HIERARCHICAL OUTRANKING METHODS FOR MULTI-CRITERIA DECISION AIDING.

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Hierarchical outranking methods for multi-criteria decision aiding

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Chapter 1

Introduction

A decision problem is a situation in which a decision maker (DM) has a finite set of possible actions and has to select an "optimal" one for the given problem. These decision problems are found in organizational decision making and planning, which are two important research topics for helping managers to support their decisions under complex and uncertain problems. This kind of problems can be found in many different fields from organizational decisions to people's daily life.

To support organizational complex decision making, Decision Support System (DSS) were originated in early 70's. DSSs are computer-based systems that aim at helping in the decision making process by collecting, organizing and analyzing business and organizational data according to the DM's goals. Since its conception, DSSs has evolved drastically [Shim et al, 2002], incorporating knowledge from different fields such as information and communication technologies (ICT), database research, artificial intelligence, decision theory, economics, cognitive science, operations research, among others [Casagrandi and Guariso, 2009, Kou et al, 2011]. The inclusion of artificial intelligent techniques lead to the so called Intelligent Decision Support Systems, where domain knowledge and the DM's preferences are included in the process of decision aiding, providing a more personalized and appropriate solution.

The widespread use of computers has made these methods available to many different kinds of users, including not only managers of companies, but also service providers or even consumers (i.e., buyers).

Real-world decision making problems generally involve multiple criteria. For instance, in several fields such as economics, engineering and environmental management; decision problems are frequently very complex because they consider different viewpoints, leading to multiple and conflicting evaluation criteria. In such situations, a unique or optimal solution may not exist but rather many solutions may be suitable.

Multiple Criteria Decision Aiding (MCDA) is a discipline that deals with multiple conflicting criteria, establishing scientific bases to elaborate recommendations according to the needs of each DM. Artificial intelligence techniques can be used to elicit and construct a knowledge model for each particular DM, as well as managing the uncertainty usually present in the data.

To represent the DM's aspirations for each of the multiple criteria, a preference model is constructed. Two main multi-criteria aggregation approaches have been proposed in MCDA: Utility-based and Outranking methods [Figueira et al, 2005]. These MCDA approaches are oriented to aggregate partial preferences into a collective preference structure. In utility-based methods, a real number is associated to each alternative representing its preferability on each criterion (called partial utility) [Dyer, 2005], whereas outranking methods build a binary preference relation for all the potential set of alternatives [Figueira et al, 2013]. Then, both MCDA approaches aggregate these partial preferences into a collective preference structure. Other non-classical approaches, such as interactive [Vanderpooten, 1989] and rule-based methods [Greco et al, 2001b] have also been very successful in this discipline.

After constructing the preference model, different methods have been designed to solve the 3 common problems in decision aiding: choice, ranking and sorting. Choice or selection problems consist of selecting a subset of the best possible alternatives as small as possible, discarding the worst alternatives; ranking problems consist of ranking the set of alternatives from the best to the worst, possibly with *ex aequo* and incomparabilities; and sorting problems in which each potential alternative must be assigned to one category from a set of predefined ordered categories.

Although MCDA methods have been under constant development in the last decades, most methods assume that all criteria must be grouped together in a common level, defining a flat structure of criteria and therefore, a unique overall recommendation is given to the DM. When the number of criteria is large, it may become cognitively difficult for the DM to analyze all of the criteria together.

Human decision making follows a comprehensible hierarchical process that implicitly presents a taxonomical structure of criteria with different levels of generality, where a large set of criteria may be decomposed into partial subsets of criteria [Mendis and Gedeon, 2012]. This taxonomical structure of criteria has the form of a tree, where the root corresponds to the most general goal of the DM, the nodes of the tree descending from the goal are intermediate sub-criteria, the nodes descending from these sub-criteria are the lower-level sub-criteria, and so on. Finally, the leaves correspond to the elementary criteria, in which the alternatives are directly evaluated. In that way, the criteria are analyzed according to the subsets defined in the hierarchy, and following the precedence relations in a bottom-up approach. For example, decision problems considering three divergent interests such as economic, environmental and social criteria may be modeled as three individual sub-problems (specialization) of a global problem (generalization). This problem modeling may help the DM to have a better picture of the problem's recommendation as a whole. For instance, analyzing in more detail the overall recommendation by evaluating the results obtained at the economical, environmental and social sub-problems.

This hierarchical analysis has been addressed in utility-based methods, but there is a lack of tools for dealing with hierarchical structure models in the outranking approach. For this reason, this Ph.D. thesis is focused on making some useful contribution to the outranking approach following this line.

Each of the two classical approaches has some advantages and drawbacks. Outranking methods have been very successful because they are based on natural realistic assumptions inspired by social choice theory. Therefore, outranking methods do not consider strong assumptions on the preference between alternatives, allowing the DM to model preference uncertainties. Considering a hierarchical structure of criteria, at each sub-problem of the hierarchy, the uncertainty or hesitation that may lead to inaccurate determination of the preference between alternatives must be considered. Even though outranking methods are very popular and have been applied in several real-world applications from different fields with successful results, up to date in very few cases a hierarchy of criteria is considered. In case that the DM is considering a large and complex set of criteria in a hierarchically structured model with different levels, it must be transformed into a flat level in such a way that a different problem statement with no criteria decomposition is finally defined.

The outranking approach builds a reflexive, non-transitive preference relation S defined on the set of potential alternatives such that for alternatives a and b, aSb means "a is at least as good as b" if there are arguments enough to claim this statement and any argument refutes it. The two best known outranking methods are ELECTRE and PROMETHEE. However, ELECTRE is the only method that strictly applies the outranking concept [Figueira et al, 2013], reason for which this thesis is focused on the ELECTRE family of methods for the case of hierarchically organized criteria.

Since the conception of the original ELECTRE (afterward called ELECTRE-I) in 1960s by Roy [1968], several variants of ELECTRE have been developed to adequate it to the different types of decision problems. These variants can be classified depending on the nature of the decision problem as follows:

- Choice or Selection: ELECTRE-I, ELECTRE IS.
- Ranking: ELECTRE-II, ELECTRE-III, ELECTRE-IV.
- Sorting: ELECTRE-TRI-B, ELECTRE-TRI-C, ELECTRE-TRI-NC.

Ranking and sorting problems are the two most common decision problems found in real-world applications involving ELECTRE methods. The introduction of hierarchical structures for ranking and sorting problems applying the ELECTRE methodology arises new questions that require of new tools to answer them. For this purpose, in this thesis the extension of the most accepted and widely applied ELECTRE versions for ranking and sorting problems are considered: ELECTRE-III and ELECTRE-TRI-B respectively.

1.1 Framework of this thesis

This thesis has been funded by the Spanish research project SHADE (TIN-2012-34369: Semantic and Hierarchical Attributes in Decision Making). The main aim of the SHADE project is the development of new techniques to solve some of the current limitations in Decision Support Systems, focusing on these three aspects: 1) management of multi-valued semantic variables with the assistance of domain ontologies, 2) management of hierarchically structured criteria in multi-criteria decision support systems based on outranking relations, and 3) automatic dynamic adaptation of the users' preferences based on the analysis of their interaction with the system. The work presented in this Ph.D. thesis is focused on solving the task 2.

The SHADE project is carried out in collaboration with the Laboratory of Intelligent Decision Support Systems (IDSS) in Poznan University of Technology. The IDSS is an important group specialized in decision support systems, integrating methodologies from operations research and artificial intelligence.

The author has been supported by a FI pre-doctoral grant from Generalitat de Catalunya.

1.2 Objectives of the thesis

The objectives of this Ph.D. thesis can be summarized as follows:

- Formalize hierarchical structures of criteria in decision problems;

- Design an outranking method for ranking problems involving subsets of related criteria defined in a hierarchy structure. The method must provide partial rankings at each sub-problem and a global ranking of the alternatives at the most general criterion.
- Design a sorting outranking method for problems involving subsets of related criteria defined in a hierarchy structure. The method must provide partial alternative assignments to predefined and possibly heterogeneous categories at each sub-problem and a global assignment at the most general criterion.
- Redefine the concept of pseudo-criteria at intermediate and root criteria allowing the possibility to define a local preference model at each node of the hierarchy. The preference model must take into account the DM's objectives, the knowledge of the sub-problem characteristics, as well as the uncertainty of the DM's preferences.
- Study the properties of the methods proposed for hierarchical structures of criteria to describe their characteristics;
- Apply the ranking and sorting methods on real-world case studies dealing with hierarchical structures of criteria. In this thesis, 3 case studies are considered including an assessment system for destination websites, the analysis of water allocation strategies in future scenarios of global change, and the analysis of touristic activities in a recommender system.

1.3 Contributions

The contributions of this Ph.D. thesis can be summarized as follows:

1. The first contribution of this Ph.D. thesis is an extension of the classical ELECTRE-III method to manage decisions with a hierarchical structure of criteria. The method proposed is called ELECTRE-III-H and it is designed to generate and propagate the partial pre-orders calculated from the bottom level up to the root following the two ELECTRE steps at all levels of the hierarchy: 1) construction of a binary outranking relation based on partial concordance and discordance indices obtained from the consideration of a given set of criteria; and 2) exploitation of the outranking relation via distillation to generate a partial pre-order of the alternatives.

   The results of this study have been published in the following journal:

   - Luis Del Vasto-Terrientes, Aida Valls, Roman Slowinski, and Piotr Zielniewicz. ELECTRE-III-H: An outranking-based decision aiding method for hierarchically structured criteria. *Expert Systems with Applications*, 42(11):4910-4926, 2015. Impact Factor: 1.965 (Q1).

Preliminary works on this topic have been presented in the following international conferences:

- Luis Del Vasto-Terrientes, Aida Valls, Roman Slowinski, Piotr Zielniewicz. Solving ranking problems with ELECTRE-III in case of hierarchical family of criteria. In *22nd International Conference on Multi-Criteria Decision Making (MCDM)*, 2013.
- Luis Del Vasto Terrientes, Aida Valls, Roman Slowinski, Piotr Zielniewicz. Extending Concordance and Discordance Relations to Hierarchical Sets of Criteria in ELECTRE-III Method. In Vicenç Torra, Yasuo Narukawa, Beatriz López, and Mateu Villaret, editors, Modeling Decisions for Artificial Intelligence - 9th International Conference, MDAI 2012, Girona, Catalonia, Spain, November 21-23, 2012. Proceedings, volume 7647 of Lecture Notes in Computer Science, pages 78–89. Springer, 2012. Core B.

2. The second contribution is the extension of the classical ELECTRE-TRI-B method following a hierarchical structure of criteria. ELECTRE-TRI-B considers a finite set of preference-ordered categories to which the alternatives are assigned. The extended method, called ELECTRE-TRI-B-H, calculates outranking relations of the alternatives with respect to the boundary profiles at all levels of the hierarchy, propagating the assignment of alternatives from the lowest level up to the root criterion. The method accepts a different set of categories for each node on the root and intermediate criteria, so that the DM receives a more representative and meaningful qualitative assessment of the alternatives depending on the nature of each criterion.

   The results of this study have been published in the following journal:

   - Luis Del Vasto-Terrientes, Aida Valls, Piotr Zielniewicz, Joan Borràs. A hierarchical multi-criteria sorting approach for recommender systems. *Journal of Intelligent Information Systems*, Accepted. DOI: 10.1007/s10844-015-0362-7. Impact Factor: 0.632 (Q3).

   Preliminary work of this topic has been presented in the following international conference:

   - Luis Del Vasto-Terrientes, Aida Valls, Piotr Zielniewicz. ELECTRE-TRI-B-H for solving hierarchically structured sorting problems. In *EURO Working Group on MCDA (EWG)*, Athens, Greece, 2014.

These two contributions permit a more detailed and flexible modeling for sorting and ranking problems where criteria are naturally organized in a hierarchy structure. Moreover, a more detailed analysis of preference relations on the set of alternatives at different levels of generality including different viewpoints, as well as in a global integrated way. This richer structure of criteria is particularly interesting for problems where the DM is evaluating a set of alternatives on the basis of large sets of diverging criteria.

3. The third contribution of this thesis is the application of the hierarchical methods proposed in real case studies with the collaboration of different partners from Catalonia (Spain), including the Science and Technology Park of Tourism and Leisure (PCT) in Vila-Seca, Universitat Pompeu Fabra (UPF) in Barcelona and Universitat Rovira i Virgili (URV) in Tarragona. The multidisciplinary character of the hierarchical methods that are proposed in this thesis has been proven with their application in 3 different fields: Website management, environmental decision making and tourism.

   The decision aiding process for these 3 real cases involved the interaction with the different DMs, representing and structuring the hierarchical decision problem with respect to the their values and needs, and the discussion about the final result taking into account the results obtained at each subproblem.

   The results of this study have been published in the following journals:

   - Luis Del Vasto-Terrientes, José Fernández-Cavia, Assumpsió Huertas, Antonio Moreno, Aida Valls. Official tourist destination websites: Hierarchical analysis and assessment with ELECTRE-III-H. *Tourism Management Perspectives*, 15:16-28, 2015. SCImago Journal Rank: 0.476 (Q2).
   - Luis Del Vasto-Terrientes, Vikas Kumar, Tzu Chi Chao, Aida Valls. A decision support system to find the best water allocation strategies in a Mediterranean river basin in future scenarios of global change. *Journal of Experimental & Theoretical Artificial Intelligence*, DOI: 10.1080/0952813X.2015.1024493. Impact Factor: 0.527 (Q4).

   The results of the real case related to tourism have been published in "A hierarchical multi-criteria sorting approach for recommender systems, *Journal of Intelligent Information Systems*".

   Preliminary works on this topic have been presented in the following international conferences:

   - Tzu Chi Chao, Luis Del Vasto-Terrientes, Aida Valls, Vikas Kumar, Marta Schuhmacher. A hierarchical decision support system to evaluate the effects of climate change in water supply in a mediterranean river basin. In *Artificial Intelligence Research and Development - Recent Advances and Applications*, CCIA, October 2014, Barcelona, Catalonia (Spain), pages 77–86, IOS Press, 2014.

- Vikas Kumar, Luis Del Vasto-Terrientes, Tzu Chi Chao, Aida Valls, Marta Schuhmacher. Application of outranking method to adaptation strategies for water supply management. In *Final SCARCE International Conference, River conservation under water scarcity: Integration of water quantity and quality in Iberian Rivers under global change*, October 2014, Tarragona, Spain, 2014.

1.4 Document Structure

The present document is divided into the following chapters:

- Chapter 2 introduces Multi-Criteria Decision Aiding, its basic concepts and the most relevant approaches are presented. This chapter also includes a brief review of the most relevant works in Social Choice Theory inspiring the study of decision aiding techniques in the last decades.
- Chapter 3 defines the concepts and notations with regard to the hierarchy of criteria. Several ranking and sorting problems from different fields in which the criteria of the decision problem can be naturally modeled in a hierarchy structure are presented.
- Chapter 4 reviews the classical ELECTRE-III ranking method and describes the extension of the ELECTRE-III method, called ELECTRE-III-H, for generating and propagating partial pre-orders at all levels of the hierarchically structured criteria.
- Chapter 5 reviews the classical ELECTRE-TRI-B sorting method and describes its extension, called ELECTRE-TRI-B-H method, for propagating assignments from the lowest level in the hierarchy up to the overall goal.
- Chapter 6 applies the ELECTRE-III-H and ELECTRE-TRI-B-H methods in 3 case studies from different fields.
- Chapter 7 presents the conclusions of the thesis and some lines of future research.

Chapter 2

Multi-Criteria Decision Aiding

Multi-Criteria Decision Aiding techniques are widely used in decision problems to find the "best possible" alternative solution, making the process more explicit, rational and efficient. MCDA is a multidisciplinary field, deriving from Operations Research, that uses mathematical approaches to deal with complex problems encountered in human activities. Nowadays it also integrates artificial intelligence and economic welfare techniques.

In decision aiding problems involving multiple criteria, a unique or optimal decision does not exist but rather many decisions may be suitable for a given problem. The decision aiding process may involve two main actors: the decision maker and the analyst [Roy and Slowinski, 2013]. The decision maker (DM) is the person that has to take the best possible decision for a given problem, whereas the analyst is a consultant that is expected to clarify the decision situation and help in the modelization stage.

In multi-criteria decision analysis, two main schools of research are acknowledged: the American or Anglo-Saxon school, commonly known as Multi-Criteria Decision Making (MCDM); and the European or French school, commonly known as MCDA. In the American conception, the decision is a matter of reproducing as faithfully as possible the DM's preference system as it truly exists in order to get as close as possible to the best decision (positivist approach), while in the European conception, the DM is helped (either by an analyst or automatic tools) to construct one or more preference models in order to study the results to which they lead (constructivist approach) [Roy, 2010]. This interaction to build and make evolve the decision-aiding process comprises several phases [Bana e Costa et al, 1999, Roy, 1996, Tsoukiàs, 2007].

In MCDM there is no distinction between the DM and the analyst so that the DM may directly use decision tools without the requirement of the presence of an analyst. On the other hand, MCDA requires the presence of both actors, each with its own role in the decision process [Tsoukiàs, 2007].

In this thesis we will be referring to the European conception of MCDA.

This chapter first introduce in Section 2.1 the basic concepts required for modeling decision problems in MCDA. Before introducing the most relevant MCDA methods, an introduction of the most relevant works in Social Choice Theory that inspired the classical MCDA approaches are presented in Section 2.2. Next, the classical MCDA approaches are presented in Section 2.3: Multi-Attribute Utility Theory from the American school in Section 2.3.1 and the outranking methods from the European school in Section 2.3.2. We focus on outranking methods, presenting the two best known methods: PROMETHEE and ELECTRE. A comparison of these two outranking methods and a discussion about a common problem concerning outranking methods handling hierarchical structures of criteria is made.

2.1 Problem modelization

The decision aiding process refers to the activities required to successfully model the decision problem defining its structure, parameters and functions with support of adequate methodological and technical tools. The model is finally subjective and represents the goals of the DM.

There are three fundamental concepts related to the decision aiding process that the model includes: alternatives, criteria and preference systems.

1. **Alternatives:** are the set of potential actions for the decision problem for which only one potential action can be applied. Alternatives are represented as follows:
   $A = \{\ a, b, c, ...\ \}$ is the finite set of alternatives and n is the number of alternatives in A.

2. **Criteria:** are tools constructed for the evaluation of alternatives that allow to compare them in terms of suitability based on the DM needs. Each criterion corresponds to a point of view considered in the decision process. Criteria are represented as follows:
   $G = \{\ g_1,\ g_2,..,\ g_m\ \}$ is the finite set of criteria, in which m is the number of criteria in G.

$g_j(a)$ represents the performance value of alternative $a \in A$ on criterion $g_j \in G$. This performance value can be of two types:

(a) Ordinal scale: The order of the values is what is important and significant. The gap between two performances does not have a clear meaning in terms of difference preferences. They can be represented in a numerical and verbal/linguistic scale.

(b) Quantitative scale: The order of the values are not only given, but also there is a clear defined quantity in a way that it gives a measure of the gap between two performances.

The performance matrix M is built for $A \times G$, where $g_j(a)$ is the performance in row a and column j.

Let us assume, without loss of generality, that all criteria are of the gain type, i.e., the greater the value, the better.

3. **Preference system:** Consists of an implicit or explicit process that assigns a preference relation between a pair of alternatives, which may include:

(a) Preference: aPb, i.e., a is preferred to b

(b) Indifference: aIb, i.e., a is indifferent to b

(c) Incomparability: aRb, i.e., a is incomparable to b

Depending on the aggregation procedure applied, the incomparability relation may not be part of the preference system.

The preference model generally fulfills the following properties:

$$\forall a, b \in A : \begin{cases} P \text{ is asymmetric, so } aPb \Rightarrow \neg(bPa) \\ I \text{ is reflexive, so } aIa \\ I \text{ is symmetric, so } aIb \Rightarrow bIa \\ R \text{ is irreflexive,so } \neg(aRa) \\ R \text{ is symmetric, so } aRb \Rightarrow bRa \end{cases}$$

When multiple conflicting criteria are considered in the decision problem, the main question that arises is "How do the DM take into account all the criteria together in order to compare, from a finite set of alternatives, each alternative to one another, and finally provide a recommendation?". This problem is called the aggregation problem, for which several mathematically explicit aggregation procedures have been developed in decision aiding. By definition, the aggregation is a procedure in which for any pair of alternatives, a clear answer to the aggregation problem is given [Roy, 2005]. The aggregation procedure must consider the possible types of dependence between criteria and the conditions under which compensation between good and bad performances are accepted or refused.

The different theoretical logic behind the different aggregation procedures has led to different MCDA approaches, each with its own informational requirements and mathematical properties [Figueira et al, 2005].

A very similar aggregation problem has been studied for a long time in voting theory framework. Nowadays, it is commonly referred to as social choice theory, as it integrates elements from welfare economics and voting theory. The results obtained in social choice have been valuable for the foundations of MCDA. In the next section, a brief introduction to the most relevant works in social choice is presented.

2.2 Social Choice foundations for MCDA

Social choice theory studies reasonable mechanisms to reflect the individual preferences from members of a society into a collective preference [Bouyssou et al, 2010]. In this discipline, the concepts of candidate and voter are generally used. The final objective is to determine the elected candidate or provide a ranking of the candidates. Seems natural to consider the elected candidate based on the "majority rule" principle, such that if candidate x gets more votes than candidate y, x must be the elected candidate. This system is generally applied in real-world cases such as democratic elections. However, it has been proven that this simple system may have conflicts when more than two candidates are running for election [May, 1952].

In social choice theory, several methods have been suggested in the literature to reflect the collective preference considering individual preferences if more than two candidates are available to choose from. The two most important methods are presented in this section: the Borda count and the Condorcet method.

For the analysis of these two social choice procedures, consider a finite set G of m voters and a finite set $A = \{a, b, c, ...\}$ of n candidates. Let us assume that each voter in G provides an ordered ranking with no ties between candidates (i.e., a total order) and the preference relation $\succ$ means "is better than" in such a way that, for instance, $\{a \succ b \succ c \succ ...n\}$ represents that a is best candidate, b second, and so on until alternative n, representing the worst candidate.

2.2.1 The Borda count

The Borda count defines a global numerical score $T(x)$ for each candidate x based on the sum of the ranks provided by each voter, such that

$$T(x) = \sum_{i=1}^{m} rank_i(x)$$

Considering $T(x)$ for candidates a and b, the Borda count method states the following possible preferences:

$$a \succ b \Leftrightarrow T(a) < T(b)$$
$$a = b \Leftrightarrow T(a) = T(b)$$

In other words, alternative b is not ranked worst than a if the Borda count of b is lower than that of a.

In this method, the numerical values of the ranks are considered as distances between candidates. This model assumes that the candidates' distances are comparable and can summed to distances on the ranking given by another voter; thus, introducing the concept of commensurability and trade-offs in the global count. For example, for 4 voters and 3 candidates a, b, and c; let us suppose that the preferences given are as follows:

2 voters have preferences $a \succ c \succ b$,
1 voter has preference $b \succ a \succ c$.

The Borda count for the candidates are $T(a)$=4, $T(b)$=7 and $T(c)$=7. Despite alternative b has been selected as the best candidate for 1 voter and none has selected c as the best, the Borda count of b and c says that these two candidates are tied. This occurs because 2 voters selected c over b. In this case, candidate a results the winner.

2.2.2 Condorcet method

The Condorcet method is based on a pairwise comparison among the candidates. It states that candidate a is better than candidate b applying the "majority rule", i.e., a is preferred to b if the number of voters ranking a over b is greater than the number of voters ranking b over a.

$$a \succ b \Leftrightarrow \forall m, |a \succ b| > |b \succ a|$$

Applying the Condorcet method to the same example provided to illustrate the Borda method, we obtain the following pairwise comparison:

Table 2.1: Condorcet pairwise matrix example

	a	b	c
a	-	2	3
b	1	-	1
c	0	2	-

According to the Condorcet matrix, candidate a defeats b twice and c thrice, so a is the Condorcet winner (b defeats a just once). Candidate b defeats c once, while c defeats b twice, so c is globally better than b.

This simple example illustrates how the Borda and Condorcet methods diverge with respect to the ranking of alternatives b and c.

Each of these methods have their own strengths and weaknesses. The Borda method always select one or more winners, while the Condorcet method may lead to unambiguous outcomes that may result in no Condorcet winner [Nehring et al, 2014]. This problem is known as the Condorcet or Voting Paradox, and occurs because even though the individual preferences of the voters are transitive, the collective preferences may be cyclic. Let us illustrate this phenomena with a classical example involving 3 candidates x, y and z; and 3 voters with the following preferences:

$$\text{Voter 1: } x \succ y \succ z,$$
$$\text{Voter 2: } y \succ z \succ x,$$
$$\text{Voter 3: } z \succ x \succ y.$$

For this case, there are majorities (exactly 2/3) that prefer x over y, y over z and z over x, violating transitivity. Furthermore, no candidate is selected as the Condorcet winner.

On the other hand, the Borda method judges the strength of a voter's preference for one candidate over another based on the number of candidates intervening between these 2 particular candidates. This implies that, the larger the number of candidates to choose from, the bigger may be the difference between candidates. Consequently, the introduction of new candidates may alter the final outcome [Risse, 2005].

These two voting oriented social choice methods have inspired the study of decision aiding methods in the last decades, resulting in the two classical approaches in the field: Multi-attribute utility based and outranking. The Multi-attribute utility based methods follow the Borda method with the aggregation of utility functions instead of the rank of the alternatives, while outranking methods follow the Condorcet method on the pairwise comparison of the alternatives. Section 2.3 studies these two main approaches, with special interest in outranking methods.

Based on the results obtained from the Borda and Condorcet methods example, it can be noticed that conceiving a "good" preference aggregation methods raises serious problems. It is well known that there is a limitation in the aggregation of ordinal information in voting methods. This was stated by Arrow [1963]. In the next section we study this problem in more detail.

2.2.3 Arrow's impossibility theorem

In social choice theory, Arrow's impossibility theorem is considered one of the most important theoretical contributions regarding aggregation procedures of ordinal information [Arrow, 1963]. This theorem states that for three or more candidates (i.e., $n \geqslant 3$) and two or more voters (i.e., $m \geqslant 2$); it is not possible to design an ordinal voting system that may impose all of the following conditions at the same time using merely ordinal preferences from the voters in a election:

1. Non-dictatorship: The aggregation procedure must take into consideration multiple voters. The preferences of a single voter may not dictate the collective decision;
2. Universality (unrestricted domain): Every configuration of rankings is admissible (no constraint on the set of admissible rankings);
3. Independence of irrelevant alternatives: The collective preference between 2 candidates should only depend on the individual preference between them. Thus, the introduction of a new candidate should not change the collective preference between the first 2 candidates;
4. Transitivity: The outcome of the aggregation procedure must be a complete ranking with possible ties;
5. Unanimity: If every individual prefers candidate x over candidate y, then so must the resulting collective preference order.

For instance, as explained in the previous section, the Borda count depends on the relative positions of alternatives (distance between alternatives in the ranking). This is, the addition or deletion of an alternative from the set results in different

Borda count of the alternatives. This may violate the independence property. For the case of the Condorcet method, the transitivity property may not be fulfilled due to the Condorcet paradox.

Sen [1970] proposed that not only ordinal information about voters' preferences among pairs of alternatives must be taken into account, as ordinal information is poor and insufficient. Moreover, cardinal information about the "utility" they derive from each one must be considered in order to achieve all 5 Arrow's conditions. However, according to Arrow [1963], the aggregation of interpersonal utilities (magnitude in each voter's mind) may seems to make no sense. In Section 2.3.1, more information about utilities is provided.

Considering the limitations shown in this theorem, it is important to study the properties of the MCDA methods.

2.3 MCDA Approaches

The multi-attribute utility theory (MAUT) and outranking methods from the American and European schools respectively, prevail nowadays in the MCDA field. MAUT and outranking methods together record a considerable number of MCDA applications in the literature. MAUT methods have some similarities to the Borda's scoring method and outranking methods to the Condorcet method [Figueira et al, 2005]. These 2 classical approaches are presented in this section, with special focus on outranking methods.

2.3.1 Multi-Attribute Utility Theory

Utility theory is a systematic approach for quantifying an individual's preferences, commonly used in economics and game theory. It represents a way of measuring the desirability of the preference of alternatives, which can be represented as goods or services [Ishizaka and Nemery, 2013b].

MAUT is founded on this approach, assigning a preference value to each alternative for all attributes or criteria [Keeney and Raiffa, 1993]. Therefore, the purpose of this approach is to associate a rating, generally a real-valued number $r_j(a)$ to alternative a on criterion j, representing the degree of "satisfaction" $\mathcal{S}$ of a on criterion g_j according to the DM's expectation and desired values. A utility function is applied to convert numerical attribute scales to value unit scales, allowing direct comparison of diverse measures. In this context, it is generally acknowledged that value functions represent the preference under certainty; and utility functions refer to preference under risk. Risky options are defined as lotteries or gambles with outcomes that depend on the occurrence from a set of mutually exclusive and exhaustive events. For example, a lottery could be defined as the flip of a fair coin, with a different outcome depending if heads or tails occurs [Dyer, 2005]. We focus on cases where no risk is involved, but for the sake of simplicity, the term *utility* is used throughout this Ph.D. thesis.

Ratings are used to compare the alternatives so that $r_j(a)$ is associated to each alternative $a \in A$ in such a way that a is judged to be preferred to b if $r_j(a) > r_j(b)$ and indifferent if $r_j(a) = r_j(b)$.

Once the real-valued function $r_j(a)$ is set to each alternative for all criteria, the aggregation of these uni-dimensional utility functions results in a global utility with a function $H : \mathcal{S}^m \to \mathcal{S}$. Several aggregation operators have been proposed, requiring the following mathematical properties:

1. Idempotency: $H(a, a, ..., a) = a, \forall a \in [0, 1]$
2. Monotonicity: $r'_j > r_j \Rightarrow H(r_1, ...r'_j, ...r_m) \geqslant H(r_1, ...r_j, ..., r_m)$
3. Commutativity: $H(r_1, r_2, r_3) = H(r_2, r_3, r_1)$
4. Compensativity: $\overset{m}{\underset{j=1}{\wedge}} r_j \leqslant H(r_1, r_2, ..., r_m) \leqslant \overset{m}{\underset{j=1}{\vee}} r_j$
5. Associativity: $H(r_1, r_2, r_3) = H(H(r_1, r_2), r_3)$
6. Decomposability: $H(r_1, r_2) = r' \Rightarrow H(r_1, r_2, ..., r_m) = H(r', r', r_3, ..., r_m)$

In Grabisch [1996], Grabisch et al [2011], the most common additive utility aggregation operators are presented:

1. Quasi-arithmetic means: Represents the family of means which include simple arithmetic mean, geometric, harmonic means, among others. It is defined as follows:

$$H(r_1, r_2, ..., r_m) = f^{-1}\left[\frac{1}{m}\sum_{j=1}^{m} f(r_j)\right]$$

It can be extended to apply the weights w_j as follows:

$$H(r_1, r_2, ..., r_m) = f^{-1}\left[\sum_{j=1}^{m} w_j f(r_j)\right]$$

2. Median: Applying the concept of median in statistics, the real-valued function r_j of the alternatives is not taken into account but their ordering. The median is referred to the middle value of this ordered list.

$$H(r_1, r_2, ..., r_m) = \begin{cases} r_{(\frac{n+1}{2})} & \text{if } n \text{ is odd,} \\ \frac{1}{2}(r_{(\frac{n}{2})} + r_{(\frac{n}{2}+1)}) & \text{if } n \text{ is even} \end{cases}$$

3. Ordered weighted averaging operators (OWA): It provides a parameterized class of mean-type aggregation operators. It establishes a trade-off between conjunctive and disjunctive model of aggregation. It generalizes other mean operators such as the max, arithmetic average median, and min. It was introduced by Yager [1988]. It is formally defined as follows:

$$H(r_1, r_2, ..., r_m) = \sum_{j=1}^{m} w_j b_j$$

where b_j is the $j - th$ largest of the rating in $r_1, ..., r_m$ and $\sum_{j=1}^{m} w_j$=1.

The global utility obtained for each alternative on A allows their comparison and the construction of a ranking of alternatives on A from the best to the worst in a complete, transitive pre-order. Following the trade-off nature of this approach, the global utility is always comparable between alternatives, so that the incomparability relation between alternatives cannot be obtained with this approach.

The steps followed in MAUT are as follows [Cho, 2003]:

1. Identify the relevant attributes,
2. Assign quantifiable criteria to each of the attributes and specify their restrictions,
3. Construct a utility function for each criterion, all providing ratings in the same range,
4. Aggregate the individual ratings using operators like presented before,
5. Evaluate the alternatives using the global utility obtained in Step 4 and choose, rank or sort them accordingly.

2.3.1.1 Strengths and weaknesses of MAUT

The strengths of the MAUT method are summarized as follows:

- In MAUT, the global utility obtained from the aggregation of the partial utilities are independent to irrelevant actions. Thus, the addition or deletion of an irrelevant alternative in set A does not change the best alternative solution;
- Preferences in MAUT are transitive. Therefore, if a priori the transitivity of the preferences is imposed, we should consider applying MAUT techniques;
- The MAUT aggregation of uni-dimensional utility functions results in fast calculation of the global utility.

The weaknesses of the MAUT method are:

- In MAUT, the role of the analyst is to estimate this function by asking the DM some well-chosen questions. However, this utility function may represent a major shortcoming of the MAUT approach because in order to build the proper utility functions, as it is neither direct nor easy for the DM to set because of the number of judgments and their complexity [Zeleny, 1982];
- MAUT is appropriate for developing preference models to address value trade-offs among multiple objectives [Keeney, 1977]. This means that MAUT allow scoring compensation, i.e., a bad score for a certain criterion are compensated with a good score of another criterion. This approach may not be convenient for some decision problems.

2.3.2 Outranking methods

The aim of outranking methods is to build a binary relation S, where aSb means "a is at least as good as b", obtained from the pairwise comparison of alternatives on set A for each criterion $j \in G$. The binary outranking relation aSb is reflexive and not necessarily a transitive relation. This relation is also denoted as $a \gtrsim b$, where $\succ$ is asymmetric and $\sim$ is symmetric. The concept of outranking methods was first proposed by Roy [1996] with the original ELECTRE method. Vincke [1992] states that the underlying idea of introducing the outranking methods is that it is better to accept a result less richer than that yielded by utility-based methods, if one can avoid mathematical hypotheses which are too strong and requiring complex information from the DM.

The outranking approach is a generalization of the dominance relation. However, the relation S is richer because the *unanimity* property of the dominance relation is weakened in such a way that not all viewpoints must be in favor of aSb to reach this conclusion, but only a sufficient evidence of this is required (majority principle). Also, preferences in outranking methods accept incomparabilities.

Outranking methods are characterized by the limited degree to which a disadvantage on a particular criterion may be compensated by advantages on other criteria [Pirlot, 1997], in comparison to MAUT that allows trade-offs of performances.

Generally, methods based on pairwise comparison of the alternatives are included within the outranking approach, in which the PROMETHEE method is very well known in the field. Every outranking method includes two phases: 1) the construction of the outranking relation, and 2) the exploitation of this relation in order to provide a recommendation to the DM [Brans and Vincke, 1985]. The next sections introduce the PROMETHEE and ELECTRE methods which are the most widespread outranking methods.

2.3.2.1 PROMETHEE

The PROMETHEE (Preference Ranking Organization MeTHod for Enrichment Evaluations) outranking method was first proposed by Brans [1982]. It builds a valued outranking relation based on a preference index $P_j(a,b) \in [0,1]$ representing the degree of preference of a over b for each criterion on G. It is calculated from the difference between the performance of the alternatives, so that $P_j(a,b) = f(g_j(a) - g_j(b))$. The closer $P_j(a,b)$ is to 0, the greater the indifference between a and b is; while the closer to 1, the greater the preference of a over b is. Note that this preference index gives a valued "preference degree" between two alternatives.

This preference index can be defined in different ways. In Brans and Vincke [1985], 6 functions that are commonly used in practical applications were presented:

1. Usual criterion: The indifference only applies when $g_j(a) = g_j(b)$. If not, then DM is indicating a strict preference of the alternative with the best performance.

2. Quasi criterion: The criterion is associated to a threshold q. If the difference between $g_j(a)$ and $g_j(b)$ do not exceeds this threshold, then a and b are indifferent. Otherwise, the alternative with the best performance is strictly preferred.

3. Criterion with linear preference: The function is associated to a threshold p. If the difference between $g_j(a)$ and $g_j(b)$ is lower than p, the DM is indicating a progressive preference of the best performance. Otherwise, it is strictly preferred.

4. Level criterion: In this function, the DM has to set the two thresholds q and p. If the difference between $g_j(a)$ and $g_j(b)$ do not exceeds q the alternatives are indifferent, between q and p there is a weak preference (0.5), and after this value becomes strict preference of the alternative with the best performance.

5. Criterion with linear preference and indifference area: In this function, a and b are considered indifferent as long as $g_j(a) - g_j(b)$ do not exceeds q and the preference increases linearly from this q until p. After p, the strict preference applies.

6. Gaussian criterion: This function (ρ) is made easily according to the experience obtained with the normal distribution in statistics.

The graphical models of these 6 functions are presented in Figure 2.1.

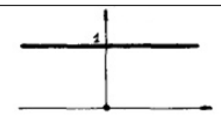
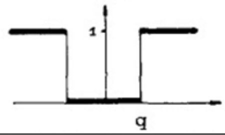
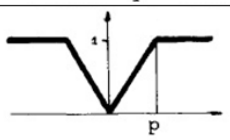
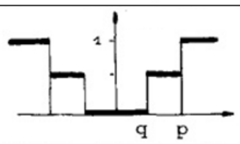
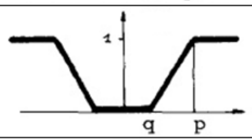
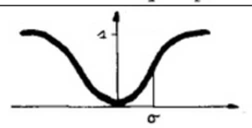
Types of criteria		Parameters
(a) Usual criterion		-
(b) Quasi criterion		q
(c) Criterion with linear preference		p
(d) Level criterion		q, p
(e) Criterion with linear preference and indifference area		q, p
(f) Gaussian criterion		σ

FIGURE 2.1: Types of criteria in PROMETHEE

Assuming that for all pairs of alternatives $(a, b) \in A$, the preference indices $P_j(a, b)$ have been calculated, the overall preference $\Pi(a, b)$ is calculated taking into account a weight w_j of each criterion j. This preference $\Pi(a, b)$ represents the weighted average of the partial preference functions $P_j(a, b)$. It is calculated as follows:

$$\Pi(a, b) = \frac{\sum_{j=1}^{m} w_j P_j(a, b)}{\sum_{j=1}^{m} w_j} \tag{2.1}$$

The preference indices Π for all pairs in A are represented as a valued graph. This graph is represented by modeling two arcs between alternatives a and b, representing $\Pi(a, b)$ and $\Pi(b, a)$ respectively. For a certain alternative we can define two concepts based on these arcs: entering flow and leaving flow. These flows, represent the origin and destination, so that for instance, the arc represented by $\Pi(a, b)$ indicates that an arrow is leaving from a to b and entering from a to b.

The leaving flow of node a is the sum of the arcs leaving a, providing a measure of the outranking character of a. It is calculated as follows:

$$\eta^+(a) = \sum_{b \in A} \Pi(a, b) \tag{2.2}$$

The entering flow of a measures the outranked character of a. It is calculated as follows:

$$\eta^-(a) = \sum_{b \in A} \Pi(b, a) \tag{2.3}$$

Using these positive and negative flows for all alternatives in A, different exploitation procedures of this graph are applied to provide the best solution depending the problem that the DM is facing (i.e., ranking, sorting or choice). The two most known PROMETHEE methods are PROMETHEE I and PROMETHEE II, applied for ranking problems.

On the one hand, for the case of PROMETHEE I, the partial pre-order is obtained from the entering and leaving flows:

- aPb: if $\eta^+(a) > \eta^+(b)$ and $\eta^-(a) < \eta^-(b)$, or
  $\eta^+(a) > \eta^+(b)$ and $\eta^-(a) = \eta^-(b)$, or
  $\eta^+(a) = \eta^+(b)$ and $\eta^-(a) < \eta^-(b)$;
- aIb: if $\eta^+(a) = \eta^+(b)$ and $\eta^-(a) = \eta^-(b)$;
- aRb: otherwise.

On the other hand, PROMETHEE II yields to a complete pre-order (no incomparabilities) by calculating the net flow score of a, which is the balance between $\eta^+(a)$ and $\eta^-(a)$, so that the greater is $\eta(a)$ the better.

$$\eta(a) = \eta^+(a) - \eta^-(a). \tag{2.4}$$

It can be easily represented in a complete pre-order as follows:

- aPb: if $\eta(a) > \eta(b)$,
- aIb: if $\eta(a) = \eta(b)$.

2.3.2.2 ELECTRE

The ELECTRE (ELimination Et Choix Traduisant la REalité) method was designed in France in the late 60s by Bernard Roy. ELECTRE methods aim at building a binary outranking relation S, where aSb means "a is at least as good as b". ELECTRE methods have been widely acknowledged as effective and efficient decision aiding tools, with successful applications in different domains [Abedi et al, 2012, Arondel and Girardin, 2000, Colson, 2000, Damaskos and Kalfakakou, 2005, Papadopoulos and Karagiannidis, 2008, Sánchez-Lozano et al, 2014, Shanian et al, 2008, Xu and Ouenniche, 2012].

ELECTRE methods establish a realistic representation of four basic situations of preference: indifference, weak preference, strict preference, and incomparability. Considering two alternatives, a and b, the four basic situations are defined as follows:

- Strict preference (aPb): it corresponds to a situation where there are clear and positive reasons in favor of one of the two alternatives,
- Weak preference (aQb): it corresponds to a situation where there are clear and positive reasons that invalidate strict preference in favor of one of the two alternatives, but they are insufficient to deduce either the strict preference in favor of the other alternative or indifference between both alternatives, thereby not allowing either of the two preceding situations to be distinguished as appropriate,
- Indifference (aIb): it corresponds to a situation where there are clear and positive reasons that justify an equivalence between the two actions,
- Incomparability (aRb): it corresponds to an absence of clear and positive reasons that would justify any of the three preceding relations. According to Roy [1991], incomparability may occur for several reasons:

- Zones of uncertainty in the DM's mind, conflicts or contradictions;
- The fact that the analyst who built the model ignores, in part, how the DM compares two alternatives;
- Imprecision, uncertainty, inaccurate determination of the maps of the criteria performances by means of which a and b are compared.

For selecting the most appropriate ELECTRE method, depending on the decision-aid context, three types of criteria may be considered: true-criteria, quasi-criteria and pseudo-criteria. We first introduce the two discrimination thresholds that may be associated to the different types of criteria and may either be fixed or dependent on the performance $g_j(a)$. The uncertainty of the DM preference model can be represented with the following intra-criteria parameters:

- indifference threshold $q_j[g_j(a)]$, below which the DM is indifferent to two alternatives in terms of their performances on criterion g_j;
- preference threshold $p_j[g_j(a)]$, above which the DM shows a clear strict preference of one alternative over the other in terms of their performances on criterion g_j.

When comparing two alternatives, $a, b \in A$, with respect to criterion $g_j \in G$, it is usually assumed that the thresholds are functions of the worst performance of the two alternatives. For the sake of simplicity, in the rest of the thesis, the notation of $q_j[g_j(a)], p_j[g_j(a)]$ is simplified as $q_j(a), p_j(a)$.

- True-criteria: This criterion model applies for $q_j(a), p_j(a) = 0$. Thus, indifference only occurs when $g_j(a) = g_j(b)$.
- Quasi-criteria: This criterion model considers indifference between small differences, such that $q_j(a) > 0$ and $q_j(a) = p_j(a)$.
- Pseudo-criteria: The most recent ELECTRE methods model criteria as pseudo-criteria [Rogers and Bruen, 1998] for handling the imprecision and uncertainty inherent to complex human evaluation processes. Consequently, the outranking relation can be interpreted as a fuzzy relation, such that $q_j(a), p_j(a) > 0$ and $q_j(a) < p_j(a)$.

In Table 2.2, the types of criteria used for each ELECTRE method and the decision problem it handles are presented.

Table 2.2: ELECTRE methods and type of criteria handled

Criteria type	Choice	Ranking	Sorting
True-criteria	I	II	-
Quasi/Pseudo-criteria	IS	III, IV	TRI-B, TRI-C, TRI-NC

For all binary relations $(a, b) \in A \times A$ on $j - th$ criterion, three relations may be established: preference, weak preference and indifference (i.e., P_j, Q_j, I_j); that can be grouped into the partial outranking relation aS_jb, so that $aP_jb \vee aQ_jb \vee aI_jb = aS_jb$. For instance, aS_jb means that a is at least as good as b on criterion j.

Then, the comprehensive outranking relation aSb is made on the basis of two social-inspired rules: the "majority opinion" and the "right to veto", so that aSb is considered to be true if there are sufficient arguments to affirm that a is not worse than b, and if there is no essential reason to refuse this assertion [Greco et al, 2010].

Considering the possible binary relations for $(a, b) \in A \times A$, the four basic situations of preferences may occur:

- aSb and $\neg(aSb)$: a is weakly or strictly preferred to b,
- bSa and $\neg(aSb)$: b is weakly or strictly preferred to a,
- aSb and bSa: a is indifferent to b,
- $\neg(aSb)$ and $\neg(aSb)$: a is incomparable to b.

In ELECTRE methods, the concepts of "majority opinion" and the "right to veto" are formalized into the definition of concordance and discordance indices. To build the outranking relations, two criteria parameters are required: the relative importance of each criterion w_j and the veto threshold $v_j[g_j(a)]$.

A weight w_j expresses the relative importance of criterion g_j, as it can be interpreted as the voting power of each criterion to the outranking relation. Let us denote W the sum of all weights of the criteria in G. The higher the intrinsic weight, the more important the criterion is. The weights of criteria do not represent substitution rates as in the case of compensatory aggregation operators. To facilitate the task of set weights in ELECTRE methods, 2 main methodologies have been defined. In case of having a set of solved examples (i.e., supervised dataset) a suitable way to find the weights is using the Robust Ordinal Regression (ROR) method [Corrente et al, 2013a]. When no solved examples are available, a well accepted procedure to help the DM to give the weights for ELECTRE methods is the Simos' procedure [Figueira and Roy, 2002]. This approach consists of associating a *playing card* with each criterion and rank these cards from the less to the most important with possibly *ex aequo* (cards or criteria with the same rank). The DM can put "white cards" between these ranks to express the relative power of each one, by making smaller or bigger the difference between ranks.

The veto threshold $v_j[g_j(a)]$ is associated to the performance $g_j(a)$, where a discordant difference in favor of one alternative greater than this value will require the DM to negate any possible outranking relationship indicated by the other criteria. Thus, the logic for the threshold $v_j(a)$ is to state that the difference between $g_j(b)$ and $g_j(a)$ must be sufficiently "large" in order to determine that aS_jb. The veto threshold $v_j[g_j(a)]$ is simplified as $v_j(a)$.

The concordance and discordance index calculations for the simplest ELECTRE methods (i.e., ELECTRE-I and ELECTRE-II) do not take into account pseudo-criteria but true-criteria (with veto threshold). ELECTRE-I was originally created for choice problems and ELECTRE-II is a result of the first modification of the original ELECTRE to rank alternatives.

First, a concordant coalition $C(a, b)$ and a discordant coalition $D(a, b)$ is calculated as follows:

$$c(a, b) = \frac{1}{W} \sum_{\forall j : g_j(a) \geqslant g_j(b)} w_j \tag{2.5}$$

$$d(a, b) = \begin{cases} 1 \text{ if } g_j(b) - g_j(a) > v_j \text{ for any } j, \\ 0 \text{ otherwise} \end{cases} \tag{2.6}$$

This concordance index $c(a, b) \in [0, 1]$ is the consensus or coalition of the agreement to the assertion aSb and the partial discordance $d_j(a, b)$ expresses a an opposition to aSb. Thus, with the combination of the overall concordance and the discordance indices, an outranking binary relation is built as follows:

$$aSb \text{ if } \begin{cases} c(a, b) \geqslant \hat{c} \text{ and } d(a, b) = 0 \end{cases} \tag{2.7}$$

where $\hat{c}$ is the concordance level required to consider that $c(a, b)$ is strong enough to support aSb. On the one hand, if $\hat{c}$=1, it is understood that all criteria must be in favor of aSb (unanimity), which is the so-called dominance relation $\triangle$ so that $\hat{c} \subseteq \triangle$ [Bouyssou, 2009]. On the other hand, if $0.5 \leqslant \hat{c} < 1$, not all criteria must be in favor of aSb but at least a sufficient majority $c(a, b)$ should be in favor, so that if the condition $c(a, b) \geqslant \hat{c}$ is fulfilled, then aSb.

Once an outranking matrix relation for $A \times A$ is generated, the exploitation procedure comes into play to provide the recommendations to the DM, which may be a set of the best alternatives for the choice problem (ELECTRE-I) or a ranking of the alternatives (ELECTRE-II). The exploitation procedure starts with the construction of a graph of the alternatives in A represented by the outranking relations [Figueira et al, 2005].

ELECTRE-I determines the best set of alternatives, called Kernel, by identifying those that fulfill the following two properties, building the kernel K of alternatives if:

1. Any alternative not in the set is outranked by at least one alternative which is in the Kernel,
2. All alternatives in the Kernel are incomparable.

The inability to produce a ranking of alternatives originated ELECTRE-II. The calculation of the concordance and the discordance indices are calculated as in ELECTRE-I. However, ELECTRE-II introduces the concepts of strong $aS^{+}b$ and weak $aS^{-}b$ binary outranking relations.

Two concordance levels s^{+} and s^{-} are then chosen to generate two outranking relations S^{1} (strong outranking) and S^{2} (weak outranking), where $s^{+} > s^{-}$ and $s^{+}, s^{-} \in [0.5, 1 - min_{j \in J} w_j]$.

The strong outranking relation is obtained as follows:

$$c(aS^{+}b) \geqslant s^{+} \text{ and } c(aS^{+}b) \geqslant c(bS^{+}a) \tag{2.8}$$

The weak outranking relation is obtained analogously:

$$c(aS^{-}b) \geqslant s^{-} \text{ and } c(aS^{-}b) \geqslant c(bS^{-}a) \tag{2.9}$$

Notice that the conditions $c(aS^{+}b) \geqslant c(bS^{+}a)$ and $c(aS^{-}b) \geqslant c(bS^{-}a)$ are applied to avoid the outranking between both alternatives, i.e., a outranking b and viceversa.

Once the strong and weak outranking relations have been calculated for all alternatives $a \in A$, the exploitation procedure is performed to rank these alternatives. This step yields to two complete pre-orders, an ascending and a descending one. The latter ranks the alternatives from the best to the worst alternative, while the former from the worst to the best. The ELECTRE-II method exploitation procedure can be summarized concisely in the following steps, as illustrated in Belton and Stewart [2002]:

1. Determine the set of alternatives, $M \subset A$ which are not strongly outranked by any other alternative in A.

2. Within M determine the subset of alternatives, say M', which is not weakly outranked by any other member of M. This defines the first set of the descending ranking.

3. Delete the alternatives in M' from A, and repeat the procedure from step 1, continuing until all alternatives have been classified. This generates the descending order.

4. Start again with A being the full set of alternatives.

5. Determine the set of alternatives say $N \subset A$, which does not strongly outrank any other alternative.

6. Within G determine the subset of alternatives, say N', which does not weakly outrank any other member of N. This defines the first set of the ascending ranking.

7. Delete the alternatives in N' from A, and repeat the procedure from step 5, continuing until all alternatives have been classified. This generates the ascending order.

Then, these two complete pre-orders are intersected to generate a partial pre-order with incomparabilities. For the case of preference modeling using true-criteria in ELECTRE-II, the incomparability relation between two alternatives is built if aSb and bSa, taking into account that the indifference relation is not considered.

The major inconvenience with these two approaches is the lack of interpretation of inaccurate and imprecise determination of data. Further extensions of ELECTRE allows to deal with this inaccuracy in the data with the pseudo-criteria, as presented in Sections 4 and 5. This allows a valued (or fuzzy) outranking relation.

2.3.2.3 Strengths and weaknesses of ELECTRE methods

The strengths and weaknesses of the classical ELECTRE methods are presented in Figueira et al [2013], Ishizaka and Nemery [2013a]. The strengths include the following:

- ELECTRE methods are able to take into account the qualitative nature of some criteria, allowing the DM to consider the original data directly, without the need to make transformations into artificial numerical scales.

- ELECTRE methods can deal with heterogeneous criteria scales, preserving the original scores of the alternatives on each criterion coded in an ordinal scale or a "weak" interval scale [Bouyssou et al, 2006], without the need for normalization techniques or the assessment of a value function. This heterogeneity of scales is usually an inconvenience for many decision support systems, which often require a common measurement scale for all criteria.

- ELECTRE follows a the non-compensatory character in the aggregation. In the ELECTRE approach, if, on a certain criterion, an alternative is strongly opposed to the assertion aSb, this fact is enough to reject the assertion aSb.
- ELECTRE methods incorporate the notion of incomparability between a pair of alternatives, referring to the case where one option is better that the other in some criteria and simultaneously is worse in other criteria, making impossible the establishment of a preference relation between them.

The main weaknesses of classical ELECTRE methods are as follows:

- When the aim is to calculate an overall score for each alternative, ELECTRE methods are not suitable and other scoring methods should be applied.
- When all the criteria are quantitative, it is better to apply another method, unless we are dealing with imperfect knowledge or a non-compensatory process should be taken into account.

2.3.2.4 Comparison between PROMETHEE and ELECTRE methods

Despite the general concept of outranking, some differences between PROMETHEE and ELECTRE methods can be found.

- The PROMETHEE valued preference index is similar to the concordance principle, but it measures the "preference degree" of a over b. The concordance principle in ELECTRE verifies if the assertion aS_jb is true using ordinal scales, distinguishing aI_jb, aP_jb, aQ_jb. A hesitation $aQ_jb \in (0,1)$ closer to 1 indicates a closer relation to indifference, whereas the closer to 0 indicates a closer relation to strict preference in favor of b over a. Thus, the difference between the performance of two alternatives cannot be considered as intensity of preference. However, for some authors the construction techniques are so similar that such different interpretations can hardly be justified [Bouyssou et al, 2006];
- The aggregation of partial preference degrees in PROMETHEE leads to a global preference index $\Pi(a,b)$ that indicates a degree of how much a is preferred over b considering all criteria. For instance, $\Pi(a,b)$=1 indicates a strong preference of a over b. In ELECTRE, the aggregation leads to a credibility degree $\rho(a,b)$ which synthesizes the strength of the coalition of criteria in favor of the assertion aSb. For instance, a $\rho(a,b)$=1 does not

indicate that a is preferred over b, as we must consider also $\rho(b, a)$. For example, $\rho(a, b)$=1 and $\rho(b, a)$=1 indicates indifference between a and b.

- ELECTRE methods rely on two indices to build a relation aSb, considering the concordance and discordance principles. PROMETHEE relies only on the preference index. Therefore, there is no "right to veto" concept in PROMETHEE;

- The exploitation procedures applied for both methods to obtain the final result are different.

2.3.3 Discussion about classical outranking methods

Although outranking methods have been very successful in several disciplines, they do not consider the organization and analysis of criteria in a hierarchical structure. For instance, in ELECTRE and PROMETHEE methods, we consider all criteria together without any consideration of a multilevel structure of sub-problems. This can be a real issue for the DM for complex decision problems involving a large set of criteria or criteria that can be naturally defined as a sub-problem of the global problem. For example, the analysis of decision problem involving economic and social impact criteria with an outranking method implies the following situations:

- Defining the relative importance of each criterion with regard to the rest of criteria. How can a DM clearly define the relative importance of an economic criterion with regard to not only other economic criteria, but also social impact criteria? Seems more intuitive to structure this problem reformulating economic and social impact related criteria as two sub-problems, so that the relative weights are defined based on common or related criteria. Then, in a more general context, determine how much important is the economic sub-problem with respect to social impact sub-problem.

- How can a DM analyze partial results to better understand the recommendations, as for example analyze the best recommendations for social impact and economics individually to have a better picture of the global results? This question cannot be solved by applying subsets of sub-criteria individually for economic and social impact criteria, obtaining a partial result for each sub-criterion, and then repeating the analysis with all criteria together in a flat level to find an overall result because the problem statement is particular for each subset of criteria.

- If the decision method is not able to work in a hierarchical way, the DM faces two strong limitations. First, when constructing the decision model;

> and second, when obtaining a simple overall result. The hypothesis of this Ph.D. thesis is that decomposing the decision problems into sub-problems, permits more flexible and realistic decision models based on the DM beliefs and needs. Hence, appropriate MCDA methods are required to deal with hierarchies of criteria.

In this thesis we address this issue to tackle, focusing on ELECTRE methods as it strictly apply the outranking concept presented originally by Roy. ELECTRE methods also apply the "discordance" concept, that may be interesting for those decision problems in which a bad performance in a certain sub-problem may not be compensated in the aggregation applying the discordance principle. For example, in a ranking problem, if alternative a has a bad performance in the economic sub-problem may not compensated with a good performance in the social impact sub-problem.

In the next chapter, we introduce the background literature about hierarchical structures of criteria in decision problems. Also, the most relevant works related to different hierarchical MCDA approaches are presented.

Chapter 3

Hierarchical structures of criteria in decision problems

As explained in Chapter 1, human decision making follows a comprehensible hierarchical process that implicitly presents a taxonomical structure of criteria with different levels of generality. Many real-world decision problems involving criteria from different nature, field or interest may be grouped into smaller related sub-problems, thus facilitating not only the setting of relative importance (i.e., weights) of each specific criterion with respect to the rest of the related criteria, but also at more general levels in the hierarchy. This yields a more natural and efficient process of making a final decision for the given problem based on the DM's needs.

A weakness of the outranking methods, including ELECTRE methods, is the lack of consideration of complex and conflicting criteria modeled in a hierarchy. Therefore, the application of outranking methods for this organization of criteria is unfeasible, giving two options to the DM: 1) Consider a different MCDA approach, which may not always be suitable depending on the decision problem taking into account the strong features that outranking methods provide (e.g., non-compensatory effect) and 2) Restructure the criteria into a flat organization. However, this results in a different problem statement with no criteria generalization and therefore, a different result may be obtained.

In this chapter we focus on the definition of hierarchical structures and their consideration in several fields, their application in ranking and sorting decision problems and the most relevant MCDA approaches applied to solve them.

First, Section 3.1 introduces some background in different fields where hierarchical structures have been considered to better organize and structure related objects since decades ago. Then, Sections 3.1.1 and 3.1.2 present complex ranking and sorting decision problems respectively, in which the DM has organized the criteria in a hierarchical structure following a tree structure. Some examples are given to illustrate how a hierarchical structure helps the DM to acquire detailed knowledge of the decision problem. Next, Section 3.2 reviews different MCDA approaches considered to solve them and finally, Section 3.3 defines the concepts and notation with regard to the hierarchical structure of criteria.

3.1 Hierarchies in decision problems

In general terms, a hierarchical structure is a system to organize related entities into several levels arranged in a treelike structure. Hierarchical structures have been studied for quite a long time in several disciplines including biology [Webster, 1979], ecology [Allen and Starr, 1988], psychology [Kozielecki, 1981], and data management [Henry, 1969].

In Kozielecki [1981], a psychological study regarding decision task modeling in hierarchical structures is presented, following the *principle of hierarchic arrangement*. The principle of hierarchic arrangement [Davenport, 1960] states that the representation of the world can be modeled in a hierarchical organization, e.g., our experience is coded in hierarchical structures. In decision tasks, hierarchical organizations may be applied to alternatives and consequences (i.e., criteria or attributes), reducing the complexity of the problem and the excessive cognitive strain. This section is focused on hierarchical organizations of criteria.

A hierarchy of criteria is a tool that enables the DM to better organize the problem based on its knowledge on the problem domain to explicitly express his/her needs. This process decomposes a complex goal into smaller problems involving subsets of criteria, enabling the DM to analyze alternatives with respect to a particular part of the problem and at different levels of generality.

In many applications, the hierarchical structure of criteria has the form of a tree, where the *root criterion* is placed at the top of the hierarchy, representing the general or overall goal of the DM. The leaves of the tree, placed at the lowest level of the hierarchy are called *elementary criteria*, represents the most specific criteria in which the DM directly evaluates the alternatives. The nodes in the hierarchy between the root and the leaves are called *intermediate criteria* or *sub-criteria*, and represents partial sub-problems of the main problem, defined on the basis of other intermediate criteria or elementary criteria directly descending from them.

This hierarchical approach is particularly suitable for complex problems with a large number of criteria or because there is a natural organization of the criteria into subgroups. In such cases it may become cognitively difficult for the DM to consider all of the criteria together [Mustajoki, 2012]. Hence, by using a hierarchical structure distinguishing different levels of generality, one may model the implicit taxonomical relations between the criteria, which divide the decision process in different steps [Matsatsinis et al, 1997].

3.1.1 Hierarchical structures in ranking problems

Several works in ranking decision problems involving hierarchical structures of criteria from a wide range of diverse disciplines are presented:

- Environmental resource management is a discipline that commonly involves conflicting interests such as economic, environmental impact and social criteria [Bobylev, 2011, Dujmovic et al, 2010, Nordstrom et al, 2010, Valls et al, 2010]. For example, in Nordstrom et al [2010], a case study of a planning process for an urban forest in Sweden is addressed. The paper evaluates three alternative strategic forest plans for areas around the urban forest in Lycksele, Sweden. The interests of four social groups are considered (timber producers, environmentalists, recreationists and reindeer herders). For each group, different sub-criteria with differing preferences are taken into account (e.g., timber producers want to maximize the fertilized area, while reindeer herders wish to minimize it).

- Complex decision models appear also in medicine [Ahsan and Bartlema, 2004, Mendis and Gedeon, 2012, Reddy et al, 2014]. In Ahsan and Bartlema [2004], a study of the public healthcare management system of Bangladesh, which operates mainly through health complexes, attempts to find the best and worst performing areas of the healthcare system. This particular case study distinguishes between different activities such as maternal care, child health or family planning. These main health activities are evaluated independently to identify their respective strengths and weaknesses. Next, an overall aggregation is performed. In Reddy et al [2014], it is presented a study for producing national guidance relating to the promotion of good health and the prevention and treatment of disease, at the Centre for Public Health (CPH) at the United Kingdom's National Institute for Health and Care Excellence (NICE). The objective is to choose the most appropriate topics for this guidance taking into account a 3-level hierarchy of criteria with 3 main sub-criteria: size of the problem, making the difference, and current variation in practice.

- Another area that is growing in popularity is the construction of rankings based on Quality Assessment, such as institution rankings [Aydin et al, 2012, Buyukozkan et al, 2011, Hsu and Pan, 2009, Torres-Salinas et al, 2011]. Complex sets of diverse criteria are used to build a ranking of alternatives taking into account different topics. For example, in Buyukozkan et al [2011], a model to evaluate perceived service quality in the healthcare sector and to evaluate the performance of pioneering Turkish hospitals in many different topics such as responsiveness, professionalism and empathy is presented, with a 3-level hierarchy of criteria. In Aydin et al [2012], the European Foundation for Quality Management (EFQM) Excellence Award evaluates organizations on the basis of three main topics: -leadership, strategy and processes- that are splitted into a 3-level hierarchy of criteria.

- In Shen et al [2012], a road safety performance evaluation for a group of European countries is presented. Several road safety performance criteria including speed, alcohol consumption and protective systems are structured hierarchically, allowing the analysis of each country's performance for each one of this criterion based on index scores.

- Sustainability assessment aims at planning and decision-making towards sustainability of resources. In Nzila et al [2012], an energy sustainability problem in Kenya is studied to eliminate energy poverty in rural households by the consideration of biogas technology, as this technology is considered as a tool for reducing the cutting of trees for charcoal or firewood as well as for combating health complications as a result of firewood smoke. Three different biogas digester plants are compared based on three main factors: environment, technical and economic. For each of these factors, 3 criteria are evaluated. In another sustainability problem, Afsordegan et al [2014] studies a wind farm location problem in the region of Catalonia (Spain). Recently, the advantages of the use of wind farms are acknowledged because of the simple installation, lack of contaminant emissions and low water consumption. However, the installation of wind farms is generally under scrutiny due to the public opinion. This study considers complex criteria such as economic, social, environmental and technical indicators that must be taken into account to find the best location to install the wind farms.

- Business management is based on strategic decisions that include complex criteria and therefore can be modeled in a hierarchical structure [Arbenz et al, 2012, Chang et al, 2015, Kilic et al, 2015, Muerza et al, 2014, Wang et al, 2004, Yang et al, 2009]. For example, in Wang et al [2004], a manufacturing chain decision problem is analyzed. The overall goal is to achieve optimal supplier efficiency with regards to a hierarchical structure of criteria, from basic indicators to four general measures of efficiency: delivery reliability, flexibility and responsiveness, cost, and assets.

3.1.2 Hierarchical structures in sorting problems

In the ELECTRE-TRI literature, sorting problems in many different fields where the DM organizes the criteria following such a hierarchical structure can be found. However, because of the limitations that present ELECTRE-TRI methods regarding the decomposition of criteria in a hierarchical structure, the problem is finally solved using a flat structure of criteria, by putting all criteria together in a unique group.

In Sánchez-Lozano et al [2014], the identification of the best plots suitable for installing photovoltaic solar farms in the Municipality of Torre Pacheco in Murcia (Spain) is studied. A Geographic Information System (GIS) provides a cartographic and alphanumeric database, including two factors of distinct nature: restrictions and criteria. The restrictions are entered into the GIS using layers defined from the current legislation (urban land, undeveloped land, special protection areas for birds, community sites, infrastructures, etc.), reducing the study area by eliminating those areas in which photovoltaic solar farms cannot be implemented. Then, the resulting areas are classified according to multiple criteria using the ELECTRE-TRI-B method. In this case study, the DM has structured the criteria into a 3-level hierarchical tree.

In financial decision making, portfolio selection and management constitutes one of the most significant domains. A finance portfolio selection is presented in Xidonas et al [2009], which entails the construction of a portfolio of equities (or securities from other asset classes) that maximizes the investor's utility. The DM has to evaluate and select the equities that are available as investment opportunities. In this case study, 4 main intermediate criteria sets are defined: (a) industry/commerce firms, (b) financial services firms, (c) banking institutions and (d) insurance firms. The ELECTRE-TRI method is then applied separately and finally, the partial results are integrated in a second stage. In this second stage, a mixed-integer multi-objective mathematical programming model is applied in order to generate the Pareto optimal portfolios.

In Arondel and Girardin [2000], an implementation of ELECTRE-TRI-B is applied in order to answer a question of researchers of the *Institut National de la Recherche Agronomique* (INRA), who assess the impact of agricultural practices on the environmental components. They were particularly interested in differentiating cropping systems as a function of their impact on groundwater quality. Cropping systems can be harmful to groundwater quality through three agricultural practices: nitrogen, pesticides and water management. Four categories of impact were defined and the total of 33 criteria are analyzed based on these three agricultural practices.

3.2 MCDA approaches for hierarchies of criteria

In the MCDA literature, very few methods consider the decomposition of decision problems using a hierarchy of criteria for ranking problems. The best known method for managing hierarchical structures is the Analytic Hierarchical Process (AHP) [Saaty, 1987], which belongs to the utility-based approach.

AHP permits the DM to focus on specific sub-criteria to find the weights of each criterion depending on its position on the hierarchy by means of pairwise comparison of criteria having the same parent, which yields the relative trade-off weights. The pairwise comparison of alternatives and criteria is based on the judgment ratio scale from 1 to 9, in which 1 represents "Equally preferred" and 9 represents "Extremely preferred". Once the comparison matrix has been given by the DM, weights or priorities are derived finding the normalized eigen vector of the matrix. This requires the matrix to be consistent (or near consistent) to obtain meaningful priorities. Then, a numerical rating is obtained for each of the decision alternatives by means of an additive aggregation operator. AHP was applied to some of the case studied mentioned above [Ahsan and Bartlema, 2004, Bobylev, 2011, Hsu and Pan, 2009, Muerza et al, 2014, Nordstrom et al, 2010, Reddy et al, 2014]. In Buyukozkan et al [2011], the Analytic Network Process (ANP) method, which is a generalization of AHP for networks instead of hierarchies, is applied. The difference between AHP and ANP, is such that ANP does not consider the alternatives as independent actions.

Despite the large literature and applications of AHP, the method has also received some critics. The consistency condition is difficult to achieve, several consistency indices have been proposed, as well as methods to obtain a transitive matrix [Bana e Costa and Vansnick, 2008]. The additive nature of the aggregation has also been posed into question because it generates rank reversals [Ishizaka and Labib, 2011], but also because it is a compensative trade-off approach, which is not appropriate in some applications.

Another weakness is the imprecision and uncertainty of the linguistic scale used for the construction of the pairwise comparison matrices. To overcome this weakness, the Fuzzy-AHP method has been proposed, which applies a range of value to incorporate possible DM's uncertainty instead of merely crisp ratio values. In Aydin et al [2012], Fuzzy-AHP is used to achieve a performance assessment of firms for EFQM Excellence Award using fuzzy scales to make pairwise comparisons. Several Fuzzy-AHP applications are presented in Mardani et al [2015] from 1994 to 2014. For ANP, a fuzzy approach has also been introduced [Chang et al, 2015].

In other complex problems, AHP and ANP are combined with other methods to treat hierarchical structures of criteria. For example, in Kilic et al [2015], Sánchez-Lozano et al [2013], AHP and ANP respectively are applied only to establish the weights of the criteria in the hierarchy.

For sorting problems, the introduction of hierarchical structures has not been taken into account with the same efforts as for ranking problems. In Ishizaka et al [2012], an AHP hierarchical sorting method, called AHPSort, is presented. The method is similar to the classical AHP method. However, instead of comparing each alternative with the rest of alternatives, each alternative is compared to profile limits indicating the minimum performance (i.e., $\in [0,1]$) needed on each criterion to belong to a certain class C from a set of predefined ordered categories previously defined by the DM. The weight calculation remain the same for this method (i.e., comparing the preference of a certain criterion with the preference of the rest of criteria in the subset). The AHPSort method follows the compensatory approach of AHP, so that if alternative a has a bad performance on criterion j with respect to a profile limit pf, a good performance of a with respect to this profile limit pf on criterion i can compensate the overall performance of a over pf. Thus, a may still be assigned to C.

There are some other utility-based approaches where aggregation operators are used to generate ratings of alternatives at different levels of generality, which may be useful for generating rankings based on these ratings. An interesting case is the method called Logic Scoring of Preference (LSP), where the operators are parametrized and can range from full conjunction, partial conjunction, partial disjunction and full disjunction. In addition, mandatory and optional criteria can be defined and treated accordingly in the different levels of the hierarchy. Some applications of this method for decision aiding are [Dujmović and De Tré, 2011, Hatch et al, 2014, Pijuan et al, 2010]. The main limitation is the complexity of the problem modeling using such high level operators on the basis of its logical properties. Moreover, all the values need to be in the same numerical scale, not allowing heterogeneity as in an outranking-based approach.

In other approaches, a flat level of criteria are finally considered despite the natural organization of the criteria. For instance, Nzila et al [2012] applied the Multi-criteria Spider-gram Cumulative Surface Area (MCSA score). However, a study presented in Dias and Domingues [2014] shows that this method may not be appropriate for a MCDA decision problem, as the results depends on the order of the criteria. In Afsordegan et al [2014], the Qualitative TOPSIS and the Condorcet-Kemeny-Young-Leveng (CKYL) are applied and compared. Both methods consider all the criteria together in the analysis.

Hierarchies of objectives are also considered in DEA (Data Envelopment Analysis) [Shen et al, 2012] and PGP (Preemptive Goal Programming) [Wang et al, 2004], but these methods concern a continuous space rather than a discrete set, considered in this thesis.

In recent years, another methodology, called Multiple Criteria Hierarchy Process (MCHP), has been proposed to deal with hierarchical structures of criteria [Corrente et al, 2012]. It can be applied to any MCDA method, including utility-based and outranking methods. For the outranking methods, this process is explained in Corrente et al [2013b] for the ELECTRE-III method. It builds crisp outranking relations (i.e., aSb=1, $\neg(aSb)$=0) at each node of the hierarchy. First, the ELECTRE-III method is applied first on the lowest level of the hierarchy to build a binary outranking preference relations for each subset of elementary criteria. Next, at upper levels, MCHP continues constructing binary outranking relations which are propagated up to the root. The preference information used to construct the outranking relations can be provided by the DM either directly (in form of outranking model parameters, like criteria weights and comparison thresholds) or indirectly (in form of pairwise comparisons of some alternatives). In the latter case, MCHP is combined with the Robust Ordinal Regression (ROR) [Corrente et al, 2013b]. The ROR takes into account all sets of outranking model parameters compatible with the preference information provided by the DM to give a solution in terms of necessary and possible outranking relations, by applying all the compatible preference models on the considered alternatives. The authors present an illustrative example regarding the evaluation of students who are competing for a scholarship based on Mathematics and Chemistry that decompose to more specific subjects. The approach based on indirect preference information relies on having a suitable set of decision examples, which may sometimes be hard to find when there is no historical data or the user is inexperienced. This is a potential shortcoming in some applications.

The main difference between the proposal presented in this thesis and the direct method described in Corrente et al [2013b] for outranking methods is such that the proposal presented in this thesis is aimed at applying the ELECTRE procedure (construction and exploitation of outranking relations) at all levels of the hierarchy.

3.3 Formalization of a hierarchical structure of criteria

This section defines the concepts and notation of the hierarchical structure of criteria considered in this thesis. The hierarchical structure of criteria distinguishes between three types of criteria depending on their level of generality in the taxonomy:

- $\mathcal{R}$ is a set composed by a unique element that is the most general criterion. This corresponds to the root node, placed at the top of the tree. This criterion represents the main goal of the DM.
- $\mathcal{E}$ is the set of the most specific criteria, called elementary criteria. They are placed at the lowest level of the hierarchical tree (i.e., the leaves). The performance of the alternatives is evaluated only in relation to these elementary criteria.
- $\mathcal{I}$ is the set of intermediate criteria (or sub-criteria). They correspond to generalizations of other sub-criteria or elementary criteria. They are placed at intermediate levels of the tree, between $\mathcal{R}$ and $\mathcal{E}$.

The grouping of elements in $\mathcal{E}$ and $\mathcal{I}$ into a more general level in the tree relies on the expert's knowledge and personal needs. Thus, the relation of the criteria forming subsets is subjective. Consider the example presented in Figure 3.1.

A x G	***Durability***	***Suitability***	***Damage cost***	***Architecture cost***	***Landscape***	***Geometry***	***Environmental preservation***
a	45	35	60	90.000.000	6	8	10
b	80	56	34	120.000.000	8	5	5
c	90	85	67	86.000.000	7	4	7
n	...	...	...	...	...	...	...

FIGURE 3.1: Example of criteria related to the evaluation of different proposal to construct a building

The DM may consider the following subsets of criteria in $\mathcal{I}$:

- Quality= { *Durability, Suitability* },
- Cost= { *Damage Cost, Architecture cost* },
- Shape= { *Landscape, Geometry* }.

Following this model, *environmental preservation* is not included in any intermediate group. Therefore, it is not considered as a criterion related to, for instance, the *quality* of the construction. Hence, we can model the hierarchy of criteria as presented in Figure 3.2.

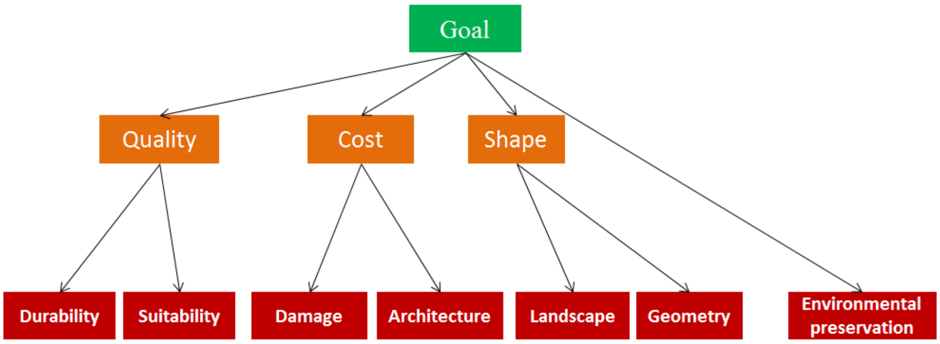


FIGURE 3.2: Example of hierarchical structure where *environmental preservation* is not related to any subgroup

However, another DM with different beliefs and considerations may suggest that the *environmental preservation* is directly related to the *quality* of the construction, as the DM may consider this criterion as a standard in quality construction because potential clients may be interested in this particular issue. The hierarchy of criteria can be modeled as presented in Figure 3.3.

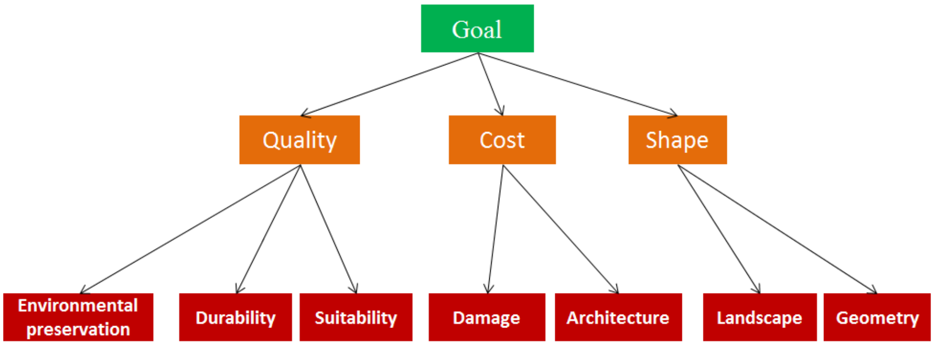


FIGURE 3.3: Example of hierarchical structure where *environmental preservation* is grouped to *quality*

The final result may be different for these two cases, as the modelization of the hierarchy of criteria is different.

Formalizing the hierarchical structure, G is redefined with respect to the definition given in Section 2.1, so that it includes not only the elementary criteria but also the more general criteria, such that $G = \mathcal{R} \cup \mathcal{I} \cup \mathcal{E}$.

Let us set the number of criteria as $m = |G|$, while $1 = |\mathcal{R}|$, $l = |\mathcal{E}|$ and $h = |\mathcal{I}|$, having that $m = 1 + l + h$.

The number of levels of the hierarchy, denoted as $\mathcal{Q}$, must be at least 3, having that $\mathcal{Q} \geqslant 3$.

Definition 3.1. A hierarchical set of criteria is structured according to the following relations:

- The root criterion in $\mathcal{R}$ does not have any parent.
- Each intermediate criterion in $\mathcal{I}$ has a unique parent $g_j \in \mathcal{R} \cup \mathcal{I}$.
- Each elementary criterion in $\mathcal{E}$ has a unique parent $g_j \in \mathcal{R} \cup \mathcal{I}$.
- Criteria in $\mathcal{I} \cup \mathcal{R}$ may have multiple direct descendants $g_j \in \mathcal{E} \cup \mathcal{I}$.

An example of this tree structure is shown in Figure 3.4:

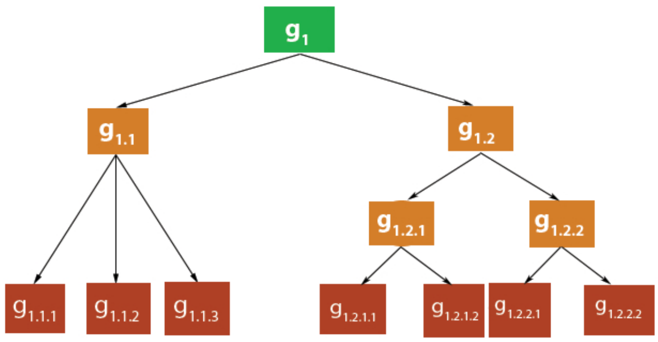


FIGURE 3.4: Hierarchical tree of the set of criteria

The structure presented in Figure 3.4 contains as elements:

- $\mathcal{R} = \{g_1\}$
- $\mathcal{I} = \{g_{1.1}, g_{1.2}, g_{1.2.1}, g_{1.2.2}\}$
- $\mathcal{E} = \{g_{1.1.1}, g_{1.1.2}, g_{1.1.3}, g_{1.2.1.1}, g_{1.2.1.2}, g_{1.2.2.1}, g_{1.2.2.2}\}$
- $G = \{g_1, g_{1.1}, g_{1.2}, g_{1.2.1}, g_{1.2.2}, g_{1.1.1}, g_{1.1.2}, g_{1.1.3}, g_{1.2.1.1}, g_{1.2.1.2}, g_{1.2.2.1}, g_{1.2.2.2}\}$

Definition 3.2. For $D \subset \mathcal{E} \cup \mathcal{I}$, being the set of direct descendants of g_i, each $g_j \in D$ may have a weight w_j that indicates its relative importance with respect to the rest of the descendants of g_i (i.e., the rest of the elements of D).

Figure 3.5 shows the hierarchical tree of criteria with weights assigned to all criteria from set G.

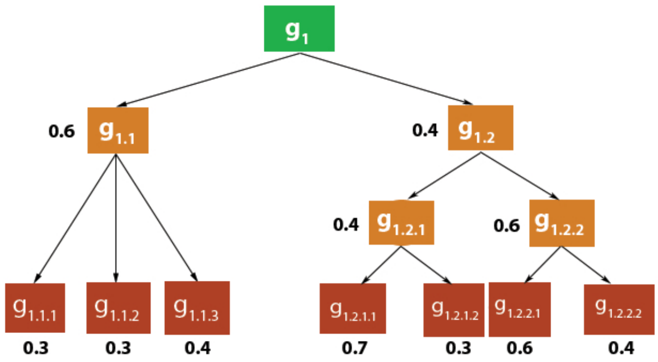


FIGURE 3.5: Hierarchical tree of criteria with their weights

Taking into account that each element in $\mathcal{I}$ represents a partial sub-problem of another sub-problem $\mathcal{I}$ or the global problem $\mathcal{R}$, the weights at each branch of the hierarchy are set based only on the subset of related criteria being analyzed. For instance, the weights of $g_{1.1.1}$, $g_{1.1.2}$ and $g_{1.1.3}$ explicitly indicates the calculation of each of these criteria with respect to its direct parent $g_{1.1}$.

The modelization of the hierarchy of criteria is flexible and subjective, making the process quite natural and easy-to-define according to his/her judgment and expertise in the domain. This structure helps the DM to acquire detailed knowledge of the complex problem, focusing first on single related sub-problems at different levels of the hierarchy.

In the rest of this thesis, we will consider this treelike hierarchy of criteria modelization to study new hierarchical outranking methods.

Chapter 4

The ELECTRE-III-H method for ranking hierarchical problems

In Chapter 3, the advantages of representing complex decision problems that can be organized in a hierarchy structure of related sub-criteria were presented. This section introduces a method for ranking a set of alternatives evaluated on multiple and conflicting criteria organized in a hierarchical structure.

In ranking problems, the DM wants to find an order structure on a set of alternatives taking into account his/her preference of one alternative over the others. This task is not straightforward when multiple criteria must be considered.

The order structure of the alternatives depends on how well they perform on particular criteria and how important each criterion is to the DM. In a hierarchy of criteria we need to generate several order structures (i.e., partial pre-orders) at each intermediate node. These orders are then aggregated at their parent node.

The main contribution of this section is a ranking method that extends the classical ELECTRE-III method by generating and propagating partial pre-orders from the bottom level up to the root of the hierarchy tree. Therefore, the main issue of extending ELECTRE-III is to allow the management of sub-criteria in terms of partial pre-orders.

We first propose an iterative procedure that maintains the two steps of the classical ELECTRE method (construction and exploitation of the outranking relation) in all intermediate nodes of the hierarchy up to the root. The classical ELECTRE-III is applied at the bottom of the tree, aggregating the most specific elementary criteria to their direct parent and obtaining the first results in the form of partial pre-orders. These partial pre-orders are interpreted as inputs for the intermediate criteria from the upper level. The partial pre-orders are aggregated with the construction of a new pairwise credibility matrix. Next, the classical exploitation process (known as distillation) is applied to generate a partial pre-order at the parent node. With this approach, the DM obtains a result (i.e., a partial pre-order) at each of the intermediate levels of the tree, in addition to the overall partial pre-order at the root level.

The second contribution is the definition of new partial concordance and discordance indices that take into account threshold values on partial pre-orders induced by criteria aggregated at intermediate levels of the hierarchy, introducing weak preferences and the right to veto. The DM can also specify the relative importance of each criterion in the context of the same subset at each level of the hierarchy.

This chapter is structured as follows. First, some other works in the aggregation of preference orderings are presented in Section 4.1. Next, Section 4.2 reviews the basic steps and formulations of the classical ELECTRE-III method. The iterative procedure is presented in Section 4.3, detailing step by step the construction and exploitation at different levels of the hierarchy, including elementary criteria level (Section 4.3.1) where the classical ELECTRE-III method is applied; and at intermediate levels (Section 4.3.2), where the new partial concordance and discordance indices are defined depending on the preference relations found in a partial pre-order. A specialization of the hierarchical procedure considering true-criteria and ELECTRE-II method for ranking problems is presented in Section 4.4. Finally, Section 4.5 studies the properties for constructing the binary relation S aggregating partial pre-orders according to new calculations of partial concordance and discordance indices, and Section 4.6 studies the rank reversal phenomena in MCDA and ELECTRE-III-H.

4.1 Aggregation of preference orderings

The aggregation of order structures are often considered in real-world decision problems because they are probably the most intuitive and effective way for representing preference judgments of alternatives, avoiding any type of scale representation [Chen et al, 2013]. However, the aggregation of order structures still represents a challenging problem nowadays. Some recent proposals are presented in this section.

Yager [2001] proposed an algorithm, called Yager's algorithm (YA), for fusing multi-agent preferences G on the basis of weak orders (i.e., complete pre-orders) of the possible actions A to a single fused ordering. YA considers the importance of the agents in terms of a simple rank ordering rather than weights. YA consists of 3 steps: 1) construction of a preference vector, which organizes actions from the best to the worst into a vector of agents organized from the most relevant to the least one. This ordering is normalized with regard to the total number of agents, so that for 2 tied actions, the next row in the column is set to null. For equally relevant agents, the actions are grouped in the same column; 2) definition of the sequence reading, which associates a number S to each cell in the preference vector in a bottom-up approach (i.e., 1 is set for the least important action of the most important agent, 2 is set for the least important action of the second most important agent, and so on), although an up-bottom approach is also possible; and 3) construction of the fused ordering, which determines the consensus of the preference orderings considering the first occurrence of the actions in an ascending manner regarding S. This approach yields a weak ordering consensus of the actions. YA is simple and intuitive, but some relevant problems may arise. For instance, the construction of the fused ordering selects the worst action according to the most important agent preference ordering so that the "majority principle" may not be fulfilled, the incomparability relation is not considered in the aggregation, among others.

An enhanced version of this approach, called EYA (Enhanced YA) is presented in Franceschini et al [2015], alleviating these problems. EYA considers the 3 steps presented in Yager [2001], following an up-bottom approach, incorporating incomparabilities in the preference orderings of the agents and not all the actions must be in the preference orderings. Also, an occurrence parameter in terms of percentage is applied to indirectly measure the level of democracy in the decision to fulfill at least a simple majority (f.i., 50%), avoiding the direct selection of actions in the first occurrence, as applied in YA. The underlying assumption is that the degree of preference of the alternatives in different preference vectors mainly depends on their relative position. EYA also results in a weak ordering. Taking into account that incomparabilities are considered as inputs and, therefore, the possibility of incomparable actions in the output seems quite natural.

In Carlsen [2015], a simple decision tool based partial orderings is studied to support remediation technologies. This study includes 5 possible options for remediation, called RO_x, where x is an option i=1, 2, ...5, and 3 polluting chemicals to remove are considered. To analyze the possible options, a first study based on simple arithmetic average is given. The author concluded that the compensation effect of this approach leads to a significant different remediation results even if the final result is similar. Hence, to solve this problem, a weak ordering based on average orders obtained from diagraming for each RO_x (i.e., the fraction of the chemicals removed in a range $\in [0, 1]$), in a Hasse diagram resulting in a partial order. The average orders are calculated using the so called Rkav for each RO_x, applied in lattice theory.

Several other works in the study of ordering based decision making are presented in Chen et al [2013], including classical social choice methods (Borda count, approval voting, etc), soft constraints, among others.

4.2 The ELECTRE-III method

ELECTRE-I and ELECTRE-II assume that all criteria are true-criteria. In true-criteria any performance difference corresponds to a difference in preference, and the indifference occurs only when two alternatives perform identically on a given criterion (Chapter 2).

The ELECTRE-III method is a ranking method designed to support pseudo-criteria $(q_j(a), p_j(a), v_j(a))$, so that the outranking relation S can be interpreted as a fuzzy relation. The incorporation of pseudo-criteria in ELECTRE-III allows establishing for all binary relations $(a, b) \in A \times A$ on $j - th$ criterion three relations: strict preference, weak preference and indifference (i.e., P_j, Q_j, I_j), unlike ELECTRE-II that only allows two relations: strict preference and indifference.

The ELECTRE-III method follows the two outranking steps: first, the construction of an outranking relation over all the possible pairs of alternatives; second, the exploitation of this outranking relation to solve the ranking decision problem [Figueira et al, 2013]. In Figure 4.1, the two outranking steps in ELECTRE-III are illustrated. For this example let us consider the following:

- $A = \{a, b, c, d, e, f\}$;
- $G = \{g_1, g_2, g_3, g_4\}$.

Additionally, the parameters are shown in Table 4.1.

Table 4.1: Parameters considered for the ranking process example

Alternative	w_j	$q_j(a)$	$p_j(a)$	$v_j(a)$
g_1	25	5	12	20
g_2	15	5	12	25
g_3	40	5	8	12
g_4	20	5	12	25

Input

A x G	g_1	g_2	g_3	g_4
a	93	43	92	86
b	82	60	74	74
c	78	80	60	70
d	70	75	68	76
e	50	90	82	61
f	92	63	90	51

Partial concordance indices

g_1

A x A	*a*	*b*	*c*	*d*	*e*	*f*
a	1	1	1	1	1	1
b	0.14	1	1	1	1	0.29
c	0	1	1	1	1	0
d	0	0	0	1	1	0
e	0	0	0	0	1	0
f	1	1	1	1	1	1

g_2

A x A	*a*	*b*	*c*	*d*	*e*	*f*
a	1	0	0	0	0	0
b	1	1	0	0	0	1
c	1	1	1	1	0.29	1
d	1	1	1	1	0	1
e	1	1	1	1	1	1
f	1	1	0	0	0	1

g_3

A x A	*a*	*b*	*c*	*d*	*e*	*f*
a	1	1	1	1	1	1
b	0	1	1	1	0	0
c	0	0	1	0	0	0
d	0	0.67	1	1	0	0
e	0	1	1	1	1	0
f	1	1	1	1	1	1

g_4

A x A	*a*	*b*	*c*	*d*	*e*	*f*
a	1	1	1	1	1	1
b	0	1	1	1	1	1
c	0	1	1	0.86	1	1
d	0.29	1	1	1	1	1
e	0	0	0.43	0	1	1
f	0	0	0	0	0.29	1

Overall concordance indices

A x A	*a*	*b*	*c*	*d*	*e*	*f*
a	1	0.85	0.85	0.85	0.85	0.85
b	0.19	1	0.85	0.85	0.45	0.42
c	0.15	0.6	1	0.57	0.49	0.35
d	0.21	0.62	0.75	1	0.45	0.35
e	0.15	0.55	0.64	0.55	1	0.35
f	0.8	0.8	0.65	0.65	0.71	1

Partial discordance indices

g_1

A x A	*a*	*b*	*c*	*d*	*e*	*f*
a	1	0	0	0	0	0
b	0	1	0	0	0	0
c	0.38	0	1	0	0	0.25
d	1	0.62	0.12	1	0	1
e	1	1	1	0.38	1	1
f	0	0	0	0	0	1

g_2

A x A	*a*	*b*	*c*	*d*	*e*	*f*
a	1	0.38	1	1	1	0.62
b	0	1	0.62	0.23	1	0
c	0	0	1	0	0	0
d	0	0	0	1	0.23	0
e	0	0	0	0	1	0
f	0	0	0.38	0	1	1

g_3

A x A	*a*	*b*	*c*	*d*	*e*	*f*
a	1	0	0	0	0	0
b	1	1	0	0	0	1
c	1	1	1	0	1	1
d	1	0	0	1	1	1
e	0.5	0	0	0	1	0
f	0	0	0	0	0	1

g_4

A x A	*a*	*b*	*c*	*d*	*e*	*f*
a	1	0	0	0	0	0
b	0	1	0	0	0	0
c	0.31	0	1	0	0	0
d	0	0	0	1	0	0
e	1	0.08	0	0.23	1	0
f	1	0.85	0.54	1	0	1

Credibility of the outranking indices

A x A	*a*	*b*	*c*	*d*	*e*	*f*
a	1	0.85	0	0	0	0.85
b	0	1	0.85	0.85	0	0
c	0	0	1	0.57	0	0
d	0	0.62	0.75	1	0	0
e	0	0	0	0.55	1	0
f	0	0.62	0.65	0	0	1

Exploitation

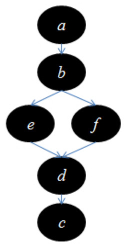


Figure 4.1: ELECTRE-III ranking process example

The calculation of the outranking relation is presented in Section 4.2.1. First, Section 4.2.1.1 shows the calculation of the partial concordance and overall concordance indices, followed by the calculation of the discordance indices in Section

4.2.1.2. Then, the calculation of the credibility of the outranking relation, which uses the overall concordance and partial discordance indices, is shown in Section 4.2.1.3. Finally, the exploitation procedure in ELECTRE-III, called distillation, is presented in Section 4.2.2.

4.2.1 Construction of the outranking relation

The outranking relation S is built taking into account the set G, which can be considered as pseudo-criteria. Given an ordered pair of alternatives $(a, b) \in A \times A$, alternative a outranks alternative b if a outperforms b on enough criteria of sufficient importance, and a is not outperformed by b with a significantly inferior performance on any single criterion. The outranking relation aSb is constructed on the basis of two tests:

- Concordance test: A sufficient majority of criteria should be in favor of the assertion aSb.
- Discordance test: If the concordance condition holds, none of the criteria in the minority should be strongly against the assertion aSb. Furthermore, very bad performance on one criterion may not be compensated by good performances on other criteria.

To determine the credibility $\rho(a, b)$ of the outranking, we calculate a partial concordance index $c_j(a, b)$ and a partial discordance index $d_j(a, b)$ for each criterion g_j.

4.2.1.1 Concordance test

Sometimes referred to as "the respect of the majority", the concordance test entails the calculation of a concordance index $c(a, b)$ that measures the strength of the coalition of criteria that support the hypothesis "a is at least as good as b". The overall concordance index is computed for each ordered pair $a, b \in A$ as follows:

$$c(a, b) = \frac{1}{W} \sum_{j=1}^{m} w_j c_j(a, b) \tag{4.1}$$

where $W = \sum_{j=1}^{m} w_j$, and the partial concordance index $c_j(a, b)$ is defined as:

$$c_j(a,b) = \begin{cases} 1 & \text{if } g_j(a) \geqslant g_j(b) - q_j(b) \\ 0 & \text{if } g_j(a) \leqslant g_j(b) - p_j(b) \\ \frac{g_j(a) - g_j(b) + p_j(b)}{p_j(b) - q_j(b)} & \text{otherwise} \end{cases} \quad (4.2)$$

4.2.1.2 Discordance test

Sometimes referred to as "the respect of minorities", the discordance test entails the calculation of discordance indices $d_j(a,b)$ that measures the strength of evidence provided by the $j-th$ criterion against the hypothesis aSb. The computation of the discordance index takes into account the criteria that disagree with the assertion aSb. In this case, each criterion is assigned a veto threshold $v_j(a)$. The partial discordance index is defined as follows:

$$d_j(a,b) = \begin{cases} 1 & \text{if } g_j(a) - g_j(b) \leqslant -v_j(a) \\ 0 & \text{if } g_j(a) - g_j(b) \geqslant -p_j(a) \\ \frac{g_j(b) - g_j(a) + p_j(a)}{v_j(a) - p_j(a)} & \text{otherwise .} \end{cases} \quad (4.3)$$

4.2.1.3 Degree of credibility of the outranking relation

The degree of credibility of outranking is calculated according to the overall concordance (4.1) and partial discordance (4.3) indices, which are combined to obtain a valued outranking relation with credibility $\rho(a,b) \in [0,1]$ defined by:

$$\rho(a,b) = \begin{cases} c(a,b) & \text{if } d_j(a,b) \leqslant c(a,b), \forall j \\ c(a,b) \prod_{j \in J_{(a,b)}} \frac{1 - d_j(a,b)}{1 - c(a,b)} & \text{otherwise} \end{cases} \quad (4.4)$$

where $J(a,b)$ is the set of criteria for which $d_j(a,b) > c(a,b)$, and the credibility of outranking is equal to the overall concordance index when there is no discordant criterion.

4.2.2 Exploitation procedure

When the credibility matrix is calculated as presented in Section 4.2.1.3, the next step in the ELECTRE-III method is the exploitation of the credibility matrix to build a partial pre-order of the alternatives in A. This procedure is known as *distillation*.

We assume that r outranking relations exist: $S_1 \subset S_2 \subset ...S_r$ (with $r > 1$). The exploitation procedure in ELECTRE-III consists in progressively refining the prescription by successively considering these relations $S_1, S_2, ..., S_r$ [Vanderpooten, 1990].

This refinement can be performed in two ways: from S_1 to S_r or vice versa. ELECTRE-III considers both possibilities and ranks the alternatives in two complete pre-orders which are constructed in two different ways. The first complete pre-order is obtained in a descending manner (descending distillation), selecting the best rated alternatives initially, and finishing with the worst. The second complete pre-order is obtained in an ascending manner (ascending distillation), selecting the worst rated alternatives initially, and finishing with the best. Both distillations make an iterated choice based on a qualification index measured from S_i.

The procedure is as follows for the descending distillation (and analogously for the ascending one). At a certain iteration of a descending distillation procedure, we construct the class C_h composed of *ex aequo* elements, having already constructed classes $C_1, ..., C_{h-1}$ (where class C_1 is the head class in the descending distillation and $A\backslash(C_1, ..., C_{h-1}) \neq \phi$). The steps for building C_h are:

0. $K_0 = A\backslash(C_1 \cup ... \cup C_{h-1}), i \leftarrow 1$.
1. Using S_i, construct K_i as the subset of actions from K_{i-1} whose qualification is maximum.
2. If $|K_i| = 1$ or $i = r$ then $C_h = K_i$ and STOP, else $i \leftarrow i + 1$ and go to step 1.

Note that two or more alternatives may belong to one distillate if they have the same qualification and none can be ranked better or worse than others. In this case, the alternatives are said to be indifferent and are assigned to the same ranking position.

An illustrative example is shown in Figure 4.2 for $A = \{a, b, c, d, e, f, g, h, i, j, k, l\}$.

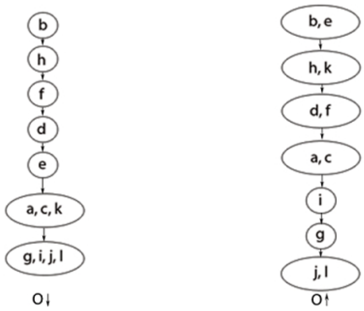


FIGURE 4.2: Complete pre-orders $O \downarrow$ and $O \uparrow$ obtained from a distillation process

The intersection of the two complete pre-orders $O \downarrow$ and $O \uparrow$ gives the final partial pre-order O, shown in Figure 4.3. This partial pre-order establishes a preference structure on the set of alternatives A. For each possible pair of alternatives, it assigns one of the following four binary relations $\{P, P^-, I, R\}$, so that for any two alternatives from set A, one may be preferred over the other, or they may be indifferent, or incomparable. The incomparability of two alternatives occurs when one of these alternatives, let us say a, is ranked better than b in $O \uparrow$, but b is ranked better than a in $O \downarrow$. For example, in Figure 4.3 alternatives e and h are incomparable in the final pre-order, because in Figure 4.2 e is ranked better than h in O↑, but h is ranked better than e in O↓ .

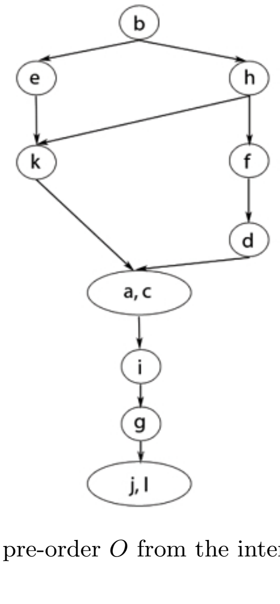


FIGURE 4.3: Partial pre-order O from the intersection of O↑ and O↓

4.3 Extension for ranking problems with a hierarchy of criteria

In this section we propose the ELECTRE-III-H method, which calculates partial pre-orders at all levels of the hierarchy of criteria.

The construction procedure we have defined is analogous to that of ELECTRE-III, presented in Section 4.2, in the sense that it also entails the calculation of the outranking relation (i.e., concordance and discordance tests) and the exploitation of this relation by distillation. However, this procedure incorporates evaluations of alternatives on intermediate criteria in the form of partial pre-orders, rather than numerical evaluations (see Section 2.3.2.2), as this is generally the case for elementary criteria. In addition, we cannot aggregate all criteria together, we have to work with groups of sub-criteria and consider their ascendants-descendants relations in the hierarchy tree. The procedure is presented in Algorithm 1.

Algorithm 1 ELECTRE-III-H Method

1: **function** ElectreIII-H(Criteria G, Alternatives A, PerformanceMatrix M)
2: $X \leftarrow$ List of $g_j \in \mathcal{I} \cup \mathcal{R}$ with descendants all in $\mathcal{E}$
3: $Y \leftarrow$ List of $g_j \in \mathcal{I} \cup \mathcal{R}$ with descendants in $\mathcal{E}$ and $\mathcal{I}$
4: $Y \leftarrow$ sortCriteriaByLevels(Y) ▷ Sort Y bottom-up
5: O = null ▷ Set of partial pre-orders
6: **for all** $x_j \in X$ **do**
7: $Z \leftarrow$ get_Children_Criteria(x_j)
8: $\rho \leftarrow$ build_Electre_III_Credibility(Z, A, M)
9: $O_j \leftarrow$ calculate_Exploitation(ρ)
10: $O = O \cup O_j$
11: **end for**
12: **for all** $y_j \in Y$ **do**
13: Z $\leftarrow$ get_Children_Criteria(y_j)
14: $\rho \leftarrow$ build_Electre_III_H_Credibility(Z, A, M, O)
15: $O_j \leftarrow$ calculate_Exploitation(ρ)
16: $O = O \cup O_j$
17: **end for**
18: return O
19: **end function**

This algorithm distinguishes between two cases that lead to the construction of two lists of criteria that are treated differently:

- List X, in line 2, contains the intermediate criteria and the root criterion whose immediate descendants are all elementary criteria (i.e., all descendants belong to $\mathcal{E}$).
- List Y, in line 3, contains the intermediate criteria that have as immediate descendants other intermediate criteria, possibly including some, but not all, elementary criteria (at least one descendant must be an intermediate criterion).

The procedure is based on a bottom-up approach, so that the criteria are analyzed from the lowest level up to the root. Each of the two lists, X and Y, undergoes a different treatment because of the differences in the information given by the criteria. Note that the difference is in the calculation of the credibility matrix ρ that depends on the definition of the concordance and discordance indices.

1. In the first stage, list X is treated (from line 6 to line 11) with the nodes placed at the bottom level of the hierarchy, i.e., the elementary criteria. This stage aggregates groups of elementary criteria by their direct ancestor x_j to obtain the first results in the form of partial pre-orders. The credibility matrix is calculated using classical ELECTRE-III indices, using the performance scores stored in matrix M (line 8). The exploitation of the credibility matrix generates a partial pre-order for each node in O_j (line 9) and stored in the set O (line 10). Note that only a subset of criteria is considered in the credibility calculation, which contains the direct descendants of the current node x_j (stored in Z in line 7).

2. In the second stage, the algorithm treats list Y (from line 12 to line 17). Then, the partial pre-orders obtained before (stored in O) are used as inputs for the upper level criteria. This requires the list to be ordered according to the precedence relations indicated by the tree structure of set G, from the lowest level up to the most general criterion (root). So, Y is sorted such that y_j has no descendant in $y_{j+1}...y_m$ (line 4). In line 14, the credibility is calculated using new formulas that will be defined in this section, which redefine the partial concordance and discordance indices in order to handle all of the binary preference relations that can be found in a partial pre-order (indifference, incomparability and preference). Therefore, in this case, both the performance matrix M and the list of partial pre-orders O are needed to calculate the credibility index according to the nature of the descendants of the current node x_j (stored in Z in line 7). Finally, the exploitation procedure of the credibility is applied (line 15), resulting in a new partial pre-order O_j for each node $g_j \in Y$.

Note that the procedure at each node is analyzed following the two steps defined in ELECTRE methods: 1) construction of a credibility matrix based on the partial concordance and partial discordance indices, using (4.4); and 2) the distillation process for exploitation of the outranking relations, which results in a partial pre-order.

All criteria in set G can be considered as pseudo-criteria, associated with indifference, preference and veto thresholds, except for the root criterion. These thresholds are defined as follows:

- The indifference $q_j(a)$, preference $p_j(a)$ and veto $v_j(a)$ thresholds referring to elementary criteria from set $\mathcal{E}$ are fixed in the same way as in the ELECTRE-III method, based on the performance of the alternatives and depending on the scale of measurement of each criterion g_j.
- The indifference $q_j(a)$, preference $p_j(a)$, and veto $v_j(a)$ thresholds referring to criteria from set $\mathcal{I}$ are functions of the difference of rank order value of the alternatives in a partial pre-order O. Having that $|A| = n$, then $0 \leqslant q_j(a) \leqslant p_j(a) \leqslant v_j(a) \leqslant n - 1$.

In the following subsections we present different steps of the method. According to Figure 4.4, ELECTRE-III-H relies on the construction and exploitation of the outranking relations for elements in $\mathcal{E}$, where the alternatives are directly evaluated. Then, for elements in $\mathcal{I}$, where partial results in the form of partial-preorders are considered as inputs; new calculations of partial concordance and discordance indices for the construction of the outranking relations are required.

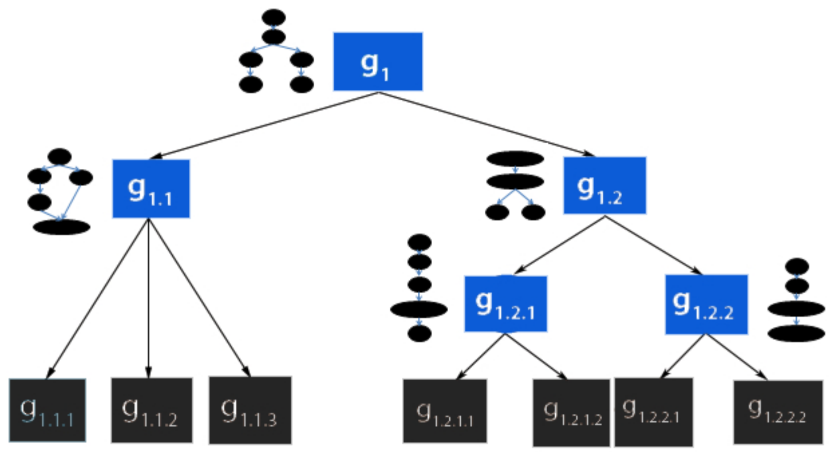


FIGURE 4.4: Hierarchical tree of the set of criteria. The nodes in black apply classical ELECTRE-III, while the blue nodes calculate new indices from partial-preorders.

4.3.1 Building and exploiting the credibility matrix with criteria in $\mathcal{E}$

The first part of the proposed algorithm corresponds to the calculation of the credibility matrix for nodes whose descendants are all elements in $\mathcal{E}$. In this case the classical ELECTRE-III method is applied to aggregate evaluations of alternatives on the elementary criteria. For example, taking into account the hierarchy in Figure 4.4, the classical ELECTRE-III is applied separately for each subset of $\mathcal{E}$ with the same ancestor: $\{g_{1.1.1}, g_{1.1.2}, g_{1.1.3}\}, \{g_{1.2.1.1}, g_{1.2.1.2}\}, \{g_{1.2.2.1}, g_{1.2.2.2}\}$.

The credibility is calculated directly from the evaluations in the performance matrix M, using the procedures explained in Section 4.2.1. This step corresponds to the first stage of Algorithm 1 (treatment of list X).

The exploitation of the credibility matrix proceeds in the usual way, as explained in Section 4.2.2. This stage results in a partial pre-order of alternatives for each immediate predecessor criterion. In our example, the results of the exploitation processes are three partial pre-orders $\{O_{1.1}, O_{1.2.1}, O_{1.2.2}\}$. These results are stored in the corresponding criterion, and are considered as evaluations of alternatives on these criteria $\{g_{1.1}, g_{1.2.1}, g_{1.2.2}\}$.

In the following section we explain how to aggregate partial pre-orders at intermediate levels of the hierarchical tree.

4.3.2 Building the credibility matrix with criteria in $\mathcal{I}$

For nodes in the hierarchy tree that have at least one direct descendant in $\mathcal{I}$, the credibility cannot be calculated in the classical way. In this second stage of Algorithm 1, which must take into account the partial pre-orders resulting from prior evaluations in the hierarchy tree, we propose a new calculation of the partial concordance and discordance indices for ELECTRE-III-H.

In the example presented in Figure 4.4, ELECTRE-III-H is applied level by level up to $\mathcal{R}$, as follows:

- $O_{1.2.1}, O_{1.2.2}$ are aggregated to obtain $O_{1.2}$
- $O_{1.1}, O_{1.2}$ are aggregated to obtain O_1

Considering any ordered pair of alternatives (a, b), the goal is to propose a method for calculating $c_j(a, b)$ and $d_j(a, b)$ from O_j. For each pair of alternatives, we have only four possible binary relations: $\{P, P^-, I, R\}$. Thus, a partial pre-order can be represented in a matrix $A \times A$ containing the type of preference relation for each pair. Let us denote this preference relation matrix as $\mathcal{M}$.

Let us take an example with a set $A = \{r, s, t, u, v, w, x, y, z\}$ of alternatives, structured in the partial pre-order O_1, as shown in Figure 4.5. Let us suppose that this partial pre-order has been generated from a subset of $\mathcal{E}$.

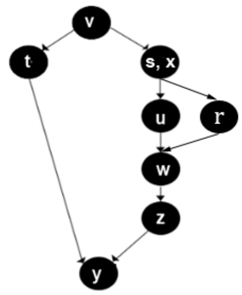


FIGURE 4.5: Example of a partial pre-order O_1 generated in $\mathcal{E}$

The corresponding preference relations are given in Table 4.2.

TABLE 4.2: Preference relation matrix $\mathcal{M}$ for partial pre-order O_1

	r	s	t	u	v	w	x	y	z	$\Gamma(\cdot)$
r	I	P^-	R	R	P^-	P	P^-	P	P	3
s	P	I	R	P	P^-	P	I	P	P	1
t	R	R	I	R	P^-	R	R	P	R	1
u	R	P^-	R	I	P^-	P	P^-	P	P	3
v	P	P	P	P	I	P	P	P	P	0
w	P^-	P^-	R	P^-	P^-	I	P^-	P	P	5
x	P	I	R	P	P^-	P	I	P	P	1
y	P^-	P^-	P^-	P^-	P^-	P^-	P^-	I	P^-	8
z	P^-	P^-	R	P^-	P^-	P^-	P^-	P	I	6

In the following subsections we explain how to calculate the partial concordance and discordance indices for each case of the four binary relations $\{P, P^-, I, R\}$. The definitions will be established using the information about the preference relations given in the preference matrix $\mathcal{M}$. The last column of Table 4.2 corresponds to the *Rank Order Value* $\Gamma(\cdot)$, defined below.

Definition 4.1. Rank Order Value. The rank order value of an alternative $a \in A$ in a partial pre-order O_j is the number of alternatives that are preferred to a in this partial pre-order. It is denoted by $\Gamma_j(\cdot)$.

The rank order value is then used to define new partial concordance and discordance indices as it represents the partial pre-order structure. This value is to be minimized. However, the indices are not only based on $\Gamma(\cdot)$ as they also take into account the actual relation of each pair of alternatives in the partial pre-order O_j.

4.3.2.1 Preference and indifference relations, P and I

The first situation we consider is when a is strictly preferred or indifferent to b in a partial pre-order O_j. Remember that the concordance index measures the support to the outranking relation defined as "a is at least as good as b", aSb. Since $S = I \vee P$, both preference aP_jb and indifference aI_jb relations in O_j indicate that O_j clearly supports aSb. Therefore, the value of partial concordance index is set to 1:

$$c_j(aP_jb) = 1 \tag{4.5}$$

$$c_j(aI_jb) = 1 \tag{4.6}$$

Following the previous rationale, when aP_b and aI_jb in O_j, we set the partial discordance index to 0:

$$d_j(aP_jb) = 0 \tag{4.7}$$

$$d_j(aI_jb) = 0 \tag{4.8}$$

4.3.2.2 Inverse preference relation, P^-

When b is preferred over a in the partial pre-order O_j, the strength of the difference between b and a must be considered to calculate the degree of concordance or discordance with respect to the outranking relation aSb. We distinguish four cases:

- **Case 1:** When the difference between the rank order values $\Gamma_j(a) - \Gamma_j(b)$ is less than or equal to the indifference threshold $q_j(a)$. In this case the concordance with aSb is maximum and the discordance is zero.

$$\text{if } \Gamma_j(a) - \Gamma_j(b) \leqslant q_j(a), \text{ then } \begin{cases} c_j(aP_j^- b) = 1 \\ d_j(aP_j^- b) = 0 \end{cases} \tag{4.9}$$

  In our example, presented in Figure 4.5, for $zP_j^- w$ and $\Gamma_j(z) = 6, \Gamma_j(w) = 5$ and threshold $q_j(z) = 1$, the partial concordance index $c_j(zP_j^- w)$=1, and the partial discordance index $d_j(zP_j^- w)$=0, as the difference of $\Gamma_j(z) - \Gamma_j(w) = q_j(z)$.

- **Case 2:** When the difference between the rank order values $\Gamma_j(a) - \Gamma_j(b)$ is greater than the indifference threshold $q_j(a)$ and less than the preference threshold $p_j(a)$, the partial concordance index decreases, and the discordance is zero. The calculation of the partial concordance index proceeds analogously to the classical ELECTRE-III in terms of the $\Gamma_j(\cdot)$ function:

$$\begin{array}{r} \text{if } \Gamma_j(a) - \Gamma_j(b) > q_j(a) \\ \text{and } \Gamma_j(a) - \Gamma_j(b) \leqslant p_j(a), \text{then} \end{array} \begin{cases} c_j(aP_j^- b) = \frac{p_j(a)-(\Gamma_j(a)-\Gamma_j(b))}{p_j(a)-q_j(a)} \\ d_j(aP_j^- b) = 0 \end{cases} \tag{4.10}$$

In the example presented in Figure 4.5, for $uP_j^- x$ and $\Gamma_j(u) = 3, \Gamma_j(x) = 1, q_j(u) = 1$ and $p_j(u) = 3$, the partial discordance index $d_j(uP^- x)$=0, whereas the partial concordance index $c_j(uP_j^- x)$ is calculated as follows:

$$c_j(uP_j^- x) = \frac{3-(3-1)}{3-1} = 0.5$$

- **Case 3:** When the difference between the rank order values $\Gamma_j(a) - \Gamma_j(b)$ is greater than the preference threshold $p_j(a)$ and less than or equal to the veto threshold $v_j(a)$, the partial concordance index is zero and the discordance increases. The calculation of the partial discordance index is analogous to the classical ELECTRE-III in terms of the $\Gamma_j(\cdot)$ function.

$$\begin{array}{r} \text{if } \Gamma_j(a) - \Gamma_j(b) > p_j(a) \\ \text{and } \Gamma_j(a) - \Gamma_j(b) \leqslant v_j(a), \text{ then} \end{array} \begin{cases} c_j(aP_j^- b) = 0 \\ d_j(aP_j^- b) = \frac{\Gamma_j(a)-\Gamma_j(b)-p_j(a)}{v_j(a)-p_j(a)} \end{cases} \tag{4.11}$$

In the example presented in Figure 4.5, for $zP_j^- x$ and $\Gamma_j(z) = 6, \Gamma_j(x) = 1, q_j(z) = 1, p_j(z) = 3$ and $v_j(z) = 6$, the partial concordance index $c_j(zP_j^- x)$=0, while the partial discordance index $d_j(zP_j^- x)$ is calculated as follows:

$$d_j(zP_j^- x) = \frac{6-1-3}{6-3} = 0.667$$

- **Case 4:** When the difference between rank order values $\Gamma_j(a) - \Gamma_j(b)$ is greater than veto threshold $v_j(a)$, the discordance is maximum and the concordance is zero.

$$\text{if } \Gamma_j(a) - \Gamma_j(b) > v_j(a), \text{ then } \begin{cases} c_j(aP_j^- b) = 0 \\ d_j(aP_j^- b) = 1 \end{cases} \tag{4.12}$$

This case corresponds to the partial concordance and discordance indices of $yP_j^- x$ in the pre-order given in Figure 4.5, where $\Gamma_j(y) = 8$, $\Gamma_j(x)$=1 and assuming that $v_j(y)$=6.

Note that, according to these definitions, the point that determines whether the difference on $\Gamma_j(a)-\Gamma_j(b)$ is in support of (some degree of concordance) or against aSb (some degree of discordance) is the value of the preference threshold p_j.

The above cases for the inverse preference relation can be summarized as follows:

$$c_j(aP_j^- b) = \begin{cases} 1 & \text{if } \Gamma_j(a) - \Gamma_j(b) \leqslant q_j(a) \\ 0 & \text{if } \Gamma_j(a) - \Gamma_j(b) > p_j(a) \\ \frac{p_j(a)-(\Gamma_j(a)-\Gamma_j(b))}{p_j(a)-q_j(a)} & \text{otherwise.} \end{cases} \tag{4.13}$$

$$d_j(aP_j^- b) = \begin{cases} 1 & \text{if } \Gamma_j(a) - \Gamma_j(b) > v_j(a) \\ 0 & \text{if } \Gamma_j(a) - \Gamma_j(b) \leqslant p_j(a) \\ \frac{\Gamma_j(a)-\Gamma_j(b)-p_j(a)}{v_j(a)-p_j(a)} & \text{otherwise.} \end{cases} \tag{4.14}$$

4.3.2.3 Incomparability relation, R

When, in partial pre-order O_j, alternative a is incomparable to alternative b, it is impossible to state whether this relation is closer to aP_jb or aI_jb or aP_j^-b; thus, the partial pre-order gives no clear support to the outranking aSb. In this case, we take into account additional information about alternatives a and b given by the function $\Gamma_j(\cdot)$. If the difference between the rank order values of $\Gamma_j(a)$ and $\Gamma_j(b)$ is negative or close to 0, then this should enforce the conviction that aR_jb could turn to aP_jb or aI_jb rather than to aP_j^-b, since the rank order value of a is less than the rank order value of b. Otherwise, if the difference between the rank order values of a and b were positive, then this should enforce the conviction that aR_jb could turn to aP_j^-b rather than to aP_jb or aI_jb.

In any case, for a pair of incomparable alternatives, the information is uncertain, so we will not set the maximum value for either concordance or discordance. Instead, we use the base values $k^c < 1$ and $k^d < 1$ for partial concordance and discordance, respectively, which are tuned as described below. To establish the base values we can consider that aR_jb could turn with an equal probability to aP_jb, aI_jb or aP_j^-b and that $S = I \vee P$, concluding that only two of the three possible relations support S. Thus, there is $\frac{2}{3}$ chance that aR_jb would confirm aSb. We therefore propose $k^c = \frac{2}{3}$ and $k^d = \frac{1}{3}$.

The proposed rules for the calculation of partial concordance and discordance indices in the case of incomparability are as follows:

$$\text{if } \Gamma_j(a) - \Gamma_j(b) \leqslant p_j(b), \text{ then } \begin{cases} c_j(aR_jb) = k^c + \delta_j^c(a,b) \\ d_j(aR_jb) = 0 \end{cases} \tag{4.15}$$

$$\text{if } \Gamma_j(a) - \Gamma_j(b) > p_j(b), \text{ then } \begin{cases} c_j(aR_jb) = 0 \\ d_j(aR_jb) = k^d + \delta_j^d(a,b) \end{cases} \tag{4.16}$$

The rationale for adding the tuning factors δ_j^c and δ_j^d to the above formulas is that the partial concordance and discordance indices in these two rules should depend on the magnitude of the difference $\Gamma_j(a) - \Gamma_j(b)$. The tuning factors δ_j^c and δ_j^d may be positive or negative, hence, they either increase or decrease the base partial concordance k^c or discordance k^d. More precisely, we propose:

- δ_j^c **for the partial concordance index:** We establish the following logical condition for concordance $c_j(aR_jb)$: "If alternative a is incomparable to alternatives b and d in a partial pre-order O_j, and $\Gamma_j(a) - \Gamma_j(b) < \Gamma_j(a) - \Gamma_j(d) \leqslant v_j(b)$ and $\leqslant v_j(d)$, then $c_j(aR_jb)$ should be greater than $c_j(aR_jd)$". According to this condition, for each pair $(a,b) \in A \times A$, such that aR_jb and $\Gamma_j(a) - \Gamma_j(b) \leqslant v_j(a)$ in O_j, we propose:

$$\delta_j^c(a,b) = \frac{(\Gamma_j(b) - \Gamma_j(a) - q_j(a)) \times \alpha}{(p_j(a) - q_j(a)) + (n-2)} \tag{4.17}$$

  where n is the number of alternatives in A, and 2 is subtracted from n in the denominator to account for the two incomparable alternatives considered (a and b). The value α has been introduced to control the maximum permitted degree of change to the original partial concordance index for incomparability.

- δ_j^d **for the partial discordance index:** We establish the following logical condition for concordance $d_j(aR_jb)$: "If alternative a is incomparable to alternatives b and d in a partial pre-order O_j, and $\Gamma_j(a) - \Gamma_j(b) > \Gamma_j(a) - \Gamma_j(d) \geqslant v_j(b)$ and $\geqslant v_j(d)$, then $d_j(aR_jb)$ should be greater than $d_j(aR_jd)$". According to this condition, for each pair $(a,b) \in A \times A$, such that aR_jb and $\Gamma_j(a) - \Gamma_j(b) > v_j(a)$ in O_j, we propose:

$$\delta_j^d(a,b) = \frac{(\Gamma_j(a) - \Gamma_j(b) - v_j(a)) \times \alpha}{(v_j(a) - p_j(a)) + (n-2)} \tag{4.18}$$

  Again, the value α controls the maximum permitted degree of change to the original partial discordance index for incomparability.

Note that α should be smaller than $\frac{1}{3}$ in order to keep the concordance and discordance indices below 1. For example, we can set α to 0.25, so that for concordance index $-0.25 \leqslant \delta_j^c(a,b) \leqslant 0.25$ and $0.42 \leqslant c_j(aR_jb) \leqslant 0.92$ and for discordance index $-0.25 \leqslant \delta_j^d(a,b) \leqslant 0.25$ and $0.08 \leqslant d_j(aR_jb) \leqslant 0.58$.

From this definition of concordance and discordance for the case of incomparability, we can find properties for the different cases determined by the thresholds.

Remark 4.2. If $\Gamma_j(a)-\Gamma_j(b) \leqslant q_j(a)$, this supports the conviction that the incomparability aR_jb could turn to aP_jb or aI_jb rather than to aP_j^-b. This corresponds to the case of strong concordance, where, according to (4.15), the partial concordance behaves as follows:

$$\text{if } \Gamma_j(a) - \Gamma_j(b) < q_j(a), \text{ then } c_j > k^c \tag{4.19}$$

$$\text{if } \Gamma_j(a) - \Gamma_j(b) = q_j(a), \text{ then } c_j = k^c \tag{4.20}$$

Remark 4.3. If $\Gamma_j(a) - \Gamma_j(b) > q_j(a)$ and $\Gamma_j(a) - \Gamma_j(b) \leqslant p_j(a)$, this supports the conviction that the incomparability aR_jb could turn to aP_jb or aI_jb, rather than to aP_j^-b. So, this is a situation of weak concordance, where:

$$\text{if } \Gamma_j(a) - \Gamma_j(b) > q_j(a) \text{ and if } \Gamma_j(a) - \Gamma_j(b) \leqslant p_j(a), \text{ then } c_j < k^c. \tag{4.21}$$

Remark 4.4. If $\Gamma_j(a) - \Gamma_j(b) > p_j(a)$ and $\Gamma_j(a) - \Gamma_j(a) \leqslant v_j(a)$, this supports the conviction that the incomparability aR_jb could turn to aP_j^-b rather than to aP_jb or aI_jb. This corresponds to situation of weak discordance, where:

$$\text{if } \Gamma_j(a) - \Gamma_j(b) > p_j(a) \text{ and if } \Gamma_j(a) - \Gamma_j(b) < v_j(a), \text{ then } d_j < k^d \tag{4.22}$$

$$\text{if } \Gamma_j(a) - \Gamma_j(b) = v_j(a), \text{ then } d_j = k^d \tag{4.23}$$

Remark 4.5. If $\Gamma_j(a) - \Gamma_j(b) > v_j(a)$, this supports the conviction that the incomparability aR_jb could turn to aP_j^-b, rather than to aP_jb or aI_jb. This is the case of strong veto, where:

$$\text{if } \Gamma_j(a) - \Gamma_j(b) > v_j(a), \text{ then } d_j > k^d \tag{4.24}$$

4.3.3 Exploitation of the credibility matrix for $\mathcal{I}$ and $\mathcal{R}$

At a given criterion g_i, using the equations presented in Section 4.3.2, we can calculate a partial concordance c_j and partial discordance d_j indices from a given partial pre-order O_j obtained in a lower level, where j is a descendant of i. If g_i has descendants that are elementary criteria ($g_j \in \mathcal{E}$), then c_j and d_j are calculated as in classical ELECTRE-III indices using (4.2) and (4.3). Overall c_i is calculated with (4.1). Then, the partial concordance indices obtained for each type of criterion are merged when the credibility matrix is calculated with (4.4).

This credibility matrix is then exploited with the distillation algorithm, as defined in Section 4.2.2. Two complete pre-orders are generated from the ascending and descending distillation chain, which are merged to generate a partial pre-order O_i.

4.4 The case of true-criteria in ELECTRE-II-H

Criterion g_j is a true-criteria if $q_j(a) = p_j(a) = 0$, such that there is no ambiguity zone or weak preference aQ_jb. In Del Vasto-Terrientes et al [2012] we considered the calculation of partial concordance and discordance indices from partial pre-orders on the basis of true-criteria. As explained in 2.3.2.2, the ELECTRE-II method also generates partial pre-orders as in ELECTRE-III but calculating partial concordance and discordance indices on the basis of true-criteria. Therefore, we can also extend to ELECTRE-II-H by redefining the partial concordance and discordance calculations from partial pre-orders using only the veto threshold. This leads to the following definitions:

1. Preference and indifference relations, P and I

$$c_j(aP_jb) = 1 \tag{4.25}$$

$$c_j(aI_jb) = 1 \tag{4.26}$$

$$d_j(aP_jb) = 0 \tag{4.27}$$

$$d_j(aI_jb) = 0 \tag{4.28}$$

2. Inverse Preference relation P^-

$$c_j(aP_j^-b) = 0 \tag{4.29}$$

$$d_j(aP_j^-b) = \begin{cases} 1 & \text{if } \Gamma_j(a) - \Gamma_j(b) > v_j(a) \\ 0 & \text{if } \Gamma_j(a) - \Gamma_j(b) \leqslant v_j(a) \end{cases} \tag{4.30}$$

3. Incomparability relation R

$$\text{if } \Gamma_j(a) - \Gamma_j(b) \leqslant v_j(b), \text{ then } \begin{cases} c_j(aR_jb) = k^c + \delta_j^c(a,b) \\ d_j(aR_jb) = 0 \end{cases} \tag{4.31}$$

$$\text{if } \Gamma_j(a) - \Gamma_j(b) > v_j(b), \text{ then } \begin{cases} c_j(aR_jb) = 0 \\ d_j(aR_jb) = k^d + \delta_j^d(a,b) \end{cases} \tag{4.32}$$

where,

$$\delta_j^c(a,b) = \frac{(\Gamma_j(b) - \Gamma_j(a)) \times \alpha}{(n-2)} \tag{4.33}$$

$$\delta_j^d(a,b) = \frac{(\Gamma_j(a) - \Gamma_j(b) - v_j(a)) \times \alpha}{(n-2)} \tag{4.34}$$

Following Algorithm 1, the ELECTRE-II-H method entails the next steps:

Algorithm 2 Hierarchical algorithm for true-criteria:

1: **function** ELECTRE-II-H(Criteria G, Alternatives A, PerformanceMatrix M)
2: $X \leftarrow$ List of $g_j \in \mathcal{I} \cup \mathcal{R}$ with descendants all in $\mathcal{E}$
3: $Y \leftarrow$ List of $g_j \in \mathcal{I} \cup \mathcal{R}$ with descendants in $\mathcal{E}$ and $\mathcal{I}$
4: $Y \leftarrow$ sortCriteriaByLevels(Y) ▷ Sort Y bottom-up
5: $O =$ null ▷ Set of partial pre-orders
6: **for all** $x_j \in X$ **do**
7: $Z \leftarrow$ get_Children_Criteria(x_j)
8: $\rho \leftarrow$ **build_Electre_II_Credibility(**Z, A, M**)**
9: $O_j \leftarrow$ **calculate_Exploitation(**ρ**)**
10: $O = O \cup O_j$
11: **end for**
12: **for all** $y_j \in Y$ **do**
13: Z $\leftarrow$ get_Children_Criteria(y_j)
14: $\rho \leftarrow$ **build_Electre_II_H_Credibility(**Z, A, M, O**)**
15: $O_j \leftarrow$ **calculate_Exploitation(**ρ**)**
16: $O = O \cup O_j$
17: **end for**
18: return O
19: **end function**

Algorithm 2 follows the same structure than Algorithm 1. The differences between Algorithm 2 for ELECTRE-II-H considering true-criteria and Algorithm 1 presented for ELECTRE-III-H and pseudo-criteria are the following:

1. In line 8, Algorithm 2 calculates the credibility matrix using Equation 2.7 using the partial concordance Equation 2.5 and the partial discordance Equation 2.6,

2. In line 9, Algorithm 2 calculates the exploitation of the credibility matrix generated in line 8 using the ELECTRE-II procedure explained in Belton and Stewart [2002] and presented in 2.3.2.2, using the strong outranking relation in Equation 2.8 and weak outranking relation in Equation 2.9, instead of the distillation,

3. In line 14, Algorithm 2 calculates the partial concordance and discordance indices using the equations provided in this section for true criteria instead of the ones of pseudo-criteria.

4. In line 15, Algorithm 2 applies the exploitation procedure as indicated for line 9, using strong and weak outranking relations.

4.5 ELECTRE-III-H Properties

In this section we study the properties of the ELECTRE-III-H method for building a valued binary outranking relation $S : A \times A$ from a set of partial pre-orders. For each pair of alternatives, a credibility value $\rho_D(a, b) \in [0, 1]$ is calculated from a set of partial pre-orders $\in D$.

Previous works have introduced a common framework and characterization for constructing outranking relations [Bouyssou et al, 1997, Greco et al, 2001a, Pirlot, 1997] or concordance and discordance measures [Bouyssou and Pirlot, 2009, Dubois et al, 2003].

In the context of this thesis, partial pre-orders in the subset of criteria D are the result of applying ELECTRE-III-H method at intermediate criteria, but the same properties fulfill if the partial pre-order is obtained from any other procedure, or even directly given by the DM.

Before the analysis of the properties of the ELECTRE-III-H method, considering the natural conditions imposed to social choice procedures and aggregation operators, in Section 4.5.2; a study of the four possible binary relations and their contribution to the assertion of the outranking relation aSb under different conditions is made in Section 4.5.1.

Let $D \subseteq G$ be a set of intermediate criteria on G that are the direct descendants of g_i, where $D = \{g_{i.1}, g_{i.2}, ..., g_{i.x}\}$. Let us assume that each element in $g_{i.j} \in D$ is associated to a weight w_j that indicates its relative importance with respect to the rest of descendants of g_i, to preference thresholds ($q_j(a)$, $p_j(a)$, and $v_j(a)$), and has a partial pre-order O_j containing the binary preference structure of the alternatives in set A. For each pair of alternatives (a, b), a binary relation $\phi_j = \{P, I, P^-, R\}$ connects them in O_j. Let us denote as $\rho_D(a, b)$ the operation to calculate the credibility index of the outranking relation aSb in the set of criteria D. We denote $aSb = true$ if $\rho_D(a, b) \geqslant \lambda$.

4.5.1 Characterization of aSb in terms of P, I, P^- and R

In this section, we study the fulfillment of the outranking relation S under different preference relations observed on the partial pre-orders that are aggregated. The conditions for holding aSb (i.e., $\rho_D(a, b) \geqslant \lambda$) are given in terms of rank order values and indifference, preference and veto thresholds.

4.5.1.1 Preference and Indifference relations, P and I

Proposition 4.6. *Given two alternatives a, $b \in A$, aSb if $\forall j$, $a\phi_j b$ where ϕ_j=$\{P \vee I\}$.*

Proof. The relations P and I fully support the outranking relation S. For all the partial pre-orders O_j, if aPb or aIb, we have

$\forall j$, $c_j(a\phi_j b) = 1$ and $d_j(a\phi_j b) = 0$,

so that:

$\forall j, d_j(a,b) < c(a,b)$, being $c(a,b)$=1.

This results in $\rho_D(a,b) = 1$.

□

4.5.1.2 Inverse Preference relation P^-

Proposition 4.7. *Given two alternatives a, $b \in A$ and O_j the partial pre-order of $g_{i.j} \in D$, $\neg(aSb)$ if $\forall j$, there is a relation $aP_j^- b$ and $\Gamma_j(a) - \Gamma_j(b) > p_j(a)$.*

Proof. Given two alternatives $a, b \in A$ and O_j the partial pre-order of $g_{i.j} \in D$, for the case of a binary relation $aP_j^- b$ in which the difference order value of a and b is greater than $p_j(a)$ the right to veto is activated, so that $d_j(aP_j^- b) > 0$ and $c_j(aP_j^- b) = 0$. Then,

$d_j(a,b) > c(a,b) = 0$, resulting in $\rho_D(a,b) = 0$.

□

Under this condition, a may not outrank b overall when b performs better on all the criteria in D.

Proposition 4.8. *Given two alternatives a, $b \in A$ and O_j the partial pre-order of $g_{i.j} \in D$, aSb if $\forall j$, $aP_j^- b$ and $\Gamma_j(a) - \Gamma_j(b) \leqslant p_j(a)$.*

Proof. Let alternatives a, $b \in A$ and O_j the partial pre-order of $g_{i.j} \in D$, for all the pairs of binary relations $aP_j^- b$ and all the differences of the order value a and b is less or equal to $p_j(a)$, then:

$\forall j$, $c_j(aP_j^- b) > 0$ and $d_j(aP_j^- b) = 0$.

So that,

$\forall j, d_j(a,b) < c(a,b) \in (0,1]$, resulting in $\rho_D(a,b) > 0$. □

4.5.1.3 Incomparability relation R

The incomparability relation gives no clear support to the outranking aSb, resulting in fuzzy outranking relations with credibility in (0, 1). Taking into account that the values of partial concordance and discordance indices are respectively in the range of $[k^c - \alpha,\ k^c + \alpha]$ and $[k^d - \alpha,\ k^d + \alpha]$, these indices do not fully agree or reject the relation aSb.

We analyze the conditions where aSb holds for incomparability relations. We assume that $\lambda \geqslant k^c - \alpha \Rightarrow aSb$. As a reminder, the partial concordance has been defined as:

$$c_j(a,b) = k^c + \frac{(\Gamma_j(b) - \Gamma_j(a) - q_j(b)) \times \alpha}{(p_j(b) - q_j(b)) + (n-2)} \tag{4.35}$$

Proposition 4.9. *Given two alternatives a, $b \in A$, aSb if $\forall j$, aR_jb and $\Gamma_j(b)$ - $\Gamma_j(a)$ - $q_j(b)$=0.*

Proof. Having alternatives a, $b \in A$, if the binary relation aR_jb holds and $\Gamma_j(b)$ - $\Gamma_j(a)$ - $q_j(b) = 0$ for all O_j in D, we have

$\forall j$, $c_j(aR_jb) = k^c = \frac{2}{3}$ and $d_j(aR_jb) = 0$.

Then,

$c(a,b) = k^c$ and $\forall j, d_j(a,b) < c(a,b)$, resulting in $\rho_D(a,b) = k^c$. □

Proposition 4.10. *Given two alternatives a, $b \in A$, $\rho_D(a,b) \in (k^c, k^c + \alpha]$ if $\forall j$, aR_jb and $\Gamma_j(a) - \Gamma_j(b) \leqslant q_j(b)$.*

Proof. Having alternatives a, $b \in A$, if the binary relation aR_jb holds and $\Gamma_j(b)$ - $\Gamma_j(a) > q_j(b)$ for all O_j in D, the numerator of the concordance indices expression is always positive, increasing the base value k^c, such that:

$\forall j,\ k^c \geqslant c_j(aR_jb) > \frac{2}{3}$ and $d_j(aR_jb) = 0$.

Then,

$\forall j, d_j(a,b) < c(a,b)$, resulting in $\rho_D(a,b) \in (k^c, k^c + \alpha]$. □

Proposition 4.11. *Given two alternatives a, $b \in A$, $\rho_D(a,b) \in [k^c - \alpha, k^c)$ if $\forall j$, aR_jb and $p_j(b) \geqslant \Gamma_j(a)$ - $\Gamma_j(b) > q_j(b)$.*

Proof. Having alternatives a, $b \in A$, if the binary relation aR_jb holds and $\Gamma_j(b)$ - $\Gamma_j(a) > q_j(b)$ and $\Gamma_j(a) - \Gamma_j(b) < p_j(b)$ for all O_j in D, the numerator of the concordance indices expression is always negative, decreasing the base value k^c, such that:

$\forall j,\ k^c - \alpha \leqslant c_j(aR_jb) < \frac{2}{3}$ and $d_j(aR_jb) = 0$.

Then,

$\forall j, d_j(a,b) < c(a,b)$, resulting in $\rho_D(a,b) \in (k^c, k^c + \alpha]$. □

Now, to analyze the next cases, let us remind that the value of the discordance index is given by the following equation:

$$dj(a,b) = k^d + \frac{(\Gamma_j(b) - \Gamma_j(a) - q_j(b)) \times \alpha}{(p_j(b) - q_j(b)) + (n-2)} \tag{4.36}$$

Proposition 4.12. *Given two alternatives a, $b \in A$, $\neg(aSb)$ if $\forall j$, aR_jb and $\Gamma_j(a)$ - $\Gamma_j(b) > p_j(b)$.*

Proof. Having alternatives a, $b \in A$, if the binary relation aR_jb holds and $\Gamma_j(a)$ - $\Gamma_j(b) > p_j(b)$ for all O_j, the difference between the rank order value of alternative a with respect to b is larger than the permitted threshold $p_j(b)$, making an opposition to the outranking relation aSb. Thus, we have:

$\forall j$, $c_j(aR_jb) = 0$ and $d_j(aR_jb) = [k^d - \alpha, k^d + \alpha]$,

then $\forall j, d_j(a,b) > c(a,b)$, resulting in $\rho_D(a,b) = 0$. □

4.5.2 Properties of ELECTRE-III-H

In the this section we study the main properties of the construction of the outranking relation for partial pre-orders in the ELECTRE-III-H method.

- **Neutrality with respect to criteria:** The credibility of aSb does not depend on the order of consideration of the criteria. For any permutation $D' = \sigma(D)$:

  $$\rho_D(a,b) = \rho_{D'}(a,b), \text{ so that } aS'b \Rightarrow aSb$$

  Proof. This property is fulfilled by $\rho_D(a,b)$ because the product and addition operators are commutative. □

- **Independence of irrelevant alternatives:** The relation aSb relies on the rank order values calculated from the preference relation matrix $\mathcal{M}$. The addition/deletion of an alternative in the set A, or even the modification of the performance of another alternative in A results in the modification of the preference relation matrix $\mathcal{M}$. Then, the independence property of the relation aSb may not be fulfilled.

  A'=$A \cup \{k\}$, then aSb does not implies $aS'b$.

  Proof. Let us consider $A = \{a, b, c\}$ where $\Gamma_j(b) < \Gamma_j(c) < \Gamma_j(a)$ and $A' = \{a, b, c, k\}$ where $\Gamma_j(b) < \Gamma_j(k) < \Gamma_j(c) < \Gamma_j(a)$.

  Let be $\beta = \Gamma_j(a) - \Gamma_j(b) = 1$ in O_j and $\beta' = \beta + 1$ in O'_j. If $q_j, p_j(a) = 0, v_j(a) = 2$, being $\beta' = v_j(a) > \beta$, then aSb and $\neg(aS'b)$. □

- **Monotonicity:** If aSb and $\Gamma(a)$ improves or $\Gamma(b)$ deteriorates in O_j, then aSb remains. The outranking relation aSb is preserved based on the improvement or deterioration of the rank order value of alternatives a and b respectively.

  Proof. Considering alternatives a and b, if aSb, the following cases may occur:

- If aP_jb and a improves or b deteriorates, then the relation between them still is aP_jb, therefore aSb holds as $c_j(a,b) = 1$,
- If aI_jb and a improves or b deteriorates, then aI_jb turns into aP_jb and aSb holds as $c_j(a,b) = 1$,
- If aP_j^-b and a improves or b deteriorates then the following cases may occur:
  * If aP_j^-b turns into aP_jb or aI_jb, then $c_j(a,b) = 1$ and aSb holds,
  * If aP_j^-b remains, the difference $\Gamma_j(a) - \Gamma_j(b)$ gets smaller, so that according to Eq. 4.13 and Eq. 4.14, $c_j(a,b)$ increases or $d_j(a,b)$ decreases respectively.
- If aR_jb and a improves and b deteriorates then the following cases may occur:
  * If aR_jb turns into aP_jb or aI_jb, then aSb holds as $c_j(a,b) = 1$,
  * If aR_jb remains but a improves and b deteriorates with respect to other alternatives in O_j, the difference $\Gamma_j(a) - \Gamma_j(b)$ get smaller, so that according to Eq. 4.17 and Eq. 4.18, $c_j(a,b)$ increases or $d_j(a,b)$ decreases respectively.

□

- **Pareto principle:** Alternative a does not outrank alternative b if b is strictly better than a on all criteria. This property is also known as Pareto efficiency or unanimity. As the $\Gamma(\cdot)$ function measures the performance of an alternative in a partial pre-order (i.e., its rank order value), we can write this property as follows:

$$\forall j,\ \Gamma_j(b) < \Gamma_j(a) - p_j(a), \text{ then } \neg(aSb)$$

Proof. By construction, in any partial pre-order, if $\forall j, \Gamma_j(b) \leqslant \Gamma_j(a) - p_j(a)$, only discordant indices $d_j(a,b) > 0$ are calculated, thus refuting aSb.

□

4.6 Rank Reversal on MCDA

Rank reversal or rank invariance principle is a phenomenon that occurs when in a decision process a ranking O is obtained from a set of alternatives A and the addition/deletion/modification of alternative(s) generates a ranking O', which reverses the rank order of some pairs of alternatives previously obtained in O. For example, let us suppose that alternatives a and b are ranked 1st and 2nd respectively in O, and a non-optimal alternative z results in a new ranking O' in which b and a are ranked 1st and 2nd respectively. The issue of rank reversals lies at the heart of many debates in MCDA [Figueira and Roy, 2009, Saaty and Sagir, 2009, Wang and Triantaphyllou, 2008, Wang and Luo, 2009].

This phenomenon has been studied and analyzed in several decision aiding methods such as AHP, DEA, TOPSIS and SAW [Wang and Luo, 2009]. This issue happens in some ELECTRE methods, in particular ELECTRE-II and ELECTRE-III. In Wang and Triantaphyllou [2008], the rank reversal is studied in ELECTRE-II and ELECTRE-III based on the following test:

Test 1: An effective MCDM method should not change the indication of the best alternative when a non-optimal alternative is replaced by another worse alternative (given that the relative importance of each decision criterion remains unchanged).

The conception of MCDA is constructivist, i.e., provide tools to the DM to take the best decision. There is no absolute truth or a pre-existing final ranking. The basic data required in ELECTRE methods are purely ordinal in nature. Given such poor data, it is not reasonable to attempt to find a pre-existing truth or ranking in which all the positions are defined in a unique and absolute way. Figueira and Roy [2009] state that the very nature of real-world problems and the fact such problems are frequently modeled using poor data (i.e., ordinal scales), makes finding the "real" ranking a rather utopian quest. The existence of a pre-existing truth, that must not be changed under the same conditions (f.i., weights, thresholds), is the basic belief of the utility theorists. This belief is commonly used to criticize methods that do not fulfill the rank invariance principle. However, several authors have indicated that in practice the rank reversal phenomena occurs frequently and is not necessarily bad [Vargas, 1994]. Roy [1972] presented an example illustrating that such phenomena can be interpreted quite naturally and suggests that forcing the independence property may not be realistic in many real-world case studies.

Figueira and Roy [2009] argue that Test 1 reveals that the data quality is too poor to permit to the method to be able to distinguish between ranking positions. This simply serves to underline the methods' limitations. Neither ELECTRE-II, nor ELECTRE-III, were presented as a means to define or identify a pre-existing order, in which the best ranked actions do not depend on the worst ones in the ranking. Thus, the test does not invalidate the method; it just shows its natural limitation of non-precise pairwise comparison. More information about the poor data quality which may result in rank reversal is given in Figueira et al [2013].

The main reason of the rank reversal is that when adding, deleting or modifying an alternative in A, the credibility matrix changes. The ELECTRE-III method does not fulfill the property of independence with respect to irrelevant actions, such that the comparison between 2 alternatives is conditioned by the remaining alternatives. If one of the remaining alternatives is, for example, modified; the exploitation procedure is applied to a different credibility matrix, which may naturally result in a different recommendation.

Another possible reason for rank reversals between two alternatives is when the way the DM compares them (i.e., discrimination thresholds) is not consistent, difficulting the determination of which one is preferred.

4.6.1 Rank Reversal in ELECTRE-III-H

From a methodological point of view, if we consider that this method follows the two steps presented in classical ELECTRE-III, including building the credibility matrix and the exploitation procedure, we may naturally expect that the rank reversal issue is replicable to ELECTRE-III-H. We must also consider that the calculation of the relation aSb from partial pre-orders does not fulfill the independence of irrelevant alternatives (Section 4.5.2) as the rank order value of the alternatives is correlated to changes of the rank order value of the remaining alternatives; thus, affecting the preference relation matrix $\mathcal{M}$ from which the partial concordance and discordance indices are calculated.

We include an empirical test to present a rank reversal case. We follow the Test 1 from Wang and Triantaphyllou [2008], which indicates that if a non-optimal alternative is replaced by another worst, the best alternative should remain. In Figure 4.6, the basic hierarchical structure of criteria for this example is presented.

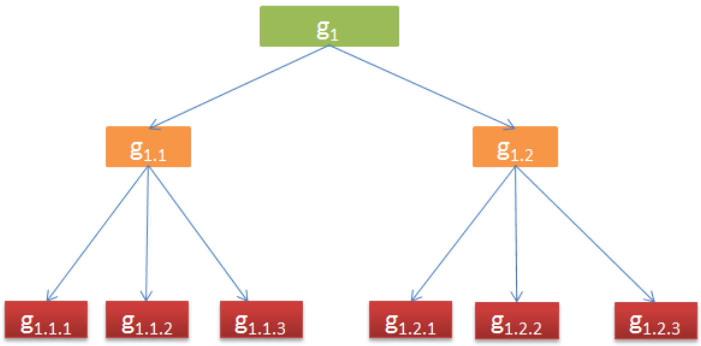


FIGURE 4.6: Hierarchy for the example Rank Reversal illustration on ELECTRE-III-H

Let us suppose that the parameters set by the DM are as follows:

- Elementary: $q_j(a) = 5$, $p_j(a) = 10$, $v_j(a) = 20$. All criteria have the same weight and are to be maximized.

- Intermediate: $q_j(a) = 0$, $p_j(a) = 1$, $v_j(a) = 3$. Weights: $g_{1.1} = 0.3$ and $g_{1.2} = 0.4$.

The empirical test comprises the evaluation from the elementary criteria, although it is only necessary to provide 2 partial pre-orders to generate global result and next, decrease the rank of a non-optimal alternative in one of the 2 initial partial pre-orders and analyze the results at the global pre-order. However, to provide a complete case, an analysis of the rank reversal in ELECTRE-III-H from the bottom level of the hierarchy up to the root is illustrated. In Table 4.3, the evaluation of the alternatives at the elementary criteria are given:

Table 4.3: Elementary criteria preference thresholds values

Alternative	$g_{1.1.1}$	$g_{1.1.2}$	$g_{1.1.3}$	$g_{1.2.1}$	$g_{1.2.2}$	$g_{1.2.3}$
a	50	70	100	50	70	100
b	**90** $\downarrow$	60	80	80	45	80
c	50	45	45	75	75	80
d	80	65	45	45	80	75
e	75	100	50	80	75	45
f	50	70	80	65	70	50
g	80	65	90	60	60	95

Using these values, ELECTRE-III-H method provides the following partial and global results:

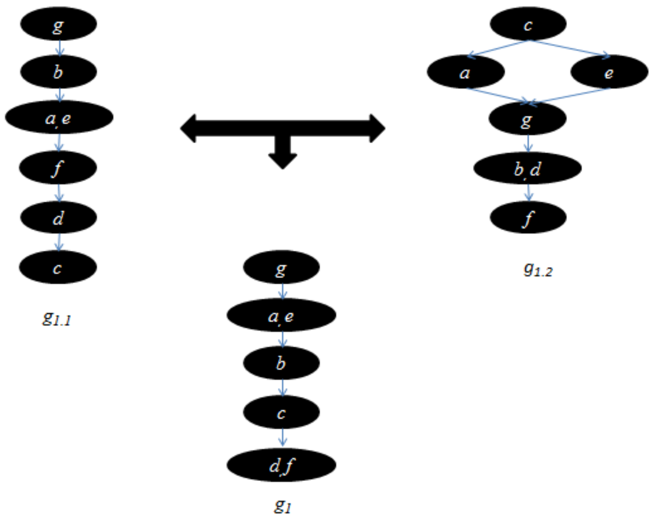


Figure 4.7: Original Result

According to the results obtained with ELECTRE-III-H, alternative g is presented as the best alternative overall, followed by a and e which are indifferent, followed by b. According to Test 1, when a non-optimal alternative is replaced by another worst, g should remain as the best alternative.

Let us suppose that we decrease the evaluation of alternative b on $g_{1.1.1}$ (see cell in bold), so that $g_{1.1.1}(b) = 40$ instead of 90. Following the statement of Test 1, decreasing the evaluation of b should not affect the rank of alternative g. In Figure 4.8, the results are shown:

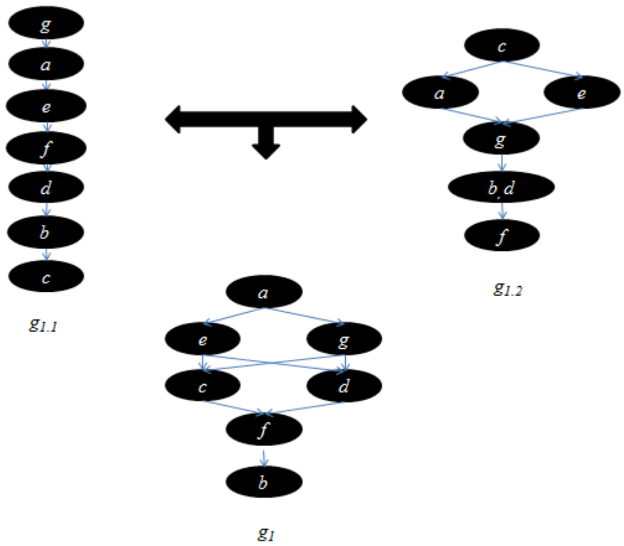
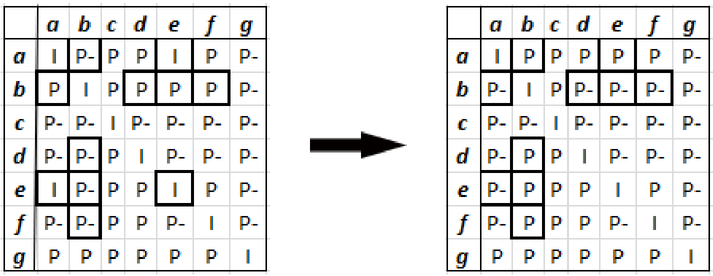


FIGURE 4.8: Modified Result

Note that on $g_{1.1}$, b clearly get worse on the partial pre-order and g remains as the best alternative. The ranking on $g_{1.2}$ remains the same, so that there is no "apparently reason" to consider a as a better alternative than g considering the Test 1 presented in Wang and Triantaphyllou [2008]. However, when the evaluation of b on $g_{1.1.1}(b)$ is decreased to 40, the rank position of alternative g and a are reversed on the final overall partial pre-order (g_1) because now a is preferred to more alternatives in $g_{1.1}$ than in the first case (f.i., a outranks e when they were indifferent in the original case).

At intermediate criteria, the calculation of partial concordance and discordance indices depends on the function $\Gamma(\cdot)$ and the binary relations (i.e., P, P^{-}, I, and R). The addition/deletion/modification of an alternative in a partial pre-order affects the preference relation matrix $\mathcal{M}$.

In Figure 4.9, the preference relation matrix $\mathcal{M}$ generated on $g_{1.1}$ are shown for both cases, the original evaluation presented in Table 4.3 and the case where alternative b is modified (performance decreases to 40).



	a	b	c	d	e	f	g
a	I	P-	P	P	I	P	P-
b	P	I	P	P	P	P	P-
c	P-	P-	I	P-	P-	P-	P-
d	P-	P-	P	I	P-	P-	P-
e	I	P-	P	P	I	P	P-
f	P-	P-	P	P	P-	I	P-
g	P	P	P	P	P	P	I

	a	b	c	d	e	f	g
a	I	P	P	P	P	P	P-
b	P-	I	P	P-	P-	P-	P-
c	P-	P-	I	P-	P-	P-	P-
d	P-	P	P	I	P-	P-	P-
e	P-	P	P	P	I	P	P-
f	P-	P	P	P	P-	I	P-
g	P	P	P	P	P	P	I

FIGURE 4.9: Preference relation matrix changes on $g_{1.1}$ for the original (left) and modified (right) test.

In Tables 4.4 and 4.5, the credibility matrix generated at the overall node is shown. Note the changes because of the modification of the performance of the alternative b on one criterion.

TABLE 4.4: Credibility matrix to generate the original g_1 partial pre-order result

	a	b	c	d	e	f	g
a	1	0,57	0,43	1	0,81	1	0,57
b	0	1	0	1	0	1	0
c	0	0	1	0,57	0	0,57	0
d	0	0	0	1	0	0,57	0
e	0,81	0,57	0,43	1	1	1	0,57
f	0	0	0	0,38	0	1	0
g	0,38	1	0	1	0,38	1	1

TABLE 4.5: Credibility matrix to generate the modified g_1 partial pre-order result

	a	b	c	d	e	f	g
a	1	1	0,43	1	0,81	1	0,57
b	0	1	0	0,57	0	0,57	0
c	0	0,57	1	0,57	0	0	0
d	0	1	0	1	0	0,57	0
e	0,38	1	0,43	1	1	1	0,57
f	0	0,38	0	0,38	0	1	0
g	0,38	1	0	1	0,38	1	1

As explained before, this issue does not invalidate the method, just shows one of its limitations. As seen in Section 2.2.3, no ordinal aggregation procedure fulfills all the 5 conditions stated by the Arrow's impossibility theorem. This is the case of the ELECTRE method, as it works as a voting-like system with ordinal data. For this specific case, the condition violated for this theorem is the independence of irrelevant alternatives.

Chapter 5

The ELECTRE-TRI-B-H method for sorting hierarchical problems

Chapter 4 solved the ranking hierarchical problem with ELECTRE-III-H method, following the formalization of the hierarchical structures of criteria presented in Section 3.3. This chapter studies the case of a sorting method for hierarchies of criteria, for which we propose an extension of the classical ELECTRE-TRI-B method.

The assignment of alternatives into predefined categories is a traditional problem known as classification or labeling. It has been mainly studied in Machine Learning and Decision Aiding [Doumpos and Grigoroundis, 2013]. Two different problems can be defined depending on whether the categories are ordered (sorting or ordered classification) or not ordered (nominal classification). In sorting, each alternative is assigned to one of the categories, which have been previously ordered from the worst to the best one, indicating several degrees of interest or suitability of the alternatives for a certain user or DM, depending on multiple conflicting criteria. For example, a professor may desire to classify their students in four categories based on their grades: Unsatisfactory, Satisfactory, Good and Excellent. Otherwise, in nominal sorting, there is not a preference relation of one category with respect to the next one. Instead, categories are used to differentiate types of alternatives. Nominal classification finds groups of elements by finding commonalities on their features and the target classes, and it has mainly been studied in Machine Learning. For example, a Human Resource department may assign the candidates in one of the following predefined categories based on skill tests: Technical, Finance and Commercial. Another classification technique is clustering, defining groups of similar alternatives, similar to the nominal classification. However, in clustering classification the groups (i.e., categories) are not known a priori. Recently, clustering techniques have been integrated into MCDA [Rocha et al, 2013].

Sorting or ordered classification is specially interesting for solving decision aiding problems because the order of the categories can be used to indicate different levels of achievement of the DM's goal. When multiple and conflicting criteria are taken into account, the DM needs formal models and operational tools to analyze this range of criteria and make the best assignment according to the preference order of the categories.

ELECTRE-TRI was designed to solve sorting problems. It was originated to solve a classification problem assigning alternatives to one of three categories: Acceptable, Unacceptable and Indeterminate. But it later was extended to sorting problems with more than three different categories. The ELECTRE-TRI approach studied in this thesis is based on the method presented in Roy [1996], which has been applied in several real problems in different fields [Arondel and Girardin, 2000, Brito et al, 2010, Cloquell-Ballester et al, 2007, Sánchez-Lozano et al, 2014, Xidonas et al, 2009]. Since the appearance of new ELECTRE methods for sorting problems, this classical ELECTRE-TRI method was renamed as ELECTRE-TRI-B, in which B stands for boundaries. In fact, this method is based on boundary profiles (or limiting profiles) that are fictive alternatives that separate two consecutive categories. The ELECTRE-TRI-B method is reviewed in Section 5.1.

To manage a taxonomical organization of the set of criteria in the form of a hierarchy in sorting problems, we consider that intermediate criteria in such a hierarchy correspond to different aspects of the decision problem (e.g., cost, distance, noise). At each of these criteria, a sorting problem must be solved. Therefore, we propose extending ELECTRE-TRI-B to handle assignments of alternatives on several levels of the hierarchy.

The first contribution in this chapter is the proposal of an adaptation of the ELECTRE-TRI-B method to manage heterogeneous sets of categories at each intermediate criteria and at root criterion aggregating previous assignments in Section 5.2. In this regard, we propose a new procedure to define profile limits in terms of different sets of categories from criteria at lower levels in the hierarchy 5.2.1. We also redefine the classical outranking construction step in terms of assignment of alternatives to categories, instead of numerical ratings 5.2.2. Next, the propagation of the assignments at each level to upper nodes in the hierarchy until the root global criterion is studied. This leads to the second contribution presented in this chapter, which is the definition of the hierarchical ELECTRE-TRI-B-H method (in Section 5.2.3).

5.1 The ELECTRE-TRI-B method

In addition to the fundamental concepts related to the decision aiding process presented in Chapter 2, which includes alternatives A, criteria G and weights W expressing a measure of relative importance of each criterion g_j, the classical ELECTRE-TRI-B method requires the introduction of the following elements:

- $B = \{ b_1, b_2,.., b_k \}$ is the finite set of profiles defining $k + 1$ categories,
- $C = \{ C_1, C_2,.., C_{k+1} \}$ is the finite set of ordered categories from the worst to the best, b_h being the upper limit of the category C_h and the lower limit of $C_{h+1}, h = 1, 2, ..., k$.

Profile limits are reference alternatives that define the limit between two consecutive categories. Alternative $a \in A$ can be represented as a vector of the form: $< g_1(a), g_2(a), ..., g_m(a) >$. Analogously, profile limit $b_h \in B$ is represented as a vector $< g_1(b_h), g_2(b_h), ..., g_m(b_h) >$. Profile b_h has the minimum value that makes it be the first alternative to be assigned to category C_{h+1}. A usual graphical representation of the profile limits in terms of the set G of criteria is given in Figure 5.1.

In general, it is assumed that the boundary profiles are directly provided by the DM. However, in case of complex scenarios, some works have proposed indirect preference elicitation tools using examples, which reduce the cognitive effort required from the DM. In Mousseau et al [2000], an ELECTRE-TRI Assistant tool is proposed, in which the parameters (boundaries, weights and thresholds) are inferred from assignment examples provided by the DM. An elicitation procedure with multiple DMs is proposed in Cailloux et al [2012], in which the common boundaries are inferred from assignment examples provided by multiple DMs, reaching a consensus among them on the boundaries and veto thresholds.

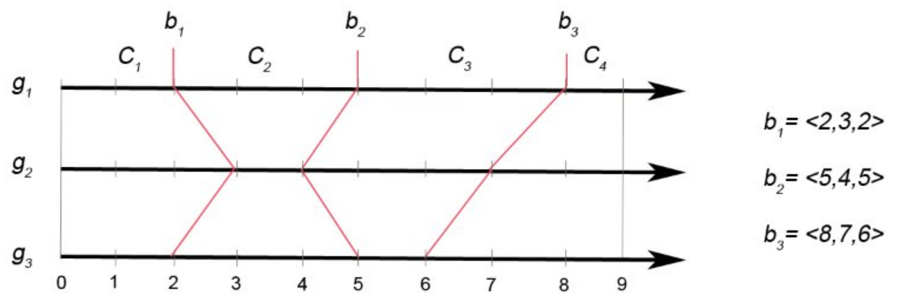


FIGURE 5.1: Graphical representation of profile limits in classical ELECTRE-TRI-B

The aim of the ELECTRE-TRI-B method is to compare alternatives to profile limits by building a fuzzy outranking relation S, where aSb_h means "a is at least as good as b_h". From this outranking relation, ELECTRE-TRI-B assigns the alternatives in A the to predefined categories C. The assertion aSb_h is considered to be true if there are sufficient arguments to affirm that a is not worse than b_h, and if there is no essential reason to refuse this assertion. These concepts are formalized in the definition of concordance (majority principle) and discordance (respect to minorities) indices with respect to aSb_h. So, aSb_h is validated if the evaluations of alternative a are at least as good as profile limit b_h for the majority of criteria and no single evaluation is strongly worst than profile limit b_h. In this way, ELECTRE-TRI-B follows a non-compensatory approach, as it was also the case of ELECTRE-III.

In order to take into account the uncertainty and imprecision associated to pairwise comparison of alternatives and profile limits on particular criteria, we use pseudo-criteria with discrimination thresholds associated to the profile limits defined in terms of b_h.

- indifference threshold $q_j(b_h)$: below which the DM is indifferent to the evaluation of alternative a and profile limit b_h on criterion g_j. For instance, if $q_j(b_h)$=2 and the difference between $g_j(b_h) - g_j(a)$=1, the DM is considering that this difference is not strong enough to establish a preference either for b_h or a. Thus, justifying an equivalence between both;
- preference threshold $p_j(b_h)$: above which the DM shows a clear strict preference in favor of alternative a over profile limit b_h on criterion g_j. For instance, if $q_j(b_h)$=2 and $p_j(b_h)$=4 and $g_j(b_h) - g_j(a)$=3, the DM considers that b_h is preferred over a but still there is a weak preference of a over b_h. This is known as a hesitation range and models the preference uncertainty on a criterion.

In addition, a veto threshold may also be associated to a criterion for a certain profile limit.

- veto threshold $v_j(b_h)$: where a discordant difference larger than the veto in favor of b_h with respect to alternative a will require the DM to negate the outranking relation aSb_h. For instance, if $v_j(b_h)$=5 and $g_j(b_h) - g_j(a) \geqslant 5$, the DM is considering that this difference prevents the assignment of a to a category higher than b_h.

ELECTRE-TRI-B assigns categories to alternatives based on two consecutive stages:

1. The construction of an outranking relation for each alternative concerning to the limits of the categories, taking into account the weights of the criteria,

2. The exploitation of these outranking relations to assign each alternative to a specific category. Two different procedures can be selected: optimistic and pessimistic.

In Figure 5.2 we illustrate the steps of the ELECTRE-TRI-B method, including the calculation and exploitation of the outranking relations. For this example let us consider the following:

- $A = \{a, b, c, d, e, f\}$,
- $G = \{g_1, g_2, g_3, g_4\}$,
- The predefined categories are *Unacceptable*, *Fair* and *Good*, so that $B = \{b_1, b_2\}$.

Additionally, the parameters are shown in Table 5.1. We assume that the performances are to maximize.

TABLE 5.1: Parameters considered for the sorting process example

Criterion	w_j	$q_j(b_h)$	$p_j(b_h)$	$v_j(b_h)$
g_1	25	5	12	20
g_2	15	5	12	25
g_3	40	5	8	12
g_4	20	5	12	25
Profile limits	g_1	g_2	g_3	g_4
b_1	45	50	60	55
b_2	70	85	85	80

Input

$A \times G$	g_1	g_2	g_3	g_4
a	93	86	92	86
b	82	60	74	74
c	78	80	60	70
d	70	75	68	76
e	50	28	20	79
f	92	63	90	51

Partial concordances (a, b_h)

g_1

	b_1	b_2
a	1	1
b	1	1
c	1	1
d	1	1
e	1	0
f	1	1

g_2

	b_1	b_2
a	1	1
b	1	0
c	1	1
d	1	0,29
e	0	0
f	1	0

g_3

	b_1	b_2
a	1	1
b	1	0
c	1	0
d	1	0
e	0	0
f	1	1

g_4

	b_1	b_2
a	1	1
b	1	0,86
c	1	0,29
d	1	1
e	1	1
f	1	0

Partial concordances (b_h, a)

g_1

	a	*b*	*c*	*d*	*e*	*f*
b_1	0	0	0	0	1	0
b_2	0	0	0,57	1	1	0

g_2

	a	*b*	*c*	*d*	*e*	*f*
b_1	0	0,29	0	0	1	0
b_2	1	1	1	1	1	1

g_3

	a	*b*	*c*	*d*	*e*	*f*
b_1	0	0	1	0	1	0
b_2	0,33	1	1	1	1	1

g_4

	a	*b*	*c*	*d*	*e*	*f*
b_1	0	0	0	0	0	1
b_2	0,86	1	1	1	1	1

Overall concordances (a, b_h)

	b_1	b_2
a	1	1
b	1	0,42
c	1	0,46
d	1	0,49
e	0,45	0,2
f	1	0,65

Overall concordances (b_h, a)

	a	*b*	*c*	*d*	*e*	*f*
b_1	0	0,04	0,4	0	0,8	0,2
b_2	0,45	0,75	0,89	1	1	0,75

Partial discordances (a, b_h)

g_1

	b_1	b_2
a	0	0
b	0	0
c	0	0
d	0	0
e	0	1
f	0	0

g_2

	b_1	b_2
a	0	0
b	0	1
c	0	0
d	0	0
e	0,77	1
f	0	0,77

g_3

	b_1	b_2
a	0	0
b	0	0,75
c	0	1
d	0	1
e	1	1
f	0	0

g_4

	b_1	b_2
a	0	0
b	0	0
c	0	0
d	0	0
e	0	0
f	0	1

Partial discordances (b_h, a)

g_1

	a	*b*	*c*	*d*	*e*	*f*
b_1	1	1	1	1	0	1
b_2	1	0	0	0	0	1

g_2

	a	*b*	*c*	*d*	*e*	*f*
b_1	1	0	1	1	0	0,08
b_2	0	0	0	0	0	0

g_3

	a	*b*	*c*	*d*	*e*	*f*
b_1	1	1	0	0	0	1
b_2	0	0	0	0	0	0

g_4

	a	*b*	*c*	*d*	*e*	*f*
b_1	1	0,54	0,2	0,69	0,92	0
b_2	0	0	0	0	0	0

(a, b_h) credibility of outranking relations

	b_1	b_2
a	1	1
b	1	0
c	1	0
d	1	0
e	0	0
f	1	0

(b_h, a) credibility of outranking relations

	a	*b*	*c*	*d*	*e*	*f*
b_1	0	0,04	0,4	0	0,8	0,2
b_2	0,45	0,75	0,89	1	1	0,75

Alternatives - profile limits relations

	b_1	b_2
a	*P*	*P*
b	*P*	P^-
c	*P*	P^-
d	*P*	P^-
e	P^-	P^-
f	*P*	P^-

Exploitation

Good: {a}
Fair: {b, c, f, d}
Unacceptable: {e}

Figure 5.2: ELECTRE-TRI-B-H method example

5.1.1 Construction of the outranking relation

In classical ELECTRE-TRI-B, the outranking relation between alternative a and the boundary profile b_h is analyzed in two directions, being aSb_h and b_hSa. The construction of each outranking relation is based on the two typical tests: the concordance test and the discordance test, as seen in Figure 5.2. With them we can calculate the credibility degree of S. This first stage is quite similar to the case of ranking with ELECTRE-III. For the sake of simplicity, we only present the calculation of aSb_h.

5.1.1.1 Concordance test

The overall concordance index $c(a, b_h)$ measures the strength of the criteria that support the outranking relation "a is at least as good as b_h" and is calculated as follows:

$$c(a, b_h) = \frac{1}{W} \sum_{j=1}^{m} w_j c_j(a, b_h) \tag{5.1}$$

where $W = \sum_{j=1}^{m} w_j$, and the partial concordance indices for each criterion are defined as:

$$c_j(a, b_h) = \begin{cases} 1 & \text{if } g_j(b_h) - g_j(a) \leqslant q_j(b_h) \\ 0 & \text{if } g_j(b_h) - g_j(a) \geqslant p_j(b_h) \\ \frac{g_j(a) - g_j(b_h) + p_j(b_h)}{p_j(b_h) - q_j(b_h)} & \text{otherwise.} \end{cases} \tag{5.2}$$

5.1.1.2 Discordance test

The computation of partial discordance indices $d(a, b_h)$ searches for criteria that disagree with the assertion aSb_h on criterion g_j. The partial discordance indices are calculated as follows:

$$d_j(a, b_h) = \begin{cases} 1 & \text{if } g_j(b_h) - g_j(a) > v_j(b_h) \\ 0 & \text{if } g_j(b_h) - g_j(a) \leqslant p_j(b_h) \\ \frac{g_j(b_h) - g_j(a) - p_j(b_h)}{v_j(b_h) - p_j(b_h)} & \text{otherwise .} \end{cases} \tag{5.3}$$

5.1.1.3 Credibility degree of the outranking relation

The degree of credibility of the outranking relation is calculated according to the overall concordance and partial discordance indices, which are combined to obtain a valued outranking relation with credibility $\rho(a, b_h) \in [0, 1]$ defined by:

$$\rho(a, b_h) = \begin{cases} c(a, b_h) & \text{if } d_j(a, b_h) \leqslant c(a, b_h), \forall j \\ c(a, b_h) \prod\limits_{j \in J_{(a, b_h)}} \frac{1 - d_j(a, b_h)}{1 - c(a, b_h)} & \text{otherwise} \end{cases} \tag{5.4}$$

where $J(a, b_h)$ is the set of criteria for which $d_j(a, b_h) > c(a, b_h)$ and the credibility of outranking is equal to the overall concordance index when there is no discordant criterion.

For sorting, the obtained fuzzy outranking relation is transformed into a crisp outranking relation S by means of the application of a cutting level λ-cut, which is considered as the smallest value of the credibility index ρ to consider that the outranking relation holds. Considering alternative a from set A and boundary profile b_h, four situations may occur:

- aSb_h and not b_hSa: aPb_h (a is strictly preferred to b_h)
- b_hSa and not aSb_h: aP^-b_h (b_h is strictly preferred to a, which can be expressed as a being inversely preferred to b_h)
- aSb_h and b_hSa: aIb_h (a is indifferent to b_h)
- Not aSb_h and not b_hSa: aRb_h (a is incomparable to b_h)

5.1.2 Exploitation procedure

The exploitation procedure for sorting analyzes the outranking relations calculated before and compares the alternatives from set A to the profiles B in order to determine to what category the alternatives should be assigned [Mousseau et al, 2000]. Each alternative is treated independently from the others. The assignment is grounded on two typical logic aggregation policies [Yu, 1992]:

- The conjunctive logic, in which an alternative can be assigned to a category when its evaluation is at least as good as the lower limit of this category. The alternative is then assigned to the highest category C_h that fulfills this condition.

- The disjunctive logic, in which an alternative can be assigned to a category, if it has, on at least one criterion, an evaluation at least as good as the lower limit of this category. The alternative is then assigned to the lowest category C_h that fulfills this condition.

With the disjunctive rule, the assignment of an action is generally better than with the conjunctive rule. The conjunctive rule is more strict and it is usually known as *pessimistic procedure*, whereas the disjunctive rule is known as *optimistic procedure*. When comparing an alternative a to the profiles, if no incomparability is found, then a is assigned to the same category by both the optimistic and the pessimistic policies. The procedures go as follows:

- Pessimistic or conjunctive assignment: This variant assigns each alternative a to the highest category C_h such that aSb_{h-1}.

  a) Compare a successively with b_r, with $r = k, k-1, ..., 1$. So, starting from the highest category.

  b) The limit b_h is the first encountered profile such that aSb_h. Assign a to category C_{h+1}.

- Optimistic or disjunctive assignment: This variant assigns each alternative a to the lowest category C_h such that b_hPa.

  a) Compare a successively with b_r, with $r = 1, 2, ..., k$. So, starting from the lowest category.

  b) The limit b_h is the first encountered profile such that b_hPa. Assign a to category C_h.

To illustrate the differences between the pessimistic and optimistic procedure, let us suppose that for the example presented in Figure 5.1, three alternatives have been evaluated. According to the performance of the alternatives, the final assignments using the pessimistic and optimistic procedure are presented in Table 5.2.

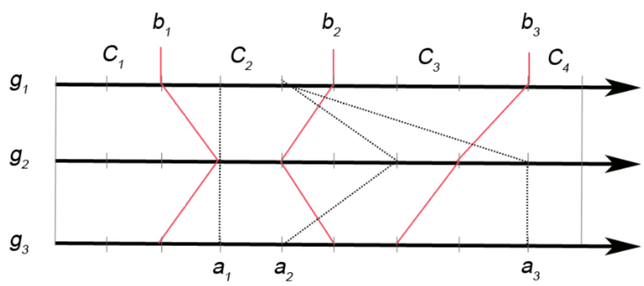


FIGURE 5.3: Example of alternatives and profile limits

TABLE 5.2: Alternatives relations with profile limits and its assignment

$A \times B$	b_1	b_2	b_3	Pessimistic assignment	Optimistic assignment
a_1	P	P^-	P^-	C_2	C_2
a_2	P	R	P^-	C_2	C_3
a_3	P	R	R	C_2	C_4

5.2 The ELECTRE-TRI-B-H method

Classical ELECTRE-TRI-B only considers a set of elementary criteria and makes a unique assignment of the alternatives to the categories of an overall criterion. In this section we propose a hierarchical version of this method, named ELECTRE-TRI-B-H, which makes an assignment of the alternatives at each of the non-elementary criteria. That is, the assignment procedure is not only computed at the root, but also at each of the intermediate sub-criteria.

First, we extend the notation of the basic ELECTRE-TRI-B method presented in Section 5.1, following the hierarchical structure defined in Section 4.2.

- For each criterion $g_j \in \mathcal{R} \cup \mathcal{I}$, $B^j = \{ b_1^j, b_2^j, .., b_{k_j}^j \}$ is the finite set of indices of the profiles defining $k_j + 1$ categories defined on this criterion,
- For each criterion $g_j \in \mathcal{R} \cup \mathcal{I}$, $C^j = \{ C_1^j, C_2^j, .., C_{k_j+1}^j \}$ is the finite set of ascending predefined categories on criterion g_j, being b_h^j the upper limit of the category C_h^j and the lower limit of $C_{h+1}^j, h = 1, 2, ..., k_j$.

Categories on nodes at $\mathcal{R}$ and $\mathcal{I}$ nodes must be defined a priori by the DM. The model that we propose in this thesis considers heterogeneous criteria, such that for a non-elementary criterion g_j, the set of categories C^j can be different from the categories of the other criteria in the hierarchy. In that way, the DM can define the most appropriate number of categories for each criterion and the most appropriate linguistic label for each of those categories. This flexibility in the definition of the criteria is a remarkable characteristic because it permits to adjust the model to each particular problem.

The method allows the introduction of relative importance of each criterion at different levels of the hierarchy. It is applied for each node g_j independently, in which the weights of $g_{j.d}$ must refer only to the other descendants of the same parent g_j.

At each non-elementary criterion, the ELECTRE-TRI-B-H method calculates the outranking relation (i.e., concordance and discordance tests) and later it performs the exploitation of this relation by either the pessimistic or optimistic procedure.

At these nodes the assignment of alternatives is established locally according to the set of categories provided by the DM. The calculation procedure is established in a bottom-up fashion, from the lowest level to the root overall criterion. The novelty of the method proposed lies on the treatment of the assignments established for a sub-criterion $g_{j.d}$ in order to make the next assignment at its parent node g_j.

Assuming that the categories C^j for all non-elementary criteria have been already provided by the DM, in Section 5.2.1 we propose a rule-based method to model the profile limits between the predefined categories. In addition we propose a definition of threshold values in terms of categories. The following sections present the process of the ELECTRE-TRI-B-H method, from the lowest level of the hierarchy as presented in Section 5.1, up to the root level as presented in 5.2.2. The complete algorithm is then given in Section 5.2.3.

5.2.1 Modeling new profile limits

In the classical ELECTRE-TRI-B method, the assignment of alternative a to a certain category results from the comparison of alternative a with respect to the lower profile of this category. To follow the same approach at intermediate criteria, we need to define profile limits at all nodes of the hierarchy. The profile limits will be referred to the immediate descendant sub-criteria values. In this section we propose a rule-based method to allow the DM to model profile limits separating the categories of the node g_j with respect to the categories of their direct descendants $g_{j.d}$. These rules establish a mapping between the assignments established at the lowest level (to $g_{j.d}$) and the assignments that must be established at the current node g_j. Then, the profiles in form of vector can be automatically obtained from these rules.

Definition 5.1. Mapping assignment rule. Considering a criterion g_j and the set D_j of the sub-criteria that are its direct descendants, $g_{j.d} \in D_j$, and being C^j and B^j the set of categories and the set of profile limits for g_j respectively, a mapping assignment rule takes the form of:

$$\textbf{if } \Psi_{j.1} \text{ and } \Psi_{j.2} \text{ ... and } \Psi_{j.|D_j|} \textbf{ then } (g_j = C^j_{h+1}),$$

where $\Psi_{j.d}$ is a disjunctive condition such as $(C^{j.d}_{h'}$ or $C^{j.d}_{h''}$ or ...), in which the subset of categories is ascending and the categories are consecutive.

The DM will give an assignment mapping rule for each category in C^j, so that i-th rule is for the assignment to category C^j_i. All those rules must fulfill the following conditions:

- Condition 1: in rule i, $\max(\Psi_{j.d}) \leqslant \min(\Psi_{j.d})$ in rule $i+1$.
- Condition 2: all the categories $C^{j.d}$ of the descendent criterion must appear in at least one rule condition $\Psi_{j.d}$.

Definition 5.2. Profile limit for sorted categorical criteria From the mapping assignment rule for category C^j_{h+1}, which is of the form: **if** $\Psi_{j.1}$ and $\Psi_{j.2}$ ... and $\Psi_{j.|D_j|}$ **then** $(g_j = C^j_{h+1})$, the profile limit b^j_h is defined as the vector:

$$b^j_h = \langle \min(\Psi_{j.1}), \min(\Psi_{j.2}), ... \min(\Psi_{j.|D_j|}) \rangle$$

By constructing the profile limit with $min(\Psi_{j.d})$, the lowest category of the condition is taken, so that the profile establishes the lowest value of criterion $g_{j.d}$ that should be assigned to C^j_{h+1}.

The following example illustrates this procedure. Let us suppose that the DM is considering the following 3 intermediate criteria in order to sort several models of cars:

- **Interior** $g_{j.1}$**:** referring to the quality of the interior accessories. The categories defined are $C^{j.1}$= {Very Poor, Poor, Medium Poor, Fair, Medium Good, Good, Very Good}.
- **Engine** $g_{j.2}$**:** referring to the acceleration, maximum speed, etc. The categories defined are $C^{j.2}$={Very Bad, Bad, Regular, Excellent}.
- **Security** $g_{j.3}$**:** referring to car crash tests score. The categories defined are $C^{j.3}$={Unreliable, Reliable}.

The overall criterion g_j defined by the DM has C^j={Bad, Acceptable, Good}. The mapping assignment rules for these three categories are the following:

Rule 1: **if** $g_{j.1}$=(Very Poor or Poor or Medium Poor) and $g_{j.2}$=(Very Bad or Bad) and $g_{j.3}$=Unreliable **then** g_j=Bad

Rule 2: **if** $g_{j.1}$=(Fair or Medium Good) and $g_{j.2}$=Regular and $g_{j.3}$=Reliable **then** g_j=Acceptable

Rule 3: **if** $g_{j.1}$=(Good or Very Good) and $g_{j.2}$=Excellent and $g_{j.3}$=Reliable **then** g_j=Good

In Figure 5.4, the modeling of profile limits in terms of categories is graphically represented.

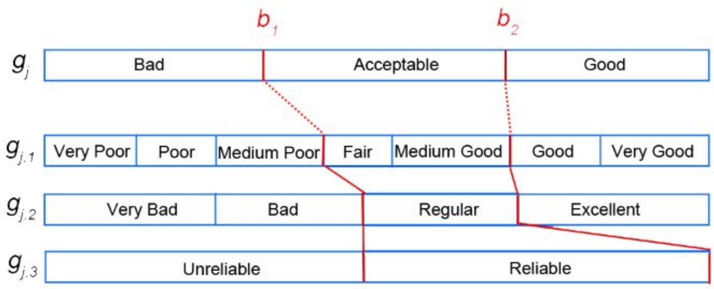


FIGURE 5.4: Graphical representation of profiles in categories, given by mapping assignment rules

The profile limits obtained from the previous rules are:

$$b_1^j = \langle C_4^{j.1}, C_3^{j.2}, C_2^{j.3} \rangle = \langle Fair, Regular, Reliable \rangle$$
$$b_2^j = \langle C_6^{j.1}, C_4^{j.2}, C_2^{j.3} \rangle = \langle Good, Excellent, Reliable \rangle$$

Note that all intermediate criteria have a different number of categories in comparison to the root criterion g_j. This example illustrates the treatment of heterogeneous sets of categories on each criterion. In the example given, $g_{j.3}$ has only two categories while g_j has three. In this case, the same category can appear in more than one mapping condition of the categories of its parent g_j. The profile limit b_1^j represents the lower limit of the category *Acceptable* and it is set to *Reliable* (for the third criterion), the same value than for b_2^j (the lower limit of the category *Good*). This means that if an alternative a is assigned to *Reliable* in $g_{j.3}$, it can be assigned either to *Acceptable* or *Good* in g_j (depending on the other conditions).

Profiles in classical ELECTRE-TRI-B are also linked to discrimination thresholds to model uncertainty in numerical scores. However, uncertainty is not only present when evaluating using quantitative scales. Qualitative scales also involve ambiguity and uncertainty, produced by the vagueness of meanings, which makes it difficult to find an exact definition of the linguistic values (i.e., categories). Issues involved in assigning uncertainty to linguistic scales are presented in several works [Chen et al, 2014, Li et al, 2009, Wang et al, 2014]. In particular, the use of heterogeneous and large sets of categories may lead to a hesitation on how to define the mapping rules to criteria at the upper level (i.e., when establishing the relation between the assignment of an alternative a to a specific category on g_j

from previous assignment rules on the subcriteria $g_{j.d}$). Based on the case presented above, the DM may consider that a car, in global terms, is *Acceptable* if the Engine is at least *Regular* and Security is *Reliable*, but may hesitate if the Interior should be at least *Fair* or *Medium Poor* because of the imprecision of those terms.

Taking this into account, we propose a new definition for the threshold value functions for indifference $q_j(b_h^j)$, preference $p_j(b_h^j)$, and veto $v_j(b_h^j)$ at the intermediate criteria. Previously we define the *Category Improvement Value* of an alternative or profile.

Definition 5.3. Category Improvement Value Φ_j. It is a function of the form $\Phi_j : A \cup B^j \longrightarrow \aleph$ that determines how many categories an alternative (or profile) may improve to get the best performance value for criterion g_j.

$$\Phi_j(x) = \begin{cases} k_{j.d} + 1 - i & \text{if } x \in A \text{ and } x \in C_i^{j.d} \\ k_{j.d} + 1 - h' + 1 & \text{if } x = b_h^j \in B^j \text{ and } b_h^j = \langle\ ...,\ C_{h'}^{j.d},\ ...\ \rangle \end{cases} \tag{5.5}$$

This value can be used to compare alternatives and profiles when they have been assigned to ordered categories. The category improvement value is a measure of performance that must be minimized.

Then, all criteria in set G can be considered as pseudo-criteria, defined as follows:

- The indifference $q_j(b_h^j)$, preference $p_j(b_h^j)$, and veto $v_j(b_h^j)$ thresholds referring to elementary criteria from set $\mathcal{E}$ are fixed in the same way as in ELECTRE-TRI-B method, based on the performance of the alternatives with respect to the profiles limits.
- The indifference $q_j(b_h^j)$, preference $p_j(b_h^j)$, and veto $v_j(b_h^j)$ thresholds referring to criteria from set $\mathcal{I}$ are defined in terms of the difference between the category improvement value of an alternative assignment to $C^{j.d}$ (i.e., partial performance aggregation from descendant nodes) and the category improvement value of the profile limits in B^j.

Having that $|C^j| = k_j + 1$, then $0 \leqslant q_j(b_h^j) \leqslant p_j(b_h^j) \leqslant v_j(b_h^j) \leqslant k_j$. In addition, for consistency, the thresholds values must fulfill the following conditions for $h > 1$:

- Condition 1: $q_j(b_h^j) \leqslant \Phi_j(b_h^j) - \Phi_j(b_{h+1}^j) + q_j(b_{h+1}^j)$
- Condition 2: $p_j(b_h^j) \leqslant \Phi_j(b_h^j) - \Phi_j(b_{h+1}^j) + p_j(b_{h+1}^j)$
- Condition 3: $v_j(b_h^j) \leqslant \Phi_j(b_h^j) - \Phi_j(b_{h+1}^j) + v_j(b_{h+1}^j)$

5.2.2 Redefinition of concordance and discordance indices in terms of category improvement values

Classical ELECTRE-TRI-B deals with numerical performance evaluations. For instance, if qualitative scales of criteria are used, each qualitative scale must be coded in an ordinal scale, as well as the discrimination and veto thresholds. One may argue that both the assignment of an alternative to a certain category and the profile limit can be coded in an ordinal scale. However, the use of the category improvement value avoids this coding, using directly category assignments without any transformation in the original scale, resulting in a transparent stage in the decision process.

Therefore, in intermediate nodes with all descendants in $\mathcal{E}$ we can directly apply the ELECTRE-TRI-B concordance and discordance measures to obtain the credibility degree of the outranking relation (see Section 5.1), because the alternatives are directly evaluated with numerical scores. For example, taking into account the hierarchy tree in Figure 3.4 and presented again below in Figure 5.5, the classical ELECTRE-TRI-B is applied for three subsets of elementary criteria separately: $\{g_{1.1.1}, g_{1.1.2}, g_{1.1.3}\}$, $\{g_{1.2.1.1}, g_{1.2.1.2}\}$ and $\{g_{1.2.2.1}, g_{1.2.2.2}\}$.

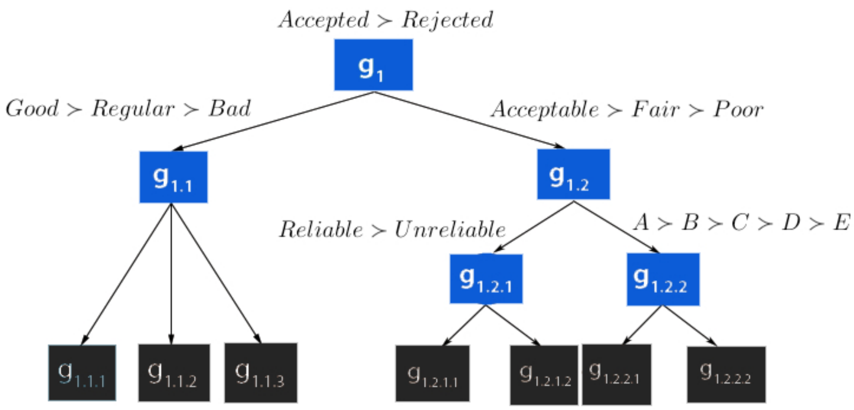


FIGURE 5.5: Hierarchical tree of the set of criteria. The nodes in black apply classical ELECTRE-TRI-B while the blue ones apply new calculations based on the category improvement value.

The exploitation of the credibility matrix proceeds in the classical way too, as explained in Section 5.1.2. This step ends with the assignment of the alternatives to one of the categories in the corresponding set $C^{1.1}$, $C^{1.2.1}$ or $C^{1.2.2}$.

For nodes on $\mathcal{I}$ and $\mathcal{R}$ in which not all their direct descendants are elements in $\mathcal{E}$, the ELECTRE-TRI-B-H procedure will also have the two classical main steps: 1) the construction of the outranking relation for each alternative with respect to the limits of the categories, and 2) the exploitation of the outranking relation to assign each alternative to a specific category. However, the construction of the outranking relations is going to be redefined to address sorting criteria (for which a previous assignment of alternatives to categories has been computed). First, the DM must follow the profile limits modeling on the criteria in $\mathcal{I}$, in form of mapping assignment rules, as presented in Section 5.2.1. From this step, the profile limits are obtained. Following the example in Figure 5.5 the profile limits $B^{1.1}$, $B^{1.2.1}$ and $B^{1.2.2}$ are calculated.

Second, the DM determines the indifference, preference and veto thresholds for each criterion. These thresholds are then used to calculate the partial concordance index $c_j(a, b_h^j)$ and discordance indices $d_j(a, b_h^j)$ in terms of the category improvement value Φ_j.

Definition 5.4. Partial concordance index for sorting criteria:

$$c_j(a, b_h^j) = \begin{cases} 1 & \text{if } \Phi_j(a) - \Phi_j(b_h^j) \leqslant q_j(b_h^j), \text{then} \\ 0 & \text{if } \Phi_j(a) - \Phi_j(b_h^j) \geqslant p_j(b_h^j), \text{then} \\ \frac{p_j(b_h^j) - (\Phi_j(a) - \Phi_j(b_h^j))}{p_j(b_h^j) - q_j(b_h^j)} & \text{otherwise.} \end{cases} \tag{5.6}$$

Definition 5.5. Partial discordance index for sorting criteria:

$$d_j(a, b_h^j) = \begin{cases} 1 & \text{if } \Phi_j(a) - \Phi_j(b_h^j) \geqslant v_j(b_h^j), \text{then} \\ 0 & \text{if } \Phi_j(a) - \Phi_j(b_h^j) \leqslant p_j(b_h^j), \text{then} \\ \frac{\Phi_j(a) - \Phi_j(b_h^j) - p_j(b_h^j)}{v_j(b_h^j) - p_j(b_h^j)} & \text{otherwise.} \end{cases} \tag{5.7}$$

Having these new definition for calculating the partial concordance indices c_j, and having their corresponding relative weights w_j, the overall concordance can be calculated with the classical equation of ELECTRE-TRI-B, Eq. (5.1). Similarly, the partial concordance indices obtained for each different type of criterion are merged when the credibility matrix is calculated with Eq. (5.4). Finally, the credibility matrix is exploited with either the pessimistic or optimistic procedures presented in Section 5.1.2. Hence, an outranking relation S that validates or invalidates the assertion aSb_h^j (and b_h^jSa) is built taking into account previous assignments of the alternatives to ordered categories. Using the previous definitions, at the parent node, the assignment of alternative a to category C_{h+1}^j relies on a majority of sub-criteria in favor of the assertion aSb_h^j (concordance test) and none of the sub-criteria in the minority should strongly oppose to the assertion aSb_h^j (discordance test). Note that we are not following a typical rule-based classification approach. Instead, the comparison between alternatives and profile limits are treated as fuzzy relations when applying the discrimination thresholds (concordance test) and veto threshold (discordance test).

When the threshold values are applied as true criterion (i.e., $q_j(b_h^j)$, $p_j(b_h^j) = 0$ and $v_j(b_h^j) = 1$), we are applying strict rules in such a way that the comparison between an alternative and the profile limits depends on the unanimity property: alternative a is assigned to C_{h+1}^j if and only if $\forall j, aSb_h^j$ without any exception (i.e., no veto activation).

For example, let us assume that alternative a_1 is assigned to *Fair*, *Bad*, and *Reliable*; and alternative a_2 is assigned to *Fair*, *Regular*, and *Reliable* (Figure 5.6).

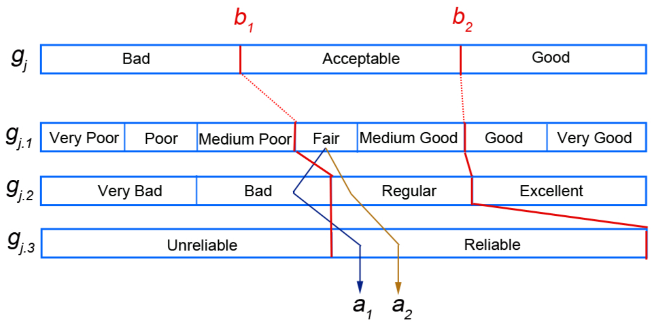


FIGURE 5.6: Alternatives and profile limits in sorting criteria

Let us assume that we first apply the model using strict rules (i.e., true criterion and $v_j(b_h^j)$=1) and second with a non-strict rule only for $b_{g_{j.2}}^1$, in which the DM has uncertainty about how to model *Acceptable* limit profile on $g_{j.2}$ (with $q_j(b_{g_{j.2}}^1)$=1 and $v_j(b_{g_{j.2}}^1)$=2). Based on the profiles generated by the rules and the assignments presented in Figure 5.6, the final assignments of the alternatives on g_j are shown in Table 5.3. Note that for this example, the assignments are the same using pessimistic or optimistic procedures, as we do not have incomparabilities.

TABLE 5.3: Alternative relations with strict and non-strict profile limits and its assignment

$A \times B$	b_1	b_2	**Assignment**
Strict profile			
a_1	P^-	P^-	*Bad*
a_2	I	P^-	*Acceptable*
Non-Strict profile			
a_1	I	P^-	*Acceptable*
a_2	I	P^-	*Acceptable*

Using strict rules and applying Eq. 5.7 on alternative a_1, the partial discordance $d_{j.2}(a_1, b_1^{g_{j.2}})$=1, thus invalidating $a_1 S b_1$, whereas $b_1 S a_1$. However, for the non-strict rule, Eq. 5.6 is applied, resulting in a partial concordance $c_{j.2}(a_1, b_1^{g_{j.2}})$=1. Then, alternative a_1 is considered indifferent to b_1 (i.e., $a_1 S b_1$ and $b_1 S a_1$).

5.2.3 ELECTRE-TRI-B-H algorithm

Assuming that the set of predefined categories and profile limits for each intermediate and root criterion of the hierarchy are already defined on the set of criteria G, the ELECTRE-TRI-B-H procedure is presented in Algorithm 3. In a similar procedure than the ranking Algorithm 1 presented in Chapter 4, this sorting algorithm distinguishes between two cases that lead to the construction of two lists of criteria that are treated differently:

- List X, in line 2, contains the intermediate criteria and the root criterion whose immediate descendants are all elementary criteria (i.e., all descendants belong to $\mathcal{E}$).
- List Y, in line 3, contains the intermediate criteria and the root criterion that have as immediate descendants other intermediate criteria, possibly including some, but not all, elementary criteria (at least one descendant must be an intermediate criterion).

Algorithm 3 ELECTRE-TRI-B-H Method

```
1: function ElectreTRI-B-H(Criteria G, Alternatives A, PerformanceMatrix M, ExploitationProcedure Proc, CuttingLevel λ)
2:     X ← List of g_j ∈ I ∪ R with descendants all in E
3:     Y ← List of g_j ∈ I ∪ R with descendants in E and I
4:     Y ← sortCriteriaByLevels(Y)                          ▷ Sort Y bottom-up
5:     Assignments = null                                    ▷ Assignments on sorting criteria
6:     for all x_j ∈ X do
7:         Z ← get_Children_Criteria(x_j)
8:         ρ ← build_Electre_TRI_B_Credibility(Z, A, M)
9:         Assignments_j ← Exploit(ρ, Proc, λ)
10:        Assignments = Assignments ∪ Assignments_j
11:    end for
12:    for all y_j ∈ Y do
13:        Z ← get_Children_Criteria(y_j)
14:        ρ ← build_Electre_TRI_B_H_Credibility(Z, A, M, Assignments)
15:        Assignments_j ← Exploit(ρ, Proc, λ)
16:        Assignments = Assignments ∪ Assignments_j
17:    end for
18:    return Assignments
19: end function
```

The hierarchical assignment procedure is based on a bottom-up approach, so that the criteria are analyzed from the lowest level up to the root. Each of the two lists, X and Y, undergoes a different treatment because of the differences in the information given by the criteria. Note that the difference is in the calculation of the credibility matrix ρ that depends on the definition of the concordance and discordance indices.

1. In the first stage, list X is treated (from line 6 to line 11) with the nodes placed at the bottom level of the hierarchy, i.e., the elementary criteria. This stage aggregates groups of elementary criteria by their direct ancestor x_j to obtain the first results in the form of assignments. The credibility matrix is calculated using classical ELECTRE-TRI-B indices, taking the performance scores stored in matrix M (line 8). The exploitation of the credibility matrix using pessimistic or optimistic procedure assigns each alternative to one of the predefined categories set by the DM for each node in $Assignments_j$ (line 9) and stored in the set $Assignments$ (line 10). Note that only a subset of criteria is considered in the credibility calculation, which contains the direct descendants of the current node x_j (stored in Z in line 7).

2. In the second stage, the algorithm treats list Y (from line 12 to line 17). Then, the assignments obtained in the previous step (stored in $Assignments$) are used as inputs for the upper level criteria. This requires the list to be ordered according to the precedence relations indicated by the tree structure of set G, from the lowest level up to the most general criterion (root). So, Y is sorted such that y_j has no descendant in $y_{j+1}...y_m$ (line 4). In line 14, the credibility is calculated using new ELECTRE-TRI-B-H formulas, which redefine the partial concordance and discordance indices between the profile limits and alternative assignments based on rules. Therefore, in this case, both the performance matrix M and the list of assignments $Assignments$ are needed to calculate the credibility index according to the nature of the descendants of the current node x_j (stored in Z in line 7). We assume that for a criterion belonging to $\mathcal{E}$, the partial concordance and discordance indices are calculated using classical ELECTRE-TRI-B formulas. Finally, the exploitation procedure of the credibility is applied (line 15) using pessimistic or optimistic procedure, resulting in a new set of assignments of alternatives $Assignments_j$ for each node $g_j \in Y$.

The ELECTRE-TRI-B-H method allows the possibility of allowing categorical elementary criteria with an ordered scale. In this case, criteria will be treated as if it was an "intermediate sorting criterion", but directly given by the DM, instead of having been calculated by the algorithm.

5.3 ELECTRE-TRI-B-H Properties

In this section we consider the properties of calculating the valued relation aSb_h^j, taking into account the independent comparison of each alternative with respect to the profile limits, leading to the assignment of each alternative to a pre-existing category.

Let $D \subseteq G$ be a set of intermediate criteria on G with direct descendants of g_i, where $D = \{g_{i.1}, g_{i.2}, ..., g_{i.x}\}$. Let us assume that each element in $g_{i.j} \in D$ is associated to a weight w_j indicating its relative importance with respect to the rest of descendants of g_i, to preference thresholds ($q_j(a)$, $p_j(a)$, and $v_j(a)$) and each alternative $a \in A$ is assigned to a category $C_h^{i.j}$.

Let us denote as $\rho_D(a, b_h^j)$ the operation to calculate the credibility index of the outranking relation aSb_h^j from the set of criteria D. We denote $aSb_h^j = true$ if $\rho_D(a, b_h^j) \geqslant \lambda$.

- **Independence of irrelevant actions:** The relation aSb_h^j only depends on the preference thresholds in $g_{i.j} \in D$ and not on the remaining alternatives, so that for the pair (a, b_h^j)

  $$A'{=}A \cup \{k\}, \text{ then } aSb_h^j \text{ implies } aS'b_h^j.$$

  Proof. Let us consider $A = \{a, b, c\}$ and $B = \{b_1^j, b_2^j\}$, where $\Phi(a) < \Phi(b_2^j) < \Phi(b_1^j)$. If alternative k is added to set A, such that $A' = \{a, b, c, k\}$; aSb_1^j and aSb_2^j implies $aS'b_1^j$ and $aS'b_2^j$, as alternative a is directly compared to profile limits b_1^j, b_2^j and the addition of alternative k does not affect these comparisons. □

- **Neutrality with respect to criteria:** The relation aSb_h^j does not depend on the order of consideration of the criteria. For any permutation $D' = \sigma(D)$:

  $$\rho_D(a, b_h^j) = \rho_{D'}(a, b_h^j), \text{ so that } aS'b_h^j \Rightarrow aSb_h^j$$

  Proof. This property is fulfilled by $\rho_D(a, b)$ because the product and addition operators are commutative. □

- **Monotonicity**: The outranking relation aSb_h^j is preserved based on the category improvement value of alternative a with respect to profile limit b_h^j.

$$\Phi_j(a) > \Phi_j^{'}(a) \text{ and } aSb_h^j \Rightarrow aS'b_h^j$$

Proof. Let us consider alternative $a \in A$ and aSb_h^j. If alternative a is improved, then $\Phi(a') \leqslant \Phi(a)$, implying aSb_h^j and $aS'b_h^j$. □

- **Linearity**: The credibility degree of the outranking relation $\rho_D(a, b_h^j)$ is equal or decreased when compared to higher profiles.

  Proof. Let us consider alternative $a \in A$ and profile limits b_1^j, b_2^j and b_3^j. Considering that $b_3^j \geq b_2^j \geq b_1^j$, if aSb_1^j, aSb_2^j and aSb_3^j, then $\rho_D(a, b_1^j) \geqslant \rho_D(a, b_2^j)$ because of the following conditions for $h > 1$:

  – Condition 1: $q_j(b_h^j) \leqslant \Phi_j(b_h^j) - \Phi_j(b_{h+1}^j) + q_j(b_{h+1}^j)$
  – Condition 2: $p_j(b_h^j) \leqslant \Phi_j(b_h^j) - \Phi_j(b_{h+1}^j) + p_j(b_{h+1}^j)$
  – Condition 3: $v_j(b_h^j) \leqslant \Phi_j(b_h^j) - \Phi_j(b_{h+1}^j) + v_j(b_{h+1}^j)$

  □

- **Pareto principle:** An alternative a must not outrank profile limit b_h^j when b_h^j is strictly better than a in all the criteria. This property is also known as Pareto efficiency or unanimity. As the $\Phi(\cdot)$ function measures the maximum degree of improvement of an alternative or a profile limit on a sorted category, and it has to be minimized, we can write this property as follows:

$$\forall j,\ \Phi_j(b_h^j) < \Phi_j(a) - p_j(a) \text{ then } \neg(aSb_h^j)$$

  Proof. By construction, in any assignment, if $\forall j, \Phi_j(a) \leqslant \Phi_j(b_h^j) - p_j(a)$, only discordant indices $d_j(a, b_h^j) > 0$ are calculated and thus, refuting aSb_h^j. □

5.3.1 Characterization of aSb_h^j

In this section we study the fulfillment of the outranking relation aSb_h^j in terms of the category improvement values of the descendant criteria that are aggregated.

Proposition 5.6. *Given an alternative $a \in A$, aSb_h^j if $\forall j$, $\Phi_j(a) \leqslant \Phi_j(b_h^j) + q_j(b_h^j)$.*

Proof. For all category improvement values of alternative a less or equal to the difference of the category improvement value of the profile limit b_h^j and the indifference threshold value $q_j(b_h^j)$, then $\forall j, c_j(a, b_h^j) = 1$ and $d_j(a, b_h^j) = 0$, so that $d_j(a, b_h^j) < c(a, b_h^j)$ and $\rho_D(a, b_h^j) = 1$. Taking into account that, for this case, the credibility index is the highest possible, we have that $\lambda \leqslant \rho_D(a, b_h^j)$. □

Proposition 5.7. *Given an alternative $a \in A$, aSb_h^j if $\forall j$, $\Phi_j(a) \leqslant \Phi_j(b_h^j)+p_j(b_h^j)$ and $c(a, b_h^j) \geqslant \lambda$.*

Proof. For all the category improvement values of alternative a less or equal to the difference of the category improvement value of profile limit b_h^j and the preference threshold value $p_j(b_h^j)$, then $\forall j, c_j(a, b_h^j) = [0, 1)$ and $d_j(a, b_h^j) = 0$. Then, $c(a, b_h^j) = [0, 1)$ and $\forall j$, $d_j(a, b_h^j) < c(a, b_h^j) = [0, 1)$. If $c(a, b_h^j) \geqslant \lambda$, then $\rho_D(a, b_h^j) = [0, 1)$. □

Proposition 5.8. *Given an alternative $a \in A$, $\neg(aSb_h^j)$ if $\exists j \Phi_j(a) > \Phi_j(b_h^j) + v_j(b_h^j)$.*

Proof. If the category improvement value of alternative $a \in A$ is greater or equal to the difference between the category improvement value of the profile limit b_h^j on criterion j and the veto threshold value $v_j(b_h^j)$, the right to veto is activated, so that $c_j(a, b_h^j) = 0$ and $d_j(a, b_h^j) = 1$. Then, $c(a, b_h^j) = 0$ and $d_j(a, b_h^j) > c(a, b_h^j) \in [0, 1)$, resulting in $\rho_D(a, b_h^j) = 0$. □

Proposition 5.9. *Given an alternative $a \in A$, $\neg(aSb_h^j)$ if $\forall j, \Phi_j(a) > \Phi_j(b_h^j) + p_j(b_h^j)$.*

Proof. For all category improvement values of alternative $a \in A$ greater to the difference between the one of the profile limit b_h^j and the preference threshold value $p_j(b_h^j)$, then $\forall c_j(a, b_h^j) = 0$ and $d_j(a, b_h^j) = [0, 1)$. Then, $c(a, b_h^j) = 0$ and $d_j(a, b_h^j) > c(a, b_h^j)$, resulting in $\rho_D(a, b_h^j) = 0$.

□

Chapter 6

Application of the hierarchical ELECTRE methods

Chapters 4 and 5 presented the ELECTRE-III-H and ELECTRE-TRI-B-H methods respectively for handling decision problems considering a hierarchy of criteria. ELECTRE methods have a long history of satisfactory real-world applications. Contributing to the legacy of real-world applications of the ELECTRE methods, the hierarchical methods proposed in this thesis have been applied to different real-world case studies with the purpose of: 1) illustrate the multidisciplinary character of the methods proposed considering their application in different fields and 2) validate the results obtained from the methods proposed performing a robustness analysis with different parameter configurations.

The real-case applications have been done with collaboration of different partners from different centers in Catalonia, Spain; including the Science and Technology Park of Tourism and Leisure (PCT) in Vila-Seca, the departments of Communication at the Universitat Pompeu Fabra (UPF) and Universitat Rovira i Virgili (URV) in Barcelona and Tarragona respectively; and the Department of Chemical Engineering also at the URV.

In this chapter we first analyze the application of the ELECTRE-III-H method for ranking official tourist destination websites in Section 6.1. Next, in Section 6.2, the integration of the ELECTRE-III-H method into an Environmental Decision Support System to find the best strategies for water allocation in future scenarios of global change in a Mediterranean basin is studied. Finally, Section 6.3 presents the integration of the ELECTRE-TRI-B-H method into a recommender system, called GoEno-Tur, to recommend activities related to wine and culture in the province of Tarragona.

The three case studies and their modelization are summarized in Table 6.1.

TABLE 6.1: Application of hierarchical ELECTRE methods to case studies

Case study	**Problem**	**# levels**	**# $\mathcal{I}$**	**# $\mathcal{E}$**	**# alts.**
Official tourist destination websites	Ranking	4	18	123	10
Water allocation strategies in future scenarios of global change	Ranking	3	3	9	48
Analysis of touristic activities in a recommender system	Sorting	3	2	6	279

6.1 Official tourist destination websites

The first application is framed in the field of website quality management. This study was conducted with the collaboration of researchers in Communication Strategies as a continuation of the research project "Online Communication for Destination Brands. Development of an Integrated Assessment Tool: Websites, Mobile Applications and Social Media (CODETUR)", CSO 2011-22691, funded by the Spanish Ministry of Economy and Competitiveness. We have worked with Dr. José Fernández-Cavia (UPF) and Dr. Assumpció Huertas (URV) on the analysis of communication features in websites of tourist destinations.

Destination Marketing Organizations (DMOs) are non-profit institutions (usually public or public/private organizations) responsible for attracting tourists and helping to commercialize hospitality and travel services based in a territory conceived as a single unit, whether a city, a region or a whole nation [Gretzel et al, 2006].

To promote destination brands, facing a global scenario, on-line tools such as websites are crucial [Choi et al, 2007, Tang and Jang, 2012]. Indeed several researchers have highlighted official websites as the most important communication tool for destinations [Lee and Gretzel, 2012]. Consequently, destination websites are very important because they can provide a huge amount of information, convey an image of the place, permit useful ways of interacting with users and also operate as a point of sale. Due to this diversity of functionalities, destination websites are complex interactive objects, which make their performance and overall quality difficult to evaluate [Law et al, 2010].

The importance of a thorough evaluation of websites for tourism destinations has been largely recognized in the last decade due to their impact and their contribution to our society. Some remarkable works that collect and review journal publications about website assessment systems have been published recently [Ip et al, 2011, Law et al, 2010]. In both articles the study is framed in the interval from 1996 to 2009.

Law et al [2010] analyzed 19 papers about the evaluation of destination websites. In this article, four evaluation approaches were identified: counting (i.e., a checklist), user judgment of the factors, numerical computation (using mainly statistical techniques), and automated evaluation (i.e., content mining and data mining tools). The technique most used in all studies was counting, followed by user judgments and numerical computation or scoring.

In the review of Ip et al [2011], they collected 30 tourism website assessment systems, which were classified into 3 main types: evaluation by phases, evaluation by features and evaluation by features and effectiveness. In evaluation by phases, five levels of website development were identified (1. Promotion, 2. Provision, 3. Processing, 4. Proactive, 5. Partnership) and each website was assigned to one of these phases. Evaluation by features consisted of defining a list of indicators that must be found (or measured) in the website, mainly focused on content and design issues. Finally, effectiveness was a more advanced dimension that considers user satisfaction, consumer intentions or expert opinions. Most of the assessment systems (around 70%) corresponded to the second model (evaluation by features). In this study it was observed that there was a lack of a standard, well-defined set of features in this area. Moreover, the evaluation of websites could be improved by incorporating theories and models from other disciplines. In fact, it can be seen that only modified versions of the Balanced Scorecard method or basic statistical techniques are used to build overall scores. However, none of the previous assessment methodologies uses the type of multi-criteria decision support system.

The feature-based website evaluation methods that appeared after 2010 still consider a small set of features and they do not seem to have incorporated more complex analysis. Li and Wang [2010, 2011] have used a model measuring 47 items. This assessment model defines five basic dimensions, namely information, communication, transaction, relationship, and technical merit. In relation to the communication of the tourism brand, they consider diverse technical and marketing factors, such as Web 2.0 components, website design, etc. We consider that although their website evaluation model is solid and useful, it leaves aside some basic website functionalities and features (i.e., quality of photographs and videos, contact with DMO, presence of a tagline or customer segmentation, home page features or accessibility from different devices), which are quite relevant for an effective communication of the brand. In recent proposals, like [Bastida and Huan, 2014, Li and Wang, 2010], we still find two important weak points: only a small number of indicators is considered and the mathematical model of analysis is based on simple additive or product averages, which does not permit a detailed comparison between websites taking into account the preferences or needs of the DMO.

There have been some other attempts to define a more extensive and complete assessment model focused on a specific part of the website, such as the home page. This is a reasonable domain restriction because the home page is the welcome point for the potential customers of a tourist destination. A recent study [Luna-Nevarez and Hyman, 2012] puts forward a methodology based on six categories: primary focus, visual and presentation style, navigation and interactivity, textual information, advertising and, finally, social media and travel aids. These variables are highly valuable but possibly the conclusions can be misleading for the users that navigate to the whole website, because we are omitting the effect of the rest of the web pages and their content and functionalities. The limitations of these recent website assessment techniques show that a more comprehensive, systematic and powerful methodology of analysis is desirable in order to assess the whole website in terms of performance, using a complete set of indicators that allow taking advantage of more refined scales. A useful evaluation method should help DMOs to extract conclusions with respect to different parameters and also to obtain comparative results with their direct competitors. The combination of the WQI assessment system with the ELECTRE-III-H decision support method goes in this direction.

In this research, building upon one of the latest destination website assessment systems, the Web Quality Index (WQI), we propose its integration with ELECTRE-III-H not only to state which website is better in terms of quality, but also to establish coherent and consistent preference relations among the different websites.

The aim of this section is to show that the ELECTRE-III-H method can be used in combination with the Web Quality Index as a powerful tool to analyze official destination websites.

6.1.1 The Web Quality Index

The *Web Quality Index* (WQI) [Fernández-Cavia et al, 2014] is an assessment system for destination websites aimed to enable communication managers and/or directors to find out if their respective websites are effective. WQI includes technical, formal and content-related aspects which affect the performance of a tourist destination website. It consists of a set of twelve parameters that are examined in each website with the aim to analyze its quality. The list of parameters and the number of indicators for each of them is given in Table 6.2.

TABLE 6.2: Web Quality Index indicators

Parameters	# indicators	Description
Home page	13	Measures the suitability and appeal of the website's home page
Content amount and quality	15	The website's content is assessed in terms of variety and suitability to the tourists' needs
Information architecture	10	Examines the manner in which the website is organized and structured in order to enable users access to information
Usability and accessibility	17	Looks into user-friendliness on the website and availability for use by people with sensory difficulties
Positioning	8	Verifies whether the website is designed to assist positioning algorithms within web search engines
Commercialization	7	Looks into the options for distributing tourist products and services through the website
Languages	6	Assesses the existence of several languages aside from the official languages of the destination in question
Brand image	12	Examines how the destination's brand image is conveyed and managed via the website's content
Persuasiveness	8	Looks into the website's persuasive capacity, that is, its capability to convince visitors that the destination is worth seeing
Interactivity	9	Examines the two-way communicative relationship between the user and the website content, between the user and the destination managers and between the user and other users
Social web	13	Studies the presence of 2.0 tools on the official destination website
Mobile communication	5	Considers whether the official destination website is adapted for mobile communication using smartphones or tablets
Total	123	

The indicators were evaluated using different measurement scales, depending on their meaning. The linguistic scales that were used are shown in Table 6.3. These linguistic labels were translated into numbers in the range of 0 to 1 depending on the number of terms, as shown in the first row of Table 6.3.

TABLE 6.3: Scales for each indicator. The first row indicates the numerical score given to each linguistic term. The rest of rows show the different sets of linguistic terms used, depending on the meaning of each indicator.

4 values	3 values	2 values
0.0 - 0.33 - 0.66 - 1.0	0.0 - 0.5 - 1.0	0.0 - 1.0
(B,R,G,VG) Bad – Regular – Good – Very Good	(B,R,G) Bad – Regular – Good	(N,Y) No – Yes
(N,F,S,M) No – Few –Sufficient – Many	(N,F,M) No – Few – Many	
(N,P,Y,E) No – Partially – Yes – Extra	(N,P,Y) No – Partially – Yes	
(L,M,H,VH) Low – Medium – High – Very High	(L,M,H) Low – Medium – High	

The fieldwork was carried out in July 2012 by two trained analysts. Each of them analyzed the whole sample in order to share criteria and identify errors.

A pilot test was applied to a sample of diverse destinations, including cities, regions and even nations. It is a convenience sample, designed to verify the use of the method in destinations of different dimensions. The destinations studied were chosen trying to achieve great variability in a small sample in order to test the viability of the methodology. It then combines Spanish and international destinations, as well as different kinds of places, such as cities, regions and nations. The inter-rater agreement for Cohen's Kappa index obtained a value of 0.81. The process was performed under the supervision of the research project director, who made the recommendations and adjustments required. The websites analyzed are shown in Table 6.4.

Table 6.4: Pilot sample of official tourist destination websites

Destination	URL
Andalusia	http://www.andalucia.org/
Catalonia	http://www20.gencat.cat/portal/site/catalunya-act
Barcelona	http://www.barcelonaturisme.com/
Madrid	http://www.esmadrid.com/
Santiago de Compostela	http://www.santiagoturismo.com/
Rías Baixas	http://www.riasbaixas.depo.es/web2009/
Stockholm	http://www.visitstockholm.com/
Wales	http://www.visitwales.co.uk/
Rome	http://www.turismoroma.it/
Switzerland	http://www.myswitzerland.com/

As an illustration, the data collected for the Interactivity parameter in the pilot test is shown in Table 6.5.

Table 6.5: Interactivity parameter

	Indicators	J1	J2	J3	J4	J5	J6	J7	J8	J9
	Scales	N,P,Y	N,P,Y	N,P,Y	N,P,Y	N,F,M	N,P,Y	N,P,Y	N,Y	N,P,Y
1	Andalusia	0	0.66	1	0	0.5	1	0.5	0	0
2	Catalonia	0	0.66	0.5	0	0	0.5	0.5	0	0
3	Barcelona	0	0.33	0.5	1	0.5	1	1	0	0
4	Madrid	0	0	1	0	1	1	1	0	0
5	Santiago	1	0.33	0.5	1	0.5	1	0.5	1	1
6	Rías Baixas	0	0	0.5	0	0	0.5	0.5	0	0
7	Stockholm	0	0	0.5	0	0.5	0.5	1	1	0
8	Wales	0	0	1	0	0.5	0.5	0	0	0
9	Rome	0	0	1	0	0	0.5	0.5	0	0
10	Switzerland	1	0.33	1	1	0.5	0.5	0.5	0	0

The nine indicators *J1* .. *J9* correspond to different features that show the degree of interactivity of the user with the website: $J1$. Multimedia visualization of the context, $J2$. Promotional multimedia tools, $J3$. Free downloads allowed, $J4$. Mobile downloads allowed, $J5$. Interactive resources, $J6$. Community of users in the destination, $J7$. Community/user feedback of the destination, $J8$. Frequently asked questions, $J9$. Chat.

6.1.2 Integration of ELECTRE-III-H and WQI

Each parameter is composed of multiple basic indicators, which were measured in binary scales (presence/absence), ternary or quaternary scales (see Table 6.3). For the analysis with ELECTRE-III-H it is convenient to have a larger set of values to be able to refine a distinction of the different levels of achievement of the objectives evaluated in each parameter. Otherwise, it is not possible to properly establish the preference relations among the websites.

The first stage correspond to the problem modeling of the WQI assessment system, in which the indicators were grouped into small subsets inside each parameter, thus identifying sub-parameters of interest. The indicators grouped in a set are strongly related, according to the expert's knowledge.

The organization of the indicators is presented in Figure 6.1. in tree form, from the most generic parameters to the most specific indicators.

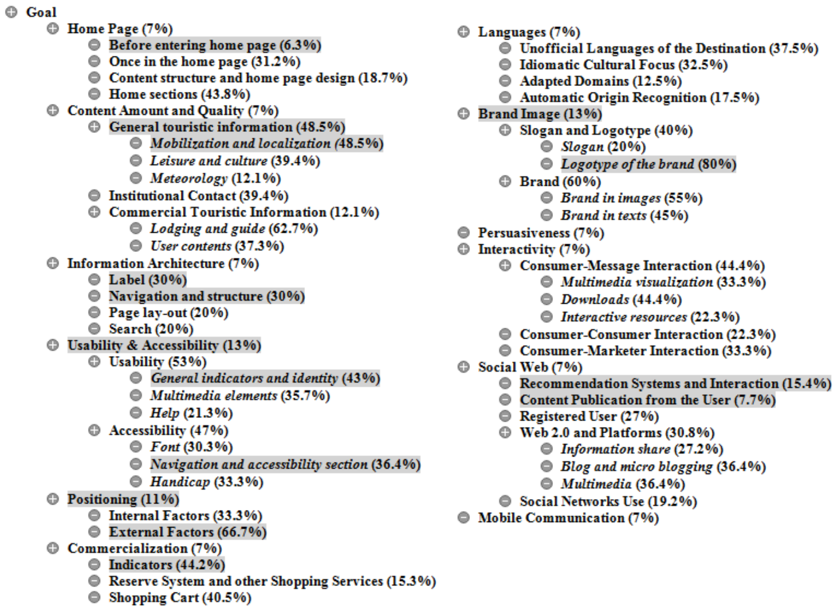


FIGURE 6.1: Graphical representation of profiles in categories, given by mapping assignment rules

According to this new organization of the parameters and indicators, the original input data values were grouped. Table 6.6 shows the values of the composed indicators used in the Interactivity parameter, obtained from Table 6.5.

TABLE 6.6: Interactivity grouped parameter

	Indicators	J1, J2	J3, J4	J5	J6, J7	J8, J9
	Group (weight)	Consumer-Message Interaction (44.4)			Consumer-Consumer (22.3)	Consumer-Marketing (33.3)
	Sub-parameter	Multimedia	Downloads	Interactive resources		
	Weights	33.3	44.4	22.3		
1	Andalusia	0.33	0.5	0.5	0.75	0
2	Catalonia	0.33	0.25	0	0.5	0
3	Barcelona	0.165	0.75	0.5	1	0
4	Madrid	0	0.5	1	1	0
5	Santiago	0.665	0.75	0.5	0.75	1
6	Rías Baixas	0	0.25	0	0.5	0
7	Stockholm	0	0.25	0.5	0.75	0.5
8	Wales	0	0.5	0.5	0.25	0
9	Rome	0	0.5	0	0.5	0
10	Switzerland	0.665	1	0.5	0.5	0

The weight of each parameter and indicator is displayed in Figure 6.1. These weights have been calculated with the Simos' procedure, presented in Chapter 2. In the problem modeling stage, the experts also provided other additional information to guide the analysis by establishing the mandatory indicators. In this study the mandatory indicators, highlighted in gray in Figure 6.1, were provided by experts with the purpose of focusing the study towards the ability to communicate the destination brand. For example, "Usability & Accessibility" is considered as mandatory when evaluating the goodness of the overall goal according to the experts, while "External Factors" is considered to be a mandatory indicator for a good "Positioning" of the website.

For the elementary indicators, all values have been normalized (from 0 to 1), for the mandatory indicators the thresholds have been set in terms of percentages: q_j=0%, p_j=10% and v_j=25% of the maximum performance allowed (i.e., 1), such that q_j=0, p_j=0.1 and v_j=0.25. These values reflect how strict the experts are with respect to mandatory elements, as there is only 0.1 of preference tolerance for evaluation differences and when the difference is larger than 0.25, the system automatically activates the veto against the worst website from the binary comparison. On the other hand, when the elementary indicator is not mandatory, the thresholds are set to q_j=10%, p_j=35% and v_j=75%, i.e., q_j=0.1, p_j=0.35 and v_j=0.75. In this case, the experts are more tolerant to differences in the performance of the websites compared. For non-elementary criteria (parameters and sub-parameters of the hierarchy), the threshold values have been specified distinguishing the mandatory and non-mandatory criteria. For these cases, the threshold values are relative to the rank order of the websites in the corresponding preference structure. Two possibilities have been defined. For mandatory parameters, we set stricter thresholds (q_j=0, p_j=1, v_j=2) than for non-indispensable (q_j=1, p_j=2, v_j=3) ones, because small performance differences of the websites are much more relevant for the decision.

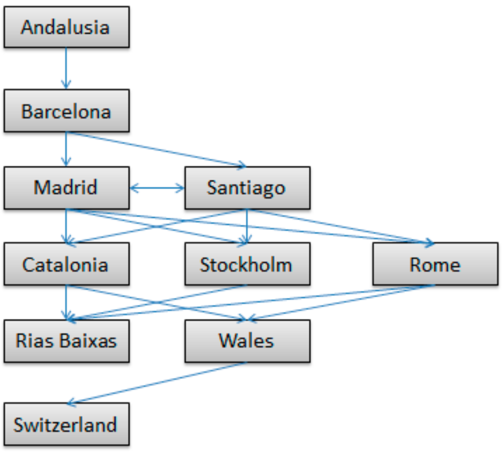
In the next Section 6.1.3, a discussion of the results obtained with ELECTRE-III-H is presented. Finally, in Section 6.1.4, a robustness test is performed for different configurations of threshold values on parameters and sub-parameters of the hierarchy.

6.1.3 Results of ELECTRE-III-H and WQI

The ELECTRE-III-H method was applied to each of the parameters in the hierarchical model presented, except "Persuasiveness" and "Mobile Communication" parameters, which are essentially elementary criteria, so that they are not calculated by ELECTRE-III-H but are directly evaluated by the experts.

Because of the large number of parameters, only the analysis of the "Home Page" is discussed in this section. The home page is of great importance in the communication of destination identity and destination brand, as the home page is the first image of the place that the tourist gets. The quality of a home page can lead users to continue surfing the Web or not. We evaluated various items related to the quality of the home page: if it asks the language before entering, if it is part of the website of the competent authority, if there is a video or presentation of the destination, if there is a sitemap, if it offers the ability to register or not or if it is linked to Web 2.0 applications. The two aspects that the experts considered essential for the quality of the home page in the communication of the destination brand were the clear identification of the destination in the home page and direct entry to the home page through its main URL.

In Figure 6.2, the results of the home page indicators are obtained using ELECTRE-III-H and WQI weighted average.



Website	WQI Score
Anda	0,94
Bcn	0,85
Mad, Sant	0,79
Stock	0,73
Rome	0,70
Cat	0,67
Switz	0,58
Wal	0,55
Rias	0,52

FIGURE 6.2: Partial pre-order and WQI obtained for Home Page

If we compare the ranking obtained using ELECTRE with the ranking obtained through statistical analysis, we observe that the first positions do not change between the two rankings. The Andalusia website possesses all the evaluated aspects, including a video about the destination, which very few other websites have. According to the results obtained in ELECTRE-III-H, Catalonia, Stockholm and Rome are incomparable websites, which indicates that these destination websites possess and lack different items. For example, Catalonia has a video on its home page, which is not the case of Stockholm or Rome. Rome allows users to register, which cannot be done in the other two websites. Stockholm has FAQs and user help, items that Rome and Catalonia do not have. Conversely, with the numerical index WQI the values for Stockholm, Catalonia and Rome are very similar, and they determine a certain order among them because they were obtained without considering imprecisions of data processing, which can lead to erroneous conclusions.

Finally, in the ELECTRE-III-H ranking we cannot detect a clear preference between Rias Baixas or Switzerland, as both websites occupy the last position and they are not mutually comparable. Now, we will concentrate on the case of the websites of Wales and Rome. The arrow between them indicates that Rome is preferred to Wales. In fact, they share some features: the URL in both websites links directly to the home page of the website, where the territory is clearly identified. Both possess the logotypes of the competent authorities, Web 2.0 applications, news, sitemap and contact details. They also share some shortcomings: they fail to ask for the language of preference and they do not have an online shop or a list of most frequently asked questions. However, Rome's website has some of the analyzed indicators that Wales does not have, like the possibility of registering and more Web 2.0 applications. Because of this, it ranks slightly higher.

The partial pre-orders of the rest of main intermediate criteria are shown in Figures 6.3, 6.4 and 6.5.

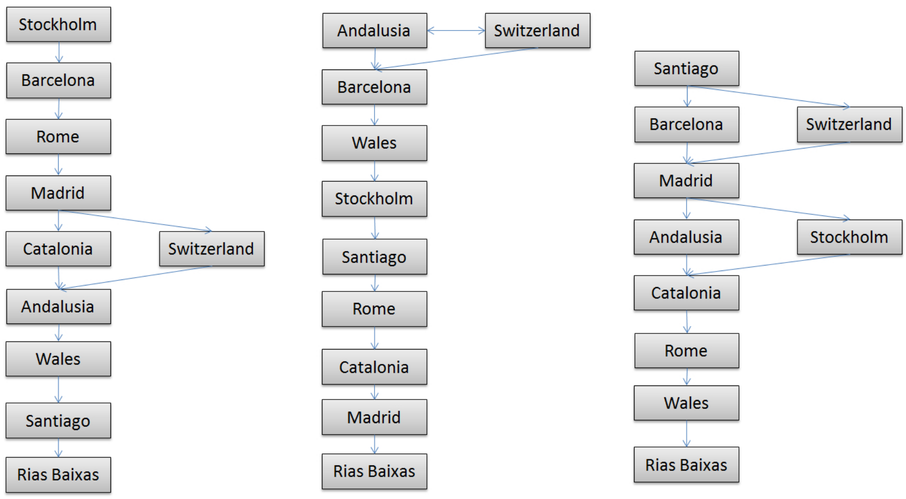


FIGURE 6.3: Partial pre-orders obtained for Usability & Accessibility, Brand Image and Interactivity respectively

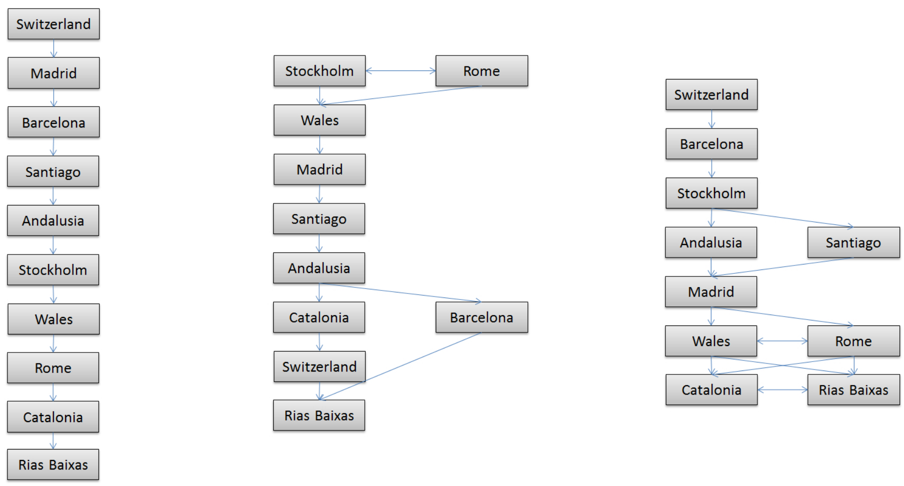


FIGURE 6.4: Partial pre-orders obtained for Content Amount and Quality, Information Architecture and Positioning respectively

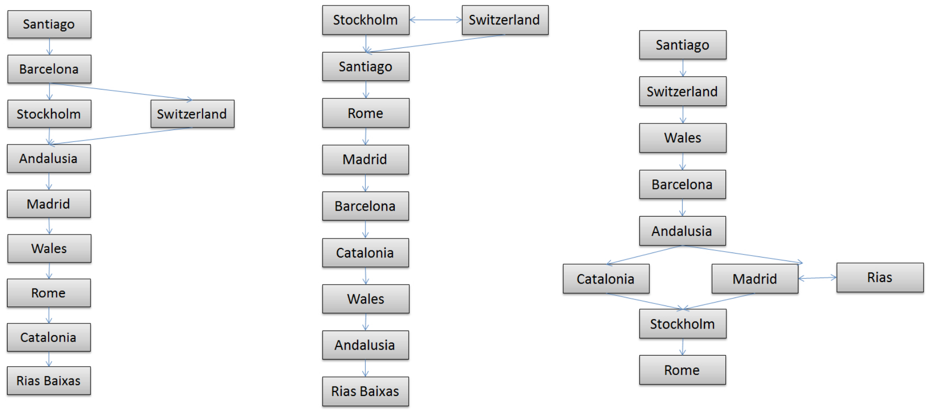


FIGURE 6.5: Partial pre-orders obtained for Commercialization, Languages and Social Web respectively

A more detailed analysis of the results obtained applying ELECTRE-III-H are explained in Del Vasto-Terrientes et al [2015a].

Finally, the global partial pre-order resulting from the aggregation of the previous partial pre-orders shown and the evaluations of "Persuasiveness" and "Mobile Communication" is presented in Figure 6.6.

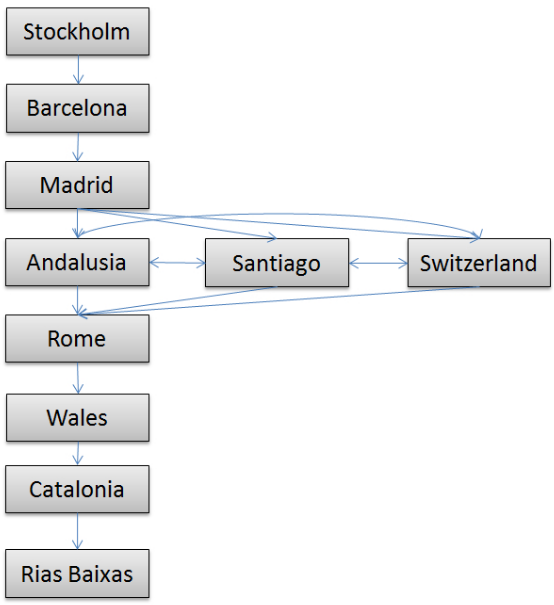


FIGURE 6.6: Global ELECTRE-III-H result

In Table 6.7 some recommendations based on the results obtained with ELECTRE-III-H can be given for each DMO manager. The names of the parameters have been shortened in the following way: Home Page (HP), Usability and Accessibility (U&A), Brand Image (BI), Interactivity (Inter), Content Amount and Quality (CAQ), Information Architecture (IA), Positioning (Pos), Commercialization (Comm), Languages (Lang), and Social Web (SW).

Table 6.7: Some recommendations for DMOs based on websites' strengths and shortcomings

Website	Urgently needs improvement	Should be reviewed	Strong features
Stockholm	SW	HP, CAQ	U&A, IA, Lang
Barcelona	-	IA, Lang, SW	U&A, BI, Pos
Madrid	BI	Pos, SW	HP, CAQ
Andalusia	Lang	U&A	HP, BI
Sant. de Comp.	U&A	BI	Inter, Comm, SW
Switzerland	HP, IA	U&A	BI, CAQ, Pos, Lang
Rome	Pos, Comm, SW	BI, Inter	U&A, IA
Wales	HP, U&A, Inter, Lang, Pos	CAQ, Comm	IA, SW
Catalonia	BI, CAQ, Pos, COMM	HP, Inter, IA	-
Rías Baixas	All parameters	SW	-

Each website has its particular weak points and it is difficult to extract general indications for DMOs. From this pilot test, it could be said that there are three parameters that should generally be improved: Home Page, because it is the entry point to the website and should have enough elements to engage the tourist; Usability & Accessibility, including facilities to identify the elements and navigation for disabled people and from different devices; and Brand Image because an appealing slogan and logotype are of great importance for capturing the attention of the user.

6.1.4 Robustness analysis

The results obtained with the ELECTRE-III-H method for this case study were validated with a robustness analysis. Three configuration scenarios of threshold values in the intermediate criteria are considered, including the "Central scenario" presented in Section 6.1.2 and detailed in Table 6.8:

Table 6.8: Robustness configuration scenarios for $\mathcal{I}$

Scenario	$q_j(a)$	$p_j(a)$	$v_j(a)$
Mandatory parameters			
Strict	0	0	2
Central	0	1	2
Tolerant	1	2	3
Non-mandatory parameters			
Strict	0	1	2
Central	1	2	3
Tolerant	2	3	4

Note that when the preference and veto thresholds are increased, we are decreasing the strength of opposition to the assertion aSb (i.e., decreasing the discordance degree). Thus, we have defined strict and tolerant settings. In order to compare the partial pre-order results, we have assigned each alternative a ranking position according to the partial pre-order. This ranking corresponds to the position of the alternatives in the partial pre-order generated in the exploitation stage. Positions depend on the number of predecessors of each alternative in the partial pre-order (i.e., the Rank Order Value, $\Gamma_j(\cdot)$). Let us remember that the evaluations of Mobile Communication and Persuasiveness are represented as elementary criteria.

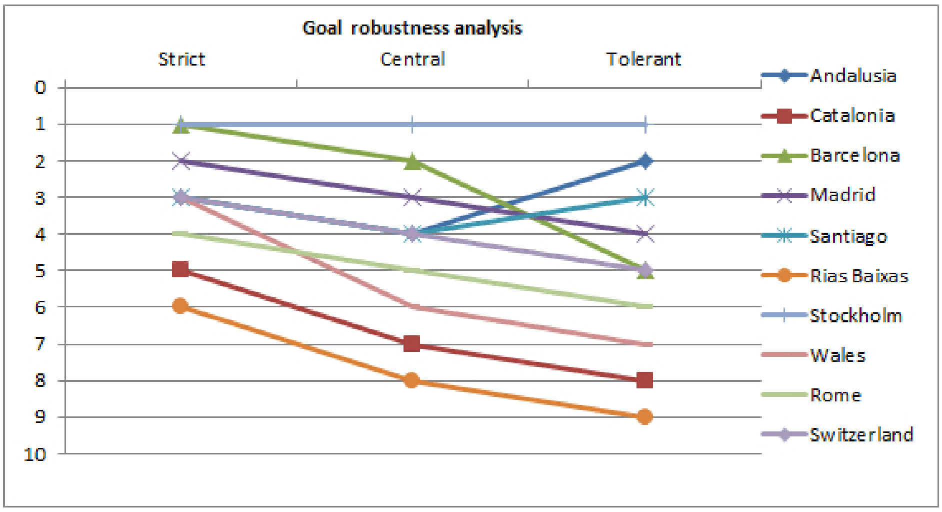


Figure 6.7: Robustness of global ELECTRE-III-H results

Figure 6.7 shows the ranking of the alternatives in order to analyze their behavior for each scenario. The ranking is relatively stable for the overall criterion in each scenario. Stockholm clearly remains the best alternative overall, whereas Rias Baixas is clearly ranked as the worst website in all scenarios. Notice that for strict parameters, the rankings yields more indifferences (i.e., ties). This is because for stricter veto thresholds, it gets more difficult to establish preferences of an alternative over another at non-elementary criteria in the hierarchy, as small differences are against the relation S.

To statistically measure the correlations between the rankings, in 6.9 the correlation based on the Spearman rho, Kendall tau and Goodman & Kruskal's gamma coefficients have been measured for the Strict and Tolerant rankings compared to that of the Central scenario.

TABLE 6.9: Rank correlations for the different configuration scenarios

Rank correlations	Strict scenario	Tolerant scenario
Spearman rho	0.47	0.82
Kendall tau	0.39	0.69
Goodman & Kruskal's gamma	0.29	0.75

For these 3 correlation measures, the values for the Tolerant scenario indicate a strong correlation while for those in Strict scenario indicate a weak correlation due to the large quantity of ties in the ranking.

6.1.5 Discussion

In Fernández-Cavia et al [2014], the authors applied a very simple statistical operator to aggregate the data collected in WQI: weighted average, entailing three relevant drawbacks in this study: the possibility of compensation, the lack of defining mandatory requirements, and the precision of the aggregation. The first refers to the compensation between high and low values when averages are calculated. In this case, low values cannot be detected and explicitly penalized. For example, the average of (0.5,0.5) is the same as the average of (0.0,1.0), but in the first case the website is performing acceptably in the two indicators, while in the second case the website has a very bad performance in the first indicator but it is perfect with respect to the second one. Second, using a weighted average we cannot define mandatory requirements or indispensable indicators that must necessarily be fulfilled in order to consider that the website is performing well in some parameters. For example, in order to evaluate Brand Image, the website must at least have a suitable logotype, while the rest of indicators are optional. Finally, the last drawback concerns the precision of the weighted average operation used in WQI, it is very sensitive to the numerical score given to each indicator. However, in this study the numbers have been introduced as mere translations of linguistic terms that initially did not correspond to an exact number but to an uncertain degree of quality. For example, in Table 6.3 we can see that "Few" and "Medium" are given the same score 0.33 in the first column, although they may have slightly different meanings for the evaluator.

This study presents a combined methodology for destination website analysis, initially based on the WQI framework and average weighted summed. The data analysis has been sophisticated with ELECTRE-III-H. Given the results of the pilot study, this combined analysis methodology represents a great advancement for this type of study. ELECTRE-III-H provides a more qualitative measuring system than simple numerical averaging, showing incompatibilities between websites because of their differential characteristics. In this way, the ELECTRE-III-H method introduces subjective elements to refine the comparison of websites and to focus it towards the fundamental aspects of destination brand communication. ELECTRE-III-H does not substitute or invalidate WQI, but it enriches and complements it, as it provides more information which enables the comparison of competitive destination websites to find the aspects in which they should be improved to overtake other destinations.

6.2 Water allocation strategies in future scenarios of global change

Increasing efforts are being made to understand the consequences of global change for society, particularly in the field of water resource management. Changes in water resources are particularly relevant in areas where water availability is a limiting factor for sustainable economic development. This is the case of the Mediterranean region, where both developed and developing countries have a common dependence on water availability to meet the needs of increasing populations and changing life styles, increasing irrigated agriculture, and increasing industry and tourism activities. Water scarcity is particularly intense in the coastal area, where the expansion of economic activities and urbanization has caused increasing water supply difficulties [Bangash et al, 2012].

This work is focused on the study of the Francolí river basin in the Mediterranean area of northern Spain (in Tarragona province) with collaboration of the Environmental analysis and management (AGA) research team from the Department of Chemical Engineering (URV).

Water management along the Francolí river and its tributaries is complex because of its low flow Mediterranean characteristic which can be subject to high interannual and seasonal variability of precipitations, with long and intense dry periods or extreme rainfall and floods [Marquès et al, 2013]. Francolí river basin has been under considerable pressure for water availability and water quality over the last decades. This river has a considerable demand of water from different sectors: household water constitutes the most important annual consumptive demand of water resources (88%) followed by industry (11%) and agriculture (1%). In particular, the city of Tarragona, located at the south of Francolí river, was solely dependent on its own water resources before 1988. Sea water intrusion in the groundwater aquifers compelled the municipalities to meet the water demand by inter-basin transfer from neighboring river basins (Ebro River and Gaià river). Moreover, Tarragona is second largest industrial area of Catalonia (North-east Spain) and most of the industries in Francolí river basin are located close to Tarragona including a large petrochemical industry. Many other small industries are situated in the upper part of the river basin. The agricultural demand varies all along the river basin depending on the crop type and cultivated area. Water demand and supply in this case study area is particularly complex and sensible to the future global change.

The necessity of developing Intelligent Environmental Decision Support Systems (IEDSS) is well-recognized in the literature [Sànchez-Marrè et al, 2008]. In this domain several inherent difficulties appear, such as the uncertainty of data intrinsic to some environmental modeling techniques, the presence of spatial relationships between the areas studied or even the temporal relationships between the current state and the past states of the environmental system must be considered in knowledge discovery and planning processes [Gibert et al, 2010]. In addition to these particular characteristics of the data, embracing a global perspective in environmental decision making implies accepting that multiple, usually conflicting criteria must be taken into account. Decisions in environmental problems usually deal with a set of diverse indicators measured on different scales and with different levels of uncertainty. Therefore, the development of IEDSS must consider the analysis of complex data. In this case, summarizing the multiple criteria into a single perspective that encompasses all of them is difficult and ineffective [Sànchez-Marrè et al, 2008].

In this work, we have built an IEDSS for evaluating and ranking different water supply strategies under different future demand scenarios.

Water management along the Francolí river and its tributaries is complex because it is a Mediterranean environment and there is limited supply of water to satisfy the demand of all sectors as well as the environmental needs [Bangash et al, 2012]. As said above, the Francolí river basin has been under considerable pressure because of water availability and water quality over the past few decades due to the population growth, climate change and increased water demand in industrial cities like Montblanc, La Riba, and Tarragona.

6.2.1 Architecture of the IEDSS

Different demand predictions have been made under different scenarios, including a neutral scenario based strictly on future statistics, and an optimistic and pessimistic scenarios. The IEDSS built [Chao et al, 2014, Del Vasto-Terrientes et al, 2015b] has two main components: Scenario construction and MCDA analysis (see Figure 6.8). The former is in charge of generating a set of actions (i.e., alternatives) and evaluate their performance on different criteria using the conditions of water supply and demand estimated for a certain future time span. The second component receives the performance matrix generated and applies ELECTRE-III-H to generate a partial pre-order to each non-elementary criterion.

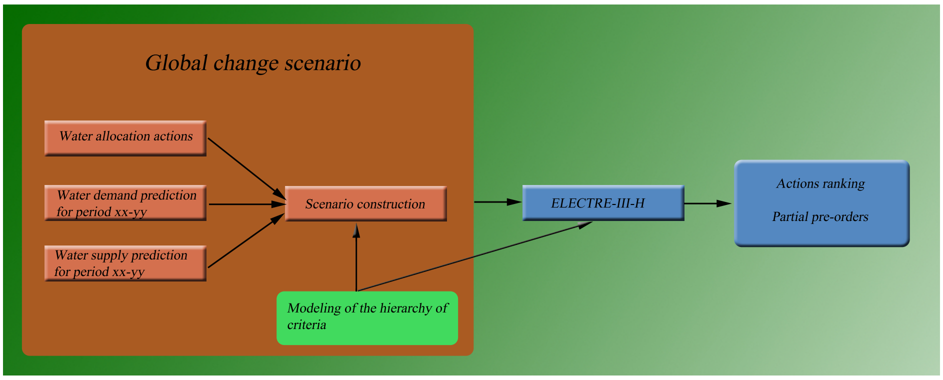


FIGURE 6.8: IEDSS architecture

The actions refer to different water allocation policies for the three sectors demanding water and considering different water supply sources. These actions are evaluated using a set of 9 criteria that are organized in a 3-level hierarchy, including environmental and economic criteria. The performance of the actions on the elementary criteria depends on the conditions of each future scenario. Each scenario is defined by the water demand and water supply estimations, which were provided by the domain experts that collaborated in this case study [Kumar et al, 2014].

Water allocation describes a process whereby an available water resource is distributed to legitimate claimants and the resulting water rights are granted, transferred, reviewed, and adapted. The allocation of water resources in river basins is one of the critical issues. The AGA research team proposed to build an IEDSS in order to make a holistic approach to water supply management at the watershed-scale considering different criteria, where different target sectors are considered together to draft possible general management strategies. It was required to distinguish the three water-demanding sectors: agriculture, domestic and industrial.

The main goal of this system is to rank the different sectorized water supply strategies under different future scenarios. Due to the shortfall in supply from primary water resources, this study also includes the use of alternative water supply sources. For each future scenario of climate change, the goal is to obtain a ranking of a set of possible actions with regards to different types of indicators, such as costs, environmental impact, water stress, etc. This led the DM to define a hierarchy of criteria, as presented in this section.

Demand predictions have been made for 3 time spans (2011-2040, 2041-2070, and 2071-2100). Due to the uncertainty in these predictions, for each time span 3 scenarios have been studied: optimistic, neutral and pessimistic.

This section explains the formalization of the water resources management problem for application to ELECTRE-III-H, in terms of actions and criteria.

Next sections explain in more detail the actions, criteria and scenario construction.

6.2.2 The actions

The set of possible water allocation policies can be more easily expressed by means of some general rules of water supply to the three sectors (industrial, domestic and agricultural). Each rule determines the percentage of the demanded water that is going to be supplied from each of the different water sources. The primary source is the water extracted from the 3 rivers (Francolí, Ebro, and Gaià). Two alternate resources have also been taken into consideration: Reclaimed water, which include the reuse of recycled domestic water in the industrial process or for the irrigation in agriculture; and Desalination, which is water obtained from processing salty marine water. The actions (i.e., rules) can be grouped under 4 main strategies:

A. Nature first: gives priority to water coming from primary sources, especially for the domestic sector.
B. Low use of alternative resources: low desalination for domestic water supply and use of reclaimed water.
C. Medium use of alternative resources: medium desalination for domestic and industrial water supply and use of reclaimed water.
D. High use of alternative resources: high desalination for domestic and industrial water supply as well as high use of reclaimed water.

For each of these 4 strategies, 12 rules have been defined for each water allocation strategy. When rules are instantiated in a certain scenario, we have a total of 48 concrete actions to compare (see Section 6.2.4).

As mentioned above, two alternative water resources are considered: water recycling and desalination. Water recycling is reusing treated waste water to meet other demands such as agricultural irrigation, industrial demand, or even other urban uses. Water reuse offers environmental benefits, conservation of precious natural resource and financial savings. The water reuse has been taken following the guidelines of regional water authority (the Catalan Agency of Water), which has a Water Reclamation Project in Tarragona, and the experts of the AGA research team. Accordingly, we assume that there will be a gradual increase in water recycling and reuse in the industrial sector. The amount of reused water in agriculture is notably smaller since there are several constraints such as geographical distance and distribution cost. Moreover, water reuse for domestic (drinking water) sector is nonviable as recycling cost is too high.

Water desalination has been contemplated only for the domestic and industrial sectors since there is no intensive centralized agriculture area in Tarragona where costly desalinated water can be used. Based on experts recommendation we assume that the domestic desalinated water supply can be up to 25% whereas the maximum for industrial water taken into account is 20%. The minimum percentage of water supply (20%) from desalination plant is based on the cost viability of the desalination plant. Table 6.10 also shows the considered values of water desalination in different sectors.

Table 6.10: Water reuse and desalination scenarios

Reuse scenarios	Industry (%)	Agriculture (%)
No reuse	0	0
Low reuse	20	10
Medium reuse	40	20
High reuse	60	30

Desalination scenarios	Industry (%)	Domestic (%)
No desalination	0	0
Low desalination	0	20
Medium desalination	10	20
High desalination	20	25

With this information, a total of 48 actions were constructed from 12 rules for each of the 4 strategies defined. For instance, in Table 6.11, the rules are defined for medium use of alternative resources. A full description of the strategies are given in Del Vasto-Terrientes et al [2015b].

The acronyms presented in the table stand for the following: LWR/MWR/HWR = Low/Medium/High water reuse, MDs = Medium Desalination, PS = Primary resource.

TABLE 6.11: Medium use of alternative resources strategy

Rule	Industry	Domestic	Agriculture
C1	LWR (20%) + MDs (10%)	80% PS + MDs (20%)	100% PS
C2	LWR (20%) + MDs (10%)	80% PS + MDs (20%)	LWR (10%)
C3	LWR (20%) + MDs (10%)	80% PS + MDs (20%)	MWR (20%)
C4	LWR (20%) + MDs (10%)	80% PS + MDs (20%)	HWR (30%)
C5	MWR (40%) + MDs (10%)	80% PS + MDs (20%)	100% PS
C6	MWR (40%) + MDs (10%)	80% PS + MDs (20%)	LWR (10%)
C7	MWR (40%) + MDs (10%)	80% PS + MDs (20%)	MWR (20%)
C8	MWR (40%) + MDs (10%)	80% PS + MDs (20%)	HWR (30%)
C9	HWR (60%) + MDs (10%)	80% PS + MDs (20%)	100% PS
C10	HWR (60%) + MDs (10%)	80% PS + MDs (20%)	LWR (10%)
C11	HWR (60%) + MDs (10%)	80% PS + MDs (20%)	MWR (20%)
C12	HWR (60%) + MDs (10%)	80% PS + MDs (20%)	HWR (30%)

6.2.3 Set of Criteria

Different sets of environmental and economic criteria and different ways of organizing the information have been studied. This section proposes a set of criteria that are of interest for the experts in order evaluate the different allocation actions and decide which are the most appropriate.

Three main perspectives have been included in the model, each one subdivided into several subcriteria. For this reason, it is especially suitable to define a hierarchical structure for the decision support system. The hierarchy of criteria that has been constructed with the help of the domain experts can be seen in Figure 6.9. It distinguishes 3 sub-goals (i.e., intermediate criteria).

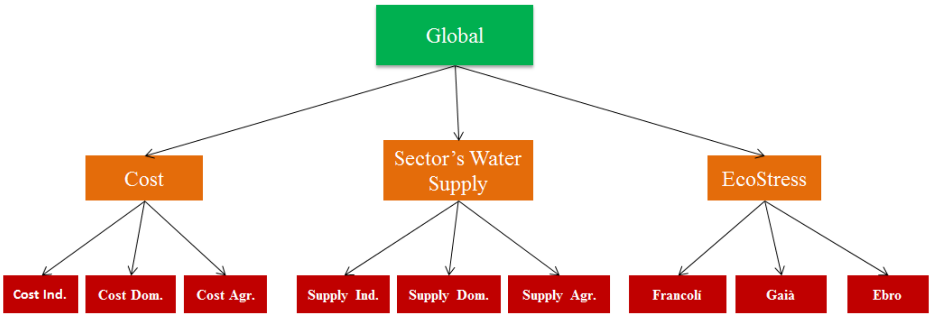


FIGURE 6.9: Hierarchical structure for the water allocation problem

The elementary criteria in the proposed model are explained as follows:

- Cost of water per sector: Each action has a cost for each demand sector that includes the cost of the primary water coming from the rivers Francolí, Ebro and Gaià as well as the cost of the alternative resources. The unit costs per each supply source are different for different sector. The cost per sector is calculated by adding the unit cost $euro/hm^3$ multiplied by the hm^3 extracted from each river and/or from water reuse and desalination.

- Water Supply per sector: It refers to the total amount of water obtained from the primary sources (rivers) to supply of water a certain sector of activity (domestic, industry or agriculture). The higher the index, the higher the environmental impact coused by each sector.
- Ecological impact per river: The EcoStress is a water use index that represents the percentage of water extracted from a river to fulfill the demand. This index gives an estimation of the ecological stress on the river. For the analysis of the current problem this index is calculated by adding the water extracted on a certain river and then dividing it by total annual water flow of this river. This index has to be minimized, i.e., the less EcoStress index the better.

The current demand is the following: Industrial 30.5 hm^3, Domestic 49.26 hm^3 and Agriculture 17.1 hm^3. In Table 6.12, we present the chart of demand variation with regards to the current water demand for the different time spans and situations:

Table 6.12: Water demand scenarios

	Optimistic			Neutral			Pessimistic		
Time span	Ind	Dom	Agr	Ind	Dom	Agr	Ind	Dom	Agr
2040	-15	-10	-25	-5	12,5	0	20	35	20
2070	-30	-25	-35	-10	15	5	20	40	30
2100	-45	-35	-50	-12,5	10	10	5	45	30

6.2.4 Construction of water allocation actions for a certain scenario

With the general allocation rules we can automatically generate a set of concrete actions for a certain future scenario of global change. We have a different scenario for each time span 2040, 2070 or 2100, and each situation: optimistic, pessimistic or neutral. This results in a total of 9 different scenarios.

The scenario is described by:

- The estimated water supply reduction in the current rivers yield. In this case study, for time span between 2011 and 2047 fan estimated reduction of 21 % water yield of the rivers [Marquès et al, 2013].

- The estimated water demand for each sector. This can be calculated using the demand variations given in Table 6.12. Global change will affect the current patterns of water demand. Due to the uncertainty in the prediction of water demand in the future, we will consider a range of values by defining three different situations, called optimistic, pessimistic and neutral demand scenarios. The demand is studied for each sector.

  Domestic: Domestic water supply and demand is not uniform and varies significantly based on location, climatic change, house characteristics, and socio-economic variables. In the current case we have considered three factors namely demographical changes, based on regional prediction by state agency IDESCAT with 2008 as a base year, technological changes and socio-economic changes as major driving force for domestic water demand.

  Agriculture: Agriculture water demand scenario is based on the following major criteria: climate change, technological changes, changes in irrigation area, cropping pattern and policy and other socio-economic factors affecting agriculture sector. For example, summer rainfall decreases but improvements in water efficiency is increased due to changes in improved farming practices and new technologies.

  Industrial Water use: Manufacturing water withdrawal has increased by a factor of 3.6 between 1950 and 2010, while water consumption in 2010 is more than 7 times higher than in 1950. A reduction in manufacturing water is possible, for instance, with the relocation of industrial production to the Far East or improvements in technology.

- A threshold in the maximum water amount that can be extracted from the rivers, in order to maintain a minimal ecological yield in the rivers.

Knowing the water demand for each sector (in hm^3) and the hm^3 that we can extract from each water source (rivers and alternative resources), we can apply one of the rules explained before and calculate the hm^3 supplied by each source to each sector. In case the maximum threshold of water extraction for a river is reached, the rest of water needed is transferred from the Ebro river (the one with larger water yield). This can be done for all the 48 rules (12 for each strategy, as presented in Table 6.11), obtaining 48 concrete water allocation actions for a certain scenario.

Once we have the water allocation actions, we can evaluate them using the 9 elementary criteria presented before. The result is a performance table, which is the input required in ELECTRE-III-H method. This table has one row for each action (i.e., alternative) and as many columns as elementary criteria, in our case 9 criteria. As an example, in Table 6.13, the performance table for the "2040 neutral" scenario is presented.

TABLE 6.13: Alternatives performance table for neutral scenario in 2040

	Cost			Water Supply			EcoStress		
	ind	dom	agr	ind	dom	agr	Francolí	Ebro	Gaià
A1	18547000,000	24675924,000	399600,000	23,18	49,79	9,99	,286	,005	,060
A2	18547000,000	24675924,000	564800,000	23,18	49,79	8,99	,259	,005	,060
A3	18547000,000	24675924,000	730000,000	23,18	49,79	7,99	,233	,005	,060
A4	18547000,000	24675924,000	895200,000	23,180	49,79	6,99	,206	,005	,060
A5	18257500,000	24675924,000	399600,000	17,39	49,79	9,99	,286	,004	,060
A6	18257500,000	24675924,000	564800,000	17,39	49,79	8,99	,259	,004	,060
A7	18257500,000	24675924,000	730000,000	17,39	49,79	7,99	,233	,004	,060
A8	18257500,000	24675924,000	895200,000	17,39	49,79	6,99	,206	,004	,060
A9	17967500,000	24675924,000	399600,000	11,59	49,79	9,99	,286	,004	,060
A10	17967500,000	24675924,000	564800,000	11,59	49,79	8,99	,259	,004	,060
A11	17967500,000	24675924,000	730000,000	11,59	49,79	7,99	,233	,004	,060
A12	17967500,000	24675924,000	895200,000	11,590	49,79	6,99	,206	,004	,060
B1	18547000,000	26014548,000	399600,000	23,18	39,83	9,99	,286	,004	,060
B2	18547000,000	26014548,000	564800,000	23,18	39,83	8,99	,259	,004	,060
B3	18547000,000	26014548,000	730000,000	23,18	39,83	7,99	,233	,004	,060
B4	18547000,000	26014548,000	895200,000	23,18	39,83	6,99	,206	,004	,060
B5	18257500,000	26014548,000	399600,000	17,39	39,83	9,99	,286	,004	,060
B6	18257500,000	26014548,000	564800,000	17,39	39,83	8,99	,259	,004	,060
B7	18257500,000	26014548,000	730000,000	17,39	39,83	7,99	,233	,004	,060
B8	18257500,000	26014548,000	895200,000	17,39	39,83	6,99	,206	,004	,060
B9	17967500,000	26014548,000	399600,000	11,59	39,83	9,99	,286	,003	,060
B10	17967500,000	26014548,000	564800,000	11,59	39,83	8,99	,259	,003	,060
B11	17967500,000	26014548,000	730000,000	11,59	39,83	7,99	,233	,003	,060
B12	17967500,000	26014548,000	895200,000	11,59	39,83	6,99	,206	,003	,060
C1	18495500,000	26014548,000	399600,000	20,29	39,83	9,99	,286	,004	,060
C2	18495500,000	26014548,000	564800,000	20,29	39,83	8,99	,259	,004	,060
C3	18495500,000	26014548,000	730000,000	20,29	39,83	7,99	,233	,004	,060
C4	18495500,000	26014548,000	895200,000	20,29	39,83	6,99	,206	,004	,060
C5	18199500,000	26014548,000	399600,000	14,49	39,83	9,99	,286	,003	,060
C6	18199500,000	26014548,000	564800,000	14,49	39,83	8,99	,259	,003	,060
C7	18199500,000	26014548,000	730000,000	14,49	39,83	7,99	,233	,003	,060
C8	18199500,000	26014548,000	895200,000	14,49	39,83	6,99	,206	,003	,060
C9	17909500,000	26014548,000	399600,000	8,69	39,83	9,99	,286	,003	,060
C10	17909500,000	26014548,000	564800,000	8,69	39,83	8,99	,259	,003	,060
C11	17909500,000	26014548,000	730000,000	8,69	39,83	7,99	,233	,003	,060
C12	17909500,000	26014548,000	895200,000	8,69	39,83	6,99	,206	,003	,060
D1	18437500,000	26349204,000	399600,000	17,39	37,34	9,99	,286	,003	,060
D2	18437500,000	26349204,000	564800,000	17,39	37,34	8,99	,259	,003	,060
D3	18437500,000	26349204,000	730000,000	17,39	37,34	7,99	,233	,003	,060
D4	18437500,000	26349204,000	895200,000	17,39	37,34	6,99	,206	,003	,060
D5	18141500,000	26349204,000	399600,000	11,59	37,34	9,99	,286	,003	,060
D6	18141500,000	26349204,000	564800,000	11,59	37,34	8,99	,259	,003	,060
D7	18141500,000	26349204,000	730000,000	11,59	37,34	7,99	,233	,003	,060
D8	18141500,000	26349204,000	895200,000	11,59	37,34	6,99	,206	,003	,060
D9	17858000,000	26349204,000	399600,000	5,8	37,34	9,99	,286	,003	,060
D10	17858000,000	26349204,000	564800,000	5,8	37,34	8,99	,259	,003	,060
D11	17858000,000	26349204,000	730000,000	5,8	37,34	7,99	,233	,003	,060
D12	17858000,000	26349204,000	895200,000	5,8	37,34	6,99	,206	,003	,060

6.2.5 Analysis of water allocation strategies

After constructing the dataset of the different future scenarios, we applied the ELECTRE-III-H method to each of them. The results of the case study are presented in this section. In this analysis, all weights of elementary criteria are equal because the experts considered that differentiating criterion priority is not appropriate, since all of them must contribute in the same proportion. The elementary criteria thresholds (indifference q_j, preference p_j and veto v_j) are the same in all scenarios. They have been calculated as a proportion κ of the mean absolute deviation (MAD) of each criterion in the neutral scenario and time span

2040. Proportions were set to κ=15% for indifference, κ=25% for preference and κ=50% for veto threshold.

First, we show the results for the 2040-neutral scenario. An example of the top elements in the partial pre-orders is illustrated in Figure 6.10. As displaying the full partial pre-order is not possible because of the large number of actions, the corresponding ranking positions have been obtained from the partial pre-orders to facilitate the comparison of the results for the different scenarios. In Table 6.14, the ranking results at intermediate criteria are shown for the neutral scenario, calculated from the partial pre-orders obtained at each criterion. Note that WS stands for Water Supply.

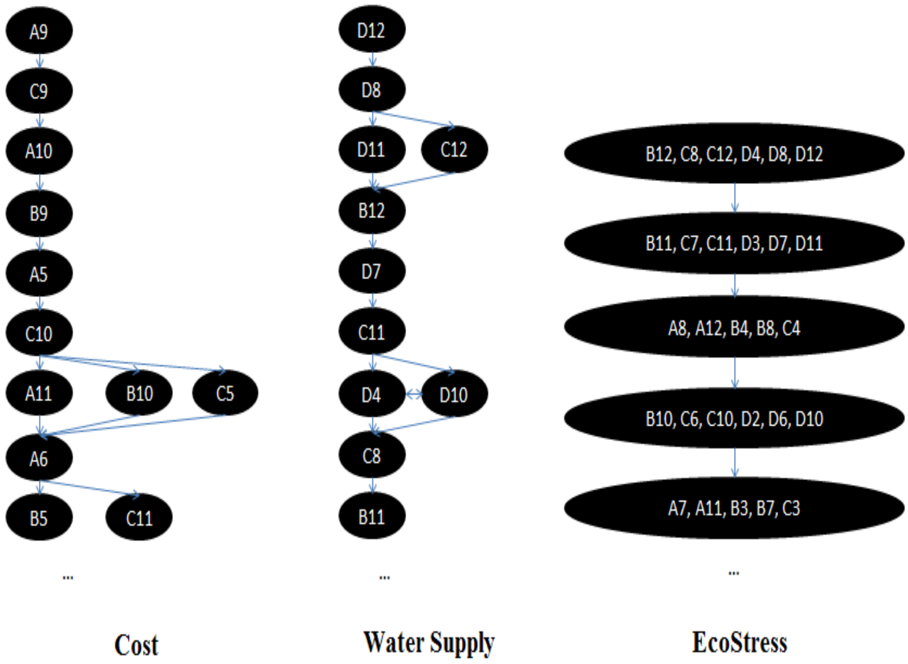


FIGURE 6.10: Partial pre-orders generated for neutral scenario 2040 (only the best positioned alternatives are displayed)

TABLE 6.14: Neutral scenario intermediate results for all time spans

	2040			2070			2100		
Alt	Cost	WS	EcoStress	Cost	WS	EcoStress	Cost	WS	EcoStress
A1	13	28	11	9	29	11	12	28	8
A2	16	27	9	11	28	9	15	27	7
A3	20	24	8	14	25	8	19	24	5
A4	22	21	7	16	22	7	22	21	3
A5	5	27	10	4	28	10	5	27	8
A6	8	25	6	6	26	6	9	25	7
A7	11	22	5	8	23	5	12	22	5
A8	17	19	3	13	20	3	18	19	3
A9	1	23	10	1	24	10	1	23	8
A10	3	21	6	2	22	6	3	21	7
A11	7	18	5	5	18	5	7	18	5
A12	12	13	3	10	14	3	13	14	3
B1	19	27	10	12	28	10	17	27	8
B2	23	25	6	15	25	6	21	24	7
B3	25	22	5	18	21	5	24	20	5
B4	27	19	3	20	19	3	27	18	3
B5	9	24	10	7	24	10	10	23	8
B6	12	16	6	9	16	6	13	16	7
B7	15	13	5	12	14	5	16	14	5
B8	23	9	3	17	10	3	23	10	3
B9	4	20	10	3	19	6	4	19	6
B10	7	12	6	5	13	4	7	13	4
B11	11	8	5	8	8	2	12	9	2
B12	16	5	3	14	5	1	19	5	1
C1	14	26	10	12	27	10	14	26	8
C2	17	23	6	15	22	6	18	21	7
C3	21	19	5	18	18	5	23	17	5
C4	26	14	3	20	14	3	26	14	3
C5	7	21	10	7	21	10	8	21	6
C6	10	14	6	9	15	6	12	15	4
C7	13	11	5	12	12	5	14	11	2
C8	19	8	3	17	8	3	22	8	1
C9	2	17	6	3	17	6	2	17	6
C10	6	10	4	5	11	4	6	11	4
C11	9	7	2	8	7	2	11	7	2
C12	14	3	1	14	3	1	17	3	1
D1	21	20	10	15	21	10	21	20	6
D2	24	15	6	17	15	6	23	15	4
D3	26	11	5	19	12	5	25	12	2
D4	27	8	3	21	8	3	28	8	1
D5	15	16	6	11	16	6	16	16	6
D6	18	9	4	14	10	4	20	10	4
D7	22	6	2	16	6	2	22	6	2
D8	25	2	1	18	2	1	24	2	1
D9	10	12	6	8	13	6	11	13	6
D10	11	8	4	10	9	4	14	9	4
D11	16	4	2	12	4	2	17	4	2
D12	17	1	1	14	1	1	20	1	1
max	27	28	11	21	29	11	28	28	8

The ranks at intermediate criteria show that no alternative performs very good at all intermediate criteria (i.e., Cost, Water Supply, EcoStress) for all time spans, because criteria measure opposite features. For example, alternative A9 is the cheapest out of the 48 available alternatives, but its Water Supply and EcoStress performance are among the worst; D8, which has a very good Water Supply performance, is an expensive solution; C9 has good cost performance but poor Water Supply performance; and B9 is very well ranked in cost but has a bad rank in Water Supply.

Because Ebro and Gaià have low variability in the amount of water that can be extracted (because Francolí is the main supply source), we can see that the EcoStress ranking has a large number of indifference relations. They correspond to cases where the actions have equivalent values on the EcoStress of the rivers. For Water Supply and Cost, the pre-order identifies more strict preference relations than EcoStress, which lead to a more linear ranking (i.e., we find less rank ties, having that in the neutral scenario Water Supply has 28 rank positions while the Cost criterion has 27, from a total of 48 options). This same behaviour is observed for each intermediate criterion in all the scenarios and time spans.

Taking this observations into account, the parameters (Table 6.15) have been set up according to the obtained rankings shown in Table 6.14. First, several rank ties occur on the EcoStress criterion as the same rank position is shared by small subsets of alternatives (around 5 or 6), indicating quite similar evaluations of alternatives in EcoStress. Thus, the preference threshold is $p_{EcoStress} = 5$ and the veto is $v_{EcoStress} = 25$ (which is about 5 rank positions). Second, for Cost and Water Supply, the ranks obtained have less rank ties, and consequently it allows the distinction of a larger set of values in $\Gamma(\cdot)$. Moreover, the parameters fixed for the Cost are stricter than for the sector water supply criterion. The reason is that a high negative comparison in the Cost evaluation should be avoided when establishing the preference relations. For Water Supply stress, the veto power has been reduced ($v_{WaterSupply} = 40$), also permitting a more relaxed measurement of concordance $p_{WaterSupply} = 20$. In that way, the final decision will be more according to the environmental criteria majority opinion, but preventing situations of high cost.

Table 6.15: Parameters at intermediate criteria

	Cost	Water supply	EcoStress
Indifference	0	0	0
Preference	10	20	5
Veto	25	40	25

6.2.6 Weight tests at intermediate criteria

After the study of the neutral scenario, we have applied 3 different tests with different weights at the intermediate level, in order to evaluate the performance water allocation actions depending on different priorizations of the criteria. Three cases of interest have been defined (see Table 6.16):

- Balanced case: the same relevance for the 3 criteria at intermediate level is given.
- Environmental-first case: we considered Water Supply and EcoStress criteria more important than the cost.
- Cost-first case: where it is more important to optimize costs than environmental impacts.

TABLE 6.16: Tests weight values

Test	Cost	Water supply	EcoStress
Balanced	33,3	33,3	33,3
Environment first	15	42,5	42,5
Cost first	75	12,5	12,5

For each of the 9 different scenarios, these 3 different weight tests have been applied, obtaining 9 different global partial pre-orders and rankings of the alternatives at the root node of the hierarchy. In Figure 6.11, the global partial pre-orders obtained for the balanced case using the optimistic scenario are shown to illustrate the results obtained using ELECTRE-III-H for the 3 time spans.

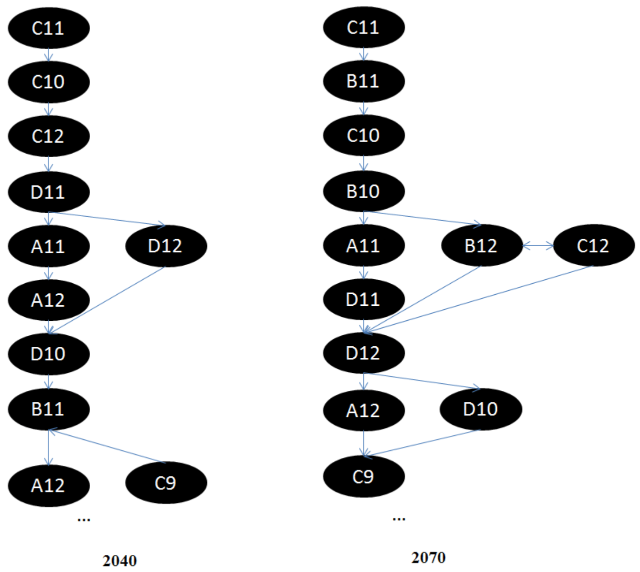


FIGURE 6.11: Global results for the balanced weight test and optimistic scenario

The better water allocation actions of all the tests are presented in Tables 6.17, 6.18, and 6.19.

TABLE 6.17: Optimistic scenario results

2040		2070		2100	
Alternative	Rank	Alternative	Rank	Alternative	Rank
Balanced					
C11	1	C11	1	C11	1
C10	2	B11	2	B11	2
C12	3	C10	3	C10	3
D11	4	B10	4	C12	4
A11	5	A11	5	B10	5
Environment first					
C12	1	D12	1	C12	1
D12	2	B12	2	B12	2
C11	3	C12	2	C11	3
D11	4	D11	3	D11	4
B12	5	C11	4	D12	4
Cost first					
C11	1	B11	1	C11	1
C10	2	C11	1	C10	2
C9	3	C10	2	B10	3
A11	4	B10	3	B11	4
C12	4	A11	4	C9	5

TABLE 6.18: Neutral scenario results

2040		2070		2100	
Alternative	Rank	Alternative	Rank	Alternative	Rank
Balanced					
C11	1	C11	1	C11	1
C12	2	C10	2	C12	2
B11	3	C12	3	B11	3
C10	4	D11	4	D10	4
B10	5	A12	5	C10	5
Environmental					
C12	1	C12	1	C12	1
B12	2	D12	2	D12	2
D12	2	C11	3	B12	3
C11	3	D11	4	C11	4
D11	4	B12	5	D11	5
Cost					
C11	1	C11	1	C11	1
C10	2	C10	2	C10	2
B10	3	C9	3	B10	3
B11	4	A11	4	B11	4
C9	4	B10	4	C12	4

TABLE 6.19: Pessimistic scenario results

2040		2070		2100	
Alternative	Rank	Alternative	Rank	Alternative	Rank
Balanced					
C11	1	C11	1	C11	1
C12	2	C10	2	C12	2
B11	3	C12	3	B11	3
D10	4	B11	4	C10	4
C10	5	D11	4	B10	5
Environment first					
C12	1	C12	1	C12	1
D12	2	D12	2	D12	2
B12	3	C11	3	B12	3
C11	4	D11	4	C11	4
D11	5	B12	5	D11	5
Cost first					
C11	1	C11	1	C11	1
C10	2	C10	2	C10	2
B10	3	C9	3	B10	3
B11	4	B10	4	B11	4
C12	4	C10	5	C12	4

These results of ELECTRE-III-H show that for all scenarios, medium and high use of alternative resources (C and D respectively) with high water reuse in the industry sector (from 9 to 12) are generally presented as the recommended alternatives. Analyzing the criteria at the intermediate level, this is due to lower EcoStress indices and water supply in comparison to strategies A and B, which propose no or low use of alternative resources alternatives.

C11 is the recommended alternative for all time spans and non-environmental scenarios. For the environmental-first scenarios, C12 is considered as the best alternative. Comparing alternatives C and D, the actions in C have high water reuse in the industry sector and medium use of alternative resources, which results in significantly better cost than actions in D that have a high use of alternative resources. This is due to a low use of desalination for industry and domestic sectors in C, which decrease the cost. However, this generally affects the EcoStress and water supply with respect to alternatives D. This behavior can be observed in the environmental-first tests, in which D12 is always between the first and second in the ranking. This is because D12 use a high desalination in the industry and domestic sectors, reducing the use of primary sources and therefore reducing also the ecological impact. The fact that the cost is not that relevant in the decision for environmental-first tests, D12 becomes a recommended alternative. Notice that this alternative is not ranked among the first 5 for cost-first and balanced tests.

Another interesting point is that giving priority to water coming from primary resources with low / medium water reuse (A1 to A8) generally results in a high EcoStress index and water supply, which has a high ecological impact and therefore, they are commonly the worst alternatives to recommend.

6.2.7 Discussion

The proposed IEDSS is useful for water managers as it gives possibility to integrate different management criteria and water allocation strategies into a single modeling framework and explore the different adaptation measures. The recommendations presented in this work have shown quite interesting results from the environmental point of view, specially because they are able to devise the trend in the future (from 2040 to 2100). With this IEDSS the managers can analyze the consequences of different water allocation actions not only in short term but also in the long term.

The different parameters of the ELECTRE-III-H method shows enough flexibility to properly model the problem according to the requirements of the DM. For example, we can define the relative importance of each aspect to be considered in the decision at different levels, and we can set more or less strictness in the action's comparison for each individual criterion (using the indifference and preference thresholds). Finally, the possibility of vetoing permits to control the compensativity effect of other MCDA methods, in which the evaluations given by minorities are always ignored in front of the majority. With ELECTRE-III-H minorities can also veto the majority if there are enough arguments to do it.

We have shown that the results obtained in this case study are robust to different time spans and to the most optimistic and pessimistic predictions. In [Chao et al, 2014], a robustness with respect to the thresholds values was studied.

6.3 Analysis of activities in a recommender system

6.3.1 Introduction

In the south of Catalonia (Spain), at the Tarragona region, wine production is a traditional activity that nowadays attracts the interest of more and more visitors. This phenomenon is known as Enotourism or Vinitourism, and attracts people that want to visit cellars, taste wines, walk in the vineyards or also do other cultural and typical activities. With the objective to improve the tourist experience and promote this kind of activities, a Web-based and mobile application recommender system is being developed at the Science and Technology Park for Tourism and Leisure (PCT). This system, called GoEno-Tur, provides personalized recommendations of touristic activities related to wine and culture in this region.

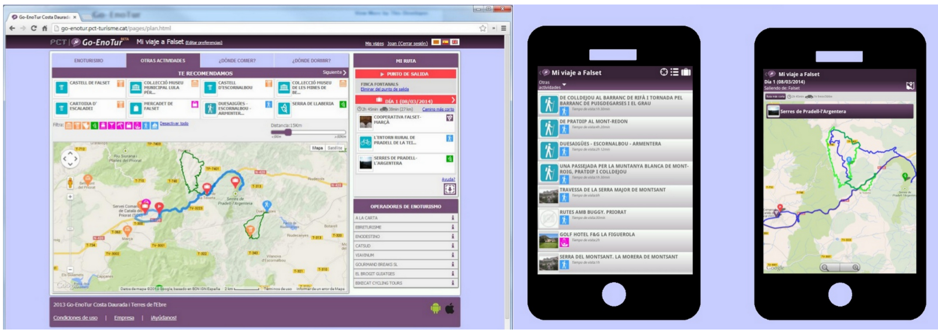


FIGURE 6.12: GoEno-Tur webpage and its mobile application

The Tarragona region has 5 recognized wine production areas with the called DO (official Designation of Origin) and several cultural locations somehow related to oenology. About 300 activities and cultural points of interest related to wine have been cataloged by the PCT team. For each user, the GoEno-Tur system selects the most adequate activities for a certain user. The proposals are given to the user in groups of 8 activities through a graphical Web interface [Borràs et al, 2012a]. The system initially starts recommending activities to each user based on his/her interests in general terms, requested during the registration stage. The user may interact through the Web interface, adding some proposals to his personal plan and discarding the others. The user may also request more proposals to the system, which displays 8 different activities per request.

In order to make a satisfactory recommendation, we need to classify the activities in different categories according to each user's interests. Three dimensions are considered. First, the system evaluates if the characteristics of the recommended activities match with the user's preferences. For this purpose, each of these activities has been associated to several tags chosen from a predefined ontology [Borràs et al, 2012a]. Tags are used to semantically explain the content of the activity and will help to provide a more personalized recommendation. For example, a certain wine cellar can be associated to *eco-making*, *vineyard*, or *bio-making*, whereas another cellar may be associated to *wine cellar*, *historic building* and *modernism*. Second, contextual information is evaluated. Context refers to additional features of the activity that may influence the user decision (f.i., distance to a location or price). Third, tourism management entities may want to promote certain activities of special interest for a certain period of time (f.i., a new museum, a temporal exhibition, a unique fair). This issue is also added as a criterion to have into account during the recommendation process.

To obtain all these indicators, the GoEnu-Tur system applies a hybrid approach including content-based, collaborative filtering and socio-demographic techniques [Borràs et al, 2012a]. Afterward, the system has to consider all these multiple criteria in order to sort the alternatives. For this purpose, we propose a hierarchical procedure for sorting. A sorting for each of the three dimensions is done, in order to evaluate separatedly the engagement of the activities with respect to the user's preferences, the context and the tourism managers strategy. Next, the system has to propagate these partial sorting results up to the root of the hierarchy obtaining a global assignment of the categories, from which the recommendations are provided.

6.3.2 Integration of ELECTRE-TRI-B-H into the GoEno-Tur recommender system

The GoEno-Tur system aims at helping the tourists of Tarragona region to easily find the most appropriate activities, specially focusing on the world of oenology. The goal of the ELECTRE-TRI-B-H method is to assign the available activities into four categories at a global level: *Unacceptable*, *Fair*, *Good*, and *Very Good*. The final recommendation will be based on the assignment of the activities to these four categories. The activities that are *Unacceptable* will never be proposed to the visitor, whereas the ones classified to *Very Good* will be the first proposals displayed. However, some intermediate ordered categorical criteria are of particular interest and may help in the recommendation procedure, see Figure 6.13. When the user discards some of the proposals and requests more alternatives, the system selects other alternatives (at least *Fair*) on the basis of the assignment of categories at intermediate nodes, for example showing the ones with good Customer Satisfaction and Context, regardless of the Touristic Strategy. Other recommendation strategies could be defined, such as promoting the Touristic Strategy in spite of the Customer Satisfaction. Therefore, the sorting at intermediate levels may be useful in the recommendation procedure.

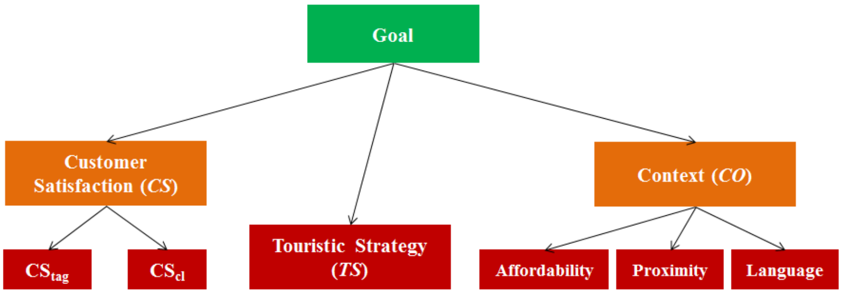


FIGURE 6.13: Hierarchy tree of criteria of the GoEno-Tur Recommender System

The hierarchy has 3 main branches. The first group of criteria evaluates the satisfaction of the user with respect to the activity description. For this purpose, each of these activities has been associated to several tags chosen from a predefined ontology [Borràs et al, 2012a]. Tags are used to semantically explain the content of the activity and will help to provide a more personalized recommendation. For example, a certain wine cellar can be associated to *eco-making*, *vineyard*, or *bio-making*, whereas another cellar may have the following tags: *wine cellar*, *historic building* and *modernism*.

The second group of criteria is related to contextual features. Context (CO) refers to additional features of the activity that also influence the user decision.

Finally, a third sorting criterion was set up by the designer of the system to include the possibility that the local tourist management entities could promote certain activities (f.i., a new museum, a temporal exhibition, a unique fair).

The decision of recommending or not an alternative is ultimately based on the following six elementary criteria:

1. Satisfaction of the Customer with the Tags (CS_{tag}): Score obtained from analyzing the interest of the user on the tags (i.e., concepts of the ontology) associated to an item. This interest is initially asked to the user when he logs into the system, and it is modified with unsupervised learning techniques that obtain implicit feedback from the user by tracking the actions of the user in the system. The items are compared to the user interest's with Content-Based recommendation techniques [Borràs et al, 2012b].

2. Satisfaction of the Customer using Collaborative Techniques (CS_{cl}): Score provided by opinions of other users that have similar interests (about the tags). Collaborative Filtering recommendation techniques are applied to obtain this number [Moreno et al, 2013].

3. Proximity: It measures the proximity between the location of the user, and the location of the item. The location of the user corresponds to the residence of the user (f.i., hotel, camping). The distance is transformed into a proximity degree in the range 0.0-1.0 using a linear utility function to be maximized.

4. Affordability: criterion measuring the satisfaction of the user, constructed by comparing the user's budget with the price of an activity. If the price is lower than the budget then the satisfaction is maximum (1.0), otherwise the satisfaction is proportional to the difference between the budget and the price.

5. Language: Boolean criterion that indicates if the activity is available in a language that is understood by the user.

6. Touristic strategy (TS): Reflects the promotion of different items according to some external expert influence. In the case of touristic activities, the local authorities can promote different activities in order to increase their visibility, or to attract people to some kind of activities that are usually less visited. The attribute considers 3 values (Low, $Medium$ and $High$). This ordinal criterion evaluates the suitability on the recommendation of the activity regardless of the user.

The model of the problem (criteria, thresholds, weights, etc), given in Table 6.20, has been constructed together with the tourism experts in order to build a realistic recommendation system that fits with the goals of the tourism destination managers in this sector.

After an empirical study, at all levels of the hierarchy a cutting threshold value of 0,65 was fixed and the pessimistic (conjunctive) procedure is applied on the exploitation procedure.

TABLE 6.20: Values of the parameters at the different nodes of the hierarchy

Criterion	Range	Weight	$q_j(b_h^j)$	$p_j(b_h^j)$	$v_j(b_h^j)$
CS_{tag}	0..1	0,2	0,03	0,07	1
CS_{cl}	0..1	0,4	0,02	0,05	0,08
TS	*Low, Medium, High*	0,3	0	0	2
Proximity	0..1	0,2	0,1	0,15	0,25
Affordability	0..1	0,2	0,1	0,15	0,2
Language	False, True	0,1	0	0	1
CS	CCC, B, BB, BBB, A, AA, AAA	0,6	0	1	2
CO	*Bad, Regular, Good*	0,4	0	0	2
Goal	*Unacceptable, Fair, Good, Very Good*	-	-	-	-

Different sets of linguistic ordered categories have been defined to each non-elementary criterion. For criterion CS, a credit-like rating scale is used to indicate the satisfaction of the users. Hence, CCC represents the lowest score, followed by CC, B, BB, BBB, A, AA, until AAA (the best). For criteria CO and *Goal*, typical linguistic scales have been chosen.

Considering that the users in the system are new, the parameters defined at intermediate criteria allow some flexibility in the comparison of the profile limits and the alternatives, meanwhile the system starts learning more about users' tastes. The veto threshold is set to 2 categories, thus the system does not apply the right to veto of any criterion unless a different of more than 2 units is found. For criterion CS, a small degree of weak preference is also set.

In order to build the profiles of the root criterion, the following set of rules have been defined by the domain experts:

Rule 1: **if** g_{CS}=CCC and g_{CO}=*Bad* **then** g_{goal}=*Unacceptable*

Rule 2: **if** g_{CS}=B and g_{CO}=*Regular* and g_{TS}=*Low* **then** g_{goal}=*Fair*

Rule 3: **if** g_{CS}=(BB *or* BBB) and g_{CO} = *Regular* and g_{TS}=*Medium* **then** g_{goal}=*Good*

Rule 4: **if** g_{CS}=(A *or* AA *or* AAA) and g_{CO}=*Good* and g_{TS}=*High* **then** g_{goal}=*Very Good*

Note that in Rule 1, the DM only takes into consideration criteria CS and CO to define an $Unacceptable$ alternative on g_{goal}. This entails that the rules defined by the DM may take into consideration only a subset of criteria to define the profile limits. The exclusion of the criterion TS in Rule 1 implies that the assignments on TS are irrelevant to determine if an alternative is assigned to $Unacceptable$ and therefore are not taken into account. Otherwise, the rest of rules include a condition on TS.

Figure 6.14 shows the representation of the profile limits in terms of categories. The corresponding vectors of the profile limits are shown in Table 6.21, obtained following Definition 5.1. Profile limit b_1 is obtained using Rule 2. Notice that Rule 1 is not used since the profile of the first category is always implicit. In this case, TS is set to low because it is the first lowest category that can be assigned to $Fair$.

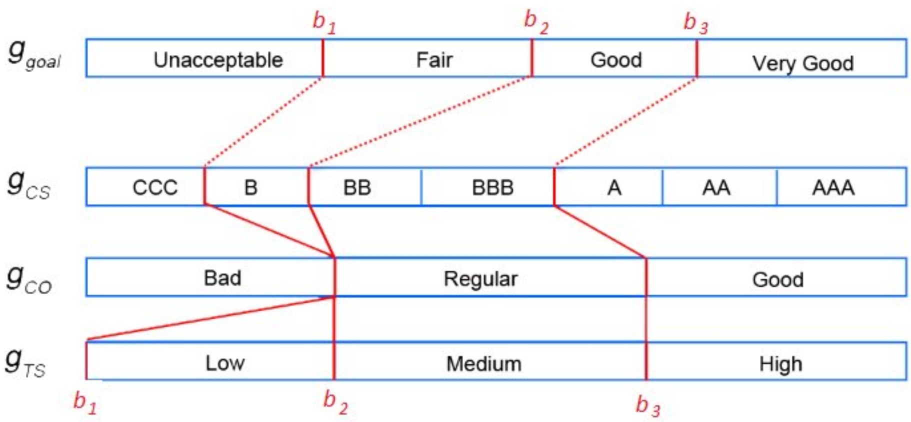


FIGURE 6.14: Graphical model of the profile limits for the tourism recommender system

TABLE 6.21: Rule-based vectors representing the profile limits

Profile limits	Vector of profile limits
b_1^{goal}	$< B,\ Regular,\ Low >$
b_2^{goal}	$< BB,\ Regular,\ Medium >$
b_3^{goal}	$< A,\ Good,\ High >$

For the ordinal categorical elementary criterion *Language*, where its direct parent is CO, the following rules have been defined:

Rule 1: **if** $g_{language}$=$false$ **then** g_{CO}=*Regular*

Rule 2: **if** $g_{language}$=$true$ **then** g_{CO}=*Good*

For this case, any rule assigns an alternative to the category *Bad* on CO, so that all alternative are considered to be at least *Regular*, which means that it may be of some interest for the user even if the language is unknown.

The following subsection presents an example of a fictive tourist called Mr. Smith. Next, a robustness analysis of the results of ELECTRE-TRI-B-H is presented.

6.3.3 Recommending to Mr. Smith

Let us consider the case of a new user in the system, named Mr. Smith, a British tourist that spends a week in a hotel at the center of Tarragona city. He has a medium travel budget. Mr. Smith's preferences are about eco-making wine cellars located in historic or unique buildings. With respect to culture, Mr. Smith tends towards Romanesque architecture and arts, and he is also very interested in castles.

ELECTRE-TRI-B-H is executed and the about 300 activities in enotourism and culture are sorted at the different levels of the hierarchy. To study the results, we will first focus on a subset of alternatives with different tags, including activities that are of interest for Mr. Smith and others that are not (Table 6.22). The performance matrix generated by the recommender system with the evaluations of the elementary criteria for Mr. Smith is shown in Table 6.23.

TABLE 6.22: Subset of alternatives and their tags

Alternative ID	Tags
151	TraditionalTowns
160	TraditionalTowns
167	Castle, Walls, TraditionalTowns
174	Tower
248	TraditionalTowns
284	Building
304	Walls, TraditionalTowns, Medieval
334	TraditionalTowns, Castle, Gothic, Renaissance, Baroque, Walls
349	Romanesque, Gothic, Carthusian, Monastery
367	Romanesque, Renaissance
371	WineCellar, WineTasting
372	Romanesque, Gothic, Baroque, Convent
375	Baroque, Chapel
2352	WineCellar, EcoMaking, BioMaking, HistoricBuilding
2354	WineCellar, ModernistCellar
2397	WineCellar, CraftMaking, UniqueBuilding, HistoricBuilding
2410	WineCellar, UniqueBuilding, HistoricBuilding
2420	WineCellar, EcoMaking, HistoricBuilding
2432	WineCellar, HistoricBuilding
2437	WineCellar, EcoMaking, HistoricBuilding
2447	WineCellar, EcoMaking
2500	WineCellar, WineTasting
2561	WineCellar, EcoMaking, CraftMaking, WineTasting, Vineyards
2582	WineCellar, EcoMaking, WineTasting
3224	WineCellar, WineTasting
3234	WineCellar, WineTasting

TABLE 6.23: Performance of the selected subset of alternatives

Alternative ID	CS_{tag}	CS_{cl}	TS	Affordability	Language	Proximity
151	0,22	0,55	High	0,02	false	0,66
160	0,22	0,55	High	0,61	true	0,00
167	0,52	0,83	High	0,93	true	0,51
174	0,22	0,58	Low	0,6	true	0,86
248	0,22	0,55	High	0,81	true	0,76
284	0,22	0,71	Low	0,23	false	0,62
304	0,22	0,64	High	0,92	false	0,65
334	0,52	0,85	High	0,89	true	0,43
349	0,52	0,87	High	0,79	false	0,5
367	0,52	0,85	Low	0,11	false	0,61
371	0,52	0,86	Low	0,54	false	0,18
372	0,52	0,87	Low	0,25	true	0,00
375	0,22	0,68	Low	0,29	false	0,74
2352	0,75	0,78	Medium	0,78	true	0,59
2354	0,75	0,57	High	0,99	true	0,61
2397	0,87	0,75	High	0,84	true	1,00
2410	0,75	0,76	High	0,90	false	0,62
2420	0,75	0,57	Medium	0,76	true	0,61
2432	0,75	0,76	High	0,63	false	0,63
2437	0,74	0,78	Medium	0,58	false	0,58
2447	0,74	0,58	Low	0,35	false	0,66
2500	0,92	0,52	Low	0,14	false	0,58
2561	0,87	0,56	Medium	1	true	0,22
2582	0,88	0,56	Low	0,65	false	0,00
3224	0,92	0,52	Low	0,66	false	0,53
3234	0,92	0,52	Low	0	false	0,56

After applying the ELECTRE-TRI-B-H method, the results of the alternatives in the subset at intermediate and root criterion are presented in Table 6.24. The alternatives are ordered with respect to the final Goal criterion. For each one, the assignments at the intermediate criteria are also given. The elementary criterion TS is also an ordinal categorical criterion, but in this case it is given directly by the DM. It has also been included in the table as it is aggregated with CS and CO for the global assessment (root).

TABLE 6.24: Subsets of assignments for each category in root criterion

Alternative ID	Global Assignment	CS	CO	TS
2352	Very Good	AA	Good	Medium
2354	Very Good	AA	Good	High
2397	Very Good	AAA	Good	High
2410	Very Good	AA	Regular	High
2420	Very Good	AA	Good	Medium
2432	Very Good	AA	Regular	High
167	Good	BBB	Regular	High
334	Good	BBB	Regular	High
349	Good	BBB	Regular	High
2437	Good	AA	Regular	Medium
2447	Good	AA	Regular	Low
2561	Good	AA	Bad	Medium
3224	Good	AA	Regular	Low
367	Fair	BBB	Bad	Low
371	Fair	BBB	Bad	Low
372	Fair	BBB	Bad	Low
2500	Fair	AA	Bad	Low
2582	Fair	AA	Bad	Low
3234	Fair	AA	Bad	Low
151	Unacceptable	CCC	Bad	High
160	Unacceptable	CCC	Bad	High
174	Unacceptable	CCC	Good	Low
248	Unacceptable	CCC	Bad	High
284	Unacceptable	CCC	Bad	Low
304	Unacceptable	CCC	Regular	High
375	Unacceptable	CCC	Regular	Low

The first observation is that the assignments for CS and CO are coherent. For example, activity 2397 has the best performance on CS_{tag} and CS_{cl}, so that it is assigned to AAA. We can also notice that for criterion CO, the alternatives with high scores in the three last columns in Table 6.23 (f.i., 174 and 2352) are assigned to *Good*; whereas alternatives are assigned to *Bad* when the performance is low (f.i., activity 2500).

More interesting is the analysis at the root level, when ordinal categories are aggregated. None of the first 6 alternatives, assigned to $Very\ Good$, is assigned to *Bad* on CO, *Low* on TS or assigned to the five worst CS categories. In a similar way, the 7 alternatives assigned to *Unacceptable* have all the worst CS degree and mainly *Bad* or *Regular* assignment on CO.

Going in more detail, we can observe how the ELECTRE-TRI-B-H method has assigned some of the wine cellars that include tags related to eco-making, historic building, or unique building to *Very Good* at the root criterion. For example, alternatives such as 2352 and 2410, related to eco-making and historical building respectively are excellent candidates to be recommended to Mr Smith. However, other cellars related to such tags are assigned to lower categories, which indicates that factors such as proximity and collaborative filters affect the final assignments. For example, the alternative 2582, related to eco-making, is assigned to *Fair* because on CO it is assigned to *Bad* by cause of the proximity.

Regarding Mr. Smith cultural preferences, all of the alternatives assigned to *Unacceptable* are neither related to Romanesque style nor castles. Moreover they have $CS = CCC$, as Rule 1 indicates. We can find some alternatives with $CO =$ *Good* or $TS =$ *High* in the *Unacceptable* category, because the CS is the most relevant criterion according to the weights given by the DM. The recommender system may definitely not recommend these alternatives, since they are not related to the main preferences of the tourist and the assignment on CO is also *Bad*, even if the DM is really interested to promote these locations (TS criterion). Other alternatives related to Romanesque, such as 367 and 372 are assigned to *Fair*, see Table 6.22. In this case we can see that Mr. Smith may be interested in these activities, in spite of the fact that the affordability and proximity are assigned to *Bad*.

Taking into account that the ELECTRE-TRI-B-H method produces a sorting at different levels of the hierarchy, the DM can apply different ways to diversify recommendations. For example, alternate *Very Good* and *Good* cellars with good culture locations, or recommend first *Very Good* and *Good* alternatives with a *High* assignment on TS only. Table 6.25 shows the number of assignments for *Very Good* and *Good* at the root level together with the values of TS. Thus, the recommender system can be configured on the basis of the combination of categories at different levels.

Table 6.25: Assignments for global results and TS

Global result	TS	#Assignments
Very Good	*High*	29
Very Good	*Medium*	6
Very Good	*Low*	0
Good	*High*	21
Good	*Medium*	39
Good	*Low*	116

6.3.4 Robustness analysis

The results of the ELECTRE-TRI-B-H method study were validated with a robustness analysis. Robustness analysis is applied when uncertainty is a factor that obstructs reliable decisions. We study how the set of recommended activities change depending on different possible configurations that the experts may set for new users, more concretely for the new user Mr. Smith. This analysis can be very helpful to refine the profile limits that are constructed using the decision rules at intermediate criteria. For this purpose, three scenarios with different configuration of the thresholds on the intermediate criteria have been defined, as shown in Table 6.26. The first scenario is named strict because no uncertainty is considered when comparing an alternative with the limiting profile of a certain category. In this case, the veto is activated if some criterion is not fulfilled exactly. This means that the assignments at intermediate criteria must fully accomplish the rules defined by the experts. The second scenario is called central, because the discrimination thresholds and right to veto are slightly increased. The veto is not applied until a difference of more than 2 units is found. This scenario is the one used to get the results shown in Section 6.3.3. In the third scenario, called tolerant, the discrimination and veto thresholds are increased with respect to the central scenario on criterion CS. As CS has 7 categories, this scenario considers that the experts are less confident with the precision of the assignments made at CS. Remember that when the preference and veto thresholds are increased, we are allowing some uncertainty margin and decreasing the strength of the opposition to the assertion aSb_h^j (i.e., decreasing the discordance degree).

TABLE 6.26: Robustness configuration parameters

Scenario	CS	CO	TS
	$q_j(b_h^j)$, $p_j(b_h^j)$, $v_j(b_h^j)$	$q_j(b_h^j)$, $p_j(b_h^j)$, $v_j(b_h^j)$	$q_j(b_h^j)$, $p_j(b_h^j)$, $v_j(b_h^j)$
Strict	0, 0, 1	0, 0, 1	0, 0, 1
Central	0, 1, 2	0, 0, 2	0, 0, 2
Tolerant	1, 2, 3	0, 0, 2	0, 0, 2

After the application of the ELECTRE-TRI-B-H method using this set of parameters, in Figure 6.15, we illustrate the number of alternatives assigned to each category in the root criterion depending on the thresholds applied. The more tolerant the parameters are, the highest the number of alternatives assigned to the better categories. For example, in the strict scenario only 8 alternatives were assigned to $Very$ *Good*, in central scenario 35 and in tolerant scenario 38. Analogously, alternatives assigned to *Unacceptable* decrease when the margin of uncertainty is increased, as we can observe in the tolerant scenario results in which no alternative is assigned to *Unacceptable*, whereas we have 15 unacceptable activities in central scenario and 104 alternatives in strict scenario.

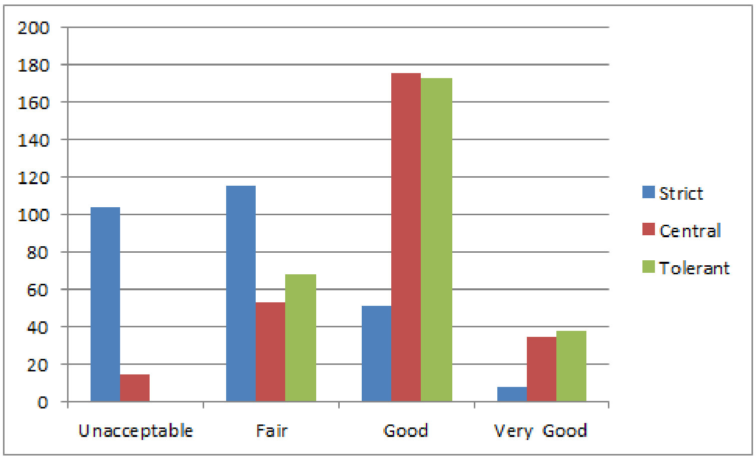


FIGURE 6.15: Number of alternative assignments for each category applying different scenarios

The stability of the assignments for this dataset in these three scenarios has also been studied. Although we cannot expect the assignments to be always the same, some degree of stability is desirable. This robustness analysis is made only at the root level, in which we aggregate the CS, CO and TS criteria, since this is the main contribution of this paper. Therefore, the analysis is based on the combinations of categories that appear in this dataset, without taking into account how many activities we have on each combination. Considering the number of categories defined for these criteria, a total of 63 possible combinations are possible ($7 \times 3 \times 3$), but only 24 appear in the dataset for this case study. We have defined that an assignment combination is "stable" when it is assigned to the same category at the root level in all the scenarios evaluated. Stable and unstable combinations are given in Table 6.27.

TABLE 6.27: Stable and unstable alternatives for the different scenarios

CS	CO	TS	Strict root	Central root	Tolerant root	Stability
AAA	$Good$	$High$	$Very\ Good$	$Very\ Good$	$Very\ Good$	Stable
AAA	$Regular$	$High$	$Good$	$Good$	$Good$	Stable
AAA	$Regular$	$Medium$	$Good$	$Good$	$Good$	Stable
AAA	Bad	$High$	$Unacceptable$	$Good$	$Good$	Unstable
AA	$Good$	$High$	$Very\ Good$	$Very\ Good$	$Very\ Good$	Stable
AA	$Good$	$Medium$	$Good$	$Very\ Good$	$Very\ Good$	Unstable
AA	$Good$	Low	$Fair$	$Fair$	$Good$	Unstable
AA	$Regular$	$High$	$Good$	$Very\ Good$	$Very\ Good$	Unstable
AA	$Regular$	$Medium$	$Good$	$Good$	$Good$	Stable
AA	$Regular$	Low	$Fair$	$Good$	$Good$	Unstable
AA	Bad	$High$	$Unacceptable$	$Good$	$Good$	Unstable
AA	Bad	$Medium$	$Unacceptable$	$Good$	$Good$	Unstable
AA	Bad	Low	$Unacceptable$	$Fair$	$Fair$	Unstable
BBB	$Good$	Low	$Fair$	$Good$	$Good$	Unstable
BBB	$Regular$	$High$	$Good$	$Good$	$Very\ Good$	Unstable
BBB	$Regular$	Low	$Fair$	$Good$	$Good$	Unstable
BBB	Bad	$High$	$Unacceptable$	$Good$	$Good$	Unstable
BBB	Bad	Low	$Unacceptable$	$Fair$	$Fair$	Unstable
CCC	$Good$	$High$	$Unacceptable$	$Unacceptable$	$Fair$	Unstable
CCC	$Good$	Low	$Unacceptable$	$Unacceptable$	$Fair$	Unstable
CCC	$Regular$	$High$	$Unacceptable$	$Unacceptable$	$Fair$	Unstable
CCC	$Regular$	Low	$Unacceptable$	$Unacceptable$	$Fair$	Unstable
CCC	Bad	$High$	$Unacceptable$	$Unacceptable$	$Fair$	Unstable
CCC	Bad	Low	$Unacceptable$	$Unacceptable$	$Fair$	Unstable

From Table 6.27 we can see that the final assignment at the root level is consistent with the profile limits defined by means of the rules (see Table 6.21). An example of a stable case is found for when an activity is $< AAA$, *Regular*, $Medium >$. Having $CS = AAA$ implies that the alternative may be assigned to $Very$ *Good* at the root; however, taking into account that the indifference and preference thresholds on CO and TS are both 0, then the *Regular* and $Medium$ categories are not allowed in the category $Very$ *Good* (see Rule 4), so that this alternative remains *Good* at the root criterion for all scenarios. The same case applies to $< AA$, *Regular*, *Medium* $>$.

Some interesting cases regarding unstable assignments are those that in the strict scenario are assigned to *Unacceptable* but in the central scenario are assigned to *Good*. For example, that is the case of $< AA$, *Bad*, *High* $>$. When the strict scenario is applied, the CO criterion activates the veto because this option has a *Bad* context, which contributes to be assigned to *Unacceptable* at the root. However, when the threshold values are increased, this right to veto is not activated and considering the good performance on the rest of criteria, the alternative is finally assigned to *Good*.

Another interesting case occurs when $CS = CCC$, where the alternatives are assigned to *Unacceptable* at the root, except for the tolerant scenario. The tolerant scenario is the only scenario that allows an indifference of 1 on CS, so that these alternatives are going to be assigned to at least $Fair$ because of the indifference with the lower profile b_1^{CS}, placed in category $Fair$.

6.3.5 Discussion

Some decision problems involve the assignment of alternatives into predefined categories depending on their performance on multiple conflicting criteria. That is the case of the hybrid recommender system GoEno-Tur, which provides personalized enotourism and cultural activities in Tarragona to the users. The large amount of available activities that can be recommended makes feasible the application of ELECTRE-TRI-B-H, classifying the alternatives to categories associated to satisfaction levels defined by the DM and allowing the modelization of a hierarchical structure of criteria with 3 main criteria: Customer-satisfaction, Touristic strategy and Context. The final recommendations are calculated with the use of intuitive decision rules at intermediate criteria using verbal labels, providing valuable and easy-to-understand information to the DM about the degree of performance of the alternatives regarding a particular subset of criteria. Besides, the calculation for the assignment of the alternatives are directly compared to the profile limits. It allows a more robust and responsive navigation system for multiple-users.

The results show the flexibility of the ELECTRE-TRI-B-H method and how this approach may help in the construction of this kind of systems. Partial assignments at intermediate levels can also be used as a tool to refine the proposals selected to be shown to the user. For instance, when new registered users request activities, very few information is known about their tastes. The DM may recommend to set more tolerant preference thresholds while the system is learning the user's preferences (e.g., castles, eco-making, etc). On the other hand, if the level of satisfaction of the provided recommendations by the system is high, the system has a robust knowledge about the user's preferences and thus, we can reduce the preference thresholds as the uncertainty about the preferences has decreased.

Chapter 7

Conclusions and future works

In complex real-world multiple criteria decision problems, a hierarchical structure of criteria facilitates more detailed analysis of the recommendations at different levels of generality. This is studied in the research project SHADE of the ITAKA group, in which this thesis is framed.

This thesis is focused on the ELECTRE outranking method, which has been very successful in real-case studies in the MCDA discipline because of their several advantages, including the ability to handling incomparability between alternatives, while it is usually treated as indifference when using a compensatory model; the management of heterogeneous scales of measurements without requiring any transformation technique that could cause a distortion of the information; the possibility to deal with uncertainty or hesitation in the preference model, avoiding strong assumptions about the preferences on the DM's mind; and the non-compensatory aggregation approach of ELECTRE may be well suited for different decision problems.

Even though ELECTRE methods provide several advantages over other MCDA approaches, a major problem is found in the problem modelization process when DMs explicitly identify subgroups of problems. ELECTRE methods do not consider the criteria in a hierarchy structure, but a flat organization. This thesis has worked with the hypothesis that a problem modelization focused on sub-goals represented in a hierarchy tree, facilitates the understanding of complex problems and permits the definition of a richer model based on the DM's knowledge of the domain.

The first contribution is the proposal of a ranking method that extends the classical ELECTRE-III method for the case of a hierarchical structure of criteria, called ELECTRE-III-H.

The hierarchical organization of criteria in ELECTRE-III-H allows the DM to design the decision model in a decomposed way, by defining locally the parameters of the preference structure (i.e., indifference, preference and veto thresholds, as well relative weights). This method, unlike other related proposals, permits upward propagation of results, following the organization of the criteria in the hierarchy tree. New concordance and discordance measures have been defined to aggregate the partial pre-orders of the alternatives calculated at a lower level of the hierarchy. The proposal is based on the concept of *Rank Order Value* ($\Gamma(\cdot)$), which encompasses the four possible binary relations that can appear in a partial pre-order: Preference, Inverse Preference, Indifference and Incomparability $\{P, P^{-}, I, R\}$. An algorithm that propagates partial-preorders from the leaves up to the root of the hierarchy has been presented.

An extension of the ELECTRE-II for the case of hierarchies of criteria considering true-criteria instead of pseudo-criteria has also been presented. The partial concordance and discordance measures are similar to that of ELECTRE-III-H but using only the veto threshold.

The following works have been published in this topic:

- Luis Del Vasto-Terrientes, Aida Valls, Roman Slowinski, and Piotr Zielniewicz. ELECTRE-III-H: An outranking-based decision aiding method for hierarchically structured criteria. *Expert Systems with Applications*, 42(11):4910-4926, 2015. Impact Factor: 1.965 (Q1).
- Luis Del Vasto-Terrientes, Aida Valls, Roman Slowinski, Piotr Zielniewicz. Solving ranking problems with ELECTRE-III in case of hierarchical family of criteria. In *22nd International Conference on Multi-Criteria Decision Making (MCDM)*, 2013.
- Luis Del Vasto Terrientes, Aida Valls, Roman Slowinski, Piotr Zielniewicz. Extending Concordance and Discordance Relations to Hierarchical Sets of Criteria in ELECTRE-III Method. In Vicenç Torra, Yasuo Narukawa, Beatriz López, and Mateu Villaret, editors, Modeling Decisions for Artificial Intelligence - 9th International Conference, MDAI 2012, Girona, Catalonia, Spain, November 21-23, 2012. Proceedings, volume 7647 of Lecture Notes in Computer Science, pages 78–89. Springer, 2012. Core B.

The second contribution is the extension of the ELECTRE-TRI-B sorting method, called ELECTRE-TRI-B-H. This extension allows sorting alternatives at different levels in a hierarchy of criteria. In this way, we may assign the alternatives to preferentially-ordered categories at different intermediate nodes, which are related to different sub-problems. When a recommender system integrates multiple sources of information, this hierarchical procedure for sorting permits to have a data categorization from different perspectives. Finally integrating all of them into a unique global categorization.

The ELECTRE-TRI-B-H method allows the possibility of dealing with different sets of categories at each criterion. The propagation of information upwards in the hierarchy requires the definition of new concordance and discordance indices based on previous assignments instead of numerical evaluations, where classical ELECTRE-TRI-B formulations are used. In addition, profile limits at upper levels must be defined in terms of the categories of their descendants. The first issue has been solved with the definition of the *Category Improvement Value* function, for comparing alternatives and profile limits. Uncertainty in preference modeling and qualitative scales is handled by means of using pseudo-criteria, considering indifference and preference discrimination thresholds in terms of the *Category Improvement Value*.

Knowledge-based decision rules have been defined to construct the profile limits of a node in terms of the categories on the descendants. One of the criticisms of the aggregation by rules is that it is too complex for an expert to define a large set of rules manually. For this reason, the method is not based on rules applied like in expert systems, but rules are only used to automatically generate profile limits between the categories of an intermediate node and its parent. Few simple rules are needed in this case. For this purpose, rules have the advantage that are more informative than mathematical functions and suitable when dealing with qualitative scales.

The following works have been published in this topic:

- Luis Del Vasto-Terrientes, Aida Valls, Piotr Zielniewicz, Joan Borràs. A hierarchical multi-criteria sorting approach for recommender systems. *Journal of Intelligent Information Systems*, Accepted. DOI: 10.1007/s10844-015-0362-7. Impact Factor: 0.632 (Q3).
- Luis Del Vasto-Terrientes, Aida Valls, Piotr Zielniewicz. ELECTRE-TRI-B-H for solving hierarchically structured sorting problems. In *EURO Working Group on MCDA (EWG)*, Athens, Greece, 2014.

It can be seen that both ranking and sorting algorithm follow the same idea, making a similar analysis of the hierarchy of criteria, introducing new tools at non-elementary nodes but maintaining the classical method at the lowest level of the hierarchy.

The third main contribution of this work is the application of the hierarchical methods to 3 real case studies from different fields, showing the multidisciplinary character of the methods proposed.

The ELECTRE-III-H ranking method has been applied to an assessment system called Web Quality Index (WQI) and a decision support system for water allocation strategies in future scenarios of global change in the Mediterranean area in Tarragona, Catalonia, Spain.

In the first case, the combined methodology allows performing a personalized evaluation of each website, depending on the dimensions, objectives and priorities stated by the destination marketing organizations (DMO). It also permits pair-to-pair assessment once the DMO manager has identified the main competitors for his/her destination, discarding all non-comparable websites, and a more qualitative measuring system than simple numerical averaging, showing incompatibilities between websites because of their differential characteristics. This type of methods in decision support are being increasingly used in other fields, such as in the comparison of industrial products, transport or environmental issues. However, they still have not been used to analyze Web resources, probably due to the lack of a clear definition of the indicators that should be assessed and how to measure them.

In the second case study, the application of ELECTRE-III-H in a decision support system for the water allocation management in the Mediterranean area of Tarragona allows not only measuring the performance of the different alternatives under several conflicting points of view (economic and environmental issues) but also in a global assessment based on different models including optimistic, neutral and pessimistic scenarios.

The following works have been published in this topic:

- Luis Del Vasto-Terrientes, José Fernández-Cavia, Assumpsió Huertas, Antonio Moreno, Aida Valls. Official tourist destination websites: Hierarchical analysis and assessment with ELECTRE-III-H. *Tourism Management Perspectives*, 15:16-28, 2015. SCImago Journal Rank: 0.476 (Q2).

- Luis Del Vasto-Terrientes, Vikas Kumar, Tzu Chi Chao, Aida Valls. A decision support system to find the best water allocation strategies in a Mediterranean river basin in future scenarios of global change. *Journal of Experimental & Theoretical Artificial Intelligence*, DOI: 10.1080/0952813X.2015.1024493. Impact Factor: 0.527 (Q4).

- Tzu Chi Chao, Luis Del Vasto-Terrientes, Aida Valls, Vikas Kumar, Marta Schuhmacher. A hierarchical decision support system to evaluate the effects of climate change in water supply in a mediterranean river basin. In

Artificial Intelligence Research and Development - Recent Advances and Applications, CCIA, October 2014, Barcelona, Catalonia (Spain), pages 77–86, IOS Press, 2014.

- Vikas Kumar, Luis Del Vasto-Terrientes, Tzu Chi Chao, Aida Valls, Marta Schuhmacher. Application of outranking method to adaptation strategies for water supply management. In *Final SCARCE International Conference, River conservation under water scarcity: Integration of water quantity and quality in Iberian Rivers under global change*, October 2014, Tarragona, Spain, 2014.

In the case of sorting, the ELECTRE-TRI-B-H method has been integrated into the GoEno-Tur recommender system, a personalized recommender system of enotourism activities in the region of Tarragona. GoEno-Tur includes customer satisfaction ratings (from collaborative and content-based filtering), contextual features and tourism management priorities. This new approach to hierarchically sorting alternatives may have a significant impact in the quality of the recommended items on real-world applications. The results of this case study show the flexibility of the method and how this approach may help in the construction of this kind of systems. Moreover, partial assignments at intermediate levels can also be used as a tool to refine the proposals selected to be shown to the user.

The study of the ELECTRE-TRI-B-H integration into the GoEno-Tur recommender system has been accepted for publication in the paper "A hierarchical multi-criteria sorting approach for recommender systems. Journal of Intelligent Information Systems", previously mentioned.

On the basis of the research done in this thesis, we can state the following conclusions.

1. The hierarchical modelization of criteria is necessary in MCDA when complex and diverging criteria are considered. Hierarchical decision aiding methods should permit:
   - A more natural, flexible and comprehensible modeling of the criteria accordingly to the DM's needs and interests.
   - The DM to prioritize criteria in terms of relative importance in a more natural way, defining at each level of the hierarchy the importance of each criterion with respect to only related criteria;
   - At intermediate levels of criteria, to set parameters to model preferences, including zones of uncertainty and hesitations (i.e., pseudo-criteria and veto thresholds);
   - A detailed analysis of the results at different levels of the hierarchy, enhancing the DM's knowledge about global result.

2. Hierarchies of criteria, which are naturally conceived in human decision making, are easily understood and constructed by DMs. Moreover, it is a generic concept with application in any field of study. Hence, taking into account the multidisciplinary character of ELECTRE methods, the hierarchical methods proposed in this thesis may be applied to any field where decision problems involves multiple criteria and a finite set of alternatives. This has been illustrated with their application to 3 real case studies from different fields. The hierarchies of criteria have shown to be very useful in the problem modeling, defining sub-groups of related elements based on the DM's knowledge in the domain.

3. The different applications also show that the proposed hierarchical methods does not constraint the hierarchy modeling to a particular form, including size or number of levels. For instance, the analysis of the assessment of tourist destination websites deals with 10 alternatives evaluated on big hierarchy with up to 18 sub-problems in 4 levels, while the IEDSS and the GoEno-Tur deal with a small hierarchy of 3 sub-problems and 3 levels each, but considering a bigger set of alternatives (i.e., 48 and 279 respectively).

4. For decision problems involving a large set of alternatives, the assignment of the alternatives to a category via ELECTRE-TRI-B-H is more adequate than the construction of partial pre-orders generated by the ELECTRE-III-H method. Ranking methods involves the binary comparison of each pair of alternatives for all criteria, whereas sorting compares each alternative directly to the profile limits, reducing the computational cost. This computational cost is calculated as follows:

    - ELECTRE-III-H: $O(n^2 \times m)$,
    - ELECTRE-TRI-B-H: $O(n \times m \times \mathcal{B})$,

    where m is the number of criteria, n the number of alternatives and $\mathcal{B} = \sum_{\forall j,} |B^j|$ (i.e., the summation of profile limits).

    Then, ELECTRE-III-H has a quadratic complexity due to the binary comparison of alternatives, while the ELECTRE-TRI-B-H has a linear complexity.

7.1 Future work

The work presented in this thesis is a contribution in the exploration of the problem presented. We believe this is an interesting and relevant field of research. Several directions of future work have been identified during this work, presented as follows:

1. Study the adaptation of an interactive tool to assist DMs in the definition of the preference thresholds for non-elementary criteria in ELECTRE-III-H and ELECTRE-TRI-B-H. The parameter elicitation is generally one of the most difficult stages in the construction of the decision problem modeling. Even if the DM has a strong knowledge of the domain and takes for granted that the parameters provided are adequate for the model, it is difficult to have a clear understanding of their consequences in the final recommendation, because hierarchical relations among the criteria imply that the final recommendation at $\mathcal{R}$ is directly affected by the partial results of their predecessors in the tree. Thus, for this type of decision problems considering hierarchies of criteria, interactive tools to assist DMs are relevant to provide robust and trustful recommendations. In this regard, a bottom-up interactive tool may be suitable based on examples (indirect elicitation). First, this tool may help the DM to define threshold parameters based on examples given by the DM and weights based on the Simos' procedure for subsets of elements in $\mathcal{E}$. Then, at an upper level in the hierarchy, depending on the decision problem (i.e., ranking or sorting), the DM may provide more examples focused on the parts of the problems. For ranking, the DM may provide examples of alternative preferences (i.e., $a \succ b$), which can be deduced by the DM from the partial pre-orders obtained at the lower level; whereas for sorting, decision rules (modeling the profile limits) may be detected from the previous assignments of the alternatives. For instance, if the DM provides an example in which $a \rightarrow C_2^j$, the rules may be defined based on the assignments of a on the predecessors of g_j.

2. An extension of the ELECTRE-III-H and ELECTRE-TRI-B-H methods for the case of group decision making (more than one stakeholder) would also be worth to study. In group decision making (GDM), an additional stage of consensus is required. Usually this consensus is achieved from an iterative process of analysis of several recommendations provided by the MCDA method, until the different members accept one. Generally, the GDM consensus is achieved considering that all the stakeholders (DMs) involved in the decision problem analyzes all the alternatives and criteria. However, taking into consideration that a hierarchy of criteria provides a specialization of criteria, grouping them into subsets of related criteria, each stakeholder may provide its individual preferences on their field of expertise (sub-problems), resulting in a partial results in terms of partial pre-orders (ranking) or alternative assignments (sorting). If more than 1 stakeholder are participating in the particular sub-problem, several iterations could be required, allowing each participant to change the parameters until some consensus of the sub-problem is reached. Then, at upper levels of the hierarchy, a consensus of the partial results (i.e., partial pre-order or alternative assignments) obtained from the different sub-problems is required. This shall lead to a new

iteration stage where the DMs must analyze the parameters of the partial results (i.e., in terms of rank order value or category improvement value), until a consensus is reached. Therefore, a bottom-up iterative approach, requiring a level by level consensus of the DMs until a global consensus is reached in the root would be an interesting line of future research.

3. A possible line of study is the extension of the ELECTRE-TRI-C method, adapting the concept of central reference actions instead of profile limits, applied for ELECTRE-TRI-B-H. In ELECTRE-TRI-C, each category is defined by a representative central action, called characteristic reference action. ELECTRE TRI-C was originally designed for decision problems where the definition of profile limits are difficult for the DM because sometimes, these boundaries may not have an objective existence. In ELECTRE-TRI-B-H, the aggregation of assignments is defined by decision rules as the assignments of the alternatives are a priori unknown. However, the ELECTRE-TRI-C method would require at each level of the hierarchy a reference action. Therefore, a possibility to define reference actions at each level of the hierarchy is a hierarchical level-by-level bottom-up approach, defining reference actions for each category at each level of the hierarchy once the assignments are given based on the previous assignments of the alternatives on the predecessors.

4. Study the extension of the ELECTRE-IS method considering hierarchies of criteria for choice problems. This thesis is focused on ranking and sorting problems because in the ELECTRE literature most real-world case studies apply ELECTRE-III and ELECTRE-TRI-B. Also, most of the recent methodologies and theoretical contributions are addressed to these types of decision problems. However, to close the gap of hierarchical decision problems applying ELECTRE methods, the study of ELECTRE-IS, which is the most "complete" choice method as it contemplates pseudo-criteria, is an interesting line of future research. Considering that different subsets of alternatives may be chosen at each sub-problem, it may difficult the construction of outranking relations as ELECTRE methods do not allow "null" alternative evaluations. For instance, considering $g_{j.1}$ and $g_{j.2}$, if alternatives a and b are chosen on $g_{j.1}$ and only alternative a is chosen in $g_{j.2}$, the classical construction of the outranking relations is not feasible. An idea for this problem may be the consideration of pseudo-criteria parameters in terms of the number of elements in $\mathcal{I}$ where alternative a has been chosen. For instance, considering 3 elements in $\mathcal{I}$, $g_{j.1}$, $g_{j.2}$ and $g_{j.3}$, where g_j is the direct parent; the DM may consider an indifference of 1. Then, if alternative a has been chosen on all the predecessors and alternative b has been chosen on 2 predecessors, we may conclude that aSb and depending on the relative importance of the 2 predecessors where alternative b has been chosen, we can also calculate the outranking relation supporting bSa.

As seen, the modeling of hierarchical structures of criteria for outranking methods opens a wide range of options to be considered for future research, including its application for choice problems and interactive tools enhancing the DM's analysis.

Bibliography

Abedi M, Torabi SA, Norouzi GH, Hamzeh M (2012) ELECTRE III: A knowledge-driven method for integration of geophysical data with geological and geochemical data in mineral prospectivity mapping. Journal of Applied Geophysics 87:9–18

Afsordegan A, Sánchez M, Agell N, Aguado J, Gamboa G (2014) A comparison of two MCDM methodologies in the selection of a windfarm location in Catalonia. In: Museros L, Pujol O, Agell N (eds) Artificial intelligence research and development, IOS Press, no. 269 in Recent advances and applications, pp 227–236

Ahsan M, Bartlema J (2004) Monitoring healthcare performance by analytic hierarchy process: a developing-country perspective. International Transactions in Operational Research 11:465–478

Allen T, Starr T (1988) Hierarchy: Perspectives for Ecological Complexity. University of Chicago Press

Arbenz P, Hummel C, Mainik G (2012) Copula based hierarchical risk aggregation through sample reordering. Insurance: Mathematics and Economics 51:122–133

Arondel C, Girardin P (2000) Sorting cropping systems on the basis of their impact on groundwater quality. European Journal of Operational Research 127:467–482

Arrow K (1963) Social Choice and Individual Values. Cowles Foundation Monographs Series, Yale University Press

Aydin S, Kahraman C, Kaya I (2012) A new fuzzy multicriteria decision making approach: An application for European Quality Award assessment. Knowledge-Based Systems 32:37–46

Bana e Costa C, Vansnick J (2008) A critical analysis of the eigenvalue method used to derive priorities in AHP. European Journal of Operational Research 187(3):1422 – 1428

Bana e Costa C, Ensslin L, Correa E, Vansnick J (1999) Decision Support Systems in action: Integrated application in a multicriteria decision aid process . European Journal of Operational Research 113(2):315 – 335

Bangash R, Passuello A, Hammond M, Schuhmacher M (2012) Water allocation assessment in low flow river under data scarce conditions: A study of hydrological simulation in Mediterranean basin . Science of The Total Environment 440:60 – 71, integrated modelling and monitoring at different river basin scales under global change

Bastida U, Huan T (2014) Performance evaluation of tourism websites' information quality of four global destination brands: Beijing, Hong Kong, Shanghai, and Taipei . Journal of Business Research 67(2):167 – 170

Belton V, Stewart T (2002) Multiple Criteria Decision Analysis: An Integrated Approach. Springer US

Bobylev N (2011) Comparative analysis of environmental impacts of selected underground construction technologies using the analytic network process. Automation in Construction 20:1030–1040

Borràs J, Moreno A, Valls A, Ferré M, Ciurana E, Salvat J, Russo A, Anton-Clavé S (2012a) Uso de técnicas de inteligencia artificial para hacer recomendaciones enoturística personalizadas en la provincia de tarragona. In: IX Congreso Nacional de Turismo y Tecnologías de la Información y las Comunicaciones (TURITEC), pp 217–230

Borràs J, Valls A, Moreno A, Isern D (2012b) Ontology-based management of uncertain preferences in user profiles. In: 14th International Conference on Information Processing and Management of Uncertainty in Knowledge-based Systems (IPMU), pp 127–136

Bouyssou D (2009) Outranking methods outranking methods. In: Floudas CA, Pardalos PM (eds) Encyclopedia of Optimization, Springer US, pp 2887–2893

Bouyssou D, Pirlot M (2009) An axiomatic analysis of concordance–discordance relations. European Journal of Operational Research 199(2):468 – 477

Bouyssou D, Pirlot M, Vincke P (1997) A General Model of Preference Aggregation. In: Karwan MH, Spronk J, Wallenius J (eds) Essays In Decision Making, Springer Berlin Heidelberg, pp 120–134

Bouyssou D, Marchant T, Pirlot M, Tsoukias A, Vincke P (2006) Evaluation and Decision Models with Multiple Criteria Stepping stones for the analyst. International Series in Operations Research & Management Science, Springer, 448 pages

Bouyssou D, Marchant T, Perny P (2010) Social Choice Theory and Multicriteria Decision Aiding, ISTE, pp 779–810

Brans J (1982) L'ingénièrie de la décision; Elaboration d'instruments d'aide à la décision. La méthode PROMETHEE. In: Nadeau R, Landry M (eds) L'aide à la décision: Nature, Instruments et Perspectives d'Avenir, Presses de l'Université Laval, Québec, Canada, pp 183–213

Brans J, Vincke P (1985) A preference ranking organization method. Management Science 31(6):647–656

Brito A, de Almeida A, Mota C (2010) A multicriteria model for risk sorting of natural gas pipelines based on ELECTRE TRI integrating Utility Theory. European Journal of Operational Research 200:812–821

Buyukozkan G, Çifçi G, Guleryuz S (2011) Strategic analysis of healthcare service quality using fuzzy AHP methodology. Expert Systems with Applications 38:9407–9424

Cailloux O, Meyer P, Mousseau V (2012) Eliciting Electre Tri category limits for a group of decision makers. European Journal of Operational Research 223:133–140

Carlsen L (2015) Partial ordering as decision support to evaluate remediation technologies. AIMS Environmental Science 2:110–121

Casagrandi R, Guariso G (2009) Impact of ICT in Environmental Sciences: A citation analysis 1990-2007. Environmental Modelling & Software 24(7):865 – 871

Chang B, Kuo C, Wu C, Tzeng G (2015) Using Fuzzy Analytic Network Process to assess the risks in enterprise resource planning system implementation . Applied Soft Computing 28:196 – 207

Chao TC, Terrientes LDV, Valls A, Kumar V, Schuhmacher M (2014) A hierarchical decision support system to evaluate the effects of climate change in water supply in a mediterranean river basin. In: Cabedo LM, Pujol O, Àgell N (eds) Artificial Intelligence Research and Development - Recent Advances and Applications, CCIA 2014, October 2014, Barcelona, Catalonia, Spain, IOS Press, Frontiers in Artificial Intelligence and Applications, vol 269, pp 77–86

Chen S, Liu J, Wang H, Augusto J (2013) Ordering based decision making - A survey . Information Fusion 14(4):521 – 531

Chen S, Liu J, Wang H, Xu Y, Augusto JC (2014) A linguistic multi-criteria decision making approach based on logical reasoning. Information Sciences 258:266–276

Cho K (2003) Multicriteria decision methods: An attempt to evaluate and unify. Mathematical and Computer Modelling 37(9–10):1099 – 1119

Choi S, Lehto XY, Oleary J (2007) What does the consumer want from a dmo website? a study of us and canadian tourists' perspectives. International Journal of Tourism Research 9(2):59–72

Cloquell-Ballester V, Monterde-Díaz R, Cloquell-Ballester V, Santamarina-Siurana M (2007) Systematic comparative and sensitivity analyses of additive and outranking techniques for supporting impact significance assessments. Environmental Impact Assessment Review 27:62–83

Colson G (2000) The OR's prize winner and the software ARGOS: How a multijudge and multicriteria ranking GDSS helps a jury to attribute a scientific award. Computers and Operations Research 27:741–755

Corrente S, Greco S, Slowinski R (2012) Multiple criteria hierarchy process in robust ordinal regression. Decision Support Systems 53:660–674

Corrente S, Greco S, Kadzinski M, Slowinski R (2013a) Robust ordinal regression in preference learning and ranking. Machine Learning 93(2-3):381–422

Corrente S, Greco S, Slowinski R (2013b) Multiple Criteria Hierarchy Process with ELECTRE and PROMETHEE. Omega 41:820–846

Damaskos X, Kalfakakou G (2005) Application of ELECTRE III and DEA Methods in the BRP of a Bank Branch Network. Yugoslav Journal of Operations Research 15(2):259–276

Davenport H (1960) Plans and the structure of behavior. by george a. miller, eugene galanter and karl h. pribram 1960. henry holt and company, new york. 226 pp. The Journal of Comparative Neurology 115(2):217–217

Del Vasto-Terrientes L, Valls A, Slowinski R, Zielniewicz P (2012) Extending Concordance and Discordance Relations to Hierarchical Sets of Criteria in ELECTRE-III Method. In: Torra V, Narukawa Y, López B, Villaret M (eds) Modeling Decisions for Artificial Intelligence - 9th International Conference, MDAI 2012, Girona, Catalonia, Spain, November 21-23, 2012. Proceedings, Springer, Lecture Notes in Computer Science, vol 7647, pp 78–89

Del Vasto-Terrientes L, Fernández-Cavia J, Huertas A, Moreno A, Valls A (2015a) Official tourist destination websites: Hierarchical analysis and assessment with ELECTRE-III-H . Tourism Management Perspectives 15:16 – 28

Del Vasto-Terrientes L, Kumar V, Chao T, Valls A (2015b) A decision support system to find the best water allocation strategies in a Mediterranean river basin in future scenarios of global change. Journal of Experimental & Theoretical Artificial Intelligence p in press

Dias L, Domingues A (2014) On multi-criteria sustainability assessment: Spidergram surface and dependence biases. Applied Energy 113(0):159 – 163

Doumpos M, Grigoroundis E (2013) Multicriteria Decision Aid and Artificial Intelligence. Wiley

Dubois D, Fargier H, Perny P, Prade H (2003) A characterization of generalized concordance rules in multicriteria decision making. International Journal of Intelligent Systems 18(7):751–774

Dujmović J, De Tré G (2011) Multicriteria methods and logic aggregation in suitability maps. International Journal of Intelligent Systems 26(10):971–1001

Dujmovic J, De Tré G, Van de Weghe N (2010) LSP suitability maps. Soft Computing 14:421–434

Dyer J (2005) MAUT - Multiattribute Utility Theory. In: Multiple Criteria Decision Analysis: State of the Art Surveys, International Series in Operations Research & Management Science, vol 78, Springer New York, pp 265–292

Fernández-Cavia J, Rovira C, Díaz-Luque P, Cavaller V (2014) Web Quality Index (WQI) for official tourist destination websites. Proposal for an assessment system . Tourism Management Perspectives 9:5 – 13

Figueira J, Roy B (2002) Determining the weights of criteria in the ELECTRE type methods with a revised Simos' procedure . European Journal of Operational Research 139(2):317 – 326

Figueira J, Roy B (2009) A note on the paper, "Ranking irregularities when evaluating alternatives by using some ELECTRE methods", by Wang and Triantaphyllou, Omega (2008) . Omega 37(3):731 – 733

Figueira J, Greco S, Ehrgott M (2005) Multiple Criteria Decision Analysis: State of the Art Surveys, vol 78. Springer

Figueira J, Greco S, Roy B, Slowinski R (2013) An Overview of ELECTRE Methods and their Recent Extensions. Journal of Multi-Criteria Decision Analysis 20(1-2):61–85

Franceschini F, Maisano D, Mastrogiacomo L (2015) A novel algorithm for fusing preference orderings by rank-ordered agents. Fuzzy Sets and Systems 266:84 – 100

Gibert K, Rodríguez-Silva G, Rodríguez-Roda I (2010) Knowledge discovery with clustering based on rules by states: A water treatment application. Environmental Modelling & Software 25(6):712 – 723

Grabisch M (1996) The application of fuzzy integrals in multicriteria decision making. European Journal of Operational Research 89(3):445 – 456

Grabisch M, Marichal J, Mesiar R, Pap E (2011) Aggregation functions: Means. Information Sciences 181(1):1 – 22

Greco S, Matarazzo B, Slowinski R (2001a) Axiomatic basis of noncompensatory preferences, communication to *Foundations of Utility and Risk Theory, FUR X*, 30 May–2 June, Torino, Italy

Greco S, Matarazzo B, Slowinski R (2001b) Rough sets theory for multicriteria decision analysis. European Journal of Operational Research 129(1):1 – 47

Greco S, Ehrgott M, Figueira J (2010) Trends in Multiple Criteria Decision Analysis. Springer, Berlin

Gretzel U, Fesenmaier D, Formica S, O'Leary J (2006) Searching for the future: Challenges faced by destination marketing organizations. Journal of Travel Research 45(2):116–126

Hatch K, Dragićević S, Dujmović J (2014) Logic scoring of preference and spatial multicriteria evaluation for urban residential land use analysis. In: Duckham M, Pebesma E, Stewart K, Frank AU (eds) Geographic Information Science, Lecture Notes in Computer Science, vol 8728, Springer International Publishing, pp 64–80

Henry W (1969) Hierarchical structure for data management. IBM Systems Journal 8(1):2–15

Hsu T, Pan F (2009) Application of Monte Carlo AHP in ranking dental quality attributes. Expert Systems with Applications 36:2310–2316

Ip C, Law R, Lee H (2011) A review of website evaluation studies in the tourism and hospitality fields from 1996 to 2009. International Journal of Tourism Research 13(3):234–265

Ishizaka A, Labib A (2011) Review of the main developments in the analytic hierarchy process. Expert Systems with Applications 38(11):14,336 – 14,345

Ishizaka A, Nemery P (2013a) Electre. In: Multi-Criteria Decision Analysis, John Wiley & Sons Ltd, pp 180–211

Ishizaka A, Nemery P (2013b) Multi-attribute utility theory. In: Multi-Criteria Decision Analysis, John Wiley & Sons Ltd, pp 81–113

Ishizaka A, Pearman C, Nemery P (2012) AHPSort: an AHP-based method for sorting problems. International Journal of Production Research 50(17):4767–4784

Keeney R (1977) The art of assessing multiattribute utility functions. Organizational Behavior and Human Performance 19(2):267 – 310

Keeney R, Raiffa H (1993) Decisions with Multiple Objectives: Preferences and Value Trade-Offs. Wiley series in probability and mathematical statistics. Applied probability and statistics, Cambridge University Press

Kilic H, Zaim S, Delen D (2015) Selecting The Best ERP system for SMEs using a combination of ANP and PROMETHEE methods . Expert Systems with Applications 42(5):2343 – 2352

Kou G, Shi Y, Wang S (2011) Multiple criteria decision making and decision support systems – guest editor's introduction. Decision Support Systems 51(2):247 – 249

Kozielecki J (1981) Psychological Decision Theory. Astrophysics and Space Science Library, Springer

Kumar V, Del Vasto-Terrientes L, Chao T, Valls A, Schuhmacher M (2014) Application of outranking method to adaptation strategies for water supply management. In: Final SCARCE International Conference, River conservation under water scarcity: Integration of water quantity and quality in Iberian Rivers under global change, Tarragona, Spain

Law R, Qi S, Buhalis D (2010) Progress in tourism management: A review of website evaluation in tourism research. Tourism Management 31:297–313

Lee W, Gretzel U (2012) Designing persuasive destination websites: A mental imagery processing perspective. Tourism Management 33(5):1270 – 1280

Li D, Liu C, Gan W (2009) A new cognitive model: Cloud model. International Journal of Intelligent Systems 24:357–375

Li X, Wang Y (2010) Evaluating the effectiveness of destination marketing organisations' websites: evidence from china. International Journal of Tourism Research 12(5):536–549

Li X, Wang Y (2011) Measuring the effectiveness of us official state tourism websites. Journal of Vacation Marketing 17(4):287–302

Luna-Nevarez C, Hyman M (2012) Common practices in destination website design. Journal of Destination Marketing and Management 1(1-2):94–106

Mardani A, Jusoh A, Zavadskas E (2015) Fuzzy multiple criteria decision-making techniques and applications - Two decades review from 1994 to 2014. Expert Systems with Applications 42(8):4126 – 4148

Marquès M, Bangash R, Kumar V, Sharp R, Schuhmacher M (2013) The impact of climate change on water provision under a low flow regime: A case study of the ecosystems services in the Francoli river basin . Journal of Hazardous Materials 263:224 – 232

Matsatsinis N, Doumpos M, Zopounidis C (1997) Knowledge acquisition and representation for expert systems in the field of financial analysis. Expert Systems with Applications 12:247–262

May K (1952) A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision. Econometrica 20(4):680–684

Mendis B, Gedeon T (2012) Complex Structured Decision Making Model: A hierarchical frame work for complex structured data. Information Sciences 194:85–106

Moreno A, Valls A, Isern D, Marin L, Borràs J (2013) SigTur/E-Destination: Ontology-based personalized recommendation of Tourism and Leisure Activities. Engineering Applications of Artificial Intelligence 26:633–651

Mousseau V, Slowinski R, Zielniewicz P (2000) A user-oriented implementation of the ELECTRE-TRI method integrating preference elicitation support. Computers and Operations Research 27:757–777

Muerza V, De Arcocha D, Larrodé E, Moreno-Jiménez J (2014) The multicriteria selection of products in technological diversification strategies: An application to the Spanish automotive industry based on AHP. Production Planning and Control 25(8):715–728

Mustajoki J (2012) Effects of imprecise weighting in hierarchical preference programming. European Journal of Operational Research 218:193–201

Nehring K, Pivato M, Puppe C (2014) The condorcet set: Majority voting over interconnected propositions. Journal of Economic Theory 151(0):268 – 303

Nordstrom E, Eriksson L, Ohman K (2010) Integrating multiple criteria decision analysis in participatory forest planning: Experience from a case study in northern Sweden. Forest Policy and Economics 12:562–574

Nzila C, Dewulf J, Spanjers H, Tuigong D, Kiriamiti H, van Langenhove H (2012) "Multi criteria sustainability assessment of biogas production in Kenya ". Applied Energy 93(0):496 – 506

Papadopoulos A, Karagiannidis A (2008) Application of the multi-criteria analysis method Electre III for the optimisation of decentralised energy systems. Omega 36:766–776

Pijuan J, Valls A, Passuello A, Schuhmacher M (2010) Evaluating the impact of sewage sludge application on agricultural soils. In: XV Congreso español sobre tecnologías y lógica fuzzy (ESTYLF), Huelva, Spain, pp 369 – 374

Pirlot M (1997) A common framework for describing some outranking methods. Journal of Multi-Criteria Decision Analysis 6(2):86–92

Reddy B, Kelly M, Thokala P, Walters S, Duenas A (2014) Prioritising public health guidance topics in the National Institute for Health and Care Excellence using the Analytic Hierarchy Process. Public Health 128(10):896 – 903

Risse M (2005) Why the count de Borda cannot beat the Marquis de Condorcet. Social Choice and Welfare 25(1):95–113

Rocha C, Dias LC, Dimas I (2013) Multicriteria classification with unknown categories: A clustering-sorting approach and an application to conflict management. Journal of Multi-Criteria Decision Analysis 20(1-2):13–27

Rogers M, Bruen M (1998) Choosing realistic values of indifference, preference and veto thresholds for use with environmental criteria within ELECTRE. European Journal of Operational Research 107:542–551

Roy B (1968) Classement et choix en présence de points de vue multiples. RAIRO - Operations Research - Recherche Operationnelle 2(V1):57–75

Roy B (1972) How Outranking Relation Helps Multiple Criteria Decision Making. Rapport de recherche, SEMA (Metra International), Direction Scientifique

Roy B (1991) The outranking approach and the foundations of electre methods. Theory and Decision 31(1):49–73

Roy B (1996) Multicriteria Methodology for Decision Aiding. Nonconvex Optimization and Its Applications, Springer

Roy B (2005) Paradigms and challenges. In: Multiple Criteria Decision Analysis: State of the Art Surveys, International Series in Operations Research & Management Science, vol 78, Springer New York, pp 3–24

Roy B (2010) Two conceptions of decision aiding. International Journal of Multicriteria Decision Making 1(1):74–79

Roy B, Slowinski R (2013) Questions guiding the choice of a multicriteria decision aiding method. EURO Journal on Decision Processes 1(1-2):69–97

Saaty R (1987) The analytic hierarchy process-what it is and how it is used. Mathematical Modelling 9(3 - 5):161 – 176

Saaty T, Sagir M (2009) An essay on rank preservation and reversal. Mathematical and Computer Modelling 49(5–6):1230 – 1243

Sánchez-Lozano J, Teruel-Solano J, Soto-Elvira P, García-Cascales M (2013) Geographical Information Systems (GIS) and Multi-Criteria Decision Making (MCDM) methods for the evaluation of solar farms locations: Case study in south-eastern Spain . Renewable and Sustainable Energy Reviews 24(0):544 – 556

Sánchez-Lozano J, Henggeler Antunes C, García-Cascales M, Dias L (2014) GIS-based photovoltaic solar farms site selection using ELECTRE-TRI: Evaluating the case for Torre Pacheco, Murcia, Southeast of Spain . Renewable Energy 66(0):478 – 494

Sànchez-Marrè M, Gibert K, Sojda R, Steyer J, Struss P, Rodríguez-Roda I, Comas J, Brilhante V, Roehl E (2008) Chapter eight intelligent environmental decision support systems. In: AJ Jakeman AR AA Voinov, Chen S (eds) Environmental Modelling, Software and Decision Support, Developments in Integrated Environmental Assessment, vol 3, Elsevier, pp 119 – 144

Sen A (1970) Collective Choice and Social Welfare. San Fransisco: Holden-Ray

Shanian A, Milani AS, Carson C, Abeyaratne RC (2008) A new application of ELECTRE III and revised Simos' procedure for group material selection under weighting uncertainty. Knowledge-Based Systems 21:709–720

Shen Y, Hermans E, Brijs T, Wets G (2012) Data Envelopment Analysis for Composite Indicators: A Multiple Layer Model. Social Indicators Research 114(2):739–756

Shim J, Warkentin M, Courtney J, Power D, Sharda R, Carlsson C (2002) Past, present, and future of decision support technology. Decision Support Systems 33(2):111 – 126

Tang L, Jang S (2012) Investigating the routes of communication on destination websites. Journal of Travel Research 51(1):94–108

Torres-Salinas D, Moreno-Torres J, Delgado-López-Cózar E, Herrera F (2011) A methodology for Institution-Field ranking based on a bidimensional analysis: The IFQ2A index. Scientometrics 88:771–786

Tsoukiàs A (2007) On the concept of decision aiding process: an operational perspective. Annals of Operations Research 154(1):3–27

Valls A, Pijuan J, Schuhmacher M, Passuello A, Nadal M, Sierra J (2010) Preference assessment for the management of sewage sludge application on agricultural soils. Multicriteria Decision Making 1:4–24

Vanderpooten D (1989) The interactive approach in MCDA: A technical framework and some basic conceptions . In: Colson G, Rodin E, de Bruyn C (eds) Models and Methods in Multiple Criteria Decision Making, International Series in Modern Applied Mathematics and Computer Science, Pergamon, Amsterdam, pp 1213 – 1220

Vanderpooten D (1990) The construction of prescriptions in outranking methods. In: Bana e Costa, Carlos (ed) Readings in Multiple Criteria Decision Aid, Springer, Berlin, pp 184–215

Vargas L (1994) Reply to Schenkerman's avoiding rank reversal in AHP decision support models . European Journal of Operational Research 74(3):420 – 425

Vincke P (1992) Multicriteria decision-aid. Wiley

Wang G, Huang S, Dismukes J (2004) Product-driven supply chain selection using integrated multi-criteria decision-making methodology. International Journal of Production Economics 91:1–15

Wang J, Peng L, Zhang H, Xiao-hong C (2014) Method of multi-criteria group decision-making based on cloud aggregation operators with linguistic information. Information Sciences 1:177–191

Wang X, Triantaphyllou E (2008) Ranking irregularities when evaluating alternatives by using some ELECTRE methods . Omega 36(1):45 – 63, special Issue Section: Papers presented at the INFORMS conference, Atlanta, 2003

Wang Y, Luo Y (2009) On rank reversal in decision analysis. Mathematical and Computer Modelling 49(5–6):1221 – 1229

Webster J (1979) Chapter 5 - HIERARCHICAL ORGANIZATION OF ECOSYSTEMS. In: Halfon E (ed) Theoretical Systems Ecology, Academic Press, pp 119 – 129

Xidonas P, Mavrotas G, Psarras J (2009) A multicriteria methodology for equity selection using financial analysis. Computers and Operations Research 36:3187–3203

Xu B, Ouenniche J (2012) Performance evaluation of competing forecasting models: A multidimensional framework based on MCDA. Expert Systems with Applications 39:8312–8324

Yager R (1988) On Ordered Weighted Averaging Aggregation Operators in Multicriteria Decision Making. IEEE Trans Syst Man Cybern 18(1):183–190

Yager R (2001) Fusion of multi-agent preference orderings. Fuzzy Sets and Systems 117(1):1 – 12

Yang C, Chuang S, Huang R (2009) Manufacturing evaluation system based on AHP/ANP approach for wafer fabricating industry. Expert Systems with Applications 36:11,369–11,377

Yu W (1992) ELECTRE TRI : Aspects méthodologiques et manuel d'utilisation. Document du LAMSADE, No.74, Université Paris-Dauphine

Zeleny M (1982) Multiple Criteria Decision Making. McGraw-Hill Series in Quantitative Methods for Management, McGraw-Hill