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Optimisation via Simulation for Healthcare Emergency Departments

Thesis submitted by **Eduardo Cesar Cabrera Flores** for the degree of Philosophiae Doctor by the Universitat Autònoma de Barcelona, under the supervision of Dr. Emilio Luque Fadón, done at the Computer Architecture and Operating Systems Department, PhD. in High Performance Computing

Barcelona, November 2013

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Supervisor	Author
Dr. Emilio Luque Fadón	**Eduardo Cesar Cabrera Flores**

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No estudio por saber más, sino por ignorar menos.
Sor Juana Inés de la Cruz

Abstract

Nowadays, many of the healthcare systems are large, complex environments and quite dynamic, specifically Emergency Departments, EDs. They are opened and working 24 hours per day throughout the year with limited resources. EDs are usually the main entrance to the hospital, and a key component of the whole healthcare system. The original mission of EDs is to primarily handle only emergency situations. However, ED visits include a wide range of illnesses and injuries, from truly emergencies to non-urgent cases. As a consequence, EDs are overcrowded. Thus, is mandatory to use extensively computer simulations of EDs to evaluate output responses. The choice of optimal simulation parameters can lead to improved functioning, but choosing a good configuration remains a challenging problem. This improvement can be achieved by modelling and simulating EDs using Agent-Based Modelling and simulation. Optimisation via simulation is an emerging field which integrates optimisation techniques into simulation analysis.

In this research a two-phase optimisation methodology for optimisation via simulation for healthcare Emergency Departments is proposed. The first phase is a coarse grained approach consisted in a global exploration step over the entire search space. This phase identifies promising regions for optimisation based on a neighbourhood structure of the problem, using either a pipeline scheme approach of an Emergency Department or the Monte Carlo heuristic plus the K-means method, or both. This first phase returns a collection of promising regions. The second phase is a fine grained approach that consists in seeking the best solution, either the optimum or a sub-optimum by performing a reduced exhaustive search in such promising regions.

This work optimises the sanitary sta configuration of an actual ED. The sanitary sta configuration comprises: doctors, triage and emergency nurses, admission personnel, and x-ray technicians, the amount, and sort of them. Sta configuration is a combinatorial and multidimensional problem, that can take a lot of time to be solved. In order to do optimisation, objective functions to minimise or maximise have to be set. Three di erent indexes were set: minimise patient length of stay (LoS); maximise number of attended patients per day (Throughput); and minimise a compound index, the product of the cost of a given sanitary sta configuration times patient length of stay (CLoS). HPC is used to run the experiments, and encouraging results were obtained. However, even with the simplified ED used in this work the search space is very large, thus, when the problem size increases, it is going to need more resources of processing in order to obtain results in a reasonable time.

Keywords: Emergency departments, modelling, simulation, agent-based, optimisation, high-performance computing.

Resumen

Actualmente, muchos de los sistemas de salud son entornos grandes, complejos y dinámicos, en particular los servicios de urgencias hospitalarios (EDs por sus siglas en inglés). Éstos abren y funcionan las 24 horas al día durante todo el año, con recursos limitados. Los EDs suelen ser la entrada principal al hospital y componente clave de todo el sistema sanitario. La misión original de los EDs es atender situaciones de emergencia. Sin embargo, los usuarios de EDs incluyen una amplia gama de enfermedades y lesiones desde casos urgentes, hasta no urgentes. Como resultado de esto, los EDs están saturados. Por lo tanto, es necesario el uso amplio de simulaciones computacionale de EDs para evaluar sus respuestas a la demanda de servicios. La elección de los parámetros de simulación óptimos puede mejorar su funcionamiento, pero la elección de una buena configuración es un gran desafío. Esta mejora se puede lograr mediante la modelización de los EDs basado en agentes y su simulación. La optimización mediante la simulación es un campo emergente que integra técnicas de optimización en el análisis de simulaciones.

En esta investigación se propone una metodología de optimización de dos fases para la optimización de EDs a través de la simulación. La primera fase es un enfoque de grano grueso que consiste en una etapa de exploración global sobre todo el espacio de búsqueda. Esta fase identifica regiones prometedoras para la optimización basado en una estructura de vecindad del problema. Esta fase utiliza ya sea un enfoque *pipeline* de EDs o la heurística de Monte Carlo más el método de K-means, o ambos. Esta primera fase devuelve una colección de regiones prometedoras. La segunda fase es un enfoque de grano fino, que consiste en la búsqueda de la mejor solución, ya sea la óptima o una subóptima mediante una búsqueda exhaustiva reducida en tales regiones prometedoras.

Este trabajo optimiza la configuración del personal sanitario de un ED existente. La configuración de su personal incluye: médicos, enfermeras de triaje y de urgencias, personal de admisión y técnicos de rayos X, cantidad y tipo de ellos. Dicha configuración es un problema combinatorio y multidimensional, que puede consumir mucho tiempo en resolverse. Específicamente tres índices diferentes se verificaron: minimizar tiempo de estancia del paciente; maximizar número de pacientes atendidos diariamente y minimizar el producto del costo de la configuración por el tiempo de estancia del paciente. HPC se utiliza para ejecutar los experimentos y se han obtenido resultados alentadores. Sin embargo, incluso con una versión simplificada de un ED utilizada en este trabajo, el espacio de búsqueda es muy grande, por lo tanto, cuando aumenta el tamaño del problema, se requerirán más recursos de cómputo para obtener resultados en un tiempo razonable.

Palabras clave: servicios de urgencias, optimización, simulación, cómputo de alto rendimiento, modelización, agentes.

Resum

Actualment, molts dels sistemes de salut són entorns grans, complexos i molt dinàmics, en particular els serveis d urgències hospitalaris (EDs per les sigles en anglès). Aquests obren i funcionen les 24 hores al dia durant tot l any, amb recursos limitats. Els EDs solen estar a l entrada principal de l hospital i un component clau de tot el sistema de salut. La missió original dels EDs és atendre situacions d emergència únicament. No obstant això, els usuaris d EDs inclouen una àmplia gamma de malalties i lesions des de casos de veritable emergència, fins als no urgents. Com a resultat d això, els EDs estan saturats. Per tant, és necessari l ús ampli de simulacions de EDs a ordinador per avaluar les seves respostes a la demanda de serveis. L elecció dels paràmetres de simulació òptims pot millorar el seu funcionament, però l elecció d una bona configuració segueix sent un gran desafiament. Aquesta millora es pot aconseguir mitjançant la modelització dels EDs basat en agents i la simulació.

L optimització a través de la simulació és un camp emergent que integra tècniques d optimització en l anàlisi de simulacions. En aquesta investigació es proposa una metodologia d optimització de dues fases per a l optimització d EDs a través de la simulació. La primera fase és un enfocament de gra gruixut que consisteix en una etapa d exploració global sobre tot l espai de cerca. Aquesta fase identifica regions prometedores per a l optimització basat en una estructura de veïnatge del problema. Aquesta fase fa servir ja sigui un enfocament pipeline d un servei d urgències o l heurística de Monte Carlo més el mètode de K-means, o ambdós. Aquesta primera fase retorna una col • lecció de regions prometedores. La segona fase és un enfocament de gra fi, que consisteix en la recerca de la millor solució, ja sigui l òptima o una sub-òptima mitjançant la realització d una recerca exhaustiva reduïda en tals regions prometedores.

En aquest treball s optimitza la configuració del personal sanitari d un ED existent. La configuració del seu personal inclou: metges, infermeres de triatge i d urgències, personal d admissió, i tècnics de raigs X, la quantitat i el tipus d ells. La configuració del personal dels EDs és un problema combinatori, que pot consumir molt de temps a resoldre. Específicament tres índex diferents: temps d estada del pacient al servei d urgències, el nombre de pacients atesos per dia i un índex compost, el producte del cost del personal sanitari configuració pel temps d estada del pacient. HPC s utilitza per executar els experiments i s han obtingut resultats encoratjadors. No obstant això, fins i tot amb una versió simplificada d un ED utilitzada en aquest treball, l espai de cerca és molt gran, per tant, quan augmenta la mida del problema, es requeriran més recursos de còmput per tal d obtenir resultats en un temps raonable.

Paraules clau: serveis d urgències, modelat, simulació, basats en agents, optimització, computació d alt rendiment.

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Dedication

To my child Leticia, and her mother Natasha Mayerhofer.

To my parents, Eduardo and Antonia.

To my siblings Thelma, Dora, Edna, Eda, and Edgar.

To my family members Constantino Cabrera (r.i.p.), Dora Figueroa, Irma, Dora, Constantino, Guadalupe, Isaías, Miguel, Josefina, Álvaro Flores (r.i.p.), Teresa Hernández (r.i.p.), Fermín, Enriqueta, Ángel, Cutberto, Miguel, Guadalupe, Francisco and Luis; aunts and uncles; cousins, especially to Jesús, Martha, Ricardo, Graciela, Guadalupe; nephews; nieces; and all my relatives.

To my new family Savia and José.

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Chapter 1

Introduction

"Estudar não é um ato de consumir idéias,
mas de criá-las e recriá-las."
Paulo Freire

1.1 Emergency Department

Health is one of the most appraised gifts for human beings; therefore, it is crucial to preserve it. Healthcare systems are characterised by high human involvement and were designed to take charge of health. Such healthcare systems have evolved along centuries, specifically during the previous decades. Hospitals, as core members of healthcare systems, are made of several independent distributed complex care units [19]; amongst other, some of these units are: cardiology, neurology, gastroenterology and emergency departments. The Emergency Department (ED) could be the most dynamic healthcare department, usually the main entrance to the hospital, and a key component of the whole healthcare system.

Emergency Departments are semi-autonomous units and responsible for managing the large influx of patients. EDs are open and staffed 24 hours per day, 365 days per year, including holidays. The original mission of EDs is to primarily handle only emergent situations. However, ED visits include a wide range of illnesses and injuries, i.e., true emergencies, urgent, semi-urgent, and non-urgent cases. In the recent years, EDs worldwide have increased their human and infrastructure resources to attend all of these cases, becoming large, complex, and dynamic units.

Healthcare and ED management is concerned with the mission of improving the healthcare delivery system, i.e., organisation, planning, coordination, staffing, evaluating and controlling of healthcare services. Their main objective is to provide affordable healthcare of the best quality.

1.2 Motivation

Nowadays, many of the healthcare systems are large and complex environments and dynamic systems, specially the Emergency Departments. The ED is a *sui* *generis* unit of hospitals. On the one hand, it is opened and working 24 hours per day throughout the year with limited resources, specially with the present financial crisis, when there are several budget reductions that could compromise healthcare systems. And on the other hand, is under a huge and growing demand of services, i.e., overcrowded. Such critical service must be provided with the best quality and effort. ED is supposed to be the unit where only severe illness and injury, emergent cases, is handled, but due to the high demand of services, it is not the case any more. As a matter of fact, ED has become a unit where urgent, non-urgent, and severe cases converge, which decrease the amount of time, quality and resources given to the patients. Therefore, it is mandatory to improve qualitatively and quantitatively the performance of such crucial department.

Moreover, High Performance Computing (HPC) has been associated, and used mainly in classical sciences as physics, astronomy and chemistry, or hard difficult engineering problems, but nowadays social sciences are also using it. The systems modelled are quite complex and demand huge amount of data space, and to preserve the data new file systems must be develop; furthermore, the simulation of such systems has long run-times on conventional computers. In addition, the models and the phenomena being modelled are inherently probabilistic. Hence, social sciences and EDs demand HPC, in order to simulate, analyse, understand and generate knowledge.

This work is interdisciplinary, since there is a relationship between health science and healthcare systems, computer science and engineering. It belongs to Computational Science applications in *Individual Oriented Behaviour*, specifically. Thus, computers are used to simulate the model, which is not a mathematical or standard one of an ED, as it is said above, in order to choice of optimal simulation staff configuration that to can lead to improved functioning.

1.3 Statement of the Problem

In the operation of Emergency Departments, it has been observed an almost steady stream of patients arriving into them, specifically non-urgent or urgent cases, but also serious ones too. The latter cases are, or at least they are supposed to be, the main target of EDs, even though all cases have to be received, and addressed. Patients can arrive either by their own or by ambulance. Moreover, there are days, periods or extreme events which modify such almost steady stream of patients and increase the demand of services that compromise the whole EDs and the *ad hoc* or ideal patient care. Nevertheless, patient input cannot be modified, i.e., it is a fact, even if it is steady or not.

EDs units are constituted by the place, the physical resources as beds and test equipment, and, finally, but the most important, people as patients and their companions, and staff members, which includes nurses, doctors, and admission staff, amongst others.

Usually, patients have the following flow in the EDs: a) arrive by walk in, if they do not require immediate care they proceed to the admission place, whereas those who need immediate attention, and those that arrive by ambulance are sent directly to a treatment area, if admission staff is busy patients wait; b) thus, patients go to a triage area, if triage nurses are busy, patients have to wait, again, but in another area -at this step patients are evaluated for the seriousness or acuity of their condition, and a priority level is assigned based on it; c) then, patients wait for a diagnosis and treatment room and a doctor; d) finally, patients could be admitted into the service or discharge. A typical ED layout is shown in Figure 1.1.

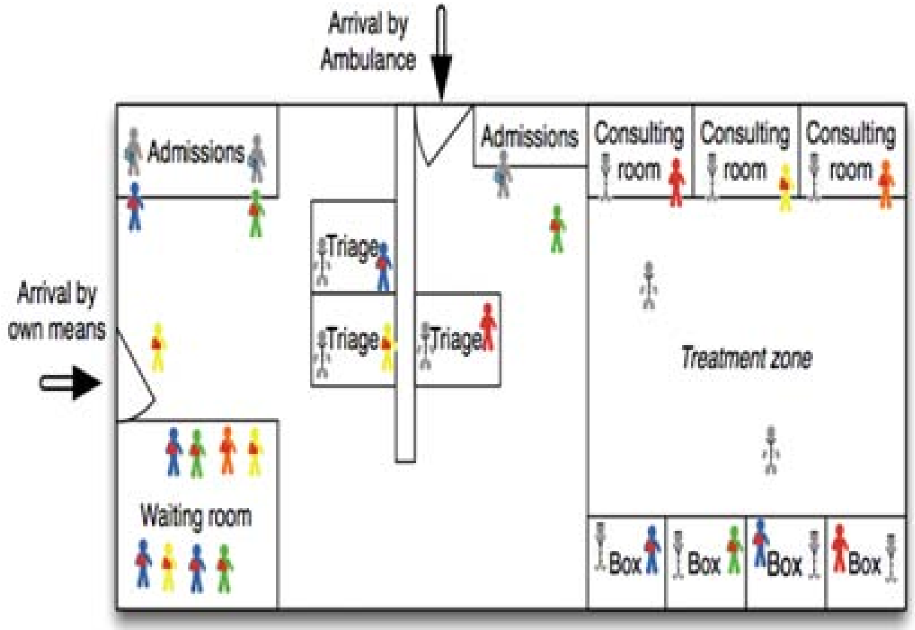


Figure 1.1: Typical layout of emergency departments

All these phases depend, not only on the stream of patients distribution, but also on the configuration of the ED staff members, i.e., the human resources of EDs, which imply a cost, associated to their salaries, as well as the costs related to screening and diagnostic tests that have to be performed to the patient. These costs have become a very important issue in the functioning of EDs, because the budget has become a major constraint for their operation (and in some cases for their existence).

In spite of such an increase, patients continue to suffer, since they do not have access to *ad hoc* healthcare, in some cases due to the inefficiencies of the EDs functioning. As a result of this, EDs are overcrowded and the length of stay (LoS) of patients has increased, whereas quality of service has decreased. Indeed, overcrowding of EDs is a worldwide issue, and a national crisis in the US [39]. Although ED overcrowding is not a new topic, it was documented in the literature for the last 20 years [11, 46, 54, 55, 58, 67], there is not a solution to this long and growing issue yet. Therefore, new techniques and paradigms should be found in order to deal with such overcrowded condition. ED managers require different and fresh solutions, because society demands not only care, quality and service, but also the best care, quality and service.

A direct solution to this issue is increasing the size of EDs, however, this straightforward solution is limited by the facility, number of staff (doctors, nurses, technicians) and services (computing, communication, radiology, laboratory), and it is not the best approach [50]. Also, healthcare managers have to maximise the use of healthcare resources, whereas being constrained by limited budget, in order to minimise patient LoS, while increase satisfaction of the patients, i.e., to optimise the performance of the ED. The resource planning of an ED is complex activity, since it is not linear, and it varies depending on time, day of week and season. The ability to simulate special situations such as seasonal increases in ED demand can be useful for the efficient use of resources.

1.4 Mathematical-Computational Modelling and Simulation of Emergency Departments

There are no standard models to characterise complex systems such as healthcare systems. Due to the absence of any formal description for **EDs**, alternative methods must be used to describe them. Simulation becomes an important tool for modelling systems including many elements as well as interdependencies amongst the elements, and/or considerable variability. Discrete event simulation (DES), system dynamics (SD) and agent-based modelling and simulation (ABMS) are the three main approaches used to simulate healthcare systems. There are a large and growing body of literature describing the use of DES and SD models in ED studies, but the use of ABMS for this purpose is few; although, healthcare systems are characterised by a high human involvement, i.e., based on human actions and interactions, that are quite difficult to model with DES, and can be more properly modelled with ABMS.

Therefore, here we use Agent-Based Model (ABM), also known as Individual oriented Modelling, since this framework describes the dynamic of the system, in which agent behaviour is complex and non-linear and the combined interaction of the agents can create a rich emergent behaviour, while showing memory [10].

1.5 Optimisation via Simulation

Computer simulations are used extensively as models of real systems to evaluate output responses. Applications of simulations are widely found in many areas including supply chain management, finance, manufacturing, engineering design and medical treatment [28, 42, 66]. Choosing optimal parameters for the simulation could lead to improved functioning, but choosing a good configuration remains a challenging problem. Historically, the parameter settings are chosen by selecting the best from a set of candidate parameter settings. Optimisation via simulation [6, 25–27, 63] is an emerging field which integrates optimisation techniques into simulation analysis. The corresponding objective function is an associated measurement of an experimental simulation. Due to the complexity of the simulation, the objective function may be difficult and expensive to evaluate. Moreover, the inaccuracy of the objective function often complicates the optimisation process.

1.6 Thesis Objectives and Methodology

The ultimate goal of this work is to devise, implement, and evaluate a methodology to do optimisation via simulation of the performance of complex and dynamic healthcare emergency department systems.

Through this methodology emergency department managers can set up strategies and management guidelines to enhance the performance of such critical system, i.e., a decision support system (DSS) for Healthcare Emergency Department (ED).

Specifically setting the hypothesis that the search space or parameter settings of emergency departments could be reduced by using a two-phase optimisation via simulation methodology.

1.6.1 Methodology

To achieve the objective previously stated, the methodology used in this thesis is based on the scientific method. The first phase is a coarse grained approach consisted in a global exploration step over the entire search space. This phase identifies promising regions for optimisation based on a neighbourhood structure of the problem. This phase uses either a pipeline scheme approach of an Emergency Department or the Monte Carlo heuristic plus the K-means method, or both. This first phase returns a collection of promising regions which are represented by hyperplanes. The second phase is a fine grained approach that consists in seeking the best solution, either the optimum or a sub-optimum by performing a "reduced exhaustive search" in such promising regions to find the optimum or a good solution. This methodology research is shown in Figure 1.2.

This research has followed an incremental development approach based on several enhancements of the original model proposed, that was published in [70], and [71]. An extension to the model of ED was published in [73].
The first version of this research was published in [12], an enhancement of the proposal of this work was published in [15], and finally, the two-phase version of the research was published in [14], and [13].

1.7 Thesis Outline

According to the objectives and the methodology described above, the outline of the remaining chapters of this dissertation is as follows.

Chapter 2: State-of-the-Art of the Healthcare Emergency Department Operations and Their Modelling.

In this chapter concepts of modelling are discussed. The framework used Agent-Based Model is also presented, as well as the model of the Emergency

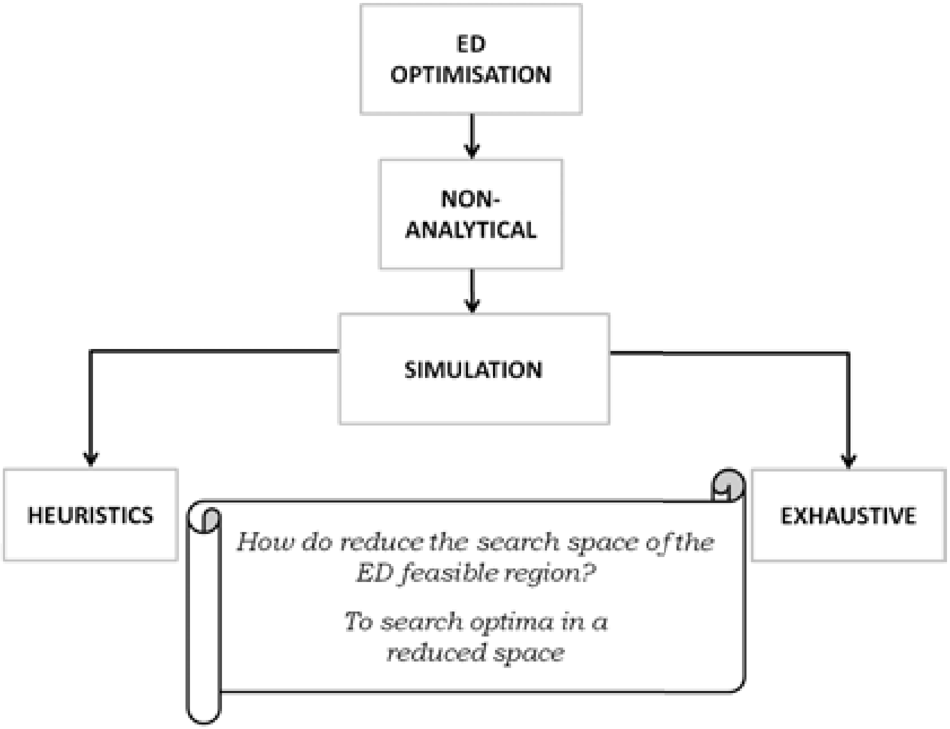


Figure 1.2: Methodology research

Department. Then topics about simulation and its importance are outlined, and finally, the simulator of the ED developed is presented.

Chapter 3: Optimisation via Simulation of Emergency Departments.
In this chapter topics of optimisation are explained. They include definitions, numerical methods, and taxonomy of optimisation approaches for single and multiple objectives. The proposed optimisation via simulation methodology is presented, as well as the cluster implementation of the optimisation proposal for emergency departments.

Chapter 4: Applications of the Proposed Optimisation via Simulation.
The evaluation of the experiment proposed in the methodology and its results are included and discussed in this chapter.

Chapter 5: Conclusions and Future Research
Concludes this dissertation and presents the open lines and future work of this research.

Chapter 2

State-of-the-Art of the Healthcare Emergency Department Operations and Their Modelling

"Publicamos para no pasarnos la vida corrigiendo los borradores."

Alfonso Reyes

In this chapter concepts about what modelling is, its purposes, and characteristics are outlined. Also, the state-of-the-art of the Healthcare Emergency Department (ED) operation and their modelling is described. The definition and vindication on using an alternative method, the Agent-Based Model (ABM), to model complex systems is presented later. The current Agent-Based Model of Healthcare Emergency Department is briefly addressed, as well as concepts of simulation and related works are discussed.

2.1 Introduction

In spite of the fact that the complexity of healthcare emergency departments has increased during the past few decades, the design of their key operational processes has been maintained somehow primitive. To address these problems, many facilities have turned to quick fixes, e.g., downsizing or adding more human or physical resources; however, most of these changes have not resulted in the desired outcome. For example, patients often experience long waiting times and encounter delays or cancellations. Most of these problems neither are caused by a lack of effort from staff nor because of their number, and cannot be resolved by, for example, working harder. Rather, it seems that the patient flow between and amongst the different healthcare departments is the source of their operational problems. Generally a healthcare facility is made up of several interdependent units, where the actions and decisions of one of them can affect the others. As such, patient flow throughout the entire system must be improved, rather than just in isolated ED units.

2.2 Operation of Emergency Departments

The task of characterising any system is difficult, especially systems where people are involved such as emergency departments. Unfortunately, oversimplification appears when attempting to categorise emergency care systems internationally. First of all, in order to characterise EDs it is mandatory defining what is meant by the term "**ED**". Without loss of generality, an ED is a healthcare system that delivers immediate, often stabilising, care for patients with emergent medical needs. This care is provided within the largest level of availability and accessibility possible, i.e., 24 hours per day, 7 days per week and 365 days per year, including holidays, with no restriction on who can access such critical service. The Emergency Department could be the most dynamic healthcare department, usually the main entrance to the hospital, and a key component of the whole healthcare system.

Emergency Departments are semi-autonomous units and responsible for managing the large influx of patients. The original mission of EDs is to primarily handle only emergency situations. However, ED visits include a wide range of illnesses and injuries, i.e., truly emergencies, urgent, semi-urgent, and non-urgent cases. In the recent years, worldwide the EDs have increased their human and infrastructure resources to attend all of those cases, becoming large, complex and dynamic units.

Healthcare and ED management is concerned with the mission of improving the healthcare delivery system, i.e., organisation, planning, coordination, staffing, evaluating and controlling of healthcare services. Its main objective is to provide healthcare of best quality, and affordable to people.

Although the diversity amongst the EDs all over the world, certain basic characteristics had proved useful for describing EDs. According to [72] when viewing EDs from the perspective of how patient care is delivered, four main characteristics can be used to describe EDs, i.e., (1) physical location, (2) physical layout, (3) time period open to patients, and (4) patient type served. These four characteristics are particularly well suited to describe a wide variety of care contexts, i.e., they represent a basic common framework to which other characteristics can be added, as to consider school or University-affiliated EDs, since they train doctors and nurses students. This training could affect time and quality of service, because such healthcare students or junior staff are less experienced than senior doctors and nurses. In what follows each of them will be discussed.

2.2.1 Physical Characteristics of Emergency Departments

Physical Location

The physical location of EDs could be one of the most basic features of emergency care. This characterisation could be subdivided into two main groups: hospital-based EDs and *independent* EDs. First ones are typically located in a general acute care hospital, but may also be found in specialty hospitals; whereas *independent* EDs can be further characterised as satellite EDs, autonomous EDs, and primary-care-based EDs.

Physical Layout

The standard physical layout of an ED is shown in Figure 2.1. It consists of the admission, the triage and the treatment zones. It is worth noting that patients might arrive to the ED by their own means or by ambulance. Emergency care may also be provided in several different layouts within a facility. Characterising EDs by physical layout distinguishes two main groups: contiguous and non-contiguous. In a contiguous ED, medical and surgical emergencies are treated in one or adjacent areas. Contiguous EDs can be further described as having or lacking triage to service. "Triage to service" does not refer to the process of patients being admitted to the hospital from the ED, but rather to the process whereby patients arriving at the ED are directed to emergency care from non-emergency medicine specialties, e.g., to a medical or surgical team. A contiguous ED with triage to service is often staffed by physicians from many different specialties, amongst others are: surgeons, internists, and cardiologists, who are employed by their respective specialty departments and who treat emergencies related to their fields.

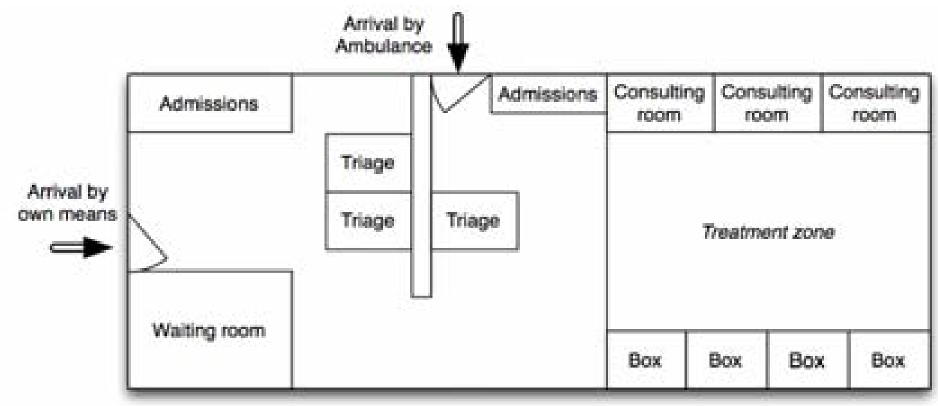


Figure 2.1: Simplified emergency department layout

2.2.2 Operational Characteristics of Emergency Departments

Time Period Open to Patients of EDs

When EDs are characterised according to when they provide emergency care, they tend to fall into four groups: full-time, part-time, seasonal or alternating. A full-time ED provides care 24 h per day, 7 days per week, 365 days per year. In contrast, a part-time ED is open less than the former. Whereas part-time EDs usually are open at least 150 of 168 hours per week and 365 days per year. Similar reasoning may be applied to seasonal EDs, which are only open during one portion of the year. Finally, alternating EDs are those which share responsibility for providing 24/7 emergency care to a population. Though each hospital may have an ED that, when considered alone, may not qualify as such due to its restricted hours of availability.

Patient Type Served by EDs

When characterising EDs by the sort of patient attended, it has been found that three main groups appear: general population EDs, adult EDs, and paediatric EDs. General population EDs serve all patients regardless of age, sex, race/ethnicity, or other major socio demographic factors. General population EDs may be further characterised as combined or separate. Combined general population EDs provide care for all patients in one common area, while separate general population EDs provide care to different groups of patients in distinct physical areas within one facility, depending upon specific patient characteristics. The most common population characteristic that distinguishes these two types of general population EDs is age, as demonstrated by children and adults being seen in separate locations within a facility. However, not all EDs primarily serve both children and adults.

Patients Flow in EDs

Usually, patients have the following flow in the **EDs** (see Figure 2.2) : **a)** arrive by walk in, if they do not require immediate care, they proceed to the admission ED unit, whereas both of those who need immediate attention, and those that arrive by ambulance are sent directly to a treatment area, if the admission staff is busy, patients have to wait; **b)** thus, patients go to an ED triage area, if triage nurses are busy patients have to wait, again, but in another ED unit -in this step patients are evaluated for the seriousness or acuity of their condition, and a priority level is assigned based on it; **c)** finally, patients wait for a diagnosis and treatment room, and a doctor. At last, patients could be admitted into the service or discharge.

Sta and Economics of EDs

Usually the configuration of the staff members of an ED consists of: doctors, nurses, admission personnel, x-ray technician, and supporting staff. These human resources imply cost, such as their salaries, as well as the costs related to the EDs' infrastructure used for the tests that have to be taken to the patient, as part of their diagnosis and treatment. As it discussed further down, the staff configuration plays a crucial role in the optimal operation of EDs.

2.3 Modelling of Emergency Departments

2.3.1 Introduction to Modelling

It is said that models are only an abstract representation of a real system. Models can be defined as a set of assumptions or approximations about how a system works, i.e., it describes the system. It is of paramount importance remembering that a model is not reality, but merely a human construction to help to better understand real world systems [45].

Modelling is an art. Models can be either mental, that are subjective, incomplete, and usually lack a formal statement (e.g., ideas and concepts), or formal, which are based on rules, and are easy transmittable, for example diagrams, and planes.

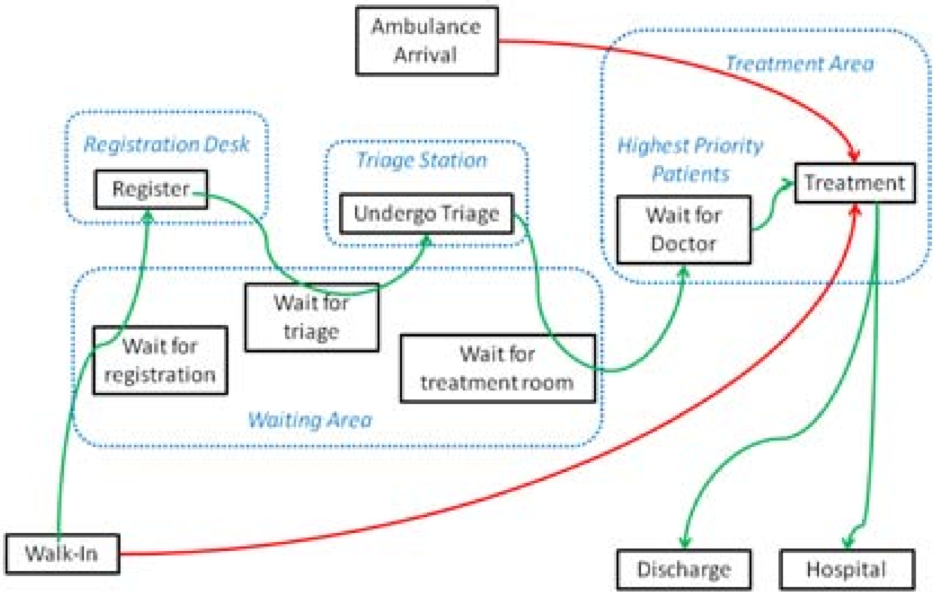


Figure 2.2: ED model by patient service

The overall modelling process is usually iterative, and comprise the following steps: 1) Determine the goals and objectives of the model; 2) Build a *conceptual* model; 3) Convert into a *specification* model; 4) Transform into a *computational* model; 5) Verify the model; 6) Validate the model.

Amongst the purposes of modelling the following can be stated: allow to study, and analyse the model instead of the real system. It is easier, faster, cheaper, and (in some cases) safer; can help to train or for educational objectives; can generate new insights; testable predictions can be made; can help to proof hypothesis; rule out a particular explanation for an experimental observation; can include wide-range of ideas or experiments.

Once the goals and objectives of the model have been stated, there are some questions that have to be examined and satisfactorily answer before carrying on the steps 2 to 6, previously stated: are the expected savings from using the model greater than the cost of developing and implementing it? Is there enough time to develop and implement the model before the recommendation is needed? Is it easier to perform an experiment on the real system (if this is possible) than to build a model of the latter?
However, answering the previous questions are not the most difficult tasks when developing the *conceptual* model, but also to identify the simplifications that ought to be made on the model without sacrificing the needed or useful accuracy of the proposed model. If unimportant details are kept out of the model, it would be easier to change it and to use it. Another key part of this *conceptual* and abstract process is how comprehensive should the model be?

The *specification* phase comprise (if it is the case) in the mathematical formulation of the model, i.e., the equations that represent it and to generate a pseudocode of the *conceptual* model of the prior step; whereas the *computational* model consist in numerical implementation, i.e., a computer program.

Verification implies that *computational* model must be consistent with the *specification* model. Finally, the model is validated if it is consistent with the system being analysed. Moreover, if an expert cannot distinguish a simulation output from the actual system output (in case we have actual observations), thus, the model is right. Nevertheless, if the output model differs from what happened in the real world, it is necessary to recall that the model is not the reality. Hence, the model still has some flaws. As an iterative process if any phase is not satisfied it has to back to the previous steps.

As mentioned in section section 2.1 the ED systems are complex [52], and difficult to analyse. Therefore, recently the use of the so-called Agent-Based Model (**ABM**) have bee suggested as a promising possibility to model the EDs. In the following section the main characteristics of the ABM will be discussed.

2.3.2 Agent-Based Model

Although, there is not a general accepted definition, it can be said that an Agent-Based Model **(ABM)** is a computational model of a heterogeneous population of agents (components of the system) and their interactions, as well as the interactions of the former with the environment. This type of modelling is used to analyse complex systems that are difficult to be tackled by classical or formal methods, which are unable to represent such systems. The result of the micro-level interactions of the components of a system can produce macro-level behaviour like cooperation, segregation, and culture, amongst others. Furthermore, the ABM framework describes the dynamics of systems in which agent behaviour is complex, stochastic, and non-linear. Also in systems in which the combined interaction of all its agents can create rich emergent behaviour, and shows memory [10]. **ABMs** are fundamentally decentralised, i.e., the behaviour of the system is defined at individual level, and the global behaviour emerges as a result of the interactions of many individuals (agents of the ABM), each one following its own behaviour rules. Hence, **ABM** is also called bottom-up modelling. An ABM agent is defined as a discrete entity with its own goals and autonomous behaviours, with a capability to adapt and modify the latter.

ABM can be a useful tool for model analysis, complementary to pure mathematical model, i.e., when a model is either not totally solved mathematically or apparently insoluble. This is the case for social science systems, where usually there is a lack of mathematical model which defines the problem.

The three fields in which ABM are most utilised are: economics, biology, and social sciences [32]. It is widely used in the latter in situations where human behaviour cannot be predicted using classical methods such as qualitative, or statistical analysis [56]. Human behaviour is also modelled with **ABMs** in the fields of psychology [69], epidemiology [21], and tourism planning [40], amongst a long list of others.

Amongst others, according to [47] the advantages of using **ABMs** to model complex systems, such as EDs, are the following: there is a natural representation of the ED components as agents; there are decisions and behaviours of the ED system that can be defined discretely (with boundaries); the ED agents can adapt and change their behaviours; the ED agents can learn and engage in dynamic strategic behaviours; the EDs agents can have dynamic relationships with other agents; ED's agents relationships can be developed and/or stopped; the spatial component of ED's agents behaviours and interactions can be fully represented.

2.3.3 Agent-Based Emergency Department Model

The Emergency Department model proposed in this thesis is a pure Agent-Based Model, and so is formed entirely of the rules governing the behaviour of the individual agents which populate the system.

The real EDs systems of interest for this thesis are the ones of hospitals of Mataro and Sabadell, especially the latter (a tertiary hospital which provides health service to a catchment area of 500,000 inhabitants, whereas its emergency department provides health service to an average of 160,000 patients/year), located in the vicinity of Barcelona. From the information obtained during the interviews carried out with ED staff and managers of those hospitals, active and passive agents were identified. The active ED agents represent people and other entities of the EDs that act upon their own initiative (*patients*, *companions of patients*, *admission personnel*, *sanitary and x-ray technicians*, *triage and emergency nurses*, *emergency doctors*, others *medical specialists*, and *social workers*). The passive agents represent ED components that are solely reactive, such as *loudspeaker system*, *patient information system*, *pneumatic pipes*, and *central diagnostic services* (*radiology service* and *laboratories*).

2.3.3.1 ED Active Agents.

The ED active agents are described by state machines, specifically Moore machines. A Moore machine has a single output for each state; transitions between states are specified by the input [53]. An example of the interaction carried out between patient and doctor active agents, represented as state machines, during the diagnosis phase is shown in Figure 2.3.

The current state of an active agent is represented by a collection of state variables, known as the state vector (**T**). Each unique combination of values for these variables defines a distinct state. Through the round of interviews at the EDs of Mataro and Sabadell hospitals an initial set of state variables for the EDs active agents has been defined, based on the minimum amount of information required to model each patient and member of staff. Some of such variables, their values and their kind of observability are shown in Table 2.1. Some of the state variables will have a potentially very large set of possible values, e.g. the symptoms or physical condition. Every time step the state machine moves to the next state is defined by the current state and the input vector as described below.

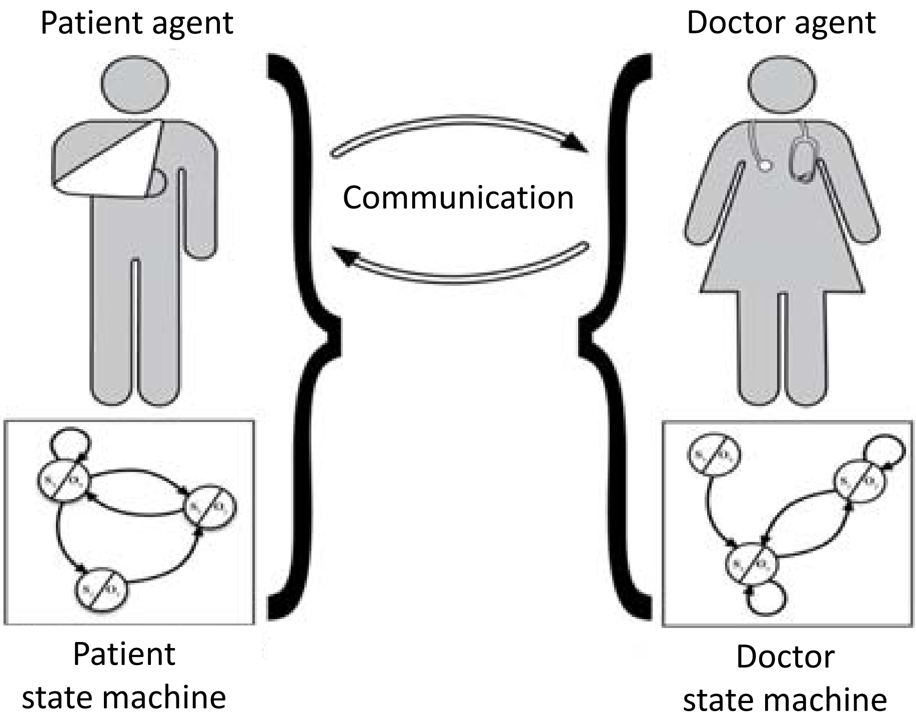


Figure 2.3: Patient and doctor active agents, and their interaction represented by state machines. S_i and O_i are the current state and the output, respectively, and the arrows represent the change of state of the agent because of the communication between patient and the doctor.

Patients are the most important active agents of the ED, and they are the reason of the existence on ED. Patients are served by their priority level, which is identified in the triage phase (mainly conducted by triage nurses) using the Spanish triage classification applied in Spanish hospitals [22, 77, 79]. This classification consists of 5 levels, with 1 being the most critical (resuscitation), and 5 being the least critical (non-urgent).

2.3.3.2 ED State Variables

In order for the Moore state machine to function, all state variables associated to an ED active agent must be enumerable in some manner (see Table 2.1). This may be achieved by using discrete variables or variables representing continuous quantities which have had their possible values divided into ranges.

A variable that is externally observable (E) indicates that any agent can discern the value of that variable merely by being within a certain proximity of the agent in question. An internally observable (I) variable is one where the agent is aware of the value of the variable, but other agents are not. An unobservable variable (N) is one which no agent, and thus nothing within the system, knows the value of it.

It is possible that a variable may have some values which are observable, and

Table 2.1: Some ED active agents state variables, and their values. **I** means that the variable is internally observable, **E** is externally observable, and **N** is unobservable.

Variable	Values	Observability
Name / Identifier	Unique per agent	**I**
Personal details (patient)	Gender; Medical history; Allergies; Origin	**I**
Location	Department entrance, Admissions, Waiting room, Triage, Consultancy room, Treatment box	**E**
Action	Idle, Requesting information from `<id>`, Giving information to `<id>`, Searching, Moving to `<location>`	**E**
Physical condition (patient)	Healthy, Hemodynamic-Constant; Barthel Index	**E / I / N**
Symptoms	None; Level 1 - Resuscitation; Level 2 - Emergent; Level 3 - Urgent ; Level 4 - Less Urgent; Level 5 Non-Urgent	**E/ I**
Level of communication	Low, Medium, High	**E**
Level of experience (doctor)	None, Resident, Junior, Senior and Consultant	**I**
Level of experience (triage and emergency nurses)	None, Low, Medium, High	**I**
Level of experience (admission personnel)	None, Low, Medium, High	**I**

others which are not or a group of values which will all appear the same to an observer, this is a partly observable variable. In the case of an agent representing a person and a variable representing their physical condition certain values may be externally observable (for instance a broken arm), others may be only internally observable (the cause of a stomach ache).

This observability is represented as implicit 1-to-location communication, each agent in the location receives, for instance, a message that another agent has a broken arm. Most agents will not respond to this input, but it is available to all as in the corresponding real life situation all people in a room would be able to see that a patient has a broken arm without the need to specifically ask this person about it.

2.3.3.3 ED Inputs, Outputs, and State Transitions

Upon each time step the state machine moves to the next state. This may represent a new state or the same one it was in before the transition. The next state the machine takes is dependent on the input during that state. The input may be described as an input vector (**I**) that contains a number of input variables, each one of which may take a number of different values. As this is a Moore machine, the output depends only on the state, so each state has its own output, although various states may have outputs that are identical. The output is described as an output vector (**O**), a collection of output variables, each with a number of defined possible values. Transitions between states are dependent on the current state at time t ($\mathbf{S}_t$) and the input at time t ($\mathbf{I}_t$). Following the transition the state machine will be in a new state ($\mathbf{S}_{t+1}$). The state machine can be represented as a state transition table, as shown in Table 2.2, where each row represents a unique state-input combination, showing the output (defined solely by the current state) and the state in the next time step (defined by the current state and the input).

Table 2.2: State transition table. $\mathbf{S}_i$ is the current state, $\mathbf{I}_j$ is the input, $\mathbf{O}_i$ is the output.

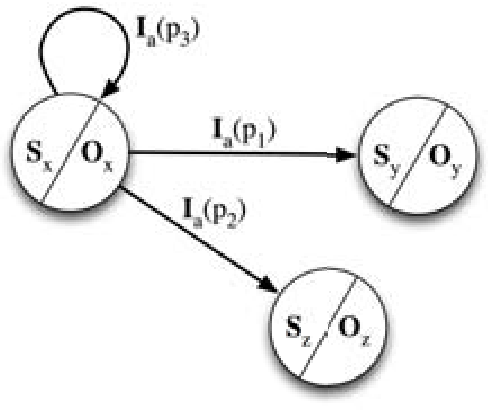
Current state / output	Input	Next state / output
$\mathbf{S}_0$ / $\mathbf{O}_0$	$\mathbf{I}_0$	$\mathbf{S}_i$ / $\mathbf{O}_i$
$\mathbf{S}_0$ / $\mathbf{O}_0$	$\mathbf{I}_1$	$\mathbf{S}_j$ / $\mathbf{O}_j$
$\mathbf{S}_0$ / $\mathbf{O}_0$	$\mathbf{I}_2$	$\mathbf{S}_k$ / $\mathbf{O}_k$
$\vdots$	$\vdots$	$\vdots$
$\mathbf{S}_x$ / $\mathbf{O}_x$	$\mathbf{I}_0$	$\mathbf{S}_y$ / $\mathbf{O}_y$
$\mathbf{S}_x$ / $\mathbf{O}_x$	$\mathbf{I}_1$	$\mathbf{S}_z$ / $\mathbf{O}_z$
$\vdots$	$\vdots$	$\vdots$

2.3.3.3.1 ED Probabilistic State Transitions In some cases the state machine representing an ED migth involve probabilistic transitions, where a given combination of a current state $\mathbf{S}_i(\mathbf{t}_i)$ and input $\mathbf{I}_j$ has more than one possible next state $\mathbf{S}_i(\mathbf{t}_{i+1})$. Which transition is made is chosen at random at the time of the transition, weights on each transition provide a means for specifying transitions that are more or less likely for a given individual. Each one of the input variable of the input vector ($\mathbf{I}$)) may take a number of different values with a certain probability. In these cases our state transition table is defined with probabilities on the input as shown in Figure 2.4b. An agent in state $\mathbf{S}_x$ receiving input $\mathbf{I}_a$ may move to either one of states $\mathbf{S}_y$, $\mathbf{S}_z$ or remain in the same state, with a probability of p_1, p_2, and p_3 respectively. One of these transitions will always occur, which is to say $p_1 + p_2 + p_3 = 1$. The state diagram would then have three different transitions for that state-input combination as shown in Figure 2.4a.

Probabilities may be different for each agent, in this way heterogeneity is provided to agents as people, since agent behaviour can be probabilistically defined external to their state.

2.3.3.4 ED Passive Agents

Passive agents represent services within the hospital ED system such as the information technology infrastructure that allows patient details to be stored, radiology services and other laboratory tests as well as special systems such as pneumatic



(a) Probabilistic state transition graph.

Current state / output	Input	Next state / output
⋮	⋮	⋮
$\mathbf{S}_x$ / $\mathbf{O}_x$	$\mathbf{I}_a(p_1)$	$\mathbf{S}_y$ / $\mathbf{O}_y$
$\mathbf{S}_x$ / $\mathbf{O}_x$	$\mathbf{I}_a(p_2)$	$\mathbf{S}_z$ / $\mathbf{O}_z$
$\mathbf{S}_x$ / $\mathbf{O}_x$	$\mathbf{I}_a(p_3)$	$\mathbf{S}_x$ / $\mathbf{O}_x$
⋮	⋮	⋮

(b) Probabilistic state transition table.

Figure 2.4: Probabilistic state transition graph, and its corresponding table. $\mathbf{I}$ and $\mathbf{O}$ represent input and output vectors, respectively, while $\mathbf{S}_i$ are the current states, and p_i is a certain probability.

tube networks that some larger hospitals use to quickly transfer samples from one part of the department to another.

2.3.3.5 ED Communication Model

The communication model represents three basic types of communication. First type is 1-to-1 communication, such between two individuals, for instance admission staff and patient, where a message has a single source and a single destination. Second is 1-to-n communication, where a message has a single source and a specific set of recipients, for example when a doctor communicates with both patient and his companion. The final type is 1-to-location communication, where a message has a single source, but is received by every agent within a certain area or location. This occurs when triage nurse call send a message to the patients of the waiting room, through the loudspeaker system.

Implicit, or passive, communication also exists, where an agent may be producing communication just be remaining in a certain area. This is the manner in which agent vision, what each agent sees, can be represented using the same model. An agent is continuously emitting messages with regard to its visible physical status and location, other agents receive these 1-to-location messages and may act upon them in certain circumstances. For instance, an agent waiting for another agent in a certain area will receive communication that the agent has entered and act upon it, representing, for instance, a nurse seeing a patient, enter a triage room and taking care of him.

Each message is comprised of a number of components. The source <src> and destination <dst> of the message, where the source is the individual and the destination is either defined as an individual, a group of individuals, or location (where all individuals within that location will receive the message). The actual content <content> of the message is the final part, creating a message tuple of the form `(<src>, <dst>, <content>)`.

The `<dst>` component of the message is the implicit destination of the message, in the real world case of 1-to-1 or 1-to-n communication is communicated via body language such as an individual facing another and making eye contact while talking. In the case of a 1-to-location message the implicit destination is the location. In some cases, a 1-to-location message is actually only meant for a certain agent, in which case the `<content>` component of the message will need to contain an explicit destination. A real world example of this is a loud speaker, all individuals within hearing distance of the loud speaker will hear the message, but if it is only directed at a certain individual their name or some other identifier will need to be used, so the specific individual knows it is for him and the remaining individuals know it is not addressed to them.

2.3.3.6 ED Environment

All actions and interactions modelled take place within certain locations, collectively known as the environment. The environment itself can be defined to different levels depending on the positional precision required of the model.

The environment in which the agents move and interact is passive and discrete. There is little distinction made between agents in the same location, a patient in the waiting room does not have any more specific sense of position than that they are in the waiting room. Certain locations may be physically distinct, but functionally identical, for instance there are usually a number of triage rooms, an agent in any one of these will act as if they are in any triage room, however they are distinct in order to represent that each available room may only be used by one nurse-patient group at a time. The environment also contains representations of the relative distances between different discrete locations, as can be seen in Figure 2.1.

2.4 Simulation

It is quite difficult, even almost impossible, to separate modelling from simulation, then why taking too much time in modelling if such model is not going to be tested? The word simulation comes from the Latin verb *simulare* and means to imitate, to simulate the operations of different kinds of some real thing or processes. Simulation is quite important since it allows us to understand the behaviour of a system, and to evaluate different strategies within a given structure. Computer simulation utilises the computers to perform experimentation on a model of the system of interest. The Figure 2.5 shows different ways to study a system [45].

Simulation is *ad hoc* or mandatory when: it is almost impossible to do experimentation in reality, because either the system does not exist or would be dangerous or quite expensive. Also, when the system cannot be interrupted, and time scale have to be changed. Amongst the advantages of doing simulation are: a) cost, experiments on real systems might be quite expensive; b) time, it is possible

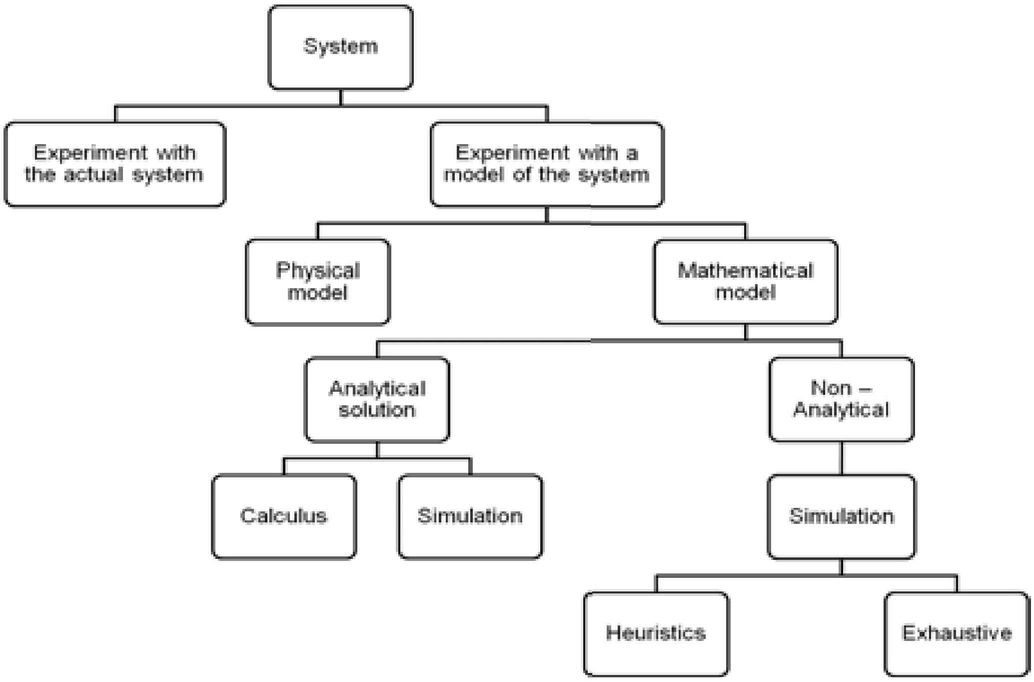


Figure 2.5: Different ways to study a system.

to simulate weeks, months, or even years in seconds; c) safety, effects of extreme conditions can be studied; d) replication, simulations are exactly replicable.

Simulation models can be characterised as: *deterministic* vs *stochastic,* is everything for sure or is there uncertainty?; *static* vs *dynamic*, does the time play an important role in the model?; and *discrete* vs *continuous,* the *state* of the system changes all the time, or only at specific or discrete times? [45]. This taxonomy is shown in Figure 2.6. A deterministic model is one where the model parameters are known or assumed, it does not contain any probabilistic component and the output is established straightforwardly once the set of input quantities and relationships in the model have been specified, whereas a stochastic model has one or more random components, and it is used wherein the cause and effect relationship is randomly determined and is generally not solved analytically; a static model is a representation of a system at a particular time or timeless, the state variables do not change while computing, in contrast, a dynamic one represents a system where the state variables evolve over time; in a discrete model its state variables change instantaneously at separated points in time, while a continuous one the state variables change continuously with respect to time [45].

Most of the time the systems, as well as the simulation, are *dynamic, discrete* and *stochastic*, which is the case of systems such as the **ED**.

2.4.1 Simulation Models for Emergency Departments

Simulation becomes an important tool for modelling systems including many elements, as well as inter-dependencies and considerable variability amongst the elements. Discrete event simulation (DES), system dynamics (SD), and agent-based

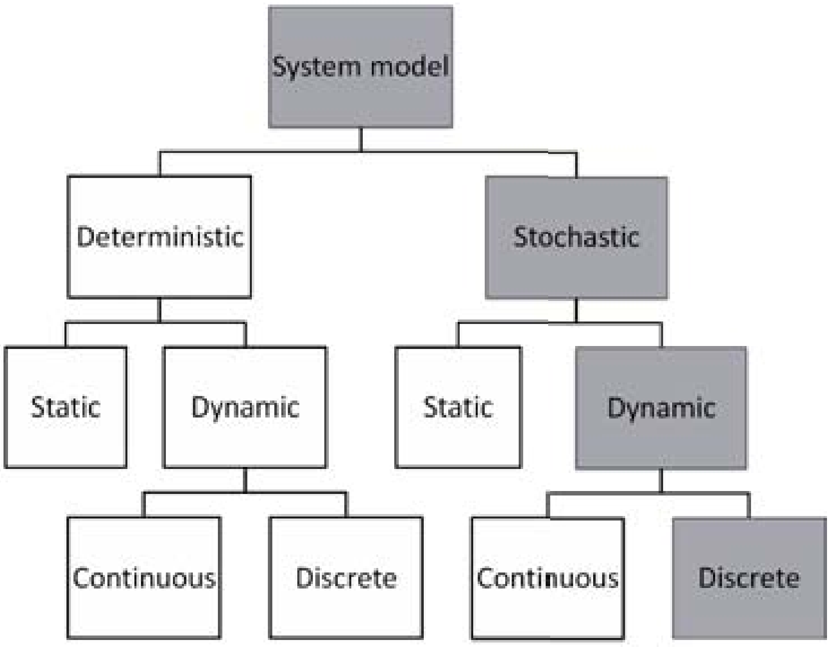


Figure 2.6: Classification of simulation models along three different levels.

modelling and simulation (ABMS) are the three main approaches used to simulate healthcare systems. DES is a technique that represents a system that can change at only a countable number of points, events, in time [45]. SD is a methodology and mathematical modelling technique which study how the information-feedback characteristics and time delays interact to influence the behaviour of the system [24]. ABM, as stated in subsection subsection 2.3.2, is a computational model of a heterogeneous population of agents (components of the system) and their interactions, as well as the interactions of such agents with the environment. There are a large and growing body of literature describing the use of DES and SD models in ED studies, but the use of ABMS for this purpose is scarce; although, healthcare systems are characterised by a high human involvement, i.e., based on human actions and interactions, that can be more properly modelled with ABMS.

2.4.2 Agent-Based Emergency Department Simulator

So far we have presented the information regarding to all the important and basic elements of the general model of and ED. It is necessary to see how agents interact and how they evolve over time. The ED simulator of this work is used as a *black box*, but the more realistic the simulator is, the better results and optimisations are. It is implemented in *NetLogo*, (version 5.0.34) an agent-based programming language and programmable modelling environment [23].

NetLogo is well suited for modelling complex systems developing over time. Modellers can give instructions to hundreds or thousands of independent agents all operating concurrently. This makes it possible to explore the connection between the micro-level behaviour of individuals and the macro-level patterns that emerge from the interaction of many individuals [2]. One of the most useful tools that *NetLogo* has is the *BehaviorSpace*, which lets you explore the *search space* of the model of possible behaviours and determine which combinations of settings cause the behaviours of interest

Taking into account the information obtained during the interviews at the ED of Sabadell hospital, we have implemented a simplified agent-based ED simulator. Two versions were implemented until now. The first one is the ED simulator v1.1, and the current ED simulator v1.2. The main difference between them is the diagnostic and treatment phase, which is more realistic in the current one, ED simulator v1.2.

The actions and interactions corresponding to the admission and triage processes have been totally implemented, but in the case of diagnostic and treatment phase, respecting to the priorities of the Sabadell Hospital ED currently only the level 1 was implemented. In such level 1 only patients with priority level 4 or 5 (less urgent, and non-urgent, respectively subsubsection 2.3.3.1) are taken care of. Nevertheless, all incoming patients are triaged by triage nurses. Once patients have been triaged, only patient type 4 and 5 are served at the stage of diagnosis-treatment phase. The rest of the patients, patient type 1, 2, and 3, are sent to a *black box* (Level 2 area in Figure 2.8), which represents the level 2 of the Sabadell Hospital ED.

The agent-based ED simulator version 1.1, which is shown in Figure 2.7, includes the following active agents: patients, admission personnel, triage nurses, and doctors. The diagnostic and treatment phase is only addressed by doctors.

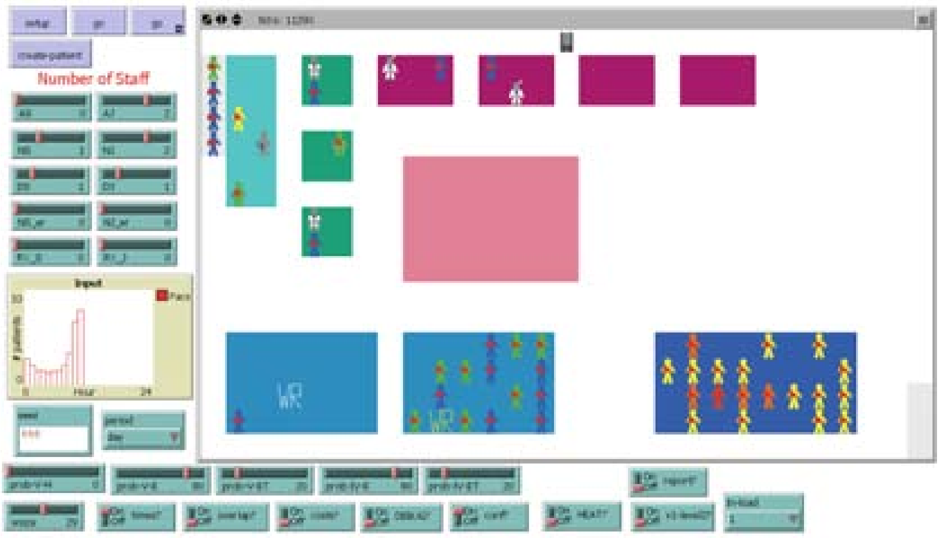


Figure 2.7: The agent-based ED simulator version 1.1. Admission personnel, triage nurses, and doctors were the sanitary staff considered.

The simple patient flow in this ED simulator is defined as follows: patients arrive to the ED on their own, and waits to be attended in the admission area. Then, patients stay in the first Waiting Room (WR) WR1, until a triage nurse call them. After the triage process patients identified as triage level 4 and triage level 5 pass to a second WR2, and stay there until a free doctor calls them to begin the diagnosis-treatment phase, depending on the patient's symptoms and physical condition, as well as prescribed diagnosis tests. At the end, patients are discharged from the ED.

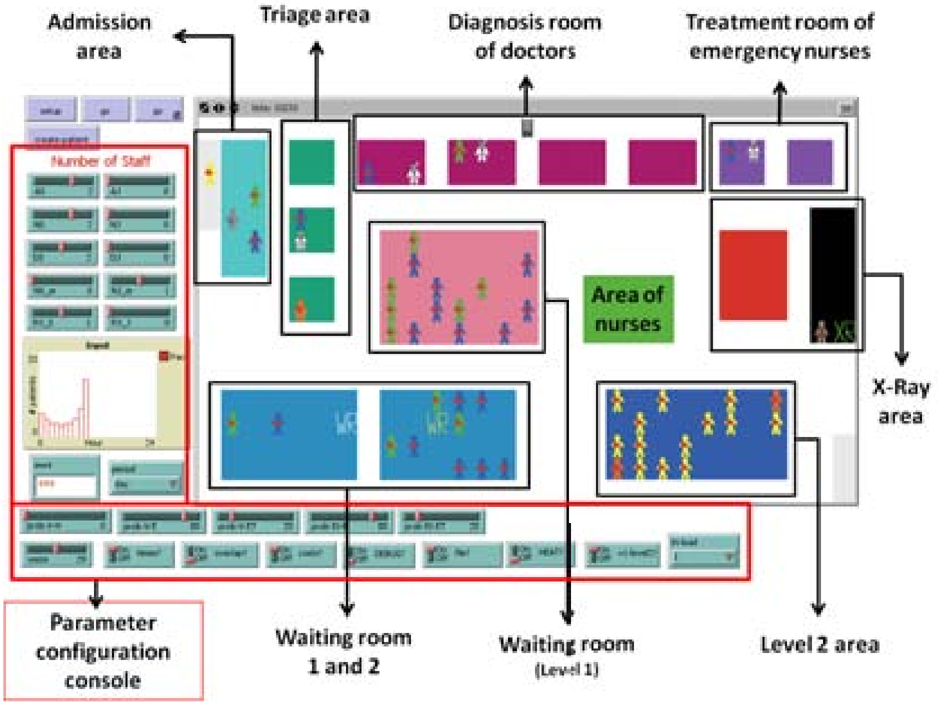


Figure 2.8: The agent-based ED simulator version 1.2, the current one. Admission personnel, triage nurses, doctors, emergency nurses, and x-ray technicians were the sanitary staff considered.

The agent-based ED simulator v1.2, which is shown in Figure 2.8, includes the following active agents: patients, admission personnel, triage nurses, emergency nurses, doctors, and x-ray technicians. The simple patient flow in this ED simulator is defined as follows: patients arrive to the ED on their own, and waits to be attended in the admission area. Then, patients stay in the first Waiting Room (WR), WR1, until a triage nurse call them. After the triage process patients identified as triage level 4 and triage level 5 pass to a second WR2, and stay there until a free doctor calls them to begin the diagnosis-treatment phase (interrogation process), depending on the patient's symptoms and physical condition, patients wait at WR (Level 1) to be attended by x-ray technician or an emergency nurse (to perform some diagnostic tests). After the diagnosis tests, patients come back to WR (Level 1) and stay there until a free doctor calls them again (treatment process). At the end, patients are discharged from the ED. Even though realistic treatment is based on the acuity of patients, in these simulators patients have the same path throughout the ED.

The simplified ED simulator is constituted of up to four diagnostic rooms, three triage rooms, two treatment rooms, two waiting rooms, an area of admissions, an area of treatment nurses, an x-ray area, the entrance, and the exit. Some parameters can be set from the graphical user interface, they are number and sort of staff members: doctors; triage and emergency nurses; x-ray technicians, and admission personnel, and senior or junior (that represents less and more expertise, respectively). Also, the input arrival patient, in percentage, as well as the maximum number of iterations can be set. Finally, also it can be selected if information about times, costs, and debugging is needed.

2.5 Discussion

The most relevant conclusions of this chapter are the following:

1. The operation and characterisation of the Healthcare Emergency Departments (ED), from the perspective of how urgent patient care is delivered, were discussed. EDs can be characterised by their: a) physical location (in a hospital unit or an independent one); b) physical layout (such as number of waiting, triage and medical rooms); c) time period open to patients (of waiting, triage and medical rooms), c) time period open to patients (24hs / 365 days per year or part time); d) patient type served (all /certain ages only); and e) type and number of staff members (admission and support personnel, nurses, doctors, medical technicians).

2. The modelling of complex and dynamic systems that lack of mathematical model which defines the problem is a difficult task, specially using classical approaches. Therefore, the use of an alternative modelling techniques such as the so-called Agent-Based Modelling, ABM, becomes an *ad hoc* strategy to study this problem. ABM is a computational model of a heterogeneous population of agents (components of the system) and of the interactions among themselves and with the environment.

3. Based on the field information obtained at Mataro and Sabadell hospitals ED's (both located in the vicinity of Barcelona) an ABM of a typical ED was proposed (see Figure 2.7). This model includes the active (all human beings interacting at the ED, i.e. patients, doctors, nurses, technicians, and all sanitary staff) and the passive (all physical reactive items of the ED, for example radiology and communication equipment) agents that are part of the ED.

4. The active agents of the ED are described by Moore machines (that have a single output for each system state under a specific input) which allows to model the dynamics of the ED (i.e., the transitions between the ED states, as well as their probabilities of occurrence).

5. The ABM of the ED proposed in this work is used as a black box simulator, and its implementation was done by using *NetLogo*, the agent-based programming language and programmable modelling environment (see Figure 2.7).

Chapter 3

Optimisation via Simulation of Emergency Departments

"Understanding is always the understanding of a smaller problem in relation to a bigger problem."

P.D. Ouspensky

3.1 Introduction

The operation of patients overcrowded Emergency Departments could be improved by, for example, adapting their layout according to their demands, by increasing the number of staff, and by modifying the level of staff expertise (junior to senior ratios), amongst other solutions. In order to find the best solution (optimal) that improves the operation of EDs one option could be to analyse, by using computational numerical algorithms, a large number of potential solutions, if not every solution, i.e., the so-called exhaustive search technique. However, this option could be impractical, because even if theoretically the exhaustive search technique guarantees to find the best solution, it could require large amount of computing time and resources. Therefore, an alternative optimisation technique is required. Such alternative optimisation technique must decrease computing time and resources used to find such best solution, i.e., the computing resources should be efficiently used. Furthermore, such best solution could be infeasible, or an approximate "good" solution might be a "better" solution, i.e., a thoughtful balancing must be done between finding the best solution, which implies to use large amount of computing time and resources, or finding an approximate "good" solution, which efficiently uses computing time and resources.

In this chapter, topics about optimisation, constraints, objectives and their classification are discussed, as well as numerical methods and their taxonomy used to do optimisation. A new optimisation via simulation algorithm is proposed, and implemented.

3.2 Optimisation

Optimisation is a common sense process, but difficult to specify neatly and rigorously; however, in general, it can be defined as the process of finding the best or optimal solution for a given problem under some conditions. Optimisation is applied in quite different fields, v.gr., in engineering the aim is to maximise the performance of a system with minimal resources and runtime, while in some industrial process the goal is to enhance the quality and efficiency of the production process. Optimisation is also applied, even in daily life, situations in which there are many cases where the maximum profit with minimal effort is search. It can also be said that optimisation is a widely used process, difficult to define and apply in order to find the truly best or optimal solution.

Mathematically an optimisation problem can be stated as:

$$\begin{aligned} &\max / \min && f(x) \\ &\text{subject to} && x \in C \end{aligned} \tag{3.2.1}$$

where x is the variable; f is a function ($f : C \to \mathbb{R}$); C is the constraint set, and $\exists\, x_0 \in C$ such that $f(x_0) \leq f(x) \quad \forall\, x \in C$ for minimisation, and $f(x_0) \geq f(x) \quad \forall\, x \in C$ for its counterpart, maximisation.

The function $f : C \to \mathbb{R}$ is known as the objective function or performance index, and it is not necessarily pure mathematical formulation, but it could be a complex algorithm. The objectives are general statements of what to optimise under some restrictions, where the optimisation process is going to apply.

Usually, the domain $C \subseteq \mathbb{R}^n$ represents the *problem* or *search space* that can be any sort of elements, e.g. arrays, numbers or equipments, amongst others. The domain is quite common specified by a set of conditions or constraints that its elements have to satisfy. These elements are known as *candidate solutions*, which define a *feasible region*. The Figure 3.1 illustrates these definitions. In this figure are shown the so-called search space, i.e., the domain where the problem is posed, and the solution exist. The traces C_1 and C_2 define the spatial constraints, which define the feasible region where the optimal solution might exist and found by using optimisation techniques.

Before stating the process of optimisation, at least, the next three elements ought to be defined [64]:

- the system description of the model,
- the objective function or the performance index, and
- the optimisation method to be used.

And last, but not least important, one more concept requires to be defined, the so-called level of optimisation, which express the degree of precision in a formal or

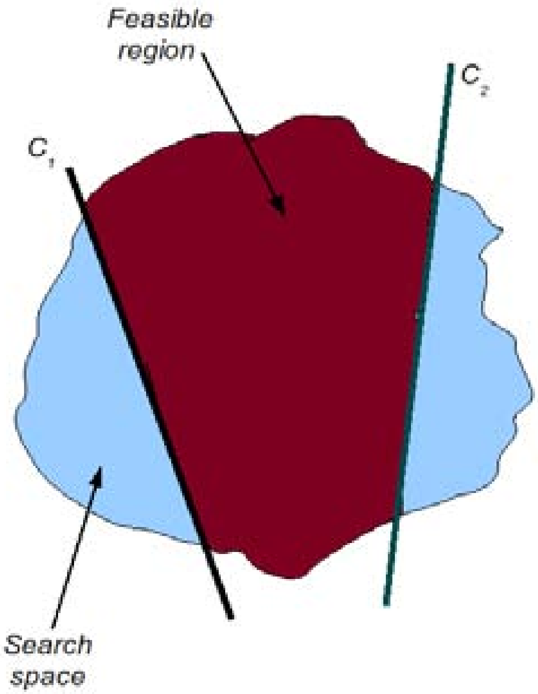


Figure 3.1: Optimisation problem under constrains C_1 and C_2. The search and feasible areas are shown.

mathematical formulation, which imposes or specifies the desired implementation. This concept shows the accuracy or practical reason rather than analytical. Sometimes a quick answer to the optimisation problem at hand is desired even with loss of generality or rigorousness, but when an "exact solution", which implies a high degree of accuracy in the computation of the objective function, optimisation techniques have to be developed or applied, as the ones shown in what follows.

3.2.1 Objective Function

The objective is a general statement about what to optimise subject to certain restrictions or constraints, which implies the degree of precision required for the searched solution, as well as about how this solution should be computed. The performance index or objective function is a rigorous mathematical expression, which allows quantitative comparisons amongst different solutions. These comparisons depend on the level or degree of the optimisation. The higher the level of optimisation, the higher the quality of the solution is. The performance index or objective function is also known, amongst others, as cost function or criterion, in economy and control theory, respectively.

Setting the objective function is a hard task, but control theory helps to assign it, without loss of generality, to one of two categories [64]: 1) time (to be minimised or maximised); and an amplitude (range or a benefit to be maximised or an error to be minimised).

3.2.2 Constraints

Constraint is a limitation, restriction, and, in general, a condition which any solution to an optimisation problem, such as the Equation 3.2.1 have to satisfy. It can be either an equality, or inequality constraint.

And, as it stated above, *feasible region* (illustrated on Figure 3.1), is composed by those *candidate solutions* that satisfy all restrictions.

Before continuing the discussion over optimisation, it is important to define what an optimum represents in the context of single and multiple objective functions. Global optimisation is about finding the best optimal possible solutions for given problems, which can be done over a single or multiple objective functions.

3.2.3 Single Objective Functions

When optimising a single function f, an optimum can be either a maximum, if it is a maximisation problem, or a minimum, when it is a minimisation problem, such as Equation 3.2.1. It can be local or global optimum as shown in Figure 3.2. The latter is an optimum of the whole domain, whereas the former is an optimum of only a subset of such domain. Usually, the maximum and minimum of a set are the greatest and least values in such set.

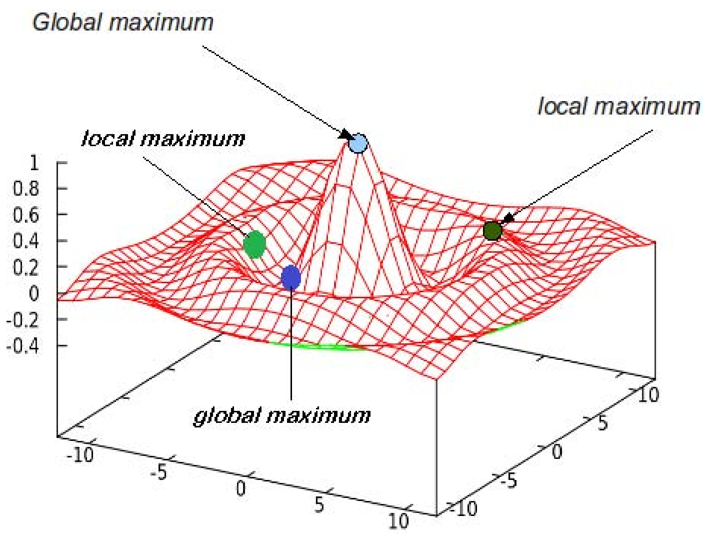


Figure 3.2: Global and local maximum.

Hence, following are the definitions for different sort of optima in single objective functions [76].

- A local maximum $\vec{x_l} \in \mathbb{X}$ of an objective function $f : \mathbb{X} \rightarrow \mathbb{R}$ is an input element with $f(\vec{x_l}) \geq f(x) \, \forall \, x$ neighbouring $\vec{x_l}$. If $\mathbb{X} \subseteq \mathbb{R}$, it can be written as in Equation 3.2.2

$$\forall \vec{x_l} \exists \varepsilon > 0 : f(\vec{x_l}) \geq f(x) \, \forall x \in \mathbb{X}, \mid x - \vec{x_l} \mid < \varepsilon \tag{3.2.2}$$

- A local minimum $\vec{x_l} \in \mathbb{X}$ of an objective function $f : \mathbb{X} \to \mathbb{R}$ is an input element with $f(\vec{x_l}) \leq f(x) \forall x$ neighbouring $\vec{x_l}$. If $\mathbb{X} \subseteq \mathbb{R}$, it can be written as in Equation 3.2.3

$$\forall \vec{x_l} \exists \varepsilon > 0 : f(\vec{x_l}) \leq f(x) \forall x \in \mathbb{X}, \mid x - \vec{x_l} \mid < \varepsilon \tag{3.2.3}$$

- A local optimum $x_l^* \in \mathbb{X}$ of one objective function $f : \mathbb{X} \to \mathbb{R}$ is either a local maximum or a local minimum.

- A global maximum $x \in \mathbb{X}$ of one objective function $f : \mathbb{X} \to \mathbb{R}$ is an input element with $f(x) \geq f(x) \forall x \in \mathbb{X}$.

- A global minimum $x \in \mathbb{X}$ of one objective function $f : \mathbb{X} \to \mathbb{R}$ is an input element with $f(x) \leq f(x) \forall x \in \mathbb{X}$.

  A global optimum $x^* \in \mathbb{X}$ of one objective function $f : \mathbb{X} \to \mathbb{R}$ is either a global maximum or a global minimum.

Even a one-dimensional function $f : \mathbb{X} = \mathbb{R} \to \mathbb{R}$ may have more than one global maximum, multiple global minima, or even both in its whole domain $\mathbb{X}$. Examples of these optima are shown in Figure 3.2.

3.2.4 Multiple Objective Functions

Even though single objective optimisation methods allow to model a large number of real problems, there are many applications where these models are unsuitable, since it is almost impossible to get a single solution that simultaneously optimises several objectives. To overcome this case multi-objective optimisation (MOO) comes into play.

Problems with two or more objectives functions, known as *multi-objective functions*, are quite common in many fields. The solution of those problems is very difficult since their objective functions tend to be in conflict with each other. Nevertheless, to simplify, many of these problems are modelled as single objective using only one of the original functions, and handle the others as constraints.

The multiple optimisation problem can be stated as follows:

$$\text{optimise} \quad [f_1(\vec{x}), f_2(\vec{x}), \cdots, f_k(\vec{x})] \tag{3.2.4}$$

subject to m inequality constraints as in Equation 3.2.5:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \cdots, m \tag{3.2.5}$$

and the p equality constraints as in Equation 3.2.6:

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \cdots, p \tag{3.2.6}$$

;where k in Equation 3.2.4 is the number of objective functions

$$\begin{aligned} &f_i : \mathbb{R}^n \to \mathbb{R} \qquad \text{and} \\ &\vec{x} = [x_1, x_2, \ldots, x_n]^T \end{aligned} \tag{3.2.7}$$

is the vector of decision variables, and it is desired to determine from amongst the set $\mathcal{PF}$ of all vectors, which one satisfy the constraints Equation 3.2.5 and Equation 3.2.6 at the particular set of values $x_1^*, x_2^*, \ldots, x_n^*$ which yield the optimum values of all the objective functions. Without loss of generality, it can be stated that the single objective function, as the one expressed in Equation 3.2.1 is a simplified form of the multiple objective functions expressed in Equation 3.2.4 [17].

The *optimality* concept for multi-objective functions is quite different to the corresponding for single one, since it is very rare that exists a single point, which at the same time, optimises all such objective functions. Hence, when dealing with multi-objective optimisation problems, instead of seeking single solutions it is usual to seek for a thoughtful balancing amongst optimality, completeness, precision, and speed.

3.2.4.1 Pareto Optimality

One of the approaches used to obtain the *optimum* of the MOO problem expressed by Equation 3.2.4 to Equation 3.2.7 is the so-called *Pareto optimality* [30], (which was originally proposed by Francis Ysidro Edgeworth in 1881 [20], and then generalised by Vilfredo Pareto in 1896 [62]), that mathematically is exposed as follows, it is said that a vector of decision variables $\vec{x}^* \in \mathcal{PF}$ is a *Pareto optimal* if $\nexists$ another $\vec{x} \in \mathcal{PF}$ such that $f_i(\vec{x}) \leq f_i(\vec{x}^*) \quad \forall i = 1, \ldots, k \quad \text{y} \quad f_j(\vec{x}) < f_j(\vec{x}^*)$ for at least one j.

This definition says that x^* is *Pareto optimal* if there exists no feasible vector of decision variables $x \in \mathcal{PF}$ which would decrease some criterion without causing a simultaneous increase in at least one other criterion. Unfortunately, this concept almost always gives not a single solution, but rather a set of solutions known as the *Pareto optimal set*. The vector x^* corresponding to the solutions included in the *Pareto optimal* set are called *non-dominated*. The plot of the objective functions whose non-dominated vectors are in the Pareto optimal set is called the *Pareto front*.

The Figure 3.3 and Figure 3.4 graphically shown the Pareto-dominance concept for a minimisation problem with two objectives (k_1, k_2). The Figure 3.3 shows the location of several solutions. The filled circles represent *non-dominated* solutions, while the non-filled ones symbolise dominated solutions. In Figure 3.4 is shown the relative distribution of the solutions in reference to a solution x. There exist solutions that are worse (in both objectives, (k_1, k_2)) than x, better (in both objectives) than x, and indifferent (better in one objective, but worse in the other).

Classical techniques for multi-objective optimisation tend to generate elements of the Pareto optimal set one at a time, this implies that a lot of trials, using different starting points, are required in order to generate lots of those elements.

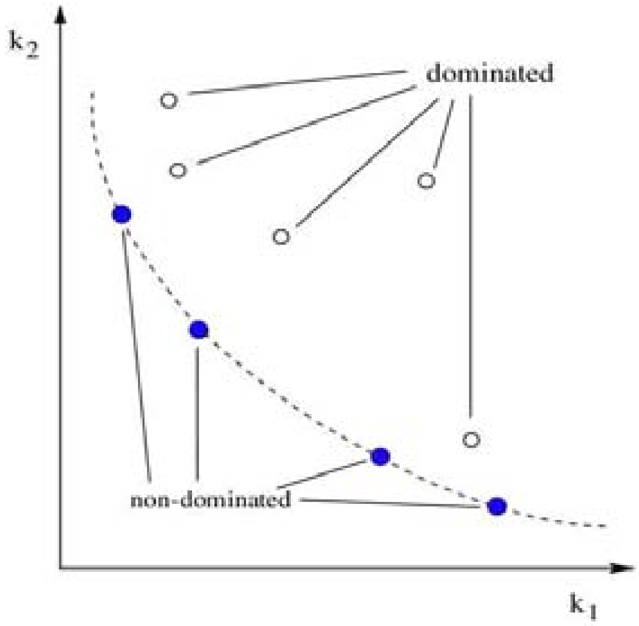


Figure 3.3: Pareto front with *non-dominated*, and dominated solutions.

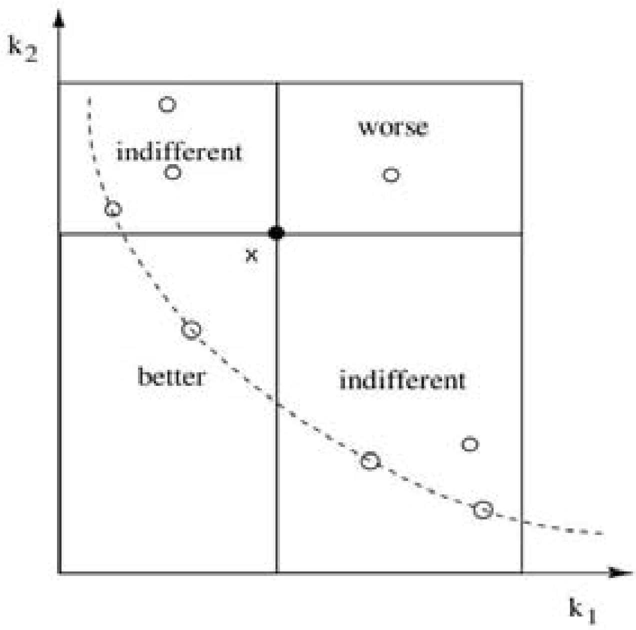


Figure 3.4: Pareto front, different sort of solutions in reference to solution x.

Moreover, most of these techniques are quite sensitive to the shape of the Pareto front, and might not work when the Pareto front is non convex. Therefore, there is needed for techniques that overcome these difficulties.

3.3 Numerical Methods in Optimisation Problems

Numerical methods belongs to numerical analysis, which is part of mathematics and computer science that creates, analyses, and implements algorithms for solving numerically the problems of continuous mathematics using computers

In most cases to solve equations such as Equation 3.2.1 or Equation 3.2.4 to Equation 3.2.7 analytically is quite difficult, unless such equations are extremely simple. Amongst others, these difficulties could be the following: it exists an analytical solution, but the order of equations is high; the geometry of the problem is very complex; there is no solution or analytical procedure for the equation; an algorithm exists to solve the equation, but it does not have polynomial time solution .

Engineers and scientists have chosen experimental physical approaches to study many of the real systems. Although, there are limitations of those approaches, some of which are inherent to the experimental method, or experimental errors, that lead to coarse accuracy of the results they obtain. Therefore, nowadays, it is almost impossible to separate the utilisation of computers throughout the design, analysis, and simulation processes of real systems.

3.3.1 Optimisation Methods

To solve equation such as Equation 3.2.5, the so-called optimisation theory is applied. Before entering into the methods some topics should be discuss. In order to choose an optimisation method some properties ought to be demanded to such method or technique that solve the model using a system of algebraic or differential equations, or any other mathematical model if it is available. However, there are some techniques available when there is no possibility of mathematical treatment. These characteristics can be divided whether the solution is analytical or numerical [64].

Amongst others, mathematical qualities are: existence of the solution, is there any solution?; uniqueness of the solution, is there only one?; necessary conditions, that have to be satisfied in all cases; sufficiency conditions, if are satisfied guarantee an extremum; absolute or local extremum, is the solution valid over a small are or over the whole search space?; and weak and strong extremum.

While computational characteristics are (more practical rather than mathematical): existence of a numerical computing method; kind of computer used; convergence, if the method uses an iterative procedure; and computing time.

Convergence is quite important property, it is a condition *sine qua non* for any numerical method. It is said that it is convergent if the numerical solution approaches the exact solution as the step size goes to zero.

Since optimisation inherently implies control, controllability and observability are defined. It is said that a system is controllable at the instant t_0 if it is possible to take from any initial state, $x(t_0)$ to any other state in finite time. And, a system is observable at time t if, with the system at state $x(t)$, it is possible to determine such state in finite time using only its outputs. Controllability and observability are dual aspects of the same problem.

Once the problem is characterised, the objective function and the constraints are set, the next step is to choose a method to solve the defined problem. Such method will depend on whether [64]: these settings are *static or dynamic;* the objective function is restricted or not restricted; these settings are linear or non-linear; and these settings are one-dimensional or multidimensional.

Without loss of generality, it can be stated that the static version is the simplified form of the dynamic method, the linear problem is the simplified form of the non-linear problem, and the one-dimensional is the simplified form of the multidimensional.

3.3.2 Taxonomy of Optimisation Methods

The optimisation methods can be broadly classified as analytical or non-analytical, as shown in Figure 3.5, that is based on the possibility to solve equations as Equation 3.2.5. The analytical method imposes the existence of the derivatives of the objective function; unfortunately, not always the function has such property. Therefore, alternative methods have to be used. The classical optimisation methods can be seen as search methods. If the size of the search space is small, or its computational search time is polynomial and not *NP-hard* or *NP-complete* [18], then an exhaustive search could be used. The so-called exhaustive search is a general problem-solving technique, which always guarantees finding a solution if it exist, used when there is not known efficient technique, but its computational cost could be high and proportional to the number of possible solutions which could be a combinatorial explosion of possible solutions. This approach could be the first approximation to tackle search problems, i.e., that the whole feasible region will be examined to find the optimum, could be used, as shown in Figure 3.1.

Also, optimisation algorithms can be divided in two other classes: deterministic and probabilistic algorithms [76]. The former is used if the search space can be time-wise explored; however, to solve a problem deterministically could be a difficult task, if the dimension of the search space is large. Therefore, a probabilistic optimisation method should be applied. The Monte Carlo-based approach is one of the most popular, even though if the yield solution, could be not the global optima, but rather an approximate good solution.

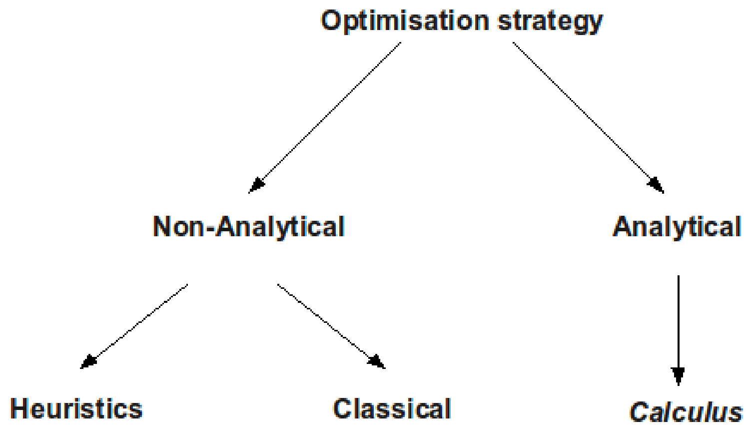


Figure 3.5: Classification of optimisation methods

3.4 Optimisation of Emergency Departments via Simulation Model

3.4.1 Optimisation via Simulation

As mentioned in section 3.1, computer simulations are used extensively and successfully to evaluate the output responses of real systems, as for example to model supply chain management, finance, manufacturing, engineering design and medical treatment [28, 42, 66]. One important feature of simulation experiments is that users can choose different system settings to try to improve the performance of their systems. Those settings are chosen by selecting the best from a set of candidate parameter settings. Therefore, it is natural to search for settings that optimise the system performance. This is called optimisation via simulation (OvS), an emerging field which integrates optimisation techniques into simulation analysis [6, 25–27, 63].

OvS is different from classical deterministic optimisation and the typical stochastic programming problem since there is no explicit form of the objective function, and function evaluations are stochastic and computationally expensive. When the parameter settings or design variables are discrete valued, thus the optimisation problems become discrete optimisation via simulation (DOvS) problems [6, 26, 34, 57]. Many DOvS algorithms are based on random search. There are at least three ways to categorize random search algorithms: by neighbourhood structure, by the number of feasible solutions, and by convergence properties. Random search algorithms typically generate candidate solutions from the neighbourhood of a selected solution in each iteration. Some algorithms have a fixed neighbourhood structure, while others change their neighbourhood structure based on the information gained through the optimisation process. In reference to the convergence properties of DOvS algorithms, they can be divided into three categories: no guaranteed convergence, locally convergent, and globally convergent. Most algorithms used in commercial software are heuristics, which provide no convergence guarantee [26, 29]. The random search algorithms in the literature are often globally convergent including, such as the stochastic ruler method [4, 78]; Andradóttir's random search methods [5, 8]; simulated annealing algorithm [3]; stochastic comparison method [31]; and nested partitions with specific implementations as in [61, 68], amongst others algorithms. Any globally convergent algorithm, which converges for an arbitrary initial approximation, without any structure assumption on the objective function requires examining every feasible solution infinitely often in order to guarantee convergence [35].

Using past information, either through visit counts or by aggregated sample means, one needs to keep a list of all visited solutions, and when a new solution is generated one needs to check if it has already been visited. The storage and checking cost can be high if the algorithm visits a large number of solutions. However, the computational and storage costs are often small compared to the cost of conducting simulation experiments in practical real-world [7]. Therefore, since simulation experiments are computationally expensive, the past information, i.e., to accumulate observations, should be used in DOvS algorithms, instead of discarding such observations.

In the context of agent-based simulation, a model and the simulator with which it is executed can include many parameters. These parameters can be specific to either the simulator, for example, the discretisation step for the modelling of time and space, or to the model. Some of the latter can be extracted from the knowledge of the field and be associated to fixed values, whereas other parameters have to be kept variable, amongst other reasons are: the knowledge of the field is generally not exhaustive, or this knowledge may not be directly compatible with the model. In this case, a common approach can be to try some values and simulate the model to see how it behaves globally. Therefore, a different approach to automate this complex process should be set.

Different methods have already been proposed to explore automatically the parameter or search space of discrete models. One of them, in the *NetLogo* platform for instance, the "*BehaviorSpace*" [23], it is sometimes called *parameter sweeping*. It lets you explore automatically and systematically the *search space* of the model of possible behaviours and determine which combinations of settings is the best. This space is a Cartesian product of values that each parameter can take. However, when we have many parameters, the search space becomes huge and the systematic exploration becomes impractical or highly computational cost. Other methods have been proposed, which differentially explores the whole search space, focusing on the most interesting areas. This is the case of the method developed in [59]. They use a "*parameter sweep infrastructure*", which is similar to the "*BehaviorSpace*" tool of *NetLogo*. However, to avoid a systematic exploration, they use "special agents", which are called searcher agents, and introduce the fitness notion. The aim of such searcher agent is to travel in the parameter space to look for the highest fitness. One drawback of such method is that those searcher agents may head for local fitness optima.

3.4.2 Optimisation via Simulation of Emergency Deparments

Amongst the previous research on the simulation of EDs the following can be included. The proposal of [75] uses ABM to simulate the work flow in ED. It focus on triage and radiology process, but not real data was used, the acuity of patients are not consider, and healthcare providers do not always serve patients in a first-come-first-serve basis. Simulation optimisation is used to improve the operation of ED in [65], using a commercial simulation package, and [1] combines simulation with optimisation, which involves a complex stochastic objective function under deterministic and stochastic set of restrictions. Other works modelling healthcare systems have focused on patient scheduling under variable pathways and stochastic process durations, the selection of an optimal mix for patient admission in order to optimise resource usage and patient throughput [37]. Also, work has been performed in evaluating patient length of stay under the effects of different ED physician staffing schedules, and the only one found until now that utilises real data is [41] or patient diversion strategies [44]. An evolutionary multi-objective optimisation approach is used in [36] for dynamic allocation of resources in hospital practice, while in [60] it was found that combining agent-based approaches and classical optimisation techniques complement each other.

3.5 Proposed Optimisation via a Simulation Model for Emergency Departments

In this section a methodology is proposed, to analyse the performance of Emergency Departments. In Figure 3.6 is shown conceptually where the niche of this research is focused: devising, developing and implementing a methodology for the optimisation via simulation filling the gap between the exhaustive search technique and heuristic approach for the MOO (see Equation 3.2.4 to Equation 3.2.7) for Emergency Departments.

3.5.1 The Optimisation Proposal for Emergency Departments

As it was stated in chapter 2, ED is a critical healthcare department, usually the main entrance to the hospital, and a key component of the whole healthcare system. EDs are semi-autonomous units that are open and staffed 24 hours per day, 365 days per year, including holidays. EDs can be described by four main characteristics: physical location; physical layout; time period open to patients, and patient type served. All of them compromised human resources, as sanitary staff, and patients as well as their companions. The original mission of EDs is to primarily handle only emergent situations. However, ED visits include a wide range of illnesses and injuries, i.e., truly emergencies, urgent, semi-urgent, and non-urgent cases. EDs have increased their resources: human, physical, equipment, infrastructure and economic, to attend all of those visits, becoming large, complex and dynamic units. The stated problem study in this research could be synthesize in the Figure 3.7.

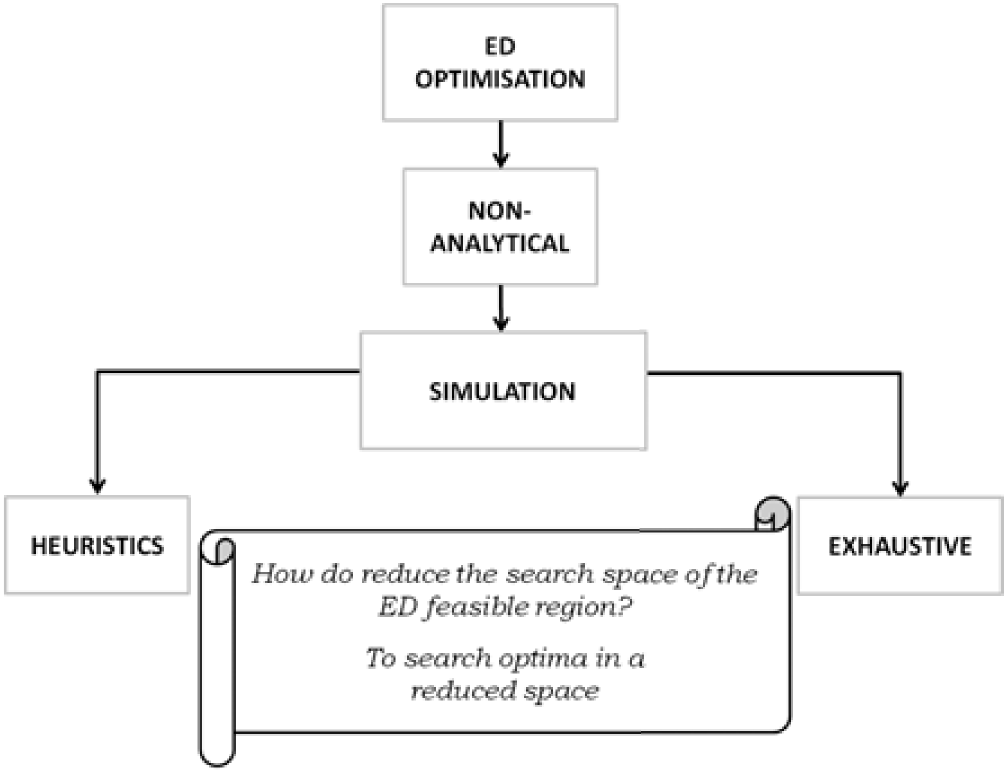


Figure 3.6: The proposed methodology

This figure shows schematically, the MOO problem of an ED, including its inputs, constraints and outputs. The decision or input variables of the ED are the resources allocation –including beds and boxes, and staff scheduling– all sanitary staff, .v.gr.: doctors, nurses, admission personnel, technicians, amongst others, that are the parameters or search space of the problem; the other (uncertain) inputs are also shown in Figure 3.7, including the most important, such as the frequency of arrival and diversity of patients; the problem constraints, that could be physical, human, equipment, and safety regulations. Also in Figure 3.7 some of the output indexes of interest are shown, v.gr., patient length of stay, resource utilisation, productivity, and patient satisfaction.

The optima solutions to the problem shown in Figure 3.7 could be found by either examining the whole feasible region, usually difficult to apply in practice since its high computational cost, or using some heuristic methods, an alternative technique that could be quicker than the former. Even though the former guarantees to find the optimum, is infeasible to apply. In contrast, the heuristic technique is a practical alternative to find an approximate and acceptable "good" solution, through a thoughtful balancing amongst optimality, completeness, precision, and lower computing time cost, which are the goals of the methodology proposed in this work, see Figure 3.6.

The optimisation via simulation methodology for EDs proposed herewith is based in a neighbourhood structure aiming to reduce the feasible region. The methodology is constituted of two phases as shown in Figure 3.8. The first phase

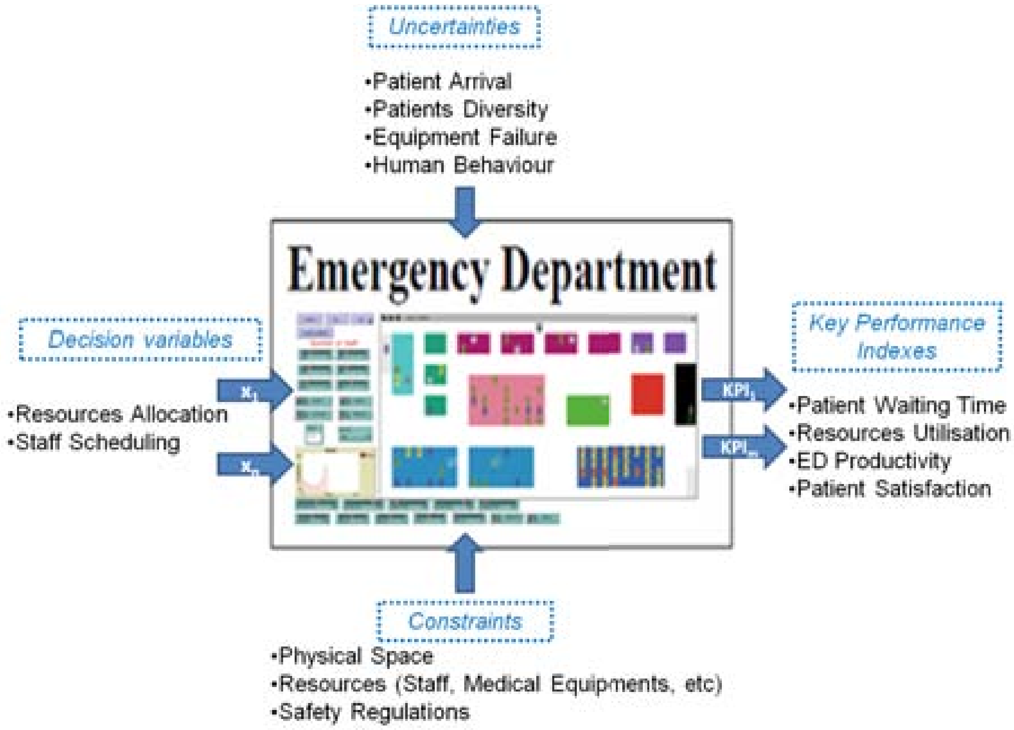


Figure 3.7: Elements of the optimisation problem of Emergency Departments.

is a coarse grained approach consisted in a global exploration step over the entire search space. This phase identifies promising regions for optimisation based on a neighbourhood structure of the problem, that uses either a pipeline scheme approach of an Emergency Department or the Monte Carlo heuristic plus the K-means method. The second phase is a fine grained approach, that consists in seeking the best solution, either the optimum or a sub-optimum lying on the Pareto frontier (discussed in subsection 3.2.3 and subsection 3.2.4) by performing a "reduced exhaustive search" in such promising regions.

3.5.2 Coarse Grained Phase

This is the first phase of the proposed optimisation via simulation methodology. This phase is a global exploration step over the entire search space. This phase identifies promising regions for optimisation based on a neighbourhood structure of the problem. This stage could be done using two alternative techniques: the pipeline scheme approach of ED, or the Monte Carlo heuristic plus the K-means methods. This first phase returns a collection of promising regions. Both approaches are discussed in the next subsections.

3.5.2.1 Pipelining Modelling Technique

Pipelining is an implementation technique whereby multiple instructions are overlapped in execution [33]. It consists on several ordered stages. Ideally, all the stages should have equal processing speed. Otherwise, the slowest stage becomes

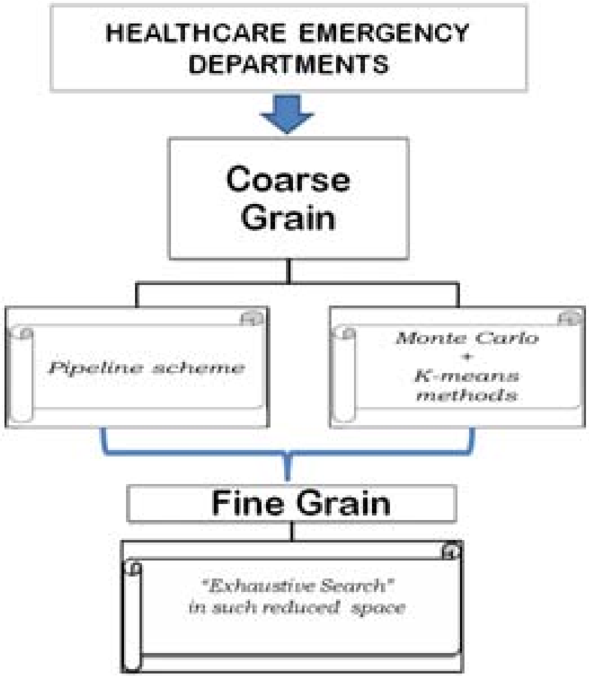


Figure 3.8: Framework of the two-phase optimisation via simulation methodology proposed.

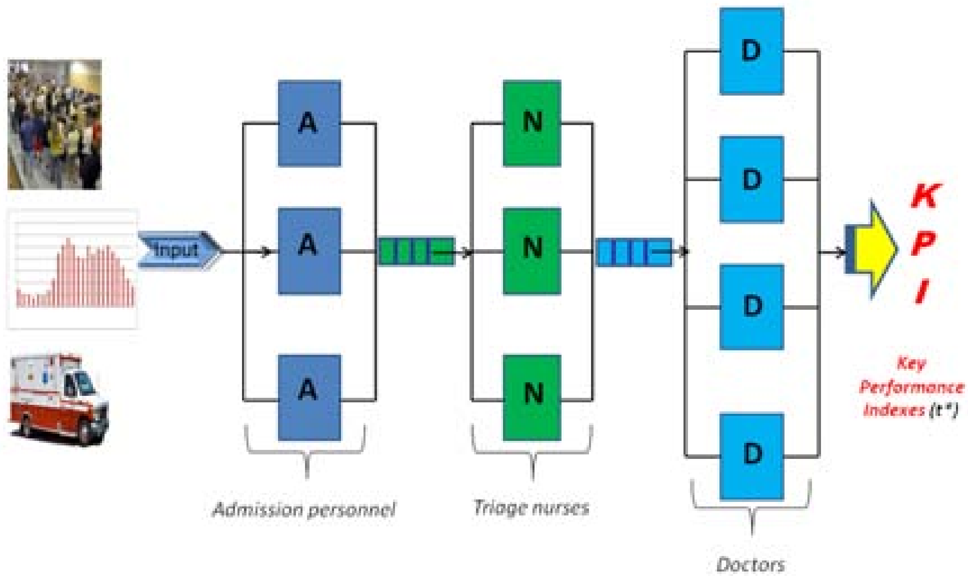
the bottleneck of the entire pipe. This bottleneck problem plus the congestion caused by improper buffering may result in many idle stages waiting for the previous ones. In a uniform-delay pipeline, all tasks have equal processing time in all stages. Ideally, all the stages can operate synchronously with full resource utilization, but, in reality, the successive stages have unequal delays. The optimal partition of the assembly line depends on a number of factors, including the quality (efficiency and capability) of the working stages, the desired processing speed, and the cost effectiveness of the entire pipe. The precedence relation of a set of subtasks $\{ T_1, T_2, \cdots, T_k\}$ for a given task T implies that some task T_j cannot start until some earlier task $T_i\,(i < j)$ finishes. The interdependencies of all subtasks form the precedence graph. With a linear precedence relation, task T_j cannot start until all earlier subtasks $\{T_i$, for all $i \leq j$ $\}$ finish. A linear pipeline can process a succession of subtasks with a linear precedence graph [38]. The pipeline time can be expressed by the following equation

$$Pipeline\,time = \frac{1}{\sum \frac{1}{S_i}} + \frac{1}{\sum \frac{1}{S_j}} + \ldots \frac{1}{\sum \frac{1}{S_k}} \tag{3.5.1}$$

where S_i (in Pipelining Modelling Technique) represents each stage of the pipeline.

3.5.2.2 Pipeline Model for Emergency Departments

The pipeline scheme (PS) model of ED is based on the patients flow in an ED, that was discussed in subsection 2.2.2. This approach is presented in Figure 3.9. It shows the reduced approximation to the patients flow in a reduced ED, where the patients arrive either by their own means or by ambulance, then go to up to three different stages, pipes, that are: admission, triage, and, finally, diagnosis and treatment stages. Therefore, the pipeline is constituted by three stages, each of them could be done in parallel up to three or four tasks in parallel, and are interconnected by buffers, that represent the waiting rooms of an ED. These buffers synchronise the stages, since them have different processing speed. Also, in the same figure is shown one of the key performance index, the approximated time (t^*) that the patients stay in the ED. All sanitary staff configurations identified in the promising region through this approach in the first phase of the methodology proposed are tested to find the optimum.



$$t^* = \frac{1}{\sum \frac{1}{time \quad A_i}} + \frac{1}{\sum \frac{1}{time \quad N_j}} + \frac{1}{\sum \frac{1}{time \quad D_k}}$$

Figure 3.9: Three-stage pipeline approach (PA) model for a reduced emergency department.

3.5.2.3 Monte Carlo Heuristic Method

Monte Carlo (MC) is a statistical sampling method used to approximate solutions to quantitative problems [43, 74]. It is widely used when: a) it is infeasible to compute an exact solution using a deterministic algorithm; b) there are many degrees of freedom, or c) there are incertitude in the input data. MC method does not suffer from the curse of dimensionality. Usually, to reduce the standard error implicit in MC method, different techniques could be used, amongst them, increasing the sample size and the variance reduction [48].

3.5.2.4 Monte Carlo Heuristic for Emergency Departments

The application of the MC method for EDs is used as an alternative technique, whenever the pipeline approach model could not be applied. The algorithm used is based on the Metropolis algorithm [51]. It randomly selects each time 25 ED sanitary staff scenarios, previously ordered by the equivalent operational patient-service time (t*) of a "single one" sanitary professional (working in parallel) of each sanitary staff configuration. The selected scenarios that have a better average mean value and standard deviation than the previous ones are accumulated, in order to reuse past information. The MC program stops when two consecutive iterations cannot be able to improve the average mean value , i.e., it becomes stationary or asymptotic, and the MC stores the last time when the average mean value was improved.

3.5.2.5 *K*-means method

K-means is a partitional clustering technique that helps to identify k clusters from a given set of n data points in d-dimensional space. Each cluster is parametrised by a vector $m^{(k)}$ called its mean [49]. It is an iterative two-step algorithm: 1) assignment step, and 2) update step. In the former, it starts with k random centres and a single cluster, and, in the latter step, such cluster is refined at each step arriving to k clusters. The time complexity for implementing k-means is $O\left(i * k * d * n\right)$, where i is the number of iterations, but according to [9] k-means method will run in expected polynomial time, where the upper bound on the expected number of iterations is $O \quad \frac{n^{34} log^4(n) k^{34} d^8}{6} \quad$, which is a polynomial in n, k, d, $\frac{1}{}$.

3.5.2.6 *K*-means Method for Emergency Departments

After MC heuristic, K-means is applied to cluster the scenarios selected by the MC program. This clusterisation allows to find both the Pareto frontier and the promising regions. This step returns a collection of promising regions, that are represented by hyperplanes delimited by the sta scenarios found in this step.

3.5.3 Fine Grained Phase

The second step of the proposed methodology is a fine grained approach. It consists of a "reduced exhaustive search". Once the feasible region of the problem is reduced in the first coarse grained phase, either by using the pipeline approach model or the MC plus the K-means methods through returning a collection of promising regions. This fine grained phase is applied to find the best solution, either the optimum or a sub-optimum lying on the Pareto frontier, by a “reduced exhaustive search”.

3.5.3.1 Reduced Exhaustive Search

The exhaustive search technique, i.e, to search in the whole feasible region, is not a “good” practical option to solve any optimisation problem, but as it was stated in subsection 3.3.2, it guarantees to find the optimal solution. In this thesis, the “reduced exhaustive search” is used to find the optimal solution in the reduced feasible space found by either the pipeline scheme or the MC plus K-means methods.

3.6 Cluster Implementation of the Optimisation Proposal for Emergency Departments

NetLogo neither does parallel runs nor does support for splitting the runs across clusters. Although its *BehaviorSpace* tool allows to run parametric runs, it suffers from load imbalance [16]. Thus a better approach must be used, and the dynamic load balancing seems the natural solution. To this end, a master-worker (M-W) framework is used.

The master-worker scheme does a dynamic load balancing of the parametric runs of the agent-based ED simulator. First of all, the master assign an equitable random initial workload to each worker, and as soon as it finished, the master assign new workload again, until the whole workload is completely done. This scheme is done intra-node. The master-worker application was implemented in C language using pthreads to launch the agent-based ED simulator, described in subsection 2.4.2.

The exhaustive search technique is used as baseline method. Then the two-phase optimisation via simulation is applied. As coarse grained phase (the first of the two-phase proposed) either the pipeline approach or MC plus K-means methods, or both are applied. This first phase returns a collection of promising regions. Finally, the fine grained phase is separately apply within the promising regions found in the previous step. The implementation of the proposed optimisation via simulation of EDs is shown schematically in Figure 3.10.

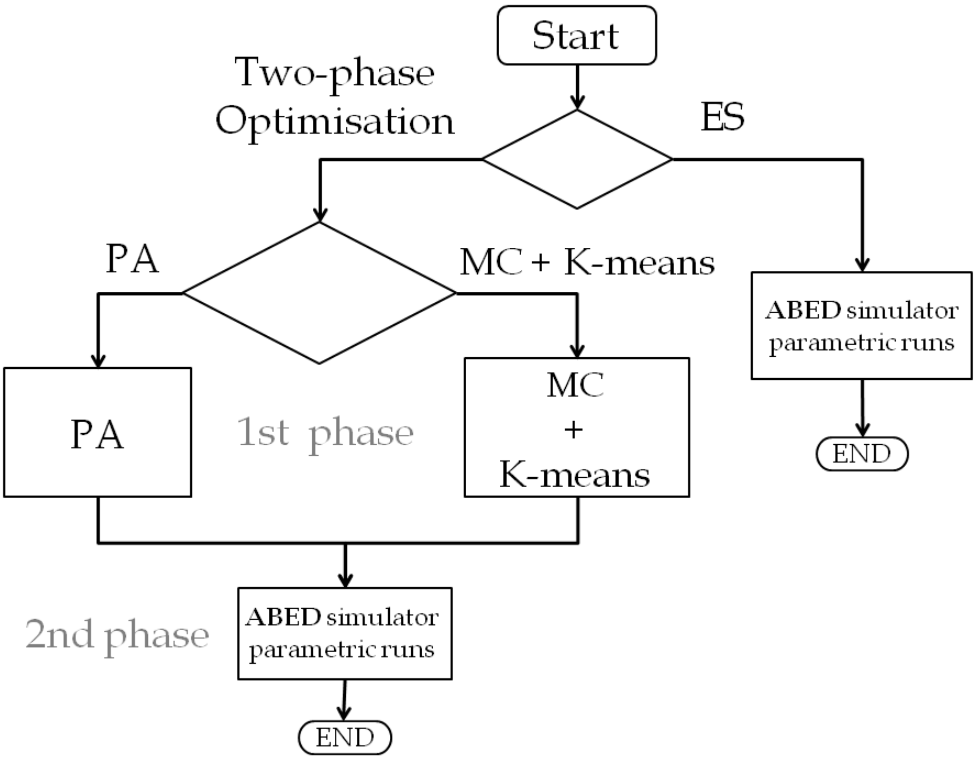


Figure 3.10: Algorithm of the optimisation via simulation of EDs methodology proposed.

The ED simulator to find the optimum sanitary staff configuration using exhaustive search technique is launched using the M-W application, which is used, as stated before, both to compare and analyse the results and performance of the methodology proposed.

The M-W application using pthreads to launch the ED simulator, described in subsection 2.4.2, was implemented in C language in order to load balancing.

The pipeline program was implemented in C++ programming language using STL, whereas the MC method was implemented in Perl programming language.

Finally, a “reduced exhaustive search” was separately applied by using the M-W application within the reduced feasible region found by either the pipeline scheme or the MC plus K-means methods, in the coarse fine grained phase.

3.7 Discussion

The most relevant conclusions of this chapter are the following:

1. The operation of patients overcrowded Emergency Departments could be improved by adapting their layout according to their demands, by increasing the number of staff, or by modifying the level of staff expertise (junior to senior ratios). In order to find the best solution (optimal) that improves the operation of EDs one option could be to analyse, by using computational

numerical algorithms, a large number of potential solutions.

2. One important feature of simulation experiments is that users can choose different system settings to try to improve the performance of their systems, by selecting the best from a set of candidate parameter settings by applying optimisation via simulation (OvS), which integrates optimisation techniques into simulation analysis.

3. In the context of agent-based simulation, a model and the simulator with which it is executed can include many parameters. These parameters can be specific to either the simulator or to the model. The latter can be extracted from the knowledge of the field and be associated to fixed values, whereas other parameters have to be kept variable, to take into account that the knowledge of the field is generally not exhaustive.

4. The "*BehaviorSpace*" in the *NetLogo* platform lets to explore automatically and systematically the *search space* of the model of possible behaviours and determine which combinations of settings is the best, but it suffers from load imbalance.

5. The MOO problem of an ED, includes its inputs, constraints and outputs. The decision or input variables of an ED are the resources allocation – including beds and boxes, and staff scheduling, all sanitary staff, .v.gr.: doctors, nurses, admission personnel, technicians, the parameters or search space of the problem; other (uncertain) inputs include the most important, such as the frequency of arrival and diversity of patients; the constraints, could be physical, human, equipment, and safety regulations; and the output the length of stay (LoS) and the number of patients attended per day.

6. The optimisation via simulation methodology for EDs proposed herewith is based in a neighbourhood structure aiming to reduce the feasible region. The methodology is constituted of two phases. The first phase is a coarse grained approach consisted in a global exploration step over the entire search space. This phase identifies promising regions for optimisation based on a neighbourhood structure of the problem, that uses either a pipeline scheme approach of an Emergency Department or the Monte Carlo heuristic plus the K-means method. The second phase is a fine grained approach, that consists in seeking the best solution, either the optimum or a sub-optimum lying on the Pareto frontier by performing a "reduced exhaustive search" in such promising regions.

7. A Master-Worker (M-W) application using pthreads to launch the ED simulator was implemented in C language in order to load balancing. This M-W application is used as the first approach to find the optimum sanitary staff configuration by using exhaustive search both to compare and analyse the results and performance of the proposal methodology. The pipeline program was implemented in C++ programming language using STL, whereas the MC method was implemented in Perl programming language. Finally, a “reduced exhaustive search” was applied by using the M-W application within the reduced feasible region found by either the pipeline scheme or the MC plus K-means methods.

Chapter 4

Applications of the Proposed Optimisation of Emergency Departments via Simulation

"It is better to be approximately right than exactly wrong."

Old adage

4.1 Introduction

The two-phase optimisation via simulation of healthcare Emergency Departments, ED, proposed in section 3.5 was applied herewith to analyse the administrative strategies leading to optimum decisions about the physical and human resources of an ED. In particular, the impact on the economics and the productivity of Sabadell Hospital ED of different sanitary staff configuration (v.gr., doctors, triage nurses, admission personnel, emergency nurses, and x-ray technicians) were analysed. It is a discrete combinatorial optimisation problem.

There are three main issues to be addressed to carry out the evaluation, namely: the *simulation models* that represent the system under study (discussed along the previous chapters); the *decision variables* and *workloads* used as inputs of the simulation models; and the *metrics* used to asses the benefits of the proposal. We have defined some significant workloads and a set of metrics to observe both functional and performance features of the proposal. This set of metrics were defined in term of three different indexes, namely: patient length of stay (LoS) in the ED; number of attended patients per day (Throughput); and a compound index, the product of the cost of a given sanitary staff configuration times patient length of stay (CLoS).

In the following sections, the *decision variables,* the *workloads*, and the *metrics* are described. Finally two case scenarios and their results are presented and discussed.

4.2 Field Information of Sabadell Hospital ED

It was found through interviews with the managers at the EDs of Sabadell hospital (which provides healthcare services to an average of 160,000 patients/year), it was found that a basic sanitary of its ED staff is composed by: doctors, triage nurses, emergency nurses, admission personnel, and x-ray technicians, as shown in Table 4.1. This table also shows some characteristics of such sanitary staff, namely: sort of staff as junior or senior (that represents less and more expertise, respectively); their respective costs (€ [1]); the operational patient-service-time (hours); and the minimum and maximum number of each kind of staff.

Table 4.1: Sabadell Hospital ED staff and their: associated expertise, costs, operational patient-service time, and number.

Sanitary sta	**Cost (€[1])**		**Time/patient (hours)**		**Number of personnel**
	Senior	Junior	Senior	Junior	min - Max
Doctor	1000	500	0.260	0.350	***1 − 4***
Triage Nurse	500	350	0.090	0.130	***1 − 3***
Emergency Nurse	500	350	0.090	0.130	***1 − 2***
Admission personnel	200	150	0.020	0.035	***1 − 3***
X-ray technician	200	150	0.020	0.035	***1 − 2***

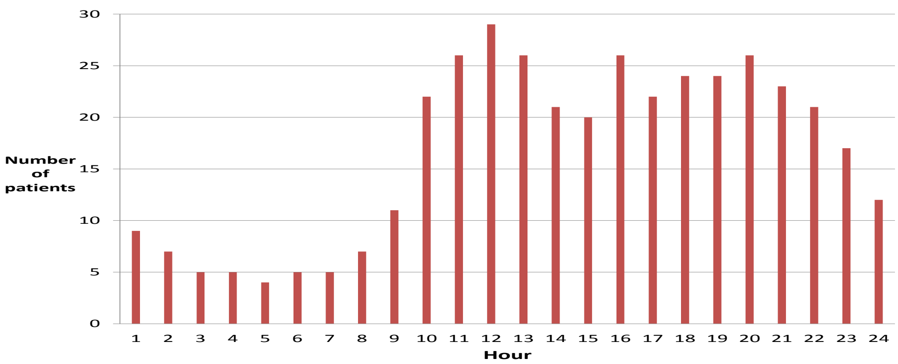


Figure 4.1: Sabadell Hospital ED average of 400 daily incoming patients and its hourly distribution (February 2010).

In reference to the Sabadell Hospital ED incoming patients, an average of four hundred of patients daily arrive to the ED of Sabadell hospital. As example, the statistics corresponding to February 2010, of this real average number of incoming patients and its hourly distribution are shown in Figure 4.1. As stated in subsubsection 2.3.3.1 all incomings patients are triaged to identify the acuity of

[1]**It is not an actual euro, could be any currency.**

them and to prioritise their urgency of attention. Thus, the average percentage, according to the priority level of urgency of attention, of the incoming patients to the ED of Sabadell hospital is as follows: triage level 1 - 1%; triage level 2 - 4%; triage level 3 - 20%; triage level 4 - 32%;and triage level 5 - 43%. Patients identified as triage level 4 and triage level 5 represent up to 75% of the total of the incoming patients to the ED of Sabadell hospital.

4.3 Decision Variables of Sabadell Hospital ED

The sanitary staff included in Table 4.1 are the *decision variables* of the ED.

Case number	$\mathbf{AD_1}$	$\mathbf{AD_2}$	$\mathbf{AD_3}$
1	AS	-	-
2	AJ	-	-
3	AS	AS	-
4	AJ	AJ	-
5	AS	AJ	-
6	AS	AS	AS
7	AJ	AJ	AJ
8	AS	AJ	AJ
9	AS	AS	AJ

Table 4.2: 9 Admission (A) personnel cases. AD_i is Admission Den_i. Where AJ means Admission personnel Junior, whereas AS means Admission personnel Senior.

Case number	$\mathbf{TR_1}$	$\mathbf{TR_2}$	$\mathbf{TR_3}$
1	NS	-	-
2	NJ	-	-
3	NS	NS	-
4	NJ	NJ	-
5	NS	NJ	-
6	NS	NS	NS
7	NJ	NJ	NJ
8	NS	NJ	NJ
9	NS	NS	NJ

Table 4.3: 9 Nurse (N) cases. TR_i represents Triage Room i. Where NJ means Triage Nurse Junior, whereas NS means Triage Nurse Senior.

Case number	$\mathbf{ENR_1}$	$\mathbf{ENR_2}$
1	ENS	-
2	ENJ	-
3	ENS	ENS
4	ENJ	ENJ
5	ENS	ENJ

Table 4.4: 5 Emergency nurse (EN) cases. $\mathrm{EN}R_i$ represents ENurse Room i. Where ENJ means Emergency Nurse Junior, whereas ENS means Emergency Nurse Senior

Case number	$\mathbf{XR_1}$	$\mathbf{XR_2}$
1	XRS	-
2	XRJ	-
3	XRS	XRS
4	XRJ	XRJ
5	XRS	XRJ

Table 4.5: 5 X-ray technician (XR) cases. XR_i represents X-ray Room i. Where XRJ means X-ray technician Junior, whereas XRS means X-ray technician Senior

The disaggregation of Table 4.1 yields Table 4.2, which includes 9 possible combinations of admission personnel (junior/senior); Table 4.3, which also includes 9 possible combinations of triage nurses (junior/senior); Table 4.4, that presents the 5 possible combinations of emergency nurse (junior/senior); Table 4.5 that shows the 5 possible combinations of x-ray technician (junior/senior); and Table 4.6, with

14 possible combinations of doctors (junior/senior) in which the examined cases for each type of staff were included. It is a discrete combinatorial problem.

Table 4.6: 14 Doctor (D) cases. DR_i represents Diagnosis Room *i*. Where DJ means Doctor Junior, whereas DS means Doctor Senior.

Case number	**DR_1**	**DR_2**	**DR_3**	**DR_4**
1	DS	-	-	-
2	DJ	-	-	-
3	DS	DS	-	-
4	DJ	DJ	-	-
5	DS	DJ	-	-
6	DS	DS	DS	-
7	DJ	DJ	DJ	-
8	DS	DJ	DJ	-
9	DS	DS	DJ	-
10	DS	DS	DS	DS
11	DJ	DJ	DJ	DJ
12	DS	DJ	DJ	DJ
13	DS	DS	DJ	DJ
14	DS	DS	DS	DJ

Table 4.2 to Table 4.6 were ordered by the sort and number of staff, whereas Table 4.7 to Table 4.11 were ordered by the equivalent operational patient-service time (t*) of a "single one" sanitary professional (working in parallel) of each sanitary staff configuration (admission personnel, nurses, doctors, and x-ray technicians). This order was obtained by applying the pipeline scheme described in subsubsection 3.5.2.2 and is graphically shown in Figure 4.2 to Figure 4.4. In these figures the index value was represented by colours, the most important values in shuch figures were the green ones.

Table 4.7: Ordering staff configuration of admission personnel according to the equivalent operational patient-service time (t*) of each staff configuration.

Case number (t*)	Old case number	**AD_1**	**AD_2**	**AD_3**	**€**	**Time (hrs)**	**t* (hrs)**
1	6	AS	AS	AS	600	0.020	0.0067
2	9	AS	AS	AJ	550	0.035	0.0078
3	8	AS	AJ	AJ	500	0.035	0.0093
4	3	AS	AS	-	400	0.020	0.001
5	7	AJ	AJ	AJ	450	0.035	0.0117
6	5	AS	AJ	-	350	0.035	0.0127
7	4	AJ	AJ	-	300	0.035	0.0175
8	1	AS	-	-	200	0.020	0.020
9	2	AJ	-	-	150	0.035	0.035

Table 4.8: Ordering staff configuration of triage nurses according to the equivalent operational patient-service time (t*) of each staff configuration.

Case number (t*)	Old case number	TR_1	TR_2	TR_3	€	Time (hrs)	t* (hrs)
1	6	NS	NS	NS	1,500	0.090	0.0067
2	9	NS	NS	NJ	1,350	0.130	0.0078
3	8	NS	NJ	NJ	1,200	0.130	0.0093
4	7	NJ	NJ	NJ	1,050	0.130	0.001
5	3	NS	NS	-	1,000	0.090	0.0117
6	5	NS	NJ	-	850	0.130	0.0127
7	4	NJ	NJ	-	700	0.130	0.0175
8	1	NS	-	-	500	0.090	0.020
9	2	NJ	-	-	350	0.130	0.035

Table 4.9: Ordering staff configuration of doctors according to the equivalent operational patient-service time (t*) of each staff configuration.

Case number (t*)	Old case number	DR_1	DR_2	DR_3	DR_4	€	Time (hrs)	t* (hrs)
1	10	DS	DS	DS	DS	4,000	0.260	0.065
2	14	DS	DS	DS	DJ	3,500	0.350	0.0695
3	13	DS	DS	DJ	DJ	3,000	0.350	0.0746
4	12	DS	DJ	DJ	DJ	2,500	0.350	0.0805
5	6	DS	DS	DS	-	3,000	0.260	0.0867
6	11	DJ	DJ	DJ	DJ	2,000	0.350	0.0875
7	9	DS	DS	DJ	-	2,500	0.350	0.0948
8	8	DS	DJ	DJ	-	2,000	0.350	0.1046
9	7	DJ	DJ	DJ	-	1,500	0.350	0.1167
10	3	DS	DS	-	-	2,000	0.260	0.130
11	5	DS	DJ	-	-	1,500	0.350	0.149
12	4	DJ	DJ	-	-	1,000	0.350	0.175
13	1	DS	-	-	-	1,000	0.260	0.260
14	2	DJ	-	-	-	500	0.350	0.350

Table 4.10: Ordering staff configuration of emergency nurses according to the equivalent operational patient-service time (t*) of each staff configuration.

Case number (t*)	Old case number	ENR_1	ENR_2	€	Time (hrs)	t* (hrs)
1	3	ENS	ENS	1,000	0.090	0.0117
2	5	ENS	ENJ	850	0.130	0.0127
3	4	ENJ	ENJ	700	0.130	0.0175
4	1	ENS	-	500	0.090	0.020
5	2	ENJ	-	350	0.130	0.035

Table 4.11: Ordering staff configuration of x-ray technicians according to the equivalent operational patient-service time (t*) of each staff configuration.

Case number (t*)	Old case number	**XR_1**	**XR_2**	**€**	**Time (hrs)**	**t* (hrs)**
1	3	XRS	XRS	400	0.020	0.001
2	5	XRS	XRJ	350	0.035	0.0127
3	4	XRJ	XRJ	300	0.035	0.0175
4	1	XRS	-	200	0.020	0.020
5	2	XRJ	-	150	0.035	0.035

In the first example, Figure 4.2 shows a 3D scattered graph, which axes were ordered by the sort and number of sanitary staff (first column/case number of Table 4.2 to Table 4.6. In this graph the green points were all scattered, and they shown lack of connectivity.

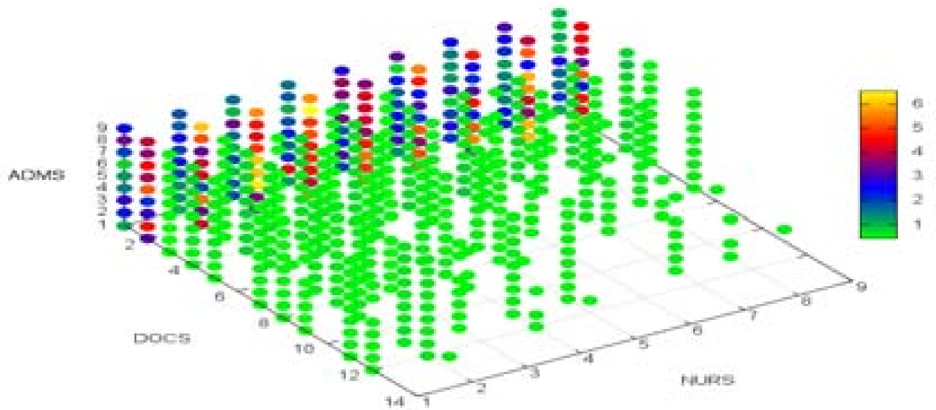


Figure 4.2: 3D scattered graph ordered by the sort and number of staff Table 4.2 to Table 4.6. The green values of interest were totally scattered.

The second example, Figure 4.3 shows the index value ordered by the cost of the sanitary staff configuration. The green points were less scattered, but blue values and others were mixed, showing a region not totally connected. Finally, the third example, Figure 4.4, shows the index value ordered by the equivalent operational patient-service time (t*) of each sanitary staff configuration of Table 4.7 to Table 4.11. This graph shows a connected and almost “non” scattered green region.

4.4 Workloads

In order to analyse the performance of the ED, the real average four hundred incoming patients daily arrive to the ED of Sabadell hospital was considered as follows. This real input was divided into four scenarios, i.e., four different workload

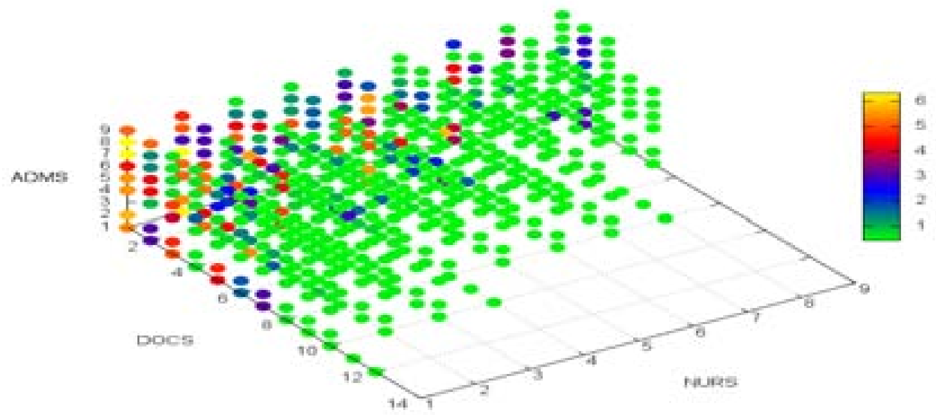


Figure 4.3: 3D scattered graph ordered by the cost of sanitary staff configuration. The green values of interest were not so scattered, but not interconnected.

scenarios, up to: 4, 9, 13, and 17 incoming patients hourly, as shown in Table 4.12 (i.e., up to 96, 216, 312, and 408, respectively for 24hrs.).

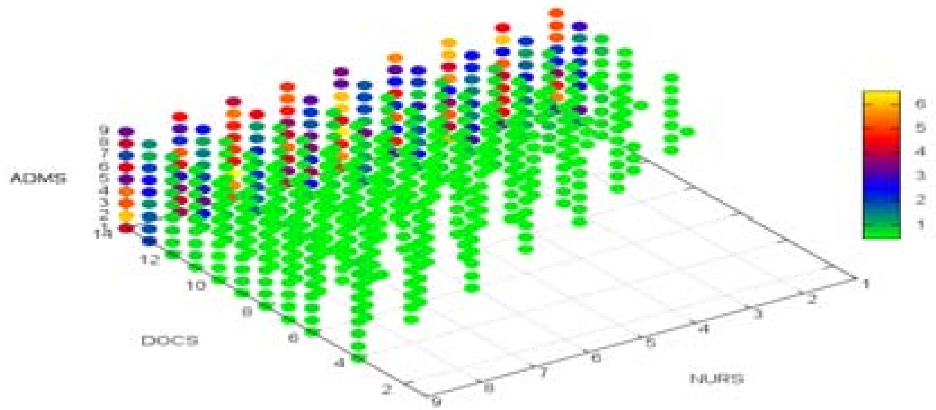


Figure 4.4: 3D scattered graph ordered by the equivalent operational patient-service time (t*) of a "single one" sanitary professional of each sanitary staff configuration Table 4.7 to Table 4.11. The green value region of interest was connected and almost "non" scattered.

These different workload scenarios were used to supply different loads to the ED, whereas the percentage of the priority level of patients was maintained.

Table 4.12: Incoming ED patients divided into four different workload scenarios, up to: 4, 9, 13, and 17 patients per hour for each scenario.

Workload scenario number	**Incoming patients (hourly)**
1	4
2	9
3	13
4	17

4.5 Evaluation Metrics

The set of metrics used in this work were: the length of stay (LoS) of the patients in the ED; the number of attended patients per day (Throughput); and a compound index, the product of the cost of a given sanitary staff configuration times patient length of stay (CLoS).

Furthermore, the computing time of each of the proposed optimisation method is measured in order to observe the gains in reducing computing time of the methodology proposed.

All simulations of the ED optimization cases analysed in this work were carried out in a Linux cluster of the CAOS Department of the UAB, which has 608 computing cores and 2.2TB of RAM, that is composed of: 9 nodes of a dual-4 core Intel Xeon E5430, 2.6GHz, 16GB RAM; 1 node of 2xdual-6 core Intel Xeon E5645, 2.4GHz, 24GB RAM; and 8 nodes of 4x16-cores AMD Opteron "Interlagos", 1.66GHz, 256 GB RAM, all in a switched 1GigE network.

4.6 Evaluation Method

The evaluation of the proposed methodology was aimed to confirm the correct operation of both the pipeline approach (PA) and the MC plus the K-means methods, described in chapter 3. To this end, we have first performed the exhaustive search (ES) to use as baseline method. The second step of this evaluation consists on applying the coarse grained phase, using either the PA, the MC plus K-means methods, or both. Finally, the fine grained phase is apply in the promising regions found in the previous step.

In order to evaluate quantitatively the proposal methodology two case studies were set. The first of them, namely case study A, was performed using the agent-based ED simulator version 1.1. This case study is further described in section 4.7. The second case or case study B was performed using the agent-based ED simulator version 1.2 (the current version). This case study is further described in section 4.8.

In both case studies, only patients identified as triage level 4 and 5 are served at the stage of diagnosis-treatment phase, the three metrics, and the four different workloads stated above were tested, and the period simulated was 24 hrs., i.e., one day of functioning of the ED, in all the experiments. Test scenarios and evaluation results of both case studies are explained in detail in the following sections.

It is important to remind that the actions and interactions corresponding to the admission and triage processes have been totally implemented, but in the case of diagnostic and treatment phase, respecting to the priorities of the Sabadell Hospital ED currently only the level 1 was implemented. In such level 1 only patients identified with priority level 4 or 5 (less urgent, and non-urgent, respectively subsubsection 2.3.3.1) were taken care of. Nevertheless, all incoming patients were triaged. Once patients have been triaged, only patients identified as triage level 4 and 5 were served at the stage of diagnosis-treatment phase.

4.7 Case Study A

The agent-based ED simulator version 1.1, which is shown in Figure 4.5, was used in this case study. In this version of the ED simulator the diagnostic and treatment phase is only addressed by doctors. The simple patient flow in this ED simulator is

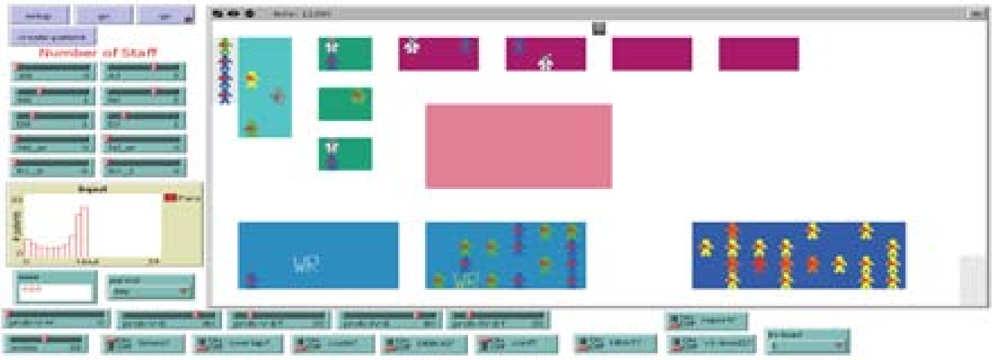


Figure 4.5: ED simulator v1.1. Admission personnel, triage nurses, and doctors were the sanitary staff considered.

defined as follows: patients arrive to the ED on their own, and waits to be attended in the admission area. Then, patients stay in the first Waiting Room (WR) WR1, until a triage nurse call them. After the triage process patients identified as triage level 4 and triage level 5 pass to a second WR2, and stay there until a free doctor calls them to begin the diagnosis-treatment phase, depending on the patient's symptoms, physical condition, and prescribed diagnosis tests. Finally, patients are discharged from the ED.

Therefore, in this case study the sanitary staff considered were: doctors, triage nurses, and admission personnel. Thus, only Table 4.2 to Table 4.6 were taken into account. As a result, 1,134 ($14D * 9N * 9A$) staff configurations were tested for each of the four workload scenarios of incoming patients stated in Table 4.12.

Finally, the three metrics above stated: LoS, Throughput, and CLoS were obtained by applying: the ES technique; the coarse grained phase, using both the PA and the MC plus K-means methods; then, the fine grained phase was applied in the reduced feasible region to find the optimum.

4.7.1 LoS Index

The first objective set was to minimise patient length of stay (LoS) in the ED, with cost configuration constraint less or equal to 3,500 €. This first index is expressed mathematically in Equation 4.7.1:

$$\begin{aligned} &\text{minimise LoS} \quad && f(D, N, A) \\ &\text{subject to} && D_{cost} + N_{cost} + A_{cost} \in Cost \leq 3,500\text{ €} \end{aligned} \tag{4.7.1}$$

It is worth noting that each of the plotted points for the following four workload scenarios were obtained running the ED simulator as many times as points are.

Each plotted point corresponds to each of the 602 staff configurations (out of 1,134) that satisfy the cost restriction.

4.7.1.1 First Workload Scenario

The results of this scenario, up to 4 patients/hour, are shown in Figure 4.6 to Figure 4.9. The ES result is shown in Figure 4.6. The red triangle was the minimum.

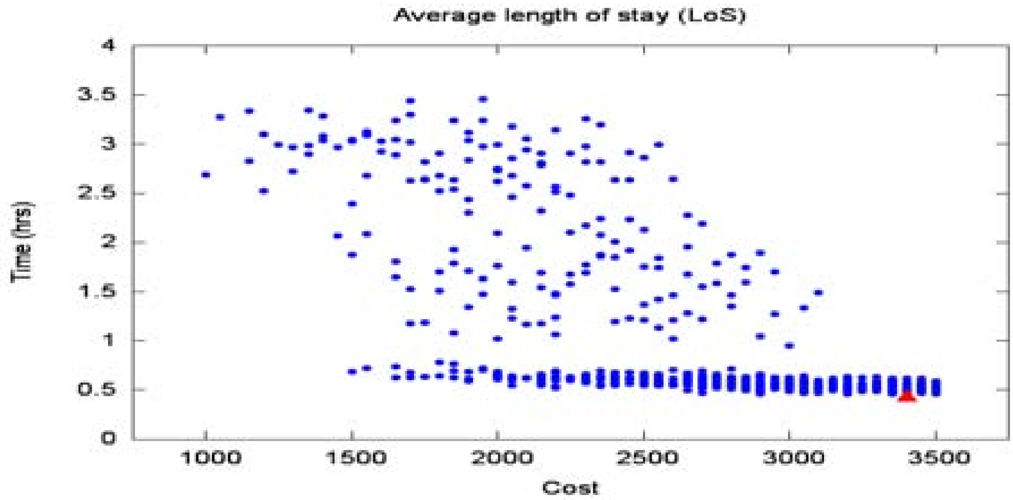


Figure 4.6: Average LoS obtained by the ES. The red triangle was the minimum.

The PA result is shown in Figure 4.7, where four regions can be clearly seen and the red triangle was the minimum.

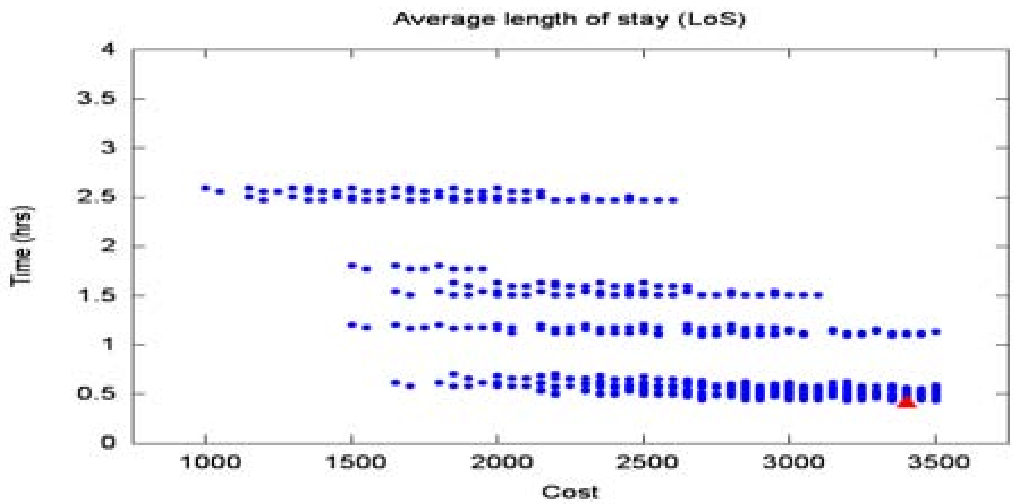


Figure 4.7: Average LoS obtained by the PA. The red triangle was the minimum.

The most important is the bottom region, in which the average minimum LoS was around 0.5 hours. There were 366 configurations (from a total of 602 in the feasible region) in this region, which is the one where the minimum was.

The MC plus the K-means methods results are shown in Figure 4.8 to Figure 4.9, respectively. The MC method found 75 configurations.

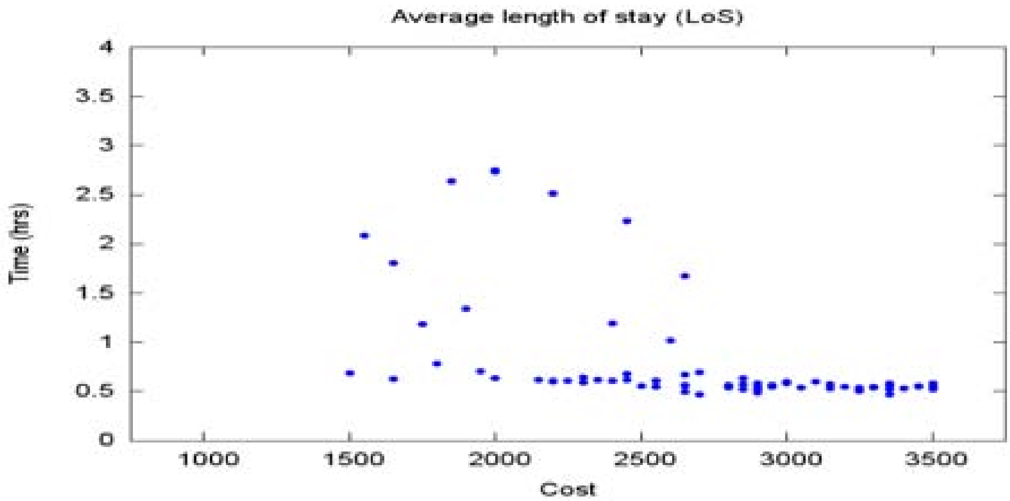


Figure 4.8: Average LoS of 75 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified three different clusters, shown in Figure 4.9. The most important cluster was the red one (at the bottom right), which delimited the region where the optimum was.

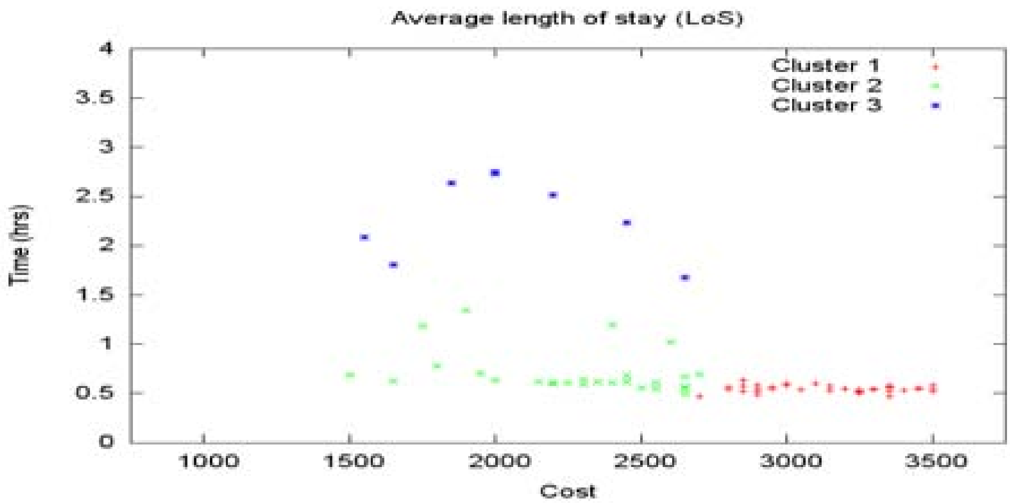


Figure 4.9: K-means identified three clusters of average LoS . The red one delimits the region where the minimum was.

The Figure 4.10 shows another way to visualise the connectivity characteristic of the reduced regions found by the proposed methodology. The axes of such graph are the equivalent operational patient-service time (t*) of a “single one” sanitary professional of each sanitary staff configuration (the first column of Table 4.7 to Table 4.9, where they were ordered by the PA Equation 3.5.1). In such figure, the points of interest were the green points, which lie in the region of interest, where the minimum was, which can be seen in black triangle. It can be seen that it was not necessary to search in the whole feasible region, but only in the green connected region.

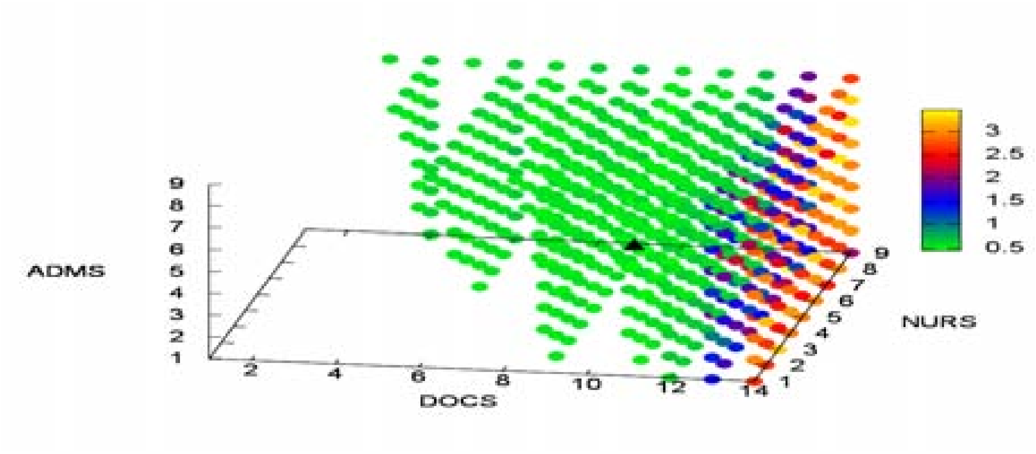


Figure 4.10: 3D scattered graph shows the average LoS index of the first workload scenario (4 patients/hour). The average LoS index in hours is represented in colour, and the minimum is the black triangle.

Finally, after separately applied both the PA and the MC plus the K-means methods, the “reduced exhaustive search” was separately performed in each reduced region identified. The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.13, where the sanitary staff configuration (doctors, nurses, and admission personnel), their associated average minimum LoS, and cost configuration are shown. The three optima independently found were the same.

Table 4.13: Optimum staff configurations that got the average minimum LoS for this workload scenario (up to 4 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in red triangle in Figure 4.6 and in black triangle in Figure 4.10.

Method	€	LoS (hrs)	D	N	A	Run time (hrs) 4 Pthreads
ES	3,400	0.44	2S	2S	2S	0.89
PA	3,400	0.43	2S	2S	2S	0.53
MC+K-means	3,400	0.44	2S	2S	2S	0.76

4.7.1.2 Second Workload Scenario

The results of this scenario, up to 9 patients/hour, are shown in Figure 4.11 to Figure 4.14. The ES result is shown in Figure 4.11, where the red triangle was the minimum.

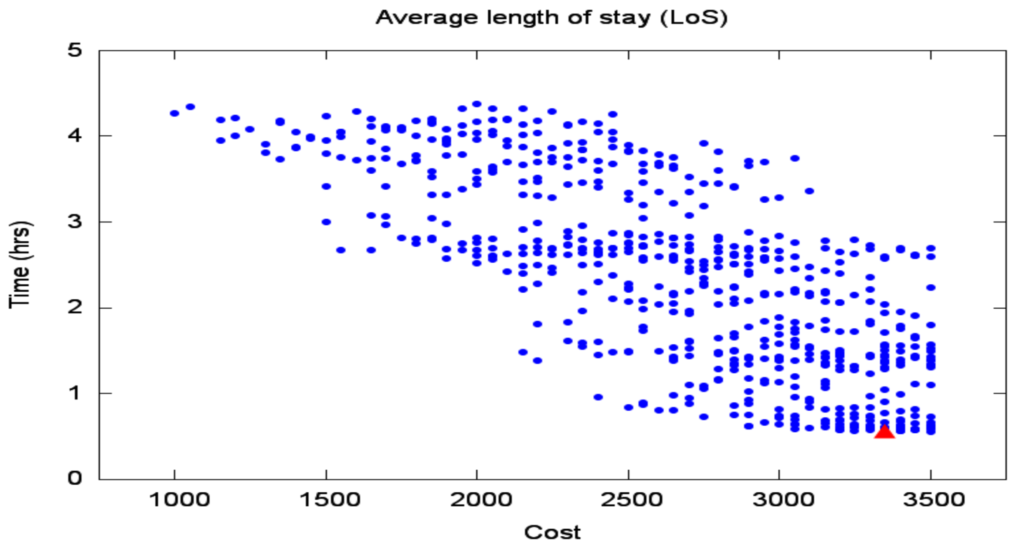


Figure 4.11: Average LoS obtained by the ES. The red triangle was the minimum.

The PA result is shown in Figure 4.12, where seven regions can be clearly seen and the red triangle is the minimum.

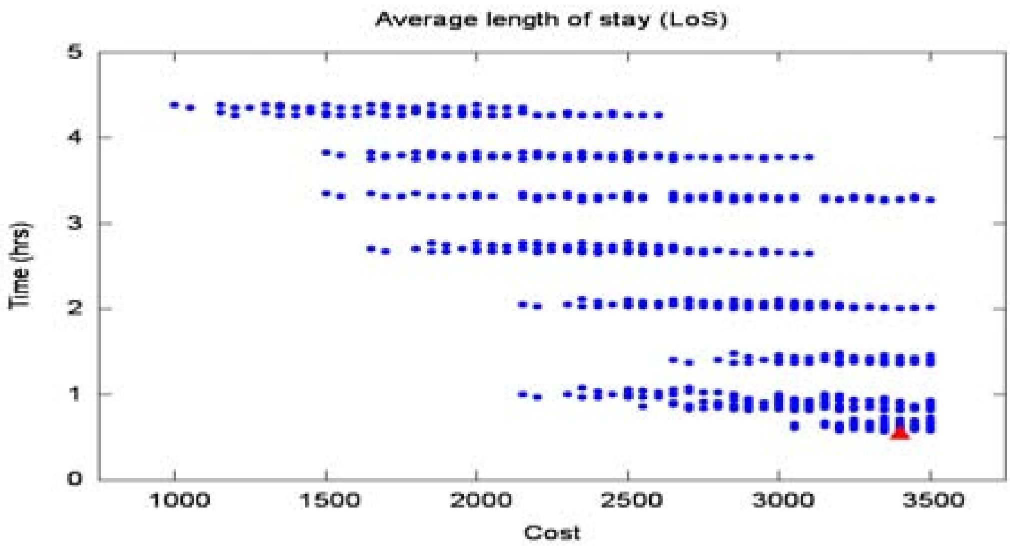


Figure 4.12: Average LoS obtained by the PA. The red triangle was the minimum.

The most important is the bottom region, in which the average minimum LoS

was around 1 hour. There were 180 configurations (from a total of 602 in the feasible region) in this region, which is the one where the minimum was.

The MC plus the K-means methods results are shown in Figure 4.13 to Figure 4.14, respectively. The MC method found 125 configurations.

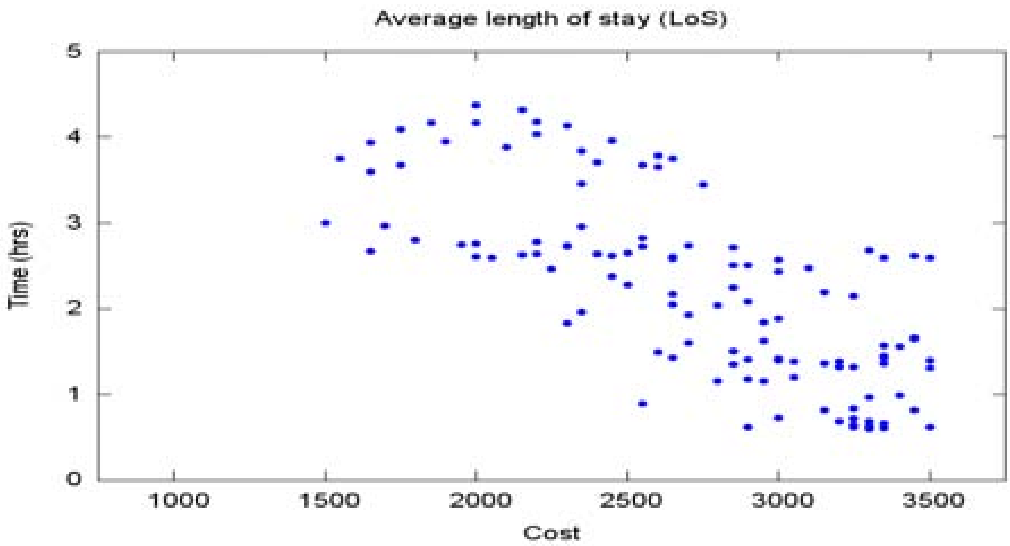


Figure 4.13: Average LoS of 125 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.14. The most important was the green cluster (at the bottom right), which delimited the region where the optimum was.

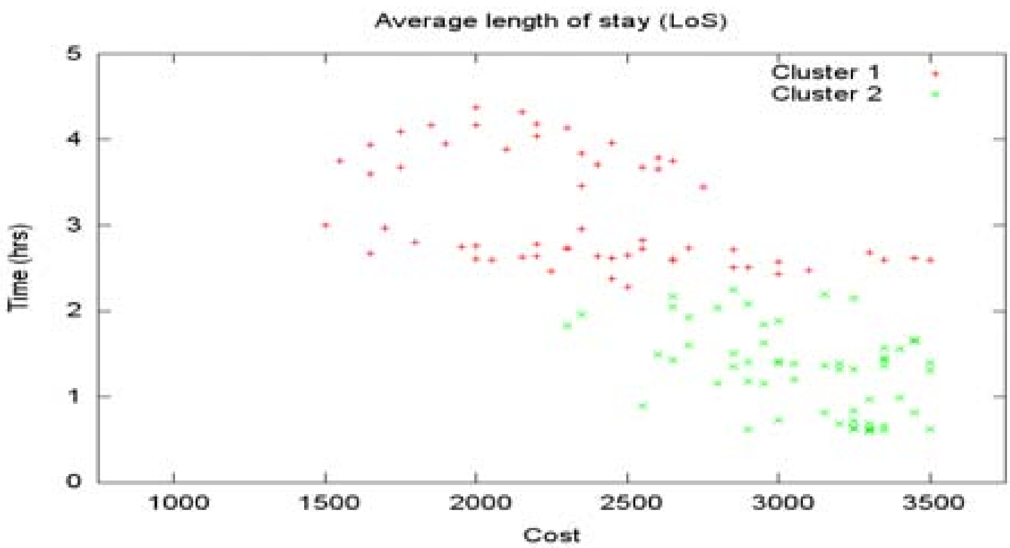


Figure 4.14: The K-means method identified two clusters of average LoS. The green one delimited the region where the minimum was.

The Figure 4.15 shows another way to visualise the connectivity characteristic of the reduced regions found by the proposed methodology. The axes of such graph are the equivalent operational patient-service time (t*) of a "single one" sanitary professional of each sanitary staff configuration (the first column of Table 4.7 to Table 4.9, where they were ordered by the PA Equation 3.5.1). In such figure, the points of interest were the green points, which lie in the region of interest, where the minimum was, which can be seen in black triangle. It can be seen that it was not necessary to search in the whole feasible region, but only in the green connected region.

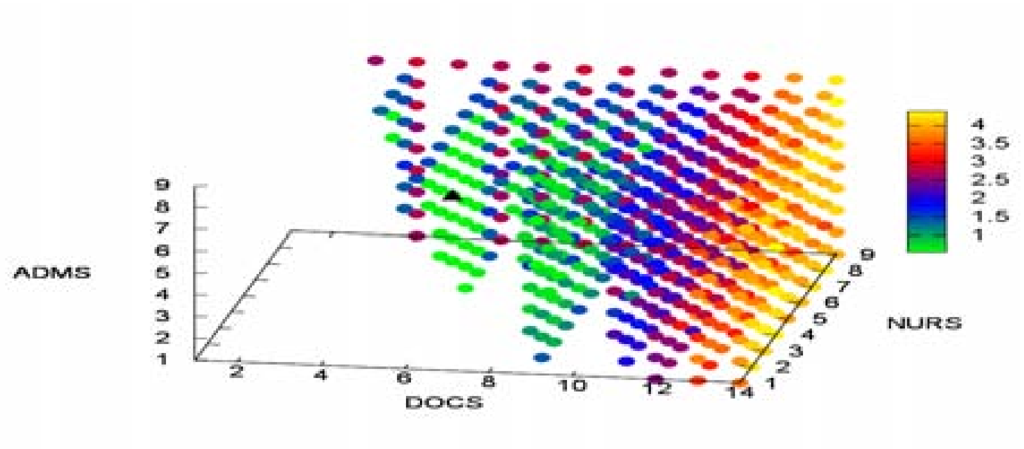


Figure 4.15: 3D scattered graph shows the average LoS index of the third workload scenario (9 patients/hour). The average LoS index is expressed in colour in hours.

Finally, after separately applied both the PA and the MC plus the K-means methods, the "reduced exhaustive search" was separately performed in each reduced region identified. The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.14, where the sanitary staff configuration (doctors, nurses, and admission personnel), their associated average minimum LoS, and cost configuration are shown. The three optima independently found were the same.

Table 4.14: Optimum staff configurations that got the average minimum LoS for this workload scenario (up to 9 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in red triangle in Figure 4.11 and in black triangle in Figure 4.15.

Method	€	LoS (hrs)	D	N	A	Run time (hrs) 4 Pthreads
ES	3,350	0.55	4J	2S	1S,1J	1.57
PA	3,350	0.55	4J	2S	1S,1J	0.39
MC+K-means	3,350	0.55	4J	2S	1S,1J	1.01

4.7.1.3 Third Workload Scenario

The results of this scenario, up to 13 patients/hour, are shown in Figure 4.16 to Figure 4.19. The ES result is shown in Figure 4.16, where the red triangle was the minimum.

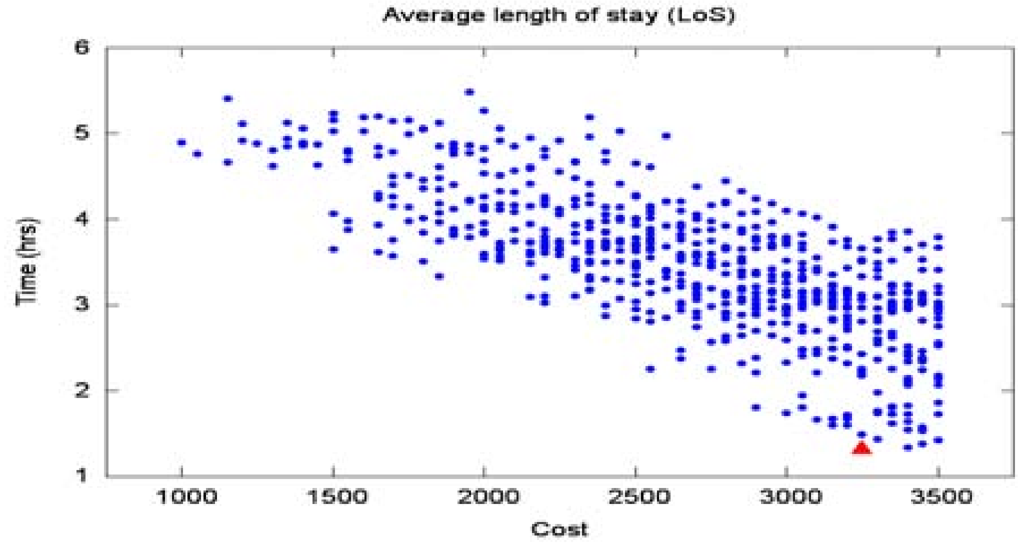


Figure 4.16: Average LoS obtained by the ES. The red triangle was the minimum.

The PA result is shown in Figure 4.17, where many regions can be clearly seen and the red triangle is the minimum.

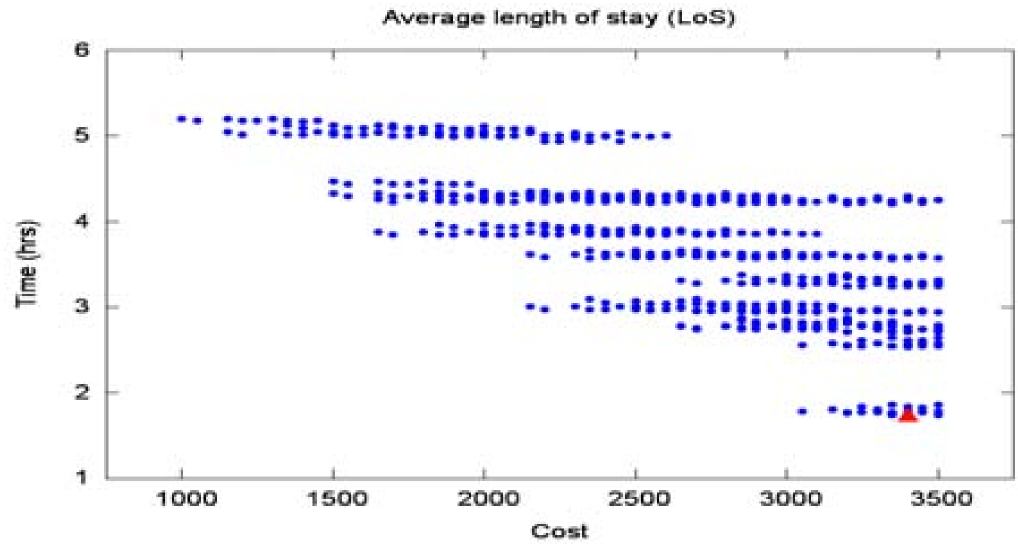


Figure 4.17: Average LoS obtained by the PA. The red triangle was the minimum.

The most important is the bottom region, in which the average minimum LoS

was less than 2 hours. There were 21 configurations (from a total of 602 in the feasible region) in this region, which is the one where the minimum was.

The MC plus the K-means methods results are shown in Figure 4.18 to Figure 4.19, respectively. The MC method found 125 configurations.

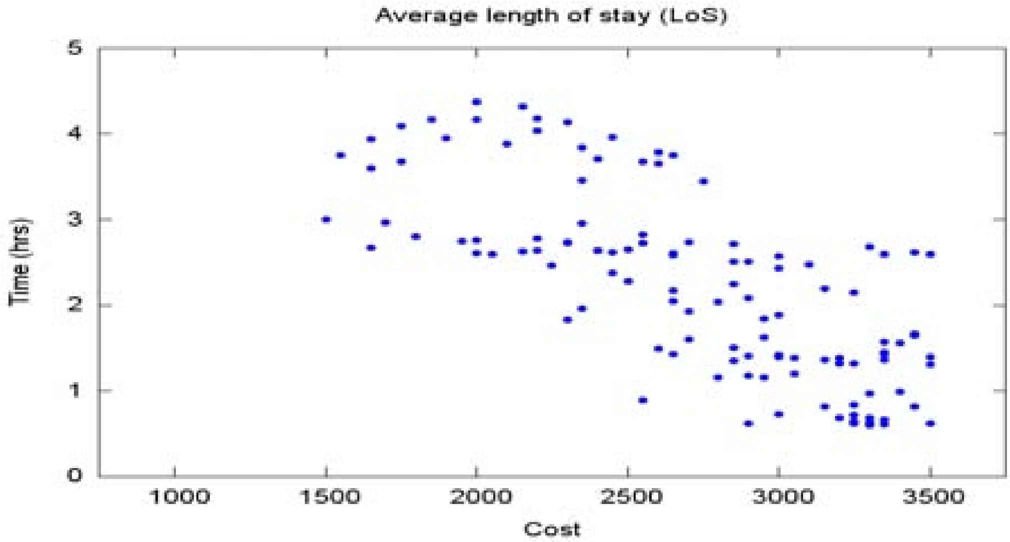


Figure 4.18: Average LoS of 125 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.19. The most important was the green cluster (at the bottom right), which delimited the region where the optimum was.

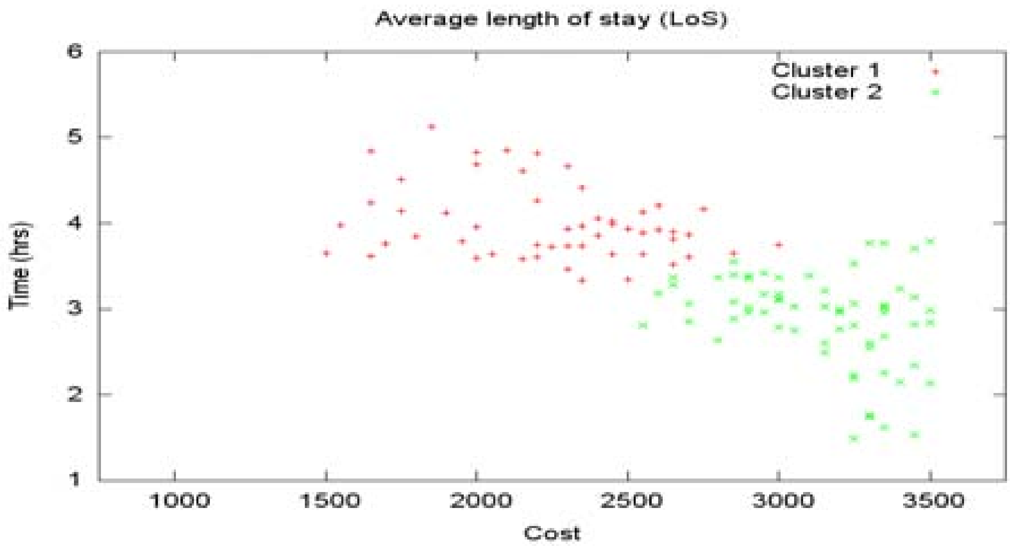


Figure 4.19: The K-means method identified two clusters of average LoS. The green one delimited the region where the minimum was.

The Figure 4.20 shows another way to visualise the connectivity characteristic of the reduced regions found by the the proposed methodology. The axes of such graph are the equivalent operational patient-service time (t*) of a "single one" sanitary professional of each sanitary staff configuration (the first column of Table 4.7 to Table 4.9, where they were ordered by the PA Equation 3.5.1). In such figure, the points of interest were the green points, which lie in the region of interest, where the minimum was, which can be seen in black triangle. It can be seen that it was not necessary to search in the whole feasible region, but only in the green connected region.

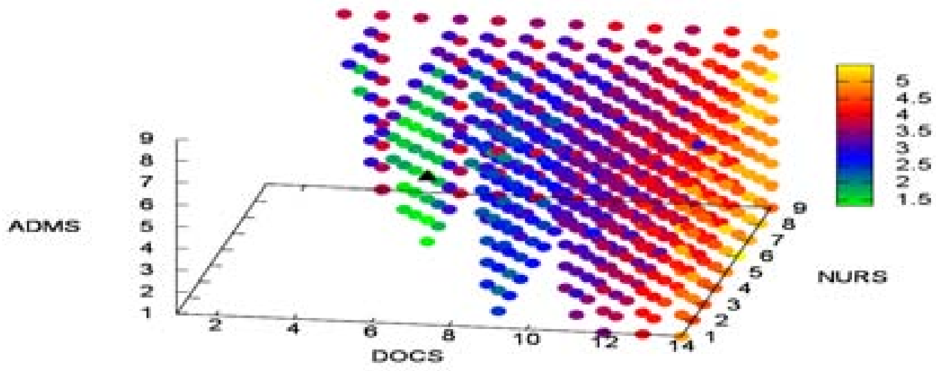


Figure 4.20: 3D scattered graph shows the average LoS index of the third workload scenario (13 patients/hour). The average LoS index is expressed in colour in hours.

Finally, after separately applied both the PA and the MC plus the K-means methods, the "reduced exhaustive search" was separately performed in each reduced region identified. The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.15, where the sanitary staff configuration (doctors, nurses, and admission personnel), their associated average minimum LoS, and cost configuration are shown. The three optima independently found were the same.

Table 4.15: Optimum staff configurations that got the average minimum LoS for this workload scenario (up to 13 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in red triangle in Figure 4.16 and in black triangle in Figure 4.20.

Method	€	LoS (hrs)	D	N	A	Run time (hrs) 4 Pthreads
ES	3,250	1.33	4J	1S,1J	2S	2.45
PA	3,250	1.33	4J	1S,1J	2S	0.16
MC+K-means	3,250	1.33	4J	1S,1J	2S	1.49

4.7.1.4 Fourth Workload Scenario

The results of this scenario, up to 17 patients/hour, are shown in Figure 4.21 to Figure 4.24. The ES result is shown in Figure 4.21, where the red triangle is the minimum.

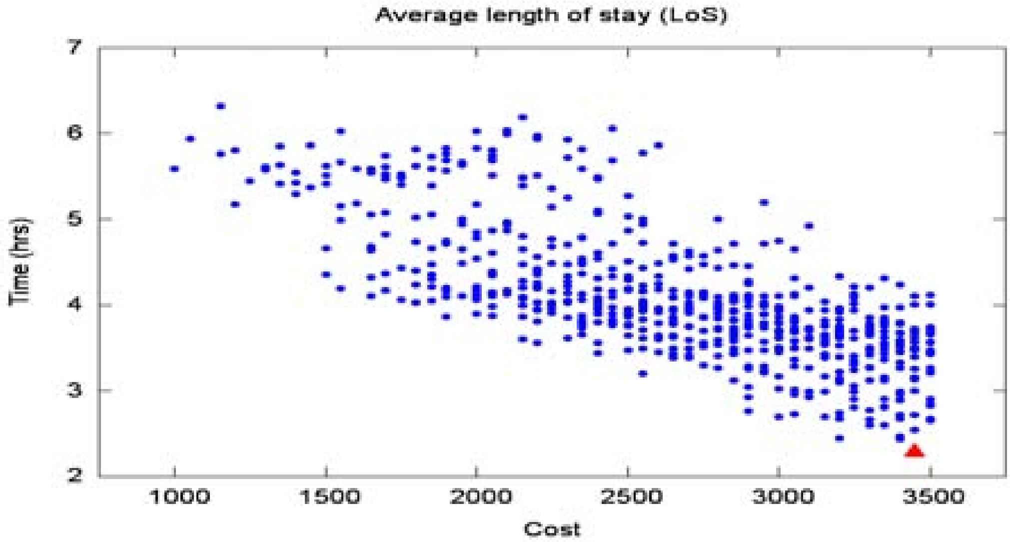


Figure 4.21: Average LoS obtained by the ES. The red triangle was the minimum.

The PA result is shown in Figure 4.22, where many regions can be clearly seen and the red triangle is the minimum.

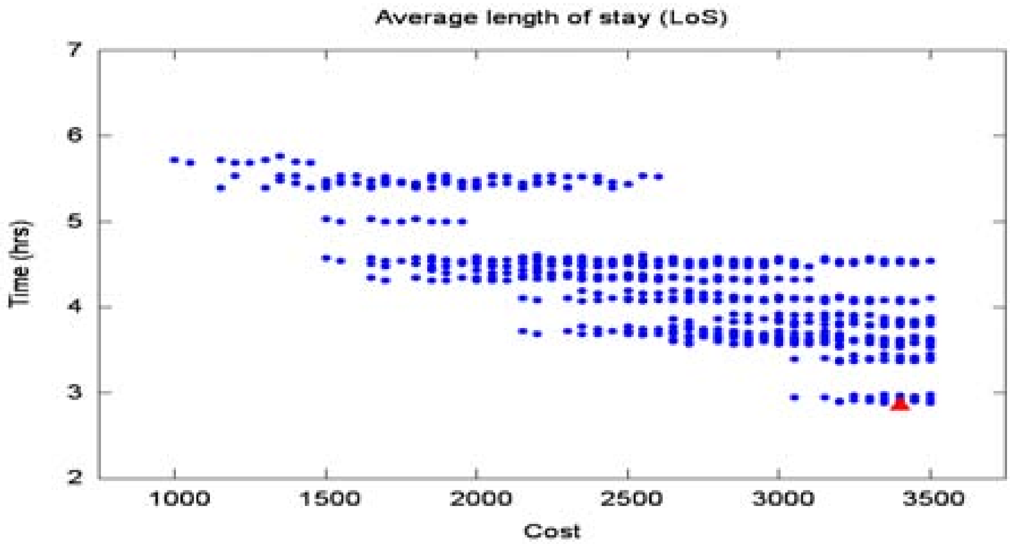


Figure 4.22: Average LoS obtained by the PA. The red triangle was the minimum.

The most important is the bottom region, in which the average minimum LoS

was around than 3 hours. There were 21 configurations (from a total of 602 in the feasible region) in this region, which is the one where the maximum was.

The MC plus the K-means methods results are shown in Figure 4.23 to Figure 4.24, respectively. The MC method found 125 configurations.

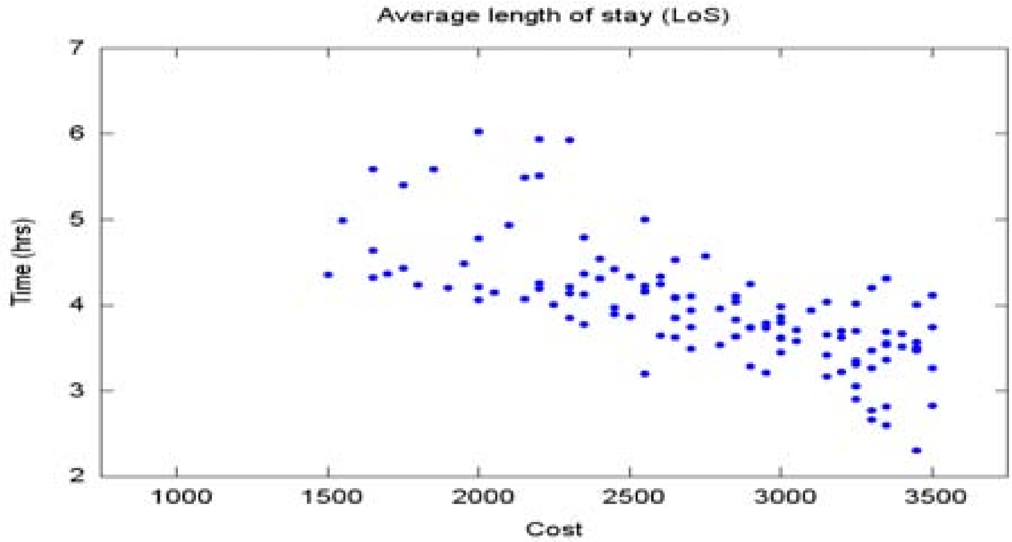


Figure 4.23: Average LoS of 125 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.24. The most important was the green cluster (at the bottom right), which delimited the region where the optimum was.

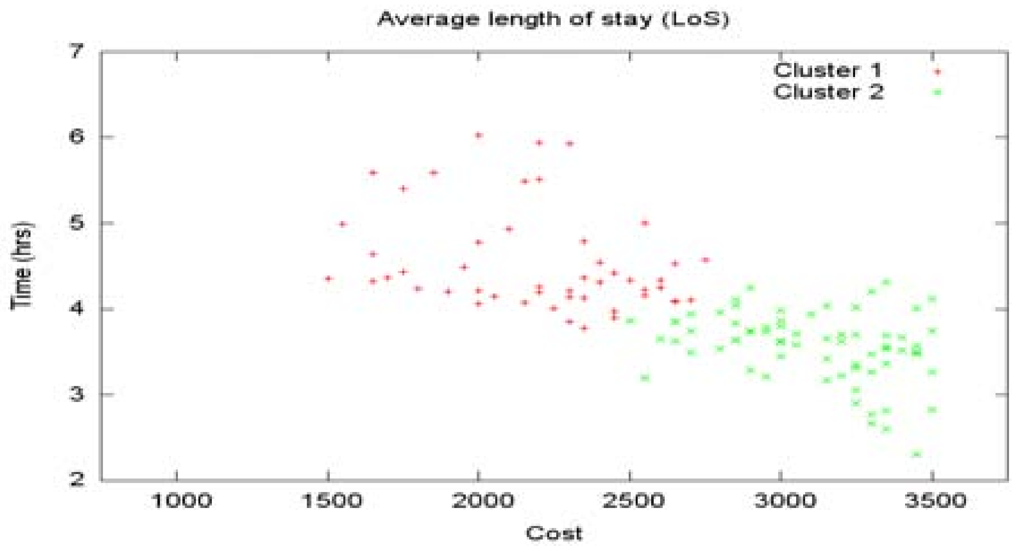


Figure 4.24: The K-means method identified two clusters of average LoS. The green one delimited the region where the minimum was.

The Figure 4.25 shows another way to visualise the connectivity characteristic of the reduced regions found by the proposed methodology. The axes of such graph are the equivalent operational patient-service time (t*) of a "single one" sanitary professional of each sanitary staff configuration (the first column of Table 4.7 to Table 4.9, where they were ordered by the PA Equation 3.5.1). In such figure, the points of interest were the green points, which lie in the region of interest, where the minimum was, which can be seen in black triangle. It can be seen that it was not necessary to search in the whole feasible region, but only in the green connected region.

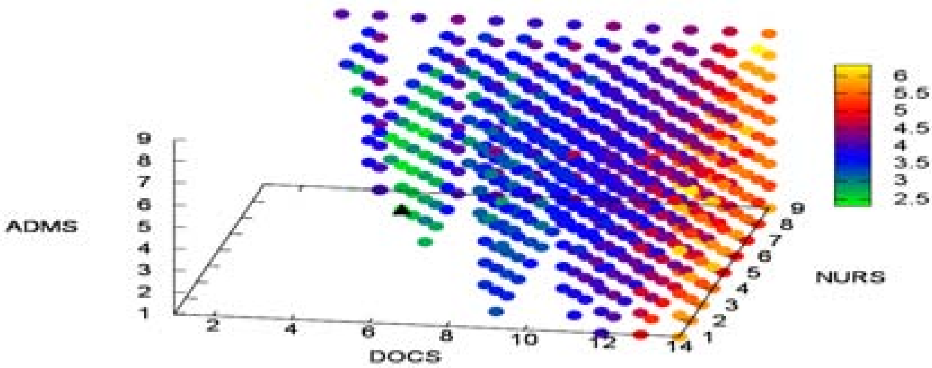


Figure 4.25: 3D scattered graph shows the average LoS index of the third workload scenario (17 patients/hour). The average LoS index is expressed in colour in hours.

Finally, after separately applied both the PA and the MC plus the K-means methods, the "reduced exhaustive search" was separately performed in each reduced region identified.The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.16, where the sanitary staff configuration (doctors, nurses, and admission personnel), their associated average minimum LoS, and cost configuration are shown. The three optima independently found were the same.

Table 4.16: Optimum staff configurations that got the average minimum LoS for this workload scenario (up to 17 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in red triangle in Figure 4.21 and in black triangle in Figure 4.25.

Method	€	LoS (hrs)	D	N	A	Run time (hrs) 4 Pthreads
ES	3,450	2.3	4J	3J	2S	3.42
PA	3,450	2.3	4J	3J	2S	0.15
MC+K-means	3,450	2.3	4J	3J	2S	2.0

4.7.2 Throughput Index

The second objective set was to maximise number of attended patients per day (Throughput) in the ED, with cost configuration constraint less or equal to 3,500 €. This index is expressed mathematically in Equation 4.7.2:

$$\begin{array}{ll} \text{Maximise patients attended} & f(D, N, A) \\ \text{subject to} & D_{cost} + N_{cost} + A_{cost} \in Cost \leq 3,500 \text{ €} \end{array} \tag{4.7.2}$$

It is worth noting that each of the plotted points were obtained running the ED simulator as many times as points are. Each plotted point corresponds to each of the 602 staff configurations (out of 1,134) that satisfy the cost restriction.

4.7.2.1 First Workload Scenario

The results of this scenario, up to 4 patients/hour, are shown in Figure 4.26 to Figure 4.29. Figure 4.26 shows ES results. The red triangles were the maxima.

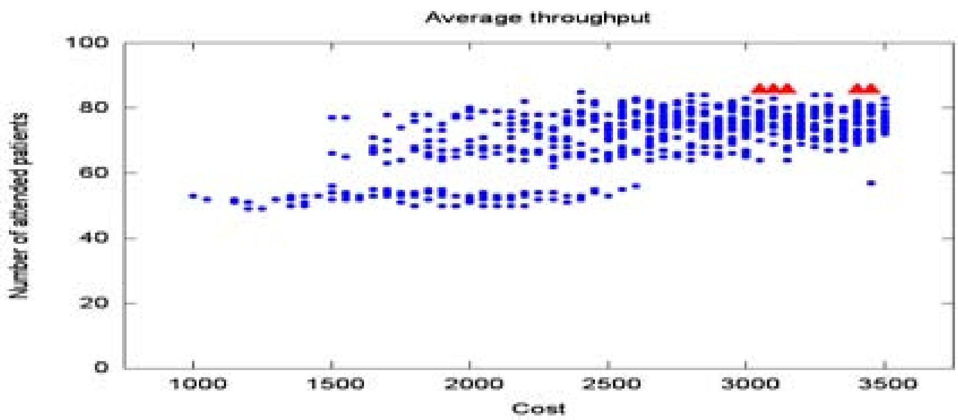


Figure 4.26: Average number of attended patients obtained by the ES method. The red triangles were the maxima.

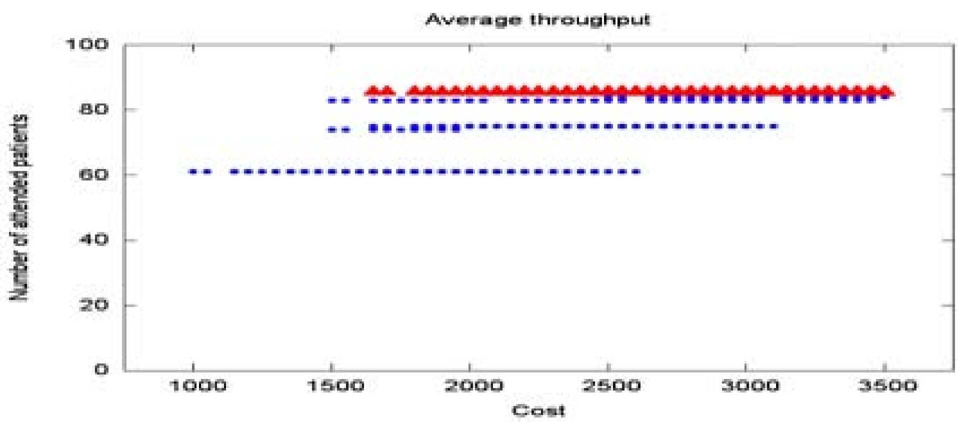


Figure 4.27: Average number of attended patients obtained by the PA. The red triangles were the maxima.

The PA result is shown in Figure 4.27, where three regions can be clearly seen and the red triangles were the maxima. The most important is the top region, in which the average maximum Throughput was more than 80 attended patients. There were 440 configurations (from a total of 602 in the feasible region) in this region, which was the one where the maxima were.

The MC plus the K-means methods results are shown in Figure 4.28 to Figure 4.29, respectively. The MC method found 50 configurations.

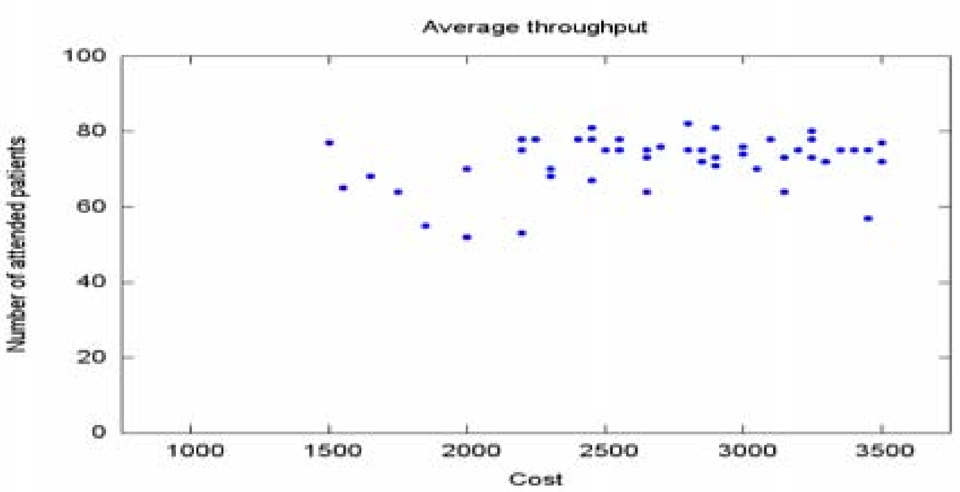


Figure 4.28: Average number of attended patients of 50 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.29. The most important was the green cluster, which delimited the region where the optima were.

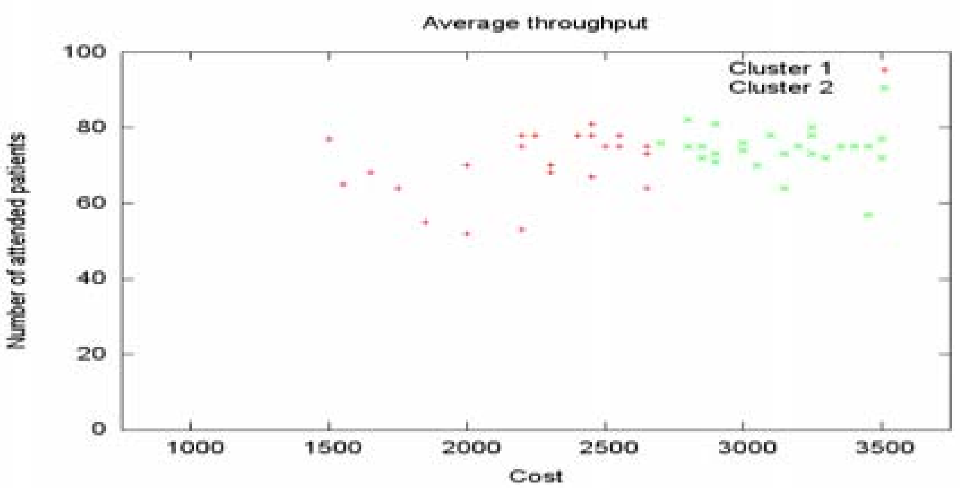


Figure 4.29: The K-means method identified two clusters of average number of attended patients. The green one delimited the region where the maxima were.

Finally, after separately applied both the PA and the MC plus the K-means methods, the "reduced exhaustive search" was separately performed in each reduced region identified. The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.17, where the sanitary staff configuration (doctors, nurses, and admission personnel), their associated average maximum Throughput, and cost configuration are shown. The three optima independently found were the same.

Table 4.17: Optimum staff configurations that got the average maximum Throughput for this workload scenario (up to 4 patients hourly), where S is Senior and J is Junior. These optimum sanitary staff configurations are shown in red triangle in Figure 4.26.

Method	€	#attended patients	D	N	A	Run time (hrs) 4 Pthreads
ES	3,050	86	2S	1S,1J	1S	0.91
PA	3,050	86	2S	1S,1J	1S	0.63
MC+K-means	3,050	86	2S	1S,1J	1S	0.73

4.7.2.2 Second Workload Scenario

The results of this scenario, up to 9 patients/hour, are shown in Figure 4.30 to Figure 4.33. The ES result is shown in Figure 4.30, where the red triangle was the maximum.

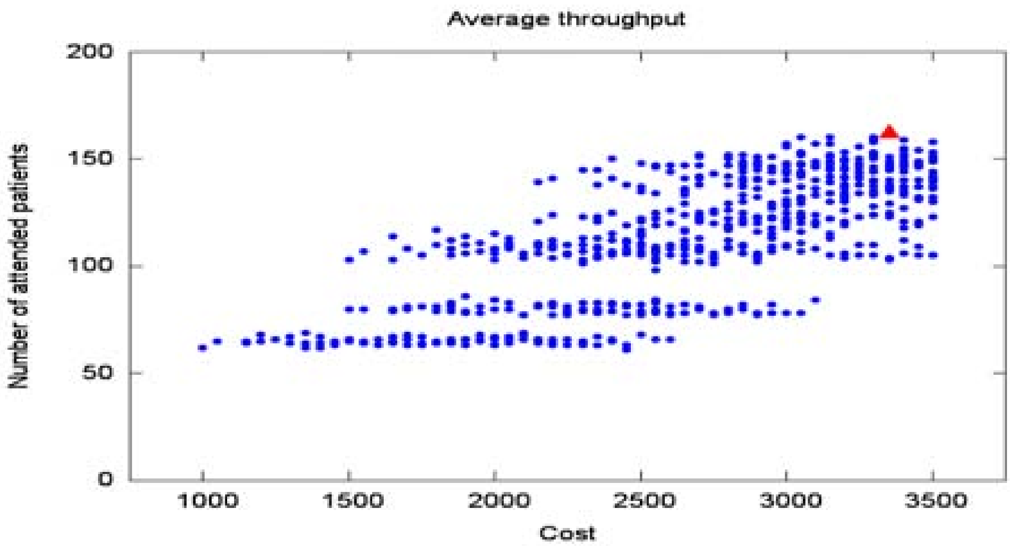


Figure 4.30: Average number of attended patients obtained by the ES method. The red triangle was the maximum.

The PA result is shown in Figure 4.31, where many regions can be seen and the red triangle was the maximum.

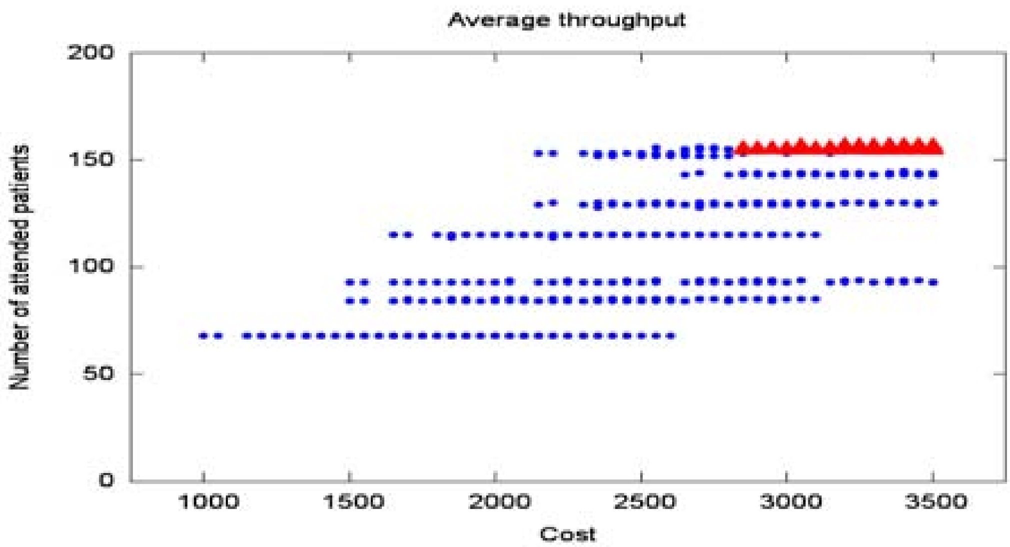


Figure 4.31: Average number of attended patients obtained by the PA. The red triangles were the maximum.

The most important is the top region, in which the average maximum Throughput was more than 150 attended patients. There were 180 configurations (from a total of 602 in the feasible region) in this region, which was the one where the maximum was.

The MC plus the K-means methods results are shown in Figure 4.32 to Figure 4.33, respectively. The MC method found 125 configurations.

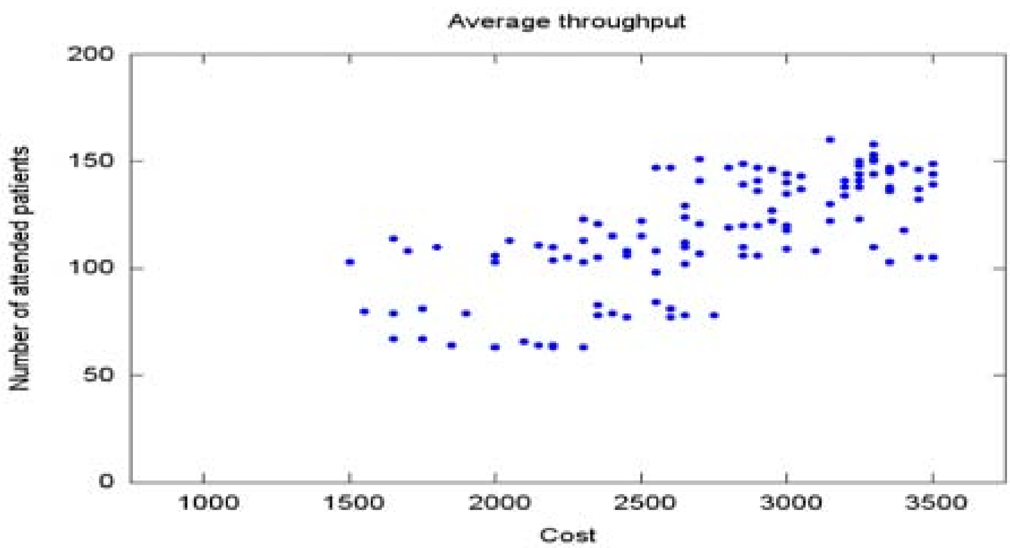


Figure 4.32: Average number of attended patients of 125 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.33. The most important was the green cluster, which delimited the region where the optimum was.

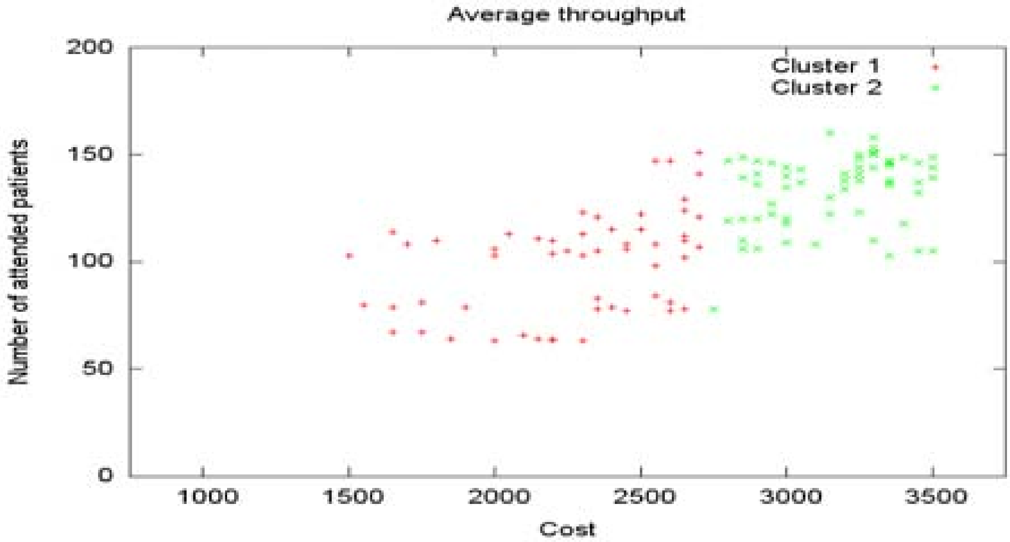


Figure 4.33: The K-means method identified two clusters of average number of attended patients. The green one delimited the region where the maximum was.

Finally, after separately applied both the PA and the MC plus the K-means methods, the "reduced exhaustive search" was separately performed in each reduced region identified. The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.18, where the sanitary staff configuration (doctors, nurses, and admission personnel), their associated average maximum Throughput, and cost configuration are shown. The three optima independently found were the same.

Table 4.18: Optimum staff configurations that got the average maximum Throughput for this workload scenario (up to 9 patients hourly), where S is Senior and J is Junior. These optimum sanitary staff configurations are shown in red triangle in Figure 4.30.

Method	€	#attended patients	D	N	A	Run time (hrs) 4 Pthreads
ES	3,350	163	1S,2J	2S	1S,1J	1.59
PA	3,350	163	1S,2J	2S	1S,1J	0.39
MC+K-means	3,350	163	1S,2J	2S	1S,1J	1.08

4.7.2.3 Third Workload Scenario

The results of this scenario, up to 13 patients/hour, are shown in Figure 4.34 to Figure 4.37. The ES result is shown in Figure 4.34, where the red triangle was the maximum.

The PA result is shown in Figure 4.35, many regions can be seen and the red triangle was the maximum. The most important is the top region, in which the average maximum Throughput was more than 200 attended patients. There were 21 configurations (from a total of 602 in the feasible region) in this region, which is the one where the maximum was.

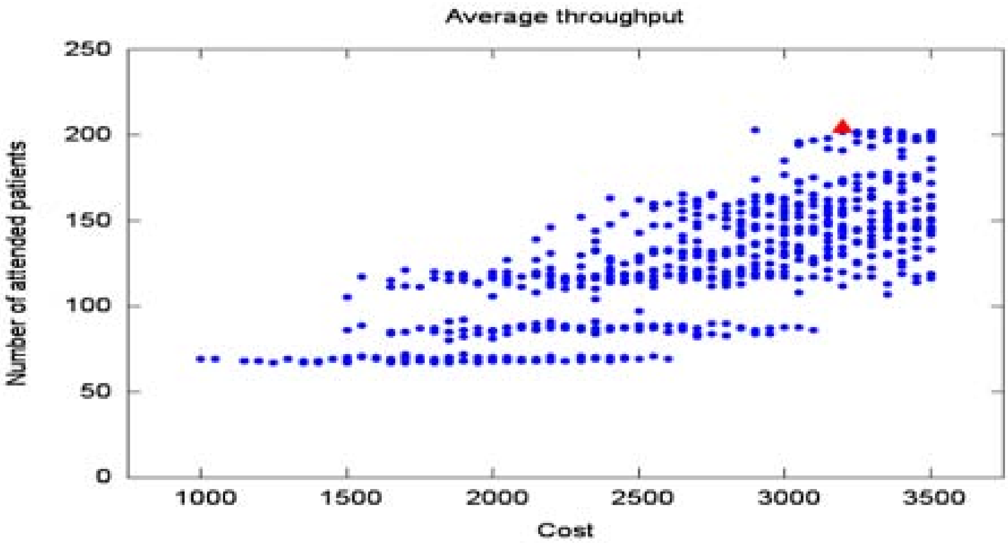


Figure 4.34: Average number of attended patients obtained by the ES method. The red triangle was the maximum.

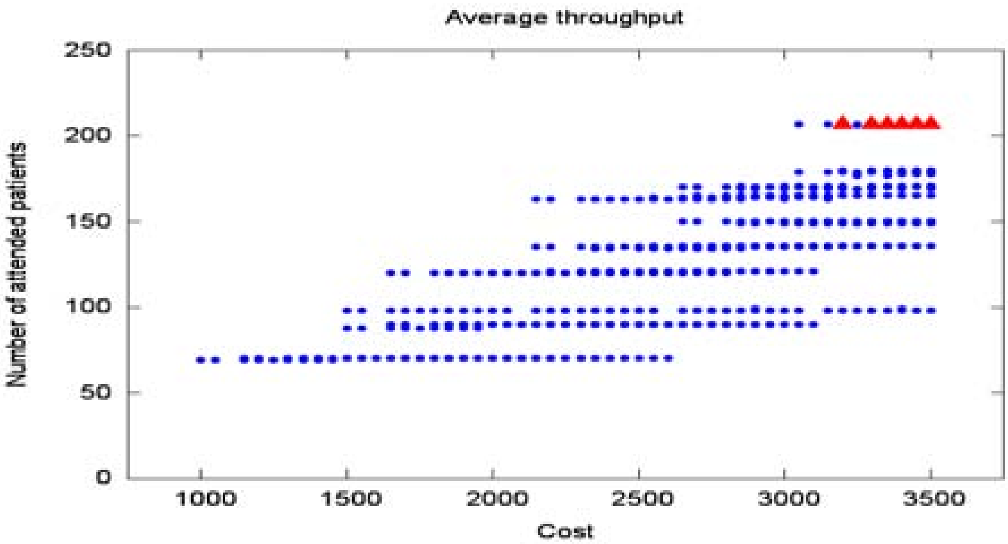


Figure 4.35: Average number of attended patients obtained by the PA. The red triangles were the maximum.

The MC plus the K-means methods results are shown in Figure 4.36 to Figure 4.37, respectively. The MC method found 275 configurations.

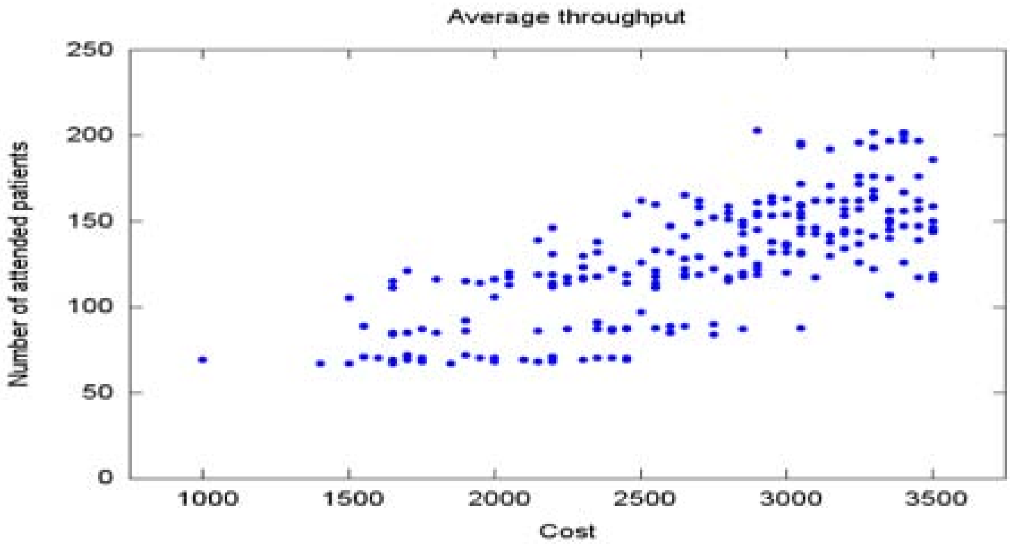


Figure 4.36: Average number of attended patients of 275 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.37, the most important was the green cluster, which delimited the region where the optimum was.

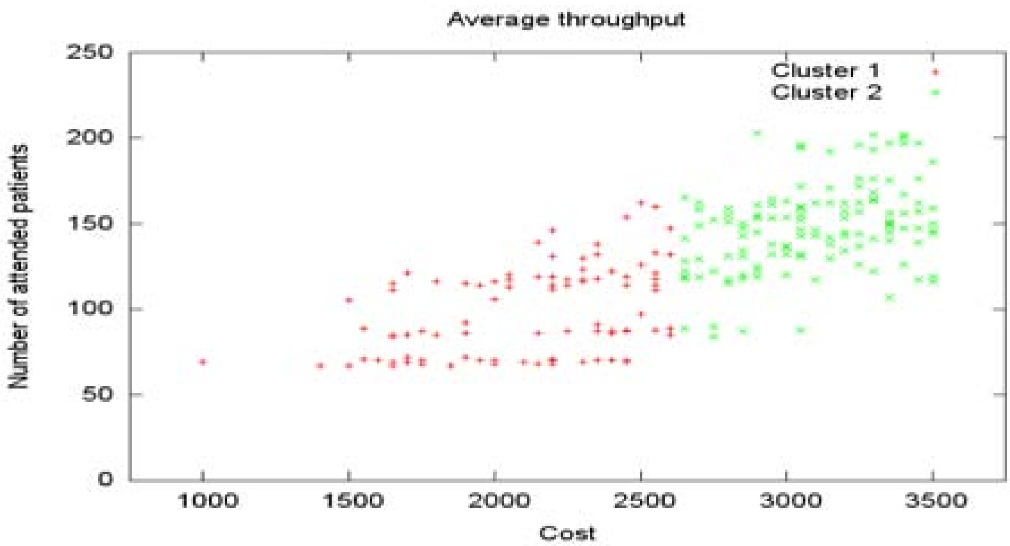


Figure 4.37: The K-means method identified two clusters of average number of attended patients. The green one delimited the region where the maximum was.

Finally, after separately applied both the PA and the MC plus the K-means methods, the "reduced exhaustive search" was separately performed in each reduced region identified. The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.19, where the sanitary staff configuration (doctors, nurses, and admission personnel), and their associated average maximum Throughput, and cost configuration are shown. The three optima independently found were the same.

Table 4.19: Optimum staff configurations that got the average maximum Throughput for this workload scenario (up to 13 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in red triangle in Figure 4.34.

Method	€	#attended patients	D	N	A	Run time (hrs) 4 Pthreads
ES	3,200	205	4J	2S	1S	2.46
PA	3,200	205	4J	2S	1S	0.13
MC+K-means	3,200	205	4J	2S	1S	1.51

4.7.2.4 Fourth Workload Scenario

The results of this scenario, up to 17 patients/hour, are shown in Figure 4.38 to Figure 4.41. The ES result is shown in Figure 4.38, where the red triangle was the maximum.

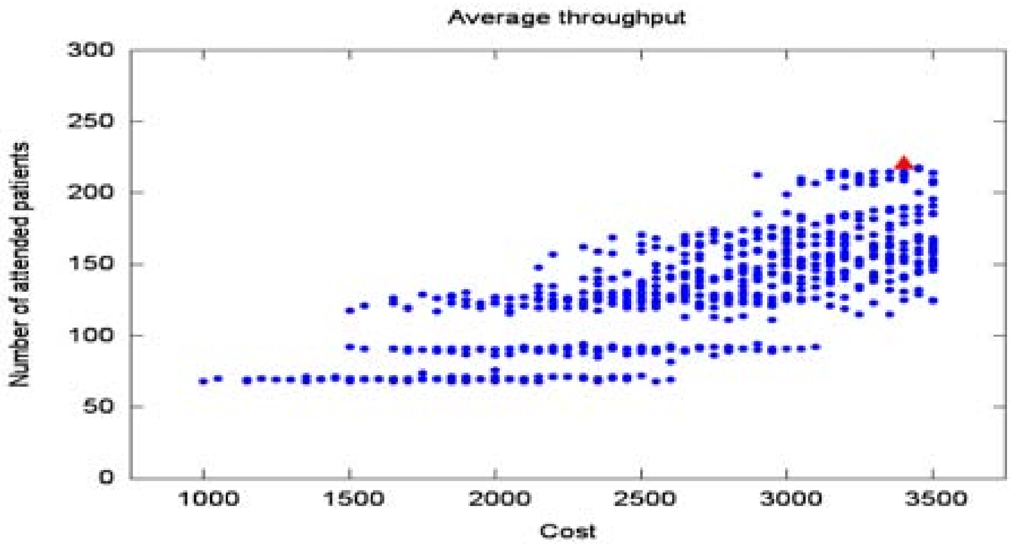


Figure 4.38: Average number of attended patients obtained by the ES method. The red triangle was the maximum.

The PA result is shown in Figure 4.39, many regions can be seen and the red triangle was the maximum. The most important is the top region, in which the average maximum Throughput was more than 200 attended patients. There were 21 configurations (from a total of 602 in the feasible region) in this region, which is the one where the maximum was.

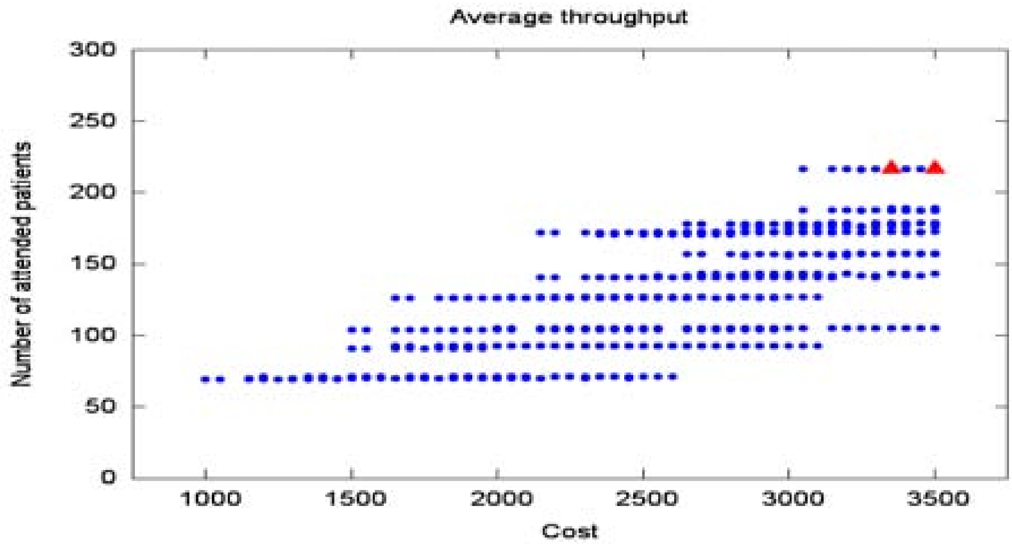


Figure 4.39: Average number of attended patients obtained by the PA. The red triangles were the maximum.

The MC plus the K-means methods results are shown in Figure 4.40 to Figure 4.41, respectively. The MC method found 125 configurations.

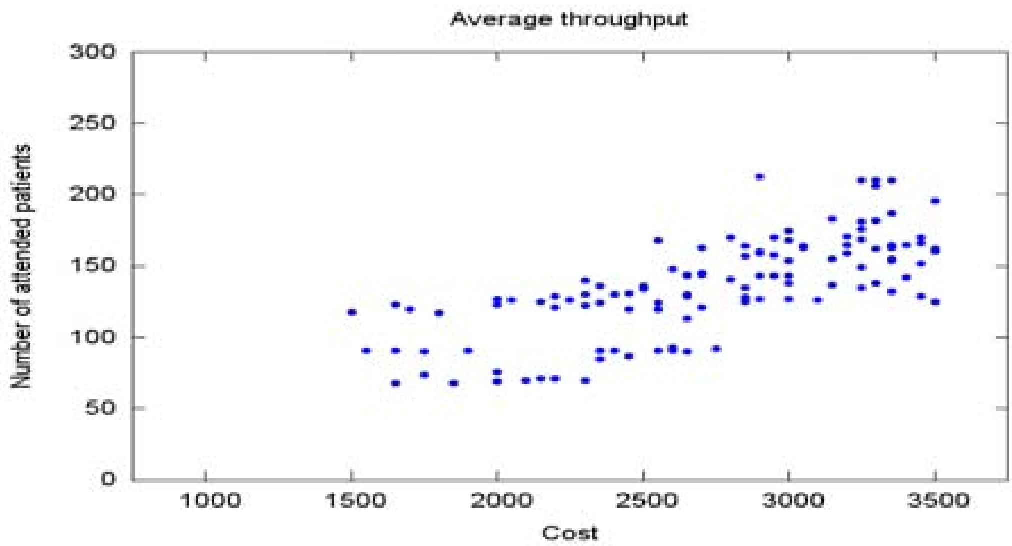


Figure 4.40: Average number of attended patients of 125 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.41, the most important was the green cluster, which delimited the region where the optimum was.

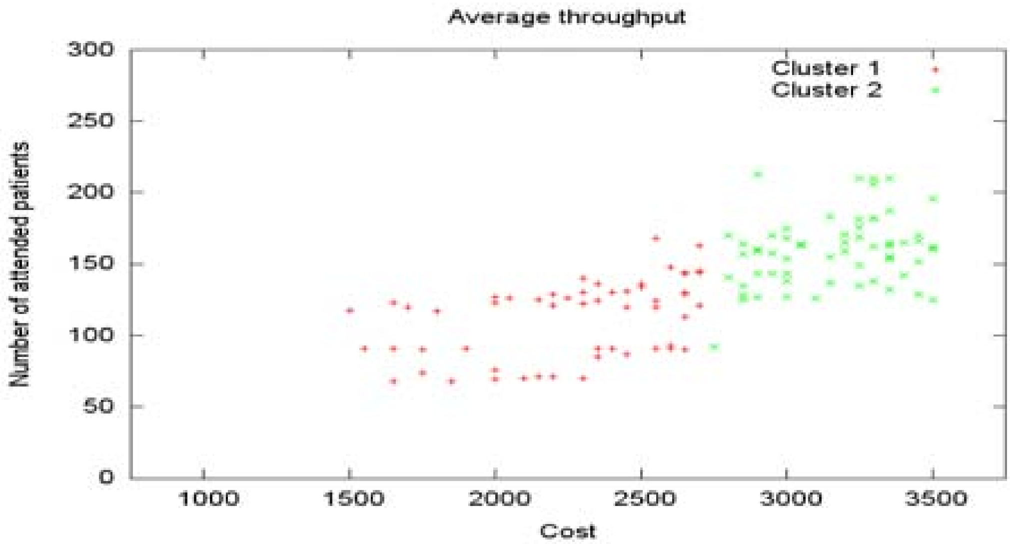


Figure 4.41: The K-means method identified two clusters of average number of attended patients. The green one delimited the region where the maximum was.

Finally, after separately applied both the PA and the MC plus the K-means methods, the "reduced exhaustive search" was separately performed in each reduced region identified. The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.20, where the sanitary staff configuration (doctors, nurses, and admission personnel), their associated average maximum Throughput, and cost configuration are shown. The three optima independently found were the same.

Table 4.20: Optimum staff configurations that got the average maximum Throughput for this workload scenario (up to 17 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in red triangle in Figure 4.38.

Method	€	#attended patients	D	N	A	Run time (hrs) 4 Pthreads
ES	3,400	221	4J	3J	1S,1J	3.43
PA	3,400	221	4J	3J	1S,1J	0.10
MC+K-means	3,400	221	4J	3J	1S,1J	2.06

4.7.3 CLoS Index

The third objective set was to minimise a compound index: *Cost* *LoS*, (CLoS), without any restriction, except the minimum and maximum number of admission personnel, nurses, and doctors stated in Table 4.2 to Table 4.6. This index is expressed mathematically in Equation 4.7.3:

$$\text{Minimise } CLoS \quad f(D, N, A) \tag{4.7.3}$$

As a consequence of not having any constraint, 1,134 ($14D * 9N * 9A$) staff configurations, that represent the whole search space, were tested for each of the four workload scenarios of incoming patients stated in Table 4.12. Each of the plotted points were obtained running the ED simulator as many times those points are, 1,134.

4.7.3.1 First Workload Scenario

The results of this scenario, up to 4 patients/hour, are shown in Figure 4.42 to Figure 4.45. The 3D scattered graphics shown the average minimum CLoS index by its colour.

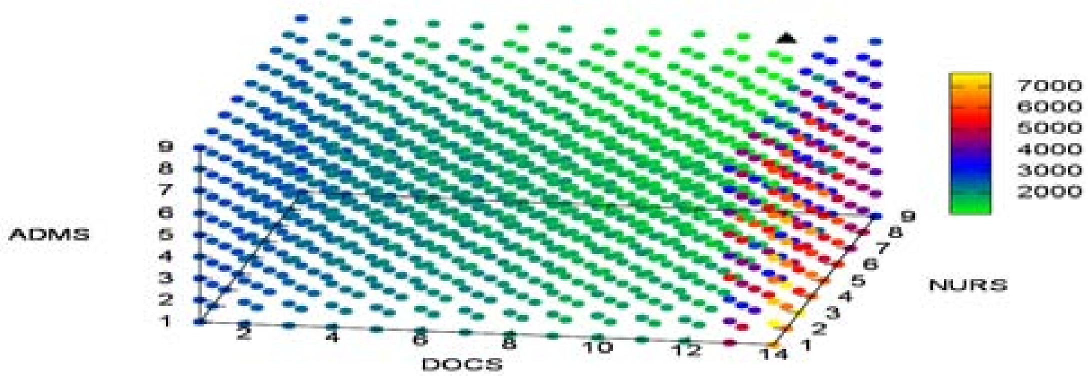


Figure 4.42: Average CLoS obtained by the ES method. The black triangle was the minimum. CLoS units are in thousands.

The green colour represents the value in the vicinity of the optimum, the lighter green colours were the most relevant. The ES result is shown in Figure 4.42, where the black triangle was the minimum.

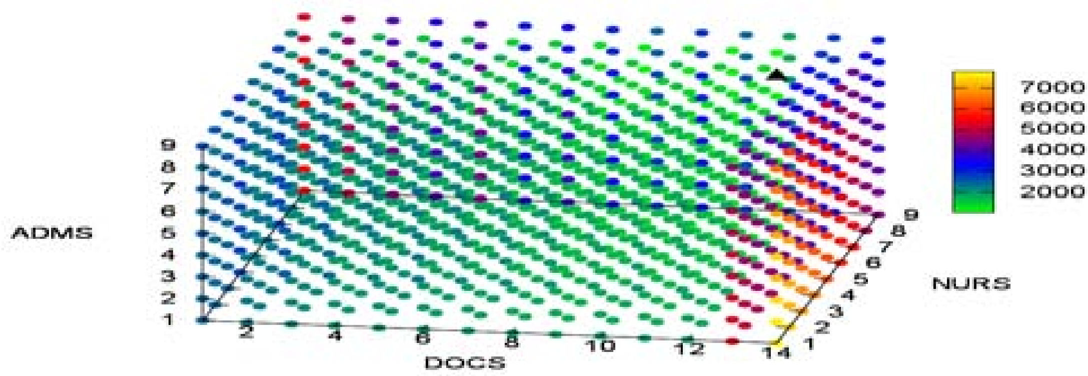


Figure 4.43: Average CLoS obtained by the PA. The black triangle was the minimum. CLoS units are in thousands.

The PA result is shown in Figure 4.43, where the black triangle was the minimum. Several regions can be seen. The most important was where the black triangle was. There were 316 configurations (from a total of 1,134) in this region, which is the one where the minimum was.

The MC plus the K-means methods results are shown in Figure 4.44 to Figure 4.45, respectively. The MC method found 75 configurations.

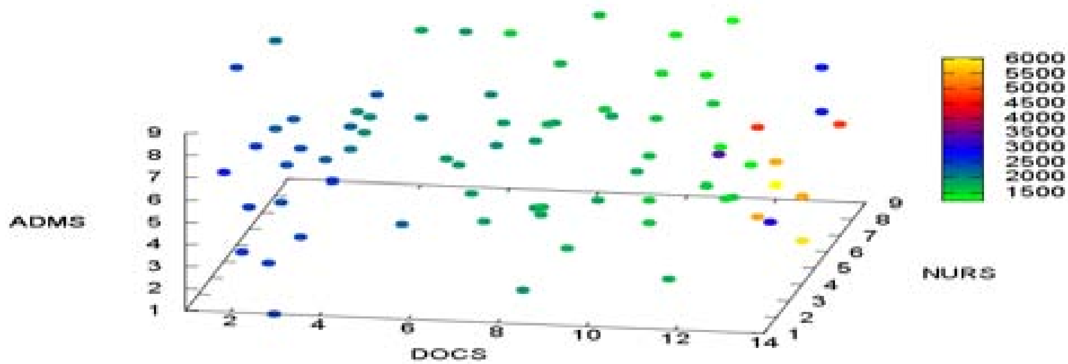


Figure 4.44: Average CLoS of 75 configurations obtained by the MC method. CLoS units are in thousands.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.45, the most important was the red cluster, which delimited the region where the optimum was.

Finally, after separately applied both the PA and the MC plus the K-means methods, the "reduced exhaustive search" was separately performed in each reduced region identified. The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.21, where the sanitary staff configuration (doctors, nurses, and admission personnel), their associated average minimum CLoS, and cost configuration are shown. The three optima independently found were the same.

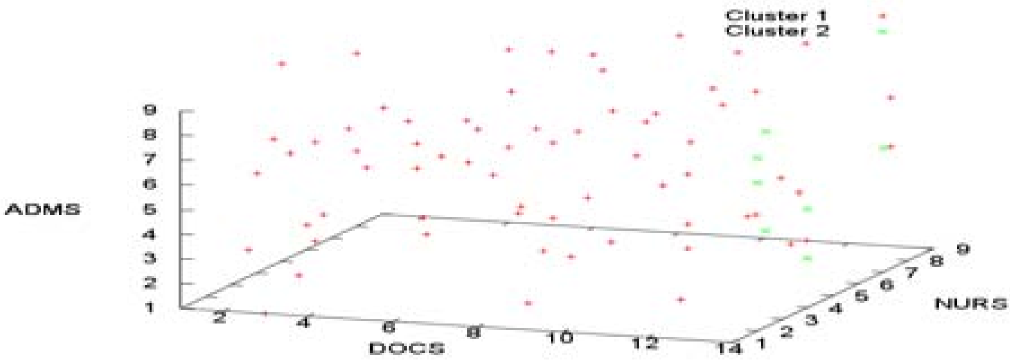


Figure 4.45: The K-means method identified two clusters of average CLoS. The red one delimited the region where the minimum was.

Table 4.21: Optimum staff configurations that got the average minimum CLoS for this workload scenario (up to 4 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in black triangle in Figure 4.42.

Method	€	CLoS	D	N	A	Run time (hrs) 32 Pthreads
ES	1,500	1.03^6	2J	1J	1J	0.46
PA	1,500	1.03^6	2J	1J	1J	0.13
MC+K-means	1,500	1.03^6	2J	1J	1J	0.35

4.7.3.2 Second Workload Scenario

The results of this scenario, up to 9 patients/hour, are shown in Figure 4.46 to Figure 4.49. The 3D scattered graphics shown the average minimum CLoS index by its colour. The green colour represents the value in the vicinity of the optimum. The ES result is shown in Figure 4.46, where the black triangle was the minimum.

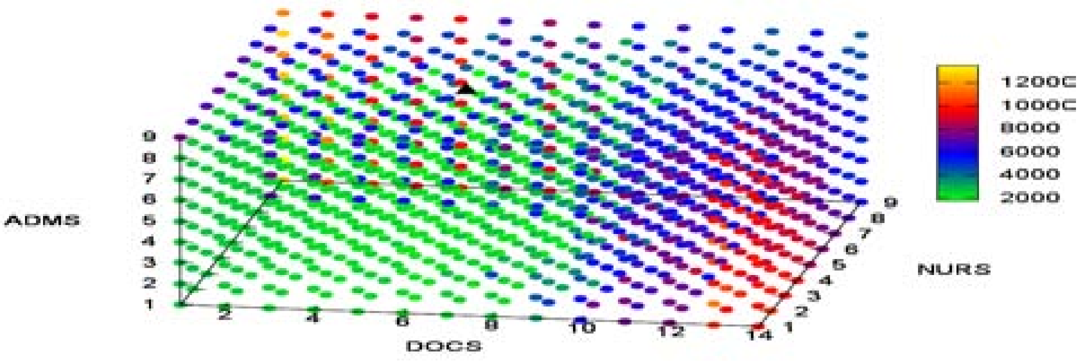


Figure 4.46: Average CLoS obtained by the ES method. The black triangle was the minimum. CLoS units are in thousands.

The PA result is shown in Figure 4.47, where the black triangle was the minimum. Several regions can be seen. The most important was where the black triangle was. There were 429 configurations (from a total of 1,134) in this region, which is the one where the minimum was.

The MC plus the K-means methods results are shown in Figure 4.48 to Figure 4.49, respectively. The MC method found 275 configurations. However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified three different clusters, shown in Figure 4.49, the most important was the red cluster, which delimited the region where the optimum was.

Finally, after separately applied both the PA and the MC plus the K-means methods, the “reduced exhaustive search” was separately performed in each reduced region identified.

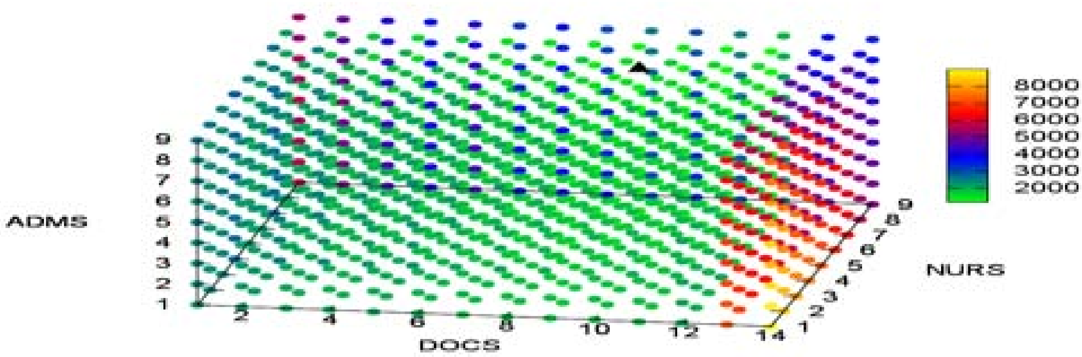


Figure 4.47: Average CLoS obtained by the PA. The black triangle was the minimum. CLoS units are in thousands.

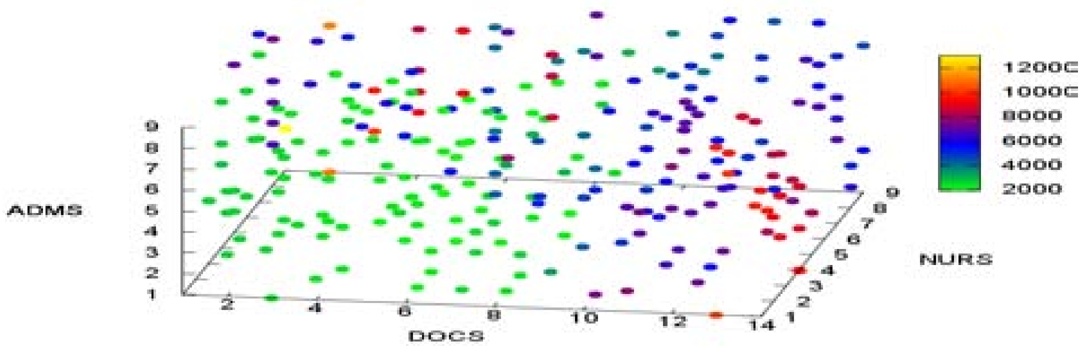


Figure 4.48: CLoS of 275 configurations obtained by the MC method. CLoS units are in thousands.

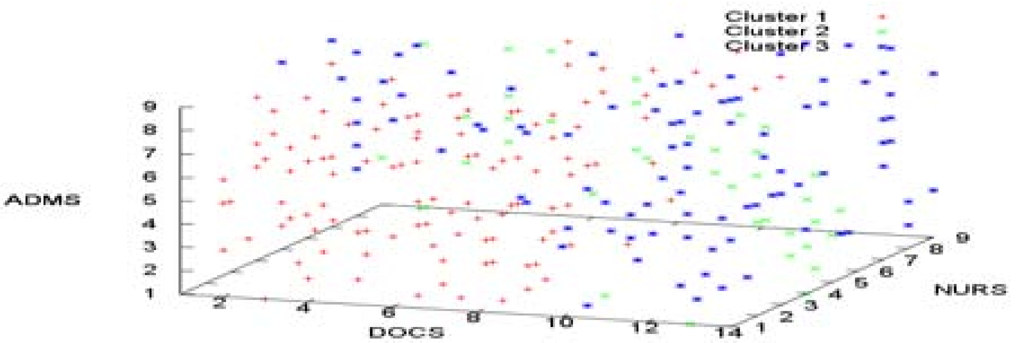


Figure 4.49: The K-means method identified three clusters of average CLoS. The red one delimited the region where the minimum was.

The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.22, where the sanitary staff configuration (doctors, nurses, and admission personnel), their associated average minimum CLoS, and cost configuration are shown. The three optima independently found were the same.

Table 4.22: Optimum staff configurations that got the average minimum CLoS for this workload scenario (up to 9 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in black triangle in Figure 4.46.

Method	€	CLoS	D	N	A	Run time (hrs) 32 Pthreads
ES	3,050	1.78^6	4J	1S,1J	1J	0.72
PA	3,050	1.78^6	4J	1S,1J	1J	0.27
MC+K-means	3,050	1.78^6	4J	1S,1J	1J	0.5

4.7.3.3 Third Workload Scenario

The results of this scenario, up to 13 patients/hour, are shown in Figure 4.50 to Figure 4.53.

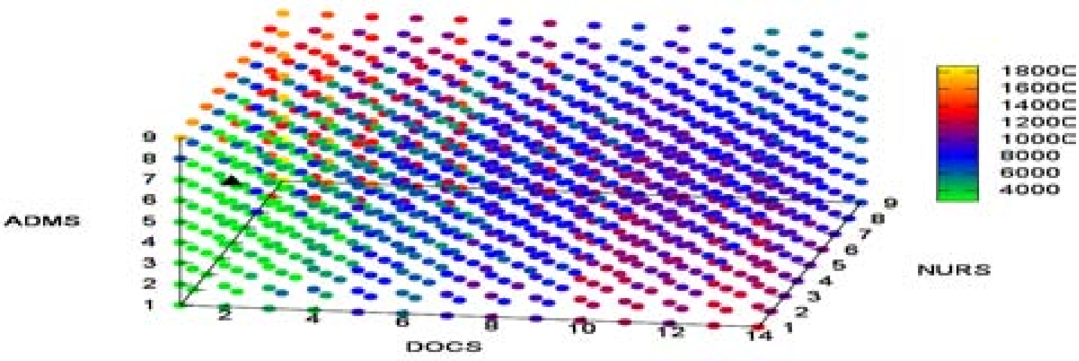


Figure 4.50: Average CLoS obtained by the ES method. The black triangle was the minimum. CLoS units are in thousands.

The 3D scattered graphics shown the average minimum CLoS index by its colour. The green colour represents the value in the vicinity of the optimum. The ES result is shown in Figure 4.50, where the black triangle was the minimum.

The PA result is shown in Figure 4.51, where the black triangle was the minimum. Several regions can be seen. The most important was where the black triangle was. There were 397 configurations (from a total of 1,134) in this region, which is the one where the minimum was.

The MC plus the K-means methods results are shown in Figure 4.52 to Figure 4.53, respectively. The MC method found 25 configurations. However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.53.

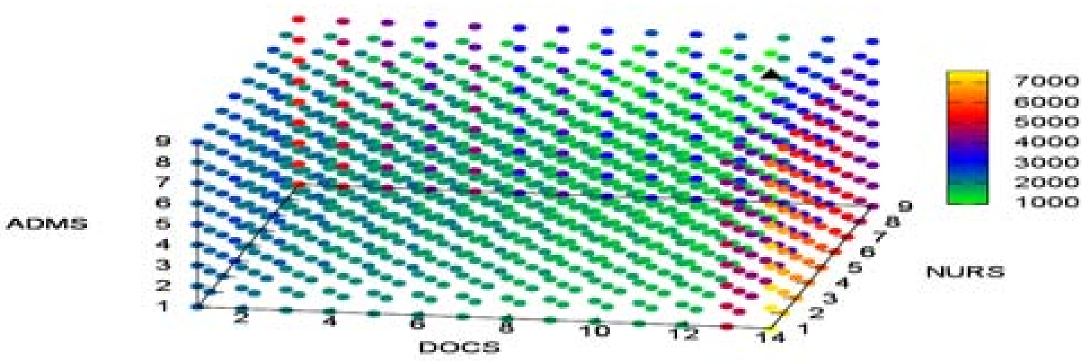


Figure 4.51: Average CLoS obtained by the PA. The black triangle was the minimum. CLoS units are in thousands.

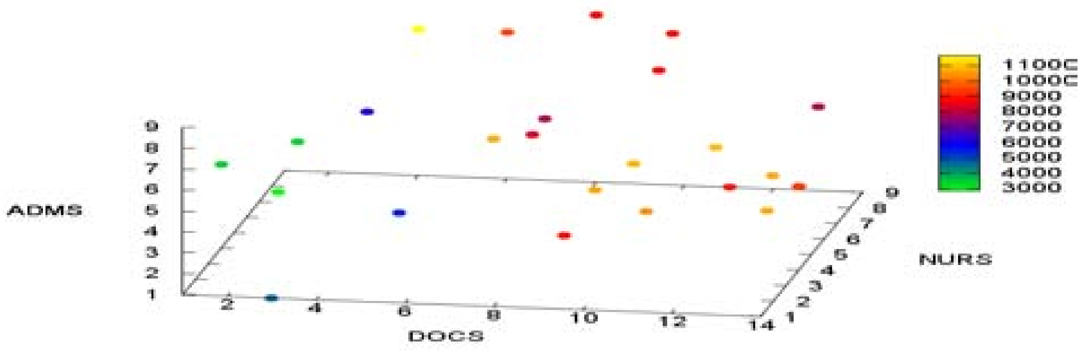


Figure 4.52: Average CLoS of 25 configurations obtained by the MC method. CLoS units are in thousands.

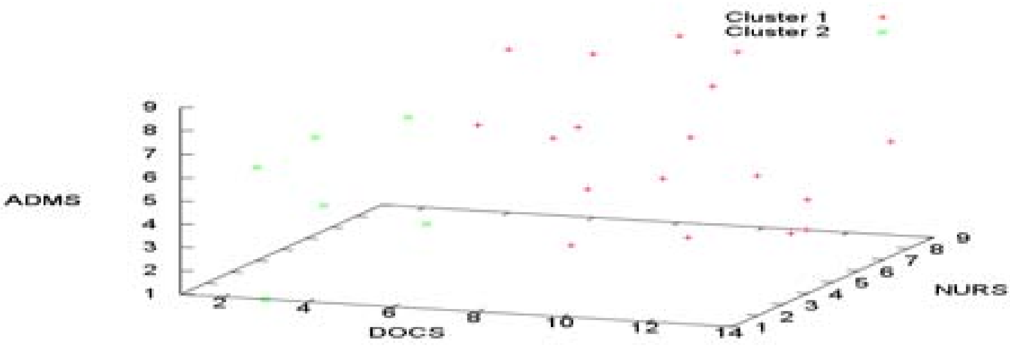


Figure 4.53: The K-means method identified two clusters of average CLoS. The green one delimited the region where the minimum was.

The most important was the green cluster, which delimited the region where the optimum was.

Finally, after separately applied both the PA and the MC plus the K-means methods, the "reduced exhaustive search" was separately performed in each reduced region identified. The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.23, where the sanitary staff configuration (doctors, nurses, and admission personnel), their associated average minimum CLoS, and cost configuration are shown. The three optima independently found were the same

Table 4.23: Optimum staff configurations that got the average minimum CLoS for this workload scenario (up to 13 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in black triangle in Figure 4.50.

Method	€	CLoS	D	N	A	Run time (hrs) 32 Pthreads
ES	5,400	2.72^6	4S	2S	2S	0.95
PA	5,400	2.72^6	4S	2S	2S	0.33
MC+K-means	5,400	2.72^6	4S	2S	2S	0.64

4.7.3.4 Fourth Workload Scenario

The results of this scenario, up to 17 patients/hour, are shown in Figure 4.54 to Figure 4.57. The 3D scattered graphics shown the average minimum CLoS index by its colour. The green colour represents the value in the vicinity of the optimum. The ES result is shown in Figure 4.54, where the black triangle was the minimum.

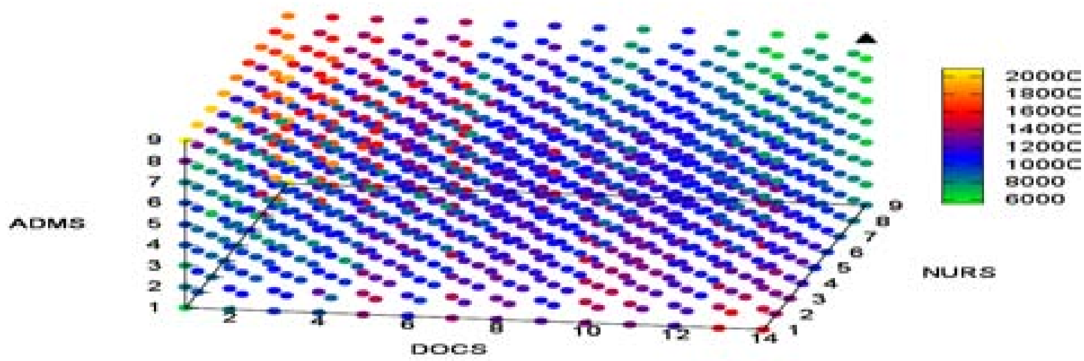


Figure 4.54: Average CLoS obtained by the ES method. The black triangle was the minimum. CLoS units are in thousands.

The PA result is shown in Figure 4.55, where the black triangle was the minimum. Several regions can be seen. The most important was where the black triangle was. There were 538 configurations (from a total of 1,134) in this region, which is the one where the minimum was.

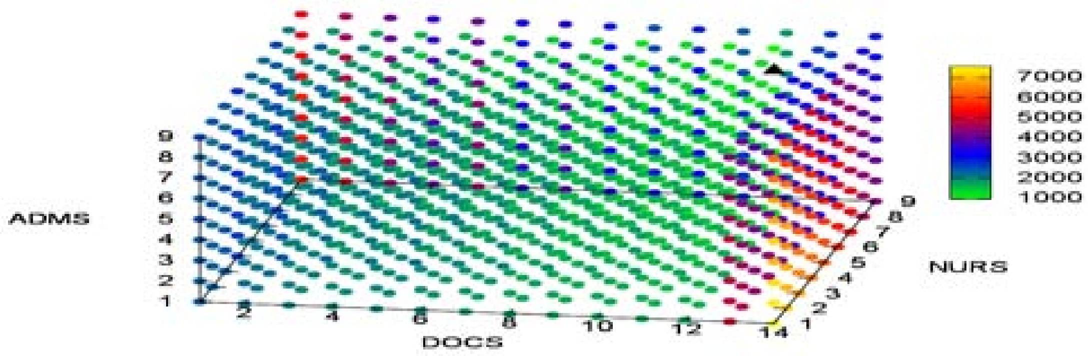


Figure 4.55: Average CLoS obtained by the PA. The black triangle was the minimum. CLoS units are in thousands.

The MC plus the K-means methods results are shown in Figure 4.54 to Figure 4.57, respectively. The MC method found 375 configurations.

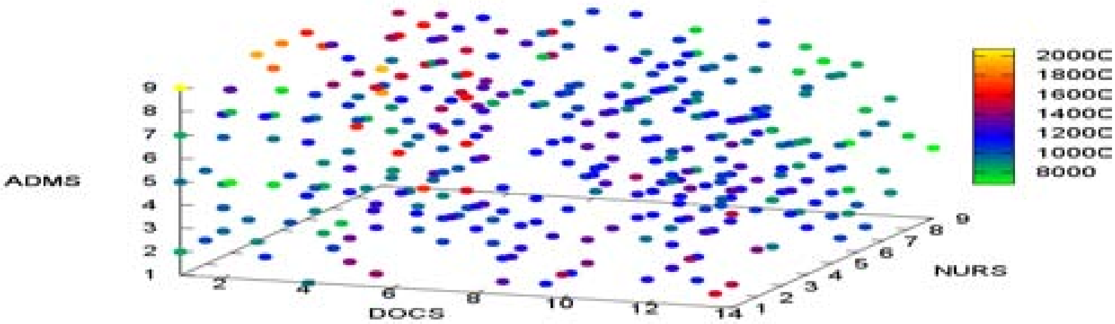


Figure 4.56: Average CLoS of 375 configurations obtained by the MC method. CLoS units are in thousands.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified three different clusters, shown in Figure 4.57, the most important was the green cluster, which delimited the region where the optimum was.

Finally, after separately applied both the PA and the MC plus the K-means methods, the "reduced exhaustive search" was separately performed in each reduced region identified. The optimum found per each method: the ES, the PA, and the MC plus the K-means methods are presented in Table 4.24, where the sanitary staff configuration (doctors, nurses, and admission personnel), their associated average minimum CLoS, and cost configuration are shown. The three optima independently found were the same

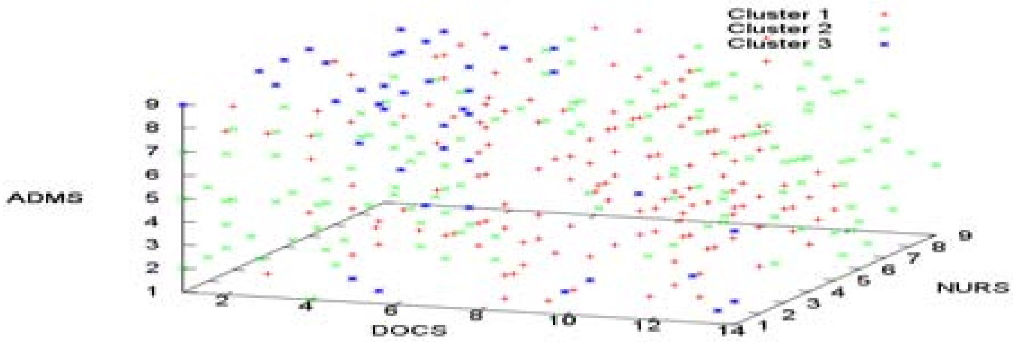


Figure 4.57: The K-means method identified three clusters of average CLoS. The green one delimited the region where the minimum was

Table 4.24: Optimum staff configurations that got the average minimum CLoS for this workload scenario (up to 17 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in black triangle in Figure 4.54.

Method	€	CLoS	D	N	A	Run time (hrs) 32 Pthreads
ES	1,000	5.59^6	1S	1S	1S	0.97
PA	1,000	5.59^6	1S	1S	1S	0.45
MC+K-means	1,000	5.59^6	1S	1S	1S	0.72

4.8 Case Study B

The agent-based ED simulator v1.2, which is shown in Figure 4.58, was used in this case study. In this version of the ED simulator the diagnosis and treatment

Figure 4.58: ED simulator v1.2. Admission personnel, triage nurses, doctors, emergency nurses, and x-ray technicians were the sanitary staff considered.

phase is more detailed, because the diagnosis tests were done by other agents, x-ray technicians or emergency nurses, or both. The simple patient flow in the such current v1.2 of the ED simulator is defined as follows: patients arrive to the ED on their own, and waits to be attended in the admission area. Then, patients stay in the first Waiting Room (WR) WR1, until a triage nurse call them. After the triage process patients identified as triage level 4 and triage level 5 pass to a second WR2, and stay there until a free doctor calls them to begin the diagnosis-treatment phase (interrogation process), depending on the patient's symptoms and physical condition, patients wait at WR (Level 1) to be attended by an x-ray technician or an emergency nurse (to perform some diagnostic tests). After tests, patients come back to WR (Level 1) and stay there until a free doctor calls them again (treatment process). At the end, patients are discharged from the ED.

It was found through interviews with the managers at the EDs of Sabadell hospital that the 100% of patients undergo interrogation phase; the 80% of patients required to perform some diagnostic tests; and only 20% need to apply a treatment. These percentages are common for both patients identified as triage level 4 and triage level 5.

Therefore, in this case study the sanitary staff considered were: admission personnel, triage nurses, emergency nurses, doctors, and x-ray technicians. It is a 5D problem. Thus, the Table 4.7 to Table 4.11 were taken into account. As a result, 28,350 ($14D * 9N * 9A * 5En * 5Xr$) staff configurations were tested for each of the four workload scenarios of incoming patients stated in Table 4.12.

Finally, the three metrics above stated: LoS, Throughput, and CLoS were performed using two methods: the exhaustive search technique (ES), and the coarse grained phase, using only the MC plus K-means methods. Finally, the fine grained phase is apply in the reduced feasible region to find the optimum.

4.8.1 LoS Index

This objective set was to minimise patient length of stay (LoS) in the ED, with cost configuration constraint less or equal to 5,500 €. This index is expressed mathematically in Equation 4.8.1:

$$\begin{aligned} &\text{Minimise LoS} && f(D, N, A, En, Xr) \\ &\text{subject to} && D_{cost} + N_{cost} + A_{cost} + En_{cost} + \\ & && Xr_{cost} \in Cost \leq 5,500\ € \end{aligned} \tag{4.8.1}$$

It is worth noting that each of the plotted points for the following four workload scenarios were obtained running the ED simulator as many times as points are. Each plotted point corresponds to each of the 23,445 staff configurations (out of 28,350) that satisfy the cost restriction.

4.8.1.1 First Workload Scenario

The results of this scenario, up to 4 patients/hour, are shown in Figure 4.59 to Figure 4.61. The ES result is shown in Figure 4.59, where the red triangle was the minimum.

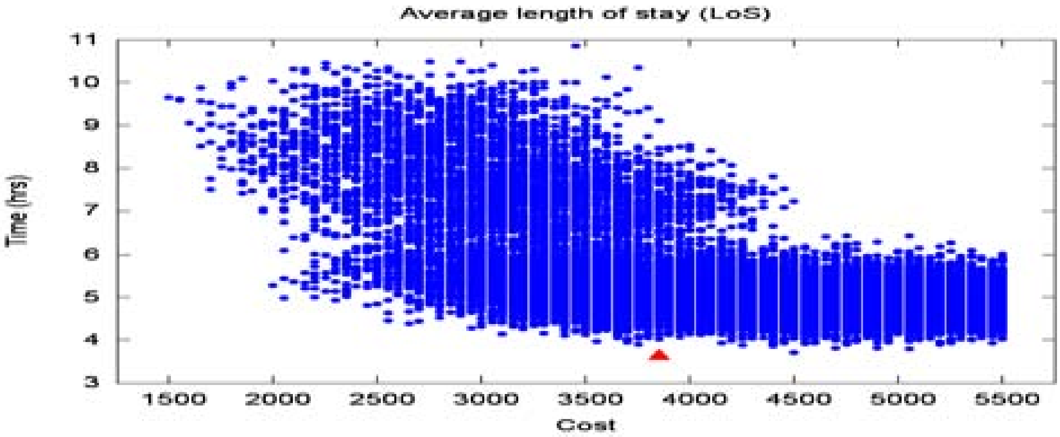


Figure 4.59: Average LoS obtained by the ES method. The red triangle was the minimum.

The MC plus the K-means methods results are shown in Figure 4.60 to Figure 4.61, respectively. The MC method found 600 configurations.

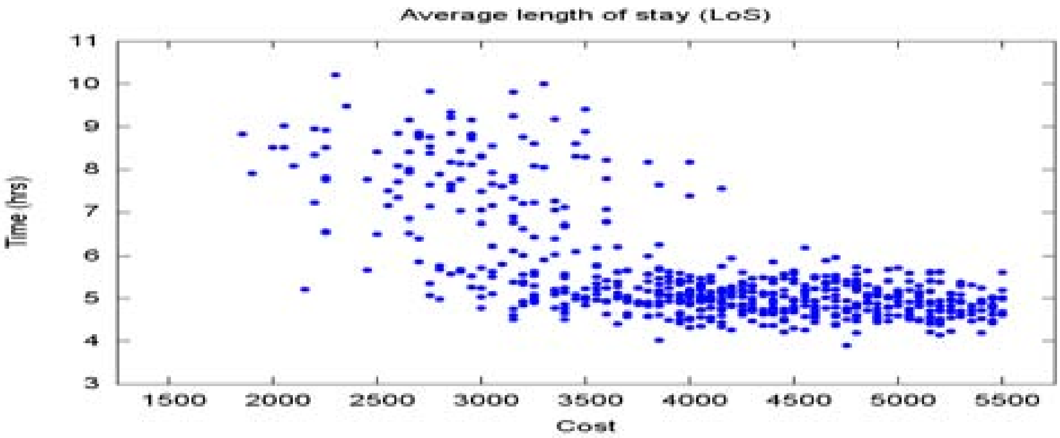


Figure 4.60: Average LoS of 600 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.61, the most important was the green cluster, which delimited the region where the optimum was.

Finally, after applied the MC plus the K-means methods, the "reduced exhaustive search" was performed in such reduced region identified. The optimum found per each method: the ES, and the MC plus the K-means methods are presented in Table 4.25, where the sanitary staff configuration (doctors, triage nurses, emergency nurses, x-ray technicians, and admission personnel), their associated average minimum LoS, and cost configuration are shown. The two optima independently found were the same.

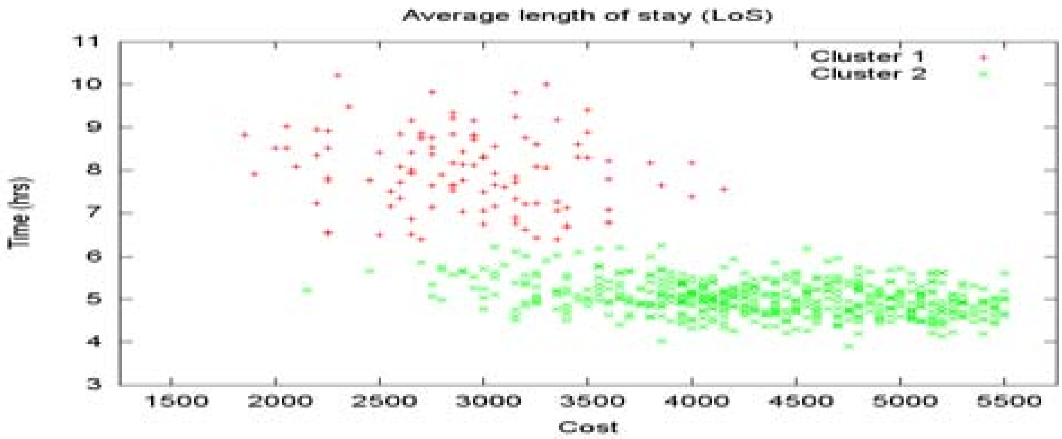


Figure 4.61: The K-means method identified two clusters of average LoS. The green one delimited the region where the minimum was..

Table 4.25: Optimum staff configurations that got the average minimum LoS for this workload scenario (up to 4 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in red triangle in Figure 4.59.

Method	€	LoS (hrs)	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	3,850	3.7	1S,3J	1J	1J	1S	1S,1J	2.5
MC+K-means	3,850	3.7	1S,3J	1J	1J	1S	1S,1J	0.84

4.8.1.2 Second Workload Scenario

The results of this scenario, up to 9 patients/hour, are shown in Figure 4.62 to Figure 4.64. The ES result is shown in Figure 4.62, where the red triangle was the minimum.

The MC plus the K-means methods results are shown in Figure 4.63 to Figure 4.64, respectively. The MC method found 150 configurations. However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.64, the most important was the red cluster, which delimited the region where the optimum was.

Finally, after applied the MC plus the K-means methods, the "reduced exhaustive search" was performed in such reduced region identified. The optimum found per each method: the ES, and the MC plus the K-means methods are presented in Table 4.26, where the sanitary staff configuration (doctors, triage nurses, emergency nurses, x-ray technicians, and admission personnel), their associated average minimum LoS, and cost configuration are shown. The two optima independently found were the same.

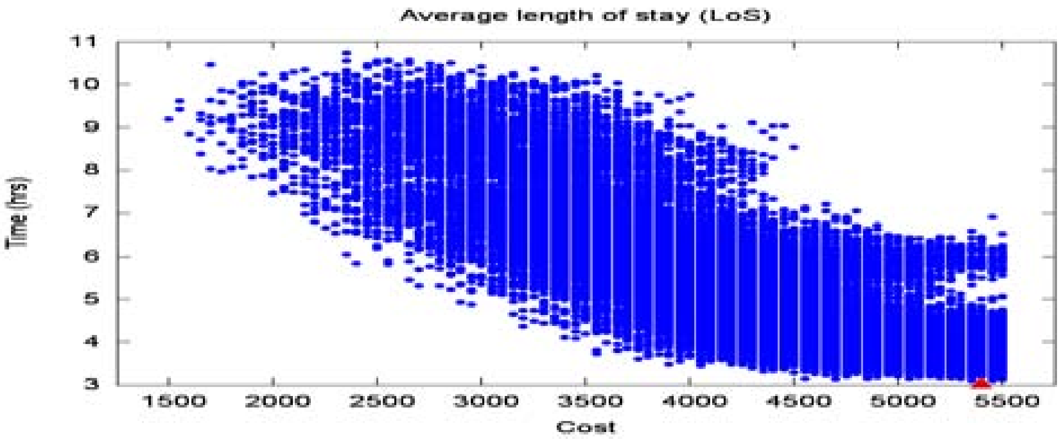


Figure 4.62: Average LoS obtained by the ES method. The red triangle was the minimum.

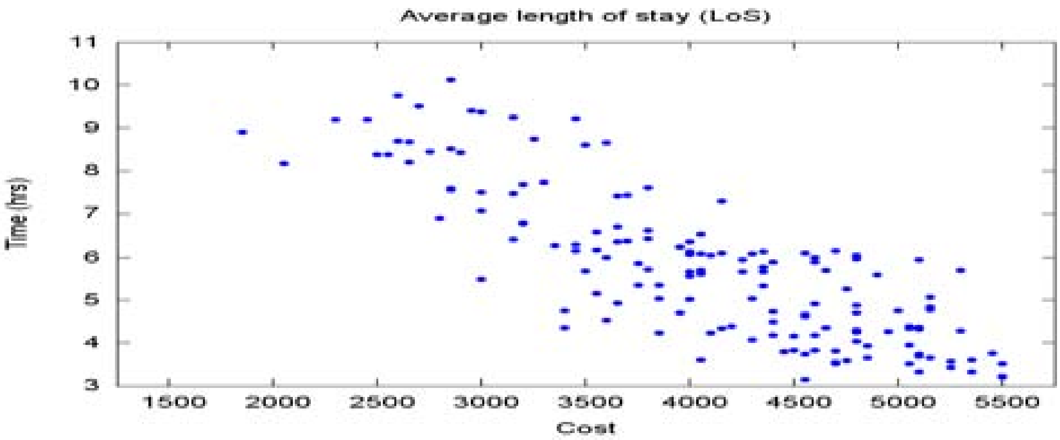


Figure 4.63: Average LoS of 150 configurations obtained by the MC method.

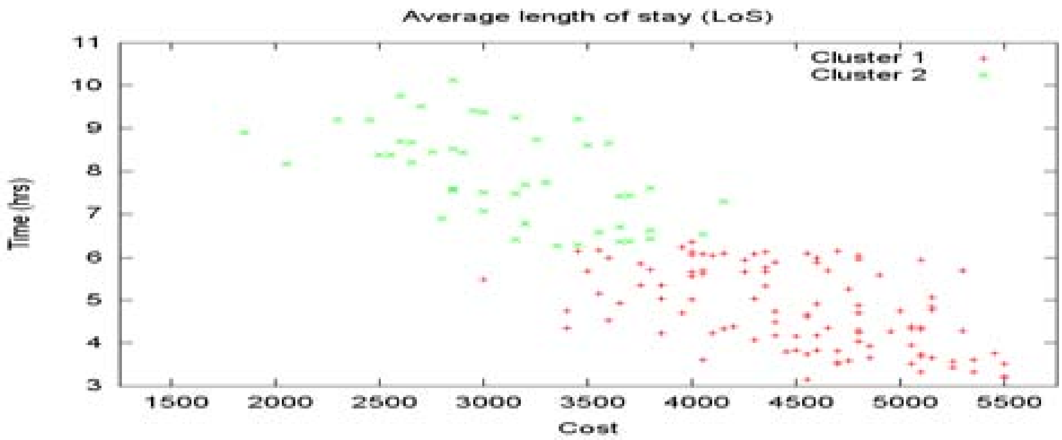


Figure 4.64: The K-means method identified two clusters of average LoS. The red one delimited the region where the minimum was.

Table 4.26: Optimum staff configurations that got the average minimum LoS for this workload scenario (up to 9 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in red triangle in Figure 4.62.

Method	€	LoS (hrs)	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	5,400	3.1	2S,2J	3J	1S,1J	1J	2J	3.07
MC+K-means	5,400	3.1	2S,2J	3J	1S,1J	1J	2J	0.57

4.8.1.3 Third Workload Scenario

The results of this scenario, up to 13 patients/hour, are shown in Figure 4.65 to Figure 4.67. The ES result is shown in Figure 4.65, where the red triangle was the minimum.

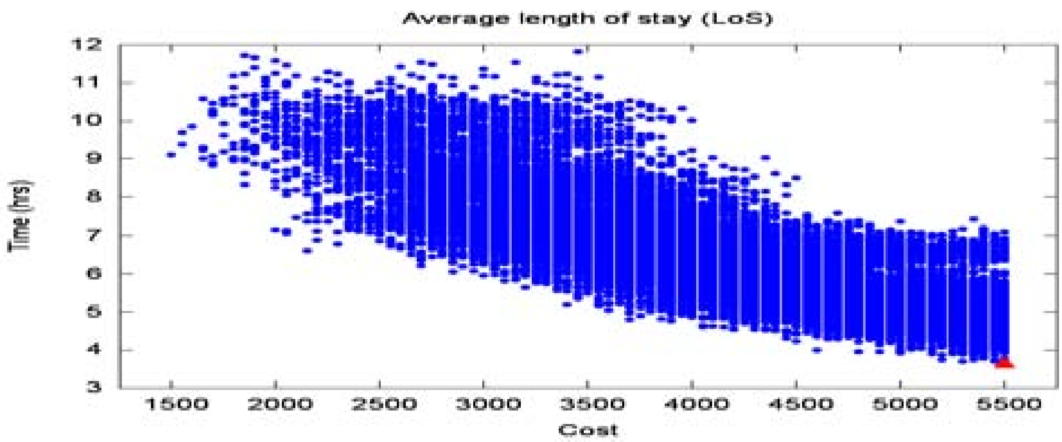


Figure 4.65: Average LoS obtained by the ES method. The red triangle was the minimum.

The MC plus the K-means methods results are shown in Figure 4.66 to Figure 4.67, respectively. The MC method found 150 configurations.

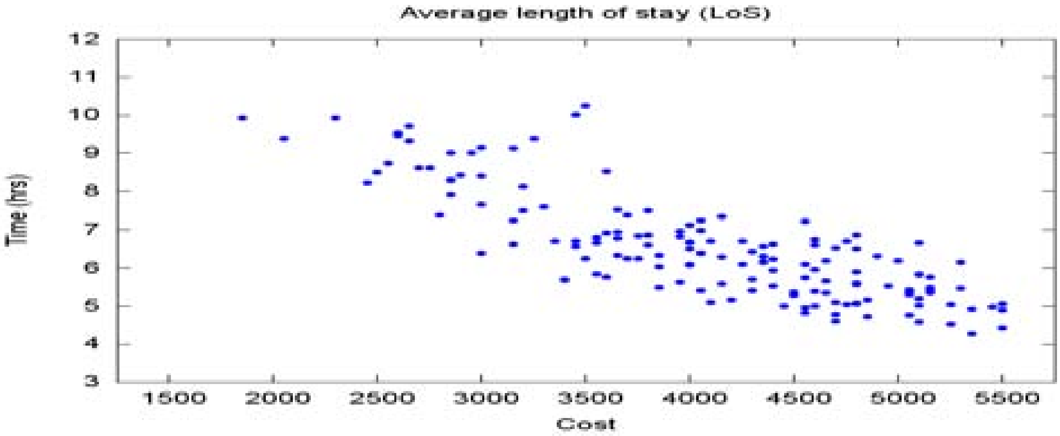


Figure 4.66: Average LoS of 150 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.67, the most important was the green cluster, which delimited the region where the optimum was.

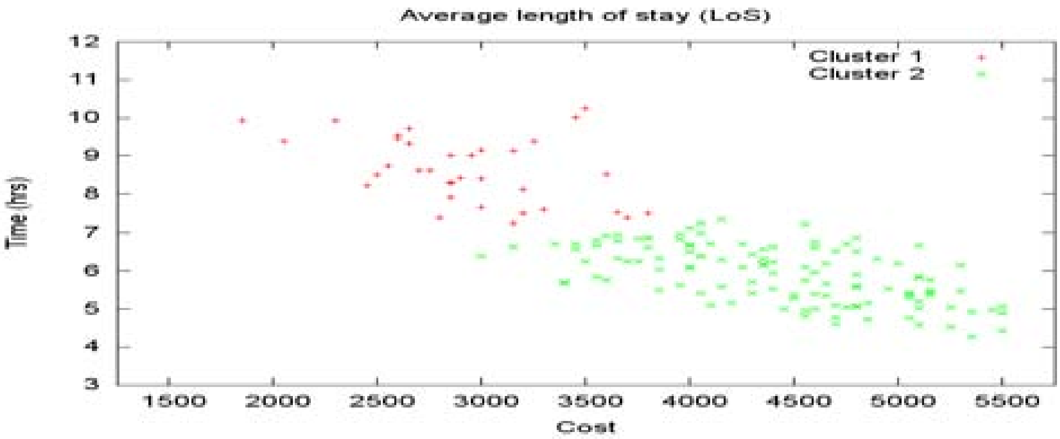


Figure 4.67: The K-means method identified two clusters of average LoS. The green one delimited the region where the minimum was.

Finally, after applied the MC plus the K-means methods, the "reduced exhaustive search" was performed in such reduced region identified. The optimum found per each method: the ES, and the MC plus the K-means methods are presented in Table 4.27, where the sanitary staff configuration (doctors, triage nurses, emergency nurses, x-ray technicians, and admission personnel), their associated average minimum LoS, and cost configuration are shown. The two optima independently found were the same.

Table 4.27: Optimum staff configurations that got the average minimum LoS for this workload scenario (up to 13 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in red triangle in Figure 4.65.

Method	€	LoS (hrs)	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	5,500	3.7	3S,1J	2S	1S,1J	1J	2J	3.72
MC+K-means	5,500	3.7	3S,1J	2S	1S,1J	1J	2J	0.56

4.8.1.4 Fourth Workload Scenario

The results of this scenario, up to 17 patients/hour, are shown in Figure 4.68 to Figure 4.70. The ES result is shown in Figure 4.68, where the red triangle was the minimum.

The MC plus the K-means methods results are shown in Figure 4.69 to Figure 4.70, respectively. The MC method found 150 configurations. However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.70, the most important was the red cluster, which delimited the region where the optimum was.

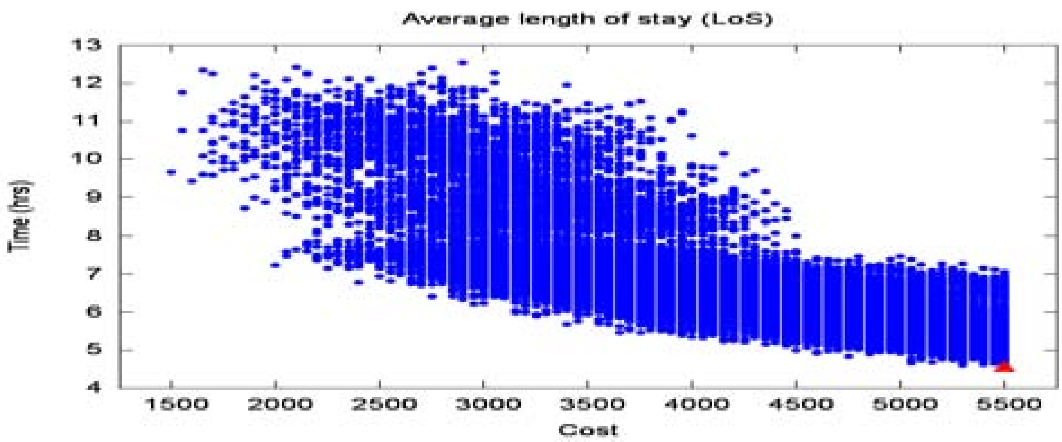


Figure 4.68: Average LoS obtained by the ES method. The red triangle was the minimum.

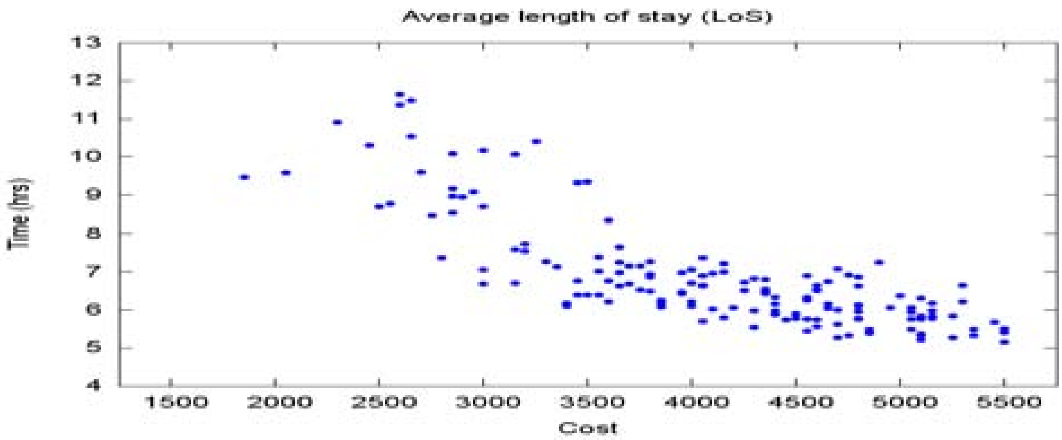


Figure 4.69: Average LoS of 150 configurations obtained by the MC method.

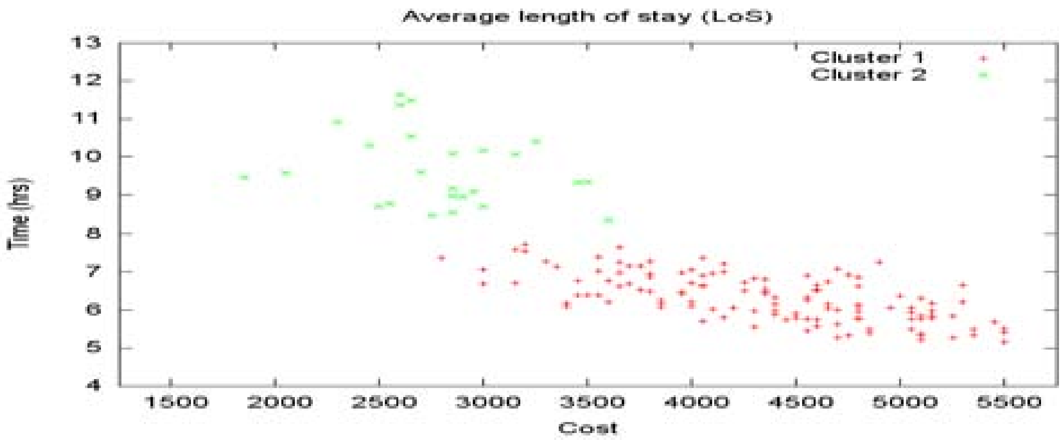


Figure 4.70: The K-means method identified two clusters of average LoS. The red one delimited the region where the minimum was.

Finally, after applied the MC plus the K-means methods, the "reduced exhaustive search" was performed in such reduced region identified. The optimum found per each method: the ES, and the MC plus the K-means methods are presented in Table 4.28, where the sanitary staff configuration (doctors, triage nurses, emergency nurses, x-ray technicians, and admission personnel), their associated average minimum LoS, and cost configuration are shown. The two optima independently found were the same.

Table 4.28: Optimum staff configurations that got the average minimum LoS for this workload scenario (up to 17 patients hourly), where S is Senior and J is Junior. This optimum sanitary staff configuration is shown in red triangle in Figure 4.68.

Method	€	LoS (hrs)	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	5,500	4.6	3S,1J	3J	2J	1S	1J	4.42
MC+K-means	5,500	4.6	3S,1J	3J	2J	1S	1J	0.62

4.8.2 Throughput Index

This objective set was to maximise number of attended patients per day (Throughput) in the ED, with cost configuration constraint less or equal to 5,500 €. This index is expressed mathematically in Equation 4.8.2:

$$\begin{aligned} &\text{Maximise patients attended} && f(D, N, A, En, Xr) \\ &\text{subject to} && D_{cost} + N_{cost} + A_{cost} + En_{cost} + \\ & && Xr_{cost} \in Cost \leq 5,500 \text{ €} \end{aligned} \tag{4.8.2}$$

It is worth noting that each of the plotted points for the following four workload scenarios were obtained running the ED simulator as many times as points are. Each plotted point corresponds to each of the 23,445 staff configurations (out of 28,350) that satisfy the cost restriction.

4.8.2.1 First Workload Scenario

The results of this scenario, up to 4 patients/hour, are shown in Figure 4.71 to Figure 4.73. The ES result is shown in Figure 4.71, where the red triangles were the maximum.

The MC plus the K-means methods results are shown in Figure 4.72 to Figure 4.73, respectively. The MC method found 1,325 configurations. However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.73, the most important was the green cluster, which delimited the region where the optimum were.

Finally, after applied the MC plus the K-means methods, the “reduced exhaustive search” was performed in such reduced region identified. The optimum found per each method: the ES and the MC plus the K-means methods are presented in Table 4.29, where the sanitary staff configuration (doctors, triage nurses, emergency nurses, x-ray technicians, and admission personnel), their associated average maximum Throughput, and cost configuration are shown. The two optima independently found were the same.

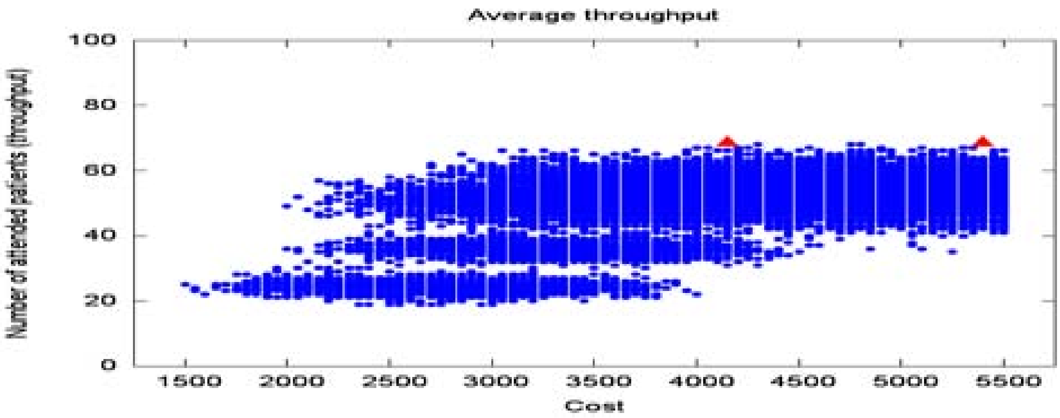


Figure 4.71: Average number of attended patients obtained by the ES method. The red triangles were the maxima.

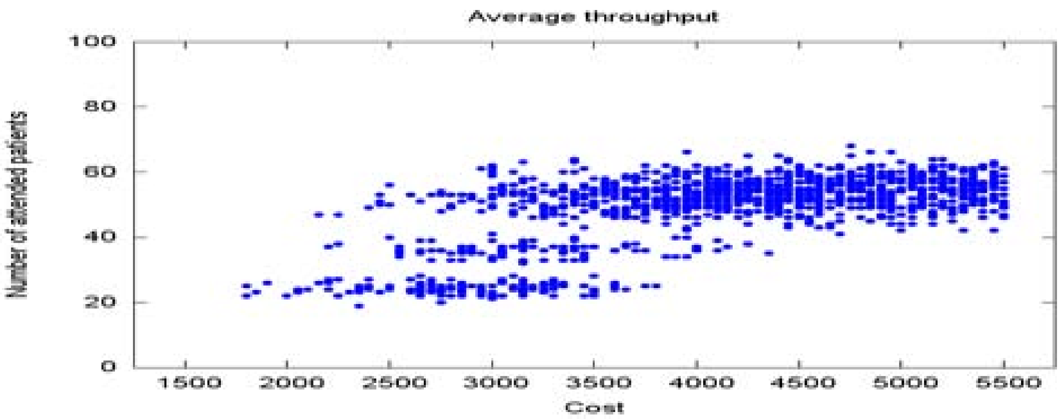


Figure 4.72: Average number of attended patients of 1,325 configurations obtained by the MC method.

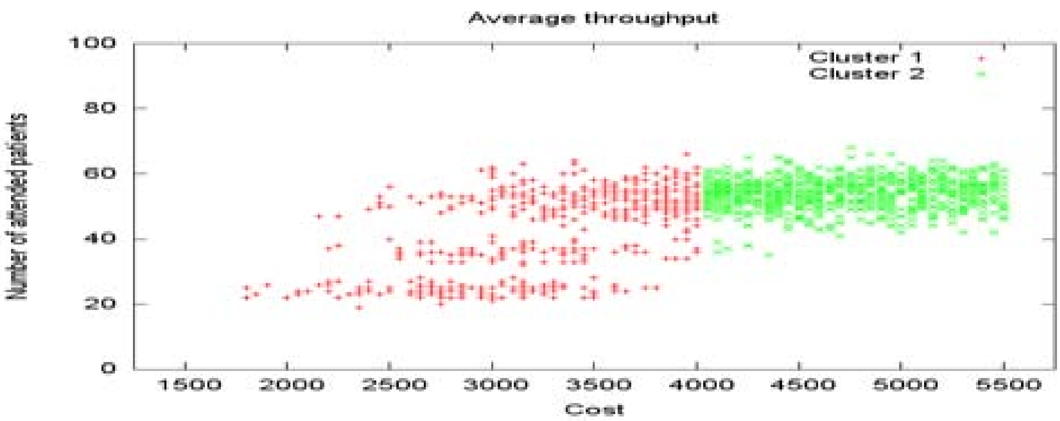


Figure 4.73: The K-means method identified two clusters of average number of attended patients. The green one delimited the region where the maxima were.

Table 4.29: Optimum staff configurations that got the average maximum Throughput for this workload scenario (up to 4 patients hourly), where S is Senior and J is Junior. These optimum sanitary staff configurations are shown in red triangle in Figure 4.71.

Method	€	#attended patients	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	5,400	69	3S	2S	1S,2J	1S	2S	2.54
MC+K-means	5,400	69	3S	2S	1S,2J	1S	2S	0.79

4.8.2.2 Second Workload Scenario

The results of this scenario, up to 9 patients/hour, are shown in Figure 4.74 to Figure 4.76. The ES result is shown in Figure 4.74, where the red triangle was the maximum.

The MC plus the K-means methods results are shown in Figure 4.75 to Figure 4.76, respectively. The MC method found 1,350 configurations. However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.76, the most important was the green cluster, which delimited the region where the optimum was.

Finally, after applied the MC plus the K-means methods, the "reduced exhaustive search" was performed in such reduced region identified. The optimum found per each method: the ES and the MC plus the K-means methods are presented in Table 4.30, where the sanitary staff configuration (doctors, triage nurses, emergency nurses, x-ray technicians, and admission personnel), their associated average maximum Throughput, and cost configuration are shown. The two optima independently found were the same.

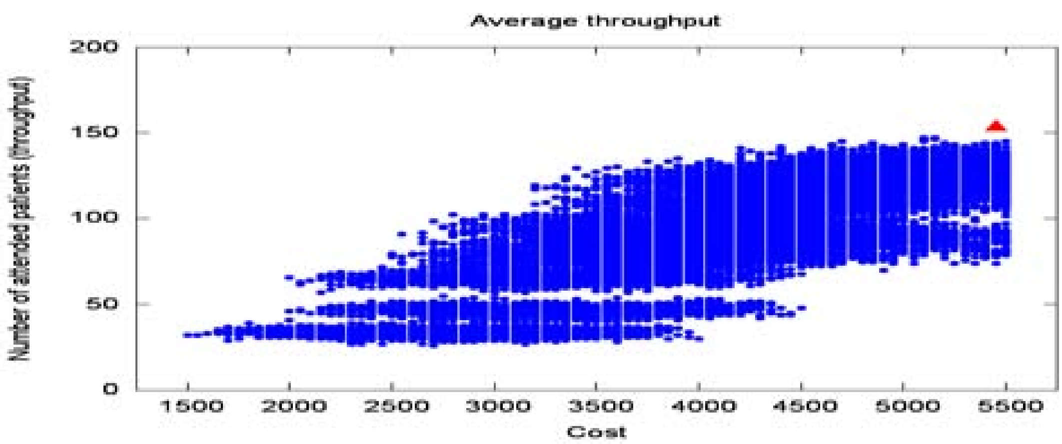


Figure 4.74: Average number of attended patients obtained by the ES method. The red triangle was the maximum.

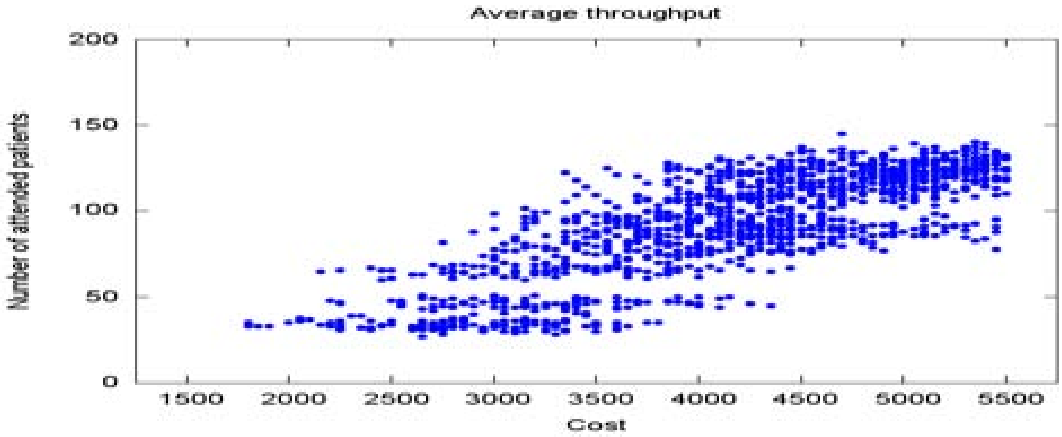


Figure 4.75: Average number of attended patients of 1,350 configurations obtained by the MC method.

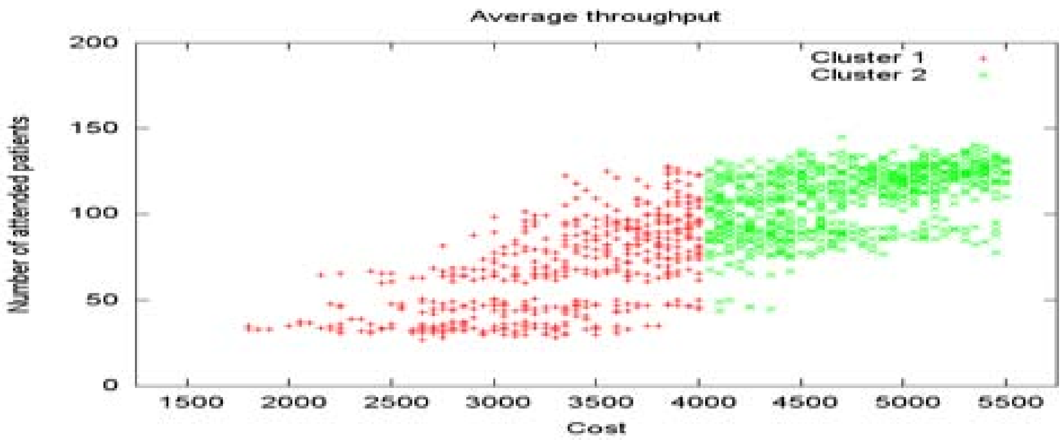


Figure 4.76: The K-means method identified two clusters of average number of attended patients. The green one delimited the region where the maximum was.

Table 4.30: Optimum staff configurations that got the average maximum Throughput for this workload scenario (up to 9 patients hourly), where S is Senior and J is Junior. These optimum sanitary staff configurations are shown in red triangle in Figure 4.74.

Method	€	#attended patients	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	5,450	154	2S,2J	2S	2S	1S,1J	1S	3.07
MC+K-means	5,450	154	2S,2J	2S	2S	1S,1J	1S	0.7

4.8.2.3 Third Workload Scenario

The results of this scenario, up to 13 patients/hour, are shown in Figure 4.77 to Figure 4.79. The ES result is shown in Figure 4.77, where the red triangle was the maximum.

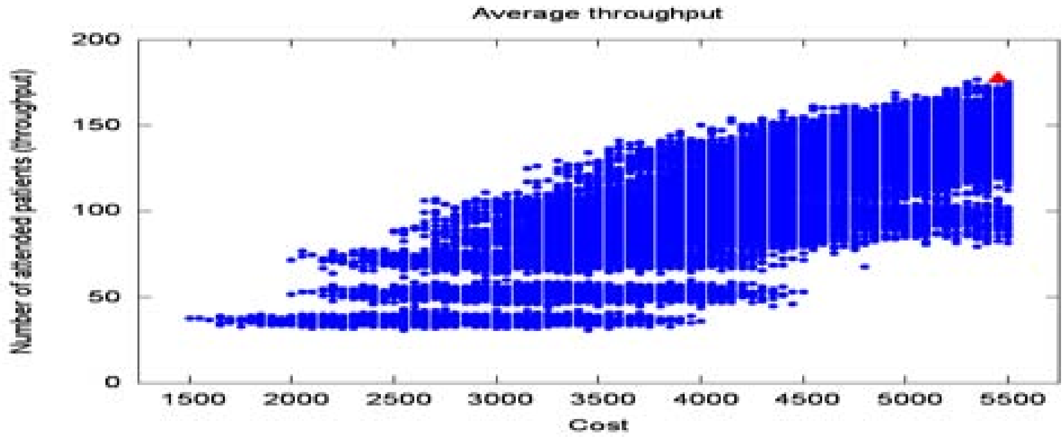


Figure 4.77: Average number of attended patients obtained by the ES method. The red triangle was the maximum.

The MC plus the K-means methods results are shown in Figure 4.78 to Figure 4.79, respectively. The MC method found 1,100 configurations.

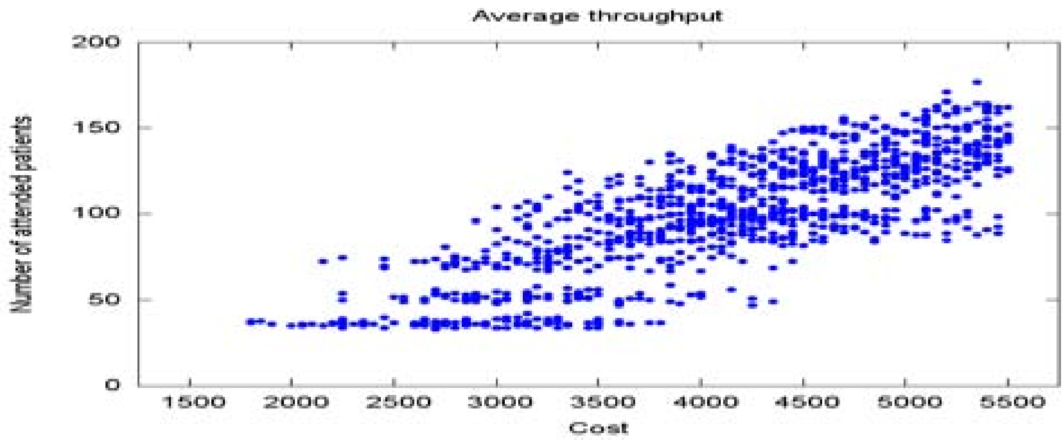


Figure 4.78: Average number of attended patients of 1,110 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.79, the most important was the green cluster, which delimited the region where the optimum was.

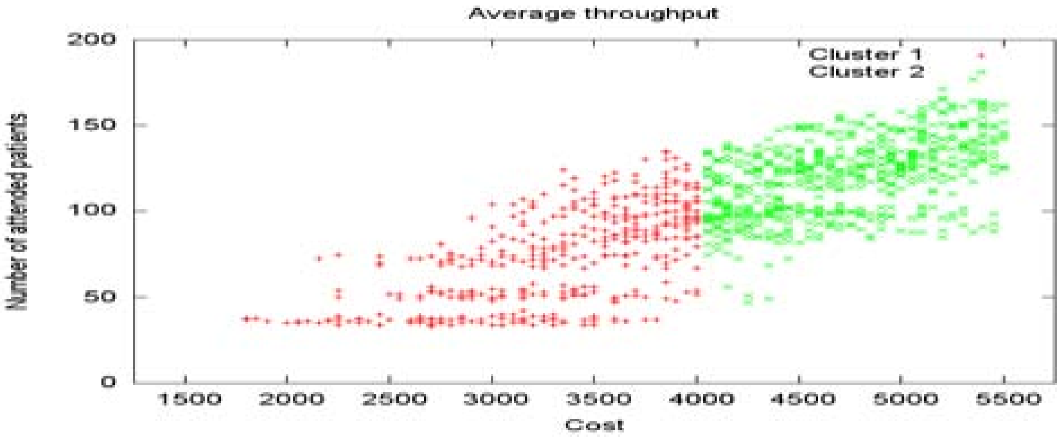


Figure 4.79: The K-means method identified two clusters of average number of attended patients. The green one delimited the region where the maximum was.

Finally, after applied the MC plus the K-means methods, the "reduced exhaustive search" was performed in such reduced region identified. The optimum found per each method: the ES and the MC plus the K-means methods are presented in Table 4.31, where the sanitary staff configuration (doctors, triage nurses, emergency nurses, x-ray technicians, and admission personnel), their associated average maximum Throughput, and cost configuration are shown. The two optima independently found were the same.

Table 4.31: Optimum staff configurations that got the average maximum Throughput for this workload scenario (up to 13 patients hourly), where S is Senior and J is Junior. These optimum sanitary staff configurations are shown in red triangle in Figure 4.77.

Method	€	#attended patients	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	5,450	178	3S,1J	2S	2S	1S,1J	1S	3.7
MC+K-means	5,450	178	3S,1J	2S	2S	1S,1J	1S	0.85

4.8.2.4 Four Workload Scenario

The results of this scenario, up to 17 patients/hour, are shown in Figure 4.80 to Figure 4.82. The ES result is shown in Figure 4.80, where the red triangle was the maximum.

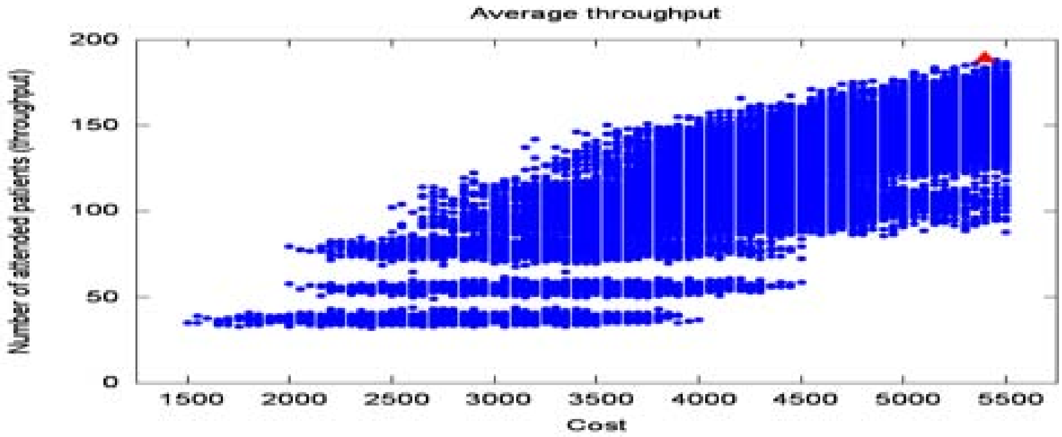


Figure 4.80: Average number of attended patients obtained by the ES method. The red triangle was the maximum.

The MC plus the K-means methods results are shown in Figure 4.81 to Figure 4.82, respectively. The MC method found 1,225 configurations.

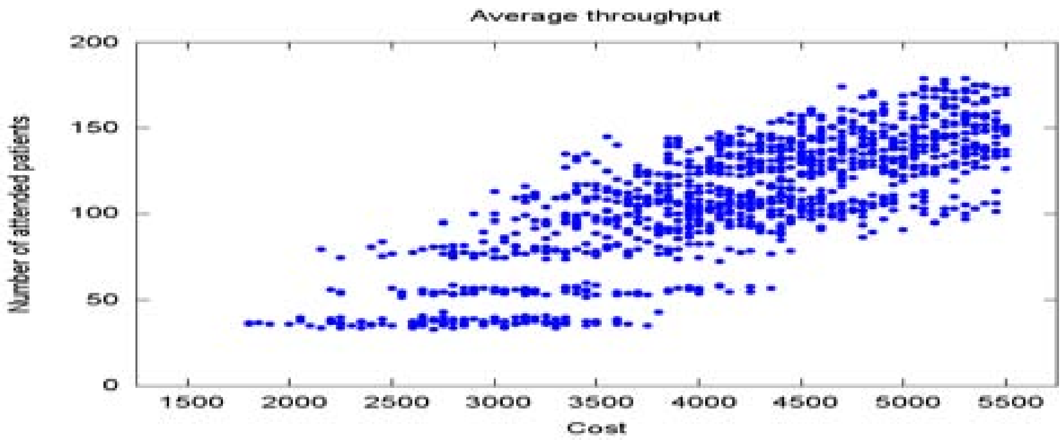


Figure 4.81: Average number of attended patients of 1,225 configurations obtained by the MC method.

However, it was difficult to get any conclusion about such region; therefore, the complementary K-means method was performed. The K-means method identified two different clusters, shown in Figure 4.82, the most important was the green cluster, which delimited the region where the optimum was.

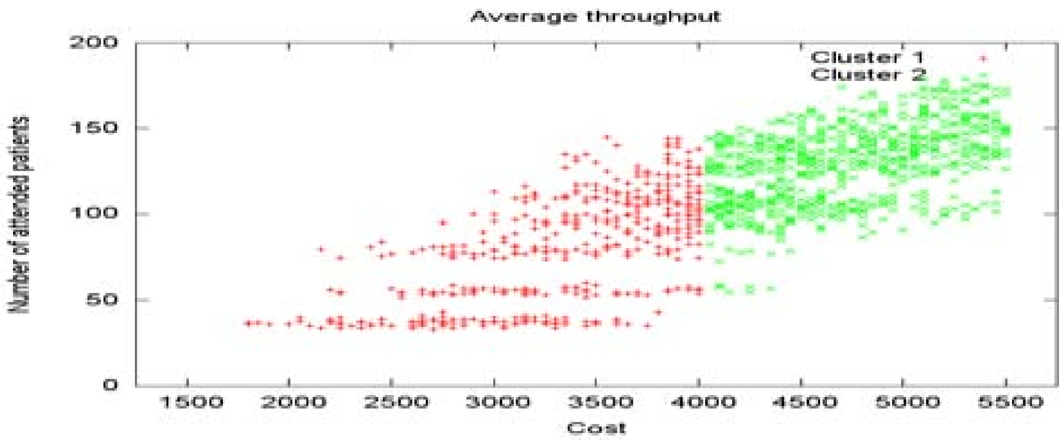


Figure 4.82: The K-means method identified two clusters of average number of attended patients. The green one delimited the region where the maximum was.

Finally, after applied the MC plus the K-means methods, the "reduced exhaustive search" was performed in such reduced region identified. The optimum found per each method: the ES and the MC plus the K-means methods are presented in Table 4.32, where the sanitary staff configuration (doctors, triage nurses, emergency nurses, x-ray technicians, and admission personnel), their associated average maximum Throughput, and cost configuration are shown. The two optima independently found were the same.

Table 4.32: Optimum staff configurations that got the average maximum Throughput for this workload scenario (up to 17 patients hourly), where S is Senior and J is Junior. These optimum sanitary staff configurations are shown in red triangle in Figure 4.80.

Method	€	#attended patients	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	5,400	190	3S,1J	1S,1J	2S,1J	1J	1J	4.41
MC+K-means	5,400	190	3S,1J	1S,1J	2S,1J	1J	1J	0.68

4.8.3 CLoS Index

This objective set was to minimise a compound index: $Cost \quad LoS$, (CLoS) in the ED, without any restriction, except the minimum and maximum number of admission personnel, nurses, doctors, emergency nurses, and x-ray technicians stated in Tabellen 4.7 bis 4.11. This index is expressed mathematically in Equation 4.8.3:

$$\text{Minimise } CLoS \quad f(D, N, A, En, Xr) \tag{4.8.3}$$

As a consequence of not having any constraint, 28,350 ($14D{*}9N{*}9A{*}5En{*}5Xr$) staff configurations, that represent the whole search space, were tested for each of the four workload scenarios of incoming patients stated in Table 4.12. The results are only shown in the following Tables.

4.8.3.1 First Workload Scenario

The optimum found per each method: the ES and the MC plus the K-means methods, after applied the "reduced exhaustive search" in the promising region identified, are presented in Table 4.33, where the sanitary staff configuration (doctors, triage nurses, admission personnel, emergency nurses, and x-ray technicians), their associated average minimum CLoS, and cost configuration are shown. The two optimum independently found were the same.

Table 4.33: Optimum staff configurations that got the average minimum CLoS for this workload scenario (up to 4 patients hourly), where S is Senior and J is Junior.

Method	€	CLoS	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	2,050	1.02^7	2S	1J	1J	1J	1S	3.73
MC+K-means	2,050	1.02^7	2S	1J	1J	1J	1S	1.08

4.8.3.2 Second Workload Scenario

The optimum found per each method: the ES and the MC plus the K-means methods, after applied the "reduced exhaustive search" in the promising region identified, are presented in Table 4.34, where the sanitary staff configuration (doctors, triage nurses, admission personnel, emergency nurses, and x-ray technicians), their associated average minimum CLoS, and cost configuration are shown. The two optimum independently found were the same.

4.8.3.3 Third Workload Scenario

The optimum found per each method: the ES and the MC plus the K-means methods, after applied the "reduced exhaustive search" in the promising region

Table 4.34: Optimum staff configurations that got the average minimum CLoS for this workload scenario (up to 9 patients hourly), where S is Senior and J is Junior.

Method	€	CLoS	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	3,550	1.32[7]	4J	2J	1S	1S	1J	4.35
MC+K-means	3,550	1.32[7]	4J	2J	1S	1S	1J	2.2

identified, are presented in Table 4.35, where the sanitary staff configuration (doctors, triage nurses, admission personnel, emergency nurses, and x-ray technicians), their associated average minimum CLoS, and cost configuration are shown. The two optimum independently found were the same.

Table 4.35: Optimum staff configurations that got the average minimum CLoS for this workload scenario (up to 13 patients hourly), where S is Senior and J is Junior.

Method	€	CLoS	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	1,500	1.37[7]	1J	1J	1J	1J	1J	5.1
MC+K-means	1,500	1.37[7]	1J	1J	1J	1J	1J	2.7

4.8.3.4 Fourth Workload Scenario

The optimum found per each method: the ES and the MC plus the K-means methods, after applied the "reduced exhaustive search" in the promising region identified, are presented in Table 4.36, where the sanitary staff configuration (doctors, triage nurses, admission personnel, emergency nurses, and x-ray technicians), their associated average minimum CLoS, and cost configuration are shown. The two optimum independently found were the same.

Table 4.36: Optimum staff configurations that got the average minimum CLoS for this workload scenario (up to 17 patients hourly), where S is Senior and J is Junior.

Method	€	CLoS	D	N	A	EN	XR	Run time (hrs) 32 Pthreads)
ES	1,500	1.45[7]	1J	1J	1J	1J	1J	5.98
MC+K-means	1,500	1.45[7]	1J	1J	1J	1J	1J	3.03

Before finishing this chapter, it is worth reminding that one of the aims of this research is to help emergency department managers in setting up strategies and management guidelines to enhance the performance of such critical system. Therefore, we must be aware not only in finding the optimum solution, but also the type of such optimum. Recovering from the first index of case study A: LoS for up to 4 incoming patient and up to 17 incoming patients. It can be noted in Figure 4.83 and Figure 4.84. In the first figure there are plenty of sanitary staff configurations nearby where the optimum were (inside the red ellipse). Thus, a sub-optimum solution would be a good solution, because choosing a sanitary staff configuration cheaper than the optimum the LoS is approximately the same.

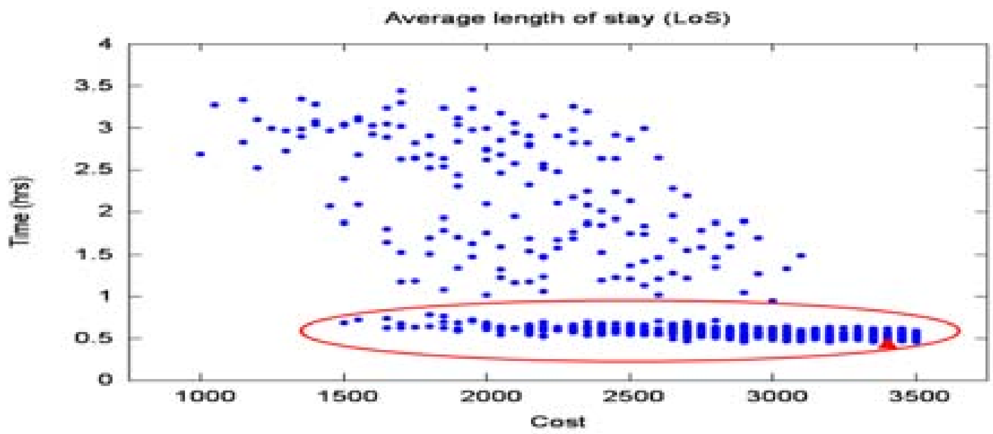


Figure 4.83: Average LoS for 4 incoming patients. The red triangle was the minimum.

On the other hand, when there are up to 17 incoming patients in Figure 4.84, it is important to carefully choose the optimum, because there are not many solutions nearby the optimum (inside the red ellipse), and selecting a sub-optimum solution would increase too much the LoS.

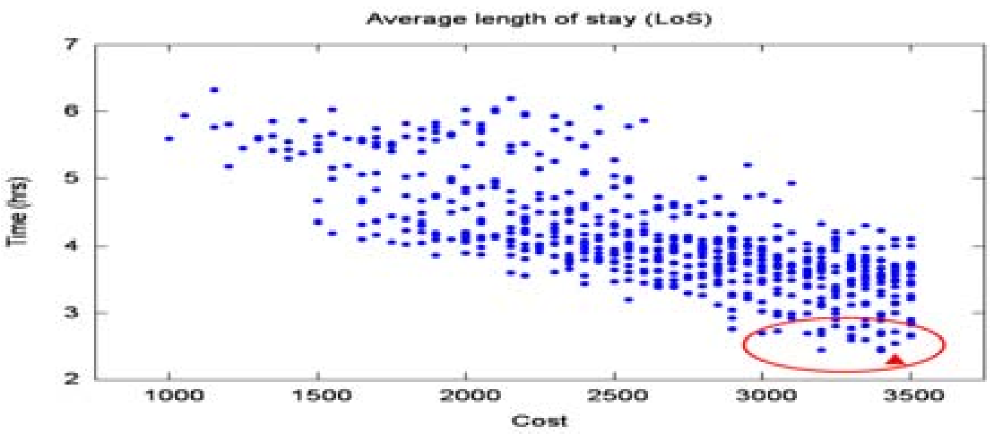


Figure 4.84: Average LoS for 17 incoming patients. The red triangle was the minimum.

4.9 Discussion

The most relevant conclusions of this chapter are the following:

1. The two-phase optimisation via simulation of healthcare Emergency Departments proposed was applied to analyse the administrative strategies leading to optimum decisions about the physical and human resources of an ED. In particular, the impact on the economics and the productivity of Sabadell Hospital ED of different sanitary staff configuration (v.gr., doctors, triage nurses, admission personnel, emergency nurses, and x-ray technicians) were analysed.

2. The evaluation of the proposal included the *simulation models*; the *decision variables* and *workloads* used as inputs of the simulation models; as well as the *metrics* used to asses the benefits of the proposal. This metrics were defined in term of three indexes: patient length of stay (LoS) in the ED; number of attended patients per day (Throughput); and a compound index, the product of the cost of a given sanitary staff configuration times patient length of stay (CLoS).

3. From interviews with the managers at the EDs of Sabadell hospital (which provides healthcare services to an average of 160,000 patients/year), it was found that a basic sanitary of its ED staff is composed by: 9 possible combinations of admission personnel (junior/senior); 9 possible combinations of triage nurses (junior/senior); 5 possible combinations of emergency nurses (junior/senior); 5 possible combinations of x-ray technicians (junior/senior); and 14 possible combinations of doctors (junior/senior) in which a set of examined cases for each type of staff were analysed as a discrete combinatorial problem.

4. In order to analyse the performance of the ED, the real average four hundred incoming patients that daily arrive to the ED of Sabadell hospital was divided into four different workload scenarios, up to: 4, 9, 13, and 17 incoming patients hourly, i.e., up to 96, 216, 312, and 408, respectively for 24hrs.

5. All simulations of the ED optimization cases analysed in this work were carried out in a Linux cluster of the CAOS Department of the UAB, which has 608 computing cores and 2.2TB of RAM, that is composed of: 9 nodes of a dual-4 core Intel Xeon E5430, 2.6GHz, 16GB RAM; 1 node of 2xdual-6 core Intel Xeon E5645, 2.4GHz, 24GB RAM; and 8 nodes of 4x16-cores AMD Opteron “Interlagos”, 1.66GHz, 256 GB RAM, all in a switched 1GigE network.

6. The evaluation of the proposed methodology aimed to confirm the correct operation of both the pipeline approach (PA) and the MC plus the K-means methods, described in chapter 3. To this end, we have first performed the exhaustive search (ES) to use as baseline method. The second step of this evaluation consisted on applying the coarse grained phase, using either the PA, the MC plus K-means methods, or both. Finally, the fine grained phase was applied in the promising regions found in the previous step.

7. To evaluate the methodology proposed, first the case study A was performed using the agent-based ED simulator version 1.1. Then the case study B was performed using the agent-based ED simulator version 1.2. In both cases, the three metrics and the four different workloads stated above were tested, and the period simulated was 24 hrs., i.e., one day of functioning of the ED, in all the experiments.

8. After separately applying for cases A and B either the pipeline approach, PA or the Monte Carlo, MC, plus the K-means methods, or both the “reduced exhaustive search”, the optimum found per each method for their associated average LoS, average Throughput, and average CLoS were the same.

9. Using the pipeline approach, PA, as the coarse grained phase of the proposed methodology and then the “reduced exhaustive search” in the promising regions previously found, our proposal obtained an improvement up to 95.6% in the computing time, whereas using the Monte Carlo, MC, plus the K-means methods and then the “reduced exhaustive search” in the promising regions previously found, our proposal obtained an improvement up to 78% in the computing time, both compared with the exhaustive search used.

10. The optimum solution not always is the best option; therefore, it is important to take into account the sort of optimum when the solutions are going to be applied into real problems.

Chapter 5

Conclusions and Future Research

"Perfection is reached, not when there is no longer anything to add, but when there is no longer anything to take away."

Antoine de Saint-Exupery

5.1 Conclusions

The most relevant conclusions of this thesis are the following:

1. The operation and characterisation of the Healthcare Emergency Departments (ED), from the perspective of how urgent patient care is delivered, were discussed. EDs can be characterised by their: a) physical location (in a hospital unit or an independent one); b) physical layout (such as number of waiting, triage and medical rooms); c) time period open to patients (of waiting, triage and medical rooms), c) time period open to patients (24hs / 365 days per year or part time); d) patient type served (all /certain ages only); and e) type and number of staff members (ddmission and support personnel, nurses, doctors, medical technicians).

2. The ABM of the ED proposed in this work is used as a black box simulator, and its implementation was done by using *NetLogo*, the agent-based programming language and programmable modelling environment

3. The optimisation via simulation methodology for EDs proposed herewith is based in a neighbourhood structure aiming to reduce the feasible region. The methodology is constituted of two phases. The first phase is a coarse grained approach consisted in a global exploration step over the entire search space. This phase identifies promising regions for optimisation based on a neighbourhood structure of the problem, that uses either a pipeline scheme approach of an Emergency Department or the Monte Carlo heuristic plus

the K-means method. The second phase is a fine grained approach, that consists in seeking the best solution, either the optimum or a sub-optimum lying on the Pareto frontier by performing a "reduced exhaustive search" in such promising regions.

4. A Master-Worker (M-W) application using pthreads to launch the ED simulator was implemented in C language in order to load balancing. This M-W application is used as the first approach to find the optimum sanitary staff configuration by using exhaustive search both to compare and analyse the results and performance of the proposal methodology. The pipeline program was implemented in C++ programming language using STL, whereas the MC method was implemented in Perl programming language. Finally, a "reduced exhaustive search" was applied by using the m-w application within the reduced feasible region found by either the pipeline scheme or the MC plus K-means methods.

5. This thesis presents a concrete example that uses a promising approach, agent-based modelling and simulation for healthcare emergency departments since its complexity and dynamic nature make them difficult to characterise. The model uses Moore state machines based agents which act and communicate within a defined layout. Two versions of the agent-based emergency department simulator were implemented in *NetLogo*, verified, and validated.

6. The evaluation of the two-phase optimisation via simulation of healthcare Emergency Departments proposed included the *simulation models*; the *decision variables* and *workloads* used as inputs of the simulation models; as well as the *metrics* used to asses the benefits of the proposal. This metrics were defined in term of three indexes: patient length of stay (LoS) in the ED; number of attended patients per day (Throughput); and a compound index, the product of the cost of a given sanitary staff configuration times patient length of stay (CLoS).

7. From interviews with the managers at the EDs of Sabadell hospital (which provides healthcare services to an average of 160,000 patients/year), it was found that a basic sanitary of its ED staff is composed by: 9 possible combinations of admission personnel (junior/senior); 9 possible combinations of triage nurses (junior/senior); 5 possible combinations of emergency nurse (junior/senior); 5 possible combinations of x-ray technician (junior/senior); and 14 possible combinations of doctors (junior/senior) in which a set of examined cases for each type of staff were analysed as a discrete combinatorial problem. In order to analyse the performance of the ED, the real average four hundred incoming patients that daily arrive to the ED of Sabadell hospital was divided into four different workload scenarios, up to: 4, 9, 13, and

17 incoming patients hourly, i.e., up to 96, 216, 312, and 408, respectively for 24hrs.

8. The evaluation of the proposed methodology aimed to confirm the correct operation of both the pipeline approach (PA) and the MC plus the K-means methods, described in chapter 3. To this end, we have first performed the exhaustive search (ES) to use as baseline method. The second step of this evaluation consisted on applying the coarse grained phase, using either the PA, the MC plus K-means methods, or both. Finally, the fine grained phase was applied in the promising regions found in the previous step.

9. To evaluate the methodology proposed, first the case study A was performed using the agent-based ED simulator version 1.1. Then the case study B was performed using the agent-based ED simulator version 1.2. In both cases, the three metrics and the four different workloads stated above were tested, and the period simulated was 24 hrs., i.e., one day of functioning of the ED, in all the experiments.

10. After separately applying for cases A and B both the pipeline approach, PA, and the Monte Carlo, MC, plus the K-means methods the "reduced exhaustive search", the optimum found per each method for their associated average LoS, average Throughput, and average CLoS were approximately the same.

11. Using as the coarse grained phase the pipeline approach, PA, of ED and then the "reduced exhaustive search" in the promising regions previously found, our proposal obtained an improvement up to 95.6% in the computing time, whereas with the Monte Carlo, MC, plus the K-means methods and then the "reduced exhaustive search" in the promising regions previously found, our proposal obtained an improvement up to 72% in the computing time, both compared with the exhaustive search used.

5.2 List of Publications

The research has as outcome the publication of the following papers:

- **E. Cabrera, M. Taboada, M. L. Iglesias, F. Epelde, and E. Luque, *Optimization of healthcare emergency departments by agent-based simulation*, in ICCS, 2011, pp. 1880 1889.**
- **E. Cabrera, M. Taboada, M. L. Iglesias, F. Epelde, and E. Luque, *Simulation optimization for healthcare emergency departments,* in ICCS, 2012, pp. 1464 1473.** which received the award of best paper.

- **E. Cabrera, M. Taboada, M. L. Iglesias, F. Epelde, and E. Luque, *Optimization of emergency departments by agent-based modeling and simulation,* in IRI, 2012, pp. 423 430.**

- **E. Cabrera, M. Taboada, M. L. Iglesias, F. Epelde, and E. Luque, *ABMS optimization for emergency departments,* in Winter Simulation Conference, 2012, p. 89.**

5.3 Future Research

- Implement a newer version of the agent-based healthcare emergency department simulator implemented in *NetLogo* or *RepastSimphony*

- Do sensitivity statistically analysis of the variables of the emergency department simulator.

- Implement the pipeline scheme for the current version of the emergency department simulator.

- Refine the MC heuristic and K-means method to find region where the optima are.

- Set more indexes together with the people from the **Emergency Department** of the Hospital of Sabadell (Parc Tauli Health Corporation).

- Connect the ED with a hospital.

- Consider a regional area in which all emergency departments of such area are connected in order to do a load balancing of the incoming patients and guarantee the best quality of service. This could be implemented as a web portal where the patient can use it, in order to decide which emergency department would be available.

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