).

The fifth IMA includes all internal drivers, which can be defined as entry systems and tools for the innovation process (Adams et al, 2006). According to the literature, internal drivers provide a competitive advantage for companies that use them formally (Bessant and Francis, 1997; Cooper et al., 2004). In that order of ideas, it is important to know and measure the timeliness in which organizations allocate resources (both financial resources and personnel) to product development and the efficiency of that process (Hinckeldeyn et al., 2015)

The organization and structure of the firm is the sixth IMA. It is generally accepted that companies can create work environments that promote the innovation process (Tidd et al., 2001). In that sense, it is necessary to know the intensity with which companies maintain their organizational structure aligned with their project management processes (Pugh et al., 1969); it is also necessary to measure the freedom that workers experience while generating ideas from experimentation, the conception of mistakes as a source of expertise (Anderson and West, 1996; Zien and Buckler, 1997) and the general creativity supporting work environment of the firm (Dul and Ceylan, 2014).

The seventh and last IMA are the external drivers. This area measures the intensity with which a company launches its products to the market (Calantone and Benedetto, 1988; Globe et al., 1973), i.e., market research, testing and development adapted to a systematic marketing program (Griffin and Page, 1993). It additionally measures the way firms reach the consumer, formal post-sale operations (Atuahene-Gima, 1995; von Zedtwitz, 2002) and technology and marketing synergy, which has been empirically demonstrated to have an influence on product performance (Huang and Tsai, 2014).

The theory of expertons

The theory of expertons (Kaufmann and Gil-Aluja, 1993; Kaufmann, 1988) suggests that in order to obtain realistic data from phenomena that are not directly measurable, an aggregated set of valuations given by experts is useful. Note that the experton is a concept issued from theories of fuzzy sets, intervals of confidence and random sets.

An experton is an extension of the probabilistic set (Hirota, 1981), where an interval of probabilities is obtained for each level $\alpha \in [0,1]$ from the valuation of several experts (Kaufmann, 1990). An experton is a generalization of a probabilistic set when cumulative probabilities are replaced by intervals, which decrease monotonically. Moreover, a probabilistic set is a generalization of a fuzzy set, which in turn is a generalization of an ordinary set. Kaufmann (1988), mathematically describes an experton as follows:

Assume E is a referential set, finite or not; r experts are asked to give their own subjective opinion about each element of E by an interval of confidence given by:

$$\forall x \in E\colon \left[a_*^j(x), a_j^*(x)\right] \subset [0,1], \tag{1}$$

where $\subset$ is the set inclusion and j is the jth expert.

Then, a statistic that concerns for each $x \in E$ the lower bounds in one way and the upper bound in the other, a cumulative complementary law $F_*(a,x)$ is established for the $a_*^j(x)$ and $F^*(a,x)$ is established for the $a_j^*(x)$. From that process, we obtain:

$$\forall x \in E, \forall x \in [0,1]\colon \tilde{A}(x) = [F_*(a,x), F^*(a,x)], \tag{2}$$

where symbol $\tilde{}$ is the nature of the concept.

The referential set E is the following experton:

$$\forall x \in E, \forall \alpha \in [0,1]\colon [F_*(a,x), F^*(a,x)] = 1. \tag{3}$$

The empty experton is then given by:

$$\forall x \in E, \colon [F_*(a,x), F^*(a,x)] = \begin{cases} 1, \alpha = 0 \\ 0, \alpha \neq 0 \end{cases}. \tag{4}$$

Finally, the experton can be reduced to a single representative value by decreasing the results' entropy. This process is useful when a final consideration or interpretation of phenomena is needed. In this case, the most common way to reduce the entropy of an experton can be obtained by calculating the mathematical expectation of the probabilistic set.

Expertons ease group decision-making by providing quantitative data retrieved directly from the dialog maintained with several experts surrounding a certain phenomenon. In general, this approach allows an interesting tool for the aggregation of information, unifying different views or expectations of groups with different interests. Moreover, the model allows recognition of the levels of distribution in the aggregate values of the characteristic membership function.

Aggregation operators in decision-making

The wide range of problems that aggregation operators reach, especially in the areas of economics, statistics and engineering, has generated increasing interest in the literature (Emrouznejad and Marra, 2014). An extensive number of applications with aggregation operators have been proposed (Beliakov et al., 2007; Torra and Narukawa, 2007; Yager et al., 2011).

One of the most common methods is the ordered weighted averaging (OWA) operator (Yager, 1988). This operator aggregates information by establishing a weighting vector that gives a specific degree of importance to the ordered arguments presented in the problem. Since its appearance, the OWA operator has been applied in numerous applications (e.g., Yager and Kacprzyk, 1997)

An interesting generalization of the OWA operator is the induced generalized ordered weighted aggregation (IGOWA) operator (Merigó and Gil-Lafuente, 2009). This aggregation operator shares the main characteristics of the OWA operator; the main difference is the introduction of order-induced variables, which generate a new reordering mechanism of the arguments (Yager and Filev, 1999), thus considering higher complexity in the attitudinal characteristics of the decision makers (Chen and Chen, 2003; Wei, 2009; Xu, 2006; Yager, 2003). The IGOWA operator can be defined as follows:

Definition 1. An IGOWA operator of dimension n is a mapping $IGOWA: R^n \rightarrow R$, associated with a weighting vector W of dimension n such that $\sum_{j=1}^{n} w_j = 1$, $w_j \in [0,1]$, a set of order-inducing variables u_i, and a parameter $\lambda \in (-\infty, \infty)$, following the next formula:

$$IGOWA(\langle u_1, a_1 \rangle, \dots, \langle u_n, a_n \rangle) = \left(\sum_{j=1}^{n} w_j {b_j}^{\lambda} \right)^{1/\lambda}, \quad (5)$$

where $(b_1, \dots, b_n)$ is $(a_1, a_2 \dots, a_n)$ reordered in decreasing values of the u_i. Note that the u_i are the order-inducing variables and the a_i are the argument variables. It has been demonstrated that the IGOWA operator is commutative, idempotent, bounded and monotonic (Merigó and Gil-Lafuente, 2009).

The IGOWA operator has been applied in financial decision-making processes (Merigó and Gil-Lafuente, 2009; Merigó and Casanovas, 2011), but the parameterized families of the operator, including the window induced generalized ordered weighted aggregation (Window-IGOWA) operator, the olympic induced generalized ordered weighted average (Olimpic-IGOWA) operator, the step induced generalized ordered weighted (S-IGOWA) operator, among others (see Merigó and Gil-Lafuente, 2009), could be employed in diverse fields of knowledge, including engineering, economics, or statistics. In our particular case, we use the IGOWA operator in a multi-criteria decision-making problem considering the aggregation of an expert's subjective opinions regarding innovation management.

Methodology

In this section, we present the methodology employed to develop the survey and the data gathering.

The survey included a total of 32 statements. Each statement can be answered by an endecadary interval of 11 equidistant positions [0,1], with 0 being complete disagreement with the statement, and 10 being complete agreement with the suggested statement (see Kaufmann, 1990). Note that all of the statements were selected from previously tested studies in innovation management diagnostics or innovation management audits (Chiesa et al., 1996; CIDEM, 2002). Note also that the 32 questions are categorized in the seven key innovation measurement areas proposed by Adams et al. (2006). The original survey was pilot tested by nine experts in innovation management: three academicians from the Universidad Michoacana de San Nicolás de Hidalgo, two academicians from the University of Barcelona and four local entrepreneurs. Seven responses were retrieved. With that information, the survey was improved.

The goal of the research was to obtain the largest pool of information possible from the manufacturing SMEs located in the city of Morelia, México. To achieve this goal, we revised the national statistical directory of economic units (INEGI, 2015) to find the total number of companies that matched the selected profile. A total of 182 active small and medium sized manufacturing firms were found. Then, a team was created and capacitated with the aim of personally surveying the 182 SMEs; this team included 39 students and three professors. The data were collected from January to July 2015. The intended respondents of the survey were the general managers of the SMEs. If the general manager was not available, then the production manager responded to the survey. From a total of 182 surveyed firms, 91 valid responses were retrieved, i.e., a response rate of 50%. From those 91 companies that adequately responded to the survey, 78% are classified as small-sized businesses and 12% as medium-sized companies. Additionally, general information was collected from participating SMEs, including type of industry, actual size of the organization and electronic address. The general characteristics of the survey are described in Table 5.1.

Table 5.1. Characteristics of the Survey

Characteristic	Value
Location	Morelia, México.
Time of the survey	January – July 2015
Estimated population*	182
Sample size	124
Number of responses	91
Response rate**	74%
Confidence level (p = q = 0.5)	96%

Source: Self-elaborated.

** Total estimate of registered manufacturing SMEs in the city of Morelia, México. (INEGI, 2015).*

***Response rate of the sample.*

Please note that the sample combines the small and medium sized manufacturing firms. Note also that a small enterprise is characterized as having a minimum of 11 employees and a maximum of 50, while medium sized manufacturing firms are characterized as having a minimum of 51 employees and a maximum of 250 employees (INEGI, 2009). A manufacturing firm is defined as an economic and legal unit under a single proprietary or controlling entity that is primarily engaged in industrial activities for public sales and has an operational structure divided into branches or a single physical location (INEGI, 2010).

Results

Validation of the survey

Two statistical analyses were applied to validate the information retrieved from the survey. The first is a reliability analysis describing the properties of the scales used within the survey. The second is a discriminant test to validate the constructed variables.

Of the diverse procedures for conducting reliability analysis (Campbell and Russo, 2001; Carmines and Zeller, 1979), in this study, we applied Cronbach's alpha coefficient of reliability. This test was applied to the 32 statements included in the survey, each of which was characterized in the seven innovation management areas described in Adams et al., 2006. Table 5.2 shows the results of the reliability test.

Table 5.2. Cronbach's alpha test results

Innovation Management Area	Elements	Cronbach's alpha*
1. Innovation Strategy	5	0.843
2. Knowledge Management	4	0.788
3. Project Management	5	0.825
4. Portfolio Management	4	0.845
5. Internal Drivers	5	0.867
6. Organization and Structure	4	0.779
7. External Drivers	5	0.516

Source: Authors calculations based on data collected in fieldwork.

**N = 91 Valid cases (100%).*

It is widely known that the Cronbach's alpha test evaluates the inter-correlation between the included elements (Hogan, 2004). In our case, the general Cronbach's alpha is 0.911 for all the elements in the survey. Regarding each area, the minimum value obtained is 0.516. Based on the minimum standard values for a Cronbach's test (Rosenthal, 1994), we determined the reliability of the elements included in the survey.

The discriminant validity test is a correlation analysis applied to the seven constructs generated by the 32 statements included in the survey, i.e., the seven IMA. The results with a significance level of 99% show that no constructs are perfectly correlated, therefore validating the information. Table 5.3 shows the obtained results.

Table 5.3. Correlation Analysis

	IS	KM	PM	PtM	ID	OS	ED
IS	-	.693**	.655**	.657**	.666**	.674**	.577**
KM		-	.738**	.719**	.721**	.649**	.574**
PM			-	.786**	.767**	.598**	.539**
PtM				-	.741**	.707**	.577**
ID					-	.698**	.634**
OS						-	.563**
ED							-

IS: Innovation Strategy, KM: Knowledge Management, PM: Project Management, PtM: Portfolio Management, ID: Internal Drivers, OS: Organization and Structure, ED: External Drivers

Source: Authors calculations based on data collected in fieldwork.

*** Correlation is significant at the 0.01 level (bilateral)*

Fuzzy Analysis

Once the results of the survey have been validated, we suggest a fuzzy analysis following two procedures. The first is the application of the theory of expertons to generate a single representative value for the opinions of the managers regarding each of the 32 statements included in the survey. The second consists of the calculation of the IGOWA operator to aggregate the results of the expertons into the seven innovation management measurement variables considered in the study.

Expertons Analysis

The objective of the experton analysis is the development of a single representative value of the 91 subjective opinions of the SME managers in relation to the 32 statements presented in the survey. The general procedure can be described as the following set of steps:

Step 1.1. From the information obtained in the survey, i.e., the subjective opinion of each manager within the interval [0,1], with 0 being complete disagreement with the statement and 1 being complete agreement with the statement, we construct the expertons. Note that the information could be presented in discrete numbers, intervals, and triplets, among others (Kaufmann and Gil-Aluja, 1993).

Step 1.2. We start by calculating the absolute frequencies, i.e., the number of experts who consider the same value for each statement of the survey. Next, we calculate the relative frequencies, i.e., divide the absolute frequencies by the total number of experts. Finally, we generate the accumulated relative frequencies (we sum from $\alpha = 1$, the relative frequencies in an accumulated way until $\alpha = 0$) (Kaufmann, 1988; Kaufmann & Gil-Aluja, 1993). The result is the experton for each α level.

Step 1.3. Once all of the opinions are aggregated, we need to calculate the expected value of the expertons. The EV is obtained by the addition of all levels of membership α, except 0, and dividing the result by 10. Following these steps, Tables 5.4 - 5.10 show the experton analysis results for the surveyed manufacturing SMEs.

Table 5.4. Expertons results for innovation strategy

Expertons	S. 1.1	S. 1.2	S. 1.3	S. 1.4	S. 1.5
0	1	1	1	1	1
0.1	1	1	1	1	1
0.2	1	1	1	1	1
0.3	0.95	0.95	0.96	0.96	0.92
0.4	0.95	0.95	0.96	0.96	0.92
0.5	0.82	0.84	0.7	0.86	0.74
0.6	0.82	0.84	0.7	0.86	0.74
0.7	0.52	0.53	0.46	0.56	0.43
0.8	0.52	0.53	0.46	0.56	0.43
0.9	0.22	0.26	0.22	0.22	0.14
1	0.22	0.26	0.22	0.22	0.14
Expected Value	0.70	0.71	0.67	0.72	0.65

Table 5.5. Expertons results for knowledge management

Expertons	S.2.1	S. 2.2	S. 2.3	S. 2.4
0.0	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.3	0.95	0.98	0.91	0.88
0.4	0.95	0.98	0.91	0.88
0.5	0.77	0.84	0.69	0.64
0.6	0.77	0.84	0.69	0.64
0.7	0.49	0.64	0.44	0.36
0.8	0.49	0.64	0.44	0.36
0.9	0.26	0.35	0.10	0.12
1.0	0.26	0.35	0.10	0.12
Expected Value	0.69	0.76	0.63	0.60

Table 5.6. Expertons results for project management

Expertons	S. 3.1	S. 3.2	S. 3.3	S. 3.4	S. 3.5
0.0	1.00	1.00	1.00	1.00	1.00
0.1	1.00	0.99	1.00	1.00	1.00
0.2	1.00	0.99	1.00	1.00	1.00
0.3	0.86	0.87	0.90	0.97	0.80
0.4	0.86	0.87	0.90	0.97	0.80
0.5	0.67	0.73	0.70	0.87	0.60
0.6	0.67	0.73	0.70	0.87	0.60
0.7	0.44	0.40	0.53	0.52	0.32
0.8	0.44	0.40	0.53	0.52	0.32
0.9	0.22	0.16	0.21	0.16	0.14
1.0	0.22	0.16	0.21	0.16	0.14
Expected Value	0.64	0.63	0.67	0.70	0.57

Table 5.7. Expertons results for portfolio management

Expertons	S.4.1	S. 4.2	S. 4.3	S. 4.4
0.0	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.3	0.86	0.81	0.89	0.82
0.4	0.86	0.81	0.89	0.82
0.5	0.65	0.55	0.74	0.71
0.6	0.65	0.55	0.74	0.71
0.7	0.35	0.33	0.31	0.47
0.8	0.35	0.33	0.31	0.47
0.9	0.12	0.14	0.10	0.18
1.0	0.12	0.14	0.10	0.18
Expected Value	0.60	0.57	0.61	0.64

Table 5.8. Expertons results for internal drivers

Expertons	S. 5.1	S. 5.2	S. 5.3	S. 5.4	S. 5.5
0.0	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	0.99	0.99
0.2	1.00	1.00	1.00	0.99	0.99
0.3	0.69	0.81	0.93	0.93	0.79
0.4	0.69	0.81	0.93	0.93	0.79
0.5	0.51	0.63	0.69	0.76	0.65
0.6	0.51	0.63	0.69	0.76	0.65
0.7	0.25	0.37	0.46	0.41	0.40
0.8	0.25	0.37	0.46	0.41	0.40
0.9	0.12	0.19	0.18	0.19	0.12
1.0	0.12	0.19	0.18	0.19	0.12
Expected Value	0.51	0.60	0.65	0.65	0.59

Table 5.9. Expertons results for organization and structure

Expertons	S. 6.1	S. 6.2	S. 6.3	S. 6.4
0.0	1.00	1.00	1.00	1.00
0.1	0.99	0.99	0.99	0.99
0.2	0.99	0.99	0.99	0.99
0.3	0.87	0.79	0.90	0.91
0.4	0.87	0.79	0.90	0.91
0.5	0.62	0.68	0.76	0.79
0.6	0.62	0.68	0.76	0.79
0.7	0.38	0.49	0.56	0.57
0.8	0.38	0.49	0.56	0.57
0.9	0.19	0.31	0.32	0.22
1.0	0.19	0.31	0.32	0.22
Expected Value	0.61	0.65	0.71	0.70

Table 5.10. Expertons results for external drivers

Expertons	S. 7.1	S. 7.2	S. 7.3	S. 7.4	S. 7.5
0.0	1.00	1.00	1.00	1.00	1.00
0.1	0.99	0.99	0.99	0.99	0.99
0.2	0.99	0.99	0.99	0.99	0.99
0.3	0.92	0.88	0.88	0.92	0.90
0.4	0.92	0.88	0.88	0.92	0.90
0.5	0.76	0.74	0.73	0.79	0.71
0.6	0.76	0.74	0.73	0.79	0.71
0.7	0.42	0.49	0.46	0.63	0.49
0.8	0.42	0.49	0.46	0.63	0.49
0.9	0.11	0.19	0.22	0.34	0.19
1.0	0.11	0.19	0.22	0.34	0.19
Expected Value	0.64	0.66	0.65	0.73	0.66

IGOWA analysis

Once we obtained the expected value for all of the expertons, the next step is the application of an aggregation operator. This calculation allows us to generate overall results for the seven innovation management areas (Adams et al., 2006): innovation strategy, knowledge management, project management, portfolio management, internal drivers, organization and structure and external drivers.

For our specific problem, we choose the IGOWA operator (Merigó and Gil-Lafuente, 2009). The main advantage of this operator is the aggregation of information considering a highly complex attitudinal character from the decision makers, which in the case of innovation management is greatly required. The steps to generate the aggregation using the IGOWLA operator can be described as follows:

Step 2.1. We first need to describe the parameters to be introduced in the aggregation process. In our case, we choose the parameter $\lambda = -3$. Table 5.11 presents the collective weights W_j assigned to the ordering of the arguments. Because of the complex attitude of the decision makers, Table 5.12 presents a set of u_i order-induced variables that take part in the reordering mechanism. Note that in our case, vectors W_j and u_i are set by the preferences of five experts in the areas of public policy. The primary intention is to have a closer approach to the specific environment of the city and the surveyed SMEs.

Table 5.11. Collective weights Wj

Innovation Measurement Area	w_1	w_2	w_3	w_4	w_5
1. Innovation Strategy	0.30	0.10	0.40	0.15	0.05
2. Knowledge Management	0.40	0.30	0.20	0.10	
3. Project Management	0.15	0.30	0.40	0.10	0.05
4. Portfolio Management	0.30	0.10	0.20	0.40	
5. Internal Drivers	0.05	0.4	0.1	0.15	0.3
6. Organization and Structure	0.4	0.3	0.2	0.1	
7. External Drivers	0.30	0.15	0.10	0.40	0.05

Source: Collected from expert opinions.

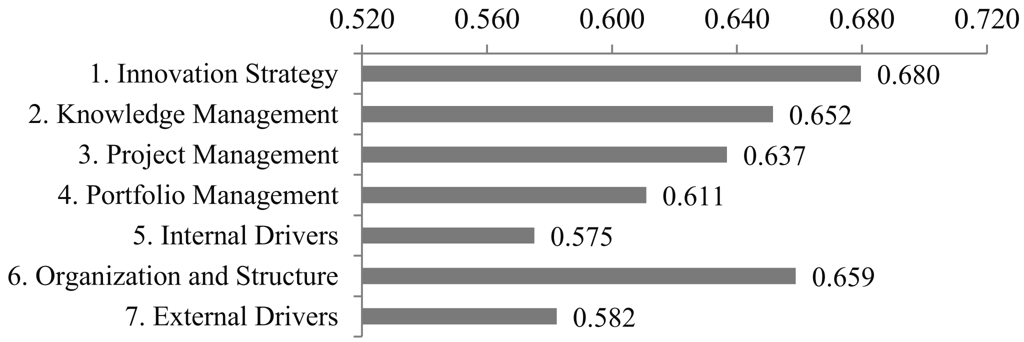
Table 5.12. Induced variables ui

Innovation Measurement Area	u_1	u_2	u_3	u_4	u_5
1. Innovation Strategy	4	7	2	1	8
2. Knowledge Management	7	4	5	6	
3. Project Management	1	7	2	3	5
4. Portfolio Management	2	3	4	1	
5. Internal Drivers	7	5	3	2	8
6. Organization and Structure	2	5	1	3	
7. External Drivers	7	1	2	5	4

Source: Collected from expert opinions.

Step 2.2. Calculate the operator following the formulation of the IGOWA operator by Eq. 5 and the information from step 2.1. Figure 5.1 presents the results for our specific problem.

Figure 5.1. IGOWA results



Step 2.3. Generate a ranking of the aggregated results. In our case, we can observe that:

$$1. innovation\ strategy \succ 6. organization\ and\ structure \succ 2. knowledge\ management \succ 3. project\ management \succ 4. portfolio\ management \succ 7. external\ drivers \succ 5. internal\ drivers$$

Note that the symbol $\succ$ means preferred in our case.

It is interesting to note that the results could be extended; however, they are subject to specific conditions. The first is the environment in which the managers of the SMEs operate, and the second is the specificities of the variables utilized in the methodological approach, i.e., the characteristics of the IGOWA operator.

Conclusions

In this paper, we present a methodological structure for innovation management measurement under a fuzzy approach. We collect data from a survey based on one of the latest innovation management frameworks and treat it using the theory of expertons and the IGOWLA operator. The main advantage of this approach is the expertise process, which is designed to work with uncertain and subjective opinions directly from the managers (experts) of the surveyed SMEs. We especially note that the flexibility of the parameters allows an accurate interpretation based on the specific conditions of the problem.

A total of 91 SMEs participated in this analysis. The survey presented 32 statements regarding the seven key innovation management measurement areas proposed by Adams et al., (2006). The results conclude that innovation strategy, organization and structure, knowledge management and project management are the most valued areas. On the other hand, portfolio management, external drivers and internal drivers are the areas with the lowest values. These results present a first approach to understanding the complex process of innovation management in the location and could be used for further analysis and policy decision-making processes.

From a managerial perspective, diverse reasons justify the need for assessing the measurement of innovation capabilities in SMEs. In a highly changing environment, the correct evaluation of companies' capabilities to introduce and maintain continuous change can signify competitive advantages in terms of cost reduction, increment on products' life cycles, increment in sales and a global market perspective. The dynamic capabilities' based view of this research also has implications for academia – first by introducing tools for the treatment of uncertainty in the field of innovation management measurement and second by assessing an empiric study of innovation management measurement frameworks that considers a broad perspective of the latest studies' key performance measurement areas.

There are several limitations to this study. One is the focused perspective used in the survey, specifically the statements that need broadening in order to obtain a better understanding of the insights of innovation management. Another limitation is the dynamism of the environment, which requires exploration in several points of time to generate accurate results. Finally, the surveyed companies were treated with the same level of importance; however, they cannot always apply to innovation management, as there are several factors that could affect the performance of highly innovative firms that need to be properly examined.

Further investigation needs to be conducted, both to minimize the observed limitations but also to improve the decision-making process by adding new tools for the treatment of subjective and uncertain information, e.g., the inclusion of interval numbers (Moore, 1966), fuzzy numbers or linguistic variables (Durbach and Stewart, 2012). Other examples include uncertain aggregation operators (Merigó et al., 2014), such as the uncertain generalized weighted average (UGWA) operator, the probabilistic weighted average (PWA) operator, and the uncertain generalized probabilistic weighted average (UGPWA), among others. The main focus for future research should be the highly changing environment of the information regarding innovation management; however, the inclusion of the latest approaches in decision-making under uncertainty can shed some light on the way we treat and understand the complexity of innovation.

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Chapter 5. New mathematical tools for business decision-making

Innovation is a highly agile and dynamic concept. As new elements are explored for its understanding and analysis, new tools must be developed for its proper quantification.

This chapter proposes three new tools addressing that objective. The first methodological tool, proposes a method to quantify the direct and indirect effect that certain resource investment could imply to the development of innovation capacities for a specific industrial sector.

The second and third tools are new aggregation operators based on the ordered weighted averaging (OWA) operator and the generalized ordered weighted logarithmic aggregation (GOWLA) operator. The new operators called: the induced generalized ordered weighted logarithmic average (IGOWLA) operator and the generalized ordered weighted logarithmic averaging distance (GOWLAD) operator; propose new methods to aggregate information in highly uncertain environments, e.g. where the preferences of the individuals change with respect to the shifting elements of the environment.

The general result is on the one hand the exploitation of the results obtained on the previous chapters, and on the other, the contribution of two new aggregation operators. Please note that nowadays aggregation operators are a highly growing and appealing field in the academia.

Quantification of the Incidence in Innovation Capacities by the Promotion of Specialized Economic Sectors[5]

Abstract

In the last decades, México has focused resources on the creation of innovation agendas for the economic development of States and Regions. Such agendas have as mission, the improvement of specific economic sectors in order to generate innovation spillovers and improve the local innovation capacities of firms. Applying the Forgotten Effects Theory, this research aims to quantify the incidence of specialized economic sectors in innovation capacities of firms. Results show high incidence of each economic sector in all the innovation capacities measurement areas, emphasizing the relevant indirect effect that the renewable energy sector presents, and the preponderant incidence of the ICT sector on Portfolio Management and Organization and Structure areas. This research presents a first step to quantify the direct and indirect effect that innovation efforts of a specific territory generate in order to influence local competitive capacities of firms.

Keywords: Economic Sectors, Welfare Indicators, Fuzzy Sets, Regional Development, Forgotten Effects Theory.

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Introduction

In the last years the Mexican Council of Science and Technology (CONACYT) has made efforts towards the concept and activities of innovation as a main trigger for regional development. In that sense, the institute, in coordination with consultative groups, leading enterprises and academia, generated 32 State Innovation Agendas and 3 large Regional Innovation Agendas. Such documents present the strategic plan, that in coordination with the public and private sectors define the main innovative activities that will be promoted in order to generate economic progress.

In the present paper the Innovation Agenda of the State of Michoacán México (Conacyt, 2015) is discussed. The research is focused on this specific State because of its economic regional relevance, strategic location, and the latent capacities that have not been adequately developed creating a wide gap between the rest of the States in the Nation.

The Innovation Agenda generated for the State of Michoacán has as objective the improvement of regional competitiveness by developing innovation capacities in forms, taking advantages of the local scientific capabilities and fostering the participation of private actors. Specifically, the vision of the Agenda is to position Michoacán by 2034 as a top five entity of the country, being an economic strategic pole for the Asian markets and a recognized generator of highly added value products, beyond the sole agricultural production. In order to achieve such objective, the State Agenda seeks to promote the development of five economic sectors (Agro-industry, Metal-mechanics, Information and Communication Technologies (ICT), Health and Renewable Energies).

The objective of the present paper is to quantify the direct and indirect incidence that the selected economic sectors have on the local innovation capacities, the results will show whether there has been an accurate selection of the selected sectors and will aid decision makers to strategically position the investment in such areas that generate a multiplicative effect, rather than those with low incidence.

In order to assess the task of identifying the role that economic sectors have in the innovation capacities of firms, we use the methodology of Forgotten Effects (Kaufmann & Gil – Aluja, 1988) and present the direct and indirect connections established between the variables selected from the Innovation Agenda of Michoacán.

The study is based on data obtained from the statistic information presented in the Innovation Agenda of Michoacán (Conacyt, 2015), however we also consulted information from the National Institute of Statistics and Geography of Mexico (INEGI, 2014).

Theoretical background

A total of 32 States integrate the United Mexican States, 31 of them are free and sovereign entities, which have the right to establish an own constitution and legislative powers. The last State is the Federal District, territory under the share dominance of the Mexican Federation and the local government entities.

The State of Michoacán, México

The State of Michoacán de Ocampo occupies the 3.0% of the National territory with a surface of 58,599 km^2, it has a total population of 4,351,037 habitants from which 2,248928 are women, and 2,102,109 are men. The State presents a distribution of 69% of urban locations and 31% of rural locations. In terms of schooling it is observed that from every 100 people: 10.7 do not have access to education, 61.8 have the primary education, 0.4 have a technical or commercial primary school, 4.8 have finalized the high school, 11.8 have concluded university, 0.5 is not specified. In the State 92% of the population is considered catholic, less than 3% of the population speaks a native language. In 2010 there were 1,066,061 private homes, from which: 87.7%, have access to potable water, 88.6%, have access to drainage 98.0% counts with electricity. The main economic indicators present that the State provides 2.5 % to the GDP being the activity of commerce the most relevant one, the primary sector provides 11.27% of the State GDP, the secondary activities provide the 19.97% to the State GDP and the third sector of economic activities provide 68.76% to the State GDP, (INEGI, 2014).

Specialized Economic Sectors

In the present research we discuss, first, the main economic sectors that the consultative group, academics, practitioners and government agents of the State of Michoacán selected to promote in order to create innovation spillovers in the region. The selected sectors are the result of the analysis of local industrial specialized and emergent areas with high growth potential. For a detailed explanation of the selection process see (Conacyt, 2015). Table 5.1 shows the industrial sectors selected by the current level of specialization in the State.

The Agro-industry sector in Michoacán represents the 11% of the total State Gross Domestic Product (GDP) and signifies the 18,08% of the manufacturing activities in the State. It comprehends 8.250 economic units and employs about 6.808 workers. In macro economic indicators, this sector provides the 1,64% of the national GDP, positioning itself in the 18th place nationwide. The main objective of the innovation agenda in this highly specialized sector is to provide added industrialization value to agriculture products, focusing on fruit pulp processing, flavor extraction and extraction of active ingredients.

Table 6.1. Selected Industrial Sectors

Industrial Sector	Level of Specialization
1. Agro-industry	High
2. Metal-mechanics	High
3. Information and Communication Technologies	Medium
4. Health	Emergent
5. Renewable Energies	Emergent

Source: Conacyt, 2015.

Metal-mechanics represents the 7% of the State GDP and the 50,12% of the manufacturing activities of the State. It comprehends about 3.304 economic units and employs over 4749 workers. This sector represents the 9,09% of the National GDP being the third place nationwide. The mission of the agenda for this specific industrial sector is to produce capital goods to articulate local productive chains specialized in machinery for agricultural products processing and compact electric energy steam and hydraulic generators.

Information and Communication Technologies represents the 2% of the State GDP, the 18,08% of the entity manufacturing activities and stands in the second place of the tertiary sector of the entity. It gathers a total amount of 891 economic units. This sector represents the 1,64% of the National GDP, being the 16th place nationwide. The objective of the innovation agenda in this medium specialized sector is to update and assimilate technologies for the participation in the international markets, focusing in image processing, marketing and social networks and automation of agriculture and renewable energy generation equipment.

The emerging health sector is a venture with high growth potential in the country. It represents the 1,9% of the State GDP and the 13,04% of the manufacturing activities in the entity. There are 161 economic units in the state. This sector represents the 1,30% of the Nation GDP positioning the entity in the 14th place nationwide. The mission of the innovation agenda for this emergent sector is to consolidate the State as an international referent in R&D, developing new medicines and medic devices. The main specialization area is Bio pharmaceutics.

The second emerging area is the renewable energies sector. It represents an estimated value of 81% in the local intern market, being the 6th place of generation of renewable energy nationwide. It has over 7% of prospected annual growth. The specific mission is the use of natural resources to generate renewable energy from geothermic, hydraulic, biomass and solar sources. It is noteworthy that from all the chosen sectors, this is the less studied one from the innovation agenda.

Innovation Measurement Areas

The innovation agenda of Michoacán aims to enhance the regional capabilities of the local firms. In order to quantify the degree of development that the chosen sectors could provide in terms of innovation spillovers, the present research takes the proposal developed by Adams et al. (2006) as the main innovation measurement reference. Adams et al. (2006) proposal is based on a review of six models and frameworks of innovation measurement (Cooper and Kleinschmidt, 1995; Chiesa et al., 1996; Goffin and Pfeiffer, 1999; Cormican and Sullivan, 2004; Burgelman et al., 2004; Verhaeghet and Kfir, 2002). In that sense, from the seven main areas outlined by the authors, a framework for measuring innovation has been adapted, taking in count recurrent and relevant factors when quantifying the structural capabilities of companies to make and maintain continuous change. Figure 6.1 shows the seven areas of innovation measurement that have been adapted from Adams et al. (2006).

Figure 6.1. Innovation Measurement Areas

<table>
<tr><td rowspan="3">5. Internal Drivers

People
Tools
Financial and physical resources</td><td colspan="3">1. Innovation Strategy

Strategic Orientation
Strategic Leadership</td><td rowspan="3">7. External Drivers

Market Research
Market Testing
Marketing and Sales</td></tr>
<tr><td>2. Knowledge Management

Knowledge repository
Ideas Generation
Information Flows</td><td>3. Project Management

Project Efficiency
Communication Tools
Collaboration</td><td>4. Portfolio Management

Balance Risk-Return
Optimal usage of Tools</td></tr>
<tr><td colspan="3">6. Organization and Structure

Culture
Structure</td></tr>
</table>

Source: Adapted from Adams et al., (2006).

Studies suggest that inefficiencies in processes reduce by maintaining an innovation strategy integrated to the culture, behavior and actions of an organization (O'Brien, 2003). Therefore, when assessing the area of innovation strategy in a firm, it is relevant to analyze factors such as long-term commitment of the administration and the direct location of resources towards innovation efforts (Cooper et al., 2004), the link between key business objectives and the leadership created by a shared and robust vision along the organizational structure (Pinto and Prescott, 1988), the risk aversion level of the board, the pro-activity of the direction and their persistence and commitment to innovation (Saleh and Wang , 1993).

The second measurement area includes the management of explicit and implicit knowledge within organizations (Davis, 1998; Nonaka, 1991), along to the process of collection and use of such information. It is favorable therefore to determine the level of "absorption capacity", understood as the ability to recognize the value of new knowledge from ideas generated within the company (Chiesa et al., 1996; Lee et al., 1996) or obtained from external connections with other companies or information resources (Atuahene-Gima, 1995; Tipping and Zeffren, 1995), also the assimilation and application of knowledge to commercial activities (Cohen and Levinthal, 1990).

Several studies address the importance of the relationship between innovation and efficiency when managing projects. Commonly, such relationship is measured in terms of cost, duration and return over investment (Chiesa et al., 1996; Adams et al., 2006). Alongside it is of interest knowing the extent of internal communication of the implicated areas when developing a new product (Damanpour, 1991), as well as the collaboration with suppliers (Bessant, 2003) and customers (Von Hippel, 1986) as they have been identified as sources that contribute to the innovation process.

Given the rapidly changing environment in which SMEs govern its production activities, the effectiveness in which an organization manages its new products portfolio is often a key determinant of competitive advantage (Bard et al., 1988). In this sense, it is important to know in which extent firms base their operations on systematized processes, guided by clear criteria, as it facilitates the optimal use of limited resources and improves the competitive position of an organization (Hall and Nauda, 1990). Furthermore, Cooper et al. (1999) state that highly competitive companies use formal tools applied consistently to all developing projects of a given portfolio.

Internal drivers, understood as entry systems and tools for the innovation process provide competitive advantage for companies that use them formally (Bessant and Francis 1997; Cooper et al., 2004). In that order of ideas, it is important to know and measure the timeliness in which organizations allocate resources (both financial and personnel) to the development of products and processes.

It is generally accepted and recognized that companies can create work environments that promote the innovation process (Dougherty and Cohen, 1995; Tidd et al., 1997). In that sense, it is necessary to know the intensity in which companies maintain their organizational structure aligned with their project management processes (Pugh et al., 1969), along to measuring the freedom that workers experience while generating ideas from experimentation and the conception of mistakes as a source of expertise (Zien and Buckler, 1997; Anderson and West, 1996).

The area of external drivers measures the intensity in which the company launches its products to the market (Calantone and di Benedetto, 1988; Globe et al., 1973), i.e. market research and testing and development, adapted to a systematic marketing program (Griffin and Page, 1993) additionally measuring the way firms reach the consumer and formal post-sale operations (Atuahene-Gima, 1995; Von-Zedtwitz, 2002).

Application of the model

Kaufmann and Gil – Aluja established the "Theory of the Forgotten Effects" (Kaufmann & Aluja, 1988). This theory allows obtaining all direct and indirect relations, with no possibility of errors, recovering the effects as it is called: “Forgotten Effects". According to the authors, all happenings that surround us are part of a system or subsystem. It means we could almost ensure that any activity is subject to a problem is a result of “causes” and “effects”. Despite a good system control, there is always the possibility of leaving voluntarily or involuntarily some causal relationships that are not always explicit, obvious or visible, and usually they are not directly perceived. It is common that there are some hidden reasons of the problems that we encounter due to effects of incidence effects on outcomes. The forgotten effects theory is an innovative and efficient approach taking into account many aspects of the relations, and which enables minimizing the errors that may occur in many processes (Gil-Lafuente, 2005).

For this study, we have two sets of elements: causes and effects (Kaufmann & Aluja, 1988). We start by valuating an occurrence of a_i over b_j with a membership functions estimated at [0,1]. Below we present two tables that include all the obtained statistical information, and have been normalized which means we divided all values of the column by the largest value. The aim is that all column values refer to the highest value, which takes the value 1. We need normalized values on base 1 because, as you can see, Tables 6.2 and 6.3 include, respectively, the variables that were used to choose the economic sectors to be enhanced, and the incidence that such criteria have on the innovation measurement areas.

Table 6.2. Chosen economic sectors evaluated by priority criteria

	EM	GR	VA	IIM	IC	HEN	SC
ICT's	0,0	1	0,1	0,6	1	1	0,3
Agro-industry	0,2	0,2	1	0,5	0,5	0,6	1
Metal-mechanics	1	0,8	0,4	1	0,7	0,8	0,3
Health	0	0,8	0	0,6	0,6	0,4	0,3
Renewable Energies	0	0,6	0	0,9	0,6	0,3	0,6

EM: Existent Markets; GR: Growth Rate; VA: Value added; IIM: Impact on Internal Markets; IC: Income; HEN: Higher Education Network; SC: Specialized Centers. Source: Retrieved from the statistics presented in Conacyt (2015).

Table 6.3. Priority criteria incidence on Innovation Measurement Areas

	IS	KM	PM	PtM	ID	OS	ED
Existent Markets	0,3	0,7	0,8	0,6	0,5	0,9	0,8
Growth Rate	0,9	0,9	0,7	0,7	1	0,6	1
Value added	0,8	0,5	0,4	0,7	0,3	0,1	0,4
Impact on Internal Markets	0,6	0,4	0,7	0,3	0,6	0,6	0,2
Income	0,2	0,6	0,7	0,5	0,9	0,4	0,1
Higher Education Network	0,4	1	0,6	0,7	0,7	0,5	0,8
Specialized Centers	1	0,9	0,8	0,7	0,4	0,9	0,7

IS: Innovation Strategy, KM: Knowledge Management, PM: Project Management, PtM: Portfolio Management, ID: Internal Drivers, OS: Organization and Structure, ED: External Drivers. Source: Retrieved from the statistics presented in Conacyt (2015).

In this stage, the priority criteria serve as a bridge between the economic sectors and the innovation measurement areas, such valuations offer enough information about the incidence displayed between the variables, however, this information is not useful to make a global study, as we should take into account the impact of economic sectors over the innovation measurement areas, hence we must perform a max-min composition between Tables 6.2 and 6.3. Table 6.4 represents the max-min composition of the five selected economic sectors and the innovation measurement areas.

Table 6.4. Max-min matrix Economic Sectors and Innovation Measurement Areas

	IS	KM	PM	PtM	ID	OS	ED
ICT's	0,9	1	0,7	0,7	1	0,6	1
Agro-industry	1	0,9	0,8	0,7	0,6	0,9	0,7
Metal-mechanics	0,8	0,8	0,8	0,7	0,8	0,9	0,8
Health	0,8	0,8	0,7	0,7	0,8	0,6	0,8
Renewable Energies	0,6	0,6	0,7	0,6	0,6	0,6	0,6

IS: Innovation Strategy, KM: Knowledge Management, PM: Project Management, PtM: Portfolio Management, ID: Internal Drivers, OS: Organization and Structure, ED: External Drivers. Source: Self – elaborated.

As it has been discussed, the direct impacts are not enough to make an overall analysis of the situation, given that causes (economic sectors) are conditioned by other causes, as well as the effects (innovation measurement areas) are affected not only by the direct causes, but also by other cross effects. Therefore it is necessary to construct two additional matrices. Table 6.5 shows the relations between the chosen economic sectors. As it is observable, each sector is 100% related to itself, but in relation to other sectors the impact is not symmetric, that is, for instance, the ICT's sector is not equally affected by the Agro-industrial that on the Health or Metal-mechanic.

Table 6.5. Relationship matrix between economic sectors

	ICT's	Agro-industry	Metal-mechanics	Health	Renewable Energies
ICT's	1	0,5	0,7	1	1
Agro-industry	0,1	1	0,1	0,4	0
Metal-mechanics	0,2	0,8	1	0,5	0,7
Health	0,3	0,2	0,1	1	0
Renewable Energies	0,3	0,9	0,7	0,5	1

Source: Self – elaborated.

On the other hand, we build another matrix that relates welfare economic indicators between each other. In Table 6.6, we show an asymmetric matrix that containing a maximum value (1) when an indicator is related by itself:

Table 6.6. Relationship matrix between Innovation Measurement Areas

	IS	KM	PM	PtM	ID	OS	ED
IS	1	0,2	0,1	0,3	0,4	0,5	0,1
KM	0,9	1	0,8	0,6	0,2	0,3	0,8
PM	0,9	0,5	1	0,7	0,2	0,3	0,1
PtM	0,9	0,7	0,4	1	0,2	0,1	0,2
ID	1	0,6	0,8	0,7	1	0,8	0,9
OS	1	0,8	0,9	0,8	0,7	1	0,8
ED	0,8	0,7	0,8	0,9	0,6	0,4	1

IS: Innovation Strategy, KM: Knowledge Management, PM: Project Management, PtM: Portfolio Management, ID: Internal Drivers, OS: Organization and Structure, ED: External Drivers. Source: Self – elaborated.

Finally, from matrices 4, 5, and 6 we calculate the final max-min composition matrix. This cause-effect matrix includes the impact of economic sectors on the innovation measurement areas; taking into account the possible effects of any variable that may have a direct effect. The results of the cumulative effects matrix are shown in Table 6.7.

Table 6.7. Final Cause - Effect matrix

	IS	KM	PM	PtM	ID	OS	ED
ICT's	1	1	0,8	0,9	1	0,8	1
Agro-industry	1	0,9	0,9	0,8	0,7	0,9	0,8
Metal-mechanics	0,9	0,8	0,9	0,8	0,8	0,9	0,8
Health	0,8	0,8	0,8	0,8	0,8	0,8	0,8
Renewable Energies	0,9	0,9	0,9	0,8	0,7	0,9	0,8

IS: Innovation Strategy, KM: Knowledge Management, PM: Project Management, PtM: Portfolio Management, ID: Internal Drivers, OS: Organization and Structure, ED: External Drivers. Source: Self – elaborated.

Results and discussion

Once the calculations and relationship matrices are analyzed, we observe that on the cumulative basis, all the selected economic sectors affect the innovation measurement areas. We can see that, either directly or indirectly, the sectors display a minimum incidence of 0,7 over 1 (the case of Agro-industry and Renewable energies over Internal Drivers). The high incidence that all the variables present in Table 6.7 show, in general, a proper selection of the selected economic sectors in terms of improving the innovation measurement areas proposed. This fact proves that, firstly, the criteria utilized to select the economic sectors has been well studied, secondly, it demonstrate that each sector can generate a favorable multiplicative effect over the innovation measurement areas.

Results show that, despite the high incidence of all the selected economic sectors, health has the least influence over the innovation measurement areas. It is observable that the influence of Metal-mechanics and Renewable energies over the innovation measurement areas is positive, and finally, results convey that ICT and Agro-industry display the most incidence over the innovation measurement areas.

The Forgotten Effects Theory allows the analysis of the direct and indirect incidence that each of the selected economic sectors has on the innovation measurement areas. Table 6.8 shows the absolute difference between the direct effect (Table 6.4) and the indirect effect (Table 6.7) of the analyzed variables.

Table 6.8. Absolute Indirect Incidence

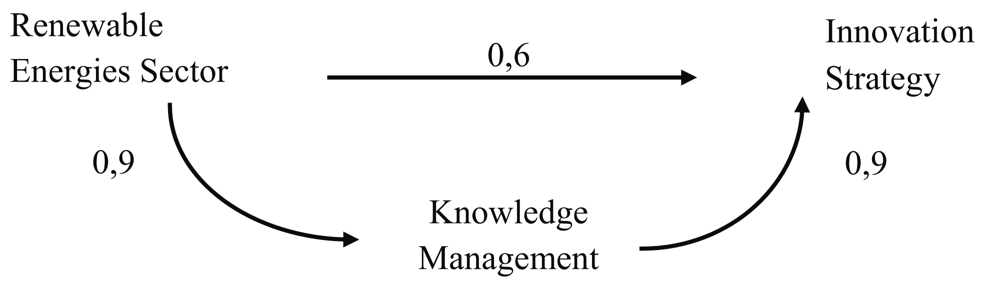
	IS	KM	PM	PtM	ID	OS	ED
ICT's	0,1	0	0,1	0,2	0	0,2	0
Agro-industry	0	0	0,1	0,1	0,1	0	0,1
Metal-mechanics	0,1	0	0,1	0,1	0	0	0
Health	0	0	0,1	0,1	0	0,2	0
Renewable Energies	0,3	0,3	0,2	0,2	0,1	0,3	0,2

IS: Innovation Strategy, KM: Knowledge Management, PM: Project Management, PtM: Portfolio Management, ID: Internal Drivers, OS: Organization and Structure, ED: External Drivers. Source: Self – elaborated.

The results obtained by the absolute indirect incidence are little, e.g. the maximum value is 0,3 (Renewable energies). Such values demonstrate that the missed expected effect of the selected sectors over the innovation measurement areas is slight; therefore, the selected sectors have been properly studied. However, results suggest that renewable energies have the most missed indirect incidence; further research is needed in order to analyze properly the factors that generate such values; nonetheless it is worthy to point out that the innovation agenda of the state does not make a robust study of the Renewable energies sector as for the rest of the selected sectors.

The effect of the Renewable energies sector over the innovation measurement areas needs to be addressed, as it reflects an opportunity of retrieving more benefits than expected. For example, the effect that Renewable energy sector has on Innovation strategy, if we only consider the direct effect (see Table 6.4) is 0.6 over 1. When we look at the direct and indirect cumulative effect the result increases to 0,9. The difference between the two values 0,3 (0,9 - 0,3) represents the isolated indirect effect. This means that the indirect effect is more important than the direct effect. In this case it is possible to know which variables are interjected, making the indirect effect more important. To find it out it is necessary to follow the max-min composition process in the calculations. In this case we have:

Figure 6.2. Indirect Effect between Renewable Energies Sector and Innovation Strategy



The primary sector is highly (0,9) linked to the Innovation strategy variable on a maximum level (0,9) with the Knowledge Management variable. In this way, it is possible to find out the more important indirect effects for each relationship. This result is coherent to the importance that the renewable energies sector has had in the State, therefore gaining high knowledge in the topic.

Conclusions

The quantification of the incidence of the direct and indirect effect of the selected economic sectors presented in the Innovation Agenda of the State of Michoacán, reflects a profound analysis of the criteria designated to evaluate the impact over the economy of the region. Moreover, there is a coherent preponderance of the support to manufacturing activities, which accompanies the vision of making the State a pole for the creation of high value added products with the TIC sector as enabler.

Data comparison and normalized max-min composition processes have been realized in order to quantify the direct and indirect effect that variables display between each other. This operation allowed us to establish a benchmark and present bounded data (between 0 and 1), which facilitates the consequent analysis of the results.

It is shown that, despite all economic sectors influence the innovation measurement areas, the impact of each sector is singular with the following order from high to low: ICT, Agro-industry, Renewable Energies, Metal-mechanics and Health. All sectors present high direct incidence values, such information supports the objectives presented in the Innovation Agenda. On the other hand it is possible to identify the Renewable energy sector as an opportunity due to its high indirect incidence on the innovation measurement areas, such indirect causal relation could allow a performance level with a multiplicative effect.

Lastly, the application of the Forgotten Effects Theory allows the prevision and development of selected economic sectors, as it gives a first insight about the relationships that link them. That information could aid decision makers to manage resources in an efficient way, hence building sustainable and effective public policies.

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Induced generalized ordered weighted logarithmic aggregation operators and their application in group decision making and innovation management[6]

Abstract

This paper introduces the induced generalized ordered weighted logarithmic aggregation (IGOWLA) operator, which is an extension of the generalized ordered weighted logarithmic aggregation (GOWLA) operator. The operator uses order-induced variables that modify the reordering process of the arguments to be aggregated. The main advantage of the induced mechanism is the consideration of the complex attitude of decision makers. We study some properties and families of the IGOWLA operator as measures for the characterization of the weighting vector. We present the general formulation of the operator and some special cases, including the induced ordered weighted logarithmic geometric averaging (IOWLGA) operator and the induced ordered weighted logarithmic aggregation (IOWLA). We also present further generalizations using quasi-arithmetic means and consider dynamic information using moving averages. Finally, we present an illustrative example of a group decision-making procedure using a multi-person analysis and the IGOWLA operator in the area of innovation management.

Keywords: OWA operator; Logarithmic aggregation operators; Induced aggregation operators; Group decision making; Innovation management.

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Introduction

Aggregation operators are becoming very popular in the literature, especially in the areas of economics, statistics and engineering (Beliakov, Pradera, & Calvo, 2007; Torra & Narukawa, 2007). Currently, the literature presents an extensive amount of aggregation operators (Aggarwal, 2015a; Beliakov, Bustince, & Calvo, 2016; Yager, Kacprzyk, & Beliakov, 2011). One of the most well-known methods of aggregation is the OWA operator. The parameterized families proposed in the OWA operator include the minimum, the maximum and the average. Since its appearance, the OWA operator has been applied extensively in diverse applications such as expert systems, group decision making, neural networks, data base systems, and fuzzy systems (Emrouznejad & Marra, 2014; Yager & Kacprzyk, 1997).

A common extension of the OWA operator developed by Yager and Filev (1999) is designated the induced ordered weighted aggregation (IOWA) operator. This extension allows a broader treatment of complex information, applying an alternative reordering process, i.e., instead of a descending ordering of the arguments, a set of order-induced variables dictates the order of the aggregation. The characteristics of the IOWA operator have attracted much attention, motivating a varied range of applications (Yager, 2003; Chiclana, Herrera-Viedma, Herrera, & Alonso, 2007). For example, Aggarwal (2015b) develops new families of IOWA operators. Spirkova (2015) uses dissimilarity functions. Chen and Chen (2003) study the use of fuzzy numbers, Wei and Zhao (2012) consider intuitionistic fuzzy information, and Meng, Chen, and Zhang (2015) analyze hesitant fuzzy sets and the Shapley framework. Xu (2006) and Xian, Zhang, and Xue (2016) develop induced aggregation operators with linguistic information and Merigó and Casanovas (2011) with distance measures.

An interesting generalization of the OWA operator results when applying quasi-arithmetic means in the aggregation process. The outcome is the quasi-arithmetic ordered weighted aggregation (Quasi-OWA) operator (Fodor, Marichal, & Roubens, 1995). This operator combines a wide range of mean operators, including the generalized mean, the OWA operator, the ordered weighted geometric (OWG) operator, and the ordered weighted quadratic averaging (OWG) operator, among others. Some of the most representative extensions of the Quasi-OWA operator are, e.g., the uncertain induced quasi-arithmetic OWA (Quasi-UIOWA) operator (Merigó &

Casanovas, 2011), the combined continuous quasi-arithmetic generalized Choquet integral aggregation operator (Zhou & Chen, 2011), and the quasi intuitionistic fuzzy ordered weighted averaging operator (Yang & Chen, 2012), among others.

When addressing dynamic information, it is useful to consider the application of partial aggregations to a sample of arguments in a changing environment. The study of moving averages and aggregation operators yields to the development of the generalized ordered weighted moving average (GOWMA) operator, introduced by Merigó and Yager (2013). This operator includes a wide range of particular cases, including geometric moving averages, quadratic moving averages, harmonic moving averages and some special cases such as the Olympic moving aggregation and the centered moving aggregation. Some extensions of this operator are the induced generalized ordered weighted moving average (IGOWMA) operator and the quasi induced ordered weighted moving average (Quasi-IOWMA).

Zhou and Chen (2010) propose a generalization of the ordered weighted geometric averaging (OWGA) operator based on an optimal model. The new operator is designated the generalized ordered weighted logarithmic aggregation (GOWLA) operator. It introduces a set of parameterized families, including the step generalized ordered weighted logarithmic averaging (Step-GOWLA) operator, the window generalized ordered weighted logarithmic averaging (Window-GOWLA) operator, and the S-GOWLA, among others. A further generalization of the GOWLA operator is that introduced by Zhou, Chen, and Liu (2012a) designated the generalized ordered weighted logarithmic proportional averaging (GOWLPA) operator. Some generalizations of this operator are the generalized hybrid logarithmic proportional averaging (GHLPA) operator and the quasi ordered weighted logarithmic partial averaging (Quasi-OWLPA) operator. Following the trend of developing aggregation operators based on optimal deviation models, Zhou, Chen, and Liu, (2012b) introduce the generalized ordered weighted exponential proportional aggregation operator (GOWEPA), which is further generalized to develop the generalized hybrid exponential proportional averaging (GHEPA) operator and the generalized hybrid exponential proportional averaging-weighted average (GHEPAWA) operator. Recently, Zhou, Tao, Chen, and Liu (2015) have introduced an additional generalization to the GOWLA designated the generalized ordered weighted logarithmic harmonic averaging (GOWLHA) operator, including the generalized hybrid logarithmic harmonic averaging (GHLHA) operator and the generalized hybrid logarithmic harmonic averaging weighted average (GHLHAWA) operator.

The aim of this paper is the presentation of the induced generalized ordered weighted logarithmic aggregation (IGOWLA) operator. It is an extension of the optimal deviation model developed by Zhou and Chen (2010), with the addition of order-induced variables that modify the reordering mechanism of the arguments; it is designed to consider a broader representation of the complex attitude of decision makers.

We study a series of properties and families of the operator such as the induced ordered weighted logarithmic geometric averaging (IOWLGA) operator, the induced ordered weighted logarithmic harmonic averaging (IOWLHA) operator, and the induced ordered weighted logarithmic aggregation (IOWLA) operator, among others. Furthermore, we present some extensions of the operator, first, using quasi-arithmetic means, obtaining the quasi induced generalized ordered weighted logarithmic aggregation operator (Quasi-IGOWLA) operator; second, utilizing moving averages, we develop the induced generalized ordered weighted logarithmic moving average (IGOWLMA) operator.

Finally, we propose an illustrative example of a multi-person decision-making analysis in the field of innovation management. The application was designed to evaluate a strategic decision-making process where a series of experts need to evaluate the performance of new concepts to develop, with a highly complex attitudinal character of the management. The results show a difference in the aggregation ranking when applying order-induced variables instead of using traditional operators. The operator can be useful for other decision-making applications in business, such as human resource management, strategic decision making and marketing.

The paper is organized as follows. In Section 2, we present basic concepts of the OWA, IOWA, Quasi-IOWA, IOWMA and GOWLA operators on which our propositions are founded. Section 3 presents the IGOWLA operator and its main concepts, properties and families. Section 4 presents the extension of the Quasi-IGOWLA operator. Section 5 introduces the IGOWLMA operator. In Section 6, we propose an illustrative application of a decision-making procedure utilizing the new operator. Finally, Section 7 summarizes the concluding remarks of the paper.

Preliminaries

In the present section, we briefly review some of the principal contributions in the field of aggregation operators. Specifically, we describe the OWA operator, the induced OWA operator, the Quasi-IOWA operator and the GOWLA operator.

The OWA operator

The ordered weighted averaging operator introduced by Yager (1988) proposes a family of aggregation operators that have been used in a plethora of applications (see, e.g., Yager & Kacprzyk, 1997). The OWA operator can be defined as follows:

Definition 1. An OWA operator is a mapping $OWA: R^n \rightarrow R$, which has an associated n vector $w_j = (w_n)^T$, where $w_j \in [0,1]$, and $\sum_{j=1}^{n} w_j = 1$. Accordingly:

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j is the jth largest of the arguments a_i.

It has been demonstrated that the OWA operator is commutative, idempotent, bounded and monotonic (Yager, 1988). Furthermore, we can obtain the ascending OWA or the descending OWA by generalizing the direction of the reordering process (Yager, 1992).

The induced OWA operator

The induced ordered weighted averaging operator, introduced by Yager and Filev (1999), presents an extension of the OWA operator. This extension allows a reordering process that is defined by order-induced variables u_i rather than the traditional ordering constructed from the values of the a_i arguments.

Definition 2. An IOWA operator of dimension n is a mapping $IOWA: R^n \rightarrow R$, associated with a weighting vector W of dimension n such that $\sum_{j=1}^{n} w_j = 1$, $w_j \in [0,1]$, and a set of order-inducing variables u_i, following the next formula:

$$IOWA(\langle u_1, a_1 \rangle, \dots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \tag{2}$$

where $(b_1, \dots, b_n)$ is $(a_1, a_2 \dots, a_n)$ reordered in decreasing values of the u_i. Note that the u_i are the order-inducing variables and the a_i are the argument variables.

The Quasi-IOWA operator

The quasi-arithmetic induced ordered weighted aggregation (Quasi-IOWA) operator presents an extension of the Quasi-OWA operator. The main difference is the reordering process; in this case, order-induced variables dictate the complex reordering of the arguments. The Quasi-IOWA operator can be defined as follows:

Definition 3. A Quasi-IOWA operator of dimension n is a mapping $QIOWA: R^n \times R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that $w_j \in [0,1]$ for all j, and $\sum_{j=1}^{n} w_j = 1$, following the next formula:

$$QIOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = g^{-1} \left\{ \sum_{j=1}^{n} w_j g(b_j) \right\}, \tag{3}$$

where b_j is the a_i value of the Quasi-IOWA pair $\langle u_i, a_i \rangle$ having the largest u_i, u_i is the order-inducing variable, a_i is the argument, and $g(b)$ is a strictly continuous monotonic function.

Note that the Quasi-IOWA can also be viewed as a generalized form of the IOWA operator by using quasi-arithmetic means. The Quasi-IOWA has a wide variety of particular cases (Merigó & Casanovas, 2009), including, e.g., the IGOWA operator, the IOWA operator, the IOWGA operator, the IOWQA operator, and the IOWHA operator.

The IOWMA operator

The induced ordered weighted moving average (IOWMA) operator introduced by Merigó and Yager (2013) is the result of the study of averages that move part or an entire set of a sample to be aggregated. Note that the IOWMA operator can be reduced to a wide range of particular cases, including the generalized moving average (GMA) operator, the generalized weighted moving average (GWMA) operator, and the generalized ordered weighted moving average (GOWMA) operator. The IOWMA operator can be defined following the next formula:

Definition 4. An induced ordered weighted moving average (IOWMA) operator of dimension m is a mapping $IOWMA: R^m \times R^m \to R$, which has an associated weighting vector W of dimension m, satisfying $\sum_{j=1+t}^{m+t} w_j = 1$ and $w_j \in [0,1]$, following the next formula:

$$IOWMA(\langle u_{1+t}, a_{1+t} \rangle, \langle u_{2+t}, a_{2+t} \rangle, \dots, \langle u_{n+t}, a_{n+t} \rangle) = \sum_{j=1+t}^{m+t} w_j b_j, \quad (4)$$

where b_j is the a_i value of the IOWMA pair $\langle u_i, a_i \rangle$ having the jth largest u_i, u_i is the order-inducing variable, a_i is the argument, m is the total number of arguments considered from the sample, and t defines the movement applied to the average from the initial analysis.

The GOWLA operator

The generalized ordered weighted logarithmic aggregation (GOWLA) operator developed by Zhou and Chen (2010) introduces a parameterized family of aggregation operators including the step-GOWLA operator, the window-GOWLA operator, the S-GOWLA operator and the GOWHLA operator. The GOWLA is the result of the construction of the optimal model:

$$minz = \sum_{j=1}^{n} w_j \left[(lny)^\lambda - \left(lna_j\right)^\lambda \right]^2, \quad (5)$$

where y is an aggregation operator of dimension n, $w = (w_1, w_2, \dots, w_n)^T$ is a weighting vector such that $w_j \in [0,1]$ for all j, and $\sum_{j=1}^{n} w_j = 1$. Note that λ is a parameter that exists as $\lambda \in (-\infty, \infty)$.

Solving the partial derivative with respect to y and $\frac{\partial y_1}{\partial y} = 0$, we obtain the generalized weighted logarithmic averaging operator (GWLA); its formulation is as follows:

$$GWLA(a_1, a_2, \dots, a_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lna_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}. \tag{6}$$

If we reorder the arguments a_i, then we can define the generalized ordered weighted logarithmic averaging operator (GOWLA) as follows:

Definition 5. A GOWLA operator of dimension n is a mapping $GOWLA: \Omega^n \rightarrow \Omega$ that is demarcated by an associated weighting vector W of dimension n, satisfying $w_j \in [0,1]$ for all j, and $\sum_{j=1}^{n} w_j = 1$ and a parameter λ that moves between $(-\infty, \infty) - \{0\}$, according to the next formula:

$$GOWLA(a_1, a_2, \dots, a_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \tag{7}$$

where b_j is the jth largest of the arguments $a_1, a_2, \dots, a_n$. Observe that $lna_j \geq 0$. In that case, $exp\left(lna_j\right) \geq exp(0)$; therefore, $a_j \geq 1$ following the notation in Zhou and Chen (2010), $\Omega = \{x | x \geq 1, x \in R\}$.

The induced GOWLA operator

Main concepts

The induced GOWLA operator (IGOWLA) is an extension of the GOWLA operator; the main distinction is the previous reordering step, i.e., The IGOWLA operator is not defined by the values of the arguments a_i but by order-induced variables u_i, which means that the position of the arguments a_i will be determined by the values of the u_i (Merigó & Gil-Lafuente, 2009). This extension enables an even more generalized ordering process, where decision making can consider wider and complex conditions. The IGOWLA operator is defined as follows:

Definition 6: An IGOWLA operator of dimension n is a mapping IGOWLA: $\Omega^n \rightarrow \Omega$ defined by an associated weighting vector W such that $w_j \in [0,1]$ and $w_j \in [0,1]$ and a set of order-inducing variables u_i, according to the next formula:

$$IGOWLA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j\left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \quad (8)$$

where λ is a parameter such that $\lambda \in (-\infty, \infty) - \{0\}$, and $(b_1, \dots, b_n)$ is $(a_1, a_2 \dots, a_n)$ reordered in decreasing values of the u_i. Observe that u_i are the order-inducing variables and a_i are the argument variables. Note that in this paper, we follow the original argument where $\Omega = \{x | x \geq 1, x \in R\}$.

Example 7.1. IGOWLA. Assume the following collection of arguments set by their respective order-inducing variables $\langle u_i, a_i\rangle$: $\langle 4,30\rangle, \langle 2,80\rangle, \langle 7,10\rangle, \langle 5,60\rangle$. Let us assume that $W = (0.1, 0.3, 0.2, 0.4)$ and $\lambda = 2$; the aggregation will result as follows:

$$b_1 = a_3 = 10, b_2 = a_4 = 60, b_3 = a_1 = 30, b_4 = a_2 = 80,$$

$$IGOWLA(\langle 4,30\rangle, \langle 2,80\rangle, \langle 7,10\rangle, \langle 5,60\rangle) = exp\left\{\left(\left(0.1 \times (ln10)\right)^2 + \left(0.3 \times (ln60)\right)^2 + \left(0.2 \times (ln30)\right)^2 + \left(0.4 \times (ln80)\right)^2\right)^{\frac{1}{2}}\right\} = 51.6158.$$

It is observable that the order-inducing variables u_i affect the order of the argument variables a_i in decreasing order.

It is possible to differentiate the operator between the descending induced generalized OWA (DIGOWA) operator, and the ascending induced generalized OWA (AIGOWA) operator. Regardless, the operators noted above are connected by the relationship of $w_j = w^*{}_{n+1-j}$, where w_j is the jth weight of the DIGOWA operator and $w^*{}_{n+1-j}$ the jth weight of the AIGOWA operator.

Proposition 1. If the weighting vector is $\sum_{j=1}^{n} w_j \neq 1$, then normalizing the weighting arguments as follows is proposed:

$$IGOWLA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = exp\left\{\left(\frac{1}{W}\left(\sum_{j=1}^{n} w_j\left(lnb_j\right)^{\lambda}\right)\right)^{\frac{1}{\lambda}}\right\}. \quad (9)$$

Note that $W = \sum_{j}^{n} w_j$.

The IGOWLA operator is a generalization of a mean operator. Therefore, it is commutative, idempotent, bounded and monotonic. These properties can be proven as follows:

Theorem 1. Commutativity: Let the function f be the IGOWLA operator. Then,

$$f(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = f(\langle u_1, e_1\rangle, \dots, \langle u_n, e_n\rangle), \quad (10)$$

where $(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle)$ is any given permutation of the arguments $(\langle u_1, e_1\rangle, \dots, \langle u_n, e_n\rangle)$.

Proof. Let

$$f(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j\left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \quad (11)$$

And

$$f(\langle u_1, e_1\rangle, \dots, \langle u_n, e_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j\left(lnd_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}. \quad (12)$$

Because $(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle)$ is a permutation of $(\langle u_1, e_1\rangle, \dots, \langle u_n, e_n\rangle)$, $b_j = d_j$, for all j, where b_j and d_j are the pairs $\langle u_i, a_i\rangle$ and $\langle u_i, e_i\rangle$ with the jth largest u_i. Therefore, $f(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = f(\langle u_1, e_1\rangle, \dots, \langle u_n, e_n\rangle)$.

The theorem is proven.

Theorem 2. Idempotency: Let f be the IGOWLA operator, and if $a_i = a$ for all i, then $f(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = a$.

Proof. Because $a_i = a$ and $\sum_{j=1}^{n} w_j = 1$, we obtain

$$f(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j\left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}$$

$$= exp\left\{\left(\sum_{j=1}^{n} w_j(lna)^{\lambda}\right)^{\frac{1}{\lambda}}\right\} \quad (13)$$

$$= exp\left\{lna\left(\sum_{j=1}^{n} w_j\right)^{\frac{1}{\lambda}}\right\} = a.$$

The theorem is proven.

Theorem 3. Boundedness: Let f be the IGOWLA operator. Therefore,

$$min\{a_i\} \le f(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) \le max\{a_i\}. \quad (14)$$

Proof. Let $max\{a_i\} = a_{max}$ and $min\{a_i\} = a_{min}$. Then,

$$f(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j\left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}$$

$$\leq exp\left\{\left(\sum_{j=1}^{n} w_j(lna_{max})^{\lambda}\right)^{\frac{1}{\lambda}}\right\} \quad (15)$$

$$\leq exp\left\{(lna_{max})\left(\sum_{j=1}^{n} w_j\right)^{\frac{1}{\lambda}}\right\} \leq a_{max},$$

and

$$f(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j\left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\} \geq exp\left\{\left(\sum_{j=1}^{n} w_j(lna_{min})^{\lambda}\right)^{\frac{1}{\lambda}}\right\} \geq exp\left\{(lna_{min})\left(\sum_{j=1}^{n} w_j\right)^{\frac{1}{\lambda}}\right\} \geq a_{min}; \quad (16)$$

then,

$$a_{min} \leq f(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) \leq a_{max}. \quad (17)$$

The theorem is proven.

Theorem 4. Monotonicity: Let f be the IGOWLA operator. If $a_i \geq c_i$ for all i, then

$$f(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) \geq f(\langle u_1, c_1\rangle, \dots, \langle u_n, c_n\rangle). \quad (18)$$

Proof. Let

$$f(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j\left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \quad (19)$$

and

$$f(\langle u_1, c_1\rangle, \ldots, \langle u_n, c_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lnd_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}. \tag{20}$$

Consider $f(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle)$. We then take the natural logarithm of $f(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle)$ two times; then, we obtain:

$$ln\left(lnf(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle)\right) = \frac{ln\left(\sum_{j=1}^{n} w_j \left(lnb_j\right)^{\lambda}\right)}{\lambda}; \tag{21}$$

accordingly, we take the partial derivative with respect to b_j

$$\frac{\partial ln(lnf)}{\partial b_j} = \frac{1}{\lambda} \cdot \frac{1}{\sum_{j=1}^{n} w_j \left(lnb_j\right)^{\lambda}} \cdot w_j \cdot \lambda\left(lnb_j\right)^{\lambda-1} \cdot \frac{1}{b_j} \geq 0. \tag{22}$$

Because $\frac{\partial ln(lnf)}{\partial b_j} \geq 0$, $ln(lnf)$ is monotonic with respect to b_j; that is, f is monotonic with respect to b_j. Because $a_i \geq c_i$ for all i, $b_j \geq d_j$ for all j. The theorem is proven.

Characterization of the weighting vector

When defining the IGOWLA operator, it is interesting to analyze the characterization of the weighting vector. Following the procedures developed by Yager (1988, 1993) and the descriptions stated by Merigó and Gil-Lafuente (2009), we can obtain the degree of orness or attitudinal character $\alpha(W)$, the entropy of dispersion $H(W)$, the balance $B(W)$ and the divergence $Div(W)$ for the induced logarithmic aggregation operators.

Due to the induced properties (Merigó & Casanovas, 2009), the attitudinal character of the IGOWLA operator can be described from two different perspectives. If we focus on the attitudinal character, then we can use the same measure as in the OWA operator (Yager, 1988) because we want to measure the complex attitude, which depends solely on the weighting vector. In this case, the formulation is as follows:

$$\alpha(W) = \left(\sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1}\right)^{\lambda}\right)^{\frac{1}{\lambda}}. \tag{23}$$

Observe that $\alpha(W) \in [0,1]$. Note that the optimistic criteria are obtained when $\alpha(W) = 1$, the pessimistic criteria are given when $\alpha(W) = 0$, and the averaging criteria are obtained when $\alpha(W) = 0.5$.

Second, if we focus on the numerical values of the aggregation, then the orness measure $\alpha(W)$ should be calculated as follows:

$$\alpha(W)^* = \left(\sum_{j=1}^{n} w_j e_j\right)^{\frac{1}{\lambda}}, \tag{24}$$

where e_j is the d_i value of the IGOWLA pair $\langle u_i, d_i\rangle$ having the jth largest u_i, u_i is the order-inducing variable, and $d_i = \left(\frac{(n-j)}{(n-1)}\right)^{\lambda}$. Note that to define the attitudinal character, we use the classical representation of the OWA operator when we do not use logarithms because it does not work well in $[0,1]$.

The dispersion measure $H(W)$, commonly utilized to analyze the type of information being used (Shannon, 1948; Yager, 1988), can be calculated by solving the next equation:

$$H(W) = -\sum_{j=1}^{n} w_j ln(w_j). \tag{25}$$

Note that if $w_j = 1$ for any j, then $H(W) = 0$, which means that the least information is being used in the operator. Conversely, if $w_j = \left(\frac{1}{n}\right)$ for all j, then a maximum amount of information is being used.

The balance of the weighting vector can also be studied from two perspectives. If we consider the attitudinal perspective from Eq. 23, we can formulate it as follows:

$$Bal(W) = \sum_{j=1}^{n} w_j \left(\frac{n+1-2j}{n-1}\right). \tag{26}$$

However, if we consider the numerical values of the aggregation, the formulation would be as follows:

$$Bal(W)^* = \sum_{j=1}^{n} w_j s_j, \tag{27}$$

where s_j is the t_i value of the IGOWLA pair $\langle u_i, t_i \rangle$ having the jth largest u_i, with u_i being the order-inducing variable and $t_i = \frac{n+1-2j}{(n-1)}$. Observe that $Bal(W) \in [-1, 1]$. For the minimum, $Bal(W) = -1$, and for the maximum, $Bal(W) = 1$. Note that this measure is applicable to any induced aggregation operators (Merigó & Gil-Lafuente, 2009; Yager & Filev, 1999).

Finally, the divergence measure of the weighting vector can be obtained by:

$$Div(W) = \sum_{j=1}^{n} w_j \left(-\alpha(W) + \frac{n-j}{n-1}\right)^2. \tag{28}$$

Observe that, here, we can also consider two perspectives, similar to Eq. 24 and Eq. 27.

Example 7.2. Characterization of the weighting vector. Following the arguments described in Example 7.1, the characterization of the weighting vector result is shown in Table 7.1:

Table 7.1. IGOWLA operator weighting vector measures

Measure	$\alpha(W)$	$\alpha(W)^*$	$H(W)$	$B(W)$	$B(W)^*$	$Div(W)$	$Div(W)^*$
Result	0.5055	0.6236	1.2799	-0.2667	0.0000	0.1404	0.1542

IGOWLA operator families

A particular group of families of the IGOWLA operator can be described when analyzing the parameter $\boldsymbol{\lambda}$. Table 7.2 presents some of the resulting cases of special interest:

Table 7.2. Families of IGOWLA operators

λ	Families	Acronym
→0	Induced ordered weighted logarithmic geometric averaging operator	IOWLGA
-1	Induced ordered weighted logarithmic harmonic averaging operator	IOWLHA
1	Induced ordered weighted logarithmic aggregation operator	IOWLA
2	Induced ordered weighted logarithmic quadratic aggregation operator	IOWLQA
3	Induced ordered weighted logarithmic cubic aggregation operator	IOWLCA
→∞	Largest of the b_j, for $j = n$	Max
→-∞	Lowest of the b_j, for $j = n$	Min

Remark 1. Let $\lambda \to 0$; then, the IGOWLA operator becomes the IOWLGA (Zhou & Chen, 2010) operator:

$$IGOWLA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = exp\left\{\prod_{j=1}^{n}\left(lnb_j\right)^{w_j}\right\}, \quad (29)$$

where $(b_1, \dots, b_n)$ is $(a_1, a_2 \dots, a_n)$ reordered in decreasing values of the u_i.

Remark 2. If $\lambda = -1$, then the IGOWLA operator is reduced to the IOWLHA operator:

$$IGOWLA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = exp\left\{\frac{1}{\sum_{j=1}^{n}\left(\frac{w_j}{lnb_j}\right)}\right\}. \quad (30)$$

Remark 3. If $\lambda = 1$, then the IGOWLA operator becomes the IOWGA (Zhou & Chen, 2010) operator:

$$IGOWLA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = \prod_{j=1}^{n} b_j^{\,w_j}. \quad (31)$$

Note that this formulation can also be presented as the IOWLA operator:

$$IGOWLA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = exp\sum_{j=1}^{n} w_j\left(lnb_j\right). \quad (32)$$

Observe that similarly, if $\lambda = 1$, then we can reduce the GOWLA operator to the OWLA operator:

$$GOWLA(a_1, a_2, \dots, a_n) = exp\sum_{j=1}^{n} w_j\left(lnb_j\right). \quad (33)$$

Furthermore, we can obtain the WLA operator; in this case, when $\lambda = 1$, we have:

$$WLA(a_1, a_2, \dots, a_n) = exp\sum_{j=1}^{n} w_i(lna_i). \quad (34)$$

Remark 4. If $\lambda = 2$, then the IGOWLA operator is reduced to the IOWLQA operator:

$$IGOWLA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = exp\left\{\sqrt{\left(\sum_{j=1}^{n} w_j\left(lnb_j\right)^2\right)}\right\}. \quad (35)$$

Remark 5. Similar to the IOWLQA operator, when $\lambda = 3$, the IGOWLA operator becomes the IOWLC operator:

$$IGOWLA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j\left(lnb_j\right)^3\right)^{\frac{1}{3}}\right\}. \quad (36)$$

Remark 6. If $\lambda \to \infty$, then the IGOWLA operator solution tends to the jth largest a_i for every pair $\langle u_i, b_i\rangle$ for all j.

$$IGOWLA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = max\{(u_i, a_i)\}. \quad (37)$$

Remark 7. If $\lambda \to -\infty$, then the IGOWLA operator solution tends to the jth lowest a_i for every pair $\langle u_i, b_i\rangle$ for all j.

$$IGOWLA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = min\{(u_i, a_i)\}. \quad (38)$$

Example 7.3. IGOWLA Families. Following the arguments described in Example 7.1, the results for each family of the IGOWLA operator are shown in Table 7.3:

Table 7.3. Families of IGOWLA operator

λ	$\to 0$	-1	1	2	3	∞	$-\infty$
Aggregation	45.93	42.44	48.98	51.61	53.84	$\to 80$	$\to 10$

The Quasi-IOWLA operator

It is possible to generate an additional generalization of the general ordered weighted averaging operators by utilizing quasi-arithmetic means instead of the ordinary means (see, e.g., (Beliakov, 2005; Merigó & Gil-Lafuente, 2009)). In the case of the IGOWLA operator, we suggest the use of a similar methodology to construct the Quasi-IOWLA operator.

Definition 7. A Quasi-WLA operator of dimension n is a mapping QWLA: $\Omega^n \rightarrow \Omega$ with an associated weighting vector W of dimension n such that $\sum_{n=1}^{j} w_i = 1$, $w_i \in [0,1]$, and a strictly monotonic continuous function $g(lnb)$, according to the next formula:

$$QWLA(a_1, a_2, \dots, a_n) = exp\ g^{-1}\left\{\left(\sum_{i=1}^{n} w_i\ g(lna_i)\right)\right\}, \tag{39}$$

where a_i are the set of arguments to be aggregated.

Note that if all of the weights of the QWLA are equal $\left(w_i = \frac{1}{n}\right)$ $\forall\ i$, then the QWLA operator becomes the quasi-arithmetic logarithmic average (QLA).

Definition 8. A Quasi-OWLA operator of dimension n is a mapping QOWLA: $\Omega^n \rightarrow \Omega$ with an associated weighting vector W of dimension n, satisfying $\sum_{n=1}^{j} w_j = 1$, and $w_j \in [0,1]$, a set of order-inducing variables u_i, and a strictly monotonic continuous function $g(lnb)$, according to the next formula:

$$QOWLA(a_1, a_2, \dots, a_n) = exp g^{-1}\left\{\sum_{j=1}^{n} w_j g\left(lnb_j\right)\right\}, \tag{40}$$

where b_j are the values a_i ordered in a decreasing manner.

Definition 9. A Quasi-IOWLA operator of dimension n is a mapping QIOWLA: $\Omega^n \rightarrow \Omega$ with an associated weighting vector W of n dimension, satisfying the condition that the sum of the weights is 1, and $w_j \in [0,1]$, a set of order-inducing variables u_i, and a strictly monotonic continuous function $g(lnb)$, according to the next formula:

$$QIOWLA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = exp g^{-1}\left\{\sum_{j=1}^{n} w_j g\left(lnb_j\right)\right\}, \tag{41}$$

where b_j are the values a_i of the Quasi-IOWA pairs $\langle u_i, a_i\rangle$ ordered in decreasing direction of their u_i values.

Note that when $g\left(lnb_j\right) = \left(lnb_j\right)^{\lambda}$, the Q-IOWLA becomes the IGOWLA operator (similarly for the Quasi-WLA, Quasi-OWLA). Therefore, all of these operators share the properties studied for the IGOWLA operator; specifically, it is bounded, idempotent and commutative. However, as shown in section 3, in some cases, it is not monotonic.

Observe that we can also distinguish between the descending Quasi-DIOWLA and the ascending Quasi-AIOWLA. The relationship found between the descending and the ascending operators is $w_j = w^*_{n+1-j}$, where $w_j$is the jth weight of the Quasi-DIOWLA operator and w^*_{n+1-j} the jth weight of the Quasi-AIOWLA operator.

The main advantage of these operators is the analysis of cases that are not considered in the classical operator, e.g., geometric aggregations, quadratic aggregations and harmonic aggregations.

Proposition 2. In case of any ties, replacing the tied arguments with the quasi-arithmetic logarithmic average operator is proposed (Merigó & Gil-Lafuente, 2009).

Moving averages with the IGOWLA operator

If we consider a dynamic perspective of a sample in the aggregation process (Merigó, Casanovas, & Palacios-Marqués, 2014; Merigó & Yager, 2013; Yager, 2008), then we can develop further generalizations of the GOWLA operator, specifically, the generalized ordered weighted logarithmic moving aggregation (GOWLMA) operator, the induced ordered weighted logarithmic moving aggregation (IOWLMA) operator and the quasi induced ordered weighted logarithmic moving aggregation (Quasi-IOWLMA) operator. The main advantage of these operators is the consideration of dynamic information, e.g., information that moves in time. The definitions can be stated as follows:

Definition 10. A GOWLMA operator of dimension m is a mapping $GOWLMA$: $\Omega^n \rightarrow \Omega$ with an associated weighting vector W of dimension m, satisfying $w_j \in [0,1]$ for all j, and $\sum_{j=1+t}^{m+t} w_j = 1$ such that:

$$GOWLMA(a_{1+t}, a_{2+t}, \ldots, a_{n+t}) = exp\left\{\left(\sum_{j=1+t}^{m+t} w_j (lnb_j)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \quad (42)$$

where b_j is the jth largest argument of the a_i, m is the total number of arguments to be introduced in the sample for the aggregation process, and t indicates the movement applied to the initial averaging analysis.

Definition 11. An IGOWLMA operator of dimension m is a mapping $IGOWLMA$: $\Omega^n \rightarrow \Omega$ with an associated weighting vector W of dimension m such that $w_j \in [0,1]$ for all j, and $\sum_{j=1+t}^{m+t} w_j = 1$ such that:

$$IGOWLMA(\langle u_{1+t}, a_{1+t}\rangle, \langle u_{2+t}, a_{2+t}\rangle, \ldots, \langle u_{n+t}, a_{n+t}\rangle) = exp\left\{\left(\sum_{j=1+t}^{m+t} w_j (lnb_j)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \quad (43)$$

where b_j is the a_i value of the IGOWLMA pair $\langle u_i, a_i\rangle$ having the jth largest u_i, with u_i being the order-inducing variable a_i the argument variable, m is the total number of arguments to be considered for the sample, and t means the movement applied to the average from the initial analysis.

Definition 12. A QIOWLMA operator of dimension m is a mapping $QIOWMLA$: $\Omega^n \rightarrow \Omega$ with an associated weighting vector W of dimension m such that $w_j \in [0,1]$ for all j, and $\sum_{j=1+t}^{m+t} w_j = 1$, a set of order-inducing variables u_i, and a strictly monotonic continuous function $g(lnb)$, according to the next formula:

$$QIOWLMA(\langle u_{1+t}, a_{1+t}\rangle, \langle u_{2+t}, a_{2+t}\rangle, \dots, \langle u_{n+t}, a_{n+t}\rangle) = exp g^{-1}\left\{\sum_{j=1+t}^{m+t} w_j g\left(lnb_j\right)\right\}, \quad (44)$$

where b_j are the values a_i of the Quasi-IOWLMA pairs $\langle u_i, a_i\rangle$ ordered in decreasing direction of their u_i values, a_i represent the argument variables, m signifies the total number of arguments to be considered in the process of aggregation, and t refers to the movement applied to the average from the initial analysis.

Note that these operators are a generalization of the IGOWLA operator; therefore, they share all of the properties and families presented in the previous sections, including the geometric, quadratic and cubic families. Observe that if the strictly continuous monotonic function $g\left(lnb_j\right)$ is equal to $\left(lnb_j\right)^{\lambda}$, then the QIOWLMA becomes the QOWLMA operator.

In particular, note that the IOWA operator becomes the OWA operator when $u_1 \geq u_2 \geq \cdots \geq u_n$; thus, we can obtain, e.g., the ordered weighted logarithmic moving aggregation operator and the quasi ordered weighted logarithmic moving aggregation operator. Additionally, recall that we can implement moving averages in the ordered weighted logarithmic moving aggregation operator and its particular cases.

Group Decision Making with the IGOWLA operator

Decision-making process

The literature on decision making exhibits a plethora of techniques and methods that can be suitable for application with the IGOWLA operator (see, e.g., (Figueira, Greco, & Ehrogott, 2005; Zhang & Xu, 2015; Zhang, Wang, Tian, & Li, 2014; Zhou, Wu, & Chen, 2014)). In this paper, we focus our attention on a decision-making application, specifically, in the selection of strategies using a multi-person analysis in the area of innovation management.

The main reason for selecting this topic is the presentation of information, which, in the case of innovation, has been stated to be imprecise and uncertain (O'Connor & Rice, 2013; Tidd, 2001). Therefore, there is the motivation to use the opinion of different decision makers or experts to find a suitable solution.

Strategic decision making in innovation management addresses diverse aspects that include not only imprecise information (Zadeh, 2006) but also a certain level of attitudinal character on the part of the decision makers, e.g., the possibility of different strategic outcomes (Loch, Solt, & Bailey, 2008), complexity and unfamiliar interactions (Bordia, Hobman, Jones, Gallois, & Callan, 2004), the lack of information (McLain, 2009), time, flexibility and control. In this sense, the use of inducing variables should aid in the complex decision-making procedure.

An interesting issue to study when addressing innovation management in a given company is the selection of the best product to develop from a portfolio of new products. The effectiveness with which an organization manages its new products portfolio is often a key determinant of competitive advantage (Bard, Balachandra, & Kaufmann, 1988). Specifically, portfolio management addresses the allocation of the scarce resources of the enterprise (money, time, people, machinery, etc.) to potential projects under uncertain conditions. The key concepts to analyze are quantity, quality, and organizational capability for new product development. Furthermore, new products must correctly align with business objectives and balance both risk and the timespan.

The process to follow in the selection of strategies with the IGOWLA operator with multi-person decision making can be summarized as follows:

Step 1. Let $A = \{A_1, A_2, \dots, A_m\}$ be a set of limited options, $S = \{S_1, S_2, \dots, S_m\}$ a set of finite states of nature, conforming to the payoff matrix, $(a_{hi})_{m\times n}$. Let $E = \{e_1, e_2, \dots, e_q\}$ be a finite set of decision makers. Assume that the selected decision makers have a different level of importance such that $X = (x_1, x_2, \dots, x_p)$ is the weighting vector for each decision maker such that $\sum_{k=1}^{p} x_k = 1$ and $x_k \in [0,1]$. Then, each decision maker must provide a pay-off matrix $(a_{hi})^k{}_{m\times n}$.

Step 2. Calculate the order-inducing variables $(u_{hi})_{m\times n}$ to be used in the payoff matrix for each alternative h and state of nature i. Find the weighting vector $W = (w_1, w_2, \dots, w_n)$, satisfying the IGOWLA operator definitions, and define the parameter λ to be applied in the aggregation. Note that the weighting vectors can be obtained using any classical method that treats probabilistic information.

Step 3. Use the weighted average to aggregate the information of the decision makers E by using the weighting vector X. The result will then be the collective payoff matrix $(a_{hi})^k{}_{m\times n}$; therefore, $a_{hi} = \sum_{k=1}^{p} x\,(a_{hi})^k$.

Step 4. Solve the IGOWLA operator as described in Eq. 8. Consider that the λ value is typically set as 1; however, any of the families described in section 3.2 can be used, depending on the problem analyzed.

Step 5. Establish a ranking of the alternatives; analyze the results of the specific problem and generate a decision-making approach.

IGOWLA Illustrative Example

This section presents an illustrative example regarding a strategic decision-making procedure in portfolio management using a multi-person analysis and the IGOWLA operator. Note that other business decision-making applications can be developed in the area of innovation management, such as knowledge management, project management, organization and structure, among others (Adams, Bessant, & Phelps, 2006).

Step 1. Assume that a company engaged in the production and marketing of fast-moving consumer goods must select a new product to develop from its portfolio of five potential enhanced beverage concepts. Thus, we have:

- A_1 Sport: vitamin C plus electrolytes
- A_2 Energy: vitamin C plus caffeine
- A_3 Recover: vitamins B5, B6 and B12
- A_4 Diet sport: vitamin C plus electrolytes, with no sugar
- A_5 AntiOx: manganese plus vitamin B3

To select the concept to be developed, the company chooses experts in different areas of the business to give their opinion. Based on the literature on innovation management, the company sets 6 key factors to be analyzed in the selection process:

- S_1 Expected benefits
- S_2 Alignment to business
- S_3 Less development costs
- S_4 Technical viability
- S_5 Minimum risk
- S_6 Less time to market

The experts are divided into three subgroups (Tables 7.4 – 7.9). The first group (Table 7.4 and 7.5) comprises two experts in the area of engineering. The second has two experts in the area of marketing and sales (Table 7.6 and 7.7). The remaining two financial experts belong to the third group (Table 7.8 and 7.9). From a scale of 1 to 100, each expert must give his or her opinion on the expected performance of each product based on the key factors selected by the direction. To correctly develop the aggregation process, we must first generate a multi-aggregation process in which the opinions of the groups can be concentrated. This process allows us to visualize the information of each group separately. Then, we need to aggregate all of the groups into a sole collective group payoff matrix. Finally, use the IGOWLA operator to generate the final results of the aggregation process and aid the board of directors in the selection of the most suitable alternative.

Step 2. Due to the complexity of the information analyzed, the administration generates a set of order-inducing variables: $U = (7, 5, 4, 2, 10, 9)$. The experts consider a weighting vector $IGOWLA = (0.1, 0.1, 0.1, 0.2, 0.1, 0.4)$.

Step 3. The weighting vectors X, which represent the importance of each expert in the analysis, are the following: the first group of experts $X_1 = (0.4, 0.6)$, the second group of experts $X_2 = (0.7, 0.3)$, and the third group of experts $X_3 = (0.5, 0.5)$. For the collective matrix, we have $X_4 = (0.3, 0.4, 0.3)$. With this information, we can obtain inter-medium results by first aggregating the opinions of the three groups of experts; the results are shown in Tables 7.10, 7.11 and 7.12. Then, we use the weighted average to aggregate the three subgroups into a collective payoff matrix. The results are shown in Table 7.13.

Step 4. Using the IGOWLA operator families, we aggregate the collective information and obtain final results. Tables 7.14 and 7.15 show the results of the aggregations.

Step 5. To generate a decision, we must establish a ranking of the performance of each product. The ordering of alternatives is presented in Table 7.16. Note that the "}" symbol represents preferred to. Additionally, note that for each of the selected aggregation operators, a different ranking can be assembled, leading to distinct decision-making processes.

Table 7.4. Payoff Matrix – Expert 1.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	80	80	100	100	100	100
A_2	20	40	40	60	20	60
A_3	80	100	80	60	80	100
A_4	80	100	80	100	80	100
A_5	40	40	60	40	60	40

Table 7.5. Payoff Matrix – Expert 2.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	100	80	100	80	100	100
A_2	20	60	60	20	20	20
A_3	80	40	40	100	60	100
A_4	80	80	100	80	100	100
A_5	40	60	60	60	40	40

Table 7.6. Payoff Matrix – Expert 3.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	100	100	80	80	100	80
A_2	20	40	60	20	60	40
A_3	100	100	100	80	40	40
A_4	100	80	80	100	80	100
A_5	40	40	40	60	40	60

Table 7.7. Payoff Matrix – Expert 4.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	80	80	100	100	80	80
A_2	60	20	40	60	40	60
A_3	60	60	80	40	100	60
A_4	100	100	80	80	100	80
A_5	60	60	40	60	40	60

Table 7.8. Payoff Matrix – Expert 5.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	80	100	80	80	100	100
A_2	20	20	60	20	40	60
A_3	60	40	60	100	100	100
A_4	80	80	80	100	80	80
A_5	60	60	40	40	40	40

Table 7.9. Payoff Matrix – Expert 6.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	100	80	80	80	100	100
A_2	60	60	40	40	20	60
A_3	60	80	40	80	100	60
A_4	80	80	100	80	80	100
A_5	60	60	40	40	40	60

Table 7.10. Payoff Matrix – Group 1 (Experts 1 and 2).

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	92.00	80.00	100.00	88.00	100.00	100.00
A_2	20.00	52.00	52.00	36.00	20.00	36.00
A_3	80.00	64.00	56.00	84.00	68.00	100.00
A_4	80.00	88.00	92.00	88.00	92.00	100.00
A_5	40.00	52.00	60.00	52.00	48.00	40.00

Table 7.11. Payoff Matrix – Group 2 (Experts 3 and 4).

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	94.00	94.00	86.00	86.00	94.00	80.00
A_2	32.00	34.00	54.00	32.00	54.00	46.00
A_3	88.00	88.00	94.00	68.00	58.00	46.00
A_4	100.00	86.00	80.00	94.00	86.00	94.00
A_5	46.00	46.00	40.00	60.00	40.00	60.00

Table 7.12. Payoff Matrix – Group 3 (Experts 5 and 6).

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	90.00	90.00	80.00	80.00	100.00	100.00
A_2	40.00	40.00	50.00	30.00	30.00	60.00
A_3	60.00	60.00	50.00	90.00	100.00	80.00
A_4	80.00	80.00	90.00	90.00	80.00	90.00
A_5	60.00	60.00	40.00	40.00	40.00	50.00

Table 7.13. Collective payoff matrix.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	92.20	88.60	88.40	84.80	97.60	92.00
A_2	30.80	41.20	52.20	32.60	36.60	47.20
A_3	77.20	72.40	69.40	79.40	73.60	72.40
A_4	88.00	84.80	86.60	91.00	86.00	94.60
A_5	48.40	52.00	46.00	51.60	42.40	51.00

Table 7.14. Aggregated Results 1.

	AM	MIN	MAX	OWA	IOWA	IGOWLA $\lambda = -1$	IGOWLA $\lambda = 1$
A_1	90.60	84.80	97.60	88.66	88.66	88.53	88.57
A_2	40.10	30.80	52.20	36.96	37.96	37.08	37.38
A_3	74.07	69.40	79.40	72.50	75.5	75.37	75.41
A_4	88.50	84.80	94.60	87.20	88.88	88.80	88.82
A_5	48.57	42.40	52.00	46.70	49.82	49.66	49.72

Table 7.15. Aggregated Results 2.

	IGOWLA $\lambda = 2$	IGOWLA $\lambda = 3$	GOWLA $\lambda = -1$	GOWLA $\lambda = 1$	GOWLA $\lambda = 2$	GOWLA $\lambda = 3$
A_1	88.59	88.61	88.53	88.57	88.59	88.61
A_2	37.53	37.69	35.99	36.32	36.49	36.66
A_3	75.43	75.45	72.39	72.43	72.44	72.46
A_4	88.84	88.85	87.12	87.15	87.16	87.17
A_5	49.75	49.77	46.45	46.54	46.58	46.62

Table 7.16. Ranking of the performance of the concepts to be developed.

	Ranking		Ranking
AM	$A_1\}A_4\}A_3\}A_5\}A_2$	IGOWLA (λ=2)	$A_4\}A_1\}A_3\}A_5\}A_2$
MIN	$A_1\}A_4\}A_3\}A_5\}A_2$	IGOWLA (λ=3)	$A_4\}A_1\}A_3\}A_5\}A_2$
MAX	$A_1\}A_4\}A_3\}A_2\}A_5$	GOWLA (λ=-1)	$A_1\}A_4\}A_3\}A_5\}A_2$
OWA	$A_1\}A_4\}A_3\}A_5\}A_2$	GOWLA (λ=1)	$A_1\}A_4\}A_3\}A_5\}A_2$
IOWA	$A_4\}A_1\}A_3\}A_5\}A_2$	GOWLA (λ=2)	$A_1\}A_4\}A_3\}A_5\}A_2$
IGOWLA (λ=-1)	$A_4\}A_1\}A_3\}A_5\}A_2$	GOWLA (λ=3)	$A_1\}A_4\}A_3\}A_5\}A_2$
IGOWLA (λ=1)	$A_4\}A_1\}A_3\}A_5\}A_2$		

The overall results show that the ordering process varies, depending on the operator used to analyze the arguments. For our specific problem, based on the opinion of six experts and the attitude of the direction, the best concepts to develop are products A_1 (Sport) and A_4 (Diet sport). It can also be observed that the induced operators display different rankings from the traditional rankings, thus indicating that the order-induced variables affect the ranking of the arguments. Note that the multi-person process can be aggregated and presented in many other ways; for this example, we assumed that the management needed the information presented as shown.

Conclusions

In this paper, we have introduced a new aggregation operator designated the IGOWLA operator, which is an extension of the optimal deviation model, the GOWLA operator, and therefore shares the same principal characteristics. The main improvement of the IGOWLA operator is the inclusion of order-induced variables in the reordering process of the arguments, thus offering a wider representation of a complex attitude of decision makers.

We have studied different properties of the IGOWLA operator, such as commutativity, idempotency, boundedness and monotonicity. We have also analyzed diverse measures for characterizing the weighting vector; specifically, we have studied the degree of orness measure, the dispersion measure, the balance measure and the divergence measure. Note that some of these measures can be calculated from two different perspectives, depending on the attitudinal character or the numerical value of the weighting vector. Furthermore, we describe several families of the IGOWLA operator based on the λ parameter, including the IOWLGA operator, the IOWLHA operator, the IOWGA operator, the IOWLA operator, the IOWLQA operator, the IOWLC operator, and the maximum and minimum IGOWLA operators.

We introduce diverse generalizations of the IGOWLA operator. First, using the notion of quasi-arithmetic means, we introduce the QWLA operator, the QOWLA operator, and the QIOWLA operator, therefore adding the option of considering geometric aggregations, quadratic aggregations and harmonic aggregations into the process. Second, considering a dynamic assessment of a sample in the aggregation process, we develop further extensions of the IGOWLA operator, including the GOWLMA operator, the IOWLMA operator and the QIOWLMA operator. One of the main advantages of this perspective is the consideration of information moving in time.

The IGOWLA operator is an approach for assisting group decision making. The operator can be applied in a wide range of scientific areas, such as statistics, economics and engineering. In the present paper, we present an illustrative example of the usage of the IGOWLA operator with a multi-person analysis to assess a strategic decision-making process in the area of innovation management, specifically in portfolio management, i.e., the selection of new products to develop. The example has proven that the IGOWLA operator is useful when the procedure involves the opinion of diverse experts with diverse backgrounds, including a complex attitudinal character of the board of directors.

Further research needs to be conducted. First, we will deepen the mathematical approach of the logarithmic properties. Second, further extensions should be developed to assess uncertain information, i.e., fuzzy numbers, linguistic variables and interval numbers, distance measures such as the Hamming or Euclidean distance, and heavy aggregations, among other complex formulations. Finally, new decision-making problems in diverse fields of knowledge should be considered.

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Logarithmic aggregation operators and distance measures[7]

Abstract

The Hamming distance is a well-known measure designed to provide insights between two strings of information. In this study we use the Hamming distance and the optimal deviation model, the generalized ordered weighted logarithmic averaging (GOWLA) operator to develop the ordered weighted logarithmic averaging distance (OWLAD) operator and the generalized ordered weighted logarithmic averaging distance (GOWLAD) operator. The main advantage of these operators is the possibility of modeling a wider and complex representation of problems under the assumption of an ideal possibility. We study the main properties, alternative formulations and families of the proposed operators. We analyze some classic measures to characterize the weighting vector and propose alternatives to deal with the logarithmic properties of the operators. Furthermore, we present some generalizations of the operators by studying their weighting vector and the lambda parameter. Finally an illustrative example regarding innovation project management measurement is proposed utilizing a multi-expert analysis and several of the newly introduced operators

Keywords: OWA operator; Logarithmic aggregation operators; Distance measures; Group decision making; Innovation management..

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Introduction

Group decision-making (GDM) techniques have gained an increasing relevance in the literature. This growing attention is mainly produced by the possibility of generating rankings of diverse alternatives for specific situations considering multiple scenarios. In our days GDM techniques have been widely combined with the theory of aggregation, producing a vast pool of contributions in diverse fields of knowledge such as artificial intelligence, fuzzy systems, imaging processing and decision sciences. This last field results of special interest to our study, as it provides a basis for combining data and obtain solutions constructed on information collected directly from decision makers, experts or stakeholders. In our days we can find a plethora of aggregation operators or aggregation functions [1–4], which have proven useful in diverse areas e.g. statistics, economics, education biology, computer science and engineering[1,2]. A classic example of an operator designed for the aggregation of information in intelligent systems is the ordered weighted average (OWA) presented in Yager [5]. The OWA allows a descending and ascending ordered aggregation mechanism, giving a result between the minimum and maximum of the values to be combined. It also provides a family of parameterized operators that have been adopted in several areas such as expert systems, data base systems, operational research and fuzzy systems [6,7].

Recently, the use of distance measurement techniques in the field of group GDM has gained special relevance. The idea of providing results based on the comparison of information retrieved from the domain experts and an ideal collection of preferences is highly appealing [8]. The current literature has vastly studied several distance measures such as the Hamming distance, the Euclidean distance and the Hausdorff distance [9–11]. From them, we focus on the Hamming distance [12], which considers the importance of each deviation value. This distance has become very popular and its applications in the field of aggregation operators are included e.g. in the ordered weighted distance (OWD) measures [13], the ordered weighted averaging distance (OWAD) operators [14], the linguistic ordered weighted averaging distance (LOWAD) operators [15], and the induced ordered weighted averaging distance (IOWAD) operators [16]. These studies have motivated researches to generate further applications, such as the intuitionistic fuzzy ordered weighted distance (IFOWD) operator [17], the fuzzy ordered distance measures presented in [18] a continuous ordered weighted distance (COWD) operator for investment selection problems [19], a probabilistic ordered weighted averaging distance operator in political management [20], the linguistic induced ordered weighted averaging distance operators for the selection of investments [21], distance measures with heavy aggregation operators (HOWAD) for strategic management [22] a linguistic continuous ordered weighted distance (LCOWD) measure for a GDM in an investment selection problem [23] and more recently the fuzzy linguistic induced ordered weighted averaging Minkowski distance (FLIOWAMD) which generalizes the Euclidean and Hamming distances for investment strategy decision making [24].

Motivated by the recent work of Zhou and Chen [25], which proposes an operator based in an optimal deviation model called the generalized ordered weighted logarithmic aggregation (GOWLA) operator, this study introduces the ordered weighted logarithmic averaging distance (OWLAD) operator and the generalized ordered weighted logarithmic averaging distance (GOWLAD) operator. These operators utilize the Hamming distance measure to provide a set of parameterized families between the maximum and the minimum, including the step-OWLAD operator, the NLHD operator, the WLHD operator, the olympic-OWLAD, the window-OWLAD operator, the median-OWLAD operator, and the centered-OWLAD, the WLGAD operator, the OWLHAD operator, the OWLAD operator, the OWLQAD operator and the OWLCAD. These families allow the assessment of complex GDM problems in which a set of optimal preferences need to be met considering diverse alternatives, scenarios and preferences. It is interesting mentioning the increasing number of studies in logarithmic aggregation operators, such as the generalized ordered weighted logarithmic proportional averaging (GOWLPA) operator [26], the generalized ordered weighted exponential proportional aggregation (GOWEPA) operator [27], and the generalized ordered weighted logarithmic harmonic averaging (GOWLHA) operator [28].

The remainder of the paper is as follows. In section 2 we present the preliminaries of this study. In section 3 we introduce the OWLAD operators, we study their main properties, alternative formulations, we propose some measures to characterize the weighting vector and introduce some families of the operator. Similarly, section 4 presents the study of the GOWLAD operators. Section 5 proposes a decision-making problem in innovation project management application, which is further assessed with a numerical example in section 6. Finally, section 7 presents our concluding comments.

Preliminaries

In order to establish the foundations of our study, this section presents a brief review of the OWA operator, the Hamming distance, the OWAD operator and the OWLA operators.

The ordered weighted averaging operator

The ordered weighted averaging (OWA) operator [5], describes a parameterized family of aggregation operators, which include the maximum, the minimum and average criteria. Applications of this operator has been widely studied in the literature [6]. It can be defined as follows:

Definition 1. An OWA operator is a mapping $OWA: R^n \rightarrow R$, with a n associated vector, characterized by having the properties $w_j \in [0,1]$, and $\sum_{j=1}^{n} w_j = 1$. According to the formula:

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j is the jth largest of the arguments a_i.

Observe that the reordering mechanism of the arguments introduces nonlinearity to the process. That property allows a shift to the conventional averaging techniques, such as the weighted average. The OWA operator is an averaging operator that displays commutativity, idempotency, boundedness and monotonicity [5]. Finally, depending on the reordering process, we can calculate the ascending OWA or the descending OWA [29].

The Hamming distance

The Hamming distance [12] has become a standard technique to measure the difference between two given parameters, elements or sets. This metric, has been applied in several domains of knowledge, some of the most recognized are fuzzy sets, artificial intelligence, operations research, and engineering. In general terms, if we assume two sets: $A = (a_1, a_2, \dots, a_n)$ and $B = (b_1, b_2, \dots, b_n)$, then the weighted Hamming distance can defined as follows:

Definition 2. A normalized Hamming distance of n dimension is a mapping WHD: $[0,1]^n \times [0,1]^n \to [0,1]$ with an associated weighting vector W of dimension n such that $w_j \in [0,1]$, and $\sum_{j=1}^{n} w_j = 1$, satisfying:

$$WHD(A, B) = \left(\sum_{i=1}^{n} w_i |a_i - b_i| \right), \tag{2}$$

where a_i and b_i are the ith arguments described in the sets A and B, respectively.

Note that the Hamming distance has the properties of non-negativity: $D(A_1, A_2) \geq 0$, commutativity: $D(A_1, A_2) = D(A_2, A_1)$, reflexivity: $D(A_1, A_2) = 0$ and triangle inequality: $D(A_1, A_2) + D(A_2, A_3) \geq D(A_1, A_3)$. Also observe that it is possible to extend the definition to all real numbers by applying $R^n \times R^n \to R$.

The ordered weighted averaging distance operator

Motivated by the application of aggregation operators to calculate the Hamming distance, Merigó and Gil-Lafuente [14] present an index named the ordered weighted averaging distance (OWAD) operator. The OWAD operators provide a parameterized family of distance aggregation operators between the maximum and the minimum. The families of the OWAD operator include the distance ordered weighted averaging (DOWAD) operator, the step-OWAD, the olympic-OWAD, the Hamming distance and the non-monotonic OWAD operator. Let P and P_k be two sets, then the OWAD operator can be defined as follows:

Definition 3. An OWAD operator of n dimension is a mapping $OWAD$: $R^n \times R^n \to R$ that has an associated weighting vector W, satisfying that $w_j \in [0,1]$, and $\sum_{j=1}^{n} w_j = 1$, according to the next formula:

$$OWAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = \sum_{i=1}^{n} w_i D_j, \qquad (3)$$

where D_j represents the jth largest of the $|x_i - y_i|$,and $x_{i,}y_i \in [0,1]$.

The ordered weighted logarithmic averaging operators

The generalized ordered weighted logarithmic aggregation (GOWLA) operator was developed by Zhou and Chen [25]. This operator is has as foundation the next optimal model:

$$minJ_1 = \sum_{j=1}^{n} w_j \left[(lny)^{\lambda} - \left(lna_j\right)^{\lambda}\right]^2, \qquad (4)$$

where y is an aggregation operator of dimension n, an associated weighting vector $w = (w_1, w_2, \dots, w_n)^T$ such that $w_j \in [0,1]$ for all j, and $\sum_{j=1}^{n} w_j = 1$. Observe that $\lambda \in (-\infty, \infty)$. By solving the partial derivative respect to y and $\frac{\partial y_1}{\partial y} = 0$, then we get the generalized weighted logarithmic averaging (GWLA) operator:

$$GWLA(a_1, a_2, \dots, a_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lna_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}. \qquad (5)$$

Reordering the arguments a_i we obtain the generalized ordered weighted logarithmic averaging (GOWLA) operator, as follows:

Definition 4. A GOWLA operator of dimension n is a mapping $GOWLA: \Omega^n \to \Omega$ with an associated weighting vector w of dimension n, such that $w_j \in [0,1]$ for all j, and $\sum_{j=1}^{n} w_j = 1$ including a parameter λ that moves between $(-\infty, \infty) - \{0\}$, satisfying the next formula:

$$GOWLA(a_1, a_2, \dots, a_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \quad (6)$$

where b_j is the jth largest of the arguments $a_1, a_2, \dots, a_n$. Note that $lna_j \geq 0$, following, $exp\left(lna_j\right) \geq exp(0)$, then $a_j \geq 1$. In the present paper, we follow the original notation described in [25], $\Omega = \{x | x \geq 1, x \in R\}$

An interesting family of the GWLA operator results when the parameter $\lambda = 1$, in this case we obtain an extension called the weighted logarithmic aggregation (WLA) operator. We can define the WLA operator as follows:

$$WLA(a_1, a_2, \dots, a_n) = exp \sum_{j=1}^{n} w_i(lna_i). \quad (7)$$

Ordered weighted logarithmic averaging distance operators

The weighted logarithmic averaging distance operator

The WLAD operator is a distance measure based on the optimal deviation model proposed by Zhou and Chen (2010). It uses the hamming distance to obtain a result between the minimum and maximum values considered in the problem.

Definition 5. A WLAD operator of dimension n is a mapping WLAD: $\Omega^n \times \Omega^n \rightarrow \Omega$ defined by an associated weighted vector W such that the sum of the weights is equal to 1 and $w_j \in [0,1]$, according to the formula:

$$WLAD(\langle x_1, y_1 \rangle, \langle x_2, y_2 \rangle, \dots, \langle x_n, y_n \rangle) = exp\left\{\sum_{j=1}^{n} w_j (ln|x_i - y_i|)\right\}, \quad (8)$$

where the argument $|x_i - y_i|$ are variables represented in the form of individual distances.

Observe that in this paper we follow the original definition [25] where $\Omega = \{x|x \geq 1, x \in R\}$. Note also, that if the individual distance$|x_i - y_i| = 0$, the aggregation process is not possible, because in logarithmic aggregation we can not use values below 1. Therefore, for individual distances below 1, we do not consider them in the aggregation, i.e. they are considered as empty.

Example 8.1. The WLAD operator. Assume the following collection of arguments: $X = (9, 24, 11, 33)$, $Y = (12, 15, 28, 23)$, and $W = (0.4, 0.1, 0.3, 0.2)$.The aggregation has as result:

$$WLAD(X, Y) = exp\{0.4 \times (ln|9 - 12|) + 0.1 \times (ln|24 - 15|) + 0.3 \times (ln|11 - 28|) + 0.2 \times (ln|33 - 23|)\} = 7.1682.$$

Observe that an alternative formulation to this approach is:

$$WLAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\sum_{j=1}^{n} w_j |ln(x_i) - ln(y_i)|\right\}. \quad (9)$$

The ordered weighted logarithmic averaging distance operator

The OWLAD operator is a generalization of the WLAD operator. The main distinctiveness is the ordering mechanism of the considered arguments. This order allows the introduction of complex decision-making processes, additionally it generates the possibility of having alternative formulations depending not only on the ascending or descending way of the ordering mechanism, but also in the system designed to solve the logarithmic distances. The properties of the OWLAD operator are commutativity, idempotency, boundedness, monotonicity and non-negativity

Definition 6. An ordered weighted logarithmic averaging distance (OWLAD) operator of dimension n is a mapping OWLAD: $\Omega^n \times \Omega^n \to \Omega$ that has an associated weighted vector W, with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, such that:

$$OWLAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\sum_{j=1}^{n} w_j ln(D_j)\right\}, \quad (10)$$

where D_j represents the jth largest of the $|x_i - y_i|$ and $|x_i - y_i|$ is the argument variable represented in the form of individual distances.

Example 8.2. The OWLAD operator. Assume the same collection of arguments mentioned in Example 8.1: $X = (9, 24, 11, 33)$, $Y = (12, 15, 28, 23)$, and $W = (0.4, 0.1, 0.3, 0.2)$. Then the aggregation will result:

$OWLAD(X, Y) = exp\{0.4 \times (ln|11 - 28|) + 0.1 \times (ln|33 - 23|) + 0.3 \times (ln|24 - 15|) + 0.2 \times (ln|9 - 12|)\} = 9.4162.$

From the ordering mechanism perspective, which differentiates this operator from the WLAD operator, two formulations can be described: the descending ordered weighted logarithmic averaging distance (DOWLAD) operator, and the ascending ordered weighted logarithmic averaging distance (AOWLAD) operator. The relation between these operators is $w_j = {w^*}_{n+1-j}$, where w_j is the jth weight of the DOWLAD operator and ${w^*}_{n+1-j}$ the jth weight of the AOWLAD operator.

Observe that in the presence of non-normalization i.e. $W = \sum_{j=1}^{n} w_j \neq 1$ in the arguments (see [1]), then the OWLAD operator can be obtained by:

$$OWLAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\frac{1}{W}\sum_{j=1}^{n} w_j ln(D_j)\right\}, \quad (11)$$

where $W = \sum_{j}^{n} w_j$.

The ordered weighted logarithmic aggregation operator displays the following properties: commutativity, idempotency, boundedness, monotonicity and non-negativity. Note that the proofs are trivial, therefore omitted. These properties can be expresses with the following theorems:

Theorem 1. Commutativity, by the ordered weighted aggregation. Let the function f be the OWLAD operator. Then,

$$f(\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle) = f(\langle c_1, d_1\rangle, \dots, \langle c_n, d_n\rangle), \qquad (12)$$

where$(\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle)$ represents any given permutation of the arguments $(\langle c_1, d_1\rangle, \dots, \langle c_n, d_n\rangle)$.

Theorem 2. Commutativity, by the distance measure. Assume f is the OWLAD operator, then

$$f(\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle) = f(\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle). \qquad (13)$$

Theorem 3. Monotonicity. Let f be the OWLAD operator; if $|x_i - y_i| \geq |c_i - d_i|$ for all i, then

$$f(\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle) \geq f(\langle c_1, d_1\rangle, \dots, \langle c_n, d_n\rangle). \qquad (14)$$

Theorem 4. Boundedness. Assume the function f is the OWLAD operator, then

$$min\{|x_i - y_i|\} \leq f(\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle) \\ \leq max\{|x_i - y_i|\}. \qquad (15)$$

Theorem 5. Idempotency. If the function f is the OWLAD operator, and if $|x_i - y_i| = a_i$, for all i, then

$$f(\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle) = a. \qquad (16)$$

Theorem 6. Non-negativity. Let the function f to be the OWLAD operator, then,

$$f(\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle) \geq 0. \qquad (17)$$

Alternative formulations of the OWLAD operators

Depending on the ordering of the arguments included in the aggregation process, four alternative formulations can be generated for the OWLAD operator, these include:

The $OWLAD^{I}$ operator can be obtained by firstly solving $|x_i - y_i|$, then finding the natural logarithm of the difference, and finally ordering the arguments in a descending direction. Following the next formula:

$$OWLAD^{I}(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\sum_{j=1}^{n} w_j ln(D_j)\right\}, \quad (18)$$

where D_j represents the jth largest of the $|x_i - y_i|$ and $|x_i - y_i|$ is the argument variable represented in the form of individual distances. Note that this first alternative formulation is equivalent to Eq. (10).

The $OWLAD^{II}$ operator is generated by finding in the first instance the natural logarithm for each argument, i.e. $ln(x_i)$, and $ln(y_i)$, secondly finding the absolute difference of the result obtained, and thirdly ordering the arguments in a descending direction. Following the next formula:

$$OWLAD^{II}(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\sum_{j=1}^{n} w_j |S_j - B_j|\right\}, \quad (19)$$

where S_j represents the jth largest of the $ln(x_i)$ and B_j represents the jth largest of the $ln(y_i)$, both arguments ordered in a descending way.

The $OWLAD^{III}$ operator is obtained when we order the arguments x_i and y_i in a descending way. Then, calculate the absolute difference of the ordered arguments. Finally obtaining the natural logarithm of the results. This sequence can be formulated as:

$$OWLAD^{III}(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\sum_{j=1}^{n} w_j ln(|E_j - M_j|)\right\}, \quad (20)$$

where E_j represents the jth largest of the x_i, and M_j represents the jth largest of the y_i, both ordered in a descending way.

The $OWLAD^{IV}$ operator is obtained when we first order the arguments x_i and y_i in a descending way, the second procedure is the calculation of the natural logarithm of the ordered arguments. The third and final process is obtaining the distance of the results. This mechanism can be formulated as:

$$OWLAD^{IV}(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\sum_{j=1}^{n} w_j \{[ln(E_j)] - [ln(M_j)]\}\right\}, \quad (21)$$

where E_j represents the jth largest of the x_i, and M_j represents the jth largest of the y_i, both ordered in a descending way.

Example 8.3.OWLAD operator alternative formulations. Following the same arguments described in Example 8.1, the results for each alternative formulation of the OWLAD operator are described in Table 8.1.

Table 8.1. Results for the alternative formulations of the OWLAD operator

$OWLAD^{I}$	$OWLAD^{II}$	$OWLAD^{III}$	$OWLAD^{IV}$
9.4162	1.7978	3.5944	1.2468

Characterization of OWLAD operators

The literature exhibits different approaches to measure, thus characterize the weights of aggregation functions. Some of the classic methods can be e.g. the degree of orness [5], the dispersion measure [5,30], the balance and the divergence. In the case of the OWLAD operator, further measures need to be developed, as the logarithmic properties of the aggregation limits the consideration of numbers between 0 and 1. Motivated by this fact, we propose a general characterization of the weighting vector and a transformation of the OWA measures into the R-scale.

Classic weighting characteristics

A first approach to characterize the weighting vector of the OWLAD operators is the use of the classic metrics, including the degree of orness or attitudinal character $\alpha(W)$, the entropy of dispersion $H(W)$, the balance $B(W)$ and the divergence $Div(W)$. Observe that these techniques do not fully consider the behavior of the logarithmic aggregation.

The first metric to be analyzed is the degree of orness. For this measure we can use the proposal found in [5] for the OWA operator. The main objective is the characterization of the complex attitude of the arguments introduced to the aggregation process, which depend exclusively on the weighting vector. The formulation can be described as:

$$\alpha(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1}\right). \tag{22}$$

Note that $\alpha(W) \in [0,1]$. Observe that an optimistic criteria is achieved when $\alpha(W) = 1$, in counterpart a pessimistic criteria is found when $\alpha(W) = 0$, lastly the averaging criteria is obtained when $\alpha(W) = 0.5$.

The second classic measure is the dispersion $H(W)$, which is utilized to study the type of information introduced in the aggregation process [5,30]. This metric can be obtained following the next:

$$H(W) = -\sum_{j=1}^{n} w_j ln(w_j). \tag{23}$$

When preforming the measure, if $w_j = 1$, for any j, then $H(W) = 0$, this means the minimum information is being used. However, if $w_j = \left(\frac{1}{n}\right)$ for all j, then the maximum of information is being used in the operator.

The third measure to be presented is the balance of the weighting vector. This metric study the way information inclines to a maximum or a minimum value. We can define it as follows:

$$Bal(W) = \sum_{j=1}^{n} w_j \left(\frac{n+1-2j}{n-1}\right). \quad (24)$$

Note that $Bal(W) \in [-1, 1]$. In case of $Bal(W) = -1$ we obtain the minimum, the maximum is obtained when $Bal(W) = 1$.

The fourth measure to be introduced is the divergence of the weighting vector. This metric can be obtained by applying the next formulation:

$$Div(W) = \sum_{j=1}^{n} w_j \left(-\alpha(W) + \frac{n-j}{n-1}\right)^2. \quad (25)$$

Example 8.4. Characterization of the OWLAD operator. Following the arguments described in Example 8.1, the characterization of the weighting vector result as shown in Table 8.2:

Table 8.2. OWLAD operator weighting vector measures

Measure	$\alpha(W)$	$H(W)$	$B(W)$	$Div(W)$
Result	0.5667	1.2799	0.1333	0.1529

A general characterization of the aggregation

Motivated by the fact that logarithms do not work in the scale $[0,1]$, we must then find additional measures to characterize the aggregation. A general approach to characterize the descending aggregation (CDA), can be represented in the next formulation:

$$CDA = \frac{b_j - b_n}{b_1 - b_n}, \quad (26)$$

where b_j is the result of the OWLAD operator and $b_1$and b_n are the largest and smallest arguments of $|x_i - y_i|$, respectively. Note that this requires the aggregation to be in a descending way. Furthermore, the dual version of this formulation can be represented as:

$$CDA + CDA^* = 1,$$

then,

$$CDA^* = 1 - \frac{b_j - b_n}{b_1 - b_n}. \quad (27)$$

If the aggregation is presented in an ascending way, then characterization of the ascending aggregation (CAA) the formulation needs to be changed in the next order:

$$CAA = \frac{b_j - b_1}{b_n - b_1}, \quad (28)$$

where b_j is the result of the OWLAD operator and $b_1$and b_n are the largest and smallest arguments of $|x_i - y_i|$, respectively. As presented for the descending formulation, the dual version of this representation can be obtained by:

$$CAA^* = 1 - \frac{b_j - b_1}{b_n - b_1}. \quad (29)$$

Example 8.5. Dual versions of the OWLAD aggregation. Following the values presented in Example 8.2: $X = (9, 24, 11, 33)$, $Y = (12, 15, 28, 23)$, and $W = (0.4, 0.1, 0.3, 0.2)$. Then, the general characterization of the aggregation and their dual versions will result as presented in Table 8.3.

Table 8.3. OWLAD operator general characterization of the aggregation

Measure	CDA	CDA*	CAA	CAA*
Result	0.4583	0.5417	0.7287	0.2713

Transformation of the OWA measures into the R-scale

An interesting mechanism to characterize the weighting vector including the logarithmic properties of the weighted logarithmic aggregation operators is the transformation of the OWA measures into the R-scale. The proposed procedure can be realized following the next steps:

Let Z be the transformation of the aggregation arguments following the next formulation:

$$Z = min + \{max - min\}\left(\frac{n-j}{n-1}\right). \quad (30)$$

Observe that the use of Z allows the transformation of the $[0,1]$ scale, into a logarithmic consistent one. Motivated by the result of this procedure, we propose the use of the Z transformation to study the degree of orness in the OWLA operator.

Step 1. Calculate the $R - \alpha(w)$, which includes the Z transformation, using the next equation:

$$R - \alpha(w) = e\left\{\sum_{j=1}^{n} w_j ln(Z_j)\right\}, \quad (31)$$

where Z is the transformation of the arguments in the aggregation. The complete formulation can be described as:

$$R - \alpha(w) = e\left\{\sum_{j=1}^{n} w_j ln\left(min(a_i) + \{max(a_i) - min(a_i)\}\left(\frac{n-j}{n-1}\right)\right)\right\}, \quad (32)$$

where a_i are the arguments $|x_i - y_i|$ of the aggregation.

Step 2. The final step is converting the result $R - \alpha(w)$ using the following:

$$x = \frac{y - min(a_i)}{\left(max(a_i) - min(a_i)\right)}, \quad (33)$$

where y is the result of the $R-\alpha(w)$ and $x \in [0,1]$. Note that the minimum is obtained when $x=0$, and the maximum when $x=1$. Also observe that we can obtain the dual of this operation by applying the next formulation:

Let x^* be the dual of the x, then

$$x + x^* = 1,$$

following

$$x^* = 1 - \frac{y - min(a_i)}{\left(max(a_i) - min(a_i)\right)} = \frac{max(a_i) - y}{max(a_i) - min(a_i)}. \quad (34)$$

Example 8.6. The degree of orness. Following the arguments defined in Example 8.2: $X = (9, 24, 11, 33)$, $Y = (12, 15, 28, 23)$, and $W = (0.4, 0.1, 0.3, 0.2)$. Then, the degree of orness considering the logarithmic scale will result as:

$$\begin{aligned} R - \alpha(w) = e\{&0.4[ln(3 + 14(1))] + 0.1[ln(3 + 14(0.6667))] \\ &+ 0.3[ln(3 + 14(0.3333))] + 0.2[ln(3 + 14(0))]\} \\ &= 9.1642. \end{aligned}$$

Therefore

$$x = \frac{9.1642 - 3}{(17 - 3)} = 0.4403,$$

and

$$x^* = \frac{17 - 9.1642}{17 - 3} = 0.5597.$$

OWLAD operator families

It is interesting to study the families of the OWLAD operators, as they represent particular cases that can be selected in accordance to the specific problems we are assessing. For the case of the OWLAD operator, several parameterized families can be described depending on the conformation of the weighting vector. These particular families include the maximum and minimum distance, the step-OWLAD operator, the normalized logarithmic

Hamming distance (NLHD), the weighted logarithmic Hamming distance (WLHD), the olympic-OWLAD, the window-OWLAD operator, the median-OWLAD operator, and the centered-OWLAD. Note that all the alternative formulations described before are also applicable for the families here presented.

Remark 1. In general terms, the maximum distance is found when $w_1 = 1$, and $w_j = 0$ for all $j \neq 1$. In counterpart, the minimum distance is achieved when $w_n = 1$, and $w_j = 0$ for all $j \neq n$.

Remark 2**.** The step-OWLAD is generated when $w_k = 1$, and $w_j = 0$ for all $j \neq k$. Observe that the step-OWLAD is reduced to the maximum distance when $k = 1$, and the minimum distance when $k = n$.

Remark 3. The NLHD is obtained when $w_j = \frac{1}{n}$, for all j. Moreover, the WLHD is generated when the ordered position of i is the same as the ordered position of j.

Remark 4. The olympic-OWLAD operator is created when $w_1 = w_n = 0$, and $w_j = \frac{1}{(n-2)}$, for all $j - \{1, n\}$. In this particular case, if $n = 3$ or $n = 4$, then the olympic-OWLAD is reduced to the median-OWLAD. For general purposes, it is also viable to represent the olympic-OWLAD operator considering that $w_j = 0$ for $j = 1, 2, \dots, k, n, n-1, \dots, n-k+1$, and for the others $w_j = \frac{1}{(n-2k)}$, where $k < \frac{n}{2}$. In this general approach, if $k = 1$, then we perform the usual olympic-OWLAD. If $k = \frac{(n-1)}{2}$, then we obtain the median-OWLAD aggregation.

Remark 5**.** There is also the possibility to consider the contrary case of the olympic-OWLAD operator. This is performed when $w_j = \left(\frac{1}{2k}\right)$ that is for $j = 1, 2, \dots, k, n, n-1, n-k+1$, and $w_j = 0$ for all the others $k < \frac{n}{2}$. In this sense, if $k = 1$, then we obtain the contrary case of the median-OWLAD.

Remark 6. Based on the S-OWA operator [31], we can further develop the S-OWLAD operator. The main characteristic of this approach is the subdivision into three classes: the “or-like”, the “and-like” and the generalized S-OWLAD operators. This last one can be created when $w_1 = \left(\frac{1}{n}\right)\left(1-(\alpha+\beta)\right)+\alpha$, $w_n = \left(\frac{1}{n}\right)\left(1-(\alpha+\beta)\right)+\beta$ and $w_j = \left(\frac{1}{n}\right)\left(1-(\alpha+\beta)\right)$ for $j = 2$ to $n-1$, where $\alpha, \beta \in [0,1]$ and $\alpha, \beta \leq 1$. In the case that $\alpha = 0$, the generalized S-OWLAD operator becomes the “and-like” S-OWLAD operator, and if $\beta = 0$, then it becomes the “or-like” S-OWLAD operator.

Remark 7. Based on Yager [32] we can describe the centered-OWLAD operator. This aggregation is created when three conditions are achieved: It is symmetric i.e. $w_j = w_{j+n-1}$, It is strongly decaying when $i < j \leq \frac{(n+1)}{2}$, then $w_i < w_j$, and when $i > j \geq \frac{(n+1)}{2}$, then $w_i < w_j$. It is inclusive if $w_j > 0$.

Several additional families could be described for the OWLAD operator following similar methodologies e.g. the ones presented in [33–35].

Generalized ordered weighted logarithmic distance operators

The generalized weighted logarithmic averaging distance operator

The GWLAD operator is a generalization of the OWLAD operator. Therefore it shares the same properties and characteristics. The GWLAD operator includes a λ parameter, which allows a wider representation of complex problems. Observe that many interesting families of the GWLAD can be developed depending on the λ value.

Definition 7. A GWLAD operator of dimension n is a mapping GWLAD: $\Omega^n \times \Omega^n \rightarrow \Omega$ with a weighted vector W of dimension n associated, satisfying that the sum of all w_j is equal to 1 and $w_j \in [0,1]$, following the formula:

$$GWLAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j(ln|x_i - y_i|)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \tag{35}$$

where $|x_i - y_i|$ are argument variables represented in the form of individual distances, and λ is a parameter such that $\lambda \in (-\infty, \infty) - \{0\}$.

Specially note that in the case that $w_j = \frac{1}{n}$ for all j, then we obtain the generalized logarithmic averaging distance operator (GLAD), which follows the next formulation:

$$GLAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\left(\frac{1}{n}\sum_{j=1}^{n} (ln|x_i - y_i|)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}. \tag{36}$$

Example 8.7. The GWLAD operator. Following the arguments defined in Example 8.2: $X = (9, 24, 11, 33)$, $Y = (12, 15, 28, 23)$, and $W = (0.4, 0.1, 0.3, 0.2)$, and a parameter $\lambda = 2$. The aggregation then results:

$$GWLAD(X, Y) = exp\left\{[0.4 \times (ln|9 - 12|)^2 + 0.1 \times (ln|24 - 15|)^2 + 0.3 \times (ln|11 - 28|)^2 + 0.2 \times (ln|33 - 23|)^2]^{1/2}\right\} = 8.2130.$$

Additionally the parameter λ in the GWLAD operator allows the study of particular cases. Table 8.4 presents some special cases that are interesting for analysis.

Table 8.4. Families of GWLAD operators

λ	Families	Acronym
$\lambda \to 0$	Weighted logarithmic geometric averaging distance operator	WLGAD
λ = -1	Weighted logarithmic harmonic averaging distance operator	WLHAD
$\lambda = 1$	Weighted logarithmic aggregation distance operator	WLAD
$\lambda = 2$	Weighted logarithmic quadratic aggregation distance operator	WLQAD
$\lambda = 3$	Weighted logarithmic cubic aggregation distance operator	WLCAD
$\lambda \to \infty$	Largest of the $\lvert x_i - y_i \rvert$	Max
$\lambda \to -\infty$	Lowest of the $\lvert x_i - y_i \rvert$	Min

Remark 8. Let $\lambda \to 0$, then the GWLAD becomes the WLGAD operator.

$$GWLAD(\langle x_1, y_1 \rangle, \langle x_2, y_2 \rangle, \dots, \langle x_n, y_n \rangle) = exp\left\{\prod_{j=1}^{n} (ln|x_i - y_i|)^{w_j}\right\}. \tag{37}$$

Remark 9. Let $\lambda = -1$, then the GWLAD operator becomes WLHAD operator.

$$GWLAD(\langle x_1, y_1 \rangle, \langle x_2, y_2 \rangle, \dots, \langle x_n, y_n \rangle) = exp\left\{\frac{1}{\sum_{j=1}^{n}\left(\frac{w_j}{ln|x_i - y_i|}\right)}\right\}. \tag{38}$$

Remark 10. Let $\lambda = 1$, then the GWLAD operator becomes the WLAD operator.

$$GWLAD(\langle x_1, y_1 \rangle, \langle x_2, y_2 \rangle, \dots, \langle x_n, y_n \rangle) = exp \sum_{j=1}^{n} w_j (ln|x_i - y_i|). \tag{39}$$

Remark 11. Let $\lambda = 2$, then the GWLAD operator becomes the WLQAD operator.

$$GWLAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\sqrt{\left(\sum_{j=1}^{n} w_j (ln|x_i - y_i|)^2\right)}\right\}. \tag{40}$$

Remark 12. Let $\lambda = 3$, then the GWLAD operator becomes the WLCAD operator.

$$GWLAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j (ln|x_i - y_i|)^3\right)^{\frac{1}{3}}\right\}. \tag{41}$$

Remark 13. Let $\lambda \to \infty$, then the GWLAD operator solution tends to the jth largest $|x_i - y_i|$.

$$GWLAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = max\{|x_i - y_i|\}. \tag{42}$$

Remark 14. Let $\lambda \to -\infty$, then the GWLAD operator solution tends to the jth lowest $|x_i - y_i|$.

$$GWLAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = min\{|x_i - y_i|\}. \tag{43}$$

Example 8.8. The GWLAD operator families. Following the arguments described in Example 8.2, the results for each family of the GWLAD operator result as shown in Table 8.5:

Table 8.5. Families of GWLAD operator

λ	$\rightarrow 0$	-1	1	2	3	∞	$-\infty$
Aggregation	6.1353	5.2601	7.1682	8.2130	9.1541	$\rightarrow 17$	$\rightarrow 3$

The generalized ordered weighted logarithmic averaging distance operator

The GOWLAD operator adds an ordering mechanism to the GWLAD operator. Therefore as a generalization of the GWLAD, it shares the same properties. The ordering mechanism allows the modeling of wider and complex problems. Also, it introduces the possibility of further alternative formulations and families depending on the λ value.

Definition 8. A GOWLAD operator of dimension n is a mapping GOWLAD: $\Omega^n \times \Omega^n \rightarrow \Omega$ defined by an associated weighted vector W of dimension n such that the sum of the weights is equal to 1 and $w_j \in [0,1]$, according to the formula:

$$GOWLAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \dots, \langle x_n, y_n\rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j\left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \tag{44}$$

where, b_j is the $|x_i - y_i|$ value of the GOWLAD $\langle x_i, y_i\rangle$, reordered in decreasing values of the $|x_i - y_i|$. The argument $|x_i - y_i|$ are variables represented in the form of individual distances, and λ is a parameter satisfying that $\lambda \in (-\infty, \infty) - \{0\}$.

Example 8.9. The GOWLAD operator. Following the arguments described in Example 8.2, where: $X = (9, 24, 11, 33)$, $Y = (12, 15, 28, 23)$. Assuming $W = (0.4, 0.1, 0.3, 0.2)$ and $\lambda = 2$, the aggregation will result as:

$$GOWLAD(X,Y) = exp\left\{\left(\left(0.4 \times (ln|11-28|)\right)^2 + \left(0.1 \times (ln|33-23|)\right)^2 + \left(0.3 \times (ln|24-15|)\right)^2 + \left(0.2 \times (ln|9-12|)\right)^2\right)^{\frac{1}{2}}\right\} = 10.2820.$$

Observe that the descending order of the arguments b_j, depend on the result of $|x_i - y_i|$.

Also note that the GOWLAD operator is a generalization of the OWLAD operator, thus the properties of commutativity, monotonicity, boundedness and idempotency are shared.

Alternative formulations of the GOWLAD operator

It is interesting to mention that the GOWLAD operator also exhibits four alternative formulations that can be generated depending on the ordering of the arguments presented, briefly we have:

The $GOWLAD^I$ operator, which is obtained by the next formulation:

$$GOWLAD^I(x_n, y_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(ln(D_j)\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \tag{45}$$

where D_j represents the jth largest of the $|x_i - y_i|$ and $|x_i - y_i|$ is the argument variable represented in the form of individual distances.

The $GOWLAD^{II}$ operator that is represented by:

$$GOWLAD^{II}(x_n, y_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(S_j - B_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \tag{46}$$

where S_j represents the jth largest of the $ln(x_i)$ and B_j represents the jth largest of the $ln(y_i)$, both arguments ordered in a descending way.

The $GOWLAD^{III}$ operator formulated as:

$$OWLAD^{III}(x_n, y_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(ln\left(\left|E_j - M_j\right|\right)\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \tag{47}$$

where E_j represents the jth largest of the x_i, and M_j represents the jth largest of the y_i, both ordered in a descending way.

The $GOWLAD^{IV}$ operator that is represented as:

$$GOWLAD^{III}(x_n, y_n) = exp\left\{\left(\sum_{j=1}^{n} w_j\{[ln(E_j)] - [ln(M_j)]\}^{\lambda}\right)^{\frac{1}{\lambda}}\right. \tag{48}$$

where E_j represents the jth largest of the x_i, and M_j represents the jth largest of the y_i, both ordered in a descending way.

Example 8.10. Alternative GOWLAD operator formulations. Following the arguments presented in Example 8.8, the results for each alternative formulation of the GOWLAD operator are described in Table 8.6.

Table 8.6. Results for the alternative formulations of the GOWLAD operator

$GOWLAD^{I}$	$GOWLAD^{II}$	$GOWLAD^{III}$	$GOWLAD^{IV}$
10.2820	1.9220	3.9026	1.2680

The GOWLAD operator families

Several particular families of the GOWLAD operator can be delimitated by the values of the parameterλ. Table 8.7 presents some representative cases of the GOWLAD operator families, including the ordered weighted logarithmic geometric averaging distance (OWLGAD) operator, the ordered weighted logarithmic harmonic averaging distance (OWLHAD) operator, the ordered weighted logarithmic aggregation distance (OWLAD) operator, the ordered weighted logarithmic quadratic aggregation distance (OWLQAD) operator, the ordered weighted logarithmic cubic aggregation distance (OWLCAD) operator, the maximum and the minimum.

Table 8.7. Families of GOWLAD operators

λ	Families	Acronym	Formula
$\lambda \to 0$	Ordered weighted logarithmic geometric averaging distance operator	OWLGAD	$GOWLAD(x_n, y_n) = exp\left\{\prod_{j=1}^{n}\left(ln(b_j)\right)^{w_j}\right\}$ (49)
λ = -1	Ordered weighted logarithmic harmonic averaging distance operator	OWLHAD	$GOWLAD(x_n, y_n) = exp\left\{\frac{1}{\sum_{j=1}^{n}\left(\frac{w_j}{lnb_j}\right)}\right\}$ (50)
λ = 1	Ordered weighted logarithmic aggregation distance operator	OWLAD	$GOWLAD(x_n, y_n) = exp\sum_{j=1}^{n} w_j(lnb_j)$ (51)
λ = 2	Ordered weighted logarithmic quadratic aggregation distance operator	OWLQAD	$GOWLAD(x_n, y_n) = exp\left\{\sqrt{\left(\sum_{j=1}^{n} w_j(lnb_j)^2\right)}\right\}$ (52)

$\lambda = 3$	Ordered weighted logarithmic cubic aggregation distance operator	OWLCAD	$GOWLAD(x_n, y_n) = exp\left\{\left(\sum_{j=1}^{n} w_j (lnb_j)^3\right)^{\frac{1}{3}}\right\}$	(53)
$\lambda \to \infty$	Largest of the b_j, for $j = n$	Max	$GOWLAD(x_n, y_n) = max\{b_j\}$	(54)
$\lambda \to -\infty$	Lowest of the b_j, for $j = n$	Min	$GOWLAD(x_n, y_n) = min\{b_j\}$	(55)

Note that for all the cases b_j is the $|x_i - y_i|$ value of the GOWLAD $\langle x_i, y_i \rangle$, reordered by the decreasing values of $|x_i - y_i|$

Group decision-making in innovation project management

The GOWLAD operator, based on the Hamming distance mechanism is applicable to a wide range of problems in decision-making procedures. However this operator can also be applicable to statistical analysis, operations, engineering and economic studies [1,2,4,36].

This paper presents a decision making application in the field of innovation project management [37]. The main motivation for using the GOWLAD operator in this area is the possibility of retrieving the opinions of several experts to select the most efficient solution for a company when managing new projects. Commonly, project management has been measured in terms of cost, duration and return over investment [37,38]. However, the GOWLAD operator opens the option to valuate uncertain and subjective factors such as the extent of internal communication of the implicated areas when developing a new product [39], as well as the collaboration with suppliers [40] and customers [41] as they have been identified as sources that contribute to the innovation process. The general process to assess a multi-person decision-making situation using the GOWLAD operator can be described as follows:

Step 1. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of limited options, $C = \{C_1, C_2, \ldots, C_m\}$ a set of finite options or alternatives, both sets conform a matrix, $(x_{hi})_{m\times n}$. Let $E = \{E_1, E_2, \ldots, E_q\}$ be a finite set of decision makers. Assume each of the decision makers have diverse levels of importance, in that case $V = (v_1, v_2, \ldots, v_p)$ represents the weighting vector of importance satisfying that $\sum_{k=1}^{p} v_k = 1$ and $x_k \in [0,1]$. At that point each decision maker must deliver a pay-off matrix $(x_{hi})^k{}_{m\times n}$.

Step 2. An ideal level of characteristics must be set to form the ideal project to be developed, see Table 8.8. In this case P is the ideal project represented by a subset, C_i represents the ith characteristic considered and $y_i \in [1,100]$; $i = 1,2, \ldots, n$, is a number between 1 and 100 for the ith characteristic. Each decision maker must here provide an ideal project ${y_i}^k$.

Table 8.8. Ideal project

	C_1	C_2	…	C_i	…	C_n
P	y_1	y_2	…	y_i	…	y_n

Step 3. Apply the weighted average (WA) to aggregate the information of the decision makers E, by using the weighting vector V. The result will then be the collective payoff matrix $(x_{hi} - y_{hi})_{m\times n}$, therefore $x_{hi} - y_{hi} = \sum_{k=1}^{p} v_k \left(x_{hi}^k - y_{hi}^k\right)$. Note that more complex aggregations could be developed if the experts' opinions are aggregated with a different method than the WA, e.g. the OWA operator.

Step 4. Solve for GOWLAD operator as described in Eq. 43. Consider that λ value is usually set in 1; however, any of the families described in section 4.4 could be used depending on the problem to be assessed.

Step 5. Establish a ranking of the evaluated options, compare the results for the specific problem assessed and compile a decision-making approach.

In order to summarize this aggregation mechanism, we propose the utilization of the next aggregation operator named the multi-person-GOWLAD (MP-GOWLAD) operator.

Definition 9. An MP-GOWLAD operator is an aggregation operator with an associated weighting vector V of dimension p satisfying that the sum of the weights is 1 and $v_k \in [0,1]$, and a weighting vector W of n dimension such that $\sum_{j=1}^{n} w_j = 1$, and $w_j \in [0,1]$, according to:

$$MP - GOWLAD\left((x_1^1, \dots, x_1^p), (y_1^1, \dots, y_1^p), \dots, (x_n^1, \dots, x_n^p), (y_n^1, \dots, y_n^p)\right) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \quad (56)$$

where, b_j is the $|x_i - y_i|$ value of the MP-GOWLAD (x_i, y_i), reordered in decreasing way of the arguments $|x_i - y_i|$. The arguments $|x_i - y_i| = \left(\sum_{k=1}^{p} v_k \left|x_i^k - y_i^k\right|\right)$, $\left|x_i^k - y_i^k\right|$ are variables given by the opinions of each expert in the form of individual distances, and λ is a parameter satisfying that $\lambda \in (-\infty, \infty) - \{0\}$. Observe that the MP-GOWLAD operator shares the properties of the GOWLAD operator.

It is interesting to mention that the MP-GOWLAD operator can be reduced to a series of particular cases following the methodology presented in Section 3.5, some interesting cases to be mentioned are the multi-person-normalized logarithmic Hamming distance (MP-NLHD) operator, the multi-person-weighted logarithmic Hamming distance (MP-WLHD) operator, the multi-person-OWLAD (MP-OWLAD) operator, the multi-person-OWLA (MP-OWLA) operator, the multi-person WLA (MP-WLA) operator.

GOWLAD operator numerical example

This section proposes an illustrative example attaining a strategic decision making procedure in innovation project management using a multi-person analysis and the GOWLAD operator. Observe that more business-decision making applications can be assessed. Especially in the area of innovation management, which has been widely described as an uncertain and subjective topic, thus an interesting area for expert decision making procedures.

Step 1. Assume that a real estate construction company must select the most adequate project to develop from their portfolio of six potential projects:

- A_1 Industrial Park
- A_2 Small multi-family housing
- A_3 Residential building
- A_4 City villas
- A_5 Commercial building
- A_6 Luxury apartments

In order to select the project to be developed, the company chooses diverse experts to valuate 6 key characteristics:

- S_1 Cost of the project
- S_2 Duration
- S_3 Return over investment (ROI)
- S_4 Expertise
- S_5 Internal communication
- S_6 External communication

A total of three experts are asked for their opinion, the results for each of the projects are shown in Tables 8.9 – 8.11. Observe all the valuations are expressed in terms of numbers between 1 and 100, being 100 the maximum valuation.

Step 2. Representing the objectives of the decision makers, each of the experts construct the ideal project to be developed. Results of this process are shown in Table 8.12.

Step 3. The weighting vector that represent the importance of each expert in the analysis is the following $V = (0.5, 0.25, 0.25)$. With this information we use the weighted average to aggregate the information into a collective matrix. The results are shown in Table 8.13.

Step 4. We apply some of the gowlad operator families; we aggregate the collective information and obtain final results. Tables 8.14 and 8.15 show the results of the aggregations.

Step 5. In order to generate complete picture of the aggregations we must establish a ranking of the performance of each project depending on the preferences of the decision makers. The ordering of alternatives is presented in Table 8.16. Note that the symbol "}" represents preferred to. Also note that for each of the selected aggregation operator, a different ranking can be generated, distinct decision making processes will results from that operation.

Table 8.9. Characteristics of the project – valuations from expert 1.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	88	56	59	95	90	64
A_2	68	88	69	96	97	96
A_3	95	62	85	99	82	79
A_4	79	62	100	72	67	79
A_5	86	82	100	96	72	58
A_6	60	93	53	59	87	73

Table 8.10. Characteristics of the project – valuations from expert 2.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	79	88	76	83	61	85
A_2	63	61	86	68	76	74
A_3	77	86	69	86	71	88
A_4	74	76	66	89	65	62
A_5	61	65	65	84	78	80
A_6	86	73	61	81	85	68

Table 8.11. Characteristics of the project – valuations from expert 3.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	75	54	75	59	39	35
A_2	33	35	50	92	96	56
A_3	93	63	64	71	38	48
A_4	48	42	70	70	55	77
A_5	61	74	94	61	49	88
A_6	77	90	86	78	35	39

Table 8.12. Ideal investment

	C_1	C_2	C_3	C_4	C_5	C_6
E_1	70	80	100	100	60	80
E_2	90	80	100	90	70	90
E_3	80	90	100	70	50	80

Table 8.13. Collective results in the form of individual distances.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	5	19	32.75	7	10	20.5
A_2	19.5	14.5	31.5	2	31.5	2
A_3	12.5	14.25	24.25	1.25	8.25	9
A_4	7.5	22	16	14.25	3.5	8.25
A_5	4	6.75	10.25	5.75	7.75	11.5
A_6	6.75	4.75	36.75	20.75	13.5	19.25

Table 8.14. Aggregated results 1.

	Max	Min	NLHD	WLHD	Step (k=3)	WLAD
A_1	32.7500	5.0000	2.5519	2.7613	3.4889	15.8196
A_2	31.5000	2.0000	2.3218	2.4974	3.4500	12.1508
A_3	24.2500	1.2500	2.1502	2.3586	3.1884	10.5759
A_4	22.0000	3.5000	2.3164	2.2806	2.7726	9.7830
A_5	11.5000	4.0000	1.9771	2.1007	2.3273	8.1718
A_6	36.7500	4.7500	2.6108	2.8433	3.6041	17.1724

Table 8.15. Aggregated results 2.

	GOWLAD -1	GOWLAD 1	GOWLAD 2	GOWLAD 3	Me-dian	Olym-pic
A_1	1.059	11.888	3.129	2.118	13.78	12.849
A_2	1.040	8.379	2.911	2.309	16.81	11.552
A_3	1.027	7.603	2.684	1.996	10.60	10.724
A_4	1.053	9.468	2.823	1.978	10.84	10.898
A_5	1.047	6.748	2.345	1.854	7.232	7.4516
A_6	1.061	12.240	3.154	2.288	16.12	13.812

Table 8.16. Ranking of the performance of the concepts to be developed.

	Ranking		Ranking
Max	$A_5\}A_4\}A_3\}A_2\}A_1\}A_6$	GOWLAD (λ=-1)	$A_3\}A_2\}A_5\}A_4\}A_1\}A_6$
Min	$A_1\}A_6\}A_5\}A_4\}A_2\}A_3$	GOWLAD (λ=1)	$A_5\}A_3\}A_2\}A_4\}A_1\}A_6$
NLHD	$A_5\}A_3\}A_4\}A_2\}A_1\}A_6$	GOWLAD (λ=2)	$A_5\}A_3\}A_4\}A_2\}A_1\}A_6$
WLHD	$A_5\}A_4\}A_3\}A_2\}A_1\}A_6$	GOWLAD (λ=3)	$A_5\}A_4\}A_3\}A_1\}A_6\}A_2$
Step (k=3)	$A_5\}A_4\}A_3\}A_2\}A_1\}A_6$	Median	$A_5\}A_3\}A_4\}A_1\}A_6\}A_2$
WLAD	$A_5\}A_4\}A_3\}A_2\}A_1\}A_6$	Olympic	$A_5\}A_3\}A_4\}A_2\}A_1\}A_6$

As is observable, the ranking changes depending on the aggregation mechanism of the chosen operator. In our example, based on the experience of three experts, the closest option to an ideal project is A_5(Commercial building) and A_3(Residential building), it can be inferred that the company has more experience in developing these real estate constructions but also it implies that the innovative characteristics of the company aligns in an adequate way to the preferences of the firm.

Conlcusions

This paper introduces a new family of ordered weighted logarithmic averaging distance operators including the ordered weighted logarithmic averaging distance OWLAD operator and the generalized ordered weighted logarithmic averaging distance GOWLAD operator. The foundation of this approach is the optimal deviation model based on the GOWLA operator. Therefore, it shares the same properties. The motivation is the extension of its characteristics to consider a wider range of complex problems. The main advantage of the ordered weighted logarithmic averaging distance operators is the introduction of distance measures, specifically the Hamming distance, to consider an optimal set of preferences and compare them to the options or alternatives to be selected by the decision makers.

Diverse properties of the OWLAD and the GOWLAD operators have been revised, such as commutativity, idempotency, boundedness, monotonicity, non-negativity and reflexivity. We have studied different classic measures to characterize the weighting vector including the degree of orness, dispersion, balance and divergence measures. Moreover, motivated by the fact that these measures fail to work with numbers between 0 and 1 we propose additional measures to characterize the aggregation, including a transformation of the OWA measures into the R-scale. We have also presented four different alternative formulations of the OWLAD and GOWLAD operators that can be addressed depending on the ordering of the arguments to be aggregated.

Several particular cases of the ordered weighted logarithmic averaging distance operators have been analyzed. Firstly, depending on the conformation of the weighting vector, the OWLAD operator can be reduced to the maximum and minimum distance, the step-OWLAD operator, the NLHD operator, the WLHD operator, the olympic-OWLAD, the window-OWLAD operator, the median-OWLAD operator, and the centered-OWLAD. Secondly, by analyzing the parameter λ the GOWLAD operator displays specific families including the maximum and the minimum, the OWLGAD operator, the OWLHAD operator, the OWLAD operator, the OWLQAD operator and the OWLCAD operator.

The OWLAD and GOWLAD operators, including their particular cases and families are designed to aid group decision-making processes. Engineering, statistics and economics are some of the scientific areas that could apply this new approach. In order to exemplify the use of the OWLAD and GOWLAD operators, we present a multi-person group decision-making problem in the area of innovation project management. The main advantage of this method is the introduction of several experts to assess a complex decision making procedure, which involves objective and subjective factors. Innovation management has been described as an uncertain series of steps and procedures; such facts make the topic interesting, moreover viable to analyze. The results in the illustrative example present different combinations of options and alternatives depending on the complex attitudinal character of the decision makers between an ideal series of characteristics and the comparison between the possible projects to realize.

Much future research is needed, firstly to address the main limitation of this study, which are multifaceted properties of the logarithms, which complicate the development of characterization measures of the weighting vector. Also, the complex decision-making processes such as innovation management that requires the development of new and robust techniques, which consider uncertain information, for example fuzzy numbers, linguistic variables and interval numbers, heavy aggregations among other complex formulations.

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Conclusion

The present Doctoral Thesis proposes a series of models for business decision-making based on innovation as driver of competitiveness.

The state of the art chapter shows the relationship between the concepts of competitiveness, innovation, Fuzzy Logic and innovation measurement. Results convey on an increasing academic interest around the topic and a growing rate of contributions and publications. However findings also reveal several limitations that are recurrently found when dealing with innovation management. Moreover, the role that uncertainty plays around the concept of innovation adds complexity to its analysis and correct measurement. These gaps open a real possibility of research, specifically, the development of new tools derived from Fuzzy Logic for the treatment of information regarding innovation.

Once the viability of the study has been established, we propose an in-depth analysis of the industrial and manufacturing network of the city of Morelia, Mexico. Two contributions are presented, both regarding the application of Moore families and Galois Lattice. Together, and combined with a series of tools derived from Fuzzy Logic, we obtain groups of companies with related characteristics. The clusters found with the application of robust mathematical models have diverse implications ranging from economies of scale to the generation of strategic alliances for the combined development of companies. The in depth knowledge of the exogenous and endogenous elements that shape the manufacturing environment of the city also enables the possibility of a better adjustment of the future business decision-making models.

The core of our study focuses on quantifying the degree of innovation capacities of small and medium-sized productive enterprises within the city of Morelia, Mexico. The proposal is a new methodological, intuitive and replicable model that applies tools derived from Fuzzy Logic for the treatment of the collected information. The empirical study comprises 32 exploratory and 5 control questions addressing 7 organizational areas of the company: innovation strategy, knowledge management, project management, portfolio management, internal drivers, organization and structure, and external drivers. The study was applied in a personal way to 217 companies. A total of 91 valid answers were obtained. Two methods were selected to treat the uncertain information, firstly the use of Expertons and secondly the use of the ordered weighted averaging (GOWA) operator. The results obtained provide a snapshot of the characteristics that shape the innovation capacities of the city, promoting the comparison and punctual analysis of the companies of the sector.

Our Doctoral Thesis includes in its fifth chapter new models for information analysis, exploitation of information and especially new mathematical tools for the treatment of information in highly uncertain environments. The new tools propose new contributions in the field of aggregation operators. In recent years this academic branch has grown exponentially, a myriad of new aggregation operators have been proposed, covering topics from financial and economic issues to artificial intelligence. The pioneer topic addressed in our proposal has as scope innovation management. It seeks to serve as a basis for the development of new aggregation operators applied to innovation measurement.

The published contributions presented in this Thesis fulfill in a formal way the main goal, intentions and objectives of our study. Much research is needed, decision-making in highly uncertain settings are an everyday increasing fact in business, politics, environment, security, etc. Therefore, new methods, approximations and complementary visions of this study are greatly needed.

It is our sincere hope that this study serves as orientation, motivation, or a modest encouragement for new research. One first step has been already materialized, the conformation of the “Red Iberoamericana para la Competitividad, Innovación y Desarrollo” (REDCID) an international research group attached to the “Programa Iberoamericano de Ciencia y Tecnología” (CYTED) is the first step towards the diffusion and replication of the ideas proposed in the present work. We continuously seek for the next step with the sole objective of making contributions in the field of regional development, business sustainable development, decision making under uncertainty and ultimately the diffusion of knowledge that serves to the progress of society.

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Appendix

Statements presented in the survey: A fuzzy methodology for innovation management measurement.

"A fuzzy methodology for innovation management measurement ", Kybernetes, Vol. 46 Iss 1 pp. 50 – 66.

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Table 9.1. Statements presented in the survey: A fuzzy methodology for innovation management measurement.

Variable	Statement
1. Innovation strategy	1.1 The management develops a strategic planning of the business, takes in count the market, its competitors and new technologies. From this strategic planning resources for innovation are considered. A surplus budget for unplanned projects is also included.
	1.2 The management treats innovation as a business process, structured by projects, and allocates specific resources to carry out the innovation process permanently.
	1.3 The management feels involved and considers innovation as a source of competitiveness for the company. The attitude towards innovation is transmitted to its workers by its personal example, mission and values, ensuring that these are understood and shared.
	1.4 The management maintains and implements a plan for professional development through internal rotation between various functions and tasks. Activities that promote learning, especially teamwork are continuously stimulated.
	1.5 Errors and failure are fully accepted as an inherent element of innovation and are assumed within a diversified portfolio of projects.

2. Knowledge management	2.1 There is a formal methodology and tools aimed to detect, structure and assimilate new knowledge. It is embodied in a corporate knowledge base, easy and accessible to all staff. It is revised constantly. 2.2. The company technologically analyzes competitors' products. It is up to date in new legislation and new patents. The company has identified external sources of knowledge and uses this information to incorporate new technologies into their products. 2.3 The company has established a benchmarking mechanism that provides insight to production technologies and organizational models of the leading companies on the world. Regular meetings are held to discuss alternative technologies and new organizational models in order to test new processes and gain experience. 2.4 The company has formal methods for the generation of new ideas they are structured and permanent. Ideas for future projects are taken into consideration and the staff that proposes innovations is rewarded.
3. Project management	3.1 The company has a detailed procedure for product development. It is aimed to minimize 'time to market' costs. This allows stage planning, with specific budget and objectives. There is an established project control system; at the end of each stage the project is reviewed. 3.2 Each project has a leader with full authority and responsibility; a team of people from various functional areas (marketing, sales, design / engineering, purchasing, production and finance) is assembled and assigned to the project, the team works full-time in the assignment. 3.3 Industrial design is used on each project since the concept stage. This is used to improve the product, its functionality, the simplification of its components and viability for manufacturing. The design is a key element of differentiation of the company.

	3.4 The company applies advanced tools for project management. This has enabled a better interaction with customers and suppliers. The company is active in the renewal of these instruments, in the development of their own instruments and the implementation of best practices. 3.5 The company has formal agreements with customers and / or suppliers to participate in the generation of new products. Collaboration tools are reviewed and redesigned continuously. Customers and suppliers are strategic elements of the business new projects.
4. Portfolio management	4.1. A multidisciplinary working group, led by the management, meets regularly to discuss the new generated ideas. The creation of new concepts is planned in terms of their life cycle, considering ranges and generations. 4.2 The company has a formal procedure for the selection of concepts according to their strategy and its internal capabilities. Technical feasibility, expected benefits, quality, price, resources and time are continuously evaluated. Market studies are considered and the challenges of the new concept on the production and marketing processes are analyzed. 4.3 There is a systematic procedure to identify the degree that uncertain elements in the external environment of the company (global, politic, economic, demographic, and technologic) affect when selecting new projects to develop. 4.4 There is a systematic use of tools for new product development. It includes the selection of the best ideas and involves key partners in order to reduce the risk of failures. There is a continuous evaluation of incorporating and improving new product development tools.

5. Internal drivers	5.1. The company has structured the allocation of human resources for new projects programs. Such programs provide facilities for the optimum performance of team activities seeking to achieve innovation goals and objectives. 5.2 The company considers the factors of innovation as a strategic resource and has formal procedures for identifying, selecting and hiring innovative staff. 5.3 The company has specific tools to detect key competencies from the staff and periodically reconsider how to improve them. 5.4 The company systematically uses tools for the definition and control of projects. The company considers these tools increase the capacity to generate new concepts, allows selecting the best ideas and to involve key partners in order to reduce the risk of failures. 5.5 The company has a clear responsibility to its defined annual budget. There is a team of internal and external collaborators whose mission is to redefine and improve production processes in line with the budget and the strategy of the company.
6. Organization and structure	6.1. The company has specific mechanisms to reward creativity, brainstorming and innovative spirit. Working in multidisciplinary teams is a common practice. There is continued feedback from the management regarding suggestions from employees. 6.2 The company has a structured and formal organizational manual. It is continuously updated and maintains a stretch alignment with the strategic objectives of the company. 6.3 The company considers the organizational climate as a strategic indicator of the business. There are formal processes for measuring organizational climate, activities and actions to promote integration and good relations among the staff.

	6.4 The company has structured programs for the integration and development of teams aimed to process and product improvement. Such teams are formally recognized in the organizational structure of the company.
7. External drivers	7.1 The company has mechanisms to identify and measure the best commercial practices employed by competing firms and worldwide references. This information is followed in order to improve the marketing processes. 7.2 The generation of new concepts does not start from fixed marketing planning. The precise definition of the distribution channel, sales and post-sales services take place during the product development process, with the possibility of a rethink everything again. 7.3 The company has a wide vision of their products, integrating services, price, communication, distribution and sale. The company constantly re-designs its marketing strategy seeking to increase the value of their products. 7.4 The companies' marketing processes have been defined to systematically provide useful market information for the development of new products, new uses or applications, and customer suggestions. The company has set up customer attention lines and has system to manage complaints that allows the identification of improvements in existing products. 7.5 The company utilizes information technologies in their marketing processes to gain competitive advantage. Information systems allow a permanent contact with customers and manage logistics. The use of IT has changed the business model. The company is active in the renewal of these instruments, the development of their own instruments and the implementation of best practices.

Complementary research I - Citation Analysis of Fuzzy Research Journals[9]

Abstract

Since the publication of the article Fuzzy Sets in Information and Control, by Lofti A. Zadeh in 1965, thousands of applications and extensions have appeared following the multi-valued logic proposed. The consolidation of fuzzy methodologies and techniques has been captured in several high quality journals. A visualization of the influence generated not only by publications, but also for universities, countries and research areas can be obtained following citations trends of journals. The purpose of the present paper is to analyze, with the utilization of bibliometric tools, the general influence that eight journals with a strong focus on fuzzy science have had in the past 50 years.

Key words: Bibliometrics, Fuzzy Research, Web of Science

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Introduction

The Theory of Fuzzy Sets [1] has been recognized as an intuitive approach capable of generating new insights to ambiguous problems in a wide variety of fields [2-4]. The capacity of bounding sets that are not clear, well defined or subjective is the main characteristic that has given the fuzzy sets theory criticism, but also notoriety.

In the past 50 years, researchers from all around the world have been publishing many studies with applications in diverse fields of knowledge, establishing fuzzy sets as a well-known science, recognized and leaded by several professional associations, such as North American Fuzzy Information Processing Society (NAFIPS), International Fuzzy Systems Association (IFSA), or the Institute of Electrical and Electronics Engineers Computational Intelligence Society (IEEE CIS).

The evolving consolidation of the fuzzy sciences have been shaped in several international scientific journals e.g. Fuzzy Sets and Systems (1978), the Journal of Japan Society for Fuzzy Theory and Intelligent Informatics (1989); the IEEE Transactions on Fuzzy Systems (1993); the International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems (1993); the Journal of Intelligent & Fuzzy Systems (1993); the Fuzzy Economic Review (1995); the International Journal of Fuzzy Systems (1999); Fuzzy Optimization and Decision Making (2002); and the Iranian Journal of Fuzzy Systems (2003).

Bibliometric analysis is a research field that analyzes publications, citations and their sources [5]. The use of information technologies has allowed a flexible and dynamic scope of a scientific field. The structured gathered material shows a broader picture of the contributions that have shaped the area.

The objective of the present research is to generate a citation analysis utilizing bibliometric tools of eight journals with a strong focus on fuzzy research. Thus offering an initial picture on the question "*who is citing the journal?*" And how does that impact the conformation of research groups, leading universities around the world, top countries that explore fuzzy techniques and prolific research areas on each of the journals selected.

Methodology

The information gathered to build the research has as foundation the core collection of the Thomson & Reuters Web of Science database (WoS). The selection of the database responds to the need for articles published in high quality journals. Some other databases such as SCOPUS and Google Scholar are available, however this paper has focused on the WoS for being considered the more complete, neutral and rigorous database.

To develop the search we have used the name of each of the studied journals: 1) "Fuzzy Sets and Systems"; 2) "Expert Systems with Applications"; 3) "Information Sciences"; 4) "Knowledge-Based Systems"; 5) "International Journal of Approximate Reasoning"; 6) "Applied Soft Computing"; 7) "International Journal of Intelligent Systems"; and 8) "International Journal of Uncertainty Fuzziness and Knowledge-Based Systems". The search was conducted for the option "All years", which include studies made from 1864 until 2016. In order to include only research studies, a search filter was used, considering only articles, reviews, letters or notes. The search was carried out in January 2016. This study classifies the material using the approach found in Merigó et al. [5]. The information is structured in three main categories: Universities, Countries and Research Areas. The top 20 results are shown for each category. The studies for all the selected journals including the filters sum a total of 242,606 articles see Table 10.1 for a descriptive analysis of the results obtained.

Table 10.1. Total Articles Citing the Studied Journals

Journal	Total Studies
Fuzzy Sets and Systems	57,671
Expert Systems with Applications	59,764
Information Sciences	58,249
Knowledge-Based Systems	14,029
International Journal of Approximate Reasoning	13,258
Applied Soft Computing	19,939
International Journal of Intelligent Systems	11,469
International Journal of Uncertainty Fuzziness and Knowledge-Based Systems"	8,227

Source: Self-elaborated.

Results

The citation analysis is a useful tool to view the sources of influence of each journal [6]. In the present research we will focus on the 20 top influencers on the categories of University, Country and Research Areas.

In general, a total of 41,074 studies from 160 universities are displayed. The Islamic Azad University leads with 3,651 studies, next we find the University of Granada (3,494), the Chinese Academy of Sciences (2,482), the Hong Kong Polytechnic University (1,594), the Harbin Institute of Technology (1,590), the Northeastern University (1,576), the Shanghai Jiao Tong University (1,506), the Indian Institute of Technology (1,488), the University of Tehran (1,457) and the Polish Academy of Sciences (1,446).

A total of 244,350 studies from 26 different countries were retrieved, from them the Peoples Republic of China leads with a total of 68,600 studies, secondly the United States of America (23,880), following Taiwan (20,124), Spain (17,377), Iran (13, 548), India (12,364), England (9,644), Turkey (8,631), Canada (8,414) and France (7,886).

There are a total of 32 Research Areas comprising 379,946 studies. The most influential area is Computer Science (155,803), Engineering (84,900), Mathematics (31,110), Operations Research Management Science (25,563), Automation Control Systems (20,207), Business Economics (10,136), Telecommunications (5,782), Physics (4,358) and Environmental Sciences Ecology (4,355). The specific results for each journal are displayed in the next Tables 10.2 – 10.9.

Table 10.2. Results for Fuzzy Sets and Systems

R	University	TS	Country	TS	Research Area	TS
1	Univ Granada	793	PR China	14957	Computer Science	38060
2	Islamic Azad Univ	790	Taiwan	5332	Engineering	18634
3	Polish Acad Sci	574	USA	5278	Mathematics	12711
4	Harbin Inst Tech	489	Spain	3966	Automation Control Systems	6844
5	Northeastern Univ	458	Iran	3025	Operations Research Management	4993
6	Natl Chiao Tung Univ	458	India	2704	Business Economics	2226
7	Univ Alberta	453	France	2226	Robotics	1171
8	Natl Cheng Kung Univ	439	Canada	2105	Environmental Sciences Ecology	1135
9	Indian Inst Tech	424	Japan	1913	Science Technology Other Topics	1048
10	Natl Taiwan Univ Sci Tech	408	England	1912	Physics	948
11	Univ Ghent	394	Turkey	1864	Instruments Instrumentation	917
12	Hong Kong Polytech Univ	383	South Korea	1860	Mechanics	822
13	Univ Tehran	382	Italy	1823	Imaging Science Photographic	768
14	Univ Oviedo	381	Poland	1689	Telecommunications	753
15	City Univ Hong Kong	371	Germany	1288	Materials Science	663
16	Chinese Acad Sci	365	Australia	1168	Water Resources	603
17	Dalian Univ Tech	354	Czech R.	1032	Energy Fuels	475
18	Sichuan Univ	322	Belgium	943	Transportation	439
19	Southeast Univ	317	Brazil	848	Geology	412
20	Shanghai Jiao Tong Univ	307	Greece	730	Remote Sensing	322

Table 10.3. Results for Expert Systems with Applications

R	University	TS	Country	TS	Research Area	TS
1	Islamic Azad Univ	1195	PR China	16834	Computer Science	31595
2	Univ Tehran	742	Taiwan	6418	Engineering	27150
3	Chinese Acad Sci	696	USA	5053	Operations Research Management	10368
4	Hong Kong Polytech Univ	625	Iran	4294	Business Economics	4039
5	Natl Taiwan Univ Sci Tech	576	Spain	3513	Automation Control Systems	3865
6	Natl Cheng Kung Univ	570	India	3313	Mathematics	3233
7	Huazhong Univ Sci Tech	528	Turkey	2986	Materials Science	1717
8	Natl Chiao Tung Univ	520	South Korea	2480	Environmental Sciences Ecology	1679
9	Harbin Inst Tech	504	England	2452	Telecommunications	1676
10	Shanghai Jiao Tong Univ	489	Malaysia	1961	Energy Fuels	1495
11	Zhejiang Univ	459	Canada	1691	Science Technology Other Topics	1196
12	Univ Malaya	446	Australia	1664	Physics	1154
13	City Univ Hong Kong	439	France	1227	Mechanics	1142
14	Indian Inst Tech	401	Italy	1216	Instruments Instrumentation	1140
15	Univ Granada	400	Brazil	1140	Information Science Library Science	1102
16	Tsinghua Univ	384	Germany	887	Math Computational Biology	896
17	Northeastern Univ	379	Greece	861	Chemistry	857
18	Amirkabir Univ Tech	376	Japan	839	Robotics	819
19	Nanyang Tech Univ	374	Poland	824	Transportation	773
20	Dalian Univ Tech	359	Singapore	714	Medical Informatics	769

Table 10.4. Results for Information Sciences

R	University	TS	Country	TS	Research Area	TS
1	Chinese Acad Sci	772	PR China	17638	Computer Science	39576
2	Islamic Azad Univ	714	USA	6919	Engineering	17946
3	Univ Granada	711	Taiwan	4325	Mathematics	8381
4	Northeastern Univ	497	Spain	3563	Automation Control Systems	5191
5	Harbin Inst Tech	494	India	2987	Operations Research Management	4289
6	Huazhong Univ Sci Tech	477	Iran	2738	Telecommunications	1889
7	Polish Acad Sci	468	Canada	2219	Business Economics	1587
8	City Univ Hong Kong	449	England	2095	Science Technology Other Topics	1260
9	Dalian Univ Tech	412	Japan	1939	Physics	1243
10	Indian Inst Tech	393	South Korea	1902	Robotics	994
11	Shanghai Jiao Tong Univ	385	France	1881	Instruments Instrumentation	856
12	Tsinghua Univ	382	Turkey	1806	Mechanics	803
13	Zhejiang Univ	372	Italy	1661	Imaging Science Photographic	728
14	Natl Chiao Tung Univ	371	Australia	1602	Mathematical Computational Biology	722
15	Univ Alberta	368	Poland	1560	Materials Science	682
16	Nanyang Tech Univ	363	Germany	1177	Environmental Sciences Ecology	603
17	Xidian Univ	348	Malaysia	938	Optics	594
18	Hong Kong Polytech Univ	336	Brazil	915	Energy Fuels	467
19	Univ Elect Sci Tech China	333	Singapore	677	Chemistry	420
20	Southeast Univ	331	Belgium	652	Information Science Library Science	408

Table 10.5. Results for Knowledge-Based Systems

R	University	TS	Country	TS	Research Area	TS
1	Chinese Acad Sci	203	PR China	4662	Computer Science	9926
2	Univ Granada	160	USA	1402	Engineering	4317
3	Islamic Azad Univ	135	Spain	999	Operations Research Management	1480
4	Cent S Univ	124	Taiwan	934	Mathematics	771
5	Shanghai Jiao Tong Univ	120	England	865	Automation Control Systems	721
6	Huazhong Univ Sci Tech	116	Australia	521	Business Economics	713
7	Sichuan Univ	110	Iran	505	Telecommunications	330
8	Southeast Univ	106	India	481	Information Science Library Science	279
9	Hong Kong Polytech Univ	103	Canada	439	Science Technology Other Topics	273
10	Harbin Inst Tech	103	South Korea	421	Robotics	249
11	Zhejiang Univ	100	France	386	Materials Science	226
12	Xi An Jiao Tong Univ	92	Japan	348	Environmental Sciences Ecology	194
13	Tsinghua Univ	92	Malaysia	344	Education Educational Research	179
14	Univ Tehran	89	Turkey	340	Medical Informatics	172
15	Natl Chiao Tung Univ	89	Italy	334	Physics	170
16	Univ Malaya	87	Germany	291	Mathematical Computational Biology	164
17	Nanjing Univ Aeronaut	85	Poland	225	Psychology	141
18	Nanyang Tech Univ	80	Brazil	224	Imaging Science Photographic	134
19	City Univ Hong Kong	80	Singapore	207	Mechanics	129
20	Tongji Univ	78	Greece	207	Social Sciences Other Topics	126

Table 10.6. Results for International Journal of Approximate Reasoning

R	University	TS	Country	TS	Research Area	TS
1	Univ Granada	518	PR China	3178	Computer Science	10264
2	Univ Ghent	169	Spain	1585	Engineering	3433
3	Univ Oviedo	165	USA	1424	Mathematics	1879
4	Polish Acad Sci	158	France	911	Operations Research Management	956
5	Univ Jaen	151	England	747	Automation Control Systems	897
6	Univ Regina	133	Italy	638	Business Economics	326
7	Southeast Univ	132	Canada	622	Science Technology Other Topics	309
8	Chinese Acad Sci	115	Taiwan	550	Robotics	269
9	Univ Alberta	111	Poland	497	Environmental Sciences Ecology	188
10	Islamic Azad Univ	102	Iran	454	Imaging Science Photographic	183
11	Univ Tech Compiegne	97	India	364	Telecommunications	156
12	Univ Toulouse 3	92	Japan	357	Physics	144
13	Univ Ostrava	89	Germany	354	Mathematical Computational Biology	134
14	Univ Kansas	86	Belgium	291	Instruments Instrumentation	103
15	Sichuan Univ	76	Czech Republic	275	Medical Informatics	98
16	Xi An Jiao Tong Univ	73	Australia	275	Mechanics	90
17	Univ Manchester	73	Brazil	236	Water Resources	82
18	Southwest Jiaotong Univ	72	Turkey	225	Remote Sensing	80
19	Tsinghua Univ	71	South Korea	162	Materials Science	80
20	Univ Politecn Madrid	70	Netherlands	159	Energy Fuels	72

Table 10.7. Results for Applied Soft Computing

R	University	TS	Country	TS	Research Area	TS
1	Islamic Azad Univ	548	PR China	6080	Computer Science	11002
2	Indian Inst Tech	270	India	2006	Engineering	8310
3	Chinese Acad Sci	261	Iran	1974	Operations Research Management	1824
4	Univ Tehran	244	Taiwan	1483	Mathematics	1537
5	Amirkabir Univ Tech	204	Spain	1192	Automation Control Systems	1490
6	Huazhong Univ Sci Tech	200	USA	1179	Energy Fuels	636
7	Dalian Univ Tech	199	Turkey	973	Materials Science	615
8	Univ Granada	197	Malaysia	772	Mechanics	600
9	Iran Univ Sci Tech	194	England	730	Science Technology Other Topics	540
10	Univ Teknol Malaysia	174	Canada	557	Telecommunications	514
11	Nanyang Tech Univ	165	Australia	542	Physics	488
12	Natl Inst Tech	159	South Korea	422	Business Economics	479
13	Univ Malaya	153	Brazil	401	Mathematical Computational Biology	410
14	Natl Taiwan Univ Sci Tech	152	France	389	Instruments Instrumentation	404
15	Northeastern Univ	151	Italy	371	Robotics	365
16	Hong Kong Polytech Univ	147	Mexico	312	Environmental Sciences Ecology	358
17	Cent S Univ	141	Poland	305	Chemistry	310
18	Natl Cheng Kung Univ	133	Japan	296	Thermodynamics	287
19	Xi An Jiao Tong Univ	127	Greece	262	Water Resources	271
20	Shanghai Jiao Tong Univ	126	Singapore	228	Optics	189

Table 10.8. Results for International Journal of Intelligent Systems

R	University	TS	Country	TS	Research Area	TS
1	Univ Granada	470	PR China	2969	Computer Science	9066
2	Iona Coll	166	USA	1537	Engineering	3053
3	Univ Jaen	160	Spain	1482	Mathematics	1266
4	Polish Acad Sci	154	Italy	567	Op Research Management Science	942
5	Southeast Univ	146	England	563	Automation Control Systems	780
6	Univ Barcelona	103	Taiwan	559	Business Economics	406
7	Univ Ghent	101	France	527	Robotics	210
8	City Univ Hong Kong	98	Canada	486	Science Technology Other Topics	191
9	Northeastern Univ	91	Poland	392	Telecommunications	156
10	Sichuan Univ	89	Australia	311	Imaging Science Photographic Tech	135
11	Cent S Univ	86	Japan	303	Environmental Sciences Ecology	119
12	Pla Univ Sci Tech	81	Iran	275	Physics	116
13	Tsinghua Univ	80	India	270	Information Science Library Science	116
14	Islamic Azad Univ	80	Germany	267	Medical Informatics	91
15	Univ Manchester	79	Turkey	250	Psychology	89
16	Shanghai Jiao Tong Univ	79	Belgium	248	Mechanics	85
17	Univ Alberta	74	South Korea	188	Mathematical Computational Biology	84
18	Csic	74	Netherlands	150	Instruments Instrumentation	84
19	Chinese Univ Hong Kong	70	Czech R.	145	Neurosciences Neurology	78
20	Chinese Acad Sci	70	Brazil	132	Social Sciences Other Topics	72

Table 10.9. Results for International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems

R	University	TS	Country	TS	Research Area	TS
1	Univ Granada	245	PR China	2282	Computer Science	6314
2	Univ Ghent	127	USA	1088	Engineering	2057
3	Slovak Univ Tech Bratislava	100	Spain	1077	Mathematics	1332
4	Polish Acad Sci	92	Taiwan	523	Operations Research Management Science	711
5	Univ Jaen	91	Italy	363	Automation Control Systems	419
6	Hebei Univ	91	France	339	Business Economics	360
7	Univ Oviedo	90	Canada	295	Telecommunications	308
8	Islamic Azad Univ	87	Iran	283	Science Technology Other Topics	133
9	Natl Cheng Kung Univ	86	England	280	Mechanics	133
10	Tsinghua Univ	85	Australia	242	Physics	95
11	Southeast Univ	84	India	239	Information Science Library Science	92
12	Univ Barcelona	78	Poland	228	Robotics	86
13	Sichuan Univ	78	Belgium	213	Environmental Sciences Ecology	79
14	Chongqing Univ Arts Sci	70	Japan	211	Medical Informatics	76
15	Univ Publ Navarra	66	Germany	195	Acoustics	57
16	Univ Rovira Virgili	63	Turkey	187	Imaging Science Photographic Technology	53
17	Tianjin Univ	63	Czech R	186	Mathematical Computational Biology	50
18	Univ Roma La Sapienza	62	Slovakia	150	Transportation	47
19	Univ Ostrava	59	South Korea	107	Instruments Instrumentation	46
20	Iona Coll	56	Austria	104	Mathematical Methods In Social Sciences	45

A deep analysis of the results shows that from the category of universities for the 8 Journals, 3 organizations appeared in the first place: University of Granada (4), the Chinese Academy of Sciences (2) and the Islamic Azad University (2).

In the category of countries The Peoples Republic of China appeared in the first position of all journals. In the second place we find the United States of America (4), Taiwan (2), India (1) and Spain (1). In the third we find the United States of America (3), Spain (3), Taiwan (1) and Iran (1).

In the category of research areas Computer Science appeared in the first position of all journals. In the second place Engineering appeared in all eight journals. In the Third place Mathematics (5) and Operations Research Management Science (3).

Conclusion

A citation analysis of eight journals indexed in the WoS strongly related to fuzzy research was conducted utilizing bibliometric analysis tools. The results are structured in 3 categories: universities, countries and research areas. The top 20 categories with more studies are displayed in each of the 8 tables presented. A total of 242,606 studies were retrieved, each of them corresponds to a contribution citing the journals selected. Results convey that in the category of Universities 3 organizations appear in the first places of the journals: University of Granada, the Chinese Academy of Sciences and the Islamic Azad University. In the category of countries the Peoples Republic of China resulted first in all the selected journals. Finally in the category of Research Areas, Computer Science resulted as first in each of the journals. The present work is a first approach to an in depth categorization of the influencers of each of the selected journals. Future research is needed; firstly expand the categories in order to include authors and years, and secondly a cross-citation analysis in order to find trends between journals. Although there are several limitations to this study, this bibliometric analysis identifies the main trends in citations for the selected journals.

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Investigadores:

España:
Anna Maria Gil-Lafuente
Agustín Torres Martínez
Víctor Gerardo Alfaro García

Cuba:
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Irene García Rondón

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