Chapter 9

Conclusions

In this thesis we have explored various aspects of the low and high energy phenomenology of the Higgs sector beyond the Standard Model including two-Higgs-doublet models, the MSSM and Left-Right models, with particular attention to the impact of the radiative corrections on processes involving top and bottom quarks. We have made clear that the interplay between the Higgs sector and the third generation of quarks at the high-energy colliders can render distinctive phenomenology (chapters 4-6), but a knowledge of the low energy experiments provides also with invaluable information (chapters 7-8 and Sec. 4.2.1) that can not be forgotten.

- First, we have presented a treatment of the supersymmetric ($\widetilde{\mathrm{QCD}}$ and $\widetilde{\mathrm{EW}}$) effects on the decay width of $H^+ \to t\bar{b}$ showing that they could even be larger than the ordinary QCD corrections, reinforcing or counterbalancing them. These calculations were made in the $b \to s\gamma$ constrained MSSM space. We made patent that the soft-SUSY breaking trilinear A_t, together with the higgsino mass parameter μ are tightly constrained after incorporating the bounds on this decay and that they may throw a helping hand in choosing relevant MSSM parameter regions. Even in this constrained case, the $\widetilde{\mathrm{QCD}}$ quantum effects are typically the dominant ones. They lie in the range 10%-50%, are slowly decoupling and of both signs. However, there are scenarios with particle masses above the LEP200 discovery range where the $\widetilde{\mathrm{EW}}$ effects, triggered by large Yukawa couplings, can be comparable to the $\widetilde{\text{QCD}}$ ones. This situation occurs for: large $\tan\beta (\gtrsim 20)$ and sbottom masses ($\gtrsim 300\,\text{GeV}$) and relatively light stops and charginos ($\approx 100-200\,\text{GeV}$). In this context the SUSY pay-off amounts to +(30-50)%, half of it being electroweak born. These effects should be visible at the Tevatron or at the LHC through measurements made with a modest precision of 20% of $\Gamma(H^+ \to t\bar{b})$ or of the branching ratio $H^+ \to \tau^+\nu_\tau$. Moreover, they could significantly modify the prediction for single top quark production (in association with a charged Higgs boson). Furthermore, we showed that a determination of $\tan\beta$ through the Higgs decay $H^+ \to \tau^+\nu_\tau$ should be possible even for Higgs bosons as heavy as $500\,\text{GeV}$.

- Second, we have studied the $\widetilde{\text{QCD}}$ effects on the supersymmetric neutral Higgs bosons decaying into quarks. We have shown that again they are large (comparable to the ordinary QCD corrections), of both signs and slowly decoupling with $m_{\tilde{g}}$, so that their inclusion in the analysis of Higgs bosons physics at the Tevatron could be of vital importance. Not only are these effects important in the Higgs decay as we first stated, but also in the production mechanisms. In fact, work is being done in the direction to obtain the corrected production cross-sections for Higgs production (both neutral and charged) at the Tevatron, since they could be of tremendous importance to interpret the experimental results. In particular at high $\tan\beta$ the fusion mechanisms to produce MSSM Higgs bosons are the leading ones and they just involve the very same interaction vertices studied in this Thesis.

- Third, we have also shown that important EW effects can be expected from general 2HDM's of non-SUSY type. However, the spectrum of Higgs bosons would be very different in both cases (SUSY and non-SUSY) and this allows to distinguish the two models. We have computed the electroweak, Yukawa driven, one-loop corrections to the unconventional top quark decay width $\Gamma(t \to H^+ b)$ for Type I and II two-Higgs-doublet models, complementing previous studies. Numerically they range approximately from -50% to +30%. These effects alter severely previous tree level analysis presented by the *Tevatron Collaboration* that placed limits on the plane $\tan\beta$-$M_{H^\pm}$, even rendering them non-applicable. In fact, They could give the clue to unravel the underlying theory behind the discovery of a charged Higgs boson since they help to distinguish the phenomenology of the MSSM and two-Higgs-doublet models.

- Fourth, from low energy phenomenology we were also able to obtain restrictions in the MSSM parameter plane $\tan\beta$-$M_{H^\pm}$ coming from semileptonic B-meson decays, thus, improving the bounds previously given. The important results are that

    1. while $\widetilde{\text{QCD}}$ effects do not allow $\mu > 0$ case, inclusion of $\widetilde{\text{EW}}$ permits that, and that
    2. in general $\widetilde{\text{EW}}$ quantum corrections do not modify severely the limits except for areas where the $\widetilde{\text{QCD}}$ contributions are *small.*

- Lastly, the limits on the couplings and masses of a Left-Right Higgs triplet have been updated. Using low-energy neutrino physics the Higgs triplet effects would be detectable with the proposed measurement of the ratio $R_{LCD} = \sigma(\nu_\mu e)/[\sigma(\overline{\nu}_\mu e) + \sigma(\nu_e e)]$ at LAMPF, and we find that in a large part of the still allowed parameter region the Left-Right tree level effects amount to corrections as large as 50%. Indeed, scenarios as natural as having $h_{ee} \sim h_{\mu\mu} \sim g$, but with $h_{e\mu} = 0$, and $m_{h^+} \lesssim m_{h^{++}} \sim 500\,GeV$ are allowed and such an experiment would put tight bounds on this couplings and masses.

A general conclusion of our work is that radiative corrections, both on low and high-energy physics phenomena (especially those related to Yukawa couplings) are a potential source of important contributions that should be taken into account to interpret experimental results and could be of essential help to unravel new physics.

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Appendix A

D-dimensional integrals

In this appendix we collect definitions and expressions for the basic integrals in momentum space that appear in the computation of one-loop Feynman diagrams, integrals that are frequently referred to the previous chapters.

Dimensional regularisation is used throughout this appendix [1]. Except when explicitly noted, the given formulae are exact for arbitrary internal masses and external momenta. Most of them are an adaptation to the $g_{\mu\nu} = \{+ - - -\}$ metric of the standard formulae of refs. [104, 213, 214].

Basic scalar functions The requirements of our set of diagrams comprehends only one-, two- and three-point functions, represented by

$$\begin{aligned} A_0(m) &= \int d^D\tilde{q}\, \frac{1}{[q^2 - m^2]}\,, && \text{(A.1)} \\ B_0\left(p, m_1, m_2\right) &= \int d^D\tilde{q}\, \frac{1}{[q^2 - m_1^2]\left[(q+p)^2 - m_2^2\right]}\,, && \text{(A.2)} \\ C_0\left(p, k, m_1, m_2, m_3\right) &= \int d^D\tilde{q}\, \frac{1}{[q^2 - m_1^2]\left[(q+p)^2 - m_2^2\right]\left[(q+p+k)^2 - m_3^2\right]}\,, && \text{(A.3)} \end{aligned}$$

using the integration measure

$$d^D\tilde{q} \equiv \mu^{(4-D)} \frac{d^Dq}{(2\pi)^D}\,. \qquad \text{(A.4)}$$

---

[1] As already noted in Chapter 3 though strictly dimensional reduction should be used in SUSY, it is not necessary in our calculations.

Tensor integrals Other tensorial structures do appear in the calculation, leading to the following two and three-point tensor integrals

$$\left[\tilde{B}_0, B_\mu, B_{\mu\nu}\right](p, m_1, m_2) = \int d^D\tilde{q}\, \frac{[q^2, q_\mu, q_\mu q_\nu]}{[q^2 - m_1^2]\left[(q+p)^2 - m_2^2\right]}\,, \tag{A.5}$$

$$\begin{aligned}&\left[\tilde{C}_0, C_\mu, C_{\mu\nu}\right] = \\ &\qquad \int d^D\tilde{q}\, \frac{[q^2, q_\mu, q_\mu q_\nu]}{[q^2 - m_1^2]\left[(q+p)^2 - m_2^2\right]\left[(q+p+k)^2 - m_3^2\right]}\,.\end{aligned} \tag{A.6}$$

From now on, unless otherwise stated, the arguments of the one-, two- and three-point functions are understood to be those of (A.1), (A.2) and (A.3) respectively.

Lorentz covariance allows us to express tensor integrals (this is true only for one-loop integrals) in terms of the scalar ones (A.1)-(A.3) and the external momenta:

$$\begin{aligned}\tilde{B}_0 &= A_0(m_2) + m_1^2 B_0(p, m_1, m_2)\ , \\ B_\mu &= p_\mu B_1(p, m_1, m_2)\ , \\ B_{\mu\nu} &= p_\mu p_\nu B_{21}(p, m_1, m_2) + g_{\mu\nu} B_{22}(p, m_1, m_2)\ , \\ \tilde{C}_0(p, k, m_1, m_2, m_3) &= B_0(k, m_2, m_3) + m_1^2 C_0(p, k, m_1, m_2, m_3)\ , \\ C_{\mu\nu} &= p_\mu p_\nu C_{21} + k_\mu k_\nu C_{22} + (p_\mu k_\nu + k_\mu p_\nu) C_{23} + g_{\mu\nu} C_{24}\,,\end{aligned} \tag{A.7}$$

defining in a unique form the Lorentz invariant functions:[2]

$$B_1 = \frac{1}{2p^2}\left[A_0(m_1) - A_0(m_2) - f_1 B_0(p, m_1, m_2)\right]\ , \tag{A.8}$$

$$\begin{aligned}B_{21} = \frac{1}{2p^2(D-1)}\Big[&(D-2)A_0(m_2) - 2m_1^2 B_0(p, m_1, m_2) \\ &- D f_1 B_1(p, m_1, m_2)\Big]\ ,\end{aligned} \tag{A.9}$$

$$B_{22} = \frac{1}{2(D-1)}\left[A_0(m_2) + 2m_1^2 B_0(p, m_1, m_2) + f_1 B_1(p, m_1, m_2)\right]\ , \tag{A.10}$$

$$\begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} = Y \begin{pmatrix} B_0(p+k, m_1, m_3) - B_0(k, m_2, m_3) - f_1 C_0 \\ B_0(p, m_1, m_2) - B_0(p+k, m_1, m_3) - f_2 C_0 \end{pmatrix}\ , \tag{A.11}$$

---

[2] Depending on p, k only through their scalar products, p^2, pk, k^2.

$$\begin{pmatrix} C_{21} \\ C_{23} \end{pmatrix} = Y \begin{pmatrix} B_1\left(p+k, m_1, m_3\right) + B_0\left(k, m_2, m_3\right) - f_1 C_{11} - 2C_{24} \\ B_1\left(p, m_1, m_2\right) - B_1\left(p+k, m_1, m_3\right) - f_2 C_{11} \end{pmatrix}, \tag{A.12}$$

$$\begin{aligned} C_{22} \;&=\; \frac{1}{2\left[p^2k^2-(pk)^2\right]}\Big\{-pk\left[B_1\left(p+k,m_1,m_3\right)-B_1\left(k,m_2,m_3\right)-f_1C_{12}\right] \\ &\qquad +p^2\left[-B_1\left(p+k,m_1,m_3\right)-f_2C_{12}-2C_{24}\right]\Big\}\,, \end{aligned} \tag{A.13}$$

$$C_{24} = \frac{1}{2(D-2)}\Big[B_0\left(k,m_2,m_3\right)+2m_1^2C_0+f_1C_{11}+f_2C_{12}\Big]\,, \tag{A.14}$$

the factors $f_{1,2}$ and the matrix Y being

$$\begin{aligned} f_1 &= p^2+m_1^2-m_2^2\,, \\ f_2 &= k^2+2pk+m_2^2-m_3^2\,, \\ Y &= \frac{1}{2\left[p^2k^2-(pk)^2\right]}\begin{pmatrix} k^2 & -pk \\ -pk & p^2 \end{pmatrix}. \end{aligned} \tag{A.15}$$

In the $D \to 4$ limit, UV divergences may be parametrized as:

$$\begin{aligned} \epsilon &= D-4\,, \\ \Delta &= \frac{2}{\epsilon}+\gamma_E-\ln(4\pi)-\ln\mu^2\,, \end{aligned} \tag{A.16}$$

where γ_E stands for the Euler constant.

$\mathbf{A_0}$, $\mathbf{B_0}$, $\mathbf{C_0}$ evaluation At the end, from eqs. (A.8)-(A.14), one is left with the evaluation of the scalar one-loop functions:

$$A_0(m) = \left(\frac{-i}{16\pi^2}\right)m^2(\Delta-1+\ln m^2)\,, \tag{A.17}$$

$$\begin{aligned} B_0\left(p,m_1,m_2\right) &= \left(\frac{-i}{16\pi^2}\right)\Big[\Delta+\ln p^2-2+\ln[(x_1-1)(x_2-1) \\ &\qquad +x_1\ln\frac{x_1}{x_1-1}+x_2\ln\frac{x_2}{x_2-1}\Big]\,, \end{aligned} \tag{A.18}$$

$$C_0\left(p,k,m_1,m_2,m_3\right) = \left(\frac{-i}{16\pi^2}\right)\frac{1}{2}\,\frac{1}{pk+p^2\xi}\sum \tag{A.19}$$

where the value of $x_{1,2}$ is

$$\begin{aligned} x_{1,2}=x_{1,2}\left(p,m_1,m_2\right) &= \frac{1}{2}+\frac{m_1^2-m_2^2}{2p^2}\pm\frac{1}{2p^2}\lambda^{1/2}\left(p^2,m_1^2,m_2^2\right)\,, \\ \lambda\left(x,y,z\right) &= \left[x-\left(\sqrt{y}-\sqrt{z}\right)^2\right]\left[x-\left(\sqrt{y}+\sqrt{z}\right)^2\right]\,, \end{aligned} \tag{A.20}$$

and $\sum$ represents the alternating sum of (complex) Spence functions

$$\begin{aligned}\sum &= Sp\left(\frac{y_1}{y_1-z_1^i}\right) - Sp\left(\frac{y_1-1}{y_1-z_1^i}\right) + Sp\left(\frac{y_1}{y_1-z_2^i}\right) - Sp\left(\frac{y_1-1}{y_1-z_2^i}\right)\\ &-Sp\left(\frac{y_2}{y_2-z_1^{ii}}\right) + Sp\left(\frac{y_2-1}{y_2-z_1^{ii}}\right) - Sp\left(\frac{y_2}{y_2-z_2^{ii}}\right) + Sp\left(\frac{y_2-1}{y_2-z_2^{ii}}\right)\\ &+Sp\left(\frac{y_3}{y_3-z_1^{iii}}\right) - Sp\left(\frac{y_3-1}{y_3-z_1^{iii}}\right) + Sp\left(\frac{y_3}{y_3-z_2^{iii}}\right) - Sp\left(\frac{y_3-1}{y_3-z_2^{iii}}\right). \qquad \text{(A.21)}\end{aligned}$$

The Spence function is defined to be

$$Sp(z) = -\int_0^1 \frac{\ln(1-zt)}{t}\, dt\,. \qquad \text{(A.22)}$$

Besides, we have set, on the one hand

$$\begin{aligned} z_{1,2}^{i} &= x_{1,2}(p, m_2, m_1)\,,\\ z_{1,2}^{ii} &= x_{1,2}(p+k, m_3, m_1)\,,\\ z_{1,2}^{iii} &= x_{1,2}(k, m_3, m_2)\,; \qquad \text{(A.23)}\end{aligned}$$

and on the other

$$y_1 = y_0 + \xi\,, \quad y_2 = \frac{y_0}{1-\xi}\,, \quad y_3 = -\frac{y_0}{\xi}\,, \quad y_0 = -\frac{1}{2}\,\frac{g+h\xi}{pk+p^2\xi}\,, \qquad \text{(A.24)}$$

where

$$g = -k^2 + m_2^2 - m_3^2\,, \qquad h = -p^2 - 2pk - m_2^2 + m_1^2\,, \qquad \text{(A.25)}$$

and ξ is a root (real for on-shell external momenta) of

$$p^2\xi^2 + 2pk\xi + k^2 = 0\,. \qquad \text{(A.26)}$$

Scalar function derivatives When dealing with the counterterms, some derivatives of two-point functions are also required. Here we use the notation

$$\frac{\partial}{\partial p^2} B_*(p, m_1, m_2) \equiv B'_*(p, m_1, m_2)\,. \qquad \text{(A.27)}$$

As we did before, we can obtain all the needed derivatives starting from the scalar function B'_0 one, whose value is:

$$\begin{aligned} B'_0\,(p, m_1, m_2) &= \left(\frac{-i}{16\pi^2}\right)\left\{\frac{1}{p^2} + \frac{1}{\lambda^{1/2}(p^2, m_1^2, m_2^2)}\right.\\ &\left.\left[x_1(x_1-1)\ln\left(\frac{x_1-1}{x_1}\right) - x_2(x_2-1)\ln\left(\frac{x_2-1}{x_2}\right)\right]\right\}\,. \qquad \text{(A.28)}\end{aligned}$$

A remarkable fact in (A.28) is the presence of a production threshold for $|p| = m_1 + m_2$ and of a pseudo-threshold at $|p| = |m_1 - m_2|$.

Integral expressions Integral forms for the scalar functions are useful to compute them in case of precision difficulties, or to derive limit expressions.

We are using the constant

$$\kappa = \frac{-i}{16\pi^2}\,, \tag{A.29}$$

and the shorthand

$$\mathcal{R}_2^2\,(z,p,m_1,m_2) = p^2z^2 - f_1z + m_1^2\,. \tag{A.30}$$

For the two-point scalar functions one has:

$$\begin{aligned}
B_0 &= \kappa\left[\Delta + \int_0^1 dz \ln \mathcal{R}_2^2\right]\,,\\
B_1 &= \kappa\left[-\frac{1}{2}\Delta - \int_0^1 dz\, z \ln \mathcal{R}_2^2\right]\,,\\
B_{21} &= \kappa\left[\int_0^1 dz\, z^2 \ln \mathcal{R}_2^2 + \frac{\Delta}{3}\right]\,,\\
B_{22} &= \kappa\left[\frac{1}{4}\left(-\frac{p^2}{3} + m_1^2 - m_2^2\right)(\Delta - 1) + \frac{1}{2}\int_0^1 dz \mathcal{R}_2^2 \ln \mathcal{R}_2^2\right]\,.
\end{aligned} \tag{A.31}$$

Correspondingly, for the three-point ones we use

$$\mathcal{R}_3^2\,(z,p,k,m_1,m_2,m_3) = p^2x^2 + k^2y^2 + 2pkxy - f_1x - f_2y + m_1^2\,, \tag{A.32}$$

to obtain

$$\begin{aligned}
C_0 &= \kappa \int_0^1 dx \int_0^x dy \mathcal{R}_3^{-2}\,,\\
[C_{11}, C_{12}] &= \kappa \int_0^1 dx \int_0^x dy\,[-x,-y]\,\mathcal{R}_3^{-2}\,,\\
[C_{21}, C_{22}, C_{23}] &= \kappa \int_0^1 dx \int_0^x dy\,\left[x^2, y^2, xy\right]\mathcal{R}_3^{-2}\,,\\
C_{24} &= \kappa\left[\frac{1}{4} + \frac{1}{2}\int_0^1 dx \int_0^x dy \ln \mathcal{R}_3^{-2}\right]\,.
\end{aligned} \tag{A.33}$$

Limit expressions Next we gather some limit forms for the scalar functions. First we consider B's:

$$\begin{aligned}
B_0(0,m_1,m_2) &= \kappa\left(\Delta-1+\frac{m_2^2}{m_2^2-m_1^2}\ln m_2^2+\frac{m_1^2}{m_1^2-m_2^2}\ln m_1^2\right)\,,\\
B_0(0,m,m) &= -2B_1(0,m,m)=\kappa\left(\Delta+\ln m^2\right)\,,\\
B_0(0,m,0) &= B_0(0,0,m)=-\kappa+B_0(0,m,m)\,,\\
B_0(p,0,0) &= -2B_1(p,0,0)=\kappa\left(\Delta-2+i\pi+\ln p^2\right)\,;
\end{aligned} \tag{A.34}$$

$$\begin{aligned}
B_1(0,m_1,m_2) &= \frac{\kappa}{2}\left[-\Delta+\frac{1}{2}+\frac{m_1^2}{m_1^2-m_2^2}-\ln m_2^2+\left(\frac{m_1^2}{m_1^2-m_2^2}\right)^2\ln\frac{m_2^2}{m_1^2}\right]\,,\\
B_1(0,m,0) &= \frac{\kappa}{2}+B_1(0,0,m)=\frac{\kappa}{2}\left(-\Delta+\frac{3}{2}-\ln m^2\right)\,;
\end{aligned} \tag{A.35}$$

$$\begin{aligned}
B_{21}(0,m_1,m_2) &= \\
&\frac{\kappa}{3}\left[\Delta-\frac{1}{3}-\frac{1}{2}\frac{m_1^2}{m_1^2-m_2^2}-\left(\frac{m_1^2}{m_1^2-m_2^2}\right)^2+\ln m_2^2+\left(\frac{m_1^2}{m_1^2-m_2^2}\right)^3\ln\frac{m_1^2}{m_2^2}\right]\,,\\
B_{21}(0,m,m) &= \frac{\kappa}{3}\left(\Delta+\ln m^2\right)\,,\\
B_{21}(0,m,0) &= -\frac{\kappa}{2}+B_{21}(0,0,m)=-\frac{\kappa}{18}+B_{21}(0,m,m)\,,\\
B_{21}(p,0,0) &= \frac{\kappa}{3}\left(\Delta-\frac{13}{6}+i\pi+\ln p^2\right)\,;
\end{aligned} \tag{A.36}$$

$$\begin{aligned}
B_{22}(0,m_1,m_2) &= \frac{\kappa}{4}\frac{1}{m_1^2-m_2^2}\left[m_1^4\left(\Delta+\ln m_1^2\right)-m_2^4\left(\Delta+\ln m_2^2\right)-\frac{3}{2}\left(m_1^4-m_2^4\right)\right]\,,\\
B_{22}(0,m,m) &= \frac{\kappa}{2}m^2\left(\Delta-1+\ln m^2\right)\,,\\
B_{22}(0,m,0) &= B_{22}(0,0,m)=-\frac{\kappa}{8}+\frac{1}{2}B_{22}(0,m,m)\,,\\
B_{22}(p,0,0) &= \frac{\kappa}{3}p^2\left(-\frac{1}{4}\Delta+\frac{2}{3}-\frac{i\pi}{4}-\frac{1}{4}\ln p^2\right)\,.
\end{aligned} \tag{A.37}$$

When looking at three-point functions, if $p^2=0$ one has

$$\begin{aligned}
&C_0(p^2=0,k,m_1,m_2,m_3)=\frac{\kappa}{2pk}\Bigg[\\
&\quad Sp\left(\frac{y_0}{y_0-z_1^i}\right)-Sp\left(\frac{y_0-1}{y_0-z_1^i}\right)+Sp\left(\frac{y_0}{y_0-z_2^i}\right)-Sp\left(\frac{y_0-1}{y_0-z_2^i}\right)\\
&-\quad Sp\left(\frac{y_0}{y_0-z_1^{ii}}\right)+Sp\left(\frac{y_0-1}{y_0-z_1^{ii}}\right)-Sp\left(\frac{y_0}{y_0-z_2^{ii}}\right)+Sp\left(\frac{y_0-1}{y_0-z_2^{ii}}\right)\Bigg]\,,
\end{aligned} \tag{A.38}$$

with

$$\begin{aligned} z^{i}_{1,2} &= x_{1,2}(k, m_2, m_3)\,, \qquad y_0 = \frac{m_1^2 - m_2^2}{2pk}\,, \\ z^{ii}_{1,2} &= x_{1,2}(p+k, m_1, m_3)\,. \end{aligned} \tag{A.39}$$

If one also has $(p+k)^2 = 0$, then

$$\begin{aligned} C_0(p,k,m_1,m_2,m_3)\Big|_{p^2=(p+k)^2=0} &= -\frac{\kappa}{k^2}\left[Sp\left(\frac{y_0}{y_0 - z_1^i}\right) - Sp\left(\frac{y_0 - 1}{y_0 - z_1^i}\right)\right. \\ + \quad Sp\left(\frac{y_0}{y_0 - z_2^i}\right) - Sp\left(\frac{y_0 - 1}{y_0 - z_2^i}\right) &\left. - Sp\left(\frac{y_0}{y_0 - z^{ii}}\right) + Sp\left(\frac{y_0 - 1}{y_0 - z^{ii}}\right)\right]\,, \end{aligned} \tag{A.40}$$

$z^i_{1,2}$ being those of (A.39), and z^{ii}, y_0

$$z^{ii} = \frac{m_1^2}{m_1^2 - m_3^2}\,, \qquad y_0 = \frac{m_2^2 - m_1^2}{k^2}\,. \tag{A.41}$$

Finally, if even $s^2 = 0$,

$$C_0(0,0,m_1,m_2,m_3) = \frac{\kappa}{m_3^2 - m_2^2}\left(\ln\frac{m_3^2}{m_2^2} + \frac{m_1^2}{m_3^2 - m_1^2}\ln\frac{m_3^2}{m_1^2} - \frac{m_1^2}{m_2^2 - m_1^2}\ln\frac{m_2^2}{m_1^2}\right)\,, \tag{A.42}$$

a symmetric expression under permutation of the masses. In this latter case, the expansions (A.11)-(A.13) are singular and no longer useful. Thus, starting again from (A.31), (A.33)

$$\begin{aligned} \tilde{C}_0(0,0,m_1,m_2,m_3) &= \kappa\left(\Delta - 1 + \frac{m_1^4}{(m_2^2 - m_1^2)(m_3^2 - m_1^2)}\ln m_1^2\right. \\ &\quad + \frac{m_2^4}{(m_1^2 - m_2^2)(m_3^2 - m_2^2)}\ln m_2^2 \\ &\quad \left. + \frac{m_3^4}{(m_1^2 - m_3^2)(m_2^2 - m_3^2)}\ln m_3^2\right)\,, \\ C_{24}(0,0,m_1,m_2,m_3) &= \frac{1}{D}\tilde{C}_0(0,0,m_1,m_2,m_3)\,. \end{aligned} \tag{A.43}$$

Some useful relations From the definitions and the integral formulae (A.31), (A.33) some relations are easily derived that show up very useful in doing selfconsistency checks of the calculation:

$$\begin{aligned} 0 &= B_0(p,m_1,m_2) + B_1(p,m_1,m_2) + B_1(p,m_2,m_1)\,, \\ 2B_{22} &= p^2 B_{21} + f_1 B_1 + m_1^2 B_0 - \frac{\kappa}{2}\left(-p^2 + m_1^2 + m_2^2\right)\,, \end{aligned}$$

$$\begin{aligned}\tilde{C}_0 &= p^2 C_{21} + k^2 C_{22} + 2pkC_{23} + 4C_{24} + \frac{\kappa}{2}\,, \\ \frac{\kappa}{2} &= p^2 C_{21} + k^2 C_{22} + 2pkC_{23} + f_1 C_{11} + f_2 C_{12} + m_1^2 C_0\,. \end{aligned} \tag{A.44}$$