Essays on Labor Markets and Macroeconomic Policy

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To my parents for their unconditional support throughout all these years

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Abstract

This thesis sheds light on several macroeconomic aspects of the labor market and economic policy. Chapter 1 analyzes whether the presence of human capital depreciation during unemployment calls for policy intervention. I argue that the latter is required because human capital depreciation during unemployment generates an externality in job creation. Chapter 2 looks at whether the prescription for conducting monetary policy changes once it is taken into account that workers' human capital depreciates during periods of unemployment. In a New Keynesian framework, I find that optimal monetary policy stays close to strict inflation targeting. Chapter 3 investigates how the effect of an increase in government spending on labor market outcomes depends on the strength of the short-run wealth effect on labor supply. I show that the role of the latter crucially depends on the degree of price and wage stickiness.

Resum

Aquesta tesi estudia diversos aspectes macroeconòmics del mercat laboral i la política econòmica. El capítol 1 analitza si la presència de depreciació del capital humà durant els períodes d'atur requereix una intervenció política. Sostinc que aquesta última és necessaria degut a que la depreciació del capital humà durant els períodes d'atur, genera una externalitat en la creació de llocs de treball. El capítol 2 analitza si la prescripció de certes polítiques monetàries canvia un cop es té en compte que el capital humà dels treballadors es deprecia durant els períodes d'atur. En un marc neokeynesià, mostro que la política monetària òptima es manté prop de l'objectiu d'inflació estricte. El capítol 3 estudia com l'efecte d'un augment de la despesa pública en el mercat de treball depèn de la força de l'efecte riquesa a curt termini sobre l'oferta de treball. Mostro que el paper d'aquest últim depèn fonamentalment del grau de rigidesa de preus i salaris.

Foreword

The 2007-2009 financial crisis has led to a deterioration of labor market outcomes in many countries around the globe. This has triggered a debate about whether or not policy intervention is required, and how effective specific policy measures are in improving labor market outcomes. The three self-contained chapters of this thesis contribute to this debate.

Chapter 1, “The Cost of Human Capital Depreciation during Unemployment”, analyzes whether the presence of human capital depreciation during unemployment calls for policy intervention. This issue seems of particular concern now that unemployment duration has increased in the Great Recession. I argue that loss of skill during unemployment generates an externality in job creation: firms ignore how their hiring decisions affect the skill composition of the unemployment pool, and hence the output produced by new hires. Overall, job creation is too low, but contrary to conventional wisdom less so in recessions than in booms. The larger share of job-seekers with eroded skills in the unemployment pool in recessions lowers the social cost of having a worker losing her skills because it decreases the expected difference in productivity between a new hire and a job-seeker with eroded skills. As a consequence, everything else equal, loss of skill during unemployment may warrant procyclical employment subsidies.

Chapter 2, “Optimal Monetary Policy in the Presence of Human Capital Depreciation during Unemployment”, is closely related to the first chapter. It looks at how the prescription for conducting monetary policy changes once it is taken into account that workers’ human capital depreciates during periods of unemployment. Human capital depreciation during unemployment is introduced into an otherwise standard New Keynesian model with search frictions in the labor market. Skill erosion has potential implications for optimal monetary policy because in its presence the flexible-price allocation is not constrained efficient. This is a consequence of a composition externality related to job creation: firms ignore how their hiring decisions affect the extent to which the unemployed workers’ skills erode, and hence the output that can be produced by new matches. Although optimal price inflation is no longer zero, strict inflation targeting is shown to stay close to the optimal policy .

Chapter 3, “A Note on Fiscal Stimulus and Labor Market Outcomes: The Role of Short-Run Wealth Effects and Wage Rigidity”, studies how the effect of an increase in government spending on labor market outcomes depends on the strength of the short-run wealth effect on labor supply. In a classical framework, the role of the short-run wealth effect in generating a positive employment response is crucial because it drives the increase in labor supply which induces this response. In a New Keynesian framework with sticky prices and wages, the short-run wealth effect plays a less important role because fiscal stimulus also increases labor demand. I find that higher degrees of wage rigidity lower the role of the short-run wealth effect in obtaining a given employment increase: more rigid wages reduce the importance of an increase in labor supply in order to limit the upward pressure on wages triggered by the increase in labor demand. Moreover, the extent to which a higher degree of wage rigidity leads to a larger increase on impact of employment in response to an increase in government spending crucially depends on the stance of monetary policy. When monetary policy is not accommodative, wage rigidity does little to increase the employment response. This in turn reinforces the importance of monetary policy in shaping the economy’s response to fiscal stimulus.

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Chapter 1

THE COST OF HUMAN CAPITAL DEPRECIATION DURING UNEMPLOYMENT

1.1 Introduction

"*...One concern we do have, of course, is the fact that more than 40 percent of the unemployed have been unemployed for six months or more. Those folks are either leaving the labor force or having their skills eroded. Although we haven't seen much sign of it yet, if that situation persists for much longer then that will reduce the human capital that is part of our growth process going forward.*" (Ben Bernanke, 2012)[1]

The Great Recession has brought back the specter of long-term unemployment. In the U.S. the average unemployment duration has increased

[1] Question and answer session of the Senate Banking Committee hearing on the 1st of March 2012 with Federal Reserve chairman Ben Bernanke testifying on monetary policy and the U.S. economy: http://www.reuters.com/article/2012/03/01/usa-fed-bernanke-idUSL2E8E13KI20120301.

from an average of around 15 weeks in the period 1960-2008 to an average of close to 40 weeks in 2012.[2] This increase has been of great concern to policy makers. One reason is the widely held belief that long unemployment spells lead to the depreciation of a worker's human capital. [3]

But does the presence of skill loss during unemployment call for policy intervention? This paper looks at this issue through the lens of an otherwise standard random search model with aggregate uncertainty in which loss of skill is introduced. Workers who had their skills eroded while being unemployed are less productive upon re-employment than workers whose skills were not affected. At the same time, I allow for learning-by-doing such that workers with eroded skills can regain their initial skill level while being employed.

In the presence of skill loss during unemployment, firms' hiring decisions do not only affect the unemployment rate but also the share of workers with eroded skills in the unemployment pool. Hiring influences workers' chance of finding a job, average unemployment duration, and thus the extent of skill erosion. For example, when firms hire less, unemployed workers have a smaller chance of finding a job, which increases their unemployment duration. Longer unemployment spells in turn raise the probability that their skills erode. As a result, a drop in hiring increases the relative share of job-seekers with eroded skills in the unemployment pool.

The skill composition of the unemployment pool determines how likely it is that job-seekers with or without eroded skills show up for job interviews. Thus, the pool's composition determines the average productivity of job candidates. Consequently, firms' hiring decisions, through their effect on job-seekers' skills, affect the output that can be generated by new matches.

Comparing hiring decisions in the laissez-faire economy and the constrained efficient economy reveals that skill erosion is a source of inef-

[2]Source: Bureau of Labor Statistics series LNS13008275.

[3]Empirical evidence for human capital depreciation is provided by e.g. Addison and Portugal (1989), Keane and Wolpin (1997), Gregory and Jukes (2001), and Neal (1995).

ficiency. This is the consequence of a composition externality related to job creation which arises in addition to the familiar congestion externality following from the search frictions. The composition externality arises because firms ignore how their hiring decisions affect the average skills of next period's job-seekers, and hence the output that can be generated by new matches. Firms neglect that through hiring they prevent workers from being unemployed and exposed to skill erosion. At the same time, firms do not take into account that by employing a worker, this worker keeps her skills or regains her skills, and hence that there is an additional job-seeker without eroded skills when the match separates. In other words, when skills erode during unemployment, there are gains from job creation which are not fully internalized.

I analyze if constrained efficiency can be attained in the absence of policy intervention. Given that I assume that wages are set every period through Nash bargaining, I examine, in the spirit of Hosios (1990), if there exists a parameter condition for workers' bargaining power which restores efficiency. I find this is not the case when aggregate uncertainty is present and workers' bargaining power is constant across states. Thus, policy intervention is required to restore constrained efficiency.

The optimal labor market policy which offsets the composition externality takes the form of a time-varying employment subsidy. This reflects that job creation in the laissez-faire economy is on average too low. Surprisingly, I find that the optimal subsidy is procyclical. Put differently, when skill erosion during unemployment is the only source of inefficiency employment should be subsidized less in recessions. This finding indicates that the composition externality matters more in booms than in recessions. The intuition behind that finding is the following. The magnitude of the composition externality hinges on the extent to which job creation affects the average skills of the unemployment pool, and hence the expected productivity of new hires. The impact of hiring on this expected productivity depends on the pool's composition. The larger the share of unemployed workers with eroded skills, the smaller the impact of having an additional unemployed worker with eroded skills in the pool. Thus, the larger share of unemployed workers with eroded skills in recessions explains why the composition externality matters less. Through the same mechanism, one can explain why the composition externality matters more during good times. In booms, the fraction of workers in the unemployment pool with eroded skills decreases. Therefore, the impact on the expected productivity of new hires of having an additional unemployed worker with eroded skills is larger. As a result, the social cost of letting a worker be unemployed and lose skills increases, which makes the composition externality more important.

Next, I look at how the presence of skill loss during unemployment changes job creation relative to an economy without skill loss. Job creation and hence labor market outcomes are expected to change relative to an economy where the unemployed are not exposed to skill erosion because its presence affects the workers' and the firms' problem. However, whether on average more or less jobs are created is not clear because of two opposing effects. On the one hand, the expected gain from job creation drops because the expected productivity of a new hire decreases relative to a world without skill loss. On the other hand, workers' outside option becomes worse when they face the possibility of losing some of their skills when being unemployed. This deterioration of their outside option leads to lower wages, which stimulates job creation. I find that the presence of skill loss can lower the average unemployment rate in the decentralized allocation. However, the unemployment rate is still too high from a social point of view because of the composition externality.

Finally, I seek to quantify the efficiency cost of human capital depreciation during unemployment. In particular, I look at the extent to which labor market outcomes should change to reach constrained efficiency. To calibrate the model to the U.S. economy I make use of its prediction that workers' wages are on average negatively affected by the length of their unemployment spells. Empirical evidence on the effect of unemployment duration on workers' wages is taken from the displacement literature. This literature investigates the effect of displacement on workers' earnings and wages.[4] The cost in terms of efficiency is considerable. When skill loss is the only source of inefficiency, restoring constrained efficiency entails a drop in the average unemployment rate of up to 1 percentage point.

The remainder of the paper is organized as follows. Section 1.2 discusses the related literature. Section 1.3 describes the model. Section 1.4 compares the job creation decision in the decentralized and the constrained efficient allocation. Next, section 1.5 explores whether constrained efficiency can be attained through the wage setting mechanism, and what the implications are for optimal labor market policy. Section 1.6 contains two exercises. The first exercise shows how labor market outcomes in the presence of skill loss change relative to an economy without skill loss. In the second exercise, the model is calibrated to the U.S. economy and it is analyzed what the cost of human capital depreciation is in terms of efficiency by looking at the extent to which labor market outcomes should change to attain constrained efficiency. Section 1.7 discusses an extension. Finally, section 1.8 concludes.

1.2 Related Literature

This paper relates to three main strands of the literature. First of all, it relates to the literature investigating how the presence of human capital depreciation during unemployment should affect the design of policy. The literature has focused in the first place on how policies providing insurance for risk-averse workers in an economy with incomplete markets are affected by the presence of skill loss. For example, Pavoni (2009) and Shimer and Werning (2006) have computed the optimal scheme for unemployment benefits in the presence of skill loss. Pavoni and Violante (2007) have derived an optimal welfare-to-work program in the presence of skill loss. Such a program consists of a mix of policy instruments targeted at the unemployed, including among others unemployment insur-

[4] See e.g. Addison and Portugal (1989), Couch and Placzek (2010), Jacobson et al. (1993), and Neal (1995).

ance and job search monitoring. Spinnewijn (2010) has analyzed how optimal training schemes for unemployed workers should be designed when workers' are exposed to both skill loss at the moment of displacement and during their unemployment spell.

My contribution to this strand of the literature lies primarily in its different focus. I focus on whether loss of skill affects the efficiency of aggregate labor market outcomes. I show that those outcomes are not constrained efficient in the presence of skill loss, and hence policy intervention is required. Moreover, I find that loss of skill during unemployment is an argument for procyclical employment subsidies.

Second, this paper relates to the literature looking at how skill erosion during unemployment affects labor market outcomes. The main findings in this literature are the following. Pissarides (1992) has shown that skill loss can be a potential explanation for the observed persistence of unemployment fluctuations. This effect has also been explored by Esteban-Pretel and Faraglia (2010) in a model with both labor market and nominal frictions. Furthermore, Ljungqvist and Sargent (1998) have found that the presence of skill loss, together with differences between welfare systems, is important to understand labor market outcomes in the U.S. versus Europe. Related work has been done by den Haan et al. (2005) and Ljungqvist and Sargent (2004, 2007, 2008). Moreover, Pissarides (1992) and Coles and Masters (2000) have shown in a framework without aggregate uncertainty that multiple equilibria can arise when unemployed workers are exposed to skill loss. Additionally, Coles and Masters (2000) provide the composition externality as an explanation for the existence of multiplicity, and have argued that those multiple equilibria are Pareto rankable.

My main focus differs from this strand of the literature, but I provide some additional insights into how loss of skills affects labor market outcomes. My paper's findings indicate that attention should be paid to the wage setting mechanism if one wants to understand how human capital depreciation during unemployment affects labor market outcomes. In particular, the influence of a worker's outside option on the wage is crucial. This is because a worker's outside option deteriorates when she faces the possibility of losing some of her human capital when being unemployed. This in turn leads to lower wages, which has a positive effect on job creation. When this positive effect is sufficiently strong, it outweighs the negative effect on job creation induced by part of the job candidates having eroded skills.[5] However, from a social point of view job creation is always too low in the laissez-faire economy because of the composition externality. Moreover, I show that taking into account that the unemployed are exposed to skill erosion does not only matter for understanding unemployment outcomes but also for understanding the welfare costs related to unemployment. Loss of skill alters those welfare costs because it gives rise to a composition externality, whose magnitude varies over the cycle.

Finally, this paper relates to other work showing that a composition externality can arise when the pool of searchers is heterogeneous. Burdett and Coles (1997, 1999) have shown that a sorting externality arises in an environment characterized by random search and two-sided heterogeneity. This externality arises because agents do not take into account how their decision to match with an agent of a particular type affects the matching possibilities of the remaining agents. Moreover, Shimer and Smith (2001) have shown that the decentralized-allocation is inefficient in an environment characterized by random search where heterogeneous agents have to decide about their search intensity. Their finding follows from workers who choose their search intensity ignoring how their search intensity affects the probability that matches of a certain type will be formed. Furthermore, Albrecht, Navarro and Vroman (2010) have shown that in a framework with random search and workers who are heterogeneous with respect to their market productivity, the decen-

[5]The importance of workers' outside option in the presence of skill loss during unemployment has also been pointed out in a recent paper by Ortego-Marti (2012). He shows that the presence of skill loss during unemployment generates more wage dispersion among otherwise identical workers in a random search model because the possibility of losing skills while being unemployed makes workers willing to accept lower wages to avoid long unemployment spells.

tralized allocation is no longer constrained efficient under the standard Hosios condition when workers' participation decision is endogeneous. Their result follows from workers not internalizing how their participation decision affects the average productivity of newly formed matches. Finally, Fernandez-Blanco and Preugschat (2011) have shown in a framework with directed search, worker heterogeneity, and imperfect information about the worker's type, that labor market outcomes are no longer constrained efficient because firms do not internalize how their hiring decisions affect the unemployment's pool composition in terms of worker types.

My paper contributes to this strand of the literature in the following aspects. First, I show that in a market characterized by random search the decentralized allocation is not constrained efficient in the presence of human capital depreciation during unemployment because the latter gives rise to a composition externality. This composition externality is driven by firms ignoring how their hiring decisions affect job-seekers' skills and hence the output that can be produced by other firms' newly formed matches. Moreover, I show that this finding hinges on both workers with and without eroded skills searching for jobs in the same market. When workers with and without eroded skills search for jobs in separate markets, with each of those markets characterized by random search, and firms choose in which market to post vacancies, the decentralized allocation is constrained efficient if the standard Hosios condition holds in each market. Finally, and in contrast to previous work, I explore the composition externality in an environment subject to aggregate shocks. Those shocks make the composition of the pool of searchers time-varying. This in turn allows me to analyze whether and how the externality's magnitude depends on the composition of the pool of searchers.

1.3 The Economy

This section outlines the model. It is an extension of a discrete-time search and matching model à la Diamond-Mortensen-Pissarides with aggregate uncertainty. In this framework unemployed workers face the risk of losing some of their human capital, making them less productive upon re-employment.[6] The longer the unemployment spell, the more likely a worker has eroded skills. At the same time the model allows for learning-by-doing such that workers can regain their human capital while being employed.[7]

1.3.1 Population and Technology

There is a continuum of infinitely-lived, risk-neutral workers on the unit interval. These workers maximize their expected discounted utility, which is defined over consumption and home production. Employed workers earn a wage w^i depending on their skills, whereas unemployed workers engage in home production which generates a value b. The latter can be thought of as the opportunity cost of working and is assumed to be the same for all workers.

Workers are heterogeneous in their skills because skill erosion during unemployment and learning-by-doing on the job makes workers' human capital depend on their employment history. To keep the analysis simple, workers' human capital can only take two values, and is either high (H) or low (L). A worker's skills determine her productivity: high-skilled workers have high productivity, whereas low-skilled workers have low productivity. The transition between skill types occurs as follows. In each period, an unemployed high-skilled worker becomes low-skilled with probability $l \in (0, 1]$. Thus the longer a worker's unemployment duration, the larger the chance that her human capital has depreciated. At the same time, when being low-skilled, she can regain her productivity while being employed through learning-by-doing. In each period, an employed low-skilled worker becomes high-skilled with probability $g \in (0, 1]$.

A large measure of risk-neutral, profit-maximizing firms employs workers. As is standard in the literature, each firm consists of a single-worker

[6] This framework abstracts from firm specific human capital. Therefore, workers only face the risk of losing some of their human capital while being unemployed and not when the match separates.

[7] In what follows I use the term "skill" and "human capital" interchangeably.

production unit. Its output depends on the worker's human capital and aggregate productivity A. The latter follows the process

$$log\left(A\right) = \rho_a log\left(A_{-1}\right) + \varepsilon \tag{1.1}$$

where ε is an *iid* shock. Given the production technology, output produced by matches with a high and low-skilled worker is determined by equations (1.2) and (1.3) respectively

$$y^H\left(A\right) = A \tag{1.2}$$

$$y^L\left(A\right) = \left(1 - \delta\right) A \tag{1.3}$$

where the skill level of a high-skilled worker is normalized to one, and that of a low-skilled worker is defined by $1 - \delta$. The parameter $\delta \in [0, 1)$ can be interpreted as the rate of human capital depreciation. When $\delta \in (0, 1)$ skill erosion is present, making some workers less productive upon re-employment.[8] When $\delta = 0$, the model boils down to the standard model.

1.3.2 Labor Market

The labor market is characterized by random search à la Diamond-Mortensen-Pissarides. I assume that all workers search in the same market. Thus when a firm opens a vacancy at cost $\kappa > 0$, both workers with and without eroded skills can apply for the job opening. Since a firm meets at most one worker at each round of interviews, which is a standard assumption in this class of models, all interviews lead to successful hiring as long as the match surplus is non-negative. In every period, the total number of interviews in the economy is determined by a matching function. This

[8] The interpretation that workers who have suffered from human capital depreciation during unemployment are less productive upon re-employment has also been used by Pissarides (1992). Alternatively, Coles and Masters (2000) and Esteban-Pretel and Faraglia (2010) assume that these workers are equally productive as workers without skill loss once a fixed training cost has been paid.

function is strictly increasing and concave in both arguments and displays constant returns to scale. It is given by

$$m(v, u) = Bv^{1-\xi}u^{\xi}$$

where B represents the efficiency of the matching process, $1 - \xi$ is the elasticity of vacancies where $\xi \in (0, 1)$, v is the total number of vacancies posted by firms, and u is the total number of job-seekers weighted by their search effectiveness. Because I assume that unemployment duration does not affect workers' search effectiveness, and normalizing search effectiveness to one, the relevant measure of job-seekers in the matching function is given by the total number of unemployed u. The latter is defined as the sum of high-skilled $\left(u^H\right)$ and low-skilled $\left(u^L\right)$ unemployed

$$u \equiv u^H + u^L \tag{1.4}$$

Labor market tightness is defined as $\theta\left(x\right) \equiv \frac{v(x)}{u(x)}$, where x denotes the state of the economy and is defined below. The probability for a firm posting a vacancy to meet a job-seeker is defined as

$$q(\theta\left(x\right)) \equiv \frac{m\left(v\left(x\right), u\left(x\right)\right)}{v\left(x\right)} = B\theta\left(x\right)^{-\xi} \tag{1.5}$$

where $q(\theta\left(x\right))$ is decreasing in labor market tightness. The probability that a job-seeker gets a job interview is given by

$$p\left(\theta\left(x\right)\right) \equiv \frac{m\left(v\left(x\right), u\left(x\right)\right)}{u\left(x\right)} = B\theta\left(x\right)^{1-\xi} \tag{1.6}$$

where $p(\theta\left(x\right))$ is increasing in labor market tightness. The job finding probability is the same for both worker types because unemployment duration has no effect on search effectiveness. When the match surplus is non-negative for both skill types, workers also have the same hiring probability.

Timing. At the beginning of the period, a shock to aggregate productivity A is realized. After observing the economy's state, firms post vacancies and hire workers. Next, production takes place using both the existing and newly hired workers. After production some workers' type changes: unemployed high-skilled workers become low-skilled with probability l, and employed low-skilled workers become high-skilled with probability g. Next exogenous separation takes place, and a fraction γ of the matches breaks up.[9]

Labor market flows and the economy's state. The unemployment pool's heterogeneity affects the economy's state x compared to the standard model. In addition to aggregate productivity A, the number of vacancies posted by firms also depends on the fraction of low-skilled job-seekers in the unemployment pool. The latter is given by

$$s(x) \equiv \frac{u^L(x)}{u(x)} \tag{1.7}$$

The number of high and low-skilled job-seekers evolve according to

$$u^H(x) = (1-l)\,\tilde{u}^H_{-1} + \gamma\left(n^H(x_{-1}) + gn^L_{-1}\right) \tag{1.8}$$

$$u^L(x) = \tilde{u}^L_{-1} + l\tilde{u}^H_{-1} + \gamma(1-g)\,n^L_{-1} \tag{1.9}$$

where $\tilde{u}^i$ are the job-seekers of type $i = \{H, L\}$ who remain unemployed after hiring takes place $\tilde{u}^i \equiv (1 - p(\theta(x)))\,u^i(x)$, and n^i is the number of workers of type i employed in a given period. Equation (1.8) shows that the high-skilled job-seekers are all previous period's unemployed high-skilled workers who have not lost their skills, and all the high-skilled workers who just got fired. The latter consists on the one hand of those who were operating in the previous period as high-skilled workers, and on the other hand of those who were low-skilled but regained their skills because of learning-by-doing. Similarly, equation (1.9) shows that the low-skilled job-seekers are last period's unemployed low-skilled workers and high-skilled workers who have lost some of their skills, and all the low-skilled workers who were employed last period but did not regain skills and just lost their job.

[9] The timing assumption in this model is standard in the business cycle literature, see e.g. Blanchard and Galí (2010). The difference compared to the standard DMP setting is that newly hired workers become productive immediately upon hiring.

High-skilled and low-skilled employment are given by

$$n^H(x) = (1-\gamma)\left[n^H(x_{-1}) + g n^L_{-1}\right] + p(\theta(x))\, u^H(x)$$

$$n^L = (1-\gamma)(1-g)\, n^L_{-1} + p(\theta(x))\, u^L(x)$$

So high-skilled employment is given by the high-skilled and low-skilled employees with regained skills who kept their job, and the high-skilled new hires. Similarly, the low-skilled employed are on the one hand those who did neither regain skills nor got fired, and on the other hand the newly hired low-skilled unemployed.

Given the labor market flows, keeping track of the fraction of low-skilled workers in the unemployment pool (equation (1.7)), implies keeping track of the distribution of worker types across employment states. However, when taking into account the definition of the total labor force, it can be seen that either the number of high or low-skilled unemployed after hiring takes place $(\tilde{u}^i)$ or the number of high or low-skilled employed (n^i) can be expressed as a function of the other three. Normalizing the total size of the labor force to one and abstracting from labor force participation decisions gives

$$1 = \tilde{u}^H + \tilde{u}^L + n^L + n^H(x)$$

As a result, workers and firms can keep track of the composition of the pool of job-seekers, by for example keeping track of $\tilde{u}^H, \tilde{u}^L$ and n^L. Therefore, the economy's state is given by $x = \left\{A, \tilde{u}^H_{-1}, \tilde{u}^L_{-1}, n^L_{-1}\right\}$.

In the following sections, I will focus on the special case where $g = l = 1$.[10] Thus a low-skilled worker's productivity is restored with probability 1 after having worked for one period, and a high-skilled worker's productivity deteriorates with probability 1 after having been out of work for one period. The latter implies that human capital depreciation during unemployment can only be avoided when a worker who loses her job in a given period finds a new one during that same period. The reason for

[10] Derivations for the generalized version of the model with $g \in (0,1]$ and $l \in (0,1]$ can be found in Appendix 1.9.6.

focusing on this specific case is because it allows me to derive analytical expressions which provide insights into this economy. Numerical analysis shows that the same insights hold in the general case.

Imposing the parameter restriction $g = l = 1$, the number of high and low-skilled job-seekers respectively is defined as

$$u^H(n_{-1}) = \gamma n_{-1} \tag{1.10}$$

$$u^L(n_{-1}) = 1 - n_{-1} \tag{1.11}$$

where n represents total employment, which is defined as $n \equiv n^H + n^L$ and evolves according to:

$$n = (1-\gamma)\, n_{-1} + p\left(\theta\left(x\right)\right)\left(1 - (1-\gamma)\, n_{-1}\right) \tag{1.12}$$

As can bee seen from equations (1.10) and (1.11), both the number of high and low-skilled job-seekers can be written as a function of previous period's employment, and hence so can be the fraction of low-skilled job-seekers in the unemployment pool. Consequently, the economy's state is now given by $x = \{A, n_{-1}\}$.

1.3.3 Firm's Problem

The firm's value of employing a worker of type $i = \{H, L\}$ is given by

$$J^i(x) \;\; = \;\; y^i(A) - w^i(x) + (1-\gamma)\,\beta E_x\left\{J^H(x')\right\} \tag{1.13}$$

where $\beta \in (0, 1)$ represents the discount factor. The firm's value of employing a worker depends on the generated output, the wage cost $w^i(x)$, where wages are set through Nash bargaining as discussed in section 1.3.5, and the continuation value of the match. The worker-firm pair keeps on producing with probability $1-\gamma$. Because $g = 1$ all low-skilled workers regain their productivity after having been employed for one period. As a result, the continuation value of the match is the value of employing a high-skilled worker.

The firm's value of posting a vacancy is given by

$$V(x) = -\kappa + q(\theta(x))\left[(1 - s(n_{-1}))\, J^H(x) + s(n_{-1})\, J^L(x)\right] + (1 - q(\theta(x)))\, \beta E_x\{V(x')\} \tag{1.14}$$

When imposing the free-entry condition $V(x) = 0$, equation (1.14) becomes

$$\frac{\kappa}{q(\theta(x))} = (1 - s(n_{-1}))\, J^H(x) + s(n_{-1})\, J^L(x) \tag{1.15}$$

This reflects that firms create jobs such that the expected hiring cost (LHS) equals the expected gain of hiring (RHS). The latter is a function of the unemployment pool's composition because the composition determines the probability that a job-seeker of a particular type shows up for the job interview.

Given free-entry, the firm's value of employing a worker can also be expressed as

$$J^i(x) \quad = \quad \frac{\kappa}{q(\theta(x))} + G^i(x) \tag{1.16}$$

where

$$G^i(x) \equiv \left(y^i(A) - w^i(x)\right) - \left(\bar{y}(x) - \bar{w}(x)\right) \tag{1.17}$$

and where $\bar{y}(x) \equiv (1 - s(n_{-1}))\, y^H(A) + s(n_{-1})\, y^L(A)$ represents the expected output of a new hire. Note that because of random matching it equals the weighted average of the output produced by each worker type. Each type's share in the unemployment pool is sufficient to determine this type's weight because all job-seekers have the same hiring probability; and $\bar{w}(x) \equiv (1 - s(n_{-1}))\, w^H(x) + s(n_{-1})\, w^L(x)$ is the expected wage cost of a new hire.

Relative to the standard model without skill loss an additional term (G^i) shows up in the firm's value of employing a worker (equation (1.16)). The intuition is straightforward. In the standard model, the value from employing a worker equals the expected hiring cost $\left(\frac{\kappa}{q(\theta(x))}\right)$ because an employee can be replaced at this cost. However, in the presence of worker heterogeneity this is no longer the case because a new hire is not necessarily of the same type. As a result, the value of employment contains the additional term G^i, capturing the gain from employing a worker of type i. This gain compares the output and wage of a given worker type to the output and wage of an average worker from the unemployment pool.

To provide some insights into this gain, expression (1.17), can be written as

$$G^H(x) = s(n_{-1})\left[J^H(x) - J^L(x)\right]$$

$$G^L(x) = -(1 - s(n_{-1}))\left[J^H(x) - J^L(x)\right]$$

First, whether the gain from employing a high-skilled worker is positive depends on the wage setting mechanism. This can be seen as follows: $J^H(x) - J^L(x) = \left(y^H(A) - y^L(A)\right) - \left(w^H(x) - w^L(x)\right)$.[11] A firm is only better off employing a high-skilled worker, i.e. $J^H(x) > J^L(x)$, when the higher productivity of a high-skilled worker is not fully offset by a wage increase, i.e. $\delta A > w^H(x) - w^L(x)$. Note that even though this implies that $G^L(x) < 0$, as long as $J^L(x) \geq 0$ firms are willing to hire the low-skilled. Second, the unemployment pool's composition also affects this benefit. This is because it is precisely a given worker type's fraction in the unemployment pool which determines how likely it is that you can replace an employee by someone of the same type. For example, when $G^H(x) > 0$ and $G^L(x) < 0$, an increase in the fraction of low-skilled searchers increases the benefits of employing a high-skilled worker because the probability that a new hire would also be high-skilled decreases. At the same time, the loss of being matched with a low-skilled worker decreases because the larger the share of low-skilled workers in the unemployment pool the more likely it would have been to be matched with a low-skilled worker.

[11] The difference between the firm's value of employing a high and a low-skilled worker only depends on the value generated during the first period of production because low-skilled workers regain their skills after one period, i.e. $g = 1$.

1.3.4 Worker's Problem

A worker of type i's value of being employed W^i is given by

$$W^i(x) = w^i(x) + \beta E_x \left\{ \begin{array}{l} (1-\gamma+\gamma p(\theta(x'))) W^H(x') \\ +\gamma(1-p(\theta(x'))) U(x') \end{array} \right\} \tag{1.18}$$

This value depends on the wage and the continuation value. The latter is made up of three parts. With probability $1-\gamma$ the match survives separation and the worker continues working at the same firm as a high-skilled worker. Also the low-skilled workers will now be high-skilled because of learning-by-doing and the probability of regaining skills being equal to one. With probability γ the match breaks up, and the worker finds a new job with probability $p(\theta)$. Given that the worker gets hired in the same period her skills are not eroded yet, and hence she continues producing as a high-skilled worker. If she does not get hired, she gets the value of being unemployed U. This value is given by

$$U(x) = b + \beta E_x \left\{ p(\theta(x')) W^L(x') + (1-p(\theta(x'))) U(x') \right\} \tag{1.19}$$

It depends on the value of home production b, and the probability of finding a job next period now as a low-skilled worker because of skill erosion during unemployment. Note that the value of being unemployed is the same for both worker types because by the time the high-skilled unemployed can start searching again for jobs they all have eroded skills.

Both value functions show that workers take into account that in the presence of skill erosion during unemployment and learning-by-doing their employment status affects their productivity. But whether workers without eroded skills are better off than workers with eroded skills depends on the wage setting mechanism. This is because the difference between the surplus from being employed as a high and low-skilled worker only depends on the wage difference. Thus workers with eroded skills are worse off when being matched only if they receive a lower wage, which can be seen as follows. Defining the surplus from being in a match as $X^i \equiv W^i(x) - U(x)$, worker type i' s surplus becomes

$$X^i(x) = w^i(x) - o(x) + (1-\gamma)\beta E_x \left\{ X^H(x') \right\} \tag{1.20}$$

where $o(x)$ represents a worker's outside option

$$o(x) \equiv b + \beta E_x \left\{ p(\theta(x')) \left[X^L(x') - \gamma X^H(x') \right] \right\} \quad (1.21)$$

The outside option reflects that a worker cannot engage in home production b and cannot search for another job. However, if she had not been employed, her skills would have deteriorated, and hence she would have been a low-skilled job-seeker. At the same time, the worker takes into account that being in a job guarantees that she keeps her productivity or regains her productivity. Thus, she keeps in mind that when the match separates she will be able to search as a high-skilled worker. Note that the outside option is the same for both worker types because $l = g = 1$.

1.3.5 Wages

Wages are renegotiated in every period. Following the literature, I assume generalized Nash bargaining between a worker and firm. Consequently, total match surplus is split between the worker and the firm so that each of them gets a fraction of the total match surplus given by their bargaining power parameter. Defining total match surplus as $M^i(x) \equiv X^i(x) + J^i(x)$, the surplus for a worker and a firm from being in a match is given by[12]

$$X^i = \eta M^i(x) \quad (1.22)$$

$$J^i(x) = (1 - \eta) M^i(x) \quad (1.23)$$

where η and $1-\eta$ measure respectively the worker's and firm's bargaining power.

The solution to the Nash bargaining problem leads to the following expression for the wage of a worker of type i

$$w^i(x) = \eta y^i(A) + (1 - \eta) o(x) \quad (1.24)$$

[12]Note that the surplus for a firm from being in a match equals the value of having a worker employed because free-entry drives the value of having a vacancy to zero.

Note that the wage difference is given by $w^H(x) - w^L(x) = \eta\delta A > 0$. So the wage difference is a function of the difference in output generated by each match.

As discussed in section 1.3.3 and 1.3.4, the wage setting mechanism plays a crucial role in determining both how employing a low-skilled worker affects firms, and how losing skills during unemployment affects workers. When both workers and firms have some bargaining power $\eta \in (0, 1)$, the following implications can be derived:

1. *Firms are strictly better off employing a high-skilled worker than employing a low-skilled worker.*

   This follows directly from combining equation (1.13) and (1.24)

   $$\begin{aligned} J^H(x) - J^L(x) &= \left(y^H(A) - y^L(A)\right) - \left(w^H(x) - w^L(x)\right) \\ &= (1-\eta)\,\delta A > 0 \end{aligned}$$

   Firms are better off employing a high-skilled worker because the output gains from employing a high-skilled instead of a low-skilled worker are not fully offset by a higher wage. This also implies that the benefits of employing a specific worker type (equation (1.17)) are given by $G^H = s(n_{-1})(1-\eta)\,\delta A > 0$ and $G^L = -(1 - s(n_{-1}))(1-\eta)\,\delta A < 0$.

2. *High-skilled workers are better off than low-skilled workers.*

   From equation (1.20) it results that the difference in terms of surplus from being employed as a high and a low-skilled worker only depends on the wage difference. Given that the wage of a high-skilled worker is strictly higher than that of a low-skilled worker when $\delta > 0$, it follows that the surplus from being employed as a high-skilled worker is strictly larger than the surplus from being employed as a low-skilled worker.

   $$X^H(x) - X^L(x) = w^H(x) - w^L(x) = \eta\delta A > 0 \qquad (1.25)$$

3. *The presence of skill loss during unemployment affects the wage of workers without eroded skills through its effect on their outside option.*

   In the presence of skill loss during unemployment a high-skilled worker takes into account that part of her outside option is searching for a job as a low-skilled worker (see equation (1.21)). Given that the surplus of being in a match as a high-skilled worker is higher than the surplus of being in a match as a low-skilled worker (see equation (1.25)), the high-skilled worker's outside option is affected by the presence of skill loss. Given that the wage depends on the outside option (see equation (1.24)), it follows that the high-skilled worker's wage will also be affected by the presence of skill loss during unemployment.

1.3.6 Equilibrium

In equilibrium, the surplus from a match with a worker of type i is given by

$$M^i(x) = y^i(A) - b + \beta E_x \left\{ \begin{array}{l} \left(1 - \gamma + \eta\gamma p\left(\theta\left(x'\right)\right)\right) M^H\left(x'\right) \\ -\eta p\left(\theta\left(x'\right)\right) M^L\left(x'\right) \end{array} \right\} \tag{1.26}$$

The above expression is obtained from combining firms' and workers' value functions (equations (1.13), (1.18) and (1.19) respectively) with the wage setting rule (equations (1.22) and (1.23)).

From equation (1.26) it follows that the surplus from a match with a low-skilled worker can be expressed as

$$M^L(x) = M^H(x) - \delta A \tag{1.27}$$

Consequently, by combining equation (1.26) for $i = H$ with equation (1.27), the surplus from a match with a high-skilled worker can be written as

$$M^H(x) = A - b + \beta E_x \left\{ \begin{array}{l} (1-\gamma)\left(1 - \eta p\left(\theta\left(x'\right)\right)\right) M^H(x') \\ +\eta p\left(\theta\left(x'\right)\right) \delta A' \end{array} \right\} \tag{1.28}$$

The equilibrium vacancy creation condition follows from combining equations (1.15), (1.23) and (1.27), and is given by

$$\frac{\kappa}{q\left(\theta\left(x\right)\right)}=\left(1-\eta\right)\left[M^{H}\left(x\right)-s\left(n_{-1}\right)\delta A\right] \tag{1.29}$$

where $s\left(n_{-1}\right)=\frac{\gamma n_{-1}}{1-(1-\gamma)n_{-1}}$.

Definition 1. *For a given state $x=\{A,n_{-1}\}$, this economy's equilibrium consists of a value for labor market tightness $\theta\left(x\right)$ which satisfies the vacancy creation condition* (*equation* (1.29))*, given the surplus from a match with a high-skilled worker* (*equation* (1.28))*, and taking into account the expressions for the job filling and finding probability* (*equations* (1.5) *and* (1.6)).

Given this period's state and the equilibrium value of labor market tightness, next period's state is determined by the law of motion for employment (equation (1.12)), and the law of motion for aggregate technology (equation (1.1)).

1.4 Skill Erosion as a Source of Inefficiency

In the previous section I have outlined the model and discussed how the presence of skill loss during unemployment affects the firms' and the workers' problem. I now turn to analyzing whether there is a cost in terms of efficiency.

When the unemployed are exposed to skill loss, and both workers with and without eroded skills search for jobs in the same labor market, a *composition effect* arises in addition to the standard congestion effect. The congestion effect follows from the presence of search frictions, and is well-understood in the literature. It refers to vacancy posting decisions affecting labor market tightness, and hence job filling and finding probabilities. The composition effect refers to today's hiring decisions affecting the output that can be generated by new matches through their influence on the skill composition of the unemployment pool. The pool's composition affects output because it determines the probability that a new match will be of a particular type.

Hiring affects the unemployment pool's composition through two channels. First, hiring affects the job finding probability, the average unemployment duration, and hence the extent to which job-seekers' skills erode. In particular, for the case where $g = l = 1$, when workers become unemployed and do not get rehired in the same period, their human capital will have depreciated by the time they face a new opportunity to find a job. Thus, hiring more workers today lowers the fraction of job-seekers with eroded skills in next period's unemployment pool. Second, hiring today implies that when this match separates, another worker of a given type will enter the unemployment pool. In particular, for the case where $g = l = 1$, all low-skilled workers will have regained their skills by the time they can become unemployed again because of learning-by-doing. As a result, hiring today implies that another high-skilled worker will be searching for a job when the match separates.

In what follows, I first solve the social planner's problem to understand how the presence of skill erosion influences job creation. Next, I compare job creation in the constrained efficient and the decentralized allocation to analyze if skill erosion during unemployment is a source of inefficiency. Finally, I discuss the role of having both workers with and without eroded skills searching for jobs in the same labor market.

1.4.1 Constrained Efficient Allocation

The social planner is subject to the same technological constraints, the same pattern of losing and regaining skills, and the same labor market frictions as in the decentralized economy. The social planner's problem consists of choosing the optimal amount of jobs to create such that the utility of the representative worker is maximized. Given workers' risk neutrality, this coincides with maximizing total output net of vacancy posting costs.

The planner's problem is given by

$$V^P(x) = \max_\theta \left[\begin{array}{l} An^H(x) + (1-\delta) An^L(x) - \kappa\theta(x)(1-(1-\gamma)n_{-1}) \\ +b(1-n) + \beta E_x \left\{ V^P(x') \right\} \end{array} \right] \tag{1.30}$$

subject to the process for aggregate technology (equation (1.1)), and the the law of motion for the endogenous state variable employment (equation (1.12)), where $n^H(x) = (1-\gamma)n_{-1} + B\theta(x)^{1-\xi}\gamma n_{-1}$ and $n^L(x) = B\theta(x)^{1-\xi}(1-n_{-1})$. Note that the relevant state for the social planner (x) is the same as in the decentralized allocation $x = \{A, n_{-1}\}$.[13]

The first order condition is

$$\frac{\kappa}{q(\theta(x))} = (1-\xi)\left[\bar{y}(x) - b + \beta E_x \left\{ \frac{\partial V^P(x')}{\partial n} \right\}\right] \tag{1.31}$$

The envelope condition for employment is

$$\frac{\partial V^P(x)}{\partial n_{-1}} = \left[\begin{array}{l} (1-\gamma+\gamma p(\theta(x)))A - p(\theta(x))(1-\delta)A + (1-\gamma)\kappa\theta(x) \\ -(1-\gamma)(1-p(\theta(x)))\left[b - \beta E_x \left\{ \frac{\partial V^P(x')}{\partial n} \right\}\right] \end{array} \right] \tag{1.32}$$

Combining the first order condition (equation (1.31)) and the envelope condition for employment (equation (1.32)) gives the following expres-

[13]This follows from the assumption that new hires become productive upon hiring in combination with the parameter restriction $l = g = 1$. As discussed in section 1.3.2, those conditions imply that the fraction of low-skilled searchers (s) can be expressed as a function of last period's employment. Moreover, those conditions imply that all employed workers will be high-skilled by the time the planner has to decide again about optimal job creation. As a result, keeping track of how many workers are employed is also sufficient to know the quality of the employed workers. If production in a given period were to take place with both last period's employed and last period's new hires, the social planner would also have to keep track of the quality of last period's new hires, since the low-skilled wouldn't have been able yet to regain their productivity. Therefore, even though the social planner needs to keep track of the total number of high-skilled and low-skilled employed workers in addition to the share of low-skilled job-seekers in the unemployment pool, this can still be done just be keeping track of last period's total employment.

sion for job creation in the constrained efficient allocation

$$\frac{\kappa}{q\left(\theta\left(x\right)\right)} = \left(1-\xi\right)\left[\bar{y}\left(x\right) - b + \beta E_x\left\{\Lambda^P\left(x'\right)\right\}\right] \tag{1.33}$$

where Λ^P represents the continuation value of the match and is given by

$$\Lambda^P\left(x\right) \equiv \Lambda_1^P\left(x\right) + \Lambda_2^P\left(x\right) + \Lambda_3^P\left(x\right) \tag{1.34}$$

where

$$\Lambda_1^P\left(x\right) \equiv \left(1-\gamma\right)\left(\frac{\kappa}{q\left(\theta\left(x\right)\right)\left(1-\xi\right)} + y^H\left(A\right) - \bar{y}\left(x\right)\right)$$

$$\Lambda_2^P\left(x\right) \equiv p\left(\theta\left(x\right)\right)\left[\begin{array}{c} \gamma\left(\frac{\kappa}{q(\theta(x))(1-\xi)} + y^H\left(A\right) - \bar{y}\left(x\right)\right) \\ -\left(\frac{\kappa}{q(\theta(x))(1-\xi)} + y^L\left(A\right) - \bar{y}\left(x\right)\right) \end{array}\right]$$

$$\Lambda_3^P\left(x\right) \equiv \left(1-\gamma\right)\left(1-\xi\right)p\left(\theta\left(x\right)\right)\left(\frac{\kappa}{q\left(\theta\left(x\right)\right)\left(1-\xi\right)}\right)$$

Job creation (equation (1.33)) is such that the expected hiring cost (LHS) equals the expected gain from job creation (RHS). As in the standard model, the expected hiring cost depends on the vacancy posting cost κ and the vacancy filling probability $q\left(\theta\right)$. The expected gain from job creation depends on the expected output produced by the new hire $\bar{y}$, the loss in home production b, and the continuation value of the match (equation (1.34)). Note that the expected gain from job creation is weighted by $1-\xi$ because the planner takes into account how posting an additional vacancy affects the vacancy filling probability.

The continuation value consists of three parts: the value when the match continues producing $\left(\Lambda_1^P\right)$, the worker's outside option $\left(\Lambda_2^P\right)$, and the congestion effect $\left(\Lambda_3^P\right)$. Even though the overall structure is the same as in the standard model, the presence of the skill loss alters the continuation value. I now discuss each part of the continuation value in detail to gain insight into how skill loss precisely influences the job creation decision.

The continuation value's first term $\left(\Lambda_1^P\right)$ captures that creating a job today guarantees that next period a match will be operating with a high-skilled worker if this match does not separate. The value of a match with a high-skilled worker is given by the savings in expected hiring costs[14], and a term representing the ouput benefit of employing a high-skilled worker. There is an output benefit related to employing a specific worker type because a new hire would not necessarily be of the same type.[15]The new hire would be an average job-seeker from the unemployment pool. Therefore, the expected output gain realized next period from hiring a worker today is defined as the output generated by this worker $\left(y^H\right)$ relative to the expected output of a new hire $(\bar{y})$.

The second term $\left(\Lambda_2^P\right)$ represents the worker's outside option and reflects that today's job creation affects next period's output through its influence on the composition of next period's unemployment pool, i.e. the composition effect. The planner takes into account that upon separation there will be a high-skilled worker searching for a job, following from employment allowing a worker to regain or keep her productivity. When this worker finds a new job in the same period, which happens with probability $p(\theta)$, a high-skilled match starts operating leading to an output gain $\left(y^H - \bar{y} > 0\right)$. At the same time, the planner also considers that if the worker had been unemployed, she would have lost skills or remained low-skilled. If this now low-skilled worker had found a job, which would have happened with probability $p(\theta)$, this worker would have produced y^L whereas if another worker had been hired, the expected output would have been $\bar{y}$, creating an output loss $\left(y^L - \bar{y} < 0\right)$. The prospect of skill loss when not being employed lowers the worker's outside option, and hence increases the continuation value of a match.

[14]The savings in expected hiring costs depend on the vacancy posting cost κ and the vacancy filling probability $q(\theta)$ weighted by $(1-\xi)$. The term $(1-\xi)$ shows up because the planner takes into account the effect on the vacancy filling probability that would be caused by posting an additional vacancy: $\frac{\partial q(\theta)}{\partial v} = -\xi \frac{q(\theta)}{v}$

[15]In the standard model with homogeneous workers the value of a match equals the savings in expected hiring costs because an employee can be replaced by an identical worker when paying this cost.

The third term $\left(\Lambda_3^P\right)$ reflects the positive congestion effect induced by having a job-seeker less when the match survives separation, making it easier for the other job seekers to get hired.[16] The value generated by this positive effect is expressed as the value of a match with an average worker, which is given by the expected hiring cost. This follows from a change in labor market tightness affecting each worker's hiring probability in exactly the same way because it is independent of her type. [17]

Thus, the continuation value shows that the presence of skill erosion during unemployment influences the job creation decision. The planner takes into account that today's job creation generates output gains in the next period through two channels: the effect on workers' skills in existing matches, reflected in Λ_1^P, and on job-seekers' skills, reflected in Λ_2^P.

The *magnitude of the composition effect* ultimately hinges on the extent to which today's job creation affects the expected productivity of new hires. Importantly, the impact of hiring on this expected productivity depends on the pool's composition. In particular, the larger the share of unemployed workers with eroded skills, the smaller the impact of having an additional unemployed worker with eroded skills in the pool. As can be seen from the continuation value, having another worker with eroded skills in the unemployment pool because of not hiring her generates an output effect given by the difference between the output that would have been produced by this worker and the expected output produced if another, random worker from the pool were to be hired. This output effect depends on the unemployment pool's composition, and hence so does the magnitude of the composition effect.

For example, an increase in the share of low-skilled workers in the unemployment pool affects the magnitude of the composition effect in two opposite ways. On the one hand, it lowers the expected productivity difference between a new hire and a low-skilled worker. Therefore,

[16]Note that $-\frac{\partial p(\theta)}{\delta u} = (1-\xi)\,\frac{p(\theta)}{u}$

[17]Note that the congestion effect following from today's job creation affecting today's labor market tightness is reflected by the expected surplus of a match with a new hire being weighted by the elasticity of vacancies in the matching function $(1-\xi)$.

it decreases the extent to which hiring affects the output of new matches by preventing workers from being unemployed and having their skills eroded. On the other hand, it increases the expected productivity difference between a new hire and a high-skilled worker. Therefore, it increases the extent to which hiring affects the output of new matches by having another high-skilled workers searching for a job when a match separates. The opposite holds for a decrease in the share of low-skilled workers in the unemployment pool.[18]

In conclusion, the social planner's problem shows that the presence of the composition effect arising in this economy influences the job creation decision because there are additional output gains related to job creation. However, the magnitude of this composition effect depends on the unemployment pool's composition. Given that this composition is state-dependent, so is the extent to which the composition effect influences job creation.

1.4.2 Constrained Efficient versus Decentralized Allocation

To detect whether skill erosion during unemployment is a source of inefficiency I compare the job creation decision in the decentralized and the constrained efficient allocation under the standard Hosios condition. This condition sets workers' bargaining power (η) equal to the elasticity of unemployment in the matching function (ξ). The reason for doing so is because in the absence of skill erosion the decentralized allocation is constrained efficient when the standard Hosios condition holds. This follows from the congestion externality being internalized when workers' bargaining power satisfies this parameter condition.

[18] The output difference between an average and a low-skilled unemployed worker is decreasing in the share of low-skilled job-seekers: $\frac{\partial\left[\bar{y}(x)-y^L(A)\right]}{\partial s(n_{-1})} = \frac{\partial[1-s(n_{-1})]\delta A}{\partial s(n_{-1})} < 0$. The output difference between a high-skilled and an average unemployed worker is increasing in the share of low-skilled job-seekers: $\frac{\partial\left[y^H(A)-\bar{y}(x)\right]}{\partial s(n_{-1})} = \frac{\partial s(n_{-1})\delta A}{\partial s(n_{-1})} > 0$

Proposition 1. *In the presence of skill erosion during unemployment, the decentralized allocation is no longer constrained efficient under the standard Hosios condition for workers' bargaining power $\eta = \xi$.*

Proof. Combining equations (1.28) and (1.29) gives the job creation equation in the decentralized allocation

$$\frac{\kappa}{q\left(\theta\left(x\right)\right)} = \left(1-\eta\right)\left[\bar{y}\left(x\right) - b + \beta E_x\left\{\Lambda^D\left(x'\right)\right\}\right] \tag{1.35}$$

where $\Lambda^D\left(x\right)$ represents the continuation value of the match and is given by

$$\Lambda^D\left(x\right) \equiv \Lambda_1^D\left(x\right) + \Lambda_2^D\left(x\right) + \Lambda_3^D\left(x\right) \tag{1.36}$$

where

$$\Lambda_1^D\left(x\right) \equiv \left(1-\gamma\right)\left(\frac{\kappa}{q\left(\theta\left(x\right)\right)\left(1-\eta\right)} + y^H\left(A\right) - \bar{y}\left(x\right)\right)$$

$$\Lambda_2^D\left(x\right) \equiv -\eta\left(1-\gamma\right)p\left(\theta\left(x\right)\right)\left(\frac{\kappa}{q\left(\theta\left(x\right)\right)\left(1-\eta\right)}\right)$$

$$\Lambda_3^D\left(x\right) \equiv \eta p\left(\theta\left(x\right)\right)\left[\gamma\left(y^H\left(A\right) - \bar{y}\left(x'\right)\right) + \left(\bar{y}\left(x\right) - y^L\left(A\right)\right)\right]$$

The job creation equation in the constrained efficient allocation is represented below in a slightly different way relative to the expression in section (1.4.1) to facilitate comparison with the job creation equation in the decentralized allocation.[19]

$$\frac{\kappa}{q\left(\theta\left(x\right)\right)} = \left(1-\xi\right)\left[\bar{y}\left(x\right) - b + \beta E_x\left\{\Lambda^P\left(x'\right)\right\}\right] \tag{1.37}$$

[19] The term reflecting the worker's outside option Λ_2^P and the term reflecting the congestion effect Λ_3^P of the continuation value of the match have been rearranged in the terms $\tilde{\Lambda}_2^P$ and $\tilde{\Lambda}_3^P$. This separates those terms which are in terms of hiring costs $\left(\tilde{\Lambda}_2^P\right)$ and those which are in terms of next period's output gains following from today's job creation affecting the skills of next period's job-seekers $\left(\tilde{\Lambda}_3^P\right)$

where the continuation value of the match is given by

$$\Lambda^{P}(x) \equiv \Lambda_{1}^{P}(x) + \tilde{\Lambda}_{2}^{P}(x) + \tilde{\Lambda}_{3}^{P}(x) \tag{1.38}$$

where

$$\Lambda_{1}^{P}(x) \equiv (1-\gamma)\left(\frac{\kappa}{q(\theta(x))(1-\xi)} + y^{H}(A) - \bar{y}(x)\right)$$

$$\tilde{\Lambda}_{2}^{P}(x) \equiv -\xi(1-\gamma)\, p(\theta(x))\left(\frac{\kappa}{q(\theta(x))(1-\xi)}\right)$$

$$\tilde{\Lambda}_{3}^{P}(x) \equiv p(\theta(x))\left[\gamma\left(y^{H}(A) - \bar{y}(x')\right) + \left(\bar{y}(x) - y^{L}(A)\right)\right]$$

By comparing equation (1.35) evaluated at $\eta = \xi$ and equation (1.37), it follows that the decentralized allocation does not replicate the constrained efficient allocation under the standard Hosios condition $\eta = \xi$. □

In the presence of skill erosion during unemployment the decentralized allocation is no longer constrained efficient under the standard Hosios condition because a *composition externality* arises which is not internalized by this condition. The composition externality refers to firms ignoring how their hiring decisions affect the skill composition of the unemployment pool, and hence the output that can be generated by other firms' newly formed matches. Comparing the term Λ_{3}^{D} and $\tilde{\Lambda}_{3}^{P}$ of the continuation values (equations (1.36) and (1.38) respectively) shows that the output gain from preventing a worker to lose skills $\left(\bar{y} - y^{L}\right)$, and in case of separation, the output gain from employment guaranteeing that a worker keeps her productivity or regains her productivity when she had eroded skills upon hiring $\left(y^{H} - \bar{y}\right)$, show up only to a fraction of the worker's bargaining power η in the decentralized allocation, whereas those output gains show up fully in the planner's allocation. This reflects that the social benefits of today's job creation are larger than the private ones.

The *magnitude of the composition externality* depends on the extent to which firms' hiring decisions affect the average skills of the unemployment pool, and hence the expected productivity of new hires. As discussed in the previous section, the impact of hiring on this expected productivity depends on the pool's composition. The larger the share of unemployed workers with eroded skills, the smaller the impact of having an additional unemployed worker with eroded skills in the pool. Given that the share of unemployed workers with eroded skills is state-dependent, so is the magnitude of the composition externality.

How the composition externality arises can be seen from the firms' job creation decision. The latter is obtained by combining equation (1.13) and (1.15)

$$\frac{\kappa}{q\left(\theta\left(x\right)\right)} = \bar{y}\left(x\right) - \bar{w}\left(x\right) + \left(1-\gamma\right)\beta E_x \left\{ \frac{\kappa}{q\left(\theta\left(x'\right)\right)} + G^H\left(x'\right) \right\} \tag{1.39}$$

The above expression shows that vacancy posting is such that the expected hiring cost (LHS) equals the expected output produced by a new hire, taking into account the expected wage cost and the continuation value (RHS). The latter shows that firms internalize how today's hiring decisions affect workers' productivity when the worker remains employed. However, it also shows that firms ignore their effect on workers' skills if the match with a worker separates or if a worker had never been hired by the firm in the first place. Those output effects are still partially accounted for in equilibrium (equation (1.35)) through the wage setting mechanism. Workers recognize that being in a job affects their skills, and hence their outside option. Therefore, as being discussed in section 1.3.5, it is reflected in the wage. Note, however, that even though those output effects are only partially internalized, the value of a match with a worker of a specific type in equilibrium is evaluated in the same way as in the planner's allocation for $\eta = \xi$. [20]

[20] This is because of the wage setting mechanism: the value of employment for the

The reason why the standard Hosios condition does not internalize both the congestion and the composition externality is the following. The social planner takes into account both the worker's outside option and the congestion effect following from having a job-seeker less if the match does not separate. The former lowers the continuation value because a matched worker cannot be employed elsewhere. The latter increases the continuation value because when a worker stays in the match, it is easier for the other searchers to find a job. What matters is the net effect. In the standard model without skill erosion during unemployment, where $\bar{y} = y^L = y^H$, the net effect can be expressed as a fraction ξ of the worker's outside option (net of the value of home production b). Therefore, the decentralized allocation is efficient when $\eta = \xi$, even though only the worker's outside option is considered up to a fraction η instead of both the outside option and the congestion effect. However, in the presence of skill erosion the net effect can no longer be expressed as a fraction ξ of the worker's outside option (net of the value of home production b). Consequently, for $\eta = \xi$ the net effect is not fully taken into account, and hence the decentralized allocation is not constrained efficient under the

firm is given by $J^i(x) = \frac{\kappa}{q(\theta(x))} + G^i(x)$ (see section 1.3.3), and the wage setting mechanism implies that $G^H = (1-\eta)\, s(n_{-1})\, \delta A = (1-\eta)\left(y^H(A) - \bar{y}(x)\right)$ and $G^L = -(1-\eta)\left(1 - s(n_{-1})\right)\delta A = (1-\eta)\left(y^L(A) - \bar{y}(x)\right)$ (see section 1.3.5). Given that $M^i(x) = \frac{J^i(x)}{(1-\eta)}$ (see expression 1.23), the value of a match with a worker of type i is just as in the social planner's allocation given by the sum of the expected hiring cost and the difference in output generated by this worker $\left(y^i\right)$ and an average worker from the unemployment pool $(\bar{y})$.

standard Hosios condition.[21]

1.4.3 The Role of a Heterogeneous Unemployment Pool

The heterogeneity of the unemployment pool plays a crucial role for the previous section's result. If workers with and without eroded skills were searching for jobs in different labor markets, the decentralized allocation would be constrained efficient as long as the standard Hosios condition would hold in each market.

Proposition 2. *When unemployed workers with and without eroded skills search for jobs in separate markets, with each of those markets characterized by random search, and firms can choose in which market to post vacancies, the decentralized allocation is constrained efficient as long as the standard Hosios condition holds in each market.*

Proof. See Appendix 1.9.2 □

Proposition 2 follows from the composition externality no longer being present when firms can decide which type of workers show up for job interviews. In this environment job creation still affects unemployed

[21] The net effect in the presence of skill erosion is given by the second and third term of expression (1.38)

$$\Upsilon(x) \equiv E_x \left\{ \begin{array}{l} -\xi(1-\gamma)\, p(x') \left(\frac{\kappa}{q(x')(1-\xi)}\right) \\ +\gamma p(x') \left(y^H(A') - \bar{y}(x')\right) + p(x') \left(\bar{y}(x') - y^L(A')\right) \end{array} \right\}$$

and the outside option net of the value of home production b is given by the second term in expression (1.34)

$$\tilde{o}(x) \equiv E_x \left\{ \begin{array}{l} -(1-\gamma)\, p(x') \left(\frac{\kappa}{q(x')(1-\xi)}\right) \\ +\gamma p(x') \left(y^H(A') - \bar{y}(x')\right) + p(x') \left(\bar{y}(x') - y^L(A')\right) \end{array} \right\}$$

Comparing both expressions shows that $\Upsilon(x) = \xi \tilde{o}(x)$ iff $\delta = 0$.

workers' skills, but it no longer affects how likely it is that job-seekers with or without eroded skills show up for job interviews. Thus, job creation no longer determines the average productivity of job candidates, and hence the output that can be generated by a new match. As a result, with segmented labor markets for workers with and without eroded skills the decentralized allocation is efficient when the standard Hosios condition holds in each market, i.e. the workers' bargaining power equals the elasticity of job-seekers in the respective labor market. However, given the absence of empirical evidence showing that workers with different unemployment durations search for jobs in separate labor markets, I consider the framework where the unemployment pool contains both workers with and without eroded skills the relevant one to focus on.

1.5 Attaining Constrained Efficiency

1.5.1 Wage Setting Mechanism

The decentralized allocation is no longer constrained efficient under the standard Hosios condition as shown in Proposition 1. However, just as in the absence of skill erosion, this condition internalizes the congestion externality.

Proposition 3. *In the presence of skill erosion during unemployment the standard Hosios condition $\eta = \xi$ internalizes the congestion externality when workers have the same hiring probability.*

Proof. See Appendix 1.9.3 □

The intuition behind Proposition 3 is the following. The same parameter condition internalizes the congestion externality because there is no interaction between the congestion effect and the unemployment pool's composition. When labor market tightness changes, the hiring probability for all workers is affected in exactly the same way because they all have the same hiring probability. Therefore, the effect of job creation on labor market tightness, and hence on the probability that a match starts producing, is independent of the unemployment pool's composition.

Next, I analyze if the standard Hosios condition can be modified such that both the congestion and the composition externality are internalized. Put differently, I examine whether there exists a parameter condition for the workers' bargaining power for which the decentralized allocation is constrained efficient.

Proposition 4. *In the presence of skill erosion during unemployment and aggregate shocks, there is no constant bargaining power η such that constrained efficiency is attained in all states of the world.*

Proof. By comparing equation (1.35) and (1.37), it follows that internalizing the composition externality requires a value of $\eta = 1$. At the same time, Proposition 3 showed that internalizing the congestion externality requires $\eta = \xi$. Because both conditions are mutually exclusive, there exists no longer a condition for the parameter η, for which the decentralized alloction replicates the constrained efficient allocation in all states of the world. □

Proposition 4 implies that some form of policy intervention is required to restore constrained efficiency. However, constrained efficiency can still be attained without policy intervention when workers' bargaining power is allowed to be state-dependent, i.e. $\eta(x)$. This can be explained as follows. The only decision which has to be made is how many vacancies to post. Therefore, the decentralized allocation is constrained efficient when the expected gain from job creation is the same as in the planner's allocation, i.e. the RHS of equation (1.35) and (1.37) are equalized. The latter depends on both the expected surplus of a new match and the share of this surplus that goes to the firm $(1 - \eta(x))$. From this it follows immediately that when the share is allowed to be state-dependent the constrained efficient allocation can be attained without policy intervention. Because even if the expected surplus of a new match differs across allocations the share that the firm receives can be adjusted such that the expected gain from job creation is the same in both allocations.[22]Given that in the literature the workers' bargaining power is considered being a constant parameter I do not explore this further, and turn to the policy implications.

1.5.2 Optimal Labor Market Policy

As discussed in the previous section, some form of policy intervention is required to restore constrained efficiency. In this section I analyze the implications for optimal labor market policy.[23] The policy maker is subject to the same technological constraints and labor market frictions as the planner. I assume throughout that the policy maker can implement a labor market policy through non-distorting revenue sources.

The optimal labor market policy takes the form of an employment subsidy. The derivation is described in Appendix 1.9.4. This subsidy is received in every period by firms when they employ a worker independently of the worker's type. Although this subsidy might be difficult to implement in practice, it sheds light on the nature of the composition externality. Note also that in the presence of an employment subsidy, workers' skills are still only affected by their employment history, leaving the trade-offs related to job creation unaffected.

Since the congestion externality is already well-understood in the literature, I focus on the case where that externality is fully internalized $(\eta = \xi)$. When only the composition externality remains, the optimal employment subsidy $\Phi(x)$ is given by

$$\Phi(x) = (1 - \xi)\, \beta E_x \{p(\theta(x'))\, \Omega(x')\} > 0 \tag{1.40}$$

[22]Note that for the same reason in the steady state there also exists a parameter condition for workers' bargaining power such that constrained efficiency is attained.

[23]See Laureys (2012b) for the implications for optimal monetary policy.

where

$$\Omega(x) \equiv \left(\bar{y}(x) - y^L(A)\right) + \gamma\left(y^H(A) - \bar{y}(x)\right) \tag{1.41}$$

First, expression (1.40) shows that employment should be subsidized to attain constrained efficiency. This reflects that not enough jobs are created in the laissez-faire economy, following from the expected social benefits of today's job creation being larger than the private ones.

Second, expression (1.40) shows that the optimal employment subsidy is state-dependent. This follows immediately from the magnitude of the composition externality varying with the economy's state, as discussed in section 1.4.2. In particular, the extent to which employment should be subsidized in a given period is a function of next period's expected economic situation. The reason is that the output gains caused by the impact of today's hiring decisions on the skill composition of the unemployment pool are only realized next period, and those gains depend on next period's expected economic situation. In particular, they are a function of both the expected fraction of low-skilled job-seekers and the expected level of technology.[24]

Third, expression (1.40) shows that the optimal subsidy is a function of the output gains caused by the impact of today's hiring decisions on the skill composition of the unemployment pool (Ω). The first term in expression (1.41) captures that hiring generates an output gain because it prevents a worker from being unemployed, and hence having an additional low-skilled job-seeker in the unemployment pool. The second term in expression (1.41) captures the output gain related to employment guaranteeing that there will be another high-skilled job-seeker when the match separates.

Finally, I expect the optimal subsidy to be decreasing in the share of workers with eroded skills in the unemployment pool because the output gains following from today's job creation affecting the skill composition

[24]The optimal subsidy also depends on the expected job finding probability. This is because ultimately today's job creation influences next period's output only to the extent that the unemployed workers get hired.

of the unemployment pool (Ω) are decreasing in it:

$$\frac{\partial\,\Omega\,(x)}{\partial s\,(n_{-1})} = \frac{\partial A\,(1-(1-\gamma)\,s\,(n_{-1}))}{\partial s\,(n_{-1})} < 0$$

Those gains are decreasing in the share of low-skilled job-seekers because the output gain from hiring and preventing workers from being unemployed and having their skills eroded has a weight of one, whereas the weight on the output gain from retraining equals the separation rate $\gamma \in (0, 1)$. As discussed in section 1.4.1, a larger share of job-seekers with eroded skills lowers the impact on output of hiring and preventing skill deterioration because it decreases the expected productivity difference between a new hire and a low-skilled worker.

1.6 Labor Market Outcomes

This section contains two different exercises. Section 1.6.2 sheds light on how the presence of skill loss changes job creation relative to an economy without skill loss, whereas in section 1.6.3 I seek to quantify the efficiency cost of skill loss during unemployment.

As discussed in section 1.3, the presence of skill loss affects the workers' and the firms' problem. Therefore, labor market outcomes are expected to change relative to an economy where the unemployed are not exposed to skill loss. However, whether on average more or less jobs will be created in the presence of skill loss is not clear because the equilibrium job creation equation (equation (1.35)) shows that there are two opposing effects at work. On the one hand, the expected gain from job creation drops because the expected output produced by a new hire decreases relative to a world without skill loss. On the other hand, there are additional expected output gains following from today's job creation affecting workers' skills. By looking at how job creation changes relative to an economy without skill loss, it can be understood which of the opposing effects dominates. This is the focus of section 1.6.2.

I perform this exercise not only for the decentralized but also for the constrained efficient allocation. Even though it is clear that job creation in the decentralized allocation is lower than in the constrained efficient allocation when the adequate policy is absent (see section 1.5), comparing labor market outcomes in both allocations provides an insight into whether the change in job creation in the decentralized allocation induced by the presence of skill loss goes in the same direction as the change in job creation in the constrained efficient allocation.

As discussed in section 1.4 and 1.5, loss of skill during unemployment generates an efficiency cost. This cost ultimately depends on the extent to which labor market outcomes should change to attain constrained efficiency, which in turn depends on the magnitude of the composition externality. To provide an insight into this cost, I perform a quantitative analysis in section 1.6.3. I calibrate the model to the U.S. economy, and look at the extent to which key labor market outcomes, such as the average unemployment rate and the job finding probability should change to attain constrained efficiency.

For both exercises I use the model's generalized version where workers lose and regain skills with some probability in every period, as outlined in section 1.3.1 and 1.3.2. For a description of the model, see Appendix 1.9.6. The model is solved numerically by taking a first-order approximation.[25]

[25] Alternatively, the model could be solved through value function iteration. The drawback of this method is the computational time following from the economy's state space being 4-dimensional. One of the advantages of solving this model with a non-linear solution method, however, is the possibility of exploring the importance of non-linearities. The latter are potentially important in an environment with skill loss as the shape of the economy's response to a given shock might depend on its size. This can be explained as follows. The initial response of the job finding probability depends on the size of the shock hitting the economy. As a result, the initial response of the average unemployment duration, and hence the change of the unemployment pool's composition also depends on it. The latter in turn affects the expected gains from job creation in future periods. Consequently, the shape of the economy's response to a shock might depend on the extent to which the shock hitting the economy affects the unemployment pool's composition. I leave the exploration of this channel for future research.

1.6.1 Calibration

The parameterization strategy is the following. The length of a period is one month. I use parameter values standard in the literature for the U.S. economy. I set the discount factor β to 0.996 implying an annual interest rate of 4%; I set the elasticity of unemployment in the matching function ξ to 0.5 following the evidence in Petrongolo and Pissarides (2001). I set $\eta = \xi = 0.5$ such that the congestion externality is internalized. The value of home production b is set to 0.71 following Hall and Milgrom (2008) and Pissarides (2009). I target the following long-run values, which correspond to the model's steady state values: a job finding probability of 35% following Fujita and Ramey (2012) and an unemployment rate of 5% following Blanchard and Galí (2010) where total unemployment is given by all those searchers who did not find a job $\tilde{u} \equiv (1 - p(\theta))\, u$. This implies an exogenous separation rate of $\gamma = \tilde{u} p(\theta) / ((1 - \tilde{u})(1 - p(\theta))) = 0.0283$, which lies in the range of values used in the literature.[26] I follow Pissarides (2009) and set $\theta = 0.72$, implying a value for matching efficiency $B = p(\theta)\, \theta^{\xi - 1} = 0.412$. Steady state aggregate productivity A is normalized to 1. Once the values of the parameters governing the skill loss process are determined, the value of the vacancy posting κ is obtained from the equilibrium conditions.

The parameters governing the skill loss process are δ, l, and g. The parameters δ and l determine the degree to which an unemployment spell erodes workers' skills: the human capital depreciation rate δ determines how many skills a high-skilled worker loses conditional upon losing, whereas the probability that a high-skilled worker will lose some of its skills in each period that she spends in unemployment depends on l. The parameter g determines how long it takes on average for a worker with eroded skills to regain those skills. Importantly, the difference in nature between the exercises performed in section 1.6.2 and 1.6.3 requires a different strategy for pinning down δ, l, and g.

Section 1.6.2 explores how the presence of skill erosion changes job

[26] For example, Fujita and Ramey (2012) target an average monthly separation rate of 0.02 , Pissarides (2009) one of 0.036, and Shimer (2005) one of 0.033

creation relative to an economy without skill erosion. This exercise needs a baseline economy. I choose the baseline economy to be an economy without skill loss, and proceed as follows. I use the parameterization strategy described above for the baseline economy. Thus, I set $\delta = 0$. Therefore, the value of the vacancy posting cost κ is obtained from the vacancy creation condition (equation (1.35)) when setting $\delta = 0$. This gives $\kappa = \frac{(1-\eta)p(\theta)(y-b)}{\theta(1-\beta(1-\gamma)(1-\eta p(\theta)))} = 0.3497$.

Next, to analyze how the presence of skill erosion during unemployment affects labor market outcomes relative to an economy without skill loss, I keep the parameter values β, ξ, η, b, γ, B, and κ fixed at their value for the baseline economy and vary only those related to the human capital process: δ, l, and g. I also keep the steady state value for aggregate technology A equal to one. However, the steady state values of unemployment $(\tilde{u})$, the job finding probability $(p\,(\theta))$ and labor market tightness (θ) will vary. Note that it is precisely this variation that will shed light on how job creation changes when skill loss is present. Finally, given that there is no quantitative aspect related to this exercise and direct empirical evidence on those three values is absent, I explore the effect of skill loss on labor market outcomes for an entire range of parameter values for δ, l and g. In particular, when I vary one of those parameters over a certain range, I keep the other parameters fixed at an arguably reasonable baseline value, namely $\delta = 0.3$, and $l = g = 0.167$, which implies that workers both lose and regain skills after six months on average.[27]

In section 1.6.3, in contrast, I parameterize the economy taking into account skill loss during unemployment. Therefore, the main difference relative to the parameterization strategy for section 1.6.2, is that now the steady state values of the unemployment rate $(\tilde{u} = 5\%)$, the job finding rate $(p\,(\theta) = 0.35)$, and the aggregate labor market tightness $(\theta = 0.72)$ are reached when skill loss is present. Thus, the value of the vacancy

[27]For those parameter values the Blanchard-Kahn (1980) conditions are satisfied which implies that the solution is stable and unique. Pissarides (1992) and Coles and Masters (2000) have shown, in a model without aggregate uncertainty, that multiple equilibria can arise in the presence of skill loss during unemployment. For a discussion about the possibility of multiple equilibria in this model see Appendix 1.9.5.

posting cost κ is now computed from the equilibirum conditions of an economy with skill loss. Given that direct empirical evidence on δ, l, and g is absent, I propose a strategy to obtain values for these parameters. This strategy is discussed in section 1.6.3.

1.6.2 Comparative Statics

Figure 1.1 shows that in the presence of skill loss the social planner lowers the average unemployment duration relative to a world without skill loss, i.e. for $\delta = 0$. The increase in job creation needed to attain this drop in the average unemployment duration also translates into a lower average unemployment rate and a smaller fraction of both low-skilled job-seekers and employed workers. This indicates that even though the expected gain from job creation drops because the expected output produced by a new hire decreases relative to a world without skill loss, this drop is more than offset by the expected output gains following from today's job creation affecting workers' skills. Put differently, the increase in the expected gains from job creation induced by the presence of skill loss outweigh the cost. Moreover, the higher δ, the lower the optimal average unemployment duration. The intuition behind this pattern is that an unemployment spell becomes more costly when the skill loss that can occur during that spell becomes more severe. As a result, it is optimal to reduce the average unemployment duration, and hence lower the chance that workers' skills erode.

The same pattern is present for all values of δ in the decentralized allocation although less pronounced. Thus, the presence of low-skilled job-seekers leads to more job creation, even though a firm can now be matched with a low-skilled worker which generates a lower value than being matched with a high-skilled worker (see section 1.3.5). This implies that the value of being matched with a high-skilled worker has to increase to the extent that it more than offsets the negative effect induced by having workers with eroded skills as potential new hires. It turns out that the value of being in a match with a high-skilled worker increases because high-skilled workers' wages decrease with δ. As discussed in section 1.3.5, and shown in the lower panel of figure 1.1, when workers face the risk of losing skills during unemployment, their outside option worsens. This lowers wages, making job creation more attractive for firms.[28] The composition externality, however, prevents job creation to reach the constrained efficient level.

Figure 1.2 shows the effect of the probability of skill loss l on labor market outcomes in the decentralized and the constrained efficient allocation relative to an economy without skill loss. Note that the latter would be attained in the limiting case where the time it takes on average for workers to lose skills goes to infinity. It can be seen that the social planner lowers average unemployment duration when skills erode faster on average. This follows directly from an unemployment spell of a given length being more costly when it is more likely that a high-skilled worker will lose her skills during that spell. The same pattern is also present in the laissez-faire economy, albeit to a lesser extent. As shown in the lower panel of figure 1.2, this can again be explained by high-skilled workers being willing to accept lower wages because of a deterioration of their outside option. Moreover, figure 1.2 shows that the share of low-skilled job-seekers increases when workers' skills erode faster on average. This implies that even though a lower average unemployment duration is desirable for higher values of l, it should not drop to the extent that it prevents an increase in the share of low-skilled job-seekers.

Figure 1.3 shows how labor market outcomes depend on the time it takes on average for low-skilled workers to regain their skills. The longer it takes on average for workers to regain their productivity, the lower average unemployment duration is in the social planner's allocation. This

[28]For this mechanism to be at work it is of course crucial that workers' bargaining power is sufficiently high. The lower workers' bargaining power, the smaller the effect of workers' outside option on wages, and hence the less likely it becomes that the positive effect on job creation induced by lower wages more than offsets the negative effect induced by having low-skilled workers searching for jobs. However, this exercise shows that when the workers' bargaining power satisfies the standard Hosios condition the positive effect is sufficiently strong to dominate the negative one.

reflects that the social cost of letting a worker lose her skills is larger when she regains her skills on average more slowly. More job creation also lowers the unemployment rate, and the fraction of low-skilled job-seekers. Despite the increase in job creation, low-skilled employment still increases as a fraction of total employment the longer it takes on average for workers to regain their skills. In the decentralized allocation labor market outcomes are similar because of the effect on a worker's outside option. As shown in the lower panel of figure 1.2, the faster a worker regains her skills on average, the smaller is the cost of skill loss in the first place, which leads to an improvement of the worker's outside option. Consequently, workers are less willing to accept a wage cut, making job creation less attractive for firms.

Next, I focus on how the presence of skill loss affects the shape of the economy's dynamic response to shocks relative to an economy without skill loss. Figures 1.4-1.6 show the response of the unemployment rate and the job finding probability to a persistent negative aggregate technology shock. The shock is such that aggregate technology decreases by 1% on impact relative to its steady state level and the autoregressive parameter ρ_a is set to 0.95. The job finding rate is represented in terms of relative deviation from its steady state, whereas the unemployment rate is represented in terms of absolute deviation from its steady state. It can be seen that in the presence of skill erosion the unemployment rate shows a humped shaped response to a negative technology shock similar to that of the economy without skill loss.[29] Moreover, the decentralized economy's response has a shape similar to that of the constrained efficient allocation.

[29] Pissarides (1992) has shown that skill loss can be a potential explanation for the observed persistence of unemployment fluctuations. The intuition behind this finding is that when the economy is hit by a negative shock, the increase in the average unemployment duration leads to a deterioration of the unemployment pool's quality. This in turn makes job creation even less attractive, leading to more persistent unemployment fluctuations. This effect has also been explored by Esteban-Pretel and Faraglia (2010) in a model with both labor market and nominal frictions. They find that even though the presence of skill loss improves the performance of the model in terms of the magnitude of the response of unemployment to a monetary shock, it does not improve the model in terms of explaining the observed persistence in unemployment.

To sum up, relative to an economy without skill erosion, steady state labor market outcomes in the laissez-faire economy change in a way similar to those in the constrained efficient allocation when skill loss is introduced: the more costly skill loss (i.e. the higher δ, the higher l, and the lower g) the shorter average unemployment duration and the lower the unemployment rate. Nevertheless, labor market outcomes differ across allocations because of the composition externality. Furthermore, both in the decentralized and the constrained efficient allocation, the presence of skill erosion does not seem to alter the shape of the economy's dynamic response to an aggregate technology shock much relative to an economy without skill erosion.

1.6.3 Cost of Skill Loss during Unemployment

Parameters of the Skill Loss Process

To analyze the cost of skill loss in terms of efficiency, the values of the parameters governing the skill loss process δ, l, and g need to be pinned down. Recall that the parameters δ and l determine the degree to which an unemployment spell erodes workers' skills: the human capital depreciation rate δ determines how many skills a high-skilled worker loses conditional upon losing, whereas the probability that a high-skilled worker will lose some of its skills in each period that she spends in unemployment depends on l. The parameter g determines how long it takes on average for a worker with eroded skills to regain those skills. Empirical evidence on those three values is absent. Therefore, I propose a strategy to pin down these parameter values by making use of two model predictions.

First, the model predicts that a productivity gap arises between an average new hire and an incumbent worker, where the latter is interpreted as being high-skilled. There is a productivity gap between new hires and incumbent workers because a fraction of the new hires has eroded skills. Over time, however, this productivity gap closes because those workers with eroded skills regain them. The training literature provides empirical evidence on those issues. Barron et al. (1997) look at how long it takes on average to become fully trained and qualified. However, the evidence depends on the type of survey used. Using employer based training survey data (the Employment Opportunity Pilot Project survey (EOPP) and the Small Business Administration survey) they find that it takes on average 20 weeks to become fully trained. However, when making use of household survey data (the PSID) they find that workers report that it takes on average 19.9 months to become fully trained. Using the EOPP, and eliminating the outliers, Cairó and Cajner (2011) find that it takes on average 13.4 weeks to become fully trained. [30]

I use the empirical evidence on how long it takes on average for a worker to be fully trained and qualified to pin down g. This provides a first insight even though the evidence refers to both the time to acquire firm specific skills and to regain those skills lost because of being unemployed. Given that the estimates for the average time it takes to become fully trained range from 13.4 weeks to 19.9 months, I set $g = 0.083$ such that it takes on average 12 months to regain skills, i.e. $1/g = 12$.

The second prediction of the model is that the presence of skill loss during unemployment translates into wage differences across workers because workers with eroded skills earn less then workers without eroded skills. Whether a worker has eroded skills depends on the length of their unemployment spell because it is precisely the spell's length which determines the chance that workers lose skills. As a result, the model predicts that on average the length of a worker's past unemployment spell lowers a worker's wage relative to the situation where this worker had not been unemployed in the first place.[31] The presence of learning-by-doing

[30]Cairó and Cajner (2011) also find that the productivity gap between an average new hire and an incumbent worker is 39.1%. However, the size of the productivity gap does not allow to pin down uniquely the values of δ and l. This is driven by the productivity of an average new hire depending both on the productivity gap between a worker with and without eroded skills and on the share of new hires with eroded skills.

[31]In this model the mere fact of displacement does not affect workers' skills. No skills are lost at the moment of becoming unemployed but only during the unemployment experience. Note that it is precisely this type of skill loss that gives rise to the composition externality. This follows from the externality's nature because it only arises when firms affect the unemployment pool's composition in terms of job-seekers' skills. If all the

in turn implies that the effect of an unemployment spell on workers' re-employment wages will faid away over time as those workers with eroded skills regain their skills.

The displacement literature has provided empirical evidence on the effect of job loss on both earnings and wages. It has been found that job loss significantly lowers both of them. In addition, those losses are documented to be long lasting. [32] The empirical evidence also shows that at least part of the wage loss is caused by the length of the unemployment spell, which is in line with the predictions of the model. For example, by using data from the Displaced Worker Survey, Addison and Portugal (1989) find that an increase in the unemployment duration by 10% reduces wages between 0.8% and 1.4%. Using the same data, Neal (1995) finds that an additional week of unemployment reduces the wages by 0.37%, implying a monthly rate of wage loss of 1.5%. By using data from the PSID, Ortego-Marti (2012) finds that an additional month of unemployment lowers wages by 1.2%.

The empirical evidence on wage losses due to displacement cannot be used to pin down uniquely the parameters δ and l. The degree to which the length of an unemployment spell lowers workers' wages is determined both by how long it takes on average before unemployment affects workers' skills (l) and if workers lose skills, how much they lose (δ). Hence,

skill loss, and hence wage loss, was driven by skill loss at the moment of displacement then there would be no composition externality. This because job-seekers' skills would be unaffected by their unemployment duration, and hence by firms' hiring decisions.

[32]For example, by using administrative data from Pennsylvania Jacobson et al. (1993) find earnings losses for workers displaced through mass layoff up to 40% in the period immediately following displacement. Six years after displacement those workers' earnings remain 25% lower than the earnings of workers who have not been displaced. Couch and Placzek (2010) obtain similar results by using administrative data from Connecticut. They find initial earnings losses of around 32%, while six years later those losses are reduced to around 14%. By using data from the Panel Study of Income Dynamics (PSID), Stevens (1997) finds that in the initial year of job loss, wages decrease by around 12%. Six years after displacement wages are still around 9% below their expected level. Gregory and Jukes (2001) report similar evidence obtained from British data. They show that an unemployment spell lowers wages by around 20% relative to the wage that would have been obtained without the unemployment experience.

a given effect of unemployment duration on wages can be caused by a range of combinations of the parameters l and δ. Thus, computing a precise value for the extent to which labor market outcomes would change if the composition externality was eliminated is not possible. Nevertheless, it is possible to provide a range of parameter combinations for δ and l in line with the empirical evidence. This in turn provides a first insight into the cost of skill loss during unemployment in terms of efficiency.

To obtain a range of parameter combinations for δ and l in line with the empirical evidence I proceed as follows. First, I pin down the time it takes on average for workers' skills to erode when they are unemployed. Given the lack of empirical evidence, I look at a range of 3 to 6 months. An average pace of skill loss of 3 months can be thought of as a natural upperbound because given the high turnover in the U.S. labor market it implies that an unemployment spell of an average length already leads to skill erosion. At the same time, 6 months can be thought of as a natural lowerbound because it implies that on average the skills of a long-term unemployed worker are eroded.[33] In order to be in line with the empirical evidence I set the value for the human capital depreciation rate δ such that I match the estimated effect of the length of an unemployment spell on wages. In particular, for a given average pace of skill loss $1/l$ and an initial guess for δ, I generate artificial employment histories and the according wage paths from the model.[34] Next, I use those panel data obtained from the model to regress wages on the length of the unemployment spell

$$ln\,(Wage) = \alpha * Length\,Spell + \varepsilon \tag{1.42}$$

I update the value of δ and repeat the same procedure until the regres-

[33]In the U.S. a worker is typically referred to as being long-term unemployed if this worker has been unemployed for at least 26 weeks.

[34]I generate an employment history and wage path for 10.000 worker by using the steady state values for the job finding probability and wages. For each worker I compute 400 periods and discard the first 150 periods to avoid an effect from the initial type and employment status of the worker. In the model, those workers who get fired and immediately rehired in the same period are not exposed to skill loss. Therefore, I compute the length of a given unemployment spell as the number of periods where the unemployed worker was exposed to skill loss.

sion coefficient equals that of the empirical finding, i.e. $\hat{\alpha} = -0.013$. Note that in the empirical literature the regression (equation (1.42)) contains additional control variables. Given that in this model workers are homogenous aside from their skill level determined by their unemployment duration, no additional controls are added. Moreover, no time fixed effects are added because the model is time-stationary.

Finally, the process for aggregate technology is set such as to match the empirical evidence on the volatility and the autocorrelation of quarterly U.S. labor productivity.[35] This implies a value for the autoregressive parameter for technology $\rho_a = 0.984$ and a standard deviation of the innovations to technology of $\sigma_a = 0.0050$. Note, however, that another process for aggregate technology could be chosen since the goal of this exercise is not to see how well the model performs in replicating the patterns observed in the data.

Findings

Table 1.1 denotes the average labor market outcomes in the decentralized versus the constrained efficient allocation for different values of the average pace of skill loss $(1/l)$ and the corresponding rate of human capital depreciation (δ). The fraction of skills that workers lose while being unemployed is relative small, ranging between 14 and 17%. Despite this relatively small loss, however, the cost in terms of efficiency generated by not fully taking into account how job creation can prevent this skill loss is considerable. In addition, the cost is higher the faster unemployed workers lose their skills on average. Looking at both the upper and lowerbound of the pace of skill loss, the unemployment rate should be between 0.92

[35] Quarterly data for seasonally-adjusted real output per person in the non-form business sector is obtained from the Bureau of Labor Statistics series PRS85006163. I restrict the sample to data in the period 1975-2005 to be in line with the period for the empirical evidence regarding the job finding probability provided by Fujita and Ramey (2012). After taking logs and detrending the data by using an HP-filter with smoothing parameter 10^5 as in Shimer (2005), the values for the standard deviation and the quarterly autocorrelation are 0.017 and 0.882 respectively.

Time for skills to erode on average in months $(1/l)$		3	4	5	6
Human capital depreciation rate (δ)		0.14	0.15	0.16	0.17
		Mean (%)			
Unemployment rate $(\tilde{u})$	D.	5.00			
	C.E.	4.08	4.22	4.32	4.40
Job finding probability $(p(\theta))$	D.	35.00			
	C.E.	39.96	39.15	38.54	38.09
Share low-skilled job-seekers (s)	D.	44.81	37.88	32.78	28.94
	C.E.	39.65	33.79	29.52	26.28
Share low-skilled employment (n^L/n)	D.	11.65	9.85	8.52	7.53
	C.E.	10.31	8.79	7.67	6.83

Table 1.1: The cost of skill loss during unemployment

and 0.60 percentage points lower to reach constrained efficiency. This would require an increase in the average job finding probability in the range of 14.17 to 8.83%, implying a drop in the average unemployment duration of between 1.5 and 1.1 weeks. In the constrained efficient allocation the share of low-skilled job-seekers would be between 13.01 and 10.12% lower relative to the share in the laissez-faire economy. Similarly, the fraction of low-skilled employed workers would decrease by between 13.00 and 10.25%. [36]

As discussed in section 1.5.2, constrained efficiency is attained when the adequate employment subsidy is implemented. Table 1.2 shows some of the properties of this subsidy. First of all, to get a better idea about the magnitude of the average optimal employment subsidy, the subsidy is expressed as a fraction of the average output per worker, which is denoted by $\tilde{\Phi}$. This shows that the optimal subsidy ranges on average between

[36] The vacancy posting cost κ varies with the values for the parameters governing the skill loss process because, as explained in section 1.6.3, the value for κ is obtained from the equilibrium conditions of the model. On average the resources spent on vacancy posting $\left(\frac{\kappa v}{A(n^H+(1-\delta)n^L)}\right)$ lies in the range of $2.67 - 2.81\%$ of total output.

8.19 and 6.00% of average ouput per worker. Second, as discussed in section 1.5.2, the optimal subsidy is time-varying. Looking at the standard deviation of the optimal subsidy relative to unemployment shows that the optimal subsidy becomes relatively less volatile the longer it takes for workers' human capital to erode. Longer periods for workers' skills to erode also imply lower volatility in the share of low-skilled job-seekers in the unemployment pool.

In section 1.5.2 I have argued that the optimal subsidy is expected to be decreasing in the share of workers with eroded skills in the unemployment pool. Simulating the model, shows that this leads the optimal employment subsidy to have a strong positive correlation with aggregate productivity, a strong negative correlation with the unemployment rate, and a strong negative correlation with the fraction of low-skilled searchers in the unemployment pool. This shows that the optimal employment subsidy is *procyclical*. The procyclicality of the subsidy reflects that the composition externality matters less in recessions than in booms. This can be explained as follows. The larger share of job-seekers with eroded skills in recessions lowers the social cost of letting a worker lose her skills because it decreases the expected productivity difference between a new hire and a job-seeker with eroded skills.[37] Put differently, the impact of letting a worker lose her skills on the average skills of the unemployment pool is smaller in recessions. As a result, in recessions firms' hiring decisions have a smaller impact on the average skills of the unemployment pool, and hence on the expected productivity of new hires. This in turn drives the procyclicality of the employment subsidy.

The labor market dynamics in the decentralized versus the constrained

[37]Note that this finding does not hinge on the source of economic fluctuations. It is important to point out, however, that when aggregate technology shocks are the driving force behind economic fluctuations, the argument for procyclical employment subsidies is reinforced. This is because, for a given skill composition of the unemployment pool, the expected productivity difference between a new hire and a worker with eroded skills is increasing in aggregate technology. Therefore, if economic fluctuations are driven by shocks in aggregate technology, there will be another channel, in addition to the unemployment pool's composition, through which the expected productivity difference between a new hire and a job-seeker with eroded skills decreases during recessions.

Time for skills to erode on average in months $(1/l)$		3	4	5	6
Human capital depreciation rate (δ)		0.14	0.15	0.16	0.17
Unemployment rate $(\tilde{u})$	mean (%)	4.08	4.22	4.32	4.40
Optimal employment subsidy (Φ)	mean	0.0808	0.0703	0.0619	0.0553
	$\tilde{\Phi}$ (%)	8.19	7.12	6.27	6.00
	vol. rel. to $\tilde{u}$	1.8480	1.3252	0.9917	0.7715
	corr. with A	0.9946	0.9969	0.9984	0.9992
	corr. with $\tilde{u}$	−0.9919	−0.9891	−0.9857	−0.9819
	corr. with s	−0.9394	−0.9245	−0.9101	−0.8958
Share of low-skilled job-seekers (s)	vol. rel to $\tilde{u}$	5.7378	5.1596	4.6600	4.2448

Note: $\tilde{\Phi}$ denotes the subsidy as a fraction of the average output per worker. The first-order approximation of the model has been simulated for 301.000 periods and the first 1.000 observations have been eliminated. The volatilities are defined as the standard deviation of the quarterly average of the monthly data, which has been detrended by using an HP-filter with smoothing parameter 10^5.

Table 1.2: The optimal employment subsidy

efficient allocation are shown in figure 1.7. The simulated series of the labor market outcomes reveal that the difference between labor market outcomes in both allocations is in the first place driven by differences in the mean.

In what has been discussed so far, the congestion externality has been fully internalized. This naturally raises the question how big the cost of skill loss during unemployment would be if in addition to the composition externality the congestion externality was present. Therefore, I analyze how the previous findings depend on the relation between the workers' bargaining power (η) and the elasticity of unemployment in the matching function (ξ).[38] Results are reported in table 1.3. First of all, I look at the

[38]Changes in the parameters ξ and η also imply a variation in the vacancy posting costs κ in order to attain the same targets for the decentralized allocation. For the first and second column of ξ and η, the total resources spent on recruitment vary between 0.7 and 1.12% of total output. In the last column however, total resources spent on recruitment are as much as 5.81%. This is driven by a sharp increase in κ which can be explained by

importance of the value of ξ while the Hosios condition still holds. Petrolongo and Pissarides (2001) have estimated a range of values for ξ. When ξ is set to a higher value, namely 0.72, the drop in the average unemployment rate needed to reach constrained efficiency is only between 0.17 and 0.12 percentage points.[39] This is not surprising since, as discussed in section 1.4.2, the expected gains from output following from job-seekers' skills being affected by their employment status is taken into account up to a fraction of the workers' bargaining power η. Therefore, the higher the workers' bargaining power, the closer average labor market outcomes are to what they would be if the composition externality was fully offset.

Second, to understand how the presence of the congestion externality affects the results two cases need to be considered. In the first case workers' bargaining power is higher than its optimal value implied by the Hosios condition, i.e. $\eta > \xi$. As a result, average job creation is too low even in the absence of skill loss during unemployment. To reach constrained efficiency the average unemployment rate should be 2.87 percentage points lower. The presence of skill loss, and hence the presence of the composition externality reinforce this pattern. Optimal unemployment is an additional 0.99 to 0.53 percentage points lower relative to the case without skill loss. In the second case workers' bargaining power is lower than its optimal value implied by the Hosios condition, i.e. $\eta < \xi$. This implies that in the absence of skill loss, too many jobs would be created, implying that the average unemployment rate should increase by 3.59 percentage points to reach constrained efficiency. The presence of skill loss only partially offsets this increases needed for the unemployment rate to be at its constrained efficient level. Optimal unemployment is between 0.84 and 0.69 percentage points lower relative to the case where only the congestion externality is present.

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vacancy posting having to be more costly to support an average unemployment rate of 5% when vacancy posting becomes more attractive because of the low bargaining power for workers.

[39]This value has been used by Shimer (2005).

		$\eta = \xi$		$\eta > \xi$			$\eta < \xi$		
Elasticity of unemployment in m.f. (ξ)		0.72			0.5			0.5	
Bargaining power workers (η)		0.72			0.72			0.3	
Time for skills to erode on av. in months $(1/l)$		3	6	/	3	6	/	3	6
Human capital depreciation rate (δ)		0.11	0.14	0	0.11	0.14	0	0.21	0.23
Congestion externality		no	no	yes	yes	yes	yes	yes	yes
Composition externality		yes	yes	no	yes	yes	no	yes	yes
Unemployment rate $(\tilde{u})$ %	D.	5.00			5.00			5.00	
	C.E.	4.83	4.88	2.13	1.14	1.60	8.59	7.75	7.90
Job finding probability $(p(\theta))$ %	D.	35.00			35.00			35.00	
	C.E.	35.85	35.58	56.48	71.06	63.61	23.18	25.24	24.84
Share low-skilled job-seekers (s) %	D.	44.81	28.94	/	50.26	28.94	/	44.81	28.94
	C.E.	43.90	28.42	/	15.11	11.15	/	56.43	39.88
Share low-skilled employment (n^L/n) %	D.	11.65	7.53	/	13.07	7.53	/	11.65	7.53
	C.E.	11.41	7.39	/	3.93	28.98	/	14.67	10.37

Table 1.3: The role of ξ and η

These results suggest that the efficiency cost generated by the congestion externality outweighs the cost generated by the composition externality because the former requires a larger change in the average unemployment rate to attain constrained efficiency than the latter. Moreover, whether unemployment should increase or decrease when both the congestion and the composition externality are present, depends in the first place on whether the congestion externality leads to too much or not enough job creation. Nevertheless, the presence of skill loss calls for a drop in the optimal unemployment rate relative to the case without skill loss.

1.7 Extension: Effect of Match Specific Productivity

In the economy analyzed so far match output is only a function of aggregate productivity and the worker's skills. When match output also depends on match specific productivity, in the spirit of Mortensen and Pissarides (1994), some matches will separate for endogenous reasons. In particular, only those matches whose productivity is above an endogenously determined threshold value will start producing. As a result, some job interviews will not result in hiring and some of the existing matches will separate.[40]

Given that some matches separate for endogenous reasons, the skill composition of the unemployment pool is affected through two channels instead of one: the vacancy posting decision and the decision on whether a match will start operating. Vacancy creation determines how many job-seekers will get a job interview. But it is the threshold value for match specific productivity which ultimately determines whether workers will be unemployed and exposed to skill erosion.[41]

[40]The model with match specific productivity for $g = l = 1$ is described in Appendix 1.10.

[41]This threshold value determines both whether a worker gets hired and whether an

Just as in the absence of match specific productivity, loss of skill generates a composition externality: firms do not take into account how both their vacancy posting decisions and their decisions concerning separation affect the skill composition of the unemployment pool, and hence the expected productivity of new hires. Thus, in the laissez-faire economy both vacancy posting and the threshold values for match specific productivity are not constrained efficient. Note that the composition externality still matters less in recessions than in booms because the larger share of job-seekers with eroded skills in recessions decreases the difference between the expected productivity of a new hire and a job-seeker with eroded skills.

In contrast to the finding in section 1.5, I find that the congestion externality is no longer internalized by the standard Hosios condition. Intuitively, this can be explained as follows. Even though all job-seekers still have the same probability of getting a job interview, they no longer have the same hiring probability. This because the threshold value for match specific productivity depends on the worker's human capital. As a result, a change in labor market tightness no longer affects the hiring probability of all workers in the same way. Consequently, there is an interaction between the congestion effect and the unemployment pool's composition.

Finally, in this framework workers with a higher unemployment duration have on average a lower hiring probability. Given that low-skilled job-seekers are less productive, match specific productivity has to be higher for those workers than for high-skilled workers for matches to have a non-negative value. Therefore, the threshold value for match specific productivity of low-skilled job-seekers is higher than that of high-skilled job-seekers. This in turn lowers the hiring probability of the low-skilled job-seekers. This prediction is in line with the empirical evidence on negative duration dependence.[42]

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existing match continues operating. This follows from the worker's and the firm's outside option being the same in the case of "not hiring" and in the case of "firing": a worker's outside option is being unemployed, while a firm's outside option is having an unfilled vacancy.

[42]However, there is a debate in the literature about the extent to which the observed

1.8 Conclusion

This paper shows that the depreciation of human capital during unemployment calls for policy intervention when both workers with and without eroded skills search for jobs in the same labor market. This is the consequence of a composition externality related to job creation: when making their hiring decisions, firms do not take into account how their decisions affect the skill composition of the unemployment pool, and hence the expected productivity of new hires. Hiring affects this composition because it prevents having additional workers with eroded skills in the unemployment pool.

constrained efficiency can be attained by implementing a procyclical employment subsidy. Employment should be subsidized because some gains from job creation are not fully taken into account, making job creation in the laissez-faire economy too low. The optimal subsidy is time-varying because the magnitude of the composition externality depends on the unemployment pool's composition. This magnitude is determined by the extent to which job creation affects the average skill composition of the unemployment pool. The impact on the latter of having an additional unemployed worker with eroded skills in the pool is smaller, the larger the fraction of workers with eroded skills in the unemployment pool. Consequently, the composition externality matters less in recessions than in booms. This in turn drives the procyclicality of the optimal employment subsidy.

Calibrating the model to the U.S. economy by making use of the empirical evidence regarding the effect of unemployment duration on workers' wages leads to the finding that the efficiency cost of human capital depreciation is considerable. If human capital depreciation was the only

decrease in the job finding probability for higher unemployment durations reflects "true" negative duration dependence. The lower job finding probability for workers with higher average unemployment duration could also result from unobserved heterogeneity. Those workers with intrinsically lower job finding probabilities make up a larger share of the long-term unemployed causing the observed negative duration dependence. See e.g. Barnichon and Figura (2011), Layard et al. (2005), and Machin and Manning (1999).

source of inefficiency, the average unemployment rate would be up to 1 percentage point lower in the constrained efficient allocation.

This paper has focused on an environment where search is random. I have shown that loss of skill during unemployment only gives rise to a composition externality when unemployed workers with and without eroded skills search for jobs in the same labor market. When workers with and without eroded skills search for jobs in separate markets, and firms can direct their vacancy posting to a particular market, loss of skill does not generate an externality. Therefore, I expect that loss of skill will give rise to a composition externality in other search environments when the environment is such that the expected productivity of a new hire depends on the unemployment pool's composition. I also expect this paper's finding that the composition externality matters less in recessions than in booms to carry over to other search environments. In particular, I expect this to happen when the environment is such that a larger share of workers with eroded skills in the unemployment pool increases the probability of hiring a worker with eroded skills, and hence decreases the expected productivity difference between a new hire and a job-seeker with eroded skills.

Finally, this paper has focused on the case where the depreciation of human capital during unemployment is the only source of inefficiency besides the search frictions characterizing the labor market. I believe that this environment is appropriate for understanding the nature of the efficiency loss generated by skill erosion. However, I also believe that introducing skill erosion into a richer environment is important because its presence might alter the efficiency loss generated by other frictions in the economy. One example is wage rigidity. This paper has emphasized that firms are still willing to hire the unemployed with eroded skills because those workers' wages are adjusted to bring them in line with their productivity. Therefore, I expect the efficiency loss generated by rigid wages to be larger once it is taken into account that the unemployed are exposed to skill erosion because firms might no longer be willing to hire unemployed workers with eroded skills. I leave the exploration of this avenue for future research.

1.9 Appendix A

1.9.1 The Economy with Separate Labor Markets

Labor Market

The labor market is characterized by two separate markets: a market for workers with eroded skills and one for workers without. So workers' skill type determines the labor market in which they are searching. Firms can decide in which market they post vacancies. Each submarket is characterized by a specific matching technology. I define market 1 as the market for the high-skilled and market 2 as the market for the low-skilled. The matching function is assumed to be the same in both markets. The number of matches in each submarket $k = \{1, 2\}$ is given by

$$m_k = Bv_k^{1-\xi} u_k^{\xi}$$

Defining labor market tightness $\theta_k(x) \equiv \frac{v_k(x)}{u_k(x)}$ the job finding probability, and the job filling probability are given by

$$q(\theta_k(x)) \equiv \frac{m(v_k(x), u_k(x))}{v_k(x)} = B\theta_k(x)^{-\xi}$$

$$p(\theta_k(x)) \equiv \frac{m(v_k(x), u_k(x))}{u_k(x)} = B\theta_k(x)^{1-\xi}$$

where $x = \{A\}$ is the economy's state. The latter is only a function of aggregate technology just as in the standard model because each submarket i contains only one type of unemployed.

Timing. The timing is the same as outlined in section 1.3.2.

Labor market flows.

Job-seekers in submarket 1

$$u_1 = \gamma n(x_{-1})$$

Job-seekers in submarket 2

$$u_2 = 1 - n(x_{-1})$$

High-skilled employment

$$n^H(x) = (1-\gamma)\, n(x_{-1}) + p(\theta_1(x))\, u_1$$

Low-skilled employment

$$n^L(x) = p(\theta_2(x))\, u_2$$

Total employment

$$n(x) = (1-\gamma)\, n(x_{-1}) + p(\theta_1(x))\, u_1 + p(\theta_2(x))\, u_2$$

The labor market flows show that in the presence of skill erosion during unemployment and learning-by-doing there is an interaction between both submarkets. The number of job-seekers in submarket 2 will not only depend on the amount of jobs created in the previous period in this submarket but also on the amount of jobs that was created in submarket 1. The reason is that workers who did not get hired in submarket 1, lose a fraction of their skills and become job-seekers in submarket 2.

Decentralized Allocation

Firm's problem

The value from having a worker of type $i = \{H, L\}$ producing is given by

$$J^i(x) = y^i(x) - w^i(x) + (1-\gamma)\, \beta E_x \left\{ J^H(x') \right\}$$

The value form having a vacancy in submarket 1 is given by

$$V_1(x) = -\kappa + q(\theta_1(x))\, J^H(x) + (1 - q(\theta_1(x)))\, \beta E_x \{V_1(x')\}$$

The value form having a vacancy in submarket 2 is given by

$$V_2(x) = -\kappa + q(\theta_2(x))\, J^H(x) + (1 - q(\theta_2(x)))\, \beta E_x \{V_2(x')\}$$

Free-entry in both submarkets implies

$$V_1(x) = V_2(x) = 0$$

Note that the value from being in a match only depends on the expected hiring cost, and no longer on the unemployment pool's composition: $J^H(x) = \frac{\kappa}{q(\theta_1(x))}$ and $J^L(x) = \frac{\kappa}{q(\theta_2(x))}$. The reason is that by posting a vacancy firms will be able to hire a worker of the same type, because the unemployment pool in each submarket is homogeneous.

Worker's problem

The value from being employed as type $i = \{H, L\}$ is given by

$$W^i(x) = w^i(x) + \beta E_x \left\{ \begin{array}{l} (1 - \gamma + \gamma p(\theta_1(x'))) W^H(x') \\ +\gamma(1 - p(\theta_1(x'))) U(x') \end{array} \right\}$$

The value from being unemployed is given by

$$U(x) = b + \beta E_x \left\{ p(\theta_2(x')) W^L(x') + (1 - p(\theta_2(x'))) U(x') \right\}$$

The worker's surplus from being in a match becomes

$$X^i(x) = w^i(x) - o(x) + (1 - \gamma) \beta E_x \left\{ X^H(x') \right\}$$

where $o(x)$ represents the worker's outside option

$$o(x) \equiv b + \beta E_x \left\{ p(\theta_2(x')) X^L(x') - \gamma p(\theta_1(x')) X^H(x') \right\}$$

Wages

Wages are determined through Nash bargaining between workers and firms in every period. When workers' bargaining power in submarket k is given by η_k, Nash bargaining implies

$$X^H(x) = \eta_1 M^H(x)$$

$$J^H(x) = (1 - \eta_1) M^H(x)$$

$$X^L(x) = \eta_2 M^L(x)$$

$$J^L(x) = (1-\eta_2) M^L(x)$$

where $M^i(x) \equiv X^i(x) + J^i(x)$ represents the surplus of being in a match with a worker of type i. The solution to the Nash bargaining problem results in the following expressions for the wage of a high-skilled and low-skilled worker respectively

$$w^H(x) = \eta_1 y^H(x) + (1-\eta_1) o(x)$$

$$w^L(x) = \eta_2 y^L(x) + (1-\eta_2) o(x) + (1-\gamma)\beta E_x \left\{ \eta_2 J^H(x') - (1-\eta_2) X^H(x') \right\}$$

Equilibrium

Submarket 1

$$\frac{\kappa}{q(\theta_1(x))} = (1-\eta_1) M^H(x)$$

$$M^H(x) = A - b + \beta E_x \left\{ \begin{array}{l} \left[1-\gamma+\eta_1\gamma p\left(\theta_1\left(x'\right)\right)\right] M^H(x') \\ -\eta_2 p\left(\theta_2\left(x'\right)\right) M^L(x') \end{array} \right\}$$

Combining both equations gives

$$\frac{\kappa}{q(\theta_1(x))} = (1-\eta_1) \left[\begin{array}{l} A-b \\ +\beta E_x \left\{ \begin{array}{l} \left[1-\gamma+\eta_1\gamma p\left(\theta_1\left(x'\right)\right)\right] \frac{\kappa}{q(\theta_1(x'))(1-\eta_1)} \\ -\eta_2 p\left(\theta_2\left(x'\right)\right) \frac{\kappa}{q(\theta_2(x'))(1-\eta_2)} \end{array} \right\} \end{array} \right] \tag{1.43}$$

Submarket 2

$$\frac{\kappa}{q(\theta_2(x))} = (1-\eta_2) M^L(x)$$

$$M^L(x) = (1-\delta) A - b + \beta E_x \left\{ \begin{array}{l} \left[1-\gamma+\eta_1\gamma p\left(\theta_1\left(x'\right)\right)\right] M^H(x') \\ -\eta_2 p\left(\theta_2\left(x'\right)\right) M^L(x') \end{array} \right\}$$

Combining both equations gives

$$\frac{\kappa}{q\left(\theta_2\left(x\right)\right)}=\left(1-\eta_2\right)\left[\begin{array}{l}\left(1-\delta\right)A-b\\ +\beta E_x\left\{\begin{array}{l}\left[1-\gamma+\eta_1\gamma p\left(\theta_1\left(x'\right)\right)\right]\frac{\kappa}{q\left(\theta_1\left(x'\right)\right)\left(1-\eta_1\right)}\\ -\eta_2 p\left(\theta_2\left(x'\right)\right)\frac{\kappa}{q\left(\theta_2\left(x'\right)\right)\left(1-\eta_2\right)}\end{array}\right\}\end{array}\right] \tag{1.44}$$

Constrained Efficient Allocation

The social planner's problem consists of choosing labor market tightness in both submarkets such that total output in the economy, net of vacancy posting costs, is maximized. The planner's problem is given by

$$V^P\left(x^P\right)=\max_{\theta_1,\theta_2}\left[\begin{array}{l}An^H\left(x^P\right)+\left(1-\delta\right)An^L\left(x^P\right)-\kappa\theta_1\left(x^P\right)\gamma n_{-1}\\ -\kappa\theta_1\left(x^P\right)\left(1-n_{-1}\right)+b\left(1-n\right)+\beta E_{x^P}\left\{V^P\left(x^{P\prime}\right)\right\}\end{array}\right]$$

with $n^H\left(x^P\right)=\left(1-\gamma\right)n_{-1}+B\theta_1\left(x^P\right)^{1-\xi}\gamma n_{-1}$
and $n^L\left(x^P\right)=B\theta_2\left(x^P\right)^{1-\xi}\left(1-n_{-1}\right)$
and subject to the law of motion for employment

$$n=\left(1-\gamma\right)n_{-1}+B\theta_1\left(x^P\right)^{1-\xi}\gamma n_{-1}+B\theta_2\left(x^P\right)^{1-\xi}\left(1-n_{-1}\right)$$

The relevant state for the social planner is given by $x^P=\{A,n_{-1}\}$.
The first order condition for labor market tightness in submarket 1 and 2 are respectively given by

$$\frac{\kappa}{q\left(\theta_1\left(x^P\right)\right)}=\left(1-\xi\right)\left[A-b+\beta E_{x^P}\left\{\frac{\partial V^P\left(x^{P\prime}\right)}{\partial n}\right\}\right]$$

$$\frac{\kappa}{q\left(\theta_2\left(x^P\right)\right)}=\left(1-\xi\right)\left[\left(1-\delta\right)A-b+\beta E_{x^P}\left\{\frac{\partial V^P\left(x^{P\prime}\right)}{\partial n}\right\}\right]$$

The envelope condition for employment is given by

$$\frac{\partial V^P\left(x^P\right)}{\partial n_{-1}} = \left[\begin{array}{l} \left[1-\gamma+\gamma p\left(\theta_1\left(x^P\right)\right)\right] A \\ -p\left(\theta_2\left(x^P\right)\right)(1-\delta) A \\ +\kappa\left(\theta_2\left(x^P\right)-\gamma\theta_1\left(x^P\right)\right) \\ -\left[1-\gamma+\gamma p\left(\theta_1\left(x^P\right)\right)-p\left(\theta_2\left(x^P\right)\right)\right]\left[b-\beta E_{x^P}\left\{\frac{\partial V^P\left(x^{P\prime}\right)}{\partial n}\right\}\right] \end{array}\right]$$

Combining the first order condition for submarket i with the envelope condition, gives the following expressions for job creation in each submarket

Submarket 1

$$\frac{\kappa}{q\left(\theta_1\left(x^P\right)\right)} = (1-\xi)\left[\begin{array}{l} A-b \\ +\beta E_{x^P}\left\{\begin{array}{l} \left[1-\gamma+\xi\gamma p\left(\theta_1\left(x^{P\prime}\right)\right)\right]\frac{\kappa}{q(\theta_1(x^{P\prime}))(1-\xi)} \\ -\xi p\left(\theta_2\left(x^{P\prime}\right)\right)\frac{\kappa}{q(\theta_2(x^{P\prime}))(1-\xi)} \end{array}\right\} \end{array}\right] \tag{1.45}$$

Submarket 2

$$\frac{\kappa}{q\left(\theta_2\left(x^P\right)\right)} = (1-\xi)\left[\begin{array}{l} (1-\delta)A-b \\ +\beta E_{x^P}\left\{\begin{array}{l} \left[1-\gamma+\xi\gamma p\left(\theta_1\left(x^{P\prime}\right)\right)\right]\frac{\kappa}{q(\theta_1(x^{P\prime}))(1-\xi)} \\ -\xi p\left(\theta_2\left(x^{P\prime}\right)\right)\frac{\kappa}{q(\theta_2(x^{P\prime}))(1-\xi)} \end{array}\right\} \end{array}\right] \tag{1.46}$$

Note that labor market tightness in each submarket is such that

$$\frac{\kappa}{q\left(\theta_2\left(x^P\right)\right)(1-\xi)} = \frac{\kappa}{q\left(\theta_1\left(x^P\right)\right)(1-\xi)} - \delta A$$

This implies that even though it is equally costly to post vacancies in submarket 1, in equilibrium labor market tightness is higher in the first submarket because there is an output gain δA related to posting in submarket 1 relative to submarket 2.

1.9.2 Proof of Proposition 2

Proof. Comparing equations (1.43) and (1.44) with equations (1.45) and (1.46), shows that the decentralized allocation is constrained efficient if the standard Hosios condition is satisfied in each submarket. □

1.9.3 Proof of Proposition 3

Proof. By comparing equation (1.35) and equation (1.37), it follows that the total effect of this period's vacancy posting on the number of matches that can be formed this period is taken into account when $\eta = \xi$. When the latter is satisfied, the expected surplus of a new match in the decentralized allocation is weighted by the elasticity of vacancies in the matching function $1 - \xi$, just as in the social planner's allocation. The effect on next period's labor market tightness following from having a job-seeker less when this period's match survives separation is also internalized for $\eta = \xi$. This can be shown as follows. For $\delta = 0$, this part of the congestion effect is internalized in the decentralized allocation because the net effect, generated by the congestion effect and a worker's outside option, can be expressed as a fraction of the worker's outside option. From the constrained efficient allocation it follows that the net effect is given by (see the second term of expression (1.38))

$$\Upsilon(x) \equiv \tilde{o}(x) + \Omega(x) = -\frac{\xi}{1-\xi}(1-\gamma)\kappa E_x\{\theta(x')\} \tag{1.47}$$

where the worker's outside option net of home production is given by (see the second term of expression (1.34))

$$\tilde{o}(x) \equiv -\left(\frac{1-\gamma}{1-\xi}\right)\kappa E_x\{\theta(x')\} \tag{1.48}$$

and where the congestion effect is given by (see the third term of expression (1.34))

$$\Omega \equiv (1-\gamma)\kappa E_x\{\theta(x')\} \tag{1.49}$$

Comparing equation (1.47) and the decentralized allocation (equation (1.35)) shows immediately that for $\eta = \xi$ the congestion effect is internalized, even though only the worker's outside option is taken into account. This follows from the net effect being a fraction ξ of the worker's outside option (net of the value of home production) (equation (1.48)), and the latter being taken into account up to a fraction η in the decentralized allocation.

For $\delta > 0$, the net effect is now given by (see the second and third term of expression (1.38))

$$\Upsilon(x) = -\frac{\xi}{1-\xi}(1-\gamma)\kappa E_x\{\theta(x')\} + E_x\{A'p(\theta(x'))[1-(1-\gamma)s(n)]\} \tag{1.50}$$

where the outside option net of home production is now given by (see the second term of expression (1.34))

$$\tilde{o}(x) = -\left(\frac{1-\gamma}{1-\xi}\right)\kappa E_x\{\theta(x')\} + E_x\{A'p(\theta(x'))[1-(1-\gamma)s(n)]\} \tag{1.51}$$

and where the congestion effect is still given by equation (1.49). The congestion effect is still expressed as a function of the match with an average job-seeker because they all have the same hiring probability, and hence a change in labor market tightness affects all job-seekers' hiring probability in the same way.

Comparing equation (1.50) and the decentralized allocation (equation (1.35)) shows that the standard Hosios condition $\eta = \xi$ still internalizes the congestion effect even though only the worker's outside option is taken into account. This follows from the fact that this part of the net effect containing the congestion effect (given by the first term in equation (1.50)) can still be expressed as a fraction ξ of part of the worker's outside option (given by the first term in equation (1.51)). This in turn follows from both part of the net effect and part of the worker's outside option being able to be expressed as a function of the hiring probability and match value of an average job-seeker.

□

1.9.4 Optimal Labor Market Policy

The Economy

Changes relative to the setup without an employment subsidy/tax

In the presence of an employment subsidy/tax only the firm's problem is affected. When employment is subsidized/taxed $\Phi(x)$, a firm's value of employing a worker of type i is given by

$$J_s^i(x) = y^i(A) - w^i(x) + \Phi(x) + (1-\gamma)\beta E_x\left\{J_s^H(x')\right\} \tag{1.52}$$

Equation (1.52) shows that the firm will receive the employment subsidy/tax $\Phi(x)$ independently of which worker type the firm employs.

Note that wages are also affected by the introduction of an employment subsidy/tax through its effect on the surplus of a match. The solution to the Nash bargaining problem is now given by

$$w^i(x) = \eta\left(y^i(A) + \Phi(x)\right) + (1-\eta)\, o(x)$$

where $o(x)$ is the worker's outside option defined by equation (1.21). It can be seen from the wage expression that in the presence of an employment subsidy $\Phi(x) > 0$, wages are higher than in the absence of it. The positive effect of employment subsidies on wages has also been emphasized by Mortensen and Pissarides (2002).

Equilibrium

The surplus of a match with a high-skilled worker (equation (1.53)) and the vacancy creation equation (equation (1.54)) can be obtained in the same way as described in section 1.3.6. However, the firm's value function is now given by equation (1.52) instead of by equation (1.13).

$$M_s^H(x) = A - b + \Phi(x) + \beta E_x\left\{\begin{array}{l}(1-\gamma)\left(1-\eta p\left(\theta\left(x'\right)\right)\right) M_s^H(x') \\ +\eta p\left(\theta\left(x'\right)\right)\delta A'\end{array}\right\} \tag{1.53}$$

$$\frac{\kappa}{q\left(\theta\left(x\right)\right)} = \left(1-\eta\right)\left[M_s^H\left(x\right) - s\left(x\right)\delta A\right] \tag{1.54}$$

Combining equation (1.53) and (1.54), job creation in the decentralized allocation in the presence of the employment subsidy is given by

$$\frac{\kappa}{q\left(x\right)\left(1-\eta\right)} = \bar{y}\left(x\right) + \Phi\left(x\right) - b + \beta E_x\left\{\Lambda^D\left(x'\right)\right\} \tag{1.55}$$

where $\Lambda^D\left(x\right)$ is defined by equation (1.36) in section 1.4.2.

Optimal Employment Subsidy/Tax

The optimal employment subsidy/tax $\Phi\left(x\right)$ internalizes both the congestion and the composition externality if job creation in the decentralized allocation (equation (1.55)) replicates job creation in the constrained efficient allocation (equation (1.33)). It follows that the optimal employment subsidy/tax is given by

$$\Phi\left(x\right) = \left[\begin{array}{l} \frac{(\eta-\xi)}{(1-\xi)(1-\eta)q(\theta(x))}\kappa \\ +\beta E_x\left\{\begin{array}{l} \frac{(\eta-\xi)(1-\gamma)(\eta p(\theta(x'))-1)}{(1-\xi)(1-\eta)q(\theta(x'))}\kappa \\ +\delta\left(1-\eta\right)p\left(\theta\left(x'\right)\right)A'\left(1-\left(1-\gamma\right)s\left(n\right)\right) \end{array}\right\} \end{array}\right] \tag{1.56}$$

1.9.5 Multiple Equilibria

Pissarides (1992) has shown that in an overlapping generations model where the unemployed are exposed to skill loss the steady state is not unique for certain parameter conditions. This finding has been confirmed by Coles and Masters (2000) in a different setup. The intuition behind this finding is that the expected gain from vacancy posting is no longer strictly decreasing in labor market tightness θ. In an economy without skill erosion during unemployment, this gain is strictly decreasing in θ because the job filling probability is a decreasing function of θ. In a model where skills erode, an increase in θ still implies a decrease in the job filling probability but there is an additional effect. Now an increase in θ also leads to an improvement in the skill distribution of the unemployment pool. The latter follows from a higher θ implying a lower average unemployment duration, and hence a smaller fraction of job-seekers with eroded skills. As pointed out by Pissarides (1992), when this effect is strong enough, an increase in θ can lead to an increase in the expected gain from vacancy posting, and hence multiple equilibria can arise.

Also in this model multiple steady state potentially arise. Evaluating the vacancy creation condition (1.29) in steady state shows that the expected gains are not necessarily strictly decreasing in θ.

1.9.6 Generalized Version of the Model

This section describes the decentralized allocation and the constrained efficient allocation when workers lose their skills with probability $0 < l \leq 1$, and regain their skills upon reemployment with probability $0 < g \leq 1$.

Decentralized Allocation

Firm's problem

Value of having a high-skilled worker employed

$$J^H(x) = y^H(A) - w^H(x) + (1-\gamma)\beta E_x\left\{J^H(x')\right\}$$

Value of having a low-skilled worker employed

$$J^L(x) = y^L(A) - w^L(x) + (1-\gamma)\beta E_x\left\{gJ^H(x') + (1-g)J^L(x')\right\}$$

Value of having a vacancy

$$V(x) = \left[\begin{array}{l} -\kappa + q(\theta(x)) \left[(1 - s(x)) J^H(x) + s(x) J^L(x) \right] \\ + (1 - q(\theta(x))) \beta E_x \{ V(x') \} \end{array} \right]$$

Imposing free entry $V(x) = 0$ leads to the following expression for vacancy creation:

$$\frac{\kappa}{q(\theta(x))} = (1 - s(x)) J^H(x) + s(x) J^L(x)$$

Worker's problem

Value of being employed as a high-skilled worker

$$W^H(x) = w^H(x) + \beta E_x \left\{ (1 - \gamma + \gamma p(x')) W^H(x') + \gamma (1 - p(x')) U^H(x') \right\}$$

Value of being employed as a low-skilled worker

$$W^L(x) = w^L(x) + \beta E_x \left\{ \begin{array}{l} (1 - \gamma + \gamma p(x')) \left[g W^H(x') + (1 - g) W^L(x') \right] \\ + \gamma (1 - p(x')) \left[g U^H(x') + (1 - g) U^L(x') \right] \end{array} \right\}$$

Value of being unemployed as a high-skilled worker

$$U^H(x) = b + \beta E_x \left\{ \begin{array}{l} l \left[p(x') W^L(x') + (1 - p(x')) U^L(x') \right] \\ + (1 - l) \left[p(x') W^H(x') + (1 - p(x')) U^H(x') \right] \end{array} \right\}$$

Value of being unemployed as a low-skilled worker

$$U^L(x) = b + \beta E_x \left\{ p(x') W^L(x') + (1 - p(x')) U^L(x') \right\}$$

Wages

Wages are set through Nash bargaining between worker and firm. Denoting the bargaining power of the workers by η, the value from being in match of a specific type for workers and firms respectively is given by

$$W^i(x) - U^i(x) = \eta M^i(x)$$
$$J^i(x) = (1-\eta) M^i(x)$$

The solution to the Nash bargaining problem leads to the following expression for wages

$$w^i(x) = \eta y^i(x) + (1-\eta) o^i(x)$$

where the outside option for each worker type are given by

$$o^H(x) \equiv b + \beta E_x \left\{ \begin{array}{l} \eta p(x') \left[(1-l) M^H(x') + l M^L(x') \right] \\ -\eta\gamma p(x') M^H(x') \\ -l \left(U^H(x') - U^L(x') \right) \end{array} \right\}$$

$$o^L(x) \equiv b + \beta E_x \left\{ \begin{array}{l} \eta p(x') M^L(x') \\ -\eta\gamma p(x') \left[g M^H(x') + (1-g) M^L(x') \right] \\ -g \left(U^H(x') - U^L(x') \right) \end{array} \right\}$$

Equilibrium

Given the match surplus from a high-skilled worker (1.57), the match surplus from a low-skilled worker (1.58), and the unemployment values (1.59) and(1.60), for a given state of the economy x, labor market tightness θ is defined by the vacancy creation condition (1.61)

$$M^H(x) = \left[\begin{array}{l} A - b \\ +\beta E_x \left\{ \begin{array}{l} \left(1-\gamma+\eta\gamma p\left(\theta\left(x'\right)\right)\right) M^H\left(x'\right) \\ -\eta p\left(\theta\left(x'\right)\right) \left[l M^L\left(x'\right) + (1-l) M^H\left(x'\right) \right] \\ +l \left(U^H\left(x'\right) - U^L(x') \right) \end{array} \right\} \end{array} \right] \tag{1.57}$$

$$M^L(x) = \left[\begin{array}{l} A(1-\delta) - b \\ +\beta E_x \left\{ \begin{array}{l} \left(1-\gamma+\eta\gamma p\left(\theta\left(x^{'}\right)\right)\right)\left(gM^H(x') + (1-g)M^L(x')\right) \\ -\eta p\left(\theta\left(x^{'}\right)\right)M^L\left(x^{'}\right) \\ +g\left(U^H(x') - U^L(x')\right) \end{array} \right\} \end{array} \right] \tag{1.58}$$

$$U^H(x) = b + \beta E_x \left\{ \begin{array}{l} \eta p\left(\theta\left(x^{'}\right)\right)\left[(1-l)M^H(x')\right. \\ \left. +lM^L(x')\right] + (1-l)U^H(x') + lU^L(x') \end{array} \right\} \tag{1.59}$$

$$U^L(x) = b + \beta E_x \left\{ \eta p\left(\theta\left(x^{'}\right)\right) M^L(x') + U^L(x') \right\} \tag{1.60}$$

$$\frac{\kappa}{q(\theta(x))} = (1-\eta)\left[(1-s(x))M^H(x) + s(x)M^L(x)\right] \tag{1.61}$$

where the law of motions of the endogenous state variables $\tilde{u}^H_{-1}, \tilde{u}^L_{-1}, n^L_{-1}$ are given by

$$\tilde{u}^H = (1-p(\theta(x)))\left[(1-l)\tilde{u}^H_{-1} + \gamma\left(1-(1-g)n^L_{-1} - \tilde{u}^H_{-1} - \tilde{u}^L_{-1}\right)\right] \tag{1.62}$$

$$\tilde{u}^L = (1-p(\theta(x)))\left[\tilde{u}^L_{-1} + l\tilde{u}^H_{-1} + \gamma(1-g)n^L_{-1}\right] \tag{1.63}$$

$$n^L = (1-\gamma)(1-g)n^L_{-1} + p(\theta(x))\left[\tilde{u}^L_{-1} + l\tilde{u}^H_{-1} + \gamma(1-g)n^L_{-1}\right] \tag{1.64}$$

Constrained Efficient Allocation

The constrained efficient allocation is obtained by solving the social planner's problem. The latter is defined as

$$V^P(x) = \max_{\theta} \begin{bmatrix} An^H(x) + (1-\delta) An^L(x) - \kappa\theta(x)(1-(1-\gamma)n(x_{-1})) \\ +b(1-n(x)) + \beta E_x \left\{ V^P(x') \right\} \end{bmatrix}$$

subject to the law of motions of the endogenous state variables $\tilde{u}^H, \tilde{u}^L$ and n^L (see equations (1.62), (1.63), and (1.64) respectively), and the process for aggregate technology (equation (1.1)), and where $n^H(x) = 1 - n^L(x) - \tilde{u}^L(x) - \tilde{u}^H(x)$; $n = n^H(x) + n^L(x)$; and the economy's state is given by $x = \left\{ A, \tilde{u}^H_{-1}, \tilde{u}^L_{-1}, n^L_{-1} \right\}$.
The first order condition for vacancy creation is

$$\frac{\kappa}{q(\theta(x))} = (1-\xi) \left[\bar{y}(x) - b + \beta E_x \left\{ \begin{array}{l} -(1-s(x)) \frac{\partial V^P(x')}{\partial u^H} \\ -s(x) \frac{\partial V^P(x')}{\partial u^L} \\ +s(x) \frac{\partial V^P(x')}{\partial n^L} \end{array} \right\} \right] \tag{1.65}$$

The envelope condition for high-skilled unemployment is

$$\frac{\partial V^P(x)}{\partial \tilde{u}^H_{-1}} = \left\{ \begin{array}{l} - \underbrace{\begin{bmatrix} (1-\gamma+\gamma p(\theta(x))) y^H(A) \\ +\gamma(1-p(\theta(x))) \beta E_x \left\{ \frac{\partial V^P(x')}{\partial \tilde{u}^H} \right\} \end{bmatrix}}_{no\ HS\ worker} \\ + \underbrace{p(\theta(x))}_{hiring} \left[(1-l) y^H(A) + l \left(y^L(A) + \beta E_x \left\{ \frac{\partial V^P(x')}{\partial n^L} \right\} \right) \right] \\ + \underbrace{(1-p(\theta(x)))}_{no\, hiring} \beta E_x \left\{ (1-l) \frac{\partial V^P(x')}{\partial \tilde{u}^H} + l \frac{\partial V^P(x')}{\partial \tilde{u}^L} \right\} \\ + (1-\gamma)(1-p(\theta(x))) b - (1-\gamma) \kappa\theta(x) \end{array} \right\} \tag{1.66}$$

The envelope condition for low-skilled unemployment is

$$\frac{\partial V^P(x)}{\partial \tilde{u}^L_{-1}} = \left\{ \begin{array}{l} -\underbrace{\left[\begin{array}{l} (1-\gamma+\gamma p(\theta(x)))\, y^H(A) \\ +\gamma(1-p(\theta(x)))\,\beta E_x \left\{ \frac{\partial V^P(x')}{\partial \tilde{u}^H} \right\} \end{array} \right]}_{no\ HS\ worker} \\ +\underbrace{p(\theta(x))}_{hiring} \left[y^L(A) + \beta E_x \left\{ \frac{\partial V^P(x')}{\partial n^L} \right\} \right] \\ +\underbrace{(1-p(\theta(x)))}_{no\ hiring} \beta E_x \left\{ \frac{\partial V^P(x')}{\partial \tilde{u}^L} \right\} \\ +(1-\gamma)(1-p(\theta(x)))\,b - (1-\gamma)\,\kappa\theta(x) \end{array} \right\} \tag{1.67}$$

The envelope condition for low-skilled employment is

$$\frac{\partial V^P(x)}{\partial n^L_{-1}} = \underbrace{(1-g)}_{no\ regaining} \left\{ \begin{array}{l} -\underbrace{\left[\begin{array}{l} (1-\gamma+\gamma p(\theta(x)))\, y^H(A) \\ +\gamma(1-p(\theta(x)))\,\beta E_x \left\{ \frac{\partial V^P(x')}{\partial \tilde{u}^H} \right\} \end{array} \right]}_{no\ HS\ worker} \\ +(1-\gamma+\gamma p(\theta(x))) \left[y^L(A) + \beta E_x \left\{ \frac{\partial V^P(x')}{\partial n^L} \right\} \right] \\ +\gamma(1-p(\theta(x)))\,\beta E_x \left\{ \frac{\partial V^P(x')}{\partial \tilde{u}^L} \right\} \end{array} \right\} \tag{1.68}$$

The constrained efficient allocation is defined by equations (1.65)-(1.68) together with the law of motions of the endogenous state variables (equations (1.62),(1.63), and (1.64)), and the process for aggregate technology (equation (1.1)).

1.10 Appendix B

1.10.1 Allowing for Match Specific Productivity

Population and Technology

The only difference relative to the assumptions outlined in the paper is that ouput also depends on match specific productivity z. The latter is assumed to be *iid*, and drawn from a distribution with the density function being denoted by $f(z)$, and the cumulative distribution function being denoted by $F(z)$. The output produced by a match with a high-skilled and low-skilled worker respectively is given by

$$y^H(z, A) = Az$$
$$y^L(z, A) = (1-\delta) Az$$

Labor Market

The overall structure of the labor market stays the same. The presence of match specific productivity alters slightly the timing of the model and the labor market flows, which are both described below.

Timing. At the begining of the period a shock to aggregate productivity A is realized. After having observed the state of the economy firms post vacancies and matching takes place. Since all matches receive an *iid* match specific productivity shock not all workers who meet a vacancy get hired and some existing matches separate endogenously. Next, production takes place with both existing and newly hired workers. After production some workers' type changes: all the low-skilled employed workers become high-skilled because of learning-by-doing, and all the high-skilled unemployed workers become low-skilled because of skill erosion during unemployment. At the end of the period, a fraction γ of the matches separates for exogenous reasons.

The timing assumption for the realization of the match specific productivity shock has become standard in the literature when it is assumed that workers become productive upon hiring, see e.g. Ravenna and Walsh (2012b), and Thomas and Zanetti (2009).

Labor market flows. The flow of high-skilled and low-skilled job-seekers is defined in the same way as in the model without match specific productivity

$$u^H(x) = \gamma n_{-1}$$
$$u^L(x) = 1 - n_{-1}$$

The flows of high-skilled and low-skilled employment are altered because only a fraction of the matches survives endogenous separation. High-skilled and low-skilled employment evolve according to the following expressions

$$n^H(x) = \lambda^H(x)\left[(1-\gamma)\,n_{-1} + p(\theta(x))\,u^H(x)\right]$$

$$n^L(x) = \lambda^L(x)\,p(\theta(x))\,u^L(x)$$

where $\lambda^i(x) \equiv 1 - F(\bar{z}^i(x))$ denotes the probability that a match with a worker of type $i = \{H, L\}$ becomes productive. This in turn depends on the distribution of match specific productivity and the threshold value $\bar{z}^i(x)$, which is defined below.

The aggregate state of the economy (x) is still given by $x = \{A, n_{-1}\}$, where the law of motion for total employment is given by

$$n = \lambda^H(x)\left[(1-\gamma)\,n_{-1} + p(\theta(x))\,\gamma n_{-1}\right] + \lambda^L(x)\,p(\theta(x))\,(1 - n_{-1}) \tag{1.69}$$

Firm's Problem

The firm's value of having a worker of type i employed with idiosyncratic match specific productivity z is given by

$$J^i(z, x) = \left[\begin{array}{l} y^i(z, A) - w^i(z, x) \\ +(1-\gamma)\,\beta E_x\left\{\int\limits_{\bar{z}^i(x')}^{\infty} J^H(k, x')\,f(k)\,dk\right\}\end{array}\right] \tag{1.70}$$

The threshold value for match specific productivity $(\bar{z}^i(x))$ for a match with a worker type i is defined as

$$J^i\left(\bar{z}^i(x), x\right) = 0$$

This implies that the threshold value is such that the value of a match with a worker of type i and match specific productivity $\bar{z}(x)$ is zero.

The firm's value of posting a vacancy is given by

$$V(x) = \left[\begin{array}{l} -\kappa + q(\theta(x)) \left(\begin{array}{c} \varsigma^H(x) \int\limits_{\bar{z}^H(x)}^{\infty} J^H(z, x) f(z)\, dz \\ +\varsigma^L(x) \int\limits_{\bar{z}^L(x)}^{\infty} J^L(z, x) f(z)\, dz \end{array}\right) \\ +(1 - \tilde{q}(\theta(x)))\beta E_x\{V(x')\} \end{array}\right] \tag{1.71}$$

where

- $\tilde{q}(\theta(x)) \equiv \tilde{q}^H(\theta(x)) + \tilde{q}^L(\theta(x))$

  It denotes the probability that a vacancy becomes a productive match.

- $\tilde{q}^i(\theta(x)) \equiv \varsigma^i(x)\, q(\theta(x))\, \lambda^i(x)$

  It denotes the probability that a vacancy becomes a productive match with a worker of type i. The latter depends on the share of this type of job-seekers in the unemployment pool $\left(\varsigma^i(x) \equiv \frac{u^i(x)}{u(x)}\right)$, the probability a vacancy and a searcher meet $(q(\theta))$, and the probability that the match specific productivity is above the threshold value (λ^i).

Note that ˜ above a variable refers to the variable's value conditional on the match becoming productive.

From the free-entry condition $V(x) = 0$, it follows that equation

(1.71) becomes

$$\frac{\kappa}{q\left(\theta\left(x\right)\right)}=\left[\begin{array}{c}\varsigma^{H}\left(x\right)\int\limits_{\bar{z}^{H}\left(x\right)}^{\infty}J^{H}\left(z,x\right)f\left(z\right)dz\\+\varsigma^{L}\left(x\right)\int\limits_{\bar{z}^{L}\left(x\right)}^{\infty}J^{L}\left(z,x\right)f\left(z\right)dz\end{array}\right]\tag{1.72}$$

Worker's Problem

A worker of type i's value function of being employed in a match with match specific productivity z is

$$W^{i}\left(z,x\right)=w^{i}\left(z,x\right)+\beta E_{x}\left\{\begin{array}{l}\left(1-\gamma+\gamma p\left(\theta\left(x^{'}\right)\right)\right)\int\limits_{\bar{z}^{H}\left(x'\right)}^{\infty}W^{i}\left(k,x'\right)f\left(k\right)dk\\+\gamma\left(1-p\left(\theta\left(x^{'}\right)\right)\right)U\left(x'\right)\end{array}\right\}$$

The value function of high and low-skilled workers of being unemployed is

$$U\left(x\right)=b+\beta E_{x}\left\{\begin{array}{l}p\left(\theta\left(x^{'}\right)\right)\int\limits_{\bar{z}^{L}\left(x'\right)}^{\infty}W^{L}\left(k,x'\right)f\left(k\right)dk\\+\left(1-p\left(\theta\left(x^{'}\right)\right)\right)U\left(x'\right)\end{array}\right\}$$

Wages

Wages are set through period-by-period Nash bargaining between worker and firm. The surplus of a match with idiosyncratic productivity z and a worker of type i is defined as

$$M^{i}\left(z,x\right)\equiv W^{i}\left(z,x\right)-U^{i}\left(x\right)+J^{i}\left(z,x\right)$$

The wage setting mechanism implies

$$X^{i}\left(z,x\right)\equiv W^{i}\left(z,x\right)-U^{i}\left(x\right)=\eta M^{i}\left(z,x\right)$$

$$J^{i}\left(z,x\right)=\left(1-\eta\right)M^{i}\left(z,x\right)$$

where η is the bargaining power of the households, and $1-\eta$ is the bargaining power of the firms.

Equilibrium

For a given state of the economy $x = \{A, n_{-1}\}$, this economy's equilibrium consists of a value for labor market tightness, and threshold values for match-specific productivity for high-skilled and low-skilled workers, such that it satisfies the vacancy creation equation (equation (1.75)) and the expression determining the threshold values for match specific productivity (equation (1.74)), given the surplus from a match with a high and low-skilled worker respectively (equation (1.73)), and taking into account the definitions for the job finding and filling probability's. Given this period's state and the equilibrium values of labor market tightness and match specific productivity, next period's state is determined by the law of motion for aggregate technology, and the law of motion for the endogenous state variable employment (equation (1.69)).

$$M^i(z,x) = \left[\begin{array}{l} y^i(z,A) - b \\ +\beta E_x \left\{ \begin{array}{l} \left(1-\gamma+\eta\gamma p\left(\theta\left(x'\right)\right)\right) \int\limits_{\bar{z}^H(x')}^{\infty} M^H(k,x') f(k)\, dk \\ -\eta p\left(\theta\left(x'\right)\right) \int\limits_{\bar{z}^L(x')}^{\infty} M^L(k,x') f(k)\, dk \end{array} \right\} \end{array} \right] \tag{1.73}$$

$$M^i\left(\bar{z}^i, x\right) = 0 \tag{1.74}$$

$$\frac{\kappa}{q(\theta(x))} = (1-\eta) \left\{ \begin{array}{l} \varsigma^H(x) \int\limits_{\bar{z}^H(x)}^{\infty} M^H(z,x) f(z)\, dz \\ +\varsigma^L(x) \int\limits_{\bar{z}^L(x)}^{\infty} M^L(z,x) f(z)\, dz \end{array} \right\} \tag{1.75}$$

1.10.2 Constrained Efficient Allocation

The social planner's problem consists of choosing labor market tightness and the threshold values for match specific productivity such that the utility of a representative worker is maximized, and is given by

$$V^P(x) = \max_{\theta,\bar{z}^H,\bar{z}^L} \left[\begin{array}{l} \tilde{y}^H(x) n^H(x) + \tilde{y}^L(x) n^L(x) \\ -\kappa\theta(x)(1-(1-\gamma)n_{-1}) \\ +b(1-n) + \beta E_x \left\{V^P(x')\right\} \end{array} \right]$$

subject to the process for aggregate technology and the law of motion for the endogenous state variable employment (equation (1.69)), and where

- $\tilde{y}^H(x) \equiv A\tilde{z}^H(x)$

  It denotes the expected output of a high-skilled match conditional on becoming productive.

- $\tilde{y}^L(x) \equiv (1-\delta)A\tilde{z}^L(x)$

  It denotes the expected output of a low-skilled match conditional on becoming productive.

- $\tilde{z}^i(x) \equiv \int\limits_{\bar{z}^i(x)}^{\infty} z \frac{f(z)}{1-F(\bar{z}^i(x))} dz$

  It denotes the expected value of match specific productivity conditional on the match becoming productive.

The first order condition for labor market tightness is given by

$$\frac{\kappa}{q(\theta(x))} = (1-\xi) \left[\begin{array}{l} \varsigma^H(x)\lambda^H(x)\tilde{y}^H(x) + \varsigma^L(x)\lambda^L(x)\tilde{y}^L(x) \\ -\left(\varsigma^H(x)\lambda^H(x) + \varsigma^L(x)\lambda^L(x)\right)\left[b - \beta E_x\left\{\frac{\partial V^P(x')}{\partial n}\right\}\right] \end{array} \right]$$

When using the notation introduced previously, this can be written as:

$$\frac{\kappa}{\tilde{q}(\theta(x))} = (1-\xi)\left[\tilde{y}^a(x) - b + \beta E_x\left\{\frac{\partial V^P(x')}{\partial n}\right\}\right] \qquad (1.76)$$

where $\tilde{y}^a(x) \equiv \tilde{y}^H(x)\frac{\tilde{q}^H(\theta(x))}{\tilde{q}(\theta(x))} + \tilde{y}^L(x)\frac{\tilde{q}^L(\theta(x))}{\tilde{q}(\theta(x))}$ reflects the expected output of a new hire.

The threshold values for match specific productivity are given by

$$A\bar{z}^{H}(x) - b + \beta E_x \left\{ \frac{\partial V^P(x')}{\partial n} \right\} = 0 \tag{1.77}$$

$$(1-\delta) A\bar{z}^{L}(x) - b + \beta E_x \left\{ \frac{\partial V^P(x')}{\partial n} \right\} = 0 \tag{1.78}$$

The envelope condition for employment is given by

$$\frac{\partial V^P(x)}{\partial n_{-1}} = \left[\begin{array}{l} \lambda^H(x)\left(1-\gamma+\gamma p(\theta(x))\right)\tilde{y}^H(x) \\ -\lambda^L(x)\, p(\theta(x))(1-\delta)\tilde{y}^L(x) \\ +(1-\gamma)\kappa\theta(x) - \zeta\left[b - \beta E_x\left\{\frac{\partial V^P(x')}{\partial n}\right\}\right] \end{array} \right] \tag{1.79}$$

where $\zeta \equiv \lambda^H(1-\gamma+\gamma p(\theta(x))) - \lambda^L p(\theta(x))$.
Combining equation (1.76) and (1.79), job creation becomes

$$\frac{\kappa}{\tilde{q}(x)(1-\xi)} = \tilde{y}^a(x) - b + \beta E_x\left\{\Lambda^{MP}(x')\right\} \tag{1.80}$$

where the continuation value of the match is defined as

$$E_x\left\{\Lambda^{MP}(x')\right\} \equiv E_x \left\{ \begin{array}{l} \underbrace{(1-\gamma)\lambda^H(x')\left(\frac{\kappa}{\tilde{q}(x')(1-\xi)} + \tilde{y}^H(x') - \tilde{y}^a(x')\right)}_{match\ survives\ separation} \\ +\gamma\lambda^H(x')\,p(x')\left(\frac{\kappa}{\tilde{q}(x')(1-\xi)} + \tilde{y}^H(x') - \tilde{y}^a(x')\right) \\ \underbrace{-\,\lambda^L(x')\,p(x')\left(\frac{\kappa}{\tilde{q}(x')(1-\xi)} + \tilde{y}^L(x') - \tilde{y}^a(x')\right)}_{outside\ option} \\ +(1-\gamma)(1-\xi)\,p(x') \\ \underbrace{*\left[\begin{array}{l} \lambda^H(x')\varsigma^H(x')\left(\frac{\kappa}{\tilde{q}(x')(1-\xi)} + \tilde{y}^H(x') - \tilde{y}^a(x')\right) \\ +\lambda^L(x')\varsigma^L(x')\left(\frac{\kappa}{\tilde{q}(x')(1-\xi)} + \tilde{y}^L(x') - \tilde{y}^a(x')\right) \end{array} \right]}_{congestion\ effect\ searchers} \end{array} \right\} \tag{1.81}$$

By combining equations (1.77) and (1.79), and equations (1.78) and (1.79), the threshold values for match specific productivity for a high-skilled and low-skilled worker respectively, are given by

$$A\bar{z}^{H}(x) - b + \beta E_{x}\left\{\Lambda^{MP}(x')\right\} = 0 \tag{1.82}$$

$$(1-\delta)\,A\bar{z}^{L}(x) - b + \beta E_{x}\left\{\Lambda^{MP}(x')\right\} = 0 \tag{1.83}$$

Overall, the social planner's job creation equation takes the same form as in the absence of match specific productivity. There are, however, three main differences.

First, the planner now takes into account that the unemployment pool's composition not only affects the average productivity of new hires but also the chance that a job interview will result in hiring. This is because the threshold value for match specific productivity depends on the worker type.

Second, the planner takes into account that the presence of match specific productivity worsens the worker's outside option even more (see second term in expression (1.81)). Now being unemployed not only lowers a worker's productivity but it also lowers a worker's re-employment prospects. This results from the threshold value of match specific productivity being higher for a low-skilled than for a high-skilled worker (equation (1.82) and (1.83)). Note that this is the case because low-skilled job-seekers are less productive, and hence match specific productivity has to be higher than for high-skilled workers for matches to have a non-negative value.

Third, the planner takes into account the congestion effect caused by having a job-seeker less in the unemployed pool next period if the match does not separate (see third term in expression (1.81)). In contrast to the model without match specific productivity, the extent to which a change in labor market tightness will affect next period's output depends on the skill composition of the unemployment pool. This follows immediately from a change in labor market tightness not having the same effect on the hiring probability of different worker types. A change in labor market tightness still affects the probability of meeting a firm in the same way for all workers. However, given that match specific productivity depends on the worker type, workers' hiring probability is not affected in the same way by a change in labor market tightness. Consequently, there is an interaction between the congestion externality and the skill composition of the unemployment pool.

1.10.3 Constrained Efficient versus Decentralized Allocation

Proposition. *In the presence of skill erosion during unemployment, the threshold value for match specific productivity is no longer constrained efficient under the standard Hosios condition* ($\eta = \xi$).

Proof.

Combining equation (1.73) and (1.75), and using the same notation as introduced previously, job creation in the decentralized allocation is determined by

$$\frac{\kappa}{\tilde{q}(x)(1-\eta)} = \tilde{y}^a(x) - b + \beta E_x\left\{\Lambda^{MD}(x')\right\} \tag{1.84}$$

where

$$E_x\left\{\Lambda^{MD}(x')\right\} \equiv E_x\left\{\begin{array}{l}\underbrace{(1-\gamma)\lambda^H(x')\left(\frac{\kappa}{\tilde{q}(x')(1-\eta)} + \tilde{y}^H(x') - \tilde{y}^a(x')\right)}_{\text{match continues producing}} \\ +\eta\gamma p(x')\lambda^H(x')\left(\frac{\kappa}{\tilde{q}(x')(1-\eta)} + \tilde{y}^H(x') - \tilde{y}^a(x')\right) \\ -\eta\lambda^L(x')p(x')\left(\frac{\kappa}{\tilde{q}(x')(1-\eta)} + \tilde{y}^L(x') - \tilde{y}^a(x')\right)\end{array}\right\} \tag{1.85}$$

and the threshold value for match specific productivity are given by

$$A\bar{z}^H(x) - b + \beta E_x\left\{\Lambda^{MD}(x')\right\} = 0 \tag{1.86}$$

$$(1-\delta) A\bar{z}^{L}(x) - b + \beta E_x \left\{ \Lambda^{MD}(x') \right\} = 0 \qquad (1.87)$$

By comparing the threshold values for match specific productivity in the decentralized (equations (1.86) and (1.87)) and the constrained efficient allocation (equations (1.82) and(1.83)) evaluated under the standard Hosios condition ($\eta = \xi$), it follows that the threshold values for match specific productivity in the decentralized allocation do not replicate those of the constrained efficient allocation in the presence of skill loss. □

Note that also the vacancy creation equation is no longer constrained efficient under the standard Hosios condition ($\eta = \xi$), which is in line with the finding in the model without match specific productivity.

In contrast to the finding in the model without match specific productivity, the congestion externality is no longer internalized under the standard Hosios condition ($\eta = \xi$). For the case without match specific productivity, this parameter condition still internalizes the congestion externality because the output effect caused by this externality can be expressed as a fraction of part of the worker's outside option. When not all workers have the same hiring probability, this is no longer the case (see third term of expression (1.81) and second and third term of expression (1.85)). Consequently, internalizing the worker's outside option up to a fraction ξ does no longer internalize the congestion externality.

1.11 Figures

Figure 1.1: Role of the human capital depreciation rate in shaping labor market outcomes

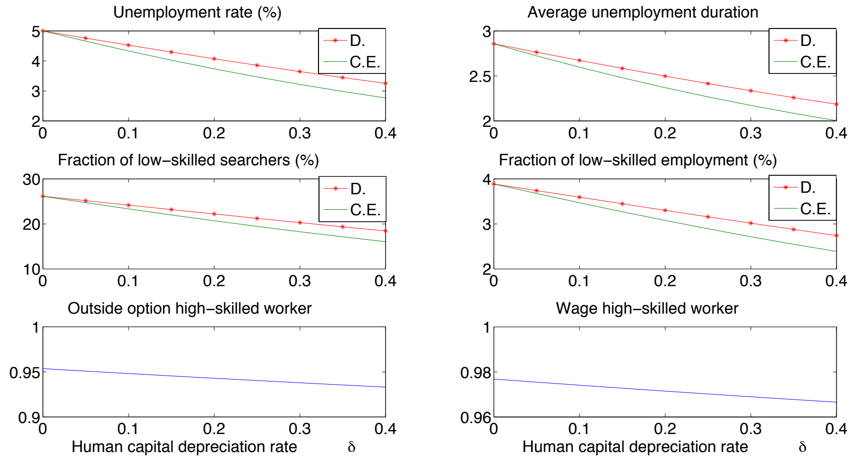


Figure 1.2: Role of the probability of skill loss in shaping labor market outcomes

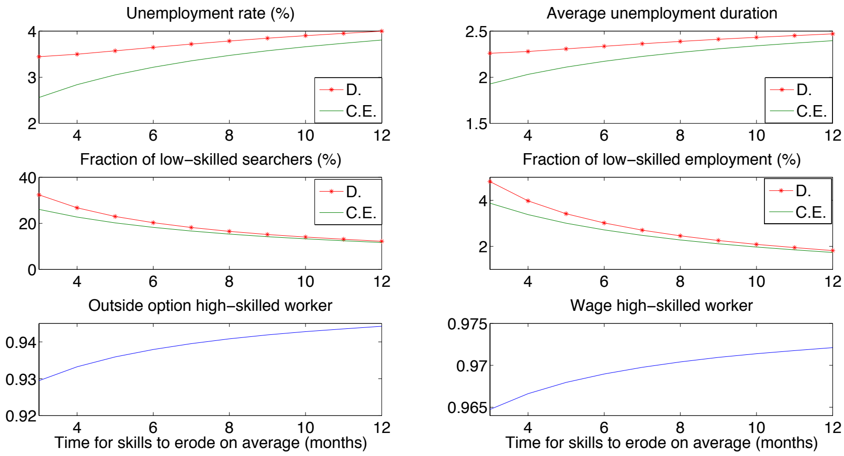


Figure 1.3: Role of the probability of regaining skills in shaping labor market outcomes

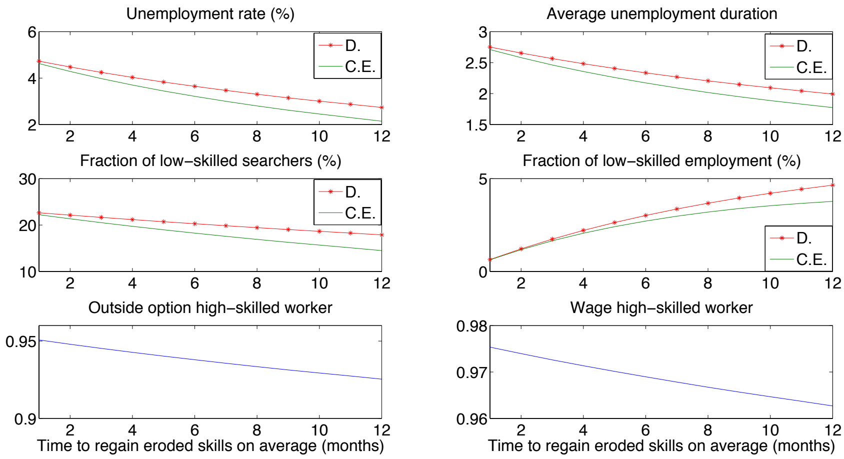


Figure 1.4: Role of the human capital depreciation rate in shaping labor market dynamics

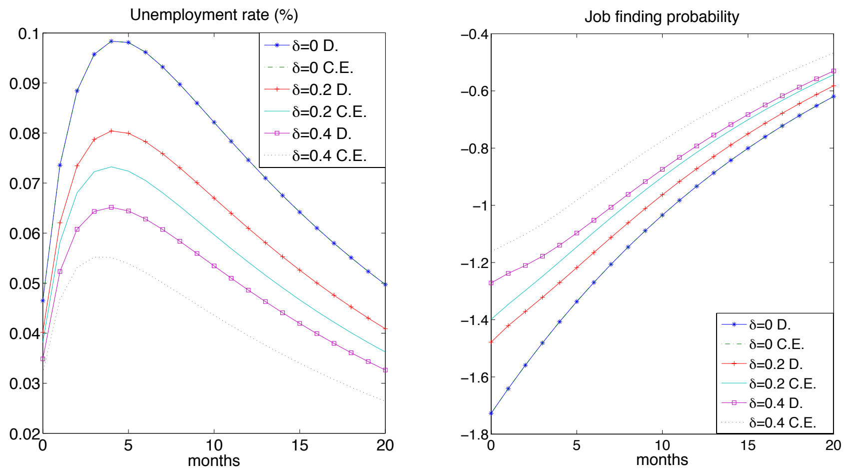


Figure 1.5: Role of the probability of skill loss in shaping labor market dynamics

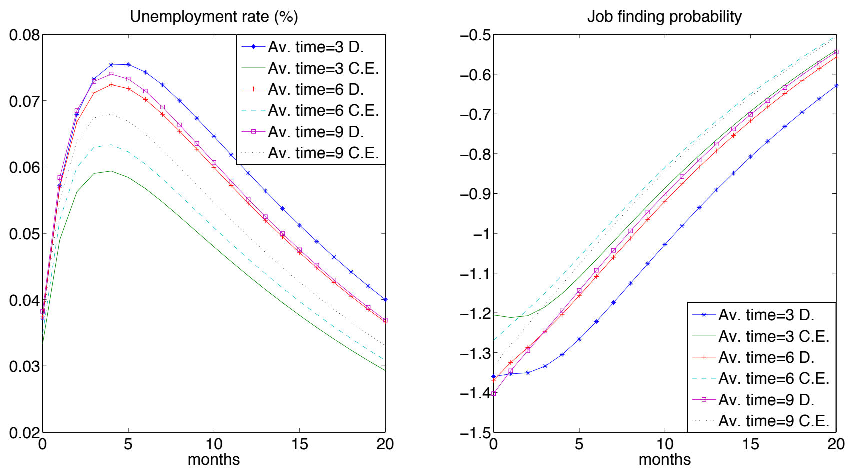


Figure 1.6: Role of the probability of regaining skills in shaping labor market dynamics

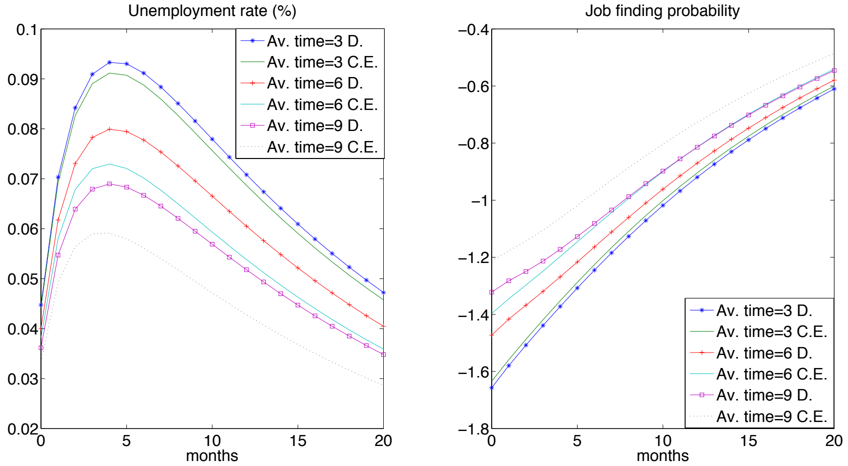
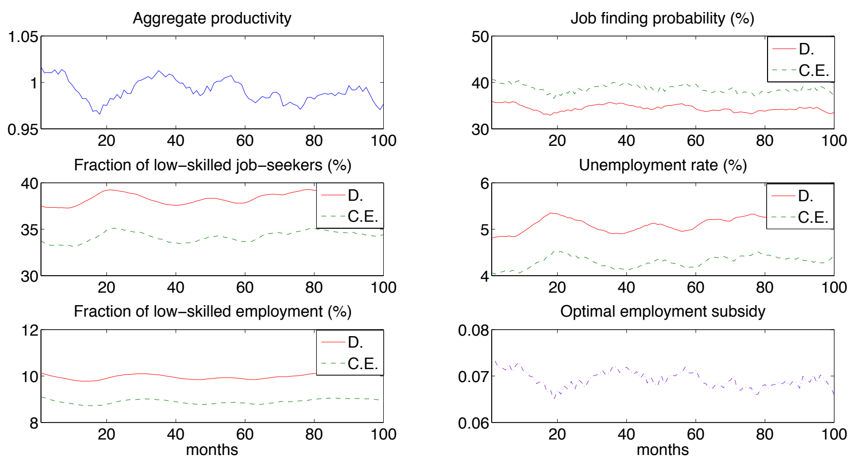


Figure 1.7: Labor market dynamics in the decentralized versus the constrained efficient allocation for $1/l = 4$



Chapter 2

OPTIMAL MONETARY POLICY IN THE PRESENCE OF HUMAN CAPITAL DEPRECIATION DURING UNEMPLOYMENT

2.1 Introduction

During the Great Recession the average unemployment duration increased substantially in many countries around the globe. In the U.S., for example, it increased from around 15 weeks in 2007 to a striking 40 weeks in 2011. One of the reasons why policy makers are concerned about those long unemployment spells is the widely held belief that workers' skills erode during periods of unemployment. This raises the question about how the presence of skill erosion during unemployment should affect the design of policy.

In this paper, I look at how the prescription for conducting monetary policy changes once it is taken into account that the unemployed are exposed to skill erosion. I introduce skill erosion during unemployment into an otherwise standard New Keynesian model with search frictions in the labor market and fully flexible wages. Skill erosion is modeled such that workers face the risk of losing a fraction of their productivity when being unemployed. So workers who have suffered from skill deterioration are less productive upon re-employment than workers who have not been affected by it. At the same time, workers can regain their initial skill level while being employed through learning-by-doing.

Optimal monetary policy is potentially affected because it might no longer be desirable from a social point of view to replicate the flexible price allocation even under the standard Hosios condition. This in contrast to the case of no skill erosion. [1] The reason is that skill loss during unemployment generates an externality in job creation. Firms ignore how their hiring decisions influence the skill composition of the unemployment pool and hence the expected productivity of new hires. As a result, the flexible price allocation is no longer constrained efficient when unemployed workers face the possibility of losing some of their skills even under the standard Hosios condition. Thus, the presence of skill erosion during unemployment generates a trade-off for the monetary policy maker.

When I analyze a calibrated model quantitatively, I find that even though optimal price inflation is no longer zero, deviations from it are almost negligible. Consequently, the prescription for the conduct of monetary policy does not change much when it is taken into account that the unemployed are exposed to skill erosion: optimal monetary policy stays close to strict inflation targeting.

This paper reinforces the literature's finding that search-related distortions in the labor market only call for small deviations from zero inflation. So far the search-related distortion on which the literature has focused is the familiar congestion externality associated with search frictions, see e.g. Faia (2009), Ravenna and Walsh (2011), and Ravenna and Walsh (2012a). This paper shows that optimal monetary policy stays close to

[1] See Thomas (2008), and Ravenna and Walsh (2011).

strict inflation targeting also in the presence of another type of search-related distortion in the labor market, namely human capital depreciation during unemployment.

The remainder of the paper is organized as follows. Section 2.2 outlines the model. Section 2.3 shows that the natural allocation is not constrained efficient in the presence of skill erosion during unemployment. Section 2.4 discusses the trade-offs faced by the monetary policy maker. Section 2.5 shows the economy's responses under the optimal monetary policy plan. Section 2.6 relates this paper's finding to the literature. Finally, section 2.7 concludes.

2.2 The Model

The economy consists of a continuum of infinitely-lived workers represented by the unit interval who form part of a representative household. The household's utility depends on the consumption of home produced goods and a variety of market goods. The latter are sold in a market characterized by monopolistic competition. The firms operating in this market adjust their prices in a staggered way. Those goods are produced by using intermediate goods, which in turn are produced by firms operating in a competitive environment. Intermediate good firms use labor as input, and recruit their workers in a market with search frictions à la Diamond-Mortensen-Pissarides. Note that the introduction of final and intermediate good firms allows for the separation of the two main frictions in the model, namely sticky prices and labor market frictions.[2]

Since the labor market is characterized by search frictions, in every period some of the household members will be unemployed. In the presence of skill erosion during unemployment, those unemployed workers face the risk of losing a fraction of their skills. At the same time, I allow for learning-by-doing such that those workers with eroded skills can

[2]This approach has been adopted by e.g. Blanchard and Galí (2010), Ravenna and Walsh (2008), Thomas (2008), and Walsh (2005)

regain them while being employed. To keep the analysis simple, workers' human capital can only take two values, and is either high (H) or low (L).[3] A worker's human capital determines her productivity: high-skilled workers have high productivity, whereas low-skilled workers have low productivity. The transition between skill types occurs as follows. In each period, an unemployed high-skilled worker becomes low-skilled with probability $l \in (0, 1]$. Thus the longer a worker's unemployment duration, the larger the chance that her human capital has depreciated. At the same time, when being low-skilled, she can regain her productivity while being employed through learning-by-doing. In each period, an employed low-skilled worker becomes high-skilled with probability $g \in (0, 1]$.

2.2.1 Labor Market

I assume that both workers with and without eroded skills search for jobs in the same market. Thus when a firm opens a vacancy at cost $\kappa > 0$, both worker types can apply to this job opening. Since a firm meets at most one worker at each round of interviews, an interview leads to successful hiring conditional on the match surplus being non-negative. In every period, the total number of interviews in the economy is determined by a matching function. This function is assumed to be strictly increasing and concave in both arguments and to display constant returns to scale. It is given by

$$m(v_t, u_t) = Bv_t^{1-\xi} u_t^{\xi}$$

where B represents the efficiency of the matching process, $1 - \xi$ is the elasticity of vacancies, v_t is the total number of vacancies posted by firms at time t, and u_t is the total number of job-seekers weighted by their search effectiveness. Because I assume that the unemployment duration does not affect workers' search effectiveness, and normalizing search effectiveness to one, the relevant measure of job-seekers in the matching function is given by the total number of unemployed. The latter is defined as the sum of high-skilled $\left(u_t^H\right)$ and low-skilled $\left(u_t^L\right)$ unemployed

[3]In what follows I use the term "skill" and "human capital" interchangeably.

workers

$$u_t \equiv u_t^H + u_t^L \tag{2.1}$$

Labor market tightness θ is defined as follows

$$\theta_t \equiv \frac{v_t}{u_t} \tag{2.2}$$

The probability for a firm posting a vacancy to meet a job-seeker is denoted by q_t and defined as

$$q_t \equiv \frac{m\left(v_t, u_t\right)}{u_t} = B{\theta_t}^{-\xi} \tag{2.3}$$

where q_t is decreasing in labor market tightness. The probability that a job-seeker gets a job interview is denoted by p_t and given by

$$p_t \equiv \frac{m\left(v_t, u_t\right)}{u_t} = B{\theta_t}^{1-\xi} \tag{2.4}$$

where p_t is increasing in labor market tightness. The job finding probability is the same for both worker types because the length of an unemployment spell has no effect on search effectiveness. When the match surplus is non-negative for both skill types, workers also have the same hiring probability. This follows from the assumption, which is standard for this representation of the labor market, that each firm meets at most one worker at each round of interviews.

The timing is as follows. At the beginning of the period hiring takes place after which both the existing and newly hired workers start producing.[4] After production some workers change type: unemployed high-skilled workers become low-skilled with probability l, and employed low-skilled workers become high-skilled with probability g. Finally, exogenous separation takes place, and a fraction γ of the matches breaks up. Given this timing, the law of motion for high and low-skilled job-seekers respectively is given by

$$u_t^H = (1-l)\left(1-p_{t-1}\right)u_{t-1}^H + \gamma\left(n_{t-1}^H + g n_{t-1}^L\right)$$

[4] This timing assumption has become standard in the business cycle literature, see e.g. Blanchard and Galí (2010).

$$u_t^L = (1 - p_{t-1})\left(u_{t-1}^L + l u_{t-1}^H\right) + \gamma (1-g)\, n_{t-1}^L$$

The above expression shows that the high-skilled searchers at time t are all the high-skilled job-seekers who remained unemployed at time $t-1$ and have not lost their skills which happens with probability $1-l$, and all the high-skilled workers who just got fired. The latter are on the one hand those who were operating at time $t-1$ as high-skilled workers, and on the other hand those who were low-skilled but regained their skills because of learning-by-doing which happens with probability g. Similarly, the low-skilled searchers at time t are previous period's unemployed low-skilled workers and high-skilled workers who have lost some of their skills, and all the low-skilled workers who were employed at time $t-1$ but did not regain skills and just lost their job.

The law of motion for high-skilled and low-skilled employment respectively is given by

$$n_t^H = (1-\gamma)\left[n_{t-1}^H + g n_{t-1}^L\right] + p_t u_t^H \tag{2.5}$$

$$n_t^L = (1-\gamma)(1-g)\, n_{t-1}^L + p_t u_t^L \tag{2.6}$$

So high-skilled employment is given by the high-skilled and low-skilled employees with regained skills who kept their job, and the high-skilled new hires. Similarly, the low-skilled employed are on the one hand those who did neither regain skills nor got fired, and on the other hand the newly hired low-skilled workers.

2.2.2 Households

I assume a representative household which consists of a continuum of infinitely-lived members represented by the unit interval. A fraction of the household members are employed, where some are high-skilled workers earning the real wage W_t^H and some are low-skilled workers earning the real wage W_t^L. Whether workers are high-skilled or low-skilled depends on their employment history. The unemployed workers generate a value b because they engage in home production.[5] The latter is assumed

[5]This approach is used by Ravenna and Walsh (2008, 2011, 2012 (a) and 2012 (b)).

to be independent of the worker's type. Following Merz (1995), I assume perfect insurance of unemployment risk. All workers pool their income, and hence they all enjoy the same total consumption. This has become the standard approach in the literature. Household's market goods' consumption C_t consists of a basket of differentiated goods defined by the Dixit-Stiglitz aggregator

$$C_t \equiv \left[\int_0^1 C_t(k) dk \right]^{\frac{\varepsilon}{1-\varepsilon}}$$

where $C_t(k)$ represents the quantity of final good k consumed by the household, and ε is the elasticity of substitution between goods. Denoting the price of the respective good by $P_t\,(k)$, and assuming that there is a continuum of differentiated goods on the unit interval, total consumption expenditure is given by $\int_0^1 P_t\,(k)\,C_t(k)dk$. Maximizing total market goods' consumption for any given level of expenditure implies that total expenditure equals $P_t C_t$, where P_t is an aggregate price index

$$P_t \equiv \left[\int_0^1 P_t(k)^{1-\varepsilon} dk \right] d^{\frac{1}{1-\varepsilon}}$$

Note that this leads to the following demand schedule for each final good

$$C_t\,(k) = \left(\frac{P_t\,(k)}{P_t} \right)^{-\varepsilon} C_t \tag{2.7}$$

The household's problem is to choose market goods' consumption and bond holdings in every period such as to maximize the following objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t\, U\left(C_t^T\right)$$

subject to the period by period budget constraint

$$P_t C_t^T + B_t \leq \left(1 + r_{t-1}^n\right) B_{t-1} + P_t \left[\, n_t^H\, W_t^H +\; n_t^L\, W_t^L + b\,(1 - n_t)\right] + T_t \tag{2.8}$$

where $\beta \in (0, 1)$ is the discount factor; $U(.)$ is the utility function which is assumed to be increasing and concave in its argument; $C_t^T \equiv C_t + b(1 - n_t)$ defines total consumption, being the sum of market goods' consumption and home production; $n_t \equiv n_t^L + n_t^H$ represents total employment, and when normalizing the size of the total labor force to one and abstracting from the labor market participation decision, total unemployment is given by $1 - n_t$; B_t are purchases of one period nominal bonds; r_t^n is the nominal interest rate which determines the return on bonds; and T_t represents the lump-sum component of income such as dividends from ownership of firms.

The household's problem gives rise to the standard Euler equation for consumption

$$U'(C_t^T) = \beta(1 + r_t^n)\ E_t\left\{U'(C_{t+1}^T)\frac{P_t}{P_{t+1}}\right\}$$

2.2.3 Intermediate Good Firms

I assume a continuum of intermediate good firms represented by the unit interval and operating in a perfectly competitive market. The intermediate good firms produce a homogeneous good which is sold at the price P_t^I to the final good firms. Each firm $j \in [0, 1]$ faces the production function

$$X_{j,t} = A_t\, n_{j,t}^e \tag{2.9}$$

where $X_{j,t}$ is the amount of the intermediate good produced by firm j, and A_t is the aggregate level of technology which follows the process

$$log\,(A_t) = \rho_a log\,(A_{t-1}) + \varepsilon_t \tag{2.10}$$

where $\varepsilon_t \sim iid\,(0, \sigma_a)$, and $n_{j,t}^e$ is firm j's effective labor input which is defined as

$$n_{j,t}^e \equiv n_{j,t}^H + (1 - \delta)n_{j,t}^L$$

The above expression implies that a worker's contribution to total output depends on the worker's skill level. The weight of a high-skilled worker is normalized to one, whereas that of a low-skilled worker is given by $1 - \delta$ where δ can be interpreted as the rate of human capital depreciation.[6]

The firm's problem consists of choosing the effective labor force, and the number of vacancies to post such as to maximize the objective function

$$E_0 \sum_{t=0}^{\infty} \beta_{0,t} \left(\frac{P_t^I}{P_t} A_t \left(n_{j,t}^H + (1-\delta)\, n_{j,t}^L \right) - n_{j,t}^H\, W_{j,t}^H - n_{j,t}^L\, W_{j,t}^L - \kappa v_{j,t} \right)$$

subject to the law of motion of high-skilled and low-skilled employment at the firm

$$n_{j,t}^H = (1-\gamma) \left(n_{j,t-1}^H + g n_{j,t-1}^L \right) + v_{j,t} q_t \,(1 - s_t) \qquad (2.11)$$

$$n_{j,t}^L = (1-\gamma)\,(1-g)\, n_{j,t-1}^L + v_{j,t} q_t s_t \qquad (2.12)$$

where $\beta_{0,t} \equiv \beta \frac{U'(C_t^T)}{U'(C_0^T)}$ is the stochastic discount factor, and where $s_t \equiv \frac{u_t^L}{u_t}$ represents the fraction of low-skilled job-seekers in the unemployment pool. The firm's profit at time t is given by the total real revenue product minus the total real cost. The latter contains two parts: the total wage cost and the spending on recruitment. By spending resources on recruitment the firm can adjust the existing workforce. Equation (2.11) shows that high-skilled employment at time t is given by last period's high-skilled workers who survived separation, last period's low-skilled workers who regained skills and survived separation, and the high-skilled new hires. Similarly, equation (2.12) shows that the number of low-skilled employed workers is given by those workers who remain employed and did not regain their skills and the low-skilled new hires. Whether the firm will end up recruiting high-skilled or low-skilled workers depends both on the probability that a vacancy gets filled (q) and the fraction of the respective job-seeker type in the unemployment pool (s).

[6] The interpretation that workers who have suffered from human capital depreciation during unemployment are less productive upon re-employment has also been used by Pissarides (1992).

I define the lagrange multiplier on constraint (2.11) and (2.12) as $\lambda_{j,t}$ and $\varphi_{j,t}$ respectively, where $\lambda_{j,t}$ represents the real marginal value of employing a high-skilled worker and $\varphi_{j,t}$ represents the real marginal value of employing a low-skilled worker. The first order conditions with respect to $v_{j,t}$, $n_{j,t}^H$ and $n_{j,t}^L$ are given by

$$\frac{\kappa}{q_t} = (1 - s_t)\,\lambda_{j,t} + s_t \varphi_{j,t} \tag{2.13}$$

$$\lambda_{j,t} = Z_t^H - W_{j,t}^H + (1-\gamma)\,E_t\left\{\beta_{t,t+1}\lambda_{j,t+1}\right\} \tag{2.14}$$

$$\varphi_{j,t} = Z_t^L - W_{j,t}^L + (1-\gamma)\,E_t\left\{\beta_{t,t+1}\left[(1-g)\,\varphi_{j,t+1} + g\lambda_{j,t+1}\right]\right\} \tag{2.15}$$

where $Z_t^H \equiv \frac{P_t^I}{P_t} A_t$ and $Z_t^L \equiv \frac{P_t^I}{P_t}(1-\delta)\,A_t$ represent the marginal revenue product of a high-skilled and low-skilled worker respectively. Note that the marginal value of having a specific worker type employed is independent of the size of the firm because of the constant returns to scale production function.

Equation (2.13) shows that a firm posts vacancies such that the expected hiring cost (LHS) equals the expected gain from vacancy posting (RHS). The latter depends on the expected real marginal value of a new hire, where the weight of each worker type is given by its share in the unemployment pool because both worker types have the same hiring probability. Equation (2.14) reflects that the real marginal value of employing a high-skilled worker equals the real marginal revenue product generated by that worker taking into account the real wage cost, and the value generated by employing that worker in period $t+1$ when the match survives separation. Just as for the high-skilled worker, the firm's real marginal value of employing a low-skilled worker depends on the real marginal revenue product generated by that worker and her wage cost. However, as can be seen from equation (2.15), the firm also takes into account that when this worker remains employed in the next period, she will have regained her skills with probability g and will generate the value of a high-skilled worker.

The total number of vacancies posted in the economy is $v_t = \int_0^1 v_{j,t}\,dj$.

2.2.4 Final Good Firms

I assume a continuum of final good firms represented by the unit interval. Each final good firm faces the production function

$$Y_{k,t} = X_{k,t} \tag{2.16}$$

where $Y_{k,t}$ is the final good produced by firm k, and $X_{k,t}$ is the amount of intermediate good used as input by firm k. So the production function implies a one to one transformation of the intermediate good into a final good.

Final good firms operate in a monopolistically competitive market. I assume sticky prices à la Calvo (1983) such that every period only a fraction $1 - \theta_p$ of the final good firms can reset their prices, whereas the remaining fraction θ_p keeps their prices unchanged. Since all firms face the same problem, all those firms who can reset their price will choose the same one. Therefore, I drop the subscript k in what follows to ease notation. Given that the firm's nominal marginal cost is the price of an intermediate good P_t^I, when a final good firm is able to reset its price, the firm chooses the optimal price $P_t^\star$ such as to maximize

$$\sum_{l=0}^{\infty} \theta_p^l E_t \left\{ \widetilde{\beta}_{t,t+l} \left(P_t^\star - P_{t+l}^I \right) Y_{t+l|t} \right\}$$

subject to the demand for the good

$$Y_{t+l|t} = Y_{t+l|t}^d = \left(\frac{P_t^\star}{P_{t+l}} \right)^{-\varepsilon} \left(C_{t+l} + \kappa v_{t+l} \right)$$

where $\widetilde{\beta}_{t,t+l} \equiv \beta \frac{U'(C_{t+l}^T)}{U'(C_t^T)} \frac{P_t}{P_{t+l}}$ is the stochastic discount factor for nominal payoffs; $Y_{t+l|t}$ is the output produced at time $t + l$ when the firm last reset its price at time t, where the latter should equal the demand for that good to ensure market clearing; and P_{t+l} is the aggregate price level at time $t + l$. Note that each final good firm's demand consists of two parts: households' demand and intermediate good firms' demand. The latter follows from the assumption that the vacancy posting cost κ is in terms of the final good. Note that the demand schedule follows from the problem of choosing the optimal consumption basket for any given level of expenditure, where it has been assumed that the price elasticity of substitution ε is the same for both households and intermediate good firms.

The optimal price setting rule for firm i resetting its price in period t is given by

$$\sum_{l=0}^{\infty} \theta_p^l E_t \left\{ \widetilde{\beta}_{t,t+l} \, Y_{t+l|t} \, \left(P_t^{\star} - \mu \, P_{t+l} \, MC_{t+l}\right) \right\} = 0 \tag{2.17}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon-1}$ is the gross desired markup, and $MC_{t+l} \equiv P_{t+l}^I / P_{t+l}$ is the real marginal cost.

2.2.5 Wages

Wages are assumed to be renegotiated in every period between the household and the firm. Following the literature, wages are set such that the surplus generated by an established employment relationship is shared between the household and the firm. The share of the surplus that each of them receives depends on their respective bargaining power. Given that all intermediate good firms face the same problem I drop the subscript j in what follows to ease notation.

The household's value, expressed in terms of consumption, of having an additional member of type i employed $(\mathcal{E}_t^i)$ is given by

$$\mathcal{E}_t^H = W_t^H + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{c} (1-\gamma+\gamma p_{t+1}) \, \mathcal{E}_{t+1}^H \\ +\gamma(1-p_{t+1}) \mathcal{U}_{t+1}^H \end{array} \right] \right\} \tag{2.18}$$

$$\mathcal{E}_t^L = W_t^L + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{l} \underbrace{g}_{regaining} \begin{pmatrix} (1-\gamma+\gamma p_{t+1})\,\mathcal{E}_{t+1}^H \\ +\gamma(1-p_{t+1})\mathcal{U}_{t+1}^H \end{pmatrix} \\ + \underbrace{(1-g)}_{no\,regaining} \begin{pmatrix} (1-\gamma+\gamma p_{t+1})\,\mathcal{E}_{t+1}^L \\ +\gamma(1-p_{t+1})\mathcal{U}_{t+1}^L \end{pmatrix} \end{array} \right] \right\} \tag{2.19}$$

The value of being employed at time t depends on the wage and next period's value. Equation (2.18) and (2.19) show that both worker types will continue being employed when the worker does not get fired or when the worker gets fired and immediately rehired. If not, the worker will be unemployed, where $\mathcal{U}_t^i$ denotes the value of being unemployed and is defined below. In the presence of learning-by-doing workers with eroded skills also take into account that being employed today enables them to regaining their skills. Thus, as can be seen from equation (2.19), next period's value for the low-skilled workers does not only depend on their employment status but also on whether they regained skills.

The household's value, expressed in terms of consumption, of having an additional member of type i unemployed at the end of the period after hiring took place ($\mathcal{U}_t^i$) is given by

$$\mathcal{U}_t^H = b + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{l} \underbrace{(1-l)}_{no\,loss} \left(p_{t+1}\mathcal{E}_{t+1}^H + (1-p_{t+1})\mathcal{U}_{t+1}^H\right) \\ + \underbrace{l}_{loss} \left(p_{t+1}\mathcal{E}_{t+1}^L + (1-p_{t+1})\mathcal{U}_{t+1}^L\right) \end{array} \right] \right\} \tag{2.20}$$

$$\mathcal{U}_t^L = b + E_t \left\{ \beta_{t,t+1} \left[p_{t+1}\mathcal{E}_{t+1}^L + (1-p_{t+1})\mathcal{U}_{t+1}^L \right] \right\}$$

The above expressions show that for both worker types the value of being unemployed is a function of the value generated through home production and next period's value. Today's unemployed workers can either become employed or remain unemployed in the next period. However, the presence of skill erosion during unemployment makes high-skilled workers take into account that being unemployed might lead to skill erosion, which can be seen from equation (2.20). If their skills erode, which happens with probability l, they will be searching for jobs as low-skilled workers.

The household's surplus, expressed in terms of consumption, for having an additional member of type i in an established employment relationship, defined as $\mathcal{H}_t^i \equiv \mathcal{E}_t^i - \mathcal{U}_t^i$, is given by

$$\mathcal{H}_t^H = W_t^H - b + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{l} (1-\gamma+\gamma p_{t+1})\, \mathcal{H}_{t+1}^H \\ -p_{t+1} \left(l\, \mathcal{H}_{t+1}^L + (1-l)\ \mathcal{H}_{t+1}^H \right) \\ +l \left(\mathcal{U}_{t+1}^H - \mathcal{U}_{t+1}^L \right) \end{array} \right] \right\} \tag{2.21}$$

$$\mathcal{H}_t^L = W_t^L - b + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{l} (1-\gamma+\gamma p_{t+1}) \left(g\, \mathcal{H}_{t+1}^H + (1-g)\, \mathcal{H}_{t+1}^L \right) \\ -p_{t+1} \mathcal{H}_{t+1}^L \\ +g \left(\mathcal{U}_{t+1}^H - \mathcal{U}_{t+1}^L \right) \end{array} \right] \right\} \tag{2.22}$$

Note that the value for the firm of having a high-skilled and low-skilled worker employed is given by λ_t and φ_t respectively.

The surplus generated by an employment relationship with a high and low-skilled worker is given by $M_t^H \equiv \mathcal{H}_t^H + \lambda_t$ and $M_t^H \equiv \mathcal{H}_t^L + \varphi_t$ respectively. Defining the household's bargaining power by η implies

$$\mathcal{H}_t^i = \eta M_t^i \tag{2.23}$$

$$\lambda_t = (1-\eta)\, M_t^H \tag{2.24}$$

$$\varphi_t = (1-\eta)\, M_t^L \tag{2.25}$$

Combining the sharing rule (equations (2.23)-(2.25)) with the expression for the household's surplus (equations (2.21) and (2.22)) and the firm's surplus (equations (2.14) and (2.15)), gives the real wage for a worker of type i

$$W_t^i = \eta Z_t^i + (1-\eta)\, \mathcal{O}_t^i \tag{2.26}$$

where $\mathcal{O}_t^i$ represents the worker's outside option

$$\mathcal{O}_t^H \equiv b + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{l} \eta p_{t+1} \left((1-l)\, M_{t+1}^H + l M_{t+1}{}^L \right) \\ -\eta\gamma p_{t+1} M_{t+1}^H \\ -l \left(\mathcal{U}_{t+1}^H - \mathcal{U}_{t+1}^L \right) \end{array} \right] \right\}$$

$$\mathcal{O}_t^L \equiv b + E_t \left\{ \beta_{t,t+1} \left[\begin{array}{l} \eta p_{t+1} M_{t+1}^L \\ -\eta\gamma p_{t+1} \left(g M_{t+1}^H + (1-g) M_{t+1}^L \right) \\ -g \left(\mathcal{U}_{t+1}^H - \mathcal{U}_{t+1}^L \right) \end{array} \right] \right\}$$

The wage is such that workers get a part, determined by their bargaining power η, of the real marginal revenue product. Moreover, workers get partially, depending on the firm's bargaining power, compensated for their outside option. A worker's outside option at time t consists of time t's home production and the possibility of searching for a job in period $t+1$. Note that high-skilled workers take into account that if they had not been employed, they could have lost a fraction of their skills with probability l. At the same time, workers' outside option also reflects that even though workers have a job today, they might have a different job next period when they get fired and immediately rehired. Note that low-skilled workers' take into account that being employed today, enables them to regain their skills. Overall, the presence of skill erosion during unemployment affects the wage because it is reflected in the worker's outside option that the worker's employment status affects her skills.

2.2.6 Equilibrium

The economy's resource constraint can be derived as follows. Aggregate demand Y_t^d is given by the sum of households' total consumption of market goods and the total resources spent on vacancy creation by firms

$$Y_t^d = C_t + \kappa v_t$$

Market clearing implies that the demand of each final good firm k has to equal its supply, i.e. $Y_{k,t} = Y_{k,t}^d$. Given the production function of the final good firms (equation (2.16)), the production function of the intermediate good firms (equation (2.9)), and the demand schedule for final goods (equation (2.7)), market clearing implies

$$\int_0^1 Y_{k,t} dk = A_t \int_0^1 n^e_{j,t} dj = A_t \left(n_t^H + (1-\delta)\, n_t^L\right) = \int_0^1 Y^d_{k,t} dk$$
$$= \left(C_t + \kappa v_t\right) \int_0^1 \left(\frac{P_{k,t}}{P_t}\right)^{-\varepsilon} dk$$

Denoting total output as $Y_t \equiv \int_0^1 Y_{k,t} dk$, the resource constraint is given by

$$Y_t = A_t n_t^H + A_t (1-\delta)\, n_t^L = (C_t + \kappa v_t)\, \Delta_t$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_{k,t}}{P_t}\right)^{-\varepsilon} dk$ is a measure of price dispersion among final good firms.

Given that all intermediate good firms face the same problem, they all behave in the same way. Therefore, equilibrium job creation is obtained by dropping subscript j and combining equation (2.13), (2.24) and (2.25)

$$\frac{\kappa}{q_t} = (1-\eta)\left((1-s_t)\, M_t^H + s_t M_t^L\right)$$

This implies that job creation is such that the expected hiring cost (LHS) equals the expected gains from job creation (RHS). The latter depends on the expected match surplus generated by a new hire, taking into account the share of the surplus that the firms will obtain $1-\eta$. For an expression of the surplus generated by a high-skilled and low-skilled worker in equilibrium, see equation (2.56) and (2.57) in Appendix 2.8.1.

Finally, total net supply of bonds in the economy is zero.

Equilibrium in this economy is defined as the path

$$\{Y_t, C_t, C_t^T, n_t^H, n_t^L, n_t, \tilde{u}_t, p_t, q_t, \theta_t, u_t, u_t^L, u_t^H, s_t, v_t, W_t^H,$$
$$W_t^L, M_t^H, M_t^L, \mathcal{U}_t^H, \mathcal{U}_t^L, P_t, P_t^\star, \Pi_t, \Delta_t, x_t, z_t, MC_t, r_t^n\}_{t=0}^{\infty}$$

that satisfies equations (2.34)-(2.60) in Appendix 2.8.1 for all $t \geq 0$, given the evolution of the exogenous shock $\{\varepsilon_t\}_{t=0}^{\infty}$, the law of motion for aggregate technology (equation (2.10)), and an expression describing the conduct of monetary policy.

2.3 Job creation in the Presence of Skill Erosion

In the absence of skill erosion during unemployment, the decentralized allocation replicates the constrained efficient allocation when the distortions following from price stickiness and monopolistic competition are offset, and when the standard Hosios (1990) condition holds. The latter refers to the parameter condition for the workers' bargaining power under which the congestion externality following from search frictions in the labor market is fully internalized, i.e. the workers' bargaining power equals the elasticity of unemployment in the matching function ($\eta = \xi$). However, in the presence of skill erosion during unemployment, those conditions no longer imply constrained efficiency.

When workers' skills erode during unemployment the economy is characterized by a composition effect. The latter arises because job creation affects the skill composition of the unemployment pool. Hiring workers prevents those workers from being unemployed, and hence exposed to skill erosion. This composition effect gives rise to an externality because firms do not take into account how their hiring decisions today affect the skill composition of the unemployment pool in the next period, and hence the productivity of new hires.[7]

To gain more insight into how the presence of skill erosion during unemployment distorts job creation, I compare the job creation decision in the constrained efficient and the decentralized allocation in the absence of sticky prices.Throughout, I focus on the special case where $g = l = 1$,

[7] See Laureys (2012a) for a more detailed discussion about the composition externality.

which implies that a worker's productivity deteriorates with probability 1 after having been out of work for one period, and is restored with probability 1 after having worked for one period.The reason for doing so is because an Euler equation for job creation can be derived such that job creation in both allocations can be directly compared.

The constrained efficient allocation is obtained by solving the problem of a benevolent social planner who is subject to the same technological constraints and labor market frictions as in the decentralized allocation. The planner's problem is outlined in Appendix 2.8.2. In the decentralized allocation in the absence of sticky prices, i.e. in the economy's natural allocation, final good firms are able to reset their price in every period. Optimal price setting implies that each final good firm sets his price in every period as a constant markup over its nominal marginal cost. Taking into account that the nominal marginal cost of each final good firm is given by the price of the intermediate good P_t^I, optimal price setting under fully flexible prices implies

$$\frac{P_t^I}{P_t} = \frac{1}{\mu}$$

Additionally, I assume that an appropriate subsidy τ, financed through lump-sum taxation, is implemented to offset the distortion related to monopolistic competition, implying that $\frac{1}{\mu(1-\tau)} = 1$. Note that the marginal revenue product of a high-skilled and low-skilled worker respectively, is now given by

$$Z_t^H = A_t$$

$$Z_t^L = (1 - \delta) A_t$$

Combining the first order conditions of the intermediate good firm's problem (equations (2.13)-(2.15)) with the relevant expressions implied by the wage setting (equations (2.24)-(2.26)), and imposing the standard Hosios condition ($\eta = \xi$), gives rise to the following expression for job creation[8]

[8]See Appendix 2.8.2 for a detailed description of the derivation.

$$\frac{\kappa}{q_t} = (1-\xi)\left[A_t(1-\delta s_t) - b + E_t\left\{\beta_{t,t+1}\Lambda^N_{t+1}\right\}\right] \tag{2.27}$$

$$E_t\left\{\Lambda^N_{t+1}\right\} \equiv E_t\left\{\begin{array}{l}(1-\gamma)\left(\frac{\kappa}{q_{t+1}(1-\xi)} - \xi\theta_{t+1}\kappa\right)\\ +(1-\gamma+\xi\gamma p_{t+1})\,\delta A_{t+1}s_{t+1}\\ +\xi p_{t+1}\delta A_{t+1}(1-s_{t+1})\end{array}\right\} \tag{2.28}$$

Job creation in the constrained efficient allocation is given by

$$\frac{\kappa}{q_t} = (1-\xi)\left[A_t(1-\delta s_t) - b + E_t\left\{\beta_{t,t+1}\Lambda^P_{t+1}\right\}\right] \tag{2.29}$$

$$E_t\left\{\Lambda^P_{t+1}\right\} \equiv E_t\left\{\begin{array}{l}(1-\gamma)\left(\frac{\kappa}{q_{t+1}(1-\xi)} - \xi\theta_{t+1}\kappa\right)\\ +(1-\gamma+\gamma p_{t+1})\,\delta A_{t+1}s_{t+1}\\ +p_{t+1}\delta A_{t+1}(1-s_{t+1})\end{array}\right\} \tag{2.30}$$

Job creation in both allocations has the same overall structure: job creation is such that the expected hiring cost (LHS) equals the expected gains from job creation (RHS). The latter are given by the expected marginal revenue product of a new hire, i.e. $\bar{Z}_t \equiv (1-s_t)\,Z^H_t + s_t Z^L_t = A_t(1-\delta s_t)$, the loss in home production b, and the continuation value of an established employment relationship. Note that the expected marginal revenue product of a new hire is given by the output generated by an average job-seeker, where the respective weights are given by that worker's share in the unemployment pool because all job-seekers have the same hiring probability. In the absence of skill erosion during unemployment, i.e. for $\delta = 0$, the continuation value consists of the savings in vacancy posting costs when the match survives separation and a term representing the net impact on output generated by both the congestion effect of having a job-seeker less in the unemployment pool when the match survives separation and the worker's outside option. This net effect is represented by the first term in expression (2.28) and (2.30). However, in the presence of skill erosion, two additional terms arise reflecting the expected future output gains related to today's job creation. Those output gains follow from today's job creation enabling workers with eroded skills to regain them and preventing high-skilled workers from losing their skills. Next, I will discuss each part in detail.

First, as can be seen from the second term in expression (2.28) and (2.30), it is taken into account that in case the new hire continues producing in period $t+1$, today's job creation generates an output gain in period $t+1$ given by the difference between the marginal revenue product generated by this worker and an average job-seeker.[9] There is an output gain related to an established employment relationship with a high-skilled worker because if another worker were to be hired this worker would not necessarily be high-skilled. The less likely it is that a new hire would be high-skilled, i.e. the lower the expected fraction of high-skilled job-seekers in the unemployment pool, the higher the expected output gain. Second, it is taken into account that employing a worker, prevents this worker from being unemployed, and hence losing some of its skills. If the worker had not been hired in period t, the worker would have found a job in period $t+1$ with probability p_{t+1}. Given that the worker would have lost some of her skills during her unemployment experience, hiring this now low-skilled worker would create an output loss. This loss is given by the difference in the output generated by a low-skilled worker and the expected output of a new hire.[10] Therefore, the expected output gain from hiring a worker, and hence preventing a worker from skill loss, is smaller the more likely it is that a new hire would be a worker with eroded skills.

Comparing both allocations shows that the natural allocation is inefficient even if the standard Hosios condition holds.[11] More precisely, the

[9]Note that $E_t\left\{Z^H_{t+1}-\bar{Z}_{t+1}\right\}=E_t\left\{\delta A_{t+1}s_{t+1}\right\}$

[10]Note that $E_t\left\{Z^L_{t+1}-\bar{Z}_{t+1}\right\}=-E_t\left\{\delta A_{t+1}\left(1-s_{t+1}\right)\right\}$

[11]In the presence of skill erosion during unemployment, the congestion externality is still offset by the standard Hosios condition, i.e $\eta=\xi$. Given that all job-seekers have the same hiring probability, a change in labor market tightness will affect all job-seekers in the same way. Therefore, there is no interaction between the congestion effect and the composition effect, enabling the same condition to offset the congestion externality. For

expected output gains from today's job creation, through its effect on the skills of next period's job-seekers, are only taken into account up to a fraction of the workers' bargaining power. This follows from firms ignoring two issues. First, a firm ignores how its job creation affects the skills of those workers who are no longer employed by the firm in period $t + 1$. Second, a firm neglects that by not hiring a worker today, there will be an additional worker with eroded skills in the unemployment pool next period. These expected output gains still partially show up in the natural allocation through the wage setting mechanism. As has been discussed in section 2.2.5, the workers' outside option reflects that their employment status affects their skills, which in turn affects the wage and ultimately job creation.

In conclusion, the decentralized allocation is no longer constrained efficient when the distortions from price stickiness, monopolistic competition, and the congestion externality are offset. This is because in the presence of skill erosion during unemployment there are gains from job creation which are not internalized. Moreover, both the economy's response to shocks and its steady state outcomes are not constrained efficient.

2.4 Optimal Monetary Policy

2.4.1 Optimal Policy Plan

The optimal monetary policy plan is derived by solving the Ramsey problem.[12] The policy maker's problem consists of maximizing the welfare of

more details see Laureys (2012a).

[12]Faia (2009) solves the Ramsey problem in a New Keynesian model with steady state distortions caused by monopolistic competition and the standard congestion externality following from search frictions in the labor market.

the representative household given by the objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t \, log \left(C_t^T \right)$$

subject to the equations (2.33)-(2.60) (in Appendix 2.8.1) describing the equilibrium conditions of the economy.

In general, the monetary authority faces a problem of the following format

$$max_{\{e_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \, \left\{ U \left(e_t, \varepsilon_t \right) + \omega_t E_t \left[f \left(e_{t-1}, e_t, e_{t+1}, \varepsilon_t \right) \right] \right\}$$

where e_t is a vector of K endogenous variables, ε_t represents the exogenous shocks to the economy, ω_t is a vector of Lagrange multipliers on the $K-1$ constraints faced by the policy maker. The latter are given by the equilibrium conditions of the economy: $E_t \left[f \left(e_{t-1}, e_t, e_{t+1}, \varepsilon_t \right) \right] = 0$. This problem gives rise to a first order condition with respect to every endogenous variable, which is of the following general form

For $t = 0$

$$U_1 \left(e_0, \varepsilon_0 \right) + \omega_0 E_0 \left[f_2 \left(e_{-1}, e_0, e_1, \varepsilon_0 \right) \right] + \beta \omega_1 E_0 \left[f_1 \left(e_0, e_1, e_2, \varepsilon_1 \right) \right] = 0 \tag{2.31}$$

For $t \geq 0$

$$U_1 \left(e_t, \varepsilon_t \right) + \omega_t E_t \left[f_2 \left(e_{t-1}, e_t, e_{t+1}, \varepsilon_t \right) \right] + \beta^{-1} \omega_{t-1} f_3 \left(e_{t-2}, e_{t-1}, e_t, \varepsilon_t \right) + \beta \omega_{t+1} E_t \left[f_1 \left(e_t, e_{t+1}, e_{t+2}, \varepsilon_{t+1} \right) \right] = 0 \tag{2.32}$$

Comparing equation (2.31) and (2.32) shows that under commitment the policy is characterized by time inconsistency. At time 0, the policy maker's optimal behavior is determined by expression (2.31). Similarly, at time 1, the policy maker's optimal behavior under commitment is given by expression (2.32). Both expressions differ because at time 0 the Lagrange multiplier $\omega_{-1} = 0$, implying that the policy maker does not need to respect what the private agents at time -1 expected the policy maker to do.

Therefore, when a policy maker reoptimizes in period 1, the optimal behavior will again be determined by expression (2.31), updated to time 1, instead of by expression (2.32). In other words, the policy maker's optimal choice for e_{t+1} in period t differs from the optimal choice for e_{t+1} made when reoptimizing in period $t+1$ because once arrived in period $t+1$ there is no need to respect what the agents at time t were expecting the policy maker to do. To ensure that the policy maker will act according to equation (2.32) in all periods, including the current one, the timeless perspective approach can be adopted. This implies that the policy is chosen before time 0, sometime in the distant past. Therefore, the current allocation satisfies condition (2.32) because it is chosen from that earlier perspective.[13]

2.4.2 Trade-offs Faced by Policy Maker

The policy maker faces four sources of inefficiency: price stickiness, monopolistic competition, the congestion externality following from labor market frictions and the composition externality which arises because of skill erosion during unemployment. Given that the policy maker has only one instrument, it will in general not be possible to eliminate all four distortions. Thus the policy maker faces a trade-off between them. Below I discuss each of those frictions and their policy implications in more detail.

Price stickiness distorts the economy in the following way. If all firms could reset their price in response to shocks they would all set their price such as to achieve their constant desired markup. As a result, the economy's average markup in the absence of price stickiness would be constant over time. However, when prices are sticky, and hence not all firms can reset their prices, the economy's average markup will vary over time in response to shocks, making it deviate from the constant frictionless markup. Therefore, aggregate demand, and hence output and employment will either by too high or too low. Moreover, price stickiness also leads to price dispersion, which in turn leads to dispersion in demand.

[13]For more details see e.g. Walsh (2003)

This generates an inefficient allocation because it is optimal for all goods to be consumed and produced in the same amount. The symmetry of the optimal allocation follows from all goods entering in a symmetric way in the utility function which is concave in the consumption of those goods, and all final good firms facing the same production function. If all the economy's distortions besides price stickiness were to be eliminated by the use of other policy instruments, the constrained efficient allocation would coincide with the natural allocation. Thus, in this case it is optimal for the policy maker to replicate the latter, which can be done through strict inflation targeting. The finding that the policy maker faces no trade-off in a New Keynesian model where price stickiness is the only distortion is a well-known result in the literature.[14] Blanchard and Galí (2007) labeled this property *the divine coincidence*.

Each of the other sources of inefficiency, however, makes the natural allocation no longer coincide with the constrained efficient allocation. Hence, from a social point of view it might no longer be optimal to conduct a zero inflation policy and replicate the natural allocation. The latter becomes inefficient when one of the other distortions is present for the following reason. First, firms operating in a market characterized by monopolistic competition have some market power which makes them charge prices above their marginal cost. As a result, demand for the final goods, and hence output and employment in the natural allocation are too low from the planner's perspective. Second, the presence of search frictions in the labor market renders job creation and output inefficient when the congestion externality is not fully internalized. This happens if the standard Hosios condition $(\eta = \xi)$ does not hold. When the only

[14] In a New Keynesian model where price stickiness is the only distortion the policy maker faces no trade-off because stabilizing inflation implies the stabilization of the welfare relevant output gap. The latter refers to the difference between the output produced in the decentralized and the constrained efficient allocation. In such an environment, and in the absence of cost-push shocks, the constrained efficient allocation always coincides with the natural allocation. Therefore replicating the natural allocation by conducting a zero inflation policy automatically leads to the stabilization of the welfare relevant output gap. See e.g. Galí (2008) for a more detailed discussion about optimal monetary policy in a standard New Keynesian framework.

source of inefficiency is the congestion externality, job creation in the natural allocation is too high or too low depending on the relation between the worker's bargaining power (η) and the elasticity of job-seekers in the matching function (ξ). On the one hand, when $\eta < \xi$ job creation is too attractive for firms, causing job creation, and hence the supply of intermediate goods being too high. On the other hand, job creation is too low when $\eta > \xi$ because job creation is not attractive enough for firms. Finally, as discussed in section 2.3, the composition externality makes job creation, and hence output and employment in the natural allocation become inefficient because there are gains related to job creation which are not internalized.

Given that offsetting the distortion from price stickiness calls for zero inflation, whereas offsetting each of the other distortions requires time-varying inflation, the policy maker faces a trade-off between the economy's distortions. Consequently, I expect optimal monetary policy to deviate from strict inflation targeting in the presence of skill erosion during unemployment, even if the distortions from monopolistic competition and the congestion effect are eliminated through the use of an adequate subsidy and by imposing the standard Hosios condition.

2.5 Dynamics

In this section I present the economy's response under the optimal monetary policy plan when the economy is subject to aggregate technology shocks.[15] To gain insight into the trade-off for the policy maker generated by the composition externality, I offset the distortions from monopolistic competition and the standard congestion externality following from search frictions. The distortion related to monopolistic competition is shut down in the same way as in section 2.3, namely by assuming the imple-

[15]The economy's behavior under the optimal monetary policy plan is compute by using Dynare. The first order conditions characterizing the Ramsey problem, as outlined in section 2.4.1, are derived after which this system of equations characterizing the equilibrium is solved by first order perturbation.

mentation of an appropriate subsidy, whereas the congestion externality is internalized by imposing the standard Hosios condition ($\eta = \xi$).

2.5.1 Calibration

The length of a period is set to one quarter. I calibrate the model to the U.S. economy. Following the literature, I set the discount discount factor β to 0.99, the elasticity of substitution ε to 6, the parameter θ_p governing the degree of price stickiness to $2/3$, and the elasticity of unemployment in the matching function to 0.5. Workers' bargaining power η is also set to 0.5 such that the standard Hosios condition holds. Following Ravenna and Walsh (2011) the value of home production b is such that the replacement ratio equals 0.54. Given the two types of workers, I use the average wage in steady state to compute the replacement ratio: $\frac{b}{\widetilde{W}} = \psi$, where ψ denotes the replacement ratio and $\widetilde{W} \equiv \frac{N^H}{N^H+N^L}W^H + \frac{N^L}{N^H+N^L}W^L$. Following Ravenna and Walsh (2011), the steady state job filling probability $q(\theta)$ is set to 0.9 . Following Blanchard and Galí (2010) I set steady state employment n to 0.95, which implies an unemployment rate $\tilde{u} \equiv (1 - p(\theta))\, u$ of 0.05. Also following Blanchard and Galí (2010), I set the steady state job finding probability $p(\theta)$ to 0.7. This implies that the separation rate $\gamma = \tilde{u}p(\theta) / ((1-\tilde{u})(1-p(\theta)))$ equals 0.12. Given these values, the value for the efficiency of the matching function B can be obtained as follows. Steady state labor market tightness is given by $\theta = p(\theta)/q(\theta) = 0.778$. This in turn implies a value of $B = p(\theta)\,\theta^{\xi-1} = 0.794$. The value for the parameter governing the vacancy posting costs κ can be computed from the equilibrium conditions once the parameters for the skill loss process are determined. Following Ravenna and Walsh (2011), I set the standard deviation of the technology shock such that the standard deviation of output is 1.82 percent conditional on a policy of price stability. The autoregressive coefficient ρ_a is set to 0.95.

The parameters governing the skill loss process are δ, l, and g. The parameters δ and l determine the degree to which an unemployment spell

Variable	Variable	Value
Discount factor	β	0.99
Elasticity of Substitution	ε	6
Price stickiness	θ_p	$2/3$
Replacement ratio	$\frac{b}{W}$	0.54
Vacancy Elasticity of Matches	$1-\xi$	0.5
Bargaining power workers	η	0.5
Employment	n	0.95
Job finding rate	$p(\theta)$	0.7
Vacancy filling rate	$q(\theta)$	0.9

Table 2.1: Parameterization

erodes workers skills: the human capital depreciation rate δ determines how many skills a high-skilled worker loses conditional upon losing, whereas the probability that a high-skilled worker will lose some of its skills in each period that she spends in unemployment depends on l. The parameter g determines how long it takes on average for a worker with eroded skills to regain those skills. Given the absence of empirical evidence on those parameter values, I look at the economy's behavior for a range of parameter values. When I vary one of those parameters over a certain range, I keep the other parameters fixed at an arguably reasonable baseline value. In particular, I set $g = l = 0.5$ such that it takes on average 2 quarters for workers to lose and regain skills, and $\delta = 0.3$ which makes workers who have suffered from skill erosion 30% less productive than before skill loss.

2.5.2 Results

Figures 2.1-2.3 show the optimal volatility of inflation as a function of the rate of human capital depreciation (δ), the time it takes on average for workers' skills to erode $(1/l)$, and the time it takes on average for workers with eroded skills to regain them $(1/g)$. These figures depict the overall pattern: the more costly skill loss (i.e. the higher δ, the higher l, and the lower g), the more volatile is inflation under the optimal policy. However, in addition to this pattern, those figures also show that for all values of the parameters governing the skill loss process the optimal volatility of inflation stays very close to zero. In other words, if skill erosion during unemployment is the only source of inefficiency, optimal monetary policy stays close to strict inflation targeting.

This is also confirmed by figure 2.4 which depicts the impulse response functions of the economy both under the Ramsey optimal policy and a zero inflation policy when the economy is hit by a persistent negative aggregate technology shock causing an initial decrease of aggregate technology of 1% relative to its steady state value. The response of the unemployment rate and the fraction of low-skilled job-seekers in the unemployment pool is expressed as the absolute deviation from its steady state level (in percentage points), while the response of the real marginal cost and inflation is expressed as the relative deviation from its steady state level (in percent). It can be seen that even though optimal price inflation is no longer zero, it stays very close to it. This immediately explains why the impulse response functions of the unemployment rate and the fraction of low-skilled job-seekers in the unemployment pool nearly coincide under both policies. Under the optimal policy the real marginal cost goes down an impact. This can be explained when looking at the nature of the externality generated by skill erosion during unemployment. In Laureys (2012a) I show that the optimal labor market policy to offset this externality takes the form of a procyclical employment subsidy, reflecting that overall job creation is too low from a social point of view but less so in recessions than in booms. As can be seen from equation 2.13, job creation depends on the real marginal revenue product which is a function of the real marginal cost. A drop on impact of the real marginal cost decreases the gains from job creation. This in turn implies that the response of the marginal cost is in line with what is expected from the findings in Laureys (2012a). However, for the specific parameter values of the skill loss process the tiny drop on impact of the real marginal cost is not enough to generate a drop on impact of price inflation.

Finally, figures 2.5-2.7 show the impulse response functions of the economy under the Ramsey optimal policy for different values of the parameters governing the skill loss process. The economy is hit by a persistent negative aggregate technology shock causing an initial decrease of aggregate technology of 1% relative to its steady state value. These figures show, in line with figures 2.1-2.3, that optimal price inflation barely deviates from zero. Despite the finding that the real marginal cost drops on impact for all parameter values in line with the discussion above, price inflation either increases or decreases on impact depending on the parameter values governing the skill loss process. This is because the response on impact of price inflation depends on the path of the real marginal cost. It can also be seen that the responses of the unemployment rate and the fraction of low-skilled job-seekers in the unemployment pool depend on the characteristics of the skill loss process.

2.6 Relation to the Literature

This paper finds that in the presence of skill erosion during unemployment, and flexible wages, optimal monetary policy stays close to strict inflation targeting. This result is in line with the finding in the literature that search-related distortions in the labor market only call for small deviations from zero inflation. In contrast, optimal monetary policy is no longer close to a zero inflation policy when labor market distortions are related to wage rigidity. Thomas (2008) builds a New Keynesian framework with labor market frictions where the Hosios condition holds such that both the steady state and the unemployment fluctuations are efficient in the natural allocation. He finds that optimal monetary policy deviates from strict inflation targeting when nominal wage bargaining is staggered instead of flexible. The reason is that by allowing for inflation real wages can be brought closer to their flexible wage counterpart. In a similar setup, Blanchard and Galí (2010) find that the presence of real wage rigidity also calls for deviations from zero inflation. Even though the policy maker can no longer bring wages closer to their flexible wage counterpart, by allowing for inflation hiring incentives can be affected. This in turn reduces the economy's welfare losses.

The literature has only focused on the search-related distortion in the labor market following from a failure of the standard Hosios condition to hold. Faia (2009) analyzes optimal monetary policy in an economy characterized by distortions from monopolistic competition, quadratic costs of price adjustment, and matching frictions in the labor market under deviations from the Hosios condition. She finds that under the Ramsey optimal policy the deviation of price inflation from zero should be larger, the higher the workers' bargaining power relative to the elasticity of unemployment in the matching function. This finding follows from the incentives for firms to post vacancies becoming smaller when the workers' bargaining power increases, which makes unemployment fluctuate above its constrained efficient level. However, those optimal deviations from zero inflation are small. Ravenna and Walsh (2011) use the linear-quadratic approach to compute optimal monetary policy in an economy with sticky prices à la Calvo, matching frictions in the labor market, and an efficient steady state. The trade-off for the policy maker, and hence the potential deviation from zero inflation is generated by the presence of shocks to workers' bargaining power. Those shocks imply a deviation from the Hosios condition, which makes job creation in the natural allocation inefficient. They find that the labor market structure has important implications for optimal monetary policy in the sense that ignoring the structure of the labor market, and hence implementing policy rules based on an incorrect perception of the nature of the welfare costs generated by labor market frictions, might lead to important welfare losses. However, they also find that zero inflation is nearly optimal.

Ravenna and Walsh (2012a) have analyzed why a zero inflation policy remains close to being the optimal policy even though the presence of search frictions in the labor market can lead to significant welfare losses. They argue that optimal monetary policy deviates little from strict inflation targeting because monetary policy is not the appropriate instrument to address the inefficiency arising from a failure of the Hosios condition. This argument is based on their finding that the optimal tax to eliminate this inefficiency is large in the steady state but moves little over the cycle.

This paper reinforces the literature's finding by showing that optimal monetary policy stays close to strict inflation targeting also in the presence of another type of search-related distortion in the labor market. The finding that optimal monetary policy stays close to strict inflation targeting in the presence of skill erosion during unemployment can be explained along the lines of Ravenna and Walsh (2012a). In Laureys (2012a) I show that, in the presence of fully flexible prices, the optimal labor market policy which restores constrained efficiency in the presence of skill loss during unemployment takes the form of a time-varying employment subsidy. However, the difference between the labor market outcomes in the presence of the optimal labor market policy and in the laissez-faire economy is in the first place driven by a difference in the steady state. This in turn explains why optimal monetary policy stays close to strict inflation targeting in the presence of skill erosion during unemployment despite the fact the natural allocation is no longer constrained efficient.

2.7 Conclusion

This paper looks at how optimal monetary policy changes once it is taken into account that workers human capital depreciates during periods of unemployment. Human capital depreciation during unemployment is introduced into an otherwise standard New Keynesian model with search frictions in the labor market. Skill erosion has potential implications for optimal monetary policy because in its presence the flexible-price allocation is not constrained efficient. This is a consequence of a composition externality related to job creation: firms ignore how their hiring decisions affect the extent to which the unemployed workers skills erode, and hence the output that can be produced by new matches. Therefore, from a social point of view it might no longer be optimal to replicate the flexible-price allocation by implementing a strict inflation targeting policy. I find that even though optimal price inflation is no longer zero, strict inflation targeting stays close to the optimal policy. This result reinforces the existing finding in the literature that search-related distortions in the labor market only call for small deviations from zero inflation. The literature, however, has only looked at search-related distortions following from the familiar congestion externality that arises in markets characterized by search frictions. My paper shows that this finding is generalized for other search-related distortions.

2.8 Appendix

2.8.1 Equilibrium

The economy's equilibrium is determined by the following equations
Evolution of aggregate technology

$$A_t = (1 - \rho_a) + \rho_a A_{t-1} + \varepsilon_t \tag{2.33}$$

Total output

$$Y_t = A_t n_t^H + A_t (1 - \delta) n_t^L \tag{2.34}$$

Total consumption

$$C_t^T = C_t + b(1 - n_t) \tag{2.35}$$

Euler equation for consumption

$$\left(C_t^T\right)^{-1} = \beta (1 + r_t^n)\ E_t \left\{ \left(C_{t+1}^T\right)^{-1} \frac{P_t}{P_{t+1}} \right\} \tag{2.36}$$

Resource constraint

$$Y_t = (C_t + \kappa v_t) \Delta_t \tag{2.37}$$

Inflation

$$\Pi_t = \frac{P_t}{P_{t-1}} \tag{2.38}$$

Price setting

$$x_t = (C_t + \kappa v_t) MC_t (1 - \tau) + \theta_p \beta E_t \left\{ \left(\frac{C_t^T}{C_{t+1}^T} \right) (\Pi_{t+1})^{\varepsilon} x_{t+1} \right\} \tag{2.39}$$

$$z_t = (C_t + \kappa v_t) + \theta_p \beta E_t \left\{ \left(\frac{C_t^T}{C_{t+1}^T} \right) (\Pi_{t+1})^{\varepsilon - 1} z_{t+1} \right\} \tag{2.40}$$

$$\frac{P_t^\star}{P_t} z_t = \frac{\varepsilon}{\varepsilon - 1} x_t \tag{2.41}$$

Law of motion aggregate price level

$$1 = \theta_p \Pi_t^{\varepsilon-1} + (1-\theta_p)\left(\frac{P_t^\star}{P_t}\right)^{1-\varepsilon} \tag{2.42}$$

Law of motion price dispersion

$$\Delta_t = (1-\theta_p)\left(\frac{P_t^\star}{P_t}\right)^{-\varepsilon} + \theta_p (\Pi_t)^\varepsilon \Delta_{t-1} \tag{2.43}$$

Job finding probability

$$p_t = B\theta_t^{1-\xi} \tag{2.44}$$

Job filling probability

$$q_t = B\theta_t^{-\xi} \tag{2.45}$$

Labor market tightness

$$\theta_t = \frac{v_t}{u_t} \tag{2.46}$$

Total number of job-seekers

$$u_t = 1 - (1-\gamma) n_t \tag{2.47}$$

Fraction of low-skilled job-seekers in the unemployment pool

$$s_t = \frac{u_t^L}{u_t} \tag{2.48}$$

Total employment

$$n_t = n_t^L + n_t^H \tag{2.49}$$

Law of motion high-skilled and low-skilled employment respectively

$$n_t^H = (1-\gamma)\left[n_{t-1}^H + g n_{t-1}^L\right] + p_t u_t^H \tag{2.50}$$

$$n_t^L = (1-\gamma)(1-g) n_{t-1}^L + p_t u_t^L \tag{2.51}$$

Law of motion high-skilled and low-skilled job-seekers respectively

$$u_t^H = (1-l)(1-p_{t-1})\,u_{t-1}^H + \gamma\left(n_{t-1}^H + g n_{t-1}^L\right) \tag{2.52}$$

$$u_t^L = (1-p_{t-1})\left(u_{t-1}^L + l u_{t-1}^H\right) + \gamma(1-g)\,n_{t-1}^L \tag{2.53}$$

The real wage of a high-skilled worker

$$W_t^H = \eta MC_t A_t + (1-\eta)\,\mathcal{O}_t^H \tag{2.54}$$

where

$$\mathcal{O}_t^H \equiv b + E_t\left\{\beta_{t,t+1}\left[\begin{array}{l} \eta p_{t+1}\left((1-l)\,M_{t+1}^H + l M_{t+1}{}^L\right) \\ -\eta\gamma p_{t+1} M_{t+1}^H \\ -l\left(\mathcal{U}_{t+1}^H - \mathcal{U}_{t+1}^L\right) \end{array}\right]\right\}$$

The real wage of a low-skilled worker

$$W_t^L = \eta MC_t(1-\delta)\,A_t + (1-\eta)\,\mathcal{O}_t^L \tag{2.55}$$

$$\mathcal{O}_t^L \equiv b + E_t\left\{\beta_{t,t+1}\left[\begin{array}{l} \eta p_{t+1} M_{t+1}^L \\ -\eta\gamma p_{t+1}\left(g M_{t+1}^H + (1-g)\,M_{t+1}^L\right) \\ -g\left(\mathcal{U}_{t+1}^H - \mathcal{U}_{t+1}^L\right) \end{array}\right]\right\}$$

Surplus generated by a high-skilled worker

$$M_t^H = \left[\begin{array}{l} MC_t A_t - b \\ +E_t\left\{\beta_{t,t+1}\left[\begin{array}{l} (1-\gamma+\eta\gamma p_{t+1})\,M_{t+1}^H \\ -\eta p_{t+1}\left(l M_{t+1}^L + (1-l)\,M_{t+1}^H\right) \\ +l\left(\mathcal{U}_{t+1}^H - \mathcal{U}_{t+1}^L\right) \end{array}\right]\right\} \end{array}\right] \tag{2.56}$$

Surplus generated by a low-skilled worker

$$M_t^L = \begin{bmatrix} MC_t\left(1-\delta\right)A_t - b \\ +E_t\left\{\beta_{t,t+1}\begin{bmatrix} \left(1-\gamma+\eta\gamma p_{t+1}\right)\left(gM_{t+1}^H + \left(1-g\right)M_{t+1}^L\right) \\ -\eta p_{t+1}M_{t+1}^L \\ +g\left(\mathcal{U}_{t+1}^H - \mathcal{U}_{t+1}^L\right) \end{bmatrix}\right\} \end{bmatrix} \tag{2.57}$$

Household's value from having an additional high-skilled and low-skilled member unemployed respectively

$$\mathcal{U}_t^H = b + E_t\left\{\beta_{t,t+1}\begin{bmatrix} \eta p_{t+1}\left(\left(1-l\right)M_{t+1}^H + lM_{t+1}^L\right) \\ +\left(1-l\right)\mathcal{U}_{t+1}^H + l\mathcal{U}_{t+1}^L \end{bmatrix}\right\} \tag{2.58}$$

$$\mathcal{U}_t^L = b + E_t\left\{\beta_{t,t+1}\left[\eta p_{t+1}M_{t+1}^L + \mathcal{U}_{t+1}^L\right]\right\} \tag{2.59}$$

Vacancy creation condition

$$\frac{\kappa}{q_t} = \left(1-\eta\right)\left(\left(1-s_t\right)M_t^H + s_t M_t^L\right) \tag{2.60}$$

The 29 endogenous variables are:

$$\left\{A_t, Y_t, C_t, C_t^T, n_t^H, n_t^L, n_t, p_t, q_t, \theta_t, u_t, u_t^L, u_t^H, s_t, v_t, W_t^H, W_t^L, M_t^H, M_t^L, \mathcal{U}_t^H, \mathcal{U}_t^L, P_t, P_t^\star, \Pi_t, \Delta_t, x_t, z_t, MC_t, r_t^n\right\}$$

To close the system, the conduct of monetary policy has to be determined.

2.8.2 Special case: l=g=1

Constrained Efficient Allocation

The constrained efficient allocation is obtained by solving the problem of a benevolent social planner. The social planner maximize the utility of the representative household and faces the same technological constraints and labor market frictions as in the decentralized economy. When choosing the optimal allocation, the social planner internalizes the effect of vacancy posting on both labor market tightness and on the quality of the labor force.The social planner's problem consist of choosing optimal labor market tightness or vacancy creation and is given by

$$V^P(n_{t-1}) = \max_{\theta_t} \left[U(C_t^T) + \beta E_t \left\{V^P(n_t)\right\}\right]$$

subject to the law of motion of total employment

$$n_t = (1-\gamma)\, n_{t-1} + B\theta_t^{1-\xi}\, (1-(1-\gamma)n_{t-1})$$

and where total consumption is given by the sum of consumption of home and non-home produced goods $C_t^T = C_t + b(1-n_t)$; where the consumption of home produced is all output produced minus that fraction of output that is used for vacancy creation $C_t = A_t\left(n_t^H + (1-\delta)n_t^L\right) - \kappa\theta_t u_t$; where high-skilled employment is given by $n_t^H = (1-\gamma)n_{t-1} + B\theta_t^{1-\xi}\,\gamma n_{t-1}$; where low-skilled employment is given by $n_t^L = B\theta_t^{1-\xi}(1-n_{t-1})$; and where the total number of job-seekers is given by $u_t = 1 - (1-\gamma)n_{t-1}$.

The first-order condition with respect to labor market tightness is

$$u'(C_t^T)\left[A_t\,(1-\delta s_t) - \frac{\kappa}{q(\theta_t)(1-\xi)} - b\right] + \beta E_t\left\{\frac{\partial V^P(n_t)}{\partial n_t}\right\} = 0$$

The envelope condition of employment is

$$\frac{\partial V^P(n_{t-1})}{\partial n_{t-1}} = u'(C_t^T)\left[\frac{\partial C_t}{\partial n_{t-1}} - b\frac{\partial n_t}{\partial n_{t-1}}\right] + \beta E_t\left\{\frac{\partial V^P(n_t)}{\partial n_t}\frac{\partial n_t}{\partial n_{t-1}}\right\}$$

Combining the above expressions gives rise to the condition for job-creation in the constrained efficient allocation given by equation (2.29) in section 2.3.

Decentralized Allocation for l=g=1

The intermediate good firm's problem is the same as the one described in section 2.2.4. For $l = g = 1$ the first order conditions become

$$\frac{\kappa}{q_t} = (1-s_t)\,\lambda_{j,t} + s_t\varphi_{j,t} \tag{2.61}$$

$$\lambda_{j,t} = Z_t^H - W_{j,t}^H + (1-\gamma) E_t \{\beta_{t,t+1}\lambda_{j,t+1}\} \tag{2.62}$$

$$\varphi_{j,t} = Z_t^L - W_{j,t}^L + (1-\gamma) E_t \{\beta_{t,t+1}\lambda_{j,t+1}\} \tag{2.63}$$

Combining the above expressions gives the following expression for firm's vacancy creation

$$\frac{\kappa}{q_t} = \bar{Z}_t - \bar{W}_{j,t} + (1-\gamma) E_t \left\{\beta_{t,t+1}\left[\frac{\kappa}{q_{t+1}} + g_{j,t+1}^H\right]\right\} \tag{2.64}$$

where

$$g_{j,t}^H \equiv \left(Z_t^H - W_{j,t}^H\right) - \left(\bar{Z}_t - \bar{W}_{j,t}\right)$$

where $\bar{Z}_t \equiv (1-s_t) Z_t^H + s_t Z_t^L$ represents the expected marginal revenue product of a new hire. It is defined as the weigthed sum of the marginal revenue product of a high-skilled and low-skilled worker. Each type's share in the unemployment pool is sufficient to determine this type's weight because all job- seekers have the same hiring probability; and where $\bar{W}_{j,t} \equiv (1-s_t) W_{j,t}^H + s_t W_{j,t}^L$ represents the expected wage cost of a new hire.

Next, the wage setting mechansim is the same as the one outlined in section 2.2.5. For $l = g = 1$ the wage of a worker of type i is given by

$$W_t^i = \eta Z_t^i + (1-\eta)\left(b + \eta E_t\left\{\beta_{t,t+1}\left[p_{t+1}M_{t+1}^L - \gamma p_{t+1}M_{t+1}^H\right]\ \right\}\right) \tag{2.65}$$

Finally, the condition for job creation in equilibrium can be obtained as follows. First, by combining equations (2.61)-(2.63) and equation (2.65), the value for a firm of having a high-skilled and a low-skilled worker employed becomes

$$\lambda_t = \frac{\kappa}{q_t} + (1-\eta)\left(Z_t^H - \bar{Z}_t\right)$$

$$\varphi_t = \frac{\kappa}{q_t} + (1-\eta)\left(Z_t^L - \bar{Z}_t\right)$$

The equilibrium wage of a worker of type i is obtained by combining the above expression with the relation between the value for the firm and the total match surplus implied by the wage setting mechanism (equation (2.24) and (2.25) in section 2.2.5) and the expression for the wage (equation (2.65))

$$W_t^i = \left[\begin{array}{l} \eta Z_t^i + (1-\eta) b \\ + (1-\eta) E_t \left\{ \beta_{t,t+1} \left[\begin{array}{l} \eta p_{t+1} \left(\frac{\kappa}{q_{t+1}(1-\eta)} + Z_{t+1}^L - \bar{Z}_{t+1} \right) \\ -\eta \gamma p_{t+1} \left(\frac{\kappa}{q_{t+1}(1-\eta)} + Z_{t+1}^H - \bar{Z}_{t+1} \right) \end{array} \right] \right\} \end{array} \right]$$

Combining the above expression for the wage with the vacancy creation condition (equation (2.64)) gives the following expression for equilibrium job creation

$$\frac{\kappa}{q_t} = (1-\eta) \left[\bar{Z}_t - b + E_t \left\{ \beta_{t,t+1} \Lambda_{t+1}^D \right\} \right] \tag{2.66}$$

where

$$E_t \left\{ \Lambda_{t+1}^D \right\} \equiv E_t \left\{ \begin{array}{l} (1-\gamma) \left(\frac{\kappa}{q_{t+1}(1-\eta)} + Z_{t+1}^H - \bar{Z}_{t+1} - \frac{\eta}{(1-\eta)} \theta_{t+1} \kappa \right) \\ + \eta p_{t+1} \left[\gamma \left(Z_{t+1}^H - \bar{Z}_{t+1} \right) + \left(\bar{Z}_{t+1} - Z_{t+1}^L \right) \right] \end{array} \right\}$$

Note that for $g = l = 1$, the economy's equilibrium is defined by equation (2.34)-(2.53) and the job creation condition described by equation (2.66), given a path for the exogenous shock $\{\varepsilon_t\}_{t=0}^{\infty}$ and the conduct of monetary policy.

2.9 Figures

Figure 2.1: Optimal volatility of inflation (in percent) as a function of the rate of human capital depreciation δ

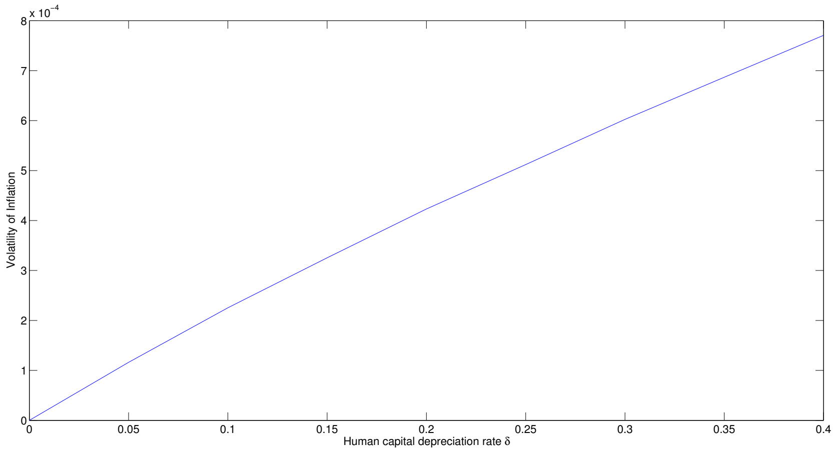


Figure 2.2: Optimal volatility of inflation (in percent) as a function of the pace of skill loss $(1/l)$

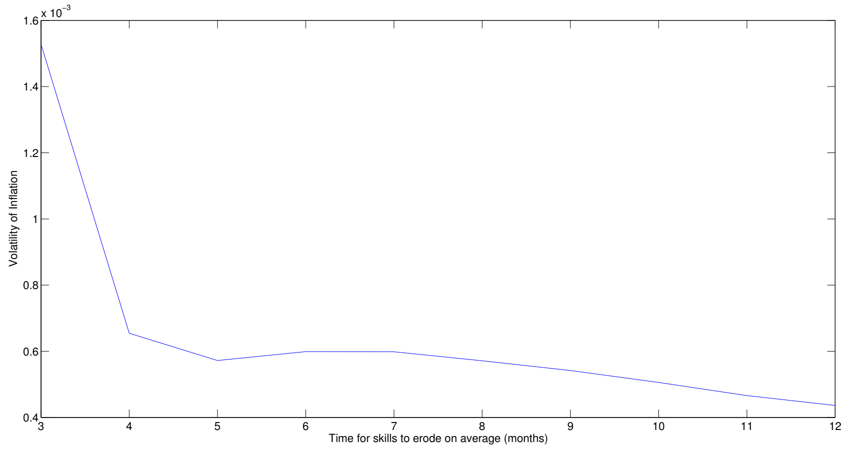


Figure 2.3: Optimal volatility of inflation (in percent) as a function of the pace of regaining skills $(1/g)$

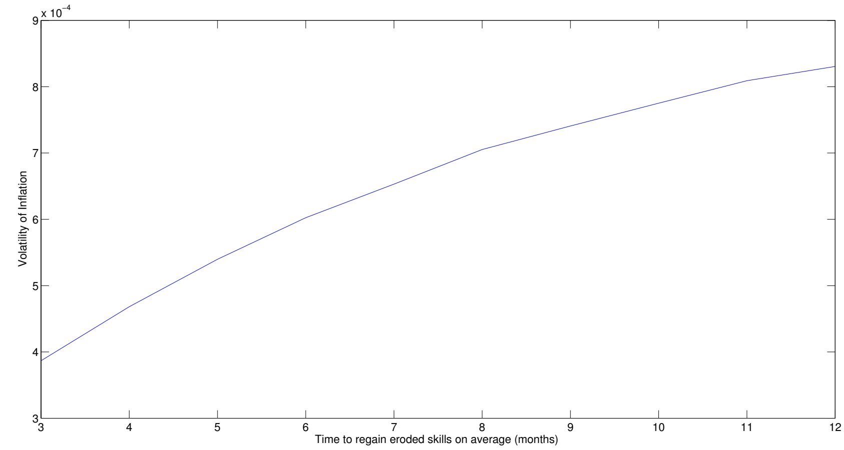


Figure 2.4: Ramsey policy versus zero inflation policy for $\delta = 0.3$ and $g = l = 0.5$

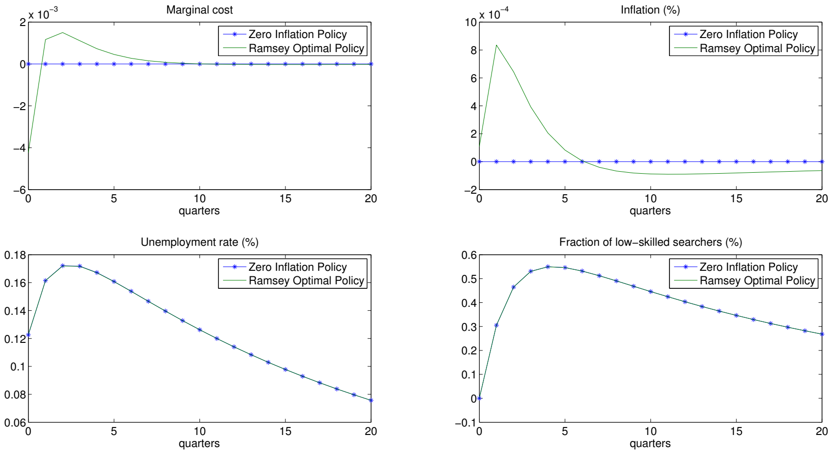


Figure 2.5: Ramsey policy for different rates of human capital depreciation, and where $g = l = 0.5$

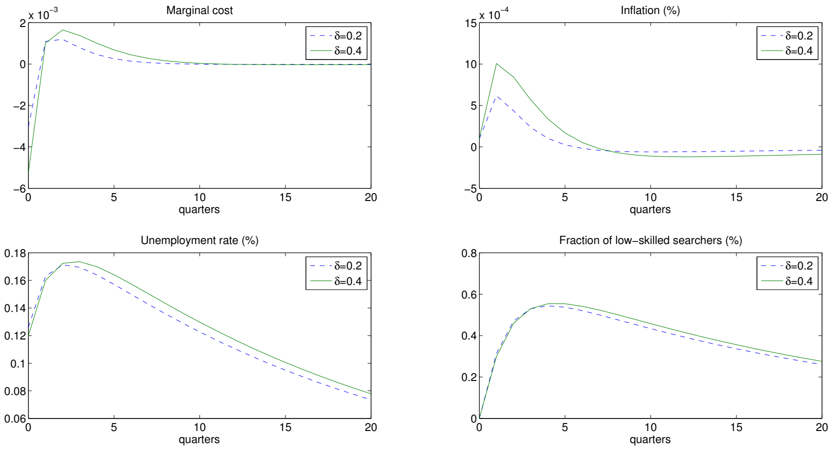


Figure 2.6: Ramsey policy for different values of the probability of skill loss, and where $\delta = 0.3$ and $g = 0.5$

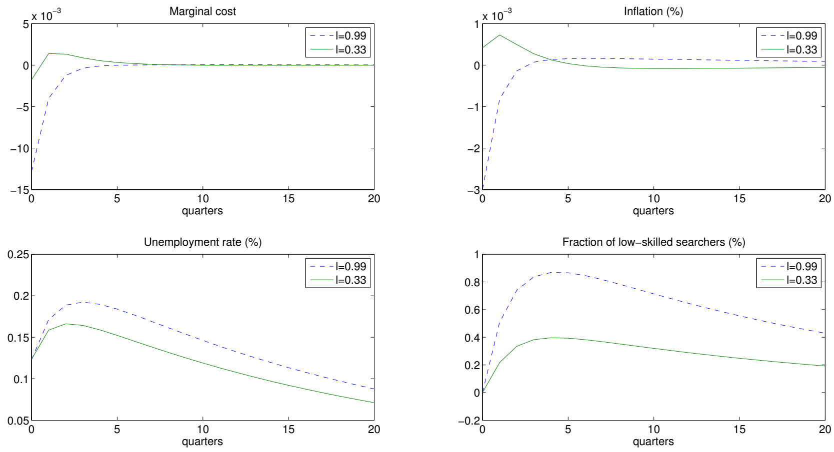
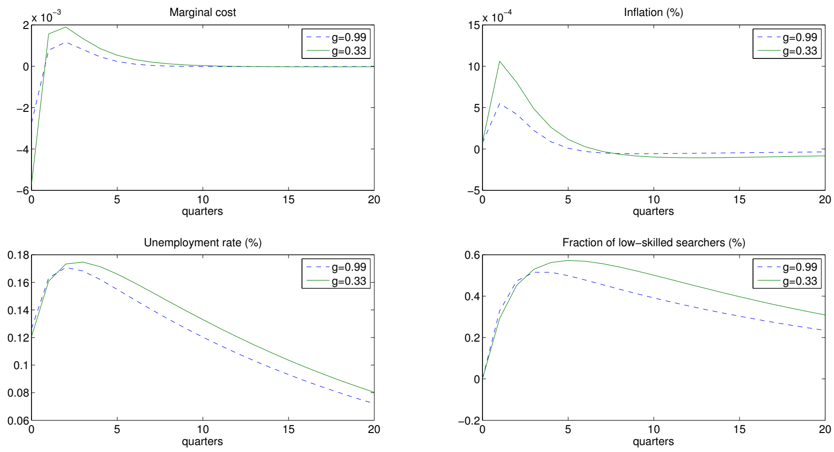


Figure 2.7: Ramsey policy for different values of the probability of regaining skills, and where $\delta = 0.3$ and $l = 0.5$



Chapter 3

A NOTE ON FISCAL STIMULUS AND LABOR MARKET OUTCOMES: THE ROLE OF SHORT-RUN WEALTH EFFECTS AND WAGE RIGIDITY

3.1 Introduction

The implementation of fiscal stimulus packages as a response to the 2007-2009 financial crisis has triggered a debate in the literature about their effect on the real economy. A large part of the literature has focused on determining the size of the fiscal output multiplier. As summarized in Hall (2009), empirical analyses based on regressions or vector autoregressions typically find output multipliers between 0.5 and 1. The size of the fiscal multiplier, however, varies across countries depending on country characteristics such as the exchange rate regime, the degree of openness, and the level of economic development as shown in empirical work by Ilzetzki, Mendoza and Végh (2013). The size of fiscal output multipliers has also been analyzed through the lens of structural models. For example, Coenen et al. (2012) compare the effect of fiscal stimulus on the economy across various DSGE models. On the one hand, when monetary policy is not accommodative, they find output multipliers in line with the empirical analyses. On the other hand, when monetary policy is accommodative, they report output multipliers well above one. The importance of the stance of monetary policy for the effect of fiscal stimulus on the economy has also been documented by e.g. Christiano, Eichenbaum and Rebelo (2011) and Woodford (2011). Less attention, however, has been paid to the effect of fiscal stimulus on labor market outcomes.

The main goal of this paper is to understand how the effect of an increase in government spending on labor market outcomes depends on the strength of the short-run wealth effect on labor supply. Models used to investigate the implications of government spending typically assume strong short-run wealth effects on labor supply. This despite the fact that the evidence points into the direction of a rather limited size of the wealth effect.[1] It has already been pointed out that the short-run wealth effect on labor supply plays an important role for the effect of government spending on the economy. In particular, Monacelli and Perotti (2009) show that depending on the intensity of the short-run wealth effect on labor supply the sign of the response of private consumption to an increase in government spending can be either positive or negative.

I use as a framework of analysis a New Keynesian model where both the goods and the labor market are characterized by monopolistic competition, a framework that has been widely used to compute the size of fiscal output multipliers. However, the standard New Keynesian framework has no measure of unemployment making it unappealing to study the impact of government spending on labor market outcomes. I overcome this problem by introducing a measure of labor force participation that allows for a definition of unemployment following Galí (2011). This interpretation of

[1]See e.g. Schmitt-Grohé and Uribe (2012).

the labor market has the advantage that it allows for a simple representation such that insights can be obtained into how the role of the strength of the short-run wealth effect in shaping the economy's response to a change in government spending depends on the degree of price and wage stickiness. To allow for varying degrees of the short-run wealth effect on labor supply I introduce preferences as in Galí, Smets and Wouters (2011).

I find that the role of the short-run wealth effect on labor supply in generating a specific increase in employment in response to an increase in government spending depends crucially on the degree of price and wage stickiness. When prices are fully flexible, the short-run wealth effect is key for generating an increase in employment. This is because it induces an increase in labor supply which drives this employment response. In the extreme case where there is no short-run wealth effect, the economy would remain unaffected. When prices are sticky, the strength of the short-run wealth effect is less important. Even in the extreme case where labor supply does not respond on impact, there is still an increase in employment driven by the increase in labor demand. Moreover, I find that the role of the short-run wealth effect in obtaining a given employment increase, is less important the higher the degree of wage rigidity. This is because more rigid wages reduce the importance of an increase in labor supply in order to limit the upward pressure on wages triggered by the increase in labor demand.

Several other papers have focused on the effect of fiscal stimulus on labor market outcomes. Monacelli, Perotti and Trigari (2010) provide empirical evidence for the U.S. economy on the size of the fiscal output and unemployment multiplier. They also document that an increase in government spending generates an increase in total hours, employment and the job finding probability, and a decrease in the separation rate. Next, they show that a standard real business cycle model with the labor market characterized by search frictions cannot replicate the size of the output multiplier, but it can replicate the decrease in the unemployment rate found in their empirical evidence when the calibration is such that the value of home production is high relative to the value of market activities. Finally, they find that a New Keynesian framework with complementarity in utility between consumption and labor and a labor market characterized by search frictions performs better in replicating the size of the multipliers. However, some of the channels, such as the labor force participation decision, remain unexplored. Brückner and Pappa (2012) provide empirical evidence on the effect of government spending on labor market outcomes for the OECD countries. They find that an increase in government spending not only increases employment and labor force participation but it can also increase the unemployment rate. They argue that a New Keynesian framework with worker heterogeneity, a labor force participation decision and a labor market characterized by search frictions can replicate the sign of the responses of labor market variables to an increase in government spending obtained in their empirical evidence. Campolmi, Faia and Winkler (2011) show, by making use of a New Keynesian framework with a labor force participation decision and a labor market characterized by search frictions, that hiring subsidies generate larger fiscal output multipliers than increases in government consumption.

The remainder of the paper is organized as follows. Section 3.2 introduces the model. Section 3.3 discusses the main driving forces of labor market outcomes. Section 3.4 analyzes the role of the short-run wealth effect on labor supply in shaping the economy's response to changes in government spending. Section 3.5 contains model simulations to gain further insights into how the effect of government spending on labor market outcomes depends on the degree of price and wage stickiness, the strength of the short-run wealth effect on labor supply, and the stance of monetary policy. Finally, section 3.6 concludes.

3.2 Model

The framework of analysis is a variant of the model of Erceg, Henderson and Levin (2000) proposed by Galí (2011). It is a standard New Keynesian model with monopolistic competition in the goods and labor market. Both prices and nominal wages are sticky à la Calvo (1983). The limitation of the EHL (2000) model for studying the effect of fiscal stimulus on labor market outcomes is the absence of unemployment. The variation proposed by Galí (2011) overcomes this limitation by introducing a measure of labor force participation which allows for an expression for the unemployment rate consistent with its empirical counterpart.

3.2.1 Households

There is a large representative household consisting of a continuum of members represented by the unit square and indexed by a pair $(i, j) \in [0, 1] \times [0, 1]$. For each household member, $i \in [0, 1]$ represents the type of labor service in which he is specialized, and $j \in [0, 1]$ determines his disutility of work. The latter equals zero when unemployed, and $\Theta_t j^{\varphi}$ when employed, where $\varphi \geq 0$ and Θ_t is an endogenous preference shifter as in Galí, Smets and Wouters (2011) as discussed below.

The household's utility depends on consumption and employment. In line with most of the literature it does not depend on government consumption. There is full risk sharing of consumption among household members following Merz (1995). Household's utility can be derived as the integral of its members' utilities and is given by

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \{N_t(i)\}\right) &\equiv E_0 \sum_{t=0}^{\infty} \beta^t \left(log C_t - \Theta_t \int_0^1 \int_0^{N_t(i)} j^{\varphi} dj\, di \right) \\ &= E_0 \sum_{t=0}^{\infty} \beta^t \left(log C_t - \Theta_t \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} di \right) \end{aligned} \tag{3.1}$$

where β is the discount factor, C_t represents consumption at time t, $N_t(i) \in [0, 1]$ is the employment rate in period t of workers specialized in type i labor, and Θ_t is a preference shifter taken as given by the household and defined as

$$\Theta_t \equiv \frac{Z_t}{\bar{C}_t}$$

where $Z_t \equiv Z_{t-1}^{\zeta} \bar{C}_t^{1-\zeta}$ with $\zeta \epsilon [0, 1)$, and $\bar{C}_t$ denotes aggregate consumption.

I introduce the endogenous preference shifter Θ_t following Galí, Smets and Wouters (2011) to allow for a varying strength of the short-run wealth effect on labor supply while preserving long-run balanced growth. This specification is similar to the one proposed by Jaimovich and Rebelo (2009) who assume non-separability of period utility in consumption and employment. The strength of the short-run wealth effect on labor supply is determined by the parameter ζ. The larger the value of ζ, the smaller the wealth effect.

Households choose the optimal path of consumption and bond holdings by maximizing expression (3.1) subject to the sequence of budget constraints

$$P_tC_t + Q_tB_t \leq B_{t-1} + \int_0^1 W_t(i)\, N_t(i)\, di + \Pi_t - T_t \tag{3.2}$$

where P_t is the price of the consumption bundle, B_t is holdings of a nominally riskless one-period bond, Q_t is the price of the bond, $W_t(i)$ is the nominal wage of labor type i, Π_t is the lump-sum component of income, and T_t are lump sum taxes.

Solving the household's problem gives the Euler equation for consumption which, after log-linearization around a zero inflation steady state, takes the following from

$$\widehat{c}_t = E_t\{\widehat{c}_{t+1}\} - \left(\widehat{i}_t - E_t\left\{\pi^p_{t+1}\right\}\right) \tag{3.3}$$

where small letters denote the log of the respective variable, a hat above a variable denotes in deviation from the respective variable's steady state, $i_t \equiv -logQ_t$ is the short term nominal intrest rate, and $\pi^p_t \equiv p_t - p_{t-1}$ is price inflation.

3.2.2 Labor Market

There is monopolistic competition in the labor market. A union representing the workers sets wages for each labor type i. Nominal wages are sticky à la Calvo (1983). In every period only a fraction $1-\theta_w$ of workers can reset their wage. Thus, θ_w can be interpreted as an index of nominal wage rigidities. Wages are set so as to maximize the household's utility subject to the labor type's demand and the household's budget constraint. Once the wage has been set, employment is determined unilaterally by firms' labor demand.

Solving for the optimal wage setting problem and log-linearizing around a zero inflation steady state leads to the following approximation of the wage setting rule [2]

$$w_t^* = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ mrs_{t+k|t} + p_{t+k} \right\}$$

where w_t^* is the newly set nominal wage, $\mu^w \equiv log\frac{\varepsilon_w}{\varepsilon_w - 1}$ is the desired wage markup with ε_w being the wage elasticity of demand for each labor type, and $mrs_{t+k|t} \equiv z_{t+k} + \varphi n_{t+k|t}$ is the marginal rate of substitution between consumption and employment at time $t+k$ for a labor type whose wage was last reset in period t.

The log-linear expression for the aggregate wage index is

$$w_t = \theta_w w_{t-1} + (1 - \theta_w)\, w_t^*$$

Defining wage inflation as $\pi_t^w \equiv w_t - w_{t-1}$, the following wage inflation equation can be derived

$$\pi_t^w = \beta E_t \left\{ \pi_{t+1}^w \right\} - \lambda_w \left(\mu_t^w - \mu^w \right)$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\varphi)}$, and

$$\mu_t^w \equiv (w_t - p_t) - mrs_t \tag{3.4}$$

denotes the average wage markup.

The model proposed by EHL (2000) cannot say anything about unemployment. In Galí (2011) this shortcoming is solved by introducing a measure of the labor force which gives rise to an expression for unemployment.

[2] For a detailed description of the wage setting problem and derivation of the equations denoted below, see EHL (2000).

The disutility of working for a household member of type (i, j), using household welfare as a criterion and taking current labor market conditions, is given by $Z_t j^{\varphi}$. Thus, it is optimal to participate in the labor market if and only if

$$\frac{W_t(i)}{P_t} \geq Z_t j^{\varphi}$$

Consequently, the marginal supplier of type i labor $L_t(i)$ is given by

$$\frac{W_t(i)}{P_t} = Z_t L_t(i)^{\varphi}$$

Taking logs and integrating over i yields

$$w_t - p_t = z_t + \varphi l_t \tag{3.5}$$

where $l_t \equiv \int_0^1 l_t(i)\, di$ is interpreted as the model's labor force, and $w_t \equiv \int_0^1 w_t(i)\, di$ is the average wage.

The unemployment rate u_t is defined as

$$u_t \equiv l_t - n_t \tag{3.6}$$

A simple relation between the wage markup and the unemployment rate can be obtained by combining the expression for the average wage markup $\mu_t^w \equiv (w_t - p_t) - (z_t + \varphi n_t)$ with equation (3.5) and (3.6)

$$u_t = \frac{\mu_t^w}{\varphi}$$

Note that the above equation implies that if wages are fully flexible, unemployment is constant, as the average wage markup μ_t^w will equal the constant desired wage markup μ^w at all times.

Figure 3.1 provides a graphical representation of the labor market.[3]

[3]The graphical representation of the labor market is based on Galí (2012).

3.2.3 Firms and Price Setting

There is a continuum of firms f represented by the unit interval, $f \epsilon [0, 1]$. Firms operate in a monopolistically competitive market and prices are sticky à la Calvo (1983). In every period only a fraction $1 - \theta_p$ of the firms can optimally reset their price. The remaining fraction θ_p keeps prices unchanged.

The production function for each firm f denoted in logs is given by

$$y_{f,t} = (1 - \alpha)\, n_{f,t} \tag{3.7}$$

where $y_{f,t}$ is log output of firm f at time t, $n_{f,t}$ is the log of labor input used by firm f at time t, and $1 - \alpha$ determines the degree of decreasing returns to labor.

Solving for the optimal price setting problem and log-linearizing around a zero inflation steady state gives the following expression for price inflation [4]

$$\pi_t^p \;=\; \beta E_t \left\{\pi_{t+1}^p\right\} - \lambda_p \left(\mu_t^p - \mu^p\right)$$

where $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)(1-\alpha)}{\theta_p(1-\alpha+\alpha\varepsilon_p)}$. The last term represents the deviation of the log of the (average) price markup at time t (μ_t^p) from the desired price markup $\mu^p \equiv log\frac{\varepsilon_p}{\varepsilon_{p-1}}$, with ε_p being the price elasticity of demand for each good type.

The average price markup at time t equals

$$\mu_t^p \;=\; mpn_t - (w_t - p_t) \tag{3.8}$$

where $mpn_t \equiv -\alpha n_t + log\,(1 - \alpha)$ is the log of the marginal product of labor.

[4] The firms' side of the model is standard in the New Keynesian literature. For a detailed analysis of the price setting problem, see Galí (2008).

3.2.4 Monetary and Fiscal Policy

I assume government consumption is financed through lump sum taxation and evolves according to

$$\widehat{g}_t = (1 - \rho_g)\, \widehat{g}_{t-1} + \varepsilon_{g,t}$$

where $\hat{g}_t$ denotes the (log) deviations of government consumption from steady state, ρ_g determines the persistence of the stimulus, and $\varepsilon_{g,t}$ represents the shock to government consumption.

In order to close the model the conduct of monetary policy should be determined. Monetary policy follows a Taylor rule of the following form

$$i_t = \rho + \phi_\pi \pi_t^p + \phi_y\, (\hat{y}_t - \hat{y}_{t-1})$$

where $\rho \equiv -log\beta$ is the discount rate.

3.3 Labor Market Outcomes

Employment is demand determined. Once nominal wages are set, employment is determined unilaterally by firms' labor demand. The latter in turn is a function of aggregate demand because, once prices are set, firms have to meet the demand for their goods. This implies that the extent to which an increase in government spending leads to an increase in employment depends on its induced increase in aggregate demand. The labor demand schedule follows from the production function and takes the following form [5]

$$n_t = \frac{1}{1-\alpha} y_t$$

Given employment, real wages are pinned down by the wage schedule which follows from rearranging expression (3.8) for the average price markup

$$w_t - p_t = mpn_t - \mu_t^p \tag{3.9}$$

[5] The interpretation of driving forces of the labor market outcomes outlined in this section follows Galí (2012).

Note that this implies that whether an increase in government spending generates an increase or a decrease in the real wage depends on the change of the marginal product of labor relative to that of the average price markup.

Labor force participation is determined by equation (3.5). This equation shows that the strength of the short-run wealth effect on labor supply is crucial for shaping the response of labor force participation to a change in government spending. For a given real wage, the stronger the short-run wealth effect, the larger the increase in labor supply.

Unemployment is defined as the difference between labor force participation and employment (equation (3.6)). Therefore, unemployment depends on both the movements in employment and labor force participation. Thus, an increase in government consumption only lowers unemployment if the increase in employment is not fully offset by the increase in labor force participation.

3.4 Role of the Short-Run Wealth Effect

To get a better understanding of how the effect of an increase in government spending on labor market outcomes depends on the short-run wealth effect on labor supply, I derive the following expression for employment

$$\hat{n}_t = \lambda_1 \hat{g}_t - \lambda_2 E_t \left\{ \sum_{j=0}^{\infty} \left(\hat{i}_{t+j} - E_{t+j} \left\{ \pi^p_{t+1+j} \right\} \right) \right\} \tag{3.10}$$

where $\lambda_1 \equiv \frac{G}{Y(1-\alpha)}$ and $\lambda_2 \equiv \frac{1}{\left(1-\frac{G}{Y}\right)(1-\alpha)}$, and the above expression is obtained by combining equation (3.3), (3.7), and (3.13) in Appendix 3.7.1. Note that the above expression holds for all values of ζ.

Expression (3.10) shows that the response of employment depends on the expected path of the real interest rate.[6] The latter matters as it affects

[6] The importance of the real rate for the response of employment in the New Keynesian framework is also emphasized in Galí (2012) .

the crowding out of private consumption, and hence the overall response of labor demand. In the presence of sticky prices, the path of the real interest rate depends on the conduct of monetary policy. Therefore, the wealth effect influences the response of employment to an increase in government spending only to the extent that it affects inflationary pressure.This, however, is in stark contrast to the extreme case where both prices and wages are fully flexible. In this case the response of employment to an increase in government spending depends directly on the strength of the short-run wealth effect, which can be seen as follows. In the presence $(\zeta = 0)$ and the absence $(\zeta = 1)$ of the short-run wealth effect on labor supply, employment is given by expressions (3.11) and (3.12), respectively:

$$\hat{n}_t = \gamma \hat{g}_t \tag{3.11}$$

$$\hat{n}_t = 0 \tag{3.12}$$

where $\gamma \equiv \frac{\frac{G}{Y}}{\left(1-\frac{G}{Y}\right)(\alpha+\varphi)+(1-\alpha)}$, and the expressions are obtained by combining equation (3.13) in Appendix 3.7.1, equation (3.7), and the labor market condition $m\hat{r}s_t = m\hat{p}n_t$. Expression (3.12) shows that when prices and wages are fully flexible government spending has no effect on employment in the absence of a short-run wealth effect on labor supply.[7]

The channel through which the wealth effect influences inflationary pressure is wages. Expression (3.4) for the wage markup can be rewritten as follows

$$w_t - p_t = \mu_t^w + z_t + \varphi n_t$$

This shows that, everything else equal, a stronger short-run wealth effect on labor supply limits the upward pressure on wages induced by the increase in labor demand. This in turn limits the inflationary pressure induced by an increase in government spending. Note that this implies that for a given stance of monetary policy the real interest rate will raise less. As a result, private consumption is crowded out less, leading to a larger increase in employment.

[7] The importance of the wealth effect for the responses of output and employment to a government spending shock is also emphasized by e.g. Baxter and King (1993) and Monacelli and Perotti (2009).

3.5 Fiscal Stimulus and Labor Market Outcomes

In this section I analyze in more detail how the economy's response to an increase in government spending depends on the strength of the short-run wealth effect on labor supply, and the degrees of price and wage rigidity. In addition to this, I look at how the results depend on the stance of monetary policy. An overview of the log-linearized equilibrium conditions is given in Appendix 3.7.1. Note that given the model's simplicity this exercise is meant to be illustrative rather than quantitatively relevant.

3.5.1 Calibration

The model is parameterized to a quarterly frequency. The values I use are standard in the literature. I set the discount factor $\beta = 0.99$; the parameter α defining the returns to labor in production is such that returns equal $2/3$. The inverse of the Frisch elasticity is set to $\varphi = 5$ implying a wage elasticity of labor supply of 0.2. The parameter ζ defining the strength of the short-run wealth effect is set to 0.5. In the baseline calibration the parameters defining price and wage stickiness take values which are standard in the literature : $\theta_p = 2/3$ and $\theta_w = 3/4$, which implies an average price duration of three quarters and an average wage duration of four quarters. The elasticity of demand for goods $\varepsilon_p = 6$ is such that the desired price markup equals 1.2. I set the value of the elasticity of demand for labor $\varepsilon_w = 4.52$ so that the steady state unemployment rate, given the value of φ, equals 0.05. The parameters of the Taylor rule for monetary policy take the standard values $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$. Following Campolmi et al. (2011), I set government spending as a fraction of total steady state output to $G/Y = 0.15$. The parameter defining the persistence of the process for government consumption is $\rho_g = 0.9$. For an overview of the assumed parameter settings, see table 1. Throughout the whole exercise I use these parameter values, unless mentioned otherwise. The shock to government consumption $\varepsilon_{g,t}$ is such that the initial increase in government spending is 1% of steady state output.

Parameter	Parameter	Value
Returns to labor	$1-\alpha$	$2/3$
Discount factor	β	0.99
Inverse Frisch elasticity of LS	φ	5
Parameter wealth effect	ζ	0.5
Price stickiness	θ_p	$2/3$
Elasticity of demand for goods	ε_p	6
Wage stickiness	θ_w	0.75
Elasticity of demand for labor	ε_w	4.52
Monetary policy rule	ϕ_π	1.5
Monetary policy rule	ϕ_y	$0.5/4$
Government spending	G/Y	0.15
Government spending	ρ_g	0.9

Table 3.1: Parameterization

3.5.2 Results

Figures 3.2-3.5 show the response of the labor market outcomes, price inflation, and the real interest rate to a persistent increase in government consumption for varying degrees of price stickiness (θ_p), nominal wage stickiness (θ_w), and the parameter governing the strength of the short-run wealth effect on labor supply (ζ). The responses are on impact with the exception of the response of the real interest rate. For the latter I show the cumulative response at the three year horizon.[8] All variables are denoted in percentage deviations from steady state with the exception of the unemployment rate and the real rate which are denoted in percentage points.

Figure 3.2 analyzes how the response of different labor market vari-

[8]It is computed as follows: $cumulative\ response\ real\ rate \equiv \sum_{j=0}^{12}\left(\hat{i}_{t+j} - E_{t+j}\left\{\pi^p_{t+1+j}\right\}\right)$

ables depends on the strength of the wealth effect and the degree of price stickiness for a given degree of wage stickiness ($\theta_w = 0.75$). It can be seen that in the presence of sticky wages, the role of the short-run wealth effect in shaping the response of employment to an increase in government spending is negligible. Even for very low degrees of price stickiness, the employment response is barely affected by the strength of the wealth effect.

Figure 3.3 shows how the response of different labor market variables depends on the strength of the wealth effect and the degree of nominal wage stickiness for a given degree of price stickiness ($\theta_p = 2/3$). I perform the same exercise in figure 3.4 for a lower degree of price stickiness ($\theta_p = 0.1$). First, in line with the findings depicted in figure 3.2, both figures show that for high degrees of wage stickiness, the role of the short-run wealth effect in shaping the response of employment is negligible. Second, both figures reveal that for low degrees of wage stickiness the wealth effect has a more pronounced effect on employment. In particular, the stronger the short-run wealth effect, the more employment increases on impact. The reason why the wealth effect only matters for low degrees of wage stickiness is the following. The increase in labor demand, induced by the increase in government spending, leads to upward pressure on wages. The less rigid wages the more important it is that labor supply increases in order to limit the upward pressure on wages. As discussed in section 3.4, the upward pressure on wages affects the inflationary pressure induced by the increase in government spending, and hence it influences the response of the real interest rate. Third, comparing figure 3.3 and 3.4 shows that the strength of the wealth effect matters especially when the degree of price stickiness is also low. This follows immediately from upward wage pressure inducing more inflationary pressure, and hence a larger increase in the real interest rate, the less sticky prices. Note that this can also be seen in the panel depicting the response of the real interest rate. For low degrees of wage stickiness, the increase in the real interest rate is smaller the stronger the wealth effect. It is, as expected, more pronounced in figure 3.4 than in figure 3.3 because of a smaller degree of price stickiness in the former.

Figures 3.2 and 3.3 also show that the strength of the wealth effect on labor supply does play an important role for the response of labor force participation. As expected from equation (3.5), the stronger the wealth effect, the larger the increase in labor force participation. Moreover, through its dependence on labor force participation, the response of unemployment is also affected by the strength of the wealth effect. A stronger wealth effect reduces the drop in the unemployment rate because of a larger increase in labor force participation.

Figure 3.5 shows how the response of different labor market variables depends on the degree of price and nominal wage stickiness for a given strength of the short-run wealth effect on labor supply ($\zeta = 0.5$). The increase in employment is larger the higher the degree of price and/or wage stickiness. This pattern is also clearly present in figures 3.2, 3.3, and 3.4. For a given degree of wage stickiness, the more sticky prices, the less an increase in government spending causes upward pressure on prices. As a result, the real interest rates raises less. This in turn limits the crowding out of private consumption, leading to a larger increase in aggregate demand and employment. Similarly, for a given degree of price stickiness, the more sticky nominal wages, the less an increase in government spending causes upward pressure on wages, and hence on prices. Therefore, the more sticky wages, the smaller the increase in the real interest rate. Moreover, figure 3.5 shows that labor force participation jumps up most for high degrees of price and low degrees of wage stickiness. This can be explained by looking at the response of the real wage. The increase in the real wage, which induces an increase in labor supply (equation (3.5)), is higher the less rigid nominal wages and the more sticky prices. This is because the more sticky prices, the less nominal wage increases can be translated into a price increase, leading to a larger decline in the average price markup. Note that the real wage's response on impact to an increase in government spending can be either positive or negative, as can be seen from figure 3.2-3.5. Whether the real wage increases or decreases on impact depends on the relative degree of price and wage stickiness.[9]

[9]Galí (2012) reports the same finding in a context where the aggregate demand shock

As discussed in section 3.3, the real wage only increases when the decline in the average price markup more than offsets the decline in the marginal product of labor. Finally, the largest drop in unemployment takes place for higher degrees of price and wage stickiness, reflecting that the movements in employment dominate those in labor supply.

In the previous exercises the conduct of monetary policy was described by a Taylor rule with a given coefficient on output and price inflation. The latter determines the extent to which the interest rate responds to the upward pressure on price inflation induced by the increase in government consumption. As a result, it affects the crowding out of private consumption and the overall response of aggregate demand to the increase in government consumption.[10] This raises the question how the above findings are affected by the stance of monetary policy. Figure 3.6 shows how the response of labor market outcomes varies depending on the degree of nominal wage stickiness and the stance of monetary policy. The latter is reflected by the coefficient of price inflation in the Taylor rule ϕ_π. As discussed above, the more sticky wages, the less upward pressure on price inflation, and hence the larger the increase in employment driven by the increase in government spending. It can be seen that the extent to which more sticky wages lead to a larger increase in employment depends on the stance of monetary policy. [11] In particular, for a given degree of wage stickiness, the increase in employment is higher when monetary policy is more accommodative, i.e. for lower values of the coefficient of price inflation in the Taylor rule ϕ_π. This is explained by the response of the real interest rate as depicted in figure 3.6. For a given degree of wage stickiness, the cumulative increase of the real interest rate is smaller, the less monetary policy responds to price inflation. Moreover, figure 3.6 shows that even though higher degrees of wage stickiness still lead to larger in-

takes the form of a preference shock.

[10]See e.g. Woodford (2011) for an extensive discussion about the effect of monetary policy on the size of the fiscal output multiplier.

[11]The importance of the stance of monetary policy in determining how the degree of wage stickiness affects the economy in response to technology and preference shocks has been emphasized by Galí (2012).

creases in employment, a more aggressive response of monetary policy to price inflation largely offsets this effect. This can again by explained by the response of the real interest rate. The more monetary policy responds to price inflation, the less the response of the real interest rate varies with the degree of wage stickiness.

3.6 Conclusion

This paper analyzes how the effect of an increase in government spending on labor market outcomes depends on the strength of the short-run wealth effect on labor supply. I find that in the presence of price stickiness the role of the strength of the short-run wealth effect on labor supply is limited, even more so for higher degrees of wage stickiness. I also document that the extent to which a higher degree of wage rigidity leads to a larger increase on impact of employment in response to an increase in government spending crucially depends on the stance of monetary policy. When monetary policy is not accommodative, wage rigidity does little to increase the employment response. This in turn reinforces the importance of monetary policy in shaping the economy's response to fiscal stimulus.

3.7 Appendix

3.7.1 Equilibrium

The equilibrium of this economy is determined by the following system of equations

Goods market clearing condition

$$\widehat{y}_t = \left(1 - \frac{G}{Y}\right)\widehat{c}_t + \frac{G}{Y}\widehat{g}_t \tag{3.13}$$

Production function

$$\widehat{y}_t = (1-\alpha)\,\hat{n}_t$$

Euler equation for consumption

$$\widehat{c}_t = E_t\left\{\widehat{c}_{t+1}\right\} - \left(\widehat{i}_t - E_t\left\{\pi^p_{t+1}\right\}\right)$$

New Keynesian Phillips curve

$$\pi^p_t = \beta E_t\left\{\pi^p_{t+1}\right\} - \lambda_p \hat{\mu}^p_t$$

Evolution of the price markup

$$\hat{\mu}^p_t = -\alpha\hat{n}_t - \widehat{rw}_t$$

where rw_t denotes the log of the real wage.

Real wage

$$\widehat{rw}_t = \pi^w_t - \pi^p_t + \widehat{rw}_{t-1}$$

Labor force

$$\widehat{rw}_t = \hat{z}_t + \varphi\hat{l}_t$$

Evolution of $\hat{z}_t$

$$\hat{z}_t = \zeta\hat{z}_{t-1} + (1-\zeta)\,\hat{c}_t$$

Unemployment

$$\widehat{u}_t = \widehat{l}_t - \widehat{n}_t$$

Wage inflation

$$\pi_t^w = \beta E_t \left\{ \pi_{t+1}^w \right\} - \lambda_w \hat{\mu}_t^w$$

Wage markup

$$\hat{\mu}_t^w = \widehat{rw}_t - (\hat{z}_t + \varphi \hat{n}_t)$$

Monetary policy rule

$$\widehat{i}_t = \phi_\pi \pi_t^p + \phi_y (\hat{y}_t - \hat{y}_{t-1})$$

Government spending process

$$\widehat{g}_t = (1 - \rho_g) \widehat{g}_{t-1} + \varepsilon_{g,t}$$

3.8 Figures

Figure 3.1: Labor Market

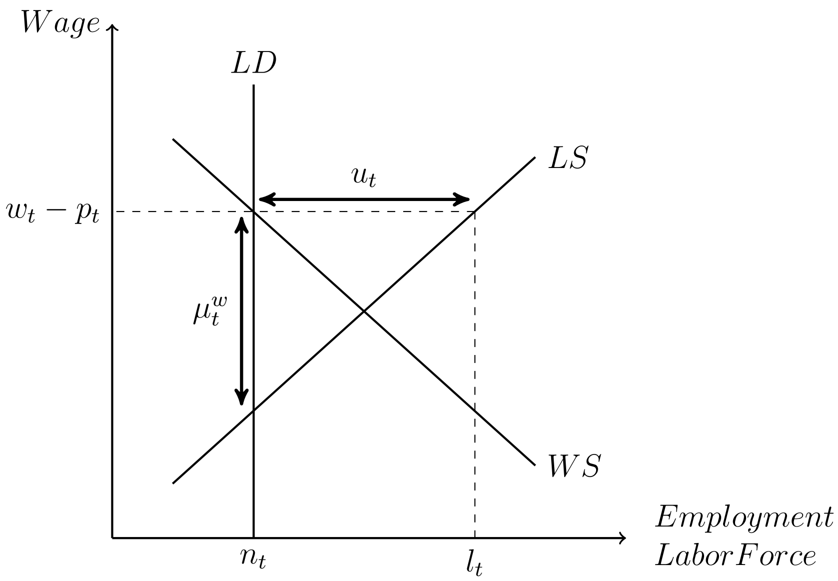


Figure 3.2: Responses to an increase in government consumption for varying θ_p and ζ

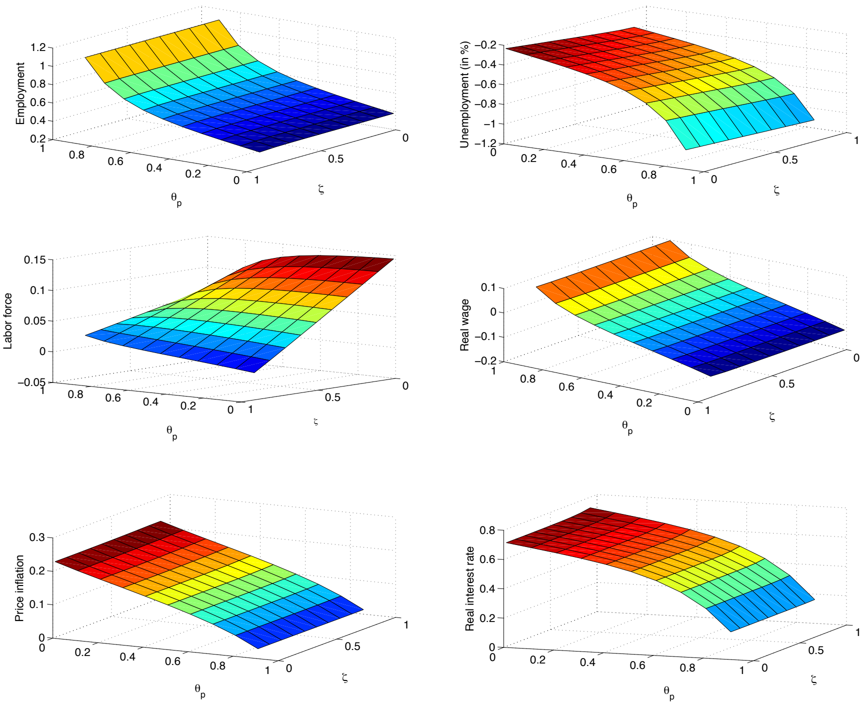


Figure 3.3: Responses to an increase in government consumption for varying θ_w and ζ

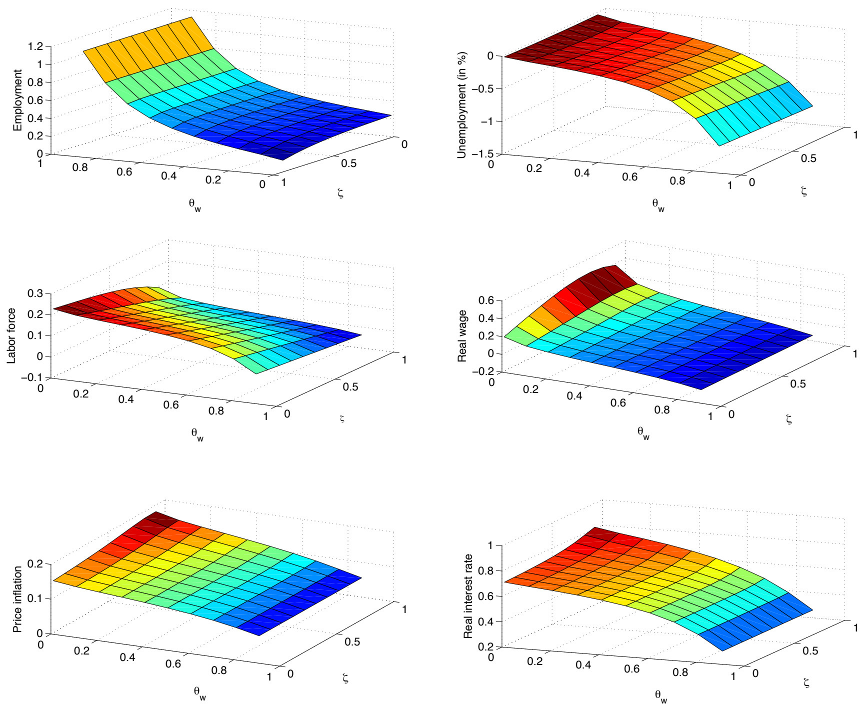


Figure 3.4: Responses to an increase in government consumption for varying θ_w and ζ for $\theta_p = 0.1$

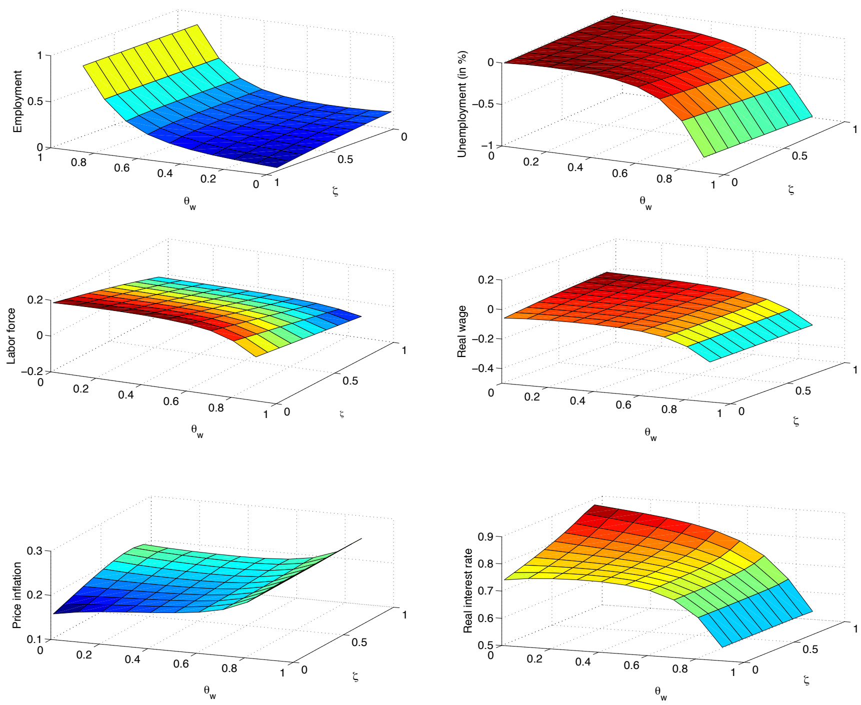


Figure 3.5: Responses to an increase in government consumption for varying θ_p and θ_w

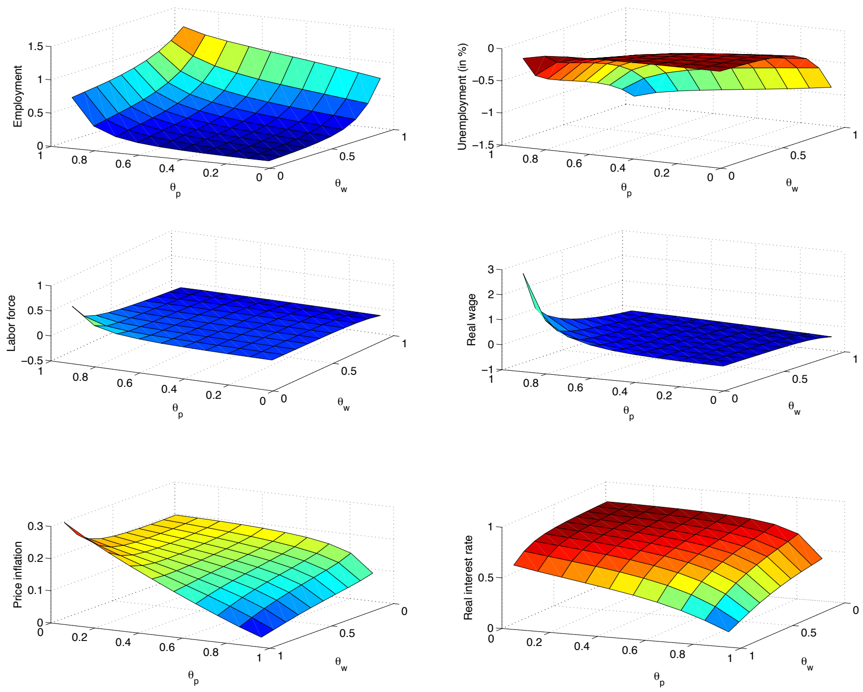
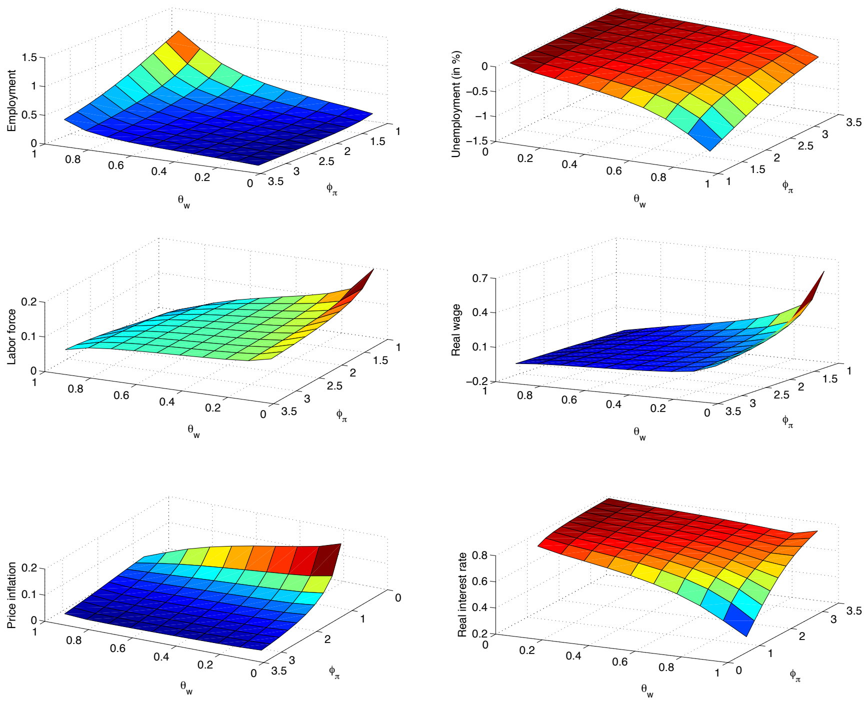


Figure 3.6: Responses to an increase in government consumption for varying θ_w and ϕ_π



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