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# Departament de Física

## Grup de Física Teòrica

Quantum effects on some low and high energy  
processes beyond the Standard Model

Memòria presentada per José Antonio Coarasa Pérez  
per a optar al grau de Doctor en Ciències Físiques per  
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CERTIFICA: Que la present memòria, que porta per títol "Quantum effects on some low and high energy processes beyond the Standard Model", ha estat realitzada sota la meua direcció per en José Antonio Coarasa Pérez i que constitueix la seva Tesis per a optar al grau de Doctor en Ciències Físicas per la Universitat Autònoma de Barcelona.

Bellaterra, abril de 1998

Joan Solà i Peracaula



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# Chapter 1

## Introduction

Elementary particle physics has made remarkable progress in the past two decades, both experimentally and theoretically. Since, the Standard Model (SM) was proposed [1–4] as the unified Theory –describing Quantum Electrodynamics, Quantum Chromodynamics and the Electroweak interactions– till its crowning with the discovery of its penultimate building block of its theoretical structure, i.e. the top quark,  $t$  [5, 6], its success describing particle physics phenomenology has been overwhelming. At a purely theoretical level the top quark existence prediction –on the grounds of requiring gauge invariance and renormalizability – since the very confirmation of the existence of the bottom quark and the measurement of its weak isospin quantum numbers [7] stood as a prove of internal consistency. Nowadays, there is no experiment which definitely contradicts this model. The only lacking building block of the SM is the Higgs boson and thus the mechanism for generating all the masses in the SM remains as yet unconfirmed. Hence, it is not clear at present whether the SM will be the last word in the phenomenology of strong and electroweak processes at the Fermi’s scale or whether it will be subsumed within a larger and more fundamental theory. This is also why our interest will be focussed in the SSB sector of the theory.

Moreover, from a theoretical point of view, the SM cannot be considered as the ultimate theory because many questions remain unanswered, among them: Why are there three independent groups  $SU(3)$ ,  $SU(2)$  and  $U(1)$ ? How can we reduce the number of free parameters? Why are there three generations of quarks and leptons? What is the origin of the symmetry

between quarks and leptons? Why do protons and electrons have exactly opposite charges? What is the origin of the hierarchy among fermion masses? . . . What's more, the SM also lacks a quantum theory of gravity.

On top of this, the recent indications for the possibility of neutrino oscillations [8–10] gives further support to the idea that the SM could be subsumed within a larger and more fundamental theory.

A model introduced to solve any of these questions will surely bring new phenomena not present in the SM phenomenology. However, the extraordinary coincidence between the SM predictions and the high precision electroweak data [11] leaves little room for such a new theory: only those models that could mimic the SM results in a low energy limit are acceptable. Among the different Theories and models to try to solve the different “drawbacks” of the SM that specially offer clues to solve the nature of the spontaneous symmetry breaking (SSB) mechanism one envisages:

- The “Top Mode” realization(s) of the SSB mechanism, i.e. SSB without fundamental Higgs bosons, but through the existence of  $t\bar{t}$  condensates [12];
- The extended Technicolor Models, where again the fundamental Higgs scalars are substituted by condensates of more fundamental particles interacting with technicolor strong forces [13, 14];
- The non-linear (chiral Lagrangian) realization of the the  $SU(2)_L \times U(1)_Y$  gauge symmetry [15, 16], which may either accommodate or dispense with the Higgs scalars;
- The supersymmetric realization of the SM, such as the Minimal Supersymmetric Standard Model (MSSM) [17–20], where a lot of new phenomenology may pop up. Hints of this new phenomenology may show up either in the form of direct or virtual effects from supersymmetric Higgs particles or from the “sparticles” themselves (i.e. the  $R$ -odd [17–19, 21] partners of the SM particles). In fact  $R$ -parity violation has become an important issue in itself lately [22–25];
- The different extensions of the Higgs sector of the Standard Model, among which we pinpoint:

- 
- The two-Higgs-doublet models (2HDM), that stand in many extensions of the SM, including the MSSM, and provide “solutions” to some of the “drawbacks” of the SM: The Strong CP problem [26], the possibility to generate sufficient CP violation to accommodate for the observed barionic asymmetry [27–29];
  - Models with Left-Right symmetry [30–34], where the left-handed neutrinos acquire a Majorana mass through the Higgs mechanism, so that through the see-saw mechanism neutrino oscillations can be explained.

In this Thesis, we will focus on the last three ways of extending the SM, and specially on the minimal supersymmetric extension of the SM, the MSSM, since it is a fully-fledged Quantum Field Theory. Moreover, from the experimental point of view it fully accommodates for the direct and indirect precision data [35] and is deserving a lot of attention nowadays due to the fact that if supersymmetry is there its effects would have to crop up in future experiments such as the Tevatron, the LHC, or LEP II. In chapter 2 we briefly give the main motivation for these models as well as the necessary notation and conventions to develop further calculations later.

On the other hand, the calculation of radiative corrections [36, 37] has played and still plays [35] an important role in the falsation of models. Loop calculations involve the full parameter set (masses and couplings) of any theory, providing information on higher energy sectors well below the threshold for its direct production. The fact that LEP was not able to produce the top quark did not preclude the LEP collaborations from having indications of its existence, and even from giving a mass determination –with the help of its radiative corrections to the  $M_W - \sin^2 \theta_W$  correlation [38]– in agreement with the Tevatron [5, 6] direct mass determination from real top quarks. The very high statistics of LEP experiments did play a fundamental role in these results. Unfortunately the same game can not be played to estimate the Higgs mass, since the one-loop Higgs radiative corrections enter that observable logarithmically on the Higgs boson mass [39] –the top effect grows quadratically with its mass.

Nowadays, as stated early, there is no significant deviations from the precision observables measured [11]. Notwithstanding, we will obtain significant information out of the potentially

large radiative corrections related with the supersymmetric strong and supersymmetric or 2HDM Yukawa sectors. To this end, we will, first, briefly review the renormalization framework needed to carry out 1-loop calculations in the MSSM in chapter 3. All our 1-loop analyses will be made in a physically well motivated renormalization scheme that is amply discussed in sec. 4.3.

A complementary approach to looking for the indirect effects –and thus exposing the existence of new physics– is offered by probing transitions which are either suppressed or forbidden in the SM. In this respect the low energy  $B$  meson phenomenology plays an important role as well, and will be taken into account.

In this Thesis we will deal mainly with the Higgs sector and their interaction with quarks.

To this end, in chapters 4 and 5 we will study the Higgs boson decaying into quarks. Specifically, in chapter 4 we will first review and depict clearly the low-energy physics constraints from the radiative  $\bar{B}^0$ -decays,  $b \rightarrow s\gamma$ , within the MSSM, so that we can use them from then on. The rest of the chapter will be dedicated to the study of the charged Higgs decaying into  $t\bar{b}$  within the relevant allowed MSSM parameter space. Thus, we will update unconstrained results having into account just supersymmetric strong effects [40,41] and what's more we will improve these calculations by adding the supersymmetric Yukawa driven electroweak effects [42,43]. In this section we also argue on the important effects [44,45] that appear in the charged Higgs boson production itself and in the expected measurement of the single top quark production cross-section and their relevance for the Tevatron II analyses. This work extends the line of research in Refs. [46–48] for a light enough charged Higgs, i.e. the charged Higgs decay of top quark [46].

The important supersymmetric strong corrections to the three MSSM neutral Higgs bosons are discussed in chapter 5. It was as early as in 1995 that we suggested [49] that these effects could also be important in the Higgs boson production mechanisms themselves an nowadays it has been shown in some partial studies for the Tevatron [50] which however require a much more detailed treatment. These studies are related to the ones stated early [44,45] and are of paramount importance for the analyses at the Tevatron II and could well be the clue to discover the Higgs sector of the MSSM.

We again use the low energy  $B$ -meson decay physics experiments, in this case the low-energy semi-tauonic  $B$ -decay,  $\bar{B} \rightarrow \tau \bar{\nu}_\tau X$  to place constraints on the MSSM  $\tan\beta$ - $M_{H^\pm}$  parameter space in chapter 7. Our previous results [51] (see also *Review of Particle Physics* 1999 [52]) are updated including the leading Yukawa electroweak supersymmetric quantum corrections to that decay. Here it is patent, once more, that quantum corrections related to the strong and/or Yukawa sectors of the MSSM may alter severely the picture obtained from just including SM strong effects [53].

Within the theoretical framework of two-Higgs-doublet models we study the charged Higgs boson decay of a top quark [54, 55], and this will be exposed in chapter 6. There, the calculation of the electroweak quantum corrections to the process  $t \rightarrow H^+ b$  is described in detail and a comparison with the SUSY case [46] is done. It gives us knowledge on the differences that appear between the phenomenology of the two extensions of the SM, letting us to distinguish between the two Higgs sectors. Moreover, we also improve the standard QCD corrected bounds in the parameter space  $\tan\beta$ - $M_{H^\pm}$  [56–58] obtained by the Tevatron Collaboration by adding the electroweak effects just calculated. These bounds are obtained by searching for an excess of taus from the cross-section  $\sigma(p\bar{p} \rightarrow t\bar{t}X \rightarrow \tau\nu_\tau X)$  with respect to  $\sigma(p\bar{p} \rightarrow t\bar{t}X \rightarrow l\nu_l X)$  ( $l = e, \mu$ ). The absence of such an excess determines an upper bound on  $\Gamma(t \rightarrow H^+ b \rightarrow \tau^+ \nu_\tau b)$  and a corresponding excluded region of the parameter space ( $\tan\beta, M_{H^\pm}$ ).

Lastly, in chapter 8 a review analysis of the constraints on the masses and couplings of the single and double charge members of Left-Right Higgs triplets [59–64] is carried out. Interestingly enough, present day experiments tolerate values of the Yukawa couplings of these scalars at the level of the standard electroweak gauge couplings. So, it is not that strange to find [64] that the proposed measurement of the ratio  $R_{LCD} = \sigma(\nu_\mu e)/[\sigma(\bar{\nu}_\mu e) + \sigma(\nu_e e)]$  at LAMPF [65] would allow to explore a large region of the parameter space inaccessible to the usual ratio  $R = \sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$  measured by the CHARM II collaboration [66].





## Chapter 2

# Physics beyond the SM

The Standard Model (SM) of high energy physics, a model in which the electromagnetism, the electroweak interactions, and Chromodynamics are unified, provides a remarkably successful description of presently known phenomena. In the last two decades the experimental high-energy frontier has advanced into the hundreds of GeV range with no confirmed deviations from Standard Model predictions and few unambiguous hints of additional structure [11]. Still, it seems quite clear that the Standard Model is a work in progress and will have to be extended to describe physics at arbitrarily high energies. Certainly a new framework will be required at the reduced Planck scale  $M_{Planck} = (8\pi G_{Newton})^{-1/2} = 2.4 \times 10^{18}$  GeV, where quantum gravitational effects become important. Based only on a proper respect for the power of Nature to surprise us, it seems nearly as obvious that new physics exists in the 16 orders of magnitude in energy between the presently explored territory and the Planck scale.

Moreover the SM does not answer all the questions that one may rise and even shows some theoretically annoying issues. To wit:

- **The Gauge problem:** Why is it that the SM has 3 symmetry groups,  $SU(3)_C \times SU(2)_L \times U(1)_Y$ , completely unrelated? To solve this “problem” Grand Unification Theories based on different groups ( $SU(5)\dots$ ) have been proposed. This kind of theories provide gauge coupling unification if Supersymmetry is also in the theory.
- **The lack of unification with gravitation:** The SM does not explain gravitation

and provides no means as to how to encompass gravitation in it. Nowadays a lot of effort is being done in this direction with *string theories*.

- **The big number of uncorrelated parameters:** There are at more than 18 parameters among gauge couplings, Yukawa couplings, mixing angles and the Higgs selfcoupling. Moreover, All the masses of the fermions together with the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the SM come from the still experimentally unconfirmed Higgs sector.
- **The spontaneous symmetry origin:** Why is there any spontaneous symmetry breaking? Why just one Higgs doublet? This questions are partly answered by supersymmetric theories in which even the spontaneous symmetry breaking could be triggered by the large yukawa coupling of the top quark driving the adequate Higgs potential coupling to be negative.
- **The problem of families:** The SM neither provides and explanation of the number of families, nor of the origin of the symmetry between quarks and leptons.
- **The problem of charge:** It does not provide either the answer to why the charge is quantized, nor why the electron and proton charge are equal.
- **Jerarquies:** This is not really a “problem” with the SM strictly speaking but with the fact that the SM should be considered an effective theory at some very high energies ( $M_{Planck}$  for example). Why should in this case be such different scales, i.e.  $M_W \approx 10^{-17} M_{Planck}$ ? Again supersymmetry provides a way out of this problem.
- **Fine-Tuning problem:** This is somehow related to the previous one, and so not strictly a problem in SM. It just states that there should be very big cancellations to provide a “light” Higgs mass (of the order of the electroweak scale) since from the calculation of the quantum effects one learns that it should be quadratically divergent with the scale of the SM encompassing theory. This problem does not exist in supersymmetric theories due to the fact that in this theories the scalar masses do not develop such divergences since there are cancellations with the added particles contributions (if

SUSY is broken the problem is kept under control provided that the soft-SUSY-breaking terms are kept light ( $\lesssim 1$  TeV).

- **$\Omega$  problem:** From cosmological analyses there seems to be a lack of matter, possibly explained with the inclusion of very low interacting particles, such as the Lightest Supersymmetric Particle that would be stable if  $R$ -parity is conserved.

This is why we have studied here the possibility of having different extensions of the Standard Model, to wit: Two Higgs Doublet Models, The Minimal Supersymmetric Standard Model and Left-Right Models. We will describe them in a nutshell to fix notation.

## 2.1 Two Higgs Doublet Models

The Two-Higgs-Doublet Models (2HDM) [67] play a special role as the simplest extension of the electroweak sector of the SM. In this class of models one enlarges the scalar sector of the SM by the introduction of another Higgs doublet, thus rendering the scalar sector in terms of

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \quad (Y = +1), \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} \quad (Y = +1). \quad (2.1)$$

The more general 2-Higgs potential may be written as:

$$\begin{aligned} V(\Phi_1, \Phi_2) = & \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 \Phi_1^\dagger \Phi_2 + \mu_{12}^{*2} \Phi_2^\dagger \Phi_1 + \\ & + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \\ & + \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_5^* (\Phi_2^\dagger \Phi_1)^2 + \\ & + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_1) + \lambda_6^* (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \\ & + \lambda_7 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_2) + \lambda_7^* (\Phi_2^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \end{aligned} \quad (2.2)$$

where  $\mu_1$ ,  $\mu_2$  and  $\lambda_i$  with  $i = 1, 2, 3, 4$  are real parameters due to lagrangian hermiticity and, only,  $\mu_{12}$  and  $\lambda_i$  with  $i=5,6,7$  may be complex.

Nevertheless, if one wants to avoid the possible Flavor Changing Neutral Currents (FCNC) coming from this lagrangian, since it is sufficient that all equally charged quarks couple to just one Higgs doublet [68], what is usually done is to impose a discrete symmetry D to account for this. One can establish two such symmetries:

$$\text{i) } \Phi_2 \rightarrow -\Phi_2, d_R \rightarrow -d_R, l_R \rightarrow -l_R$$

$$\text{ii) } \Phi_2 \rightarrow -\Phi_2.$$

With the first one all “up” (“down”) fermions are couple to  $\Phi_2$  ( $\Phi_1$ ) and the resulting model is known as a Type II Model; while with the second one, all fermions are coupled to  $\Phi_1$  and the model is named Type I.

The restrictions derived from such symmetries on the potential render

$$\lambda_6 = \lambda_7 = \mu_{12} = 0 \tag{2.3}$$

and, moreover, in that case  $\lambda_5$  may be chosen real reabsorbing any phase in the Higgs doublets definitions. Thus, the resulting Type I and II models have 7 parameters coming from the Higgs potential.

After spontaneous symmetry breaking, triggered by the two Higgs doublets acquiring a vacuum expectation value (VEV):

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

one is left with two CP-even (scalar) Higgs bosons  $h^0, H^0$ , a CP-odd (pseudoscalar) Higgs boson  $A^0$  and a pair of charged Higgs bosons  $H^\pm$ . The new –apart from  $M_W^2 = \frac{g^2}{2}(v_1^2 + v_2^2)$ – parameters of these models consist of:

- the masses of the Higgs particles,  $M_{h^0}, M_{H^0}, M_{A^0}$  and  $M_{H^\pm}$  (with the convention  $M_{h^0} < M_{H^0}$ ),
- the ratio of the two vacuum expectation values

$$\tan \beta \equiv \frac{\langle H_2^0 \rangle}{\langle H_1^0 \rangle} \equiv \frac{v_2}{v_1}, \tag{2.4}$$

- and the mixing angle  $\alpha$  between the two CP-even states.

In this models, the mass eigenstates (Goldstone and physical fields respectively) may be expressed:

$$\begin{aligned}
G^\pm &= \cos \beta \phi_1^\pm + \sin \beta \phi_2^\pm \\
G^0 &= \sqrt{2}(\cos \beta \operatorname{Im}\phi_1^0 + \sin \beta \operatorname{Im}\phi_2^0) \\
H^\pm &= -\sin \beta \phi_1^\pm + \cos \beta \phi_2^\pm \\
A^0 &= \sqrt{2}(-\sin \beta \operatorname{Im}\phi_1^0 + \cos \beta \operatorname{Im}\phi_2^0) \\
H^0 &= \sqrt{2}(\cos \alpha (\operatorname{Re}\phi_1^0 - v_1) + \sin \alpha (\operatorname{Re}\phi_2^0 - v_2)) \\
h^0 &= \sqrt{2}(-\sin \alpha (\operatorname{Re}\phi_1^0 - v_1) + \cos \alpha (\operatorname{Re}\phi_2^0 - v_2))
\end{aligned}$$

For both models (Type I and Type II) the Yukawa couplings with the matter fermions (we illustrate just for the third quark family) may be written:

$$\lambda_t \equiv \frac{h_t}{g} = \frac{m_t}{\sqrt{2} M_W \sin \beta} \quad , \quad \lambda_b^{\{I, II\}} \equiv \frac{h_b}{g} = \frac{m_b}{\sqrt{2} M_W \{\sin \beta, \cos \beta\}} . \quad (2.5)$$

Type II models do appear in specific extensions of the SM, such as the Minimal Supersymmetric Standard Model (MSSM) which is currently under intensive study both theoretically and experimentally and that will be discussed in Sec.2.2.1.

### 2.1.1 The Higgs Feynman Rules

Let us now define a few parameters that will allow us to treat the two models, Type I and Type II, as a single model and giving the feynman rules that will be used later in chapter 6.

$$\begin{aligned}
a_I &\equiv -\cot \beta & a_{II} &\equiv +\tan \beta \\
R_I &\equiv \frac{\sin \alpha}{\sin \beta} & R_{II} &\equiv \frac{\cos \alpha}{\cos \beta} \\
r_I &\equiv \frac{\cos \alpha}{\sin \beta} & r_{II} &\equiv -\frac{\sin \alpha}{\cos \beta}
\end{aligned} \quad (2.6)$$

The interaction lagrangian between Higgs bosons and fermions for a Type j Model reads [67], using eq. 2.6:

$$\begin{aligned}
\mathcal{L}_{Hff} = & -\frac{g}{2M_W}\bar{D}M_D D(H^0 R_j + h^0 r_j) + \frac{ig}{2M_W}\bar{D}M_D\gamma_5 D a_j \\
& -\frac{g}{2M_W}\bar{U}M_U U(H^0 \frac{\sin\alpha}{\sin\beta} + h^0 \frac{\cos\alpha}{\sin\beta}) + \frac{ig}{2M_W}\bar{D}M_D\gamma_5 D \cot\beta \\
& +\frac{g}{\sqrt{2}M_W}\left(\bar{U}[M_U \cot\beta \hat{V}_{CKM} P_L + \hat{V}_{CKM} M_D a_j]DH^+ + h.c.\right) \quad (2.7)
\end{aligned}$$

We will still need the interaction among a charged Higgs and two other Higgs bosons, to this end we define the matrix  $M_{AB}^{H^+}$ , where the first index runs among the charged Higgs and Goldstone boson ( $H_A = (H_1, H_2) \equiv (H^-, G^-)$ ), and the second one among the neutral Higgs eigenstates ( $H_B = (H_1, \dots, H_4) \equiv (H^0, h^0, A^0, G^0)$ ):

$$\begin{aligned}
M_{H^-\{H_0, h_0\}}^{H^+} &= \frac{M_{A^0}^2 - m_{\{H_0, h_0\}}^2}{M_W^2} \cot 2\beta \{\sin(\alpha - \beta), \cos(\alpha - \beta)\} + \\
& \frac{M_{A^0}^2 - M_{H^\pm}^2 - m_{\{H_0, h_0\}}^2}{M_W^2} / 2 \{\cos(\beta - \alpha), \sin(\beta - \alpha)\}, \\
M_{G^-\{H^0, h^0\}}^{H^+} &= \frac{i\{\sin(\alpha - \beta), \cos(\alpha - \beta)\}(M_{H^\pm}^2 - m_{\{H_0, h_0\}}^2)}{2M_W} \\
M_{G^- A^0}^{H^+} &= \frac{i(M_{A^0}^2 - M_{H^\pm}^2)}{2M_W} \quad (2.8)
\end{aligned}$$

The combinations that are not listed are just zero. Using this definition the interaction among  $H^\pm$  and two other Higgs bosons simply reads:

$$\mathcal{L}_{H^\pm H_z H_t} = g(M_{AB}^{H^+} H^+ H_A H_B + h.c.) \quad (2.9)$$

## 2.2 Supersymmetry

We will not study here the formal structure of supersymmetric theories, this has been done in detail long time ago [69], but we will introduce here the notation and the feeling on what it is.

Supersymmetry (SUSY) is just a symmetry that relates fermions and bosons, and in general, different spin particles among them, such that their properties are related. For example, in a N=1 supersymmetric conserving model there would be as many fermions as bosons. N designs usually the number of supersymmetries introduced in the theory, or equivalently the

number of “relations” between particles of different spins. Supersymmetry organizes particles in “multiplets”, either chiral superfields for matter particles or vector superfields for gauge particles. Each superfield contains several fields describing usual particles with equal properties (mass, charges...) but for their spin. The following quantum number,  $R$ -parity, is defined:

$$R = (-1)^{2S+L+3B}, S \equiv \text{spin}, L \equiv \text{lepton number}, B \equiv \text{barion number},$$

to distinguish the usual SM field that have  $R = +1$  from the supersymmetric partners that have  $R = -1$ .

### 2.2.1 The Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) is the minimal Supersymmetric extension of the Standard Model. It is introduced by means of a N=1 SUSY on the Standard Model with the minimum number of new particles. It conserves  $R$ -parity, so that  $R$ -odd (supersymmetric) particles can only be created in pairs.

In a supersymmetric extension of the Standard Model each of the known fundamental particles must be in either a chiral or gauge supermultiplet and have a superpartner with spin differing by 1/2 unit. The first step in understanding the exciting phenomenological consequences of this prediction is to decide how the known particles fit into supermultiplets, and to give them appropriate names. The names for the spin-0 partners of the quarks and leptons are constructed by prepending an “s”, which is short for scalar. Thus generically they are called *squarks* and *sleptons* (short for “scalar quark” and “scalar lepton”). The left-handed and right-handed pieces of the quarks and leptons are separate two-component Weyl fermions with different gauge transformation properties in the Standard Model, so each must have its own complex scalar partner. The symbols for the squarks and sleptons are the same as for the corresponding fermion, but with a tilde used to denote the superpartner of a Standard Model particle. For example, the superpartners of the left-handed and right-handed parts of the electron Dirac field are called left- and right-handed selectrons, and are denoted  $\tilde{e}_L$  and  $\tilde{e}_R$ . It is important to keep in mind that the “handedness” here does not refer to the

helicity of the selectrons (they are spin-0 particles) but to that of their superpartners. A similar nomenclature applies for smuons and staus:  $\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R$ . In the Standard Model the neutrinos are always left-handed, so the sneutrinos are denoted generically by  $\tilde{\nu}$ , with a possible subscript indicating which lepton flavor they carry:  $\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$ . Finally, a complete list of the squarks is  $\tilde{q}_L, \tilde{q}_R$  with  $q = u, d, s, c, b, t$ . The gauge interactions of each of these squark and slepton field are the same as for the corresponding Standard Model fermion; for instance, a left-handed squark like  $\tilde{u}_L$  will couple to the  $W$  boson while  $\tilde{u}_R$  will not.

It seems clear that the Higgs scalar boson must reside in a chiral supermultiplet, since it has spin 0. Actually, it turns out that one chiral supermultiplet is not enough. In fact, because of the structure of supersymmetric theories, only a  $Y = +1/2$  Higgs chiral supermultiplet can have the Yukawa couplings necessary to give masses to charge  $+2/3$  up-type quarks (up, charm, top), and only a  $Y = -1/2$  Higgs can have the Yukawa couplings necessary to give masses to charge  $-1/3$  down-type quarks (down, strange, bottom) and to charged leptons. We will call the  $SU(2)_L$ -doublet complex scalar fields corresponding to these two cases  $H_2$  and  $H_1$ :

$$\hat{H}_1 = \begin{pmatrix} \hat{H}_1^0 \\ \hat{H}_1^- \end{pmatrix}, \quad \hat{H}_2 = \begin{pmatrix} \hat{H}_2^+ \\ \hat{H}_2^0 \end{pmatrix}, \quad (2.10)$$

respectively<sup>1</sup>. The weak isospin components of  $H_2$  with  $T_3 = (+1/2, -1/2)$  have electric charges 1, 0 respectively, and are denoted  $(H_2^+, H_2^0)$ . Similarly, the  $SU(2)_L$ -doublet complex scalar  $H_1$  has  $T_3 = (+1/2, -1/2)$  components  $(H_1^0, H_1^-)$ . The neutral scalar that corresponds to the physical Standard Model Higgs boson is in a linear combination of  $H_2^0$  and  $H_1^0$ . The generic nomenclature for a spin-1/2 superpartner is to append “-ino” to the name of the Standard Model particle, so the fermionic partners of the Higgs scalars are called higgsinos. They are denoted by  $\tilde{H}_2, \tilde{H}_1$  for the  $SU(2)_L$ -doublet left-handed Weyl spinor fields, with weak isospin components  $\tilde{H}_2^+, \tilde{H}_2^0$  and  $\tilde{H}_1^0, \tilde{H}_1^-$ .

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<sup>1</sup>notice that this Higgs boson content is the one of a Type II Higgs doublet model but with (see eq. 2.1)

$$\hat{H}_1 = \begin{pmatrix} \hat{H}_1^0 \\ \hat{H}_1^- \end{pmatrix} = \begin{pmatrix} \phi_1^{0*} \\ -\phi_1^- \end{pmatrix}, \quad \hat{H}_2 = \begin{pmatrix} \hat{H}_2^+ \\ \hat{H}_2^0 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}. \quad (2.11)$$



Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks ( $\times 3$ families)	$\hat{Q}$	$(\tilde{u}_L \ \tilde{d}_L)$	$(u_L \ d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{3})$
	$\hat{U}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{4}{3})$
	$\hat{D}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3})$
sleptons, leptons ( $\times 3$ families)	$\hat{L}$	$(\tilde{\nu} \ \tilde{e}_L)$	$(\nu \ e_L)$	$(\mathbf{1}, \mathbf{2}, -1)$
	$\hat{R}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs, higgsinos	$H_2$	$(H_2^+ \ H_2^0)$	$(\tilde{H}_2^+ \ \tilde{H}_2^0)$	$(\mathbf{1}, \mathbf{2}, +1)$
	$H_1$	$(H_1^0 \ H_1^-)$	$(\tilde{H}_1^0 \ \tilde{H}_1^-)$	$(\mathbf{1}, \mathbf{2}, -1)$

Table 2.1: *Organization of matter fields in Chiral supermultiplets in the Minimal Supersymmetric Standard Model.*

We have now found all of the chiral supermultiplets of a minimal phenomenologically viable extension of the Standard Model. They are summarized in Table 2.1, classified according to their transformation properties under the Standard Model gauge group  $SU(3)_C \times SU(2)_L \times U(1)_Y$ , which combines  $u_L, d_L$  and  $\nu, e_L$  degrees of freedom into  $SU(2)_L$  doublets. Here we have followed the standard convention that all chiral supermultiplets are defined in terms of left-handed Weyl spinors, so that the *conjugates* of the right-handed quarks and leptons (and their superpartners) appear in Table 2.1. It is useful also to have a symbol for each of the chiral supermultiplets as a whole; these are indicated in the second column of Table 2.1. Thus for example  $\hat{Q}$  stands for the  $SU(2)_L$ -doublet chiral supermultiplet containing  $\tilde{u}_L, u_L$  (with weak isospin component  $T_3 = +1/2$ ), and  $\tilde{d}_L, d_L$  (with  $T_3 = -1/2$ ), while  $\hat{U}$  stands for the  $SU(2)_L$ -singlet supermultiplet containing  $\tilde{u}_R^*, u_R^\dagger$ . There are three families for each of the quark and lepton supermultiplets, but we have used first-family representatives in Table 2.1. Below, a family index  $i = 1, 2, 3$  will be affixed to the chiral supermultiplet names ( $\hat{Q}_i, \hat{U}_i, \dots$ ) when needed, e.g.  $(\hat{R}_1, \hat{R}_2, \hat{R}_3) = (\hat{e}, \hat{\mu}, \hat{\tau})$ . The bar on  $\hat{U}, \hat{D}, \hat{R}$  fields is part of the name, and does not denote any kind of conjugation. It is interesting to note that the Higgs chiral supermultiplet  $H_2$  (containing  $H_2^0, H_1^-, \tilde{H}_2^0, \tilde{H}_1^-$ ) has exactly the same Standard Model gauge quantum numbers as the left-handed slepton and leptons  $L_i$ , e.g.  $(\tilde{\nu},$

Names	spin 1/2	spin 1	$SU(3)_C, SU(2)_L, U(1)_Y$
gluino, gluon	$\tilde{g}$	$g$	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\tilde{W}^\pm \tilde{W}^0$	$W^\pm W^0$	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson	$\tilde{B}^0$	$B^0$	$(\mathbf{1}, \mathbf{1}, 0)$

Table 2.2: *Gauge supermultiplets in the Minimal Supersymmetric Standard Model.*

$\tilde{e}_L, \nu, e_L$ ). Naively one might therefore suppose that we could have been more economical in our assignment by taking a neutrino and a Higgs scalar to be superpartners, instead of putting them in separate supermultiplets. This would amount to the proposal that the Higgs boson and a sneutrino should be the same particle. This is a nice try which played a key role in some of the first attempts to connect supersymmetry to phenomenology, [70] but it is now known not to work. Many insoluble phenomenological problems would result, including lepton number violation and a mass for at least one of the neutrinos in gross violation of experimental bounds [52]. Therefore, all of the superpartners of Standard Model particles are really new particles, and cannot be identified with some other Standard Model state.

The vector bosons of the Standard Model clearly must reside in gauge supermultiplets. Their fermionic superpartners are generically referred to as *gauginos*. The  $SU(3)_C$  color gauge interactions of QCD are mediated by the gluon, whose spin-1/2 color-octet supersymmetric partner is the gluino. As usual, a tilde is used to denote the supersymmetric partner of a Standard Model state, so the symbols for the gluon and gluino are  $g$  and  $\tilde{g}$  respectively. The electroweak gauge symmetry  $SU(2)_L \times U(1)_Y$  has associated with it spin-1 gauge bosons  $W^+, W^0, W^-$  and  $B^0$ , with spin-1/2 superpartners  $\tilde{W}^+, \tilde{W}^0, \tilde{W}^-$  and  $\tilde{B}^0$ , called *winos* and *bino*. After electroweak symmetry breaking, the  $W^0, B^0$  gauge eigenstates mix to give mass eigenstates  $Z^0$  and  $\gamma$ . The corresponding gaugino mixtures of  $\tilde{W}^0$  and  $\tilde{B}^0$  are called *zino* ( $\tilde{Z}^0$ ) and *photino* ( $\tilde{\gamma}$ ); if supersymmetry were unbroken, they would be mass eigenstates with masses  $m_Z$  and 0. Table 2.2 summarizes the gauge supermultiplets of a minimal supersymmetric extension of the Standard Model.