The chiral and gauge supermultiplets in Tables 2.1 and 2.2 make up the particle content of the Minimal Supersymmetric Standard Model (MSSM). The most obvious and unfortunate feature of this theory is that none of the superpartners of the Standard Model particles has been discovered as of this writing. If supersymmetry were unbroken, then there would have to be selectrons $\tilde{e}_L$ and $\tilde{e}_R$ with masses exactly equal to $m_e = 0.511...$ MeV. A similar statement applies to each of the other sleptons and squarks, and there would also have to be a massless gluino and photino. These particles would be extraordinarily easy to detect. Clearly, therefore, *supersymmetry must be a broken symmetry* in the vacuum state chosen by nature. Thus, it is not that odd not to have seen any of the SUSY particles, since they all receive their mass from the SUSY-breaking sector which is completely new.

The mass splittings between the known Standard Model particles and their superpartners are just determined by the parameters that break supersymmetry $m_{\rm soft}$. This tells us that the superpartner masses cannot be too large. Otherwise, we would lose our successful cure for the hierarchy problem since the $m^2_{\rm soft}$ corrections to the Higgs scalar (mass)^2 would be unnaturally large compared to the electroweak breaking scale of 174 GeV.

The MSSM Lagrangian

In this section we single out the specific pieces of the MSSM Interaction Lagrangian involved in the processes under study. Although the Lagrangian of the MSSM is well-known [17–19,21], it is always useful to project explicitly the relevant pieces and to cast them in a most suitable form for specific purposes. As a matter of fact, we have produced a complete set of Feynman rules for the MSSM using an algebraic computer code based on MATHEMATICA [71][2]. Here we limit ourselves to quote the Lagrangian interactions affecting the process-dependent parts of the processes under study and omit the interaction pieces needed to compute the universal counterterm structures, whose SM parts are well-known [36, 37, 72] and the corresponding SUSY contributions are also available in the literature since long time ago [73–76]. All our interactions are expressed in the mass-eigenstate basis.

[2]We have corrected several mistakes in the subset of rules presented in Ref. [67] involving sparticles and Higgses.

The full MSSM lagrangian can be written:

$$\begin{aligned}\mathcal{L}_{\text{MSSM}} &= \mathcal{L}_{\text{Kinetik}} + \mathcal{L}_{\text{Gauge}} - V_{\tilde{G}\psi\tilde{\psi}} - V_D - V_Y - \sum_i \left|\frac{\partial W(\varphi)}{\partial\varphi_i}\right|^2 \\ & -V^I_{\text{soft}} - m_1^2\, H_1^\dagger H_1 - m_2^2\, H_2^\dagger H_2 - m_{12}^2\,\left(H_1 H_2 + H_1^\dagger H_2^\dagger\right) \\ & -\tfrac{1}{2} m_{\tilde{g}}\,\psi_{\tilde{g}}^a\psi_{\tilde{g}}^a - \tfrac{1}{2} M\,\tilde{w}_i\tilde{w}_i - \tfrac{1}{2}M'\,\tilde{B}^0\tilde{B}^0 \\ & -m^2_{\tilde{L}}\,\tilde{L}^*\tilde{L} - m^2_{\tilde{R}}\,\tilde{R}^*\tilde{R} - m^2_{\tilde{Q}}\,\tilde{Q}^*\tilde{Q} - m^2_{\tilde{U}}\,\tilde{U}^*\tilde{U} - m^2_{\tilde{D}}\,\tilde{D}^*\tilde{D}\end{aligned} \tag{2.12}$$

were:

- $\mathcal{L}_{\text{Kinetik}}$ are the usual kinetic terms;
- $\mathcal{L}_{\text{Gauge}}$ stand for the standard gauge interactions;
- $$V_{\tilde{G}\psi\tilde{\psi}} = i\sqrt{2} g_a \varphi_k \bar{\lambda}^a \left(T^a\right)_{kl} \bar{\psi}_l + \text{ h.c.}$$

  are the gaugino interactions with (φ, ψ) the spin 0 and spin 1/2 components of a chiral superfield respectively, T^a a generator of the gauge symmetry, λ_a the gaugino field and g^a its coupling constant;
- $$V_D = \frac{1}{2}\sum D^a D^a \ ,$$

  with

  $$D^a = g^a \varphi_i^* \left(T^a\right)_{ij} \varphi_j \ ,$$

  provide the D-terms, related to the gauge structure of the theory, that do not contain neither gauge bosons nor gauginos and where again φ_i stand for the scalar components of the superfields;
- $$W = \epsilon_{ij}\left[f\hat{H}_1^i\hat{L}^j\hat{\bar{R}} + h_d\hat{H}_1^i\hat{Q}^j\hat{\bar{D}} + h_u\hat{H}_2^j\hat{\bar{Q}}^i\hat{U} - \mu\hat{H}_1^i\hat{H}_2^j\right] \ . \tag{2.13}$$

  is the MSSM superpotential [17–19, 21]
- V_Y are the yukawa interactions obtained from eq. 2.13 by replacing two of the superfields by its fermionic field content and the third by its scalar field content;
- $$V^{\text{I}}_{\text{soft}} = \frac{g}{\sqrt{2}M_W\cos\beta}\epsilon_{ij}\left[m_l A_l H_1^i\tilde{L}^j\tilde{R} + m_d A_d H_1^i\tilde{Q}^j\tilde{D} - m_u A_u H_2^i\tilde{Q}\tilde{U}\right] + \text{ h.c.} \quad .$$

are soft-SUSY-Breaking interaction terms, where the trilinear Soft-SUSY-Breaking couplings A_f can play an important role specially for the third generation interactions and masses, and they are in the source of the large value of the bottom quark mass renormalization effects (see Sec. 4.4.4);

- and the rest are the Soft-SUSY breaking masses.

From this lagrangian the full MSSM spectrum and the interaction among the different particles can be obtained. We briefly describe the necessary SUSY formalism, the particles (giving the recipes to obtain the mass-eigenstates) and its interaction entering our computations:

- As explained earlier two Higgs doublets are needed in the MSSM, and this makes the Higgs sector of the MSSM a particular expression of the Type II two-Higgs-doublet-model discussed earlier (Cf. Sec. 2.1). The ratio of the two VEV of the two neutral components of the corresponding scalar Higgs doublets (v_1 and v_2), responsible for giving mass to the fermions (down and up respectively):

$$\tan\beta = \frac{v_2}{v_1}\,, \tag{2.14}$$

is a most relevant parameter throughout our analysis.

From the lagrangian eq. 2.12 the Higgs potential can be extracted:

$$\begin{aligned} V &= m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 - m_{12}^2\,\left(\epsilon_{ij}\,H_1^i\,H_2^j + \text{h.c.}\right) \\ &+ \frac{1}{8}(g^2+g'^2)\,\left(|H_1|^2 - |H_2|^2\right)^2 + \frac{1}{2}\,g^2\,|H_1^\dagger\,H_2|^2\,. \end{aligned} \tag{2.15}$$

It is clear from this potential that only three parameters enter the Higgs potential, i.e, with only $M_W^2 = \frac{1}{2}(v_1^2+v_2^2)$, $\tan\beta$, and a mass (we will choose $M_{H^\pm}$) the physical inputs of the Higgs sector in the MSSM are defined. Thus, the rest of the masses, M_{A^0},M_{H^0}, M_{h^0}, and the neutral mixing angle, α, may be expressed at tree level as a function of this three parameters:

$$\begin{aligned} M_{A^0}^2 &= M_{H^\pm}^2 - M_W^2\ , \\ M_{H^0,h^0}^2 &= \frac{1}{2}\left(M_{A^0}^2 + M_Z^2 \pm \sqrt{\left(M_{A^0}^2+M_Z^2\right)^2 - 4\,M_{A^0}^2\,M_Z^2\cos^2 2\beta}\right)\ , \end{aligned} \tag{2.16}$$

$$\cos 2\alpha = -\cos 2\beta \left(\frac{M^2_{A^0} - M^2_Z}{M^2_{H^0} - M^2_{h^0}} \right) \ , \ \sin 2\alpha = -\sin 2\beta \left(\frac{M^2_{H^0} + M^2_{h^0}}{M^2_{H^0} - M^2_{h^0}} \right) \ . \tag{2.17}$$

These relations are no longer true beyond tree level and, in fact, since the corrections to the light Higgs boson mass and the α CP even mixing angle may be large and allow regions not allowed at tree level, we will use the one loop relations found in [77–81].

- The fermionic partners of the weak-eigenstate gauge bosons and Higgs bosons, called gauginos, $\tilde{B}$, $\tilde{W}$, and higgsinos, $\tilde{H}$, respectively. From them we construct the fermionic mass-eigenstates, the so-called charginos and neutralinos, by, first, forming the following three sets of two-component Weyl spinors:

$$\Gamma^+_i = \{-i\tilde{W}^+, \tilde{H}^+_2\} \ , \quad \Gamma^-_i = \{-i\tilde{W}^-, \tilde{H}^-_1\} \ , \tag{2.18}$$

$$\Gamma^0_\alpha = \{-i\tilde{B}^0, -i\tilde{W}^0_3, \tilde{H}^0_2, \tilde{H}^0_1\} \ , \tag{2.19}$$

which get mixed up when the neutral Higgs fields acquire nonvanishing VEV's and then diagonalizing the resulting "ino" mass Lagrangian

$$\begin{aligned}
\mathcal{L}_M \ &= \ - < \Gamma^+| \begin{pmatrix} M & \sqrt{2} M_W c_\beta \\ \sqrt{2} M_W s_\beta & \mu \end{pmatrix} |\Gamma^- > \\
&- \ \frac{1}{2} < \Gamma^0| \begin{pmatrix} M' & 0 & M_Z s_\beta s_W & -M_Z c_\beta s_W \\ 0 & M & -M_Z s_\beta c_W & M_Z c_\beta c_W \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & 0 & -\mu \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & -\mu & 0 \end{pmatrix} |\Gamma^0 > \\
&+ \ h.c. \ ,
\end{aligned} \tag{2.20}$$

where we remark the presence of the parameter μ introduced above and of the soft SUSY-breaking Majorana masses M and M', usually related as $M'/M = (5/3) \ \tan^2\theta_W$, and where $c_\beta = \cos\beta$ and $s_\beta = \sin\beta$. The corresponding mass-eigenstates[3] (charginos and neutralinos) are the following:

$$\Psi^+_i = \begin{pmatrix} U_{ij}\Gamma^+_j \\ V^*_{ij}\bar{\Gamma}^-_j \end{pmatrix} \quad , \quad \Psi^-_i = C\bar{\Psi}_i^{-T} = \begin{pmatrix} V_{ij}\Gamma^-_j \\ U^*_{ij}\bar{\Gamma}^+_j \end{pmatrix} \ , \tag{2.21}$$

[3]We use the following notation: first Latin indices a,b,...=1,2 are reserved for sfermions, middle Latin indices i,j,...=1,2 for charginos, and first Greek indices $\alpha, \beta, ... = 1, \ldots, 4$ for neutralinos.

and

$$\Psi^0_\alpha = \begin{pmatrix} N_{\alpha\beta}\Gamma^0_\beta \\ N^*_{\alpha\beta}\bar{\Gamma}^0_\beta \end{pmatrix} = C\bar{\Psi}^{0T}_\alpha \ , \tag{2.22}$$

where the matrices U, V, N are defined through

$$U^*\mathcal{M}V^\dagger = diag\{M_1, M_2\} \quad , \quad N^*\mathcal{M}^0 N^\dagger = diag\{M^0_1, \ldots, M^0_4\} \ . \tag{2.23}$$

In practice, we have performed the calculations with real matrices U, V and N, so we have been using unphysical mass-eigenstates (associated to non-positively definite chargino-neutralino masses). The transition from our unphysical mass-eigenstate basis $\{\Psi\} \equiv \{\Psi^\pm_i, \Psi^0_\alpha\}$ into the physical mass-eigenstate basis $\{\chi\} \equiv \{\chi^\pm_i, \chi^0_\alpha\}$ can be done by introducing a set of ϵ parameters as follows: for every chargino-neutralino Ψ whose mass matrix eigenvalue is M_i, M_α the proper physical state, χ, is given by

$$\chi = \begin{cases} \Psi & \text{if} \ \ \epsilon = 1 \\ \pm\gamma_5\,\Psi & \text{if} \ \ \epsilon = -1\,, \end{cases} \tag{2.24}$$

and the physical masses for charginos and neutralinos are $m_{\chi^\pm_i} = \epsilon\, M_i$ and $m_{\chi^0_\alpha} = \epsilon\, M^0_\alpha$, respectively. Needless to say, in this real formalism one is supposed to propagate the ϵ parameters accordingly in all the relevant couplings, as shown in detail in Ref. [82]. This procedure is entirely equivalent [83] to use complex diagonalization matrices insuring that physical states are characterized by a set of positive-definite mass eigenvalues.and for this reason we have maintained the complex notation in all our formulae in Section 4. Whereas for computations with real sparticles the distinction matters [82], for virtual sparticles the ϵ parameters cancel out, and so one could use either basis $\{\Psi\}$ or $\{\chi\}$ without the inclusion of the ϵ coefficients. We have stressed here the differences between the two bases just to make clear what are the physical chargino-neutralino states, when they are referred to in the text.

Among the gauginos we also have the strongly interacting gluinos, $\tilde{g}^r$ $(r = 1, \ldots, 8)$, which are the fermionic partners of the gluons.

- As for the scalar partners of quarks and leptons, they are called squarks, $\tilde{q}$, and sleptons, $\tilde{l}$, respectively. We will use the third quark-squark generation $(t,b)-(\tilde{t},\tilde{b})$ as a generic fermion-sfermion generation. The squark mass-eigenstates, $\tilde{q}_a=\{\tilde{q}_1,\tilde{q}_2\}$, if we neglect intergenerational mixing, are obtained from the weak-eigenstate ones $\tilde{q}'_a=\{\tilde{q}'_1\equiv\tilde{q}_L,\ \tilde{q}'_2\equiv\tilde{q}_R\}$, through

$$\begin{aligned}\tilde{q}'_a &= \sum_b R^{(q)}_{ab}\tilde{q}_b,\\ R^{(q)} &= \begin{pmatrix}\cos\theta_q & -\sin\theta_q\\ \sin\theta_q & \cos\theta_q\end{pmatrix} \qquad (q=t,b)\,.\end{aligned} \tag{2.25}$$

The rotation matrices in (2.25) diagonalize the corresponding stop and sbottom mass matrices:

$$\mathcal{M}^2_{\tilde{q}}=\begin{pmatrix}M^2_{\tilde{q}_L}+m_q^2+\cos 2\beta(T_3^{q_L}-Q_q s_W^2)M_Z^2 & m_q M^q_{LR}\\ m_q M^q_{LR} & M^2_{\tilde{q}_R}+m_q^2+\cos 2\beta\, Q_q s_W^2 M_Z^2\end{pmatrix}, \tag{2.26}$$

$$R^{(q)\dagger}\mathcal{M}^2_{\tilde{q}}R^{(q)}=diag\{m^2_{\tilde{q}_2},m^2_{\tilde{q}_1}\} \qquad (m_{\tilde{q}_2}\geq m_{\tilde{q}_1})\,, \tag{2.27}$$

with $T_3^{q_L}$ the third component of weak isospin, Q the electric charge, and $M_{\tilde{q}_{L,R}}$ the soft SUSY-breaking squark masses [17–19,21]. (By $SU(2)_L$-gauge invariance, we must have $M_{\tilde{t}_L}=M_{\tilde{b}_L}$, whereas $M_{\tilde{t}_R}$, $M_{\tilde{b}_R}$ are in general independent parameters.) The mixing angle on eq.(2.25) is given by

$$\tan 2\theta_q=\frac{2\,m_q\,M^q_{LR}}{M^2_{\tilde{q}_L}-M^2_{\tilde{q}_R}+\cos 2\beta(T_3^{q_L}-2Q_q s_W^2)M_Z^2}\,, \tag{2.28}$$

where

$$M^t_{LR}=A_t-\mu\cot\beta\,,\qquad M^b_{LR}=A_b-\mu\tan\beta\,, \tag{2.29}$$

are, respectively, the stop and sbottom off-diagonal mixing terms on eq.(2.26). Furthermore, μ is the SUSY Higgs mass parameter in the superpotential, and $A_{t,b}$ are the trilinear soft SUSY-breaking parameters. We shall assume (see eq. 2.51) that $|A_{t,b}|\lesssim 3M_{\tilde{Q}}$, where $M_{\tilde{Q}}$ is the average soft SUSY-breaking mass appearing in the mass matrix (2.26);

this relation roughly corresponds to the necessary, though not sufficient, condition for the absence of colour-breaking minima [84–87].

The charged slepton mass-eigenstates can be obtained in a similar way after straightforward substitutions in the mass matrices, with the only proviso that there is no $\tilde{\nu}_R$, so that $\tilde{\nu}_L$ is itself the sneutrino mass-eigenstate, hence $R^{(\tilde{\nu})}_{ab} = 0$ unless $a = b = 1$ where $R^{(\tilde{\nu})}_{11} = 1$.

Next let us describe the relevant pieces of the MSSM interaction Lagrangian involving the fields defined above.

- **fermion–sfermion–(chargino or neutralino)**

  After translating the allowed quark-squark-higgsino/gaugino interactions into the mass-eigenstate basis, one finds

$$\begin{aligned}
\mathcal{L}_{\Psi q\tilde{q}} &= g \sum_{a=1,2}\sum_{i=1,2} \Big(-\tilde{t}^*_a \bar{\Psi}^-_i \left(A^{(t)}_{+ai} P_L + A^{(t)}_{-ai} P_R \right) b \\
&\qquad\qquad - \tilde{b}^*_a \bar{\Psi}^+_i \left(A^{(b)}_{+ai} P_L + A^{(b)}_{-ai} P_R \right) t \Big) \\
&+ \frac{g}{\sqrt{2}} \sum_{a=1,2}\sum_{\alpha=1,\dots,4} \Big(-\tilde{t}^*_a \bar{\Psi}^0_\alpha \left(A^{(t)}_{+a\alpha} P_L + A^{(t)}_{-a\alpha} P_R \right) t \\
&\qquad\qquad + \tilde{b}^*_a \bar{\Psi}^0_\alpha \left(A^{(b)}_{+a\alpha} P_L + A^{(b)}_{-a\alpha} P_R \right) b \Big) \\
&+ \text{h.c.}
\end{aligned} \tag{2.30}$$

  where $A^{(t)}_{\pm ai}$, $A^{(b)}_{\pm ai}$, $A^{(t)}_{\pm a\alpha}$, $A^{(b)}_{\pm a\alpha}$ are

$$\begin{aligned}
A^{(t)}_{+ai} &= R^{(t)*}_{1a} U^*_{i1} - \lambda_t R^{(t)*}_{2a} U^*_{i2}\,, \\
A^{(t)}_{-ai} &= -\lambda_b R^{(t)*}_{1a} V_{i2}\,, \\
A^{(t)}_{+a\alpha} &= R^{(t)*}_{1a} \left(N^*_{\alpha 2} + Y_L \tan\theta_W N^*_{\alpha 1} \right) + \sqrt{2}\lambda_t R^{(t)*}_{2a} N^*_{\alpha 3}\,, \\
A^{(t)}_{-a\alpha} &= \sqrt{2}\lambda_t R^{(t)*}_{1a} N_{\alpha 3} - Y^t_R \tan\theta_W R^{(t)*}_{2a} N_{\alpha 1}\,, \\
A^{(b)}_{+ai} &= R^{(b)*}_{1a} V^*_{i1} - \lambda_b R^{(b)*}_{2a} V^*_{i2}\,, \\
A^{(b)}_{-ai} &= -\lambda_t R^{(b)*}_{1a} U_{i2}\,, \\
A^{(b)}_{+a\alpha} &= R^{(b)*}_{1a} \left(N^*_{\alpha 2} - Y_L \tan\theta_W N^*_{\alpha 1} \right) - \sqrt{2}\lambda_b R^{(b)*}_{2a} N^*_{\alpha 4}\,, \\
A^{(b)}_{-a\alpha} &= -\sqrt{2}\lambda_b R^{(b)*}_{1a} N_{\alpha 4} + Y^b_R \tan\theta_W R^{(b)*}_{2a} N_{\alpha 1}\,.
\end{aligned} \tag{2.31}$$

with Y_L and $Y_R^{t,b}$ the weak hypercharges of the left-handed $SU(2)_L$ doublet and right-handed singlet fermion, and λ_t and λ_b are – Cf. eq.(2.5) – the potentially significant Yukawa couplings normalized to the $SU(2)_L$ gauge coupling constant g.

- **quark–squark–gluino**

$$\mathcal{L}_{\tilde{g}q\tilde{q}} = -\frac{g_s}{\sqrt{2}}\tilde{q}^*_{a,k}\,\bar{\tilde{g}}_r\,(\lambda^r)_{kl}\left(R^{(q)*}_{1a}P_L - R^{(q)*}_{2a}P_R\right)q_l + \text{ h.c.} \tag{2.32}$$

where λ^r are the Gell-Mann matrices. This is just the SUSY-QCD Lagrangian written in the squark mass-eigenstate basis.

- **squark–squark–Higgs** For the charged Higgs we have:

$$\mathcal{L}_{H^\pm\tilde{q}\tilde{q}} = \frac{g}{\sqrt{2}M_W}H^-\,\tilde{b}^*_a G_{ab}\,\tilde{t}_b \;+\; \text{h.c.} \tag{2.33}$$

where we have introduced the coupling matrix

$$\begin{aligned} G_{ab} &= R^{(t)}_{cb}R^{*(b)}_{da}\,g_{cd} \\ g_{cd} &= \begin{pmatrix} m_b^2\tan\beta + m_t^2\cot\beta - M_W^2\sin 2\beta & m_b\,(\mu + A_b\tan\beta) \\ m_t\,(\mu + A_t\cot\beta) & m_t m_b\,(\tan\beta + \cot\beta) \end{pmatrix}. \end{aligned} \tag{2.34}$$

while the interaction Lagrangian between neutral higgses and squarks, in compact notation, can be cast as follows:

$$\mathcal{L}_{\Phi\tilde{q}\tilde{q}} = \frac{-g}{2M_W}\Phi^i\tilde{q}^*_a G^{(q)}_{i\;ab}\tilde{q}_b\,, \tag{2.35}$$

where we have introduced the mass-eigenstate coupling matrices

$$G^{(q)}_i = R^{(q)\dagger}\,\hat{G}^{(q)}_i\,R^{(q)}\,, \tag{2.36}$$

related to the corresponding weak-eigenstate coupling matrices, $\hat{G}^{(q)}_i$, by means of the rotation matrices $R^{(q)}$. For the $t\,\bar{t}$ final states, we have

$$\hat{G}^{(t)}_1\,[\cot\beta] \;=\; \begin{pmatrix} 0 & -im_t\,(\mu + A_t\cot\beta) \\ im_t\,(\mu + A_t\cot\beta) & 0 \end{pmatrix},$$

$$\hat{G}_2^{(t)}\left[c_\alpha, s_\alpha, s_\beta\right] = \begin{pmatrix} -2M_Z^2\left(T_3^{(t)} - Q^{(t)}s_W^2\right)s_{\alpha+\beta} + \frac{2m_t^2 c_\alpha}{s_\beta} & \frac{m_t}{s_\beta}\left(\mu s_\alpha + A_t c_\alpha\right) \\ \frac{m_t}{s_\beta}\left(\mu s_\alpha + A_t c_\alpha\right) & -2M_Z^2 Q^{(t)} s_W^2 s_{\alpha+\beta} + \frac{2m_t^2 c_\alpha}{s_\beta} \end{pmatrix},$$

$$\hat{G}_3^{(t)}\left[c_\alpha, s_\alpha, s_\beta\right] = \begin{pmatrix} 2M_Z^2\left(T_3^{(t)} - Q^{(t)}s_W^2\right)c_{\alpha+\beta} + \frac{2m_t^2 s_\alpha}{s_\beta} & \frac{m_t}{s_\beta}\left(-\mu c_\alpha + A_t s_\alpha\right) \\ \frac{m_t}{s_\beta}\left(-\mu c_\alpha + A_t s_\alpha\right) & 2M_Z^2 Q^{(t)} s_W^2 c_{\alpha+\beta} + \frac{2m_t^2 s_\alpha}{s_\beta} \end{pmatrix}, \tag{2.37}$$

with $c_\alpha \equiv \cos\alpha, s_\beta \equiv \sin\beta,\ s_{\alpha+\beta} \equiv \sin(\alpha+\beta)$ etc. and $Q^{(q)}, T_3^{(q)}$ the electric charge and 3rd component of weak isospin. For the $b\,\bar{b}$ final states, the following replacements are to be performed with respect to the $\hat{G}_i^{(t)}[...]$ in eq.(2.37):

$$\begin{aligned} \hat{G}_1^{(t)} &\rightarrow \hat{G}_1^{(b)}\left[\tan\beta\right], \\ \hat{G}_2^{(t)} &\rightarrow \hat{G}_2^{(b)}\left[s_\alpha, c_\alpha, -c_\beta\right], \\ \hat{G}_3^{(t)} &\rightarrow \hat{G}_3^{(b)}\left[s_\alpha, c_\alpha, c_\beta\right]. \end{aligned} \tag{2.38}$$

- **chargino–neutralino–charged Higgs**

$$\mathcal{L}_{H^\pm\Psi^\mp\Psi^0} = -g\,H^-\bar{\Psi}_\alpha^0\left(\cos\beta\,Q_{\alpha i}^L P_L + \sin\beta\,Q_{\alpha i}^R P_R\right)\Psi_i^+ \;+\; \text{h.c.} \tag{2.39}$$

with

$$\left\{\begin{aligned} Q_{\alpha i}^L &= U_{i1}^* N_{\alpha 3}^* + \tfrac{1}{\sqrt{2}}\left(N_{\alpha 2}^* + \tan\theta_W N_{\alpha 1}^*\right)U_{i2}^* \\ Q_{\alpha i}^R &= V_{i1} N_{\alpha 4} - \tfrac{1}{\sqrt{2}}\left(N_{\alpha 2} + \tan\theta_W N_{\alpha 1}\right)V_{i2}\,. \end{aligned}\right. \tag{2.40}$$

- **Gauge interactions**

In our calculation we only need the sparticle interactions with the $W^\pm$:

-quarks

$$\mathcal{L}_{W^\pm qq} = \frac{g}{\sqrt{2}}\,\bar{t}\gamma^\mu P_L b\,W_\mu^+ + \text{ h.c.} \tag{2.41}$$

-squarks

$$\mathcal{L}_{W^\pm\tilde{q}\tilde{q}} = i\frac{g}{\sqrt{2}}R_{1a}^{(t)*}R_{1b}^{(b)}W_\mu^+\tilde{t}_a^*\overset{\leftrightarrow}{\partial^\mu}\,\tilde{b}_b + \text{ h.c.} \tag{2.42}$$

-charginos and neutralinos

$$\mathcal{L}_{W^\pm\Psi^\mp\Psi^0} = g\,\bar{\Psi}^0_\alpha\gamma^\mu\left(C^L_{\alpha i}\epsilon_\alpha\epsilon_i P_L + C^R_{\alpha i}P_R\right)\Psi^+_i\,W^-_\mu + \text{ h.c.} \tag{2.43}$$

$$\begin{cases} C^L_{\alpha i} &=& \frac{1}{\sqrt{2}}N_{\alpha 3}U^*_{i2} - N_{\alpha 2}U^*_{i1} \\ C^R_{\alpha i} &=& -\frac{1}{\sqrt{2}}N^*_{\alpha 4}V_{i2} - N^*_{\alpha 2}V_{i1}\,. \end{cases} \tag{2.44}$$

Constraints to MSSM

The MSSM reproduces the behaviour of the SM up to energies probed so far [35] and this is why it is still a phenomenologically appealing model. In this section, we will give a summary of the limits placed on the supersymmetric parameters coming either from direct and indirect experimental limits or theoretical limits. We will use them except where explicitly stated for demonstrational purposes. Apart from the limits given here the ones coming from semi-tauonic B-decays will be discussed in chapter 7 together with the limits coming from $BR(t \to H^+ b)$ [48].

Direct experimental limits. The most stringent bound to the MSSM parameter space is the LEP II bound to the mass of charged particles beyond the SM. At present [88–90] this limit is roughly

$$M_{\text{charged}} \gtrsim 90\,\text{GeV}\ . \tag{2.45}$$

Specific searches for Supersymmetric particles are being performed at LEP II, negative neutralino searches rise up a limit on neutralino masses of [89]

$$M_{\chi^0_1} \gtrsim 30\,\text{GeV}\ , \tag{2.46}$$

it turns out that after translating this limit to the $\mu - M$ parameters it is less restrictive than the one obtained for the charginos from (2.45).

Actual Higgs searches at LEP II imply that, for the MSSM neutral Higgs sector [91]

$$M_{h^0} > 72.2\,\text{GeV}\ ,\ M_{A^0} > 76.1\,\text{GeV}\ . \tag{2.47}$$

Notice that without the MSSM relations there is no model independent bound on M_{A^0} from LEP [92]. Actual fits to the MSSM parameter space project a preferred value for the charged Higgs mass of $M_{H^\pm} \simeq 120\,\mathrm{GeV}$ [93].

Hadron colliders bounds are not so restrictive as those from e^+e^- machines. Most bounds on squark and gluino masses are obtained by supposing squark mass unification in simple models, such as mSUGRA. At present the limits on squarks (1st and 2nd generation) and gluino masses are [52]

$$m_{\tilde{q}} > 176\,\mathrm{GeV}\ ,\ m_{\tilde{g}} > 173\,\mathrm{GeV}\ . \tag{2.48}$$

Indirect experimental limits. Indirect limits on sparticle masses are obtained from the EW precision data. We apply these limits through all our computations by computing the contribution of sparticles to these observables and requiring that they satisfy the bounds from EW measurements. We require new contributions to the ρ parameter to be smaller than present experimental error on it, namely

$$\delta\rho_{\mathrm{new}} < 0.003\ . \tag{2.49}$$

We notice that as $\delta\rho_{\mathrm{new}}$ is also the main contribution from sparticle contributions to Δr [94], new contributions to this parameter are also below experimental constrains. Also the corrections in the α- and G_F-on-shell renormalization schemes will not differ significantly.

Theoretical restrictions. In the MSSM there should exist a light neutral scalar Higgs boson h^0. Tree-level analysis put this bound to the Z mass, however the existence of large radiative corrections to the Higgs bosons mass relations grow this limit up to $\sim 130\,\mathrm{GeV}$. Recently the two-loop radiative corrections to Higgs mass relations in the MSSM have been performed [95–97], and the present upper limit on M_{h^0} is

$$M_{h^0} \leq 130 - 135\,\mathrm{GeV}\ . \tag{2.50}$$

The two numbers in (2.50) have been computed by different groups [95–97] and there is a great interest in make them match [97]. It is very important to know as precise as possible this limit, as by means of a possible Run III of the Tevatron collider (TEV33, at the same energy, but higher luminosity) either a h^0 should be found, or on the contrary a lower limit to its mass in the ballpark of 130 GeV will be put. Thus it is of extreme importance to have both, a very precise prediction for the bound (2.50), and a very precise analysis of the Tevatron data. Of course if the MSSM is extended in some way this limit can be evaded, though not to values larger of $\sim 200\,\mathrm{GeV}$ [98,99].

To avoid colour-breaking minima in the MSSM there is also a necessary condition on squark trilinear coupling (A). The approximate (necessary) condition of absence of colour-breaking minima can be written,

$$A_q^2 < 3\,(m_{\tilde{t}}^2 + m_{\tilde{b}}^2 + M_H^2 + \mu^2)\,, \tag{2.51}$$

where $m_{\tilde{q}}$ is of the order of the average squark masses for $\tilde{q} = \tilde{t}, \tilde{b}$ [84–87].

Whatever the spectrum of the MSSM is, it should comply with the benefits that SUSY introduces into the SM which apply the following condition is fulfilled:

$$M_{\mathrm{SUSY}} \lesssim \mathcal{O}(1\ \mathrm{TeV})\ . \tag{2.52}$$

If supersymmetric particle were heavier than the TeV scale then problems with GUT's appear. This statement does not mean that SUSY would not exist, but that then the SM would not gain practical benefit from the inclusion of SUSY, part of the motivation for SUSY would be lost.

A similar upper bound is obtained when making cosmological analyses, in these type of analyses one supposes the neutralino to be part of the cold dark matter of the universe, and requires its annihilation rate to be sufficiently small to account for the maximum of cold dark matter allowed for cosmological models, while at the same time sufficiently large so that its presence does not becomes overwhelming. Astronomical observations also restrict the parameters of SUSY models, usually in the lower range of the mass parameters (see e.g. [100]).

For the various RGE analysis to hold the couplings of the MSSM should be perturbative all the way from the unification scale to the EW scale. This implies, among other restrictions, that top and bottom Yukawa couplings should be below certain limits. In terms of $\tan\beta$ this amounts it to be confined in the approximate interval

$$.5 \lesssim \tan\beta \lesssim 70 \ . \tag{2.53}$$

2.3 Left-Right Symmetric Models

Let us now consider the minimal left-right symmetric model as described in [33]. The main interest for such models [30–34] lies in part in their providing of a see-saw mechanism for neutrino masses that automatically can explain neutrino oscillations, for which there seems to be further indications [8, 9]. Below, we provide a brief description of the structure of the model needed in chapter 8. In this model the EW gauge group is enlarged to be $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and the Left (Right) components of the fermion fields are assumed to transform as $\Psi_{a\ L} \equiv (1/2,\ 0)$ and $\Psi_{a\ R} \equiv (0,\ 1/2)$ under the $SU(2)_L \times SU(2)_R$ subsymmetry.

The Higgs sector of the model consists of the bi-doublet field $\phi \equiv (1/2,\ 1/2,\ 0)$ and triplet Higgs fields: $\Delta_L(1,\ 0,\ +2) \oplus \Delta_R(0,\ 1,\ +2)$. in which we will be specially interested.

2.3.1 Lagrangian and Feynman rules

The interaction with lepton of the scalar triplet introduced in these models can be written as:

$$\mathcal{L} = i \sum_{i,j=e,\mu,\tau} g_{ij} \left(\Psi_{iL}^T \, \mathcal{C} \tau_2 \Delta \Psi_{jL} \right) + \text{h.c.} \ , \tag{2.54}$$

where $\mathcal{C}$ is the charge conjugation matrix, Ψ_{iL} are the standard lepton doublets and Δ is the scalar triplet written in the usual matrix form,

$$\Delta = \begin{pmatrix} h^+/\sqrt{2} & h^{++} \\ h^0 & -h^+/\sqrt{2} \end{pmatrix} \ .$$

Obviously, the interactions described by the lagrangian of eq. (2.54) do not conserve the lepton family numbers in general. They do conserve, however, the *total* lepton number, L, if the value $L = -2$ is assigned to the scalar triplet Δ.

We will assume the coupling constants g_{ij} to be real. From the Lagrangian 2.54 and paying special attention to the leptonic number flow (as indicated in Fig. 2.1) the relevant

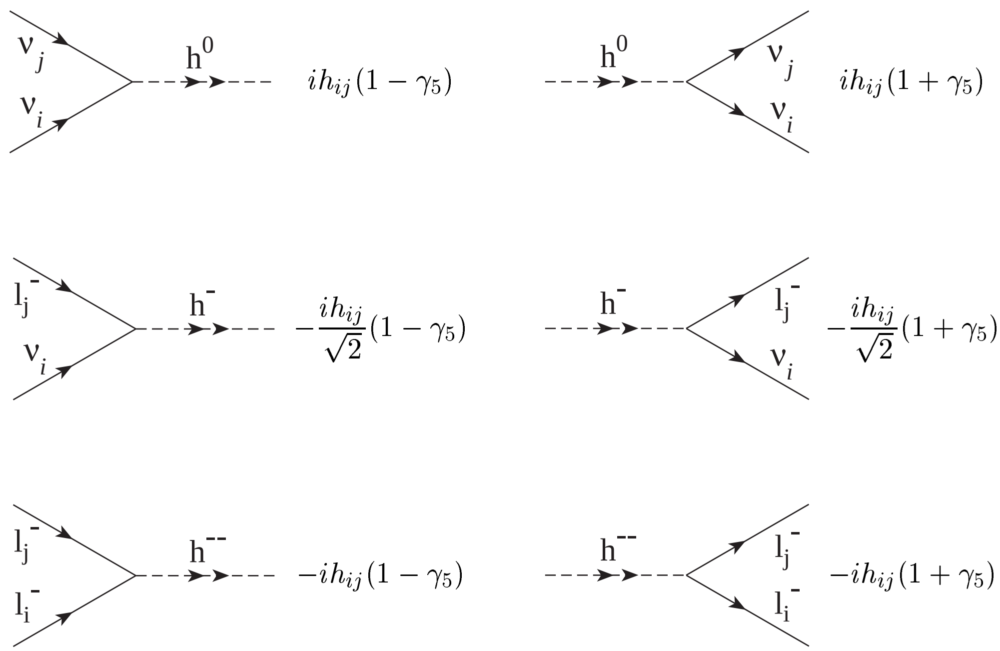


Figure 2.1: *Feynman rules corresponding to the interactions described by the lagrangian of eq. (2.54). Each arrow indicates one unit of total lepton number.*

Feynman rules may be derived. We show them in Fig. 2.1 in terms of $h_{ij} \equiv (g_{ij} + g_{ji})/2$. (Notice that, by definition, $h_{ij} = h_{ji}$.)

Chapter 3

Renormalization and Radiative Corrections

When one evaluates an observable beyond tree level in perturbation theory –radiative correction of the observable– what one usually encounters is that the just calculated quantum correction, thought to be small a priori, is divergent. Renormalization provides a framework with which to interpret this results. We will not give here a detailed study of renormalization [36, 101–104] but we will give a recipe and some details on the needed formulæ for the following chapters.

3.1 Renormalization

Any tree level lagrangian involves a certain number of free parameters that are not to be fixed by the theory, but by experiment. The definition of these parameters and their relation to measurable quantities is the context of a **renormalization scheme**. The parameters (or combinations of them) have to be fixed with specific experiments together with a the previous calculation of the results in a given theory. After doing this, thus, defining the physical inputs, other observables can be calculated and compared to experiment to verify or falsify the theory.

When computing in higher order of perturbation theory, not only is the prediction for the observable testing the model changed from that coming of a tree level study, but also the relations between the formal input parameters and their defining experiments. Moreover, the procedure of defining the input parameters and test a model is further obscured by the appearance of divergences in the calculated observables.

As hinted above renormalization provides a procedure to be able to extract physical results in these cases. In the recipe given to renormalize a theory a lot of conventions have to be used –this does not mean at all that the physical results may change with the different ways of renormalizing a theory. This set of conventions, that comprise for example giving a procedure to **regularise** the infinities –that is, to give sense to the different infinities appearing in the calculation– or specifying how to measure the formal parameters of the theory, are what is called a **renormalization scheme**.

We will use what is usually called the On-shell Renormalization Scheme [101,105,106][1]. Our way to give sense to infinities will be by means of dimensional regularisation, though other are possible and sometimes necessary (dimensional reduction...). In fact, strictly dimensional reduction should be used in SUSY, but it is not necessary in our calculations. In Appendix A we collect the regularised integrals that pop up in our calculations.

In practice, starting with the bare lagrangian, $\mathcal{L}_0 = \mathcal{L}(g_0^i,\ m_0^i,\ \phi_0^i)$ written in terms of bare fields and coupling constants (denoted with a 0 subscript), what one must do is to write the bare quantities in terms of the "renormalized", $(g_0^i,\ m_0^i)$ and the counterterms $(\delta g^i,\ \delta m^i)$:

$$\begin{aligned} g_0^i &= g^i + \delta g^i \\ m_0^i &= m^i + \delta m^i\,. \end{aligned} \tag{3.1}$$

to obtain:

$$\mathcal{L}_0 = \mathcal{L}\left(g^i,\, m^i,\, \phi^i\right) + \delta\mathcal{L}\left(\delta g^i,\, \delta m^i\right)\,. \tag{3.2}$$

Then, it is clear that if one uses the new expansion (3.2) of the bare lagrangian in the calculations –notice that the first term in the sum, the one which has the renormalized parameters, is formally identical to the bare lagrangian and– it will generate diagrams with the same divergences as the original. But now one can try to cancel these infinities with

[1] For a comprehensive exposition, see e.g. Refs. [107,108].

the help of the contributions that arise from the counterterm lagrangian $\delta\mathcal{L}$, defining the value for the counterterms in a convenient way, that is giving appropriate renormalization conditions for the counterterms –choosing the scheme. Since the effects to be compensated appear for the first time at one-loop, the counterterms will always be one order higher in the coupling constant with regard to the corresponding physical lagrangian terms. The theory being renormalizable, and once the (finite in number) counterterms have been fixed a finite prediction is guaranteed for all the physical observables.

Nevertheless we will also require minimal multiplicative renormalization, in the line of Refs. [109, 110], for the fields: $\phi_0^i = (Z^i)^{\frac{1}{2}}\,\hat{\phi}^i = \left(1+\frac{1}{2}\delta Z^i\right)\hat{\phi}^i$. This is not necessary but it is convenient since it may give finite and gauge invariant Green Functions by adjusting the new field "counterterms" (wave function renormalization constants).

3.2 The on-shell scheme in the MSSM

We will outline the steps explained in [101, 105, 106] and applied in [36, 37] to obtain the procedure to renormalize the MSSM for the physical processes discussed in this thesis.

Out of the MSSM lagrangian we are only interested in those terms which only involve standard fields since we do not have external supersymmetric particles. For the sake of simplicity we point out the u and d quark pairs from all the matter fermions. Whenever a 0 index is used, it will be denoting a bare parameter or field. Some parts of this lagrangian correspond identically to those of the SM, while the parts that involve the Higgs doublets suppose some obvious modifications.

The expression of the R-even sector in the bare lagrangian is given by:

$$\begin{aligned}\mathcal{L}_R^{\rm MSSM} &= -\frac{1}{4}W^0_{\mu\nu}W^{0\,\mu\nu} - \frac{1}{4}B^0_{\mu\nu}B^{0\,\mu\nu} + \sum_{i=1,2}\left(D^0_\mu H^0_i\right)^\dagger\left(D^{0\,\mu}H^0_i\right) + \bar{Q}^0\gamma^\mu iD^0_\mu Q^0 \\ &+ \bar{U}^0\gamma^\mu iD^0_\mu U^0 + \bar{D}^0\gamma^\mu iD^0_\mu D^0 \\ &+ \left(+h^0_u H_2^{0\,T} i\sigma_2\bar{Q}^0 U^0 - h^0_d H_1^{0\,T} i\sigma_2\bar{Q}^0 D^0 + \text{ h.c.}\right) - \mathcal{V}\left(H^0_1,\, H^0_2\right) \end{aligned} \tag{3.3}$$

In order to get finite Green functions, the minimum procedure consists on the introduction of field renormalization constants, $Z = 1 + \delta Z$, one for each representation of the gauge group that appears in the lagrangian. In this way we replace the bare fields with:

$$\begin{aligned}
W_\mu^{0\,a} &= \left(Z_2^W\right)^{1/2} W_\mu^a \,, \\
B_\mu^0 &= \left(Z_2^B\right)^{1/2} B_\mu \,, \\
Q^0 &= \left(Z^Q\right)^{1/2} Q \,, \\
U^0 &= \left(Z^U\right)^{1/2} U \,, \\
D^0 &= \left(Z^D\right)^{1/2} D \,, \\
H_i^0 &= \left(Z^{H_i}\right)^{1/2} H_i \,.
\end{aligned} \tag{3.4}$$

Simultaneously, the bare coupling constants are replaced by their expressions in terms of renormalized couplings and counterterms:

$$\begin{aligned}
g^0 &= Z_1^W \left(Z_2^W\right)^{-3/2} g \,, \\
g^{'0} &= Z_1^B \left(Z_2^B\right)^{-3/2} g' \,, \\
v_i^0 &= \left(Z^{H_i}\right)^{1/2} \left(v_i - \delta v_i\right) \,, \\
h_{u,d}^0 &= \left(Z^{H_{2,1}}\right)^{-1/2} Z_1^{u,d} \, h_{u,d} \,, \\
m_i^{0\,2} &= m_i^2 + \delta m_i^2 \,, \\
m_{12}^{0\,2} &= m_{12}^2 + \delta m_{12}^2 .
\end{aligned} \tag{3.5}$$

Note that we have used a common renormalization constant for every weak isospin doublet –in this way the counterterm Lagrangian, $\delta\mathcal{L}$, as well as the various Green's functions are automatically gauge-invariant [36, 37, 72]–, so that if we require the pole of the propagator of the down component to have a residue equal to 1, then we can not do the same with the up component. A first solution to this could be to add by hand an extra finite wave function renormalization on the up doublet components. In our case though, what we do is to introduce extra renormalization counterterms for these fields. Now we modify the notation used up to now in order to make it more general: we will have two counterterms δZ_L^f instead of δZ^Q, and in the same way, we will call $\delta Z^{U,D}$ to the former δZ_R^f. Illustrating it all with the third generation quarks, we have:

$$\begin{pmatrix} t_L^0 \\ b_L^0 \end{pmatrix} \to Z_L^{1/2} \begin{pmatrix} t_L \\ b_L \end{pmatrix} \to \begin{pmatrix} (Z_L^t)^{1/2} t_L \\ (Z_L^b)^{1/2} b_L \end{pmatrix} ,$$
$$b_R^0 \to (Z_R^b)^{1/2} b_R \,, \quad t_R^0 \to (Z_R^t)^{1/2} t_R \,. \tag{3.6}$$

Here $Z_i = 1 + \delta Z_i$ are the doublet (Z_L) and singlet ($Z_R^{t,b}$) field renormalization constants for the top and bottom quarks. What we mean is that although in the minimal field renormalization scheme there is only one fundamental constant, Z_L, per matter doublet, it is useful to work with $Z_L^b = Z_L$ and Z_L^t, where the latter differs from the former by a *finite* renormalization effect [36,37,72].

The rotation used to get the physical gauge fields (of definite mass and charge) can now be applied to the renormalized lagrangian, resulting in identical expressions to those of the tree level, but entailing physical instead of bare masses. The very same rotation has to be applied to the counterterm lagrangian, so the combinations $\delta Z_i^{\gamma,\,Z}$ and $\delta Z_i^{\gamma Z}$ are obtained in terms of $\delta Z_i^{W,\,B}$:

$$\begin{pmatrix} \delta Z_i^\gamma \\ \delta Z_i^Z \end{pmatrix} = \begin{pmatrix} s_\theta^2 & c_\theta^2 \\ c_\theta^2 & s_\theta^2 \end{pmatrix} \begin{pmatrix} \delta Z_i^W \\ \delta Z_i^B \end{pmatrix} , \tag{3.7}$$

$$\delta Z_i^{\gamma Z} = \frac{s_\theta c_\theta}{c_\theta^2 - s_\theta^2} \left(\delta Z_i^Z - \delta Z_i^\gamma \right) . \tag{3.8}$$

Other linear usually used for the fermions are:

$$\delta Z_{V,A}^f = \frac{\delta Z_L^f \pm \delta Z_R^f}{2} , \tag{3.9}$$

whereas in the Higgs sector we are going to need a counterterm directly related to the wave function renormalization of the $H^\pm$

$$\delta Z_{H^\pm} = \sin\beta^2\, \delta Z_{H_1} + \cos\beta^2\, \delta Z_{H_2} \,. \tag{3.10}$$

The SM -and in general any gauge theory- does not hold mass terms for the gauge bosons nor for the fermions in the unbroken phase of the bare lagrangian. Thus, the so-called mass counterterms associated with M_W, M_Z, m_f, are not true mass counterterms, in the sense that they do not result from the shifts (3.5), (3.4) in the bare lagrangian. It is the spontaneous breakdown of the symmetry what generates effective masses for the gauge boson and fermion fields. After rewriting the lagrangian around the true vacuum, certain combinations of the original counterterms appear, together with the effective masses, as coefficients of the bilinear gauge and fermion terms. These particular combinations may be defined to be the usual mass counterterms. Their concrete expressions in the case of the gauge bosons are,

$$\begin{aligned}
\frac{\delta M_W^2}{M_W^2} &= 2\delta Z_1^W - 3\delta Z_2^W + \cos\beta^2\,\delta Z^{H_1} + \sin\beta^2\,\delta Z^{H_2} - \frac{\delta v_1^2 + \delta v_2^2}{v^2}\,, \\
\frac{\delta M_Z^2}{M_Z^2} &= 2\delta Z_1^Z - 3\delta Z_2^Z + \cos\beta^2\,\delta Z^{H_1} + \sin\beta^2\,\delta Z^{H_2} - \frac{\delta v_1^2 + \delta v_2^2}{v^2}\,, \\
\frac{\delta M_\gamma^2}{M_\gamma^2} &= 0\,,
\end{aligned} \tag{3.11}$$

whereas for the fermions,

$$\frac{\delta m_{u,d}}{m_{u,d}} = \delta Z_1^{u,d} - \frac{\delta v_{2,1}}{v_{2,1}}\,. \tag{3.12}$$

Making all of this substitutions, the bare lagrangian is made up of a term formally identical to (3.3), but where renormalized expressions are substituting for the bare ones, plus a counterterm lagrangian, $\delta\mathcal{L}$. We must fix the value of this counterterms imposing adequate conditions for the on-shell scheme.

3.2.1 Renormalization conditions

The counterterms to fix are $\delta Z_i^{W,B}$, δZ_i^H, δv_i, δm_i^2, δm_{12}^2 plus three more for each massive fermion, $\delta Z_{L,R}^f$, δZ_1^f, and one for each neutrino, δZ_L^ν. As a whole, for N_g fermionic generations, this means $11 + 10N_g$ counterterms and the same conditions to determine them.

- A first condition is used to guarantee that no linear Higgs field terms appear in the renormalized one-loop potential, or equivalently, that the v_i still represent the true position of the minimum. This point is accomplished when the renormalized one-point Green function of neutral higgses vanishes. Diagrammatically this is realised cancelling out the tadpole loop contribution with the associated counterterm δt_i The expression for these counterterms, in terms of the original ones, is obtained by selecting the linear

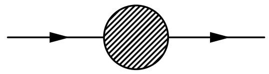
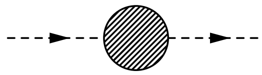
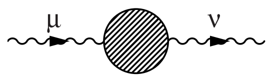
fermion	 $\equiv \frac{i}{\not{k}-m-\Sigma(k^2)} \equiv \frac{i}{\not{k}-m} + \frac{i}{\not{k}-m}\left(-i\Sigma(k^2)\right)\frac{i}{\not{k}-m}$
scalar	 $\equiv \frac{i}{k^2-m^2-\Sigma(k^2)} \equiv \frac{i}{k^2-m^2} + \frac{i}{k^2-m^2}\left(-i\Sigma(k^2)\right)\frac{i}{k^2-m^2}$
gauge boson	 $\equiv \frac{-ig_{\mu\nu}}{k^2-m^2-\Sigma(k^2)} \equiv \frac{-ig_{\mu\nu}}{k^2-m^2} + \frac{-ig_{\mu\alpha}}{k^2-m^2}\left(+ig^{\alpha\beta}\Sigma(k^2)\right)\frac{-ig_{\beta\nu}}{k^2-m^2}$

Table 3.1: *Self-energies sign conventions for the various kind of particles. The gauge bosons are dealt with in the Feynman gauge.*

ϕ_i^0 coefficients of the counterterm lagrangian. They are complex enough and not worth including, their effect is reduced to prevent from including any tadpole diagram in the calculations.

- In the on-shell scheme, the renormalized masses are made to coincide with the physical mass values, determined by the single-pole position in the propagator of the particle. This restriction is used to fix the mass counterterms. As we have previously mentioned, these are combinations of the original counterterms in (3.5). Our sign conventions for these self-energies are shown in table 3.1 and are driven by the prescription that the unrenormalized self-energy always adds up to the bare mass parameter (or the squared mass, depending on the kind of particle), that is, the mass parameter counterterm is *minus* the unrenormalized self-energy, i.e.

$$m^0 + \mathrm{Re}\left(\Sigma(k^2)\right) = m + \delta m + \mathrm{Re}\left(\Sigma(k^2) = 0\right) \ , \ \delta m = -\mathrm{Re}\left(\Sigma(k^2)\right) \ .$$

Defining $\hat{\Sigma}^*$ as renormalized selfenergies, the formulas relating amputated two-point Green functions and self-energies, are:

$$\begin{aligned} D^{-1}_{V\,\mu\nu}(k^2) &= ig_{\mu\nu}\left[k^2 - M_W^2 - \hat{\Sigma}_V(k^2)\right] , \\ S_f^{-1}(k) &= -i\left[\not{k} - m_f - \hat{\Sigma}_f(k)\right] , \end{aligned}$$

$$S_H^{-1}(k^2) \;=\; -i\left[k^2 - m_H^2 - \hat{\Sigma}_H(k^2)\right]\,, \tag{3.13}$$

where we have already replaced bare masses with the physical ones, advancing the effect of eq. (3.19). The renormalized self-energies $\hat{\Sigma}^*$ are made up of bare self-energies, and the corresponding mass and wave function counterterms:

$$\begin{aligned}
\hat{\Sigma}_\gamma(k^2) &= \Sigma_\gamma(k^2) - \delta Z_2^\gamma k^2\,,\\
\hat{\Sigma}_{\rm W}(k^2) &= \Sigma_{\rm W}(k^2) + \delta M_W^2 - \delta Z_2^W\left(k^2 - M_W^2\right)\,,\\
\hat{\Sigma}_{\rm Z}(k^2) &= \Sigma_{\rm Z}(k^2) + \delta M_Z^2 - \delta Z_2^Z\left(k^2 - M_Z^2\right)\,,\\
\hat{\Sigma}_{H^\pm}(k^2) &= \Sigma_{H^\pm}(k^2) + \delta M_{H^2} - \delta Z_{H^\pm}\left(k^2 - M_Z^2\right)\,.
\end{aligned} \tag{3.14}$$

Moreover, we will have to keep the mixture γ-Z, and (as a new feature with respect to the SM) $H^\pm$-$W^\pm$:

$$\begin{aligned}
\hat{\Sigma}_{\gamma{\rm Z}}(k^2) &= \Sigma_{\gamma{\rm Z}}(k^2) - \delta Z_2^{\gamma Z}\, k^2 + \left(\delta Z_1^{\gamma Z} - \delta Z_2^{\gamma Z}\right) M_Z^2\,,\\
\hat{\Sigma}_{\rm HW}(k^2) &= \Sigma_{\rm HW}(k^2) - \delta Z_{\rm HW} M_W^2\,.
\end{aligned} \tag{3.15}$$

In the last expression we have used the definition (see eq. 4.14 in Sec. 4.3.1)

$$\delta Z_{\rm HW} = \sin\!\beta\cos\!\beta\left[\delta Z_{H_2} - \delta Z_{H_1} + \frac{\delta v_2}{v_2} - \frac{\delta v_1}{v_1}\right]\,. \tag{3.16}$$

Finally, for the fermions, it is convenient to decompose previously the self-energy in the form

$$\Sigma_f(k) = \not{k}\Sigma_f^L(k^2)P_L + \not{k}\Sigma_f^R(k^2)P_R + m_f\Sigma_f^S(k^2)\,, \tag{3.17}$$

so that $\hat{\Sigma}_f$ can be written as

$$\begin{aligned}
\hat{\Sigma}_f \;=\;& \not{k}\left(\Sigma_f^L - \delta Z_L^f\right)P_L + \not{k}\left(\Sigma_f^R - \delta Z_R^f\right)P_R\\
&+ m_f\left[\Sigma_f^S + \frac{\delta m_f}{m_f} + \frac{1}{2}\left(\delta Z_L^f + \delta Z_R^f\right)\right]\,.
\end{aligned} \tag{3.18}$$

Using this definitions given above, the on-shell renormalization conditions for the masses read:

$$\Re\,\hat{\Sigma}_{\mathrm{W}}(M_W^2) = 0\,,$$
$$\Re\,\hat{\Sigma}_{\mathrm{Z}}(M_Z^2) = 0\,, \qquad (3.19)$$
$$\Re\,\hat{\Sigma}_f(m_f[2]) = 0\,.$$

This means two equations for the gauge bosons and two more for each massive fermion, given the fact that the chiral character of the fermionic self-energy allows to separate two independent components in the equation (3.19).

- In the SM, at low energies, there exists a residual U(1) symmetry, which can be identified with QED. The nearest renormalization conditions to those of the QED in the SM are:

$$\hat{\Gamma}^{\mu}_{\gamma ee} = (k^2 = 0,\ \not{p} = \not{q} = m_e) = ie\gamma^{\mu}\,,$$
$$\Re\ \frac{\partial}{\partial k^2}\hat{\Sigma}^{\gamma}(k^2)\Big|_{k^2=0} = 0\,, \qquad (3.20)$$
$$\Re\,\hat{\Sigma}^{\gamma \mathrm{Z}}(0) = 0\,.$$

The first condition guarantees that the electromagnetic coupling constant at low energies, defined through the renormalized amputated irreducible Green function associated to the vertex $ee\gamma$, $\hat{\Gamma}^{\mu}_{\gamma ee}$, corresponds to that of the QED. The second condition absorbs any finite photon wave function renormalization, setting to 1 the residue of the pole of the propagator; while the third one cancels the mixing term between the photon and the Z for an on-shell photon.

- We are also demanding for a residue=1 of the fermion propagator pole, for every fermionic specie:

$$\Re\ \frac{1}{\not{k} - m_f}\hat{\Sigma}_f(k^2)\Big|_{k^2=m_f^2} = 0\,. \qquad (3.21)$$

On the whole we have already defined $7 + 10N_g$ conditions. There are four conditions left that depend, as the ones referring to the tadpole diagrams, on the Higgs sector. That is three more conditions than in the SM, due to the presence of a higher number of scalar degrees of freedom. Let us remind that at tree level there are only two new parameters with respect to the SM in this sector.

Three counterterms are not enough to impose on-shell conditions to the four distinct Higgs boson masses. Therefore, one of the Higgs must be singled out from the rest, and following Sec. 4.3 it will be $H^\pm$.

- On-shell mass and propagator residue renormalization for the $H^\pm$ boson:

$$\begin{aligned} &\Re\,\hat{\Sigma}_{H^\pm}(M_{H^\pm}^2) = 0\,, \\ &\Re\,\frac{\partial}{\partial k^2}\hat{\Sigma}_{H^\pm}(k^2)\Big|_{k^2=M_{H^\pm}^2} = 0\,. \end{aligned} \tag{3.22}$$

- Vanishing $H^\pm$-$W^\pm$ mixing on the $H^\pm$ mass shell:

$$\Re\,\hat{\Sigma}_{\rm HW}(M_{H^\pm}^2) = 0\,. \tag{3.23}$$

- Renormalization of the $\tan\beta$ parameter. The simplest condition to pose referring to $\tan\beta$ consists in insisting on the validity of $\tan\beta = v_2/v_1$ for the one-loop values of Higgs potential minima. This is achieved when:

$$\frac{\delta v_1}{v_1} = \frac{\delta v_2}{v_2}\,. \tag{3.24}$$

  At the end of the day, a physical process will be needed to fix the value of $\tan\beta$, which means that process dependent terms will be engaged in the renormalization condition, so, we will prefer, as further explained in Sec. 4.3, to use the decay $H^+ \to \tau\nu_\tau$ to define $\tan\beta$.

With the complete set of renormalization conditions we have introduced, one may obtain expressions for the counterterms depending only on loop diagrams (self-energies and vertices). From this point on, all free parameters will be given in terms of finite, measured quantities, and one will be able to start calculating in perturbation theory. Let us give the explicit forms of the needed counterterms.

First of all, there are gauge related counterterms. Mass counterterms are expressed as:

$$\begin{aligned}\delta M_W^2 &= -\Sigma_W(M_W^2)\,,\\ \delta M_Z^2 &= -\Sigma_Z(M_Z^2)\,.\end{aligned} \tag{3.25}$$

introducing the notation $\Sigma'(k) \equiv \partial\Sigma(k)/\partial p^2$, the counterterms δZ_i^W are given by:

$$\begin{aligned}\delta Z_2^W &= \Sigma'_\gamma(0) - 2\frac{c_\theta}{s_\theta}\frac{\Sigma_{\gamma Z}(0)}{M_Z^2} + \frac{c_\theta^2}{s_\theta^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right)\,,\\ \delta Z_1^W &= \delta Z_2^W - \frac{1}{s_\theta c_\theta}\frac{\Sigma_{\gamma Z}(0)}{M_Z^2}\,.\end{aligned} \tag{3.26}$$

In the case of the fermions, we have both mass and wave function counterterms:

$$\begin{aligned}\frac{\delta m_f}{m_f} &= -\left[\Sigma_f^S({m_f}^2) + \frac{\Sigma_f^L(m_f^2) + \Sigma_f^R(m_f^2)}{2}\right]\,,\\ \delta Z_{L,R}^f &= \Sigma_f^{L,R}(m_f^2) + m_f^2\left[\Sigma_f^{L'}(m_f^2) + \Sigma_f^{R'}(m_f^2) + 2\Sigma_f^{S'}(m_f^2)\right]\,,\end{aligned} \tag{3.27}$$

for which we have used the decomposition 3.17), and the shortened notation $\frac{\partial}{\partial k^2}\Sigma(k^2) = \Sigma'(k^2)$.

To finish with, we get the following relationships in the Higgs sector

$$\begin{aligned}\delta M_{H^\pm}^2 &= -\Sigma_{H^\pm}(M_{H^\pm})\,,\\ \delta Z_{H^\pm} &= \Sigma'_{H^\pm}(M_{H^\pm})\,,\\ \delta Z_{\rm HW} &= \frac{\Sigma_{\rm HW}(M_{H^\pm})}{M_W^2}\end{aligned} \tag{3.28}$$

for the mass, wave function and $H^\pm$-$W^\pm$ mixed counterterms respectively (see Sec. 4.3.1). It is clear that with these "settings" the neutral Higgs fields will undergo an additional finite wave function renormalization.

In fact, we will make use of both the α or the G_F parametrizations. In the "α-scheme", the structure constant $\alpha \equiv \alpha_{\rm em}(q^2 = 0)$ and the masses of the gauge bosons, fermions and scalars are the renormalized parameters: $(\alpha, M_W, M_Z, M_H, m_f, M_{SUSY}, ...)$ –M_{SUSY} standing for the collection of renormalized sparticle masses. Similarly, the "G_F-scheme" is characterized by the set of inputs $(G_F, M_W, M_Z, M_H, m_f, M_{SUSY}, ...)$. Beyond lowest order, the relation between the two on-shell schemes is given by

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 s_W^2}(1 + \Delta r^{MSSM})\,, \tag{3.29}$$

where Δr^{MSSM} is the prediction of the parameter Δr in the MSSM[2].

[2] A dedicated study of Δr^{MSSM} has been presented in Ref [94, 111, 112].