CHAPTER 2: Vector Smooth Transition Regression Models for US GDP and the Composite index of Leading Indicators

1 Introduction

Much effort has been devoted to evaluate how well time series models represent real US output and to connect the evolution of this series with the business-cycles phenomenon. Additionally, many authors have tried to improve the accuracy of univariate models of output in two ways. First, by using nonlinear models that try to capture the possible nonlinearities in output dynamics. Second, by including certain variables that incorporate additional information about output features. In this paper, we are interested in examining the (possible nonlinear) accuracy of the Composite index of Leading Indicators (CLI) information to anticipate both future output changes and turning points.

Until the 1990s, the emphasis of this literature has been on linear models. Univariate linear models of output have basically followed extensions of the seminal analysis of Box and Jenkins (1976). The most significant multivariate linear analysis for examining the accuracy of CLI at anticipating output features are the work of Auerbach (1982), Braun and Zarnowitz (1989), and Diebold and Rudebusch (1991).

However, during the current decade, several studies have found evidence in favor of nonlinear behavior in output. Hamilton (1989), Teräsvirta and Anderson (1992), Tiao and Tsay (1994), Potter (1995), Teräsvirta (1995), and Pesaran and Potter (1997) propose alternative univariate nonlinear approaches to US output. On the other hand, Granger, Teräsvirta and Anderson (1993), Estrella and Mishkin (1998), and Filardo (1992, 1999), examine with nonlinear multivariate models the predictive performance of CLI.

All of these studies suffer at least one of the following drawbacks. Firstly, linear models implicitly impose strong symmetry properties. Secondly, univariate models lose the leading information that CLI may incorporate into the system. Thirdly, non-vector autoregressive models fail to capture the dynamic interactions among the variables in the model. To our knowledge, only Hamilton and Perez-Quiros (1996) propose a nonlinear VAR model for analyzing the accuracy of those variables.

Our contribution to the previous literature is twofold. First, we propose a vector autoregressive extension of the STR model developed by Granger and Teräsvirta (1993). By analogy, we call it Vector Smooth Transition Regression (VSTR) model. The primary principle for estimation is maximum likelihood. This approach leads to simple linearity and model selection tests. In line with Eitrheim and Teräsvirta (1996), we also extend to the VAR context the tests for examining the accuracy of VSTR models. Finally, we analyze the most recent model selection techniques in order to formally select one model from the family of VSTR. Thus, we focus the multiple-equation STR models in an alternative view to Weise (1999) and Rothman et al. (2001).

Second, we contribute to the empirical literature of CLI in nonlinear models by applying the VSTR methodology to model the nonlinear features of GDP and CLI together. We find that a logistic-VSTR is more accurate than any other nonlinear VSTR specification. In anticipating growth, these gains come basically from recessionary periods. In replicating the US business-cycle phases, our proposed model is unequivocally the best.

The plan of the paper is as follows. The baseline model is presented in section 2.

Section 3 deals with linearity tests, model selection procedures and a brief discussion about the techniques used for comparing the accuracy of the nonlinear models. Empirical results are considered in Section 4. The final section contains concluding remarks and suggests directions for future research.

2 The baseline model

Consider the following vector autoregressive generalization of the STR model:

$$\begin{aligned} y_t &= \beta_y' A_t + (\widetilde{\beta}_y' A_t) F_y(D_{ty}) + u_{yt}, \\ x_t &= \beta_x' A_t + (\widetilde{\beta}_x' A_t) F_x(D_{tx}) + u_{xt}, \end{aligned} \tag{1}$$

where $A_t = (1, y_{t-1}, x_{t-1}, ..., y_{t-p}, x_{t-p})' = (1, X_t')'$, $\beta_y' = (\eta_y, a_1, b_1, ..., a_p, b_p)$, $\beta_x' = (\eta_x, c_1, d_1, ..., c_p, d_p)$, $\widetilde{\beta}_y' = (\widetilde{\eta}_y, \widetilde{a}_1, \widetilde{b}_1, ..., \widetilde{a}_p, \widetilde{b}_p)$, $\widetilde{\beta}_x' = (\widetilde{\eta}_x, \widetilde{c}_1, \widetilde{d}_1, ..., \widetilde{c}_p, \widetilde{d}_p)$, and

$$U_t = (u_{yt}, u_{xt})' \sim N\left[0, \Omega\right]. \tag{2}$$

The key component of a VSTR model is the *transition function* F. By convention, it is bounded between zero and one. If F is zero, then the baseline model becomes a linear VAR (VARa), with parameters β_y and β_x. On the other hand, if F is one, then the VSTR model becomes another linear VAR (VARb), with parameters $\beta_y + \widetilde{\beta}_y$ and $\beta_x + \widetilde{\beta}_x$. Hence, F may be interpreted as a filtering rule that locates the model between these two extreme regimes. This section presents a brief discussion about the economic interpretation of VSTR models, depending on the form of the transition function.

2.1 Logistic transition function

In this case, F is the following monotonically increasing function:

$$F_i(D_{ti}) = \frac{1}{1 + e^{-\gamma_i D_{ti}}}, \tag{3}$$

where γ_i is the *smoothness parameter*, and $i = y, x$. We refer to D_{ti} as *switching expression* which may present two alternative forms. First, D_{ti} may be the difference between a proposed *transition variable* z_{ti}, which is usually a lagged value of y and x, and an estimated *threshold* g_i, that is

$$D_{ti} = z_{ti} - g_i. \tag{4}$$

We call a logistic VSTR model with switching expression (4) Logistic VSTR (LVSTR(z_{ty}, z_{tx})). Note that, as γ_i approaches infinity, F_i converges to the Heaviside function. In this extreme case, the baseline model generalizes to a VAR the SETAR model proposed by Tsay (1989).

Second, D_{ti} may be the weighted average of the q_i lagged deviations from a linear path:

$$D_{ti} = \sum_{j=1}^{q_i} w_{ij} \widehat{u}_{i,t-j}, \tag{5}$$

where $\sum_{j=1}^{q} w_{ij} = 1$, and $\widehat{u}_i$ is the estimated residual of the i-th equation from a linear path. Similarly, a logistic VSTR model with D_{ti} as in (5), represents the LVSTR-Deviated (LVSTR-D(q_y, q_x)) models.

Applied to GDP and CLI rates of growth, logistic models have a nice economic interpretation. Assume that $\widetilde{\beta}$ and γ are both greater than zero. In logistic models, VARa (F close to zero) is interpreted as the linear path which models extreme recessionary periods whereas VARb (F close to one) can be seen as the linear model associated with great expansions. To see this, note that in extreme contractions (expansions) the transition variable is lower (higher) enough than the threshold in LVSTR models, and the actual GDP is less (greater) enough than a linear path in LVSTR-D models for keeping the transition function close to zero (one). Thus, the transition function locates the model either near to or far from recessions, depending on the switching expression's values.

2.2 Exponential transition function

Consider the exponential transition function

$$F_i(D_{ti}) = 1 - e^{-\gamma_i D_{ti}}, \tag{6}$$

where $i = y, x$. Assume the following alternative forms for the switching expression. First, let D_{ti} be the squared difference between the transition variable and the threshold,

$$D_{ti} = (z_{ti} - g_i)^2. \tag{7}$$

Let us denote an exponential model with switching expression (7) as Exponential VSTR (EVSTR(z_{ty}, z_{tx})).

Second, let D_{ti} be the weighted sum of the q lagged squared deviations from a linear path

$$D_{ti} = \sum_{j=1}^{q_i} w_{ij} \widehat{u}_{i,t-j}^2, \tag{8}$$

where w_{ij} and $\widehat{u}_i$ are the same as in (5). We refer to these model as EVSTR-Deviated (EVSTR-D(q_y, q_x)).

Applied to GDP and CLI, exponential models have different economic interpretation to logistic models. Now, VARa can be associated with a middle ground, whereas troughs and peaks have similar dynamic structures represented in VARb. That is to say, if either the transition variable is different to the threshold in the EVSTR, or the model deviates from a linear path in the EVSTR-D, then F becomes different to zero, and the model smoothly approximates from the middle ground to any of the extreme situations represented by VARb ($F = 1$).

3 Specification of VSTR models

The aim of this section is to describe a battery of model selection rules in order to obtain one nonlinear specification from the set of VSTR models described in Section 2. Note that, since we base the estimation of VSTR upon the maximum likelihood principle, any test may be carried out through simple likelihood ratio tests. Additionally, to economize on notation we restrict the analysis to the case of $z_{ty} = z_{tx} = z_t$, and $q_y = q_x = q$.[1]

In the spirit of the seminal methodology in Tsay (1989), Figure 1 describes the procedure for modelling VSTR systems. First, we specify a linear VAR and its maximum lag length using standard linear techniques. Second, we apply linearity tests to the auxiliary regressions described in Table 1. This requires that we have to select a priori a set of variables to include in the switching expression. In deviated models, this implies selecting the maximum value of q. In the remaining cases, the natural way of selecting the candidates for being transition variables is to try with lagged values of y and x.[2] Third,

[1] In deviated models, this implies that the system is deviated from the linear path according to the same number of lagged deviations for both GDP and CLI. In the remaining cases, this implies that the same transition variable locates the entire system between regimes.

[2] Both maximum value of q ($qmax$) and maximum lag of x and y ($lagmax$), depend upon the frequency of the data. For example, with montly data, it is advisable to try for $qmax = lagmax = 12$, whereas with

once linearity is rejected, we decide the specific form of the transition function with the sequence of model selection tests appearing in Table 2.[3]

Up to this point, the VAR generalization of STR models is straightforward. However, note that the sequence of tests described above finds as nonlinear models as rejections of linearity. Teräsvirta (1994) suggests that in such case the selected model should be the one with the smallest p-value in the linearity test. However, this procedure involves two drawbacks. First, we may find appropriate estimates and forecasts of the nonlinear model even if linearity is weakly rejected. Second, it is not clear what to do in case of similar p-values. The remain of the section tries to guard against these drawbacks by basing the decision upon an a posteriori evaluation of the accuracy of the estimated nonlinear models at capturing the nonlinearities of GDP and business-cycles.

3.1 Testing the adequacy of VSTR models

Eitrheim and Teräsvirta (1996) propose three kind of tests for evaluating the adequacy of the estimated single-equation nonlinear model. Specifically, they consider that a model with Serially Independent errors (test SI), with No Remaining Nonlinearity (test NRN), and with Parameter Constancy (tests PC) may be considered as adequate for fitting the

quarterly data may be enough to try for $qmax = lagmax = 4$.

[3]Linearity and model selection tests are based upon linear approximations of VSTR models about $\gamma = 0$, in line with Luukkoven et al. (1988). In the case of logistic models that include the transition variable belonging to the set of explanatory variables, the identification problem is avoided by using a third order linear approximation. In deviated models, a second order approximation is necessary for discriminating between logistic and exponential models. In the remaining cases, identification is achieved with a first order Taylor approximation.

data. This section extends these tests to a multiple-equations framework.

To derive the test SI, we consider the alternative baseline model that takes into account the possible serial dependence of errors:

$$Y_t = G(\varphi_t, \Psi) + U_t, \tag{9}$$

where $Y_t = (y_t, x_t)'$, $\varphi_t = (1, y_{t-1}, ..., y_{t-p}, x_{t-1}, ..., x_{t-p})'$, $G(\varphi_t, \Psi) = (G_y(\varphi_t, \Psi_y), G_x(\varphi_t, \Psi_x))'$, Ψ_i is the $(\frac{g}{2} \times 1)$ vector of unknown parameters contained in both the autoregressive lags and in the transition function. Instead of (2), errors are assumed to evolve as

$$U_t = \Phi(L)U_t + \varsigma_t, \ \varsigma_t \sim N[0, \Gamma]. \tag{10}$$

Here, $\Phi(L) = (\Phi_1 L + ... + \Phi_r L^r)$ indicates a (2×2) matrix polynomial in the lag operator L, and Γ is a (2×2) matrix of constant parameters. The null hypothesis of serial independence of errors is $H_0 : \Phi_1 = \cdots = \Phi_r = 0$. As we show in the Appendix, the statistic

$$LM = \frac{1}{T} m'_\Phi \left(M_{\Phi\Phi} - M_{\Phi\Psi} \left(M_{\Psi\Psi}\right)^{-1} M'_{\Phi\Psi} \right)^{-1} m_\Phi, \tag{11}$$

follows under the null a χ^2 distribution with $4r$ degrees of freedom.[4]

[4] A tilde above any expression refers to its maximum likelihood estimate under the null. Let $\widetilde{V}_t$ be the $(2r\times1)$ matrix $(\widetilde{v}'_{yt}, \widetilde{v}'_{xt})'$, where $\widetilde{v}_{it} = (\widetilde{u}_{i(t-1)}, ..., \widetilde{u}_{i(t-r)})'$, and let $\widetilde{Z}_t$ be the $(\frac{g}{2}\times2)$ matrix $(\widetilde{z}_{yt}, \widetilde{z}_{xt})$, where $\widetilde{z}_{it}$ is $\partial\widetilde{G}_i/\partial\Psi_i = \partial G_i(\varphi_t, \widetilde{\Psi}_i)/\partial\Psi_i$, with $i = y, x$. Expression $\partial\widetilde{G}_i/\partial\Psi_i$ is composed by three vectors. The first is $\partial\widetilde{G}_i/\partial\alpha_i = A_i$. The second is $\partial\widetilde{G}_i/\partial\widetilde{\alpha}_i = \widetilde{A}_i$. The third term depends on the selected transition function. If this corresponds to non-deviated models, then $\partial\widetilde{G}_i/\partial\gamma_i$ and $\partial\widetilde{G}_i/\partial g_i$ appear in Eitrheim and Teräsvirta (1996). However, if the transition function corresponds to deviated models, then the last expression is zero. Thus, the following identities hold: $m_\Phi = \sum \left(\widetilde{\Gamma}^{-1}\widetilde{U}_t \otimes \widetilde{V}_t\right)$, $M_{\Phi\Phi} = \frac{1}{T}\sum \left(\widetilde{\Gamma}^{-1} \otimes \widetilde{V}_t\widetilde{V}'_t\right)$, $M_{\Phi\Psi} = \frac{1}{T}\sum \left(\widetilde{\Gamma}^{-1}\widetilde{Z}_t \otimes \widetilde{V}_t\right)$, and $M_{\Psi\Psi} = \frac{1}{T}\sum \left(\widetilde{Z}_t\widetilde{\Gamma}^{-1}\widetilde{Z}_t\right)$, where $\otimes$ denotes the Kronecker product and T is the sample size.

To obtain the test NRN it is useful to rewrite the baseline model allowing for additive misspecification as follows:

$$\begin{aligned} y_t &= \beta_y' A_{yt} + (\widetilde{\beta}_y' A_{yt}) F_y^1(D_{ty}^1) + (\widetilde{\theta}_y' A_t) F_y^2(D_{ty}^2) + u_{yt}, \\ x_t &= \beta_x' A_{xt} + (\widetilde{\beta}_x' A_{xt}) F_x^1(D_{tx}^1) + (\widetilde{\theta}_x' A_t) F_x^2(D_{tx}^2) + u_{xt}, \end{aligned} \tag{12}$$

where F is the transition function analyzed in previous sections. Following the usual linear approximations of F_i^2, Table 4 describes a test which has power against an omitted additive VSTR component.[5]

Finally, the tests PC are obtained under the assumption that the transition function has constant parameters, whereas both β_i and $\widetilde{\beta}_i$ may change over time. We assume that the change may be possibly non-monotonic and not necessarily symmetric. This may be modelled as follows: $\beta_i(t) = \beta_i + \lambda_{1i} H_i$ and $\widetilde{\beta}_i = \widetilde{\beta}_i + \lambda_{2i} H_i$, where $i = y$ and x, and

$$H_i = (1 + \exp\{-\gamma_i(t^k + s_{i(k-1)} t^{k-1} + ... + s_{i1} t + s_{i0})\})^{-1} - 0.5. \tag{13}$$

Substracting one-half is useful just in deriving the tests. After linear approximations of H_i Table 5 describes a test against the null of time-varying parameters.

3.2 Investigating the predictive accuracy

In this section, we propose a method for checking the accuracy of the nonlinear models at forecasting output growth and turning points. The former may be checked by using the

[5]Following Eitrheim and Teräsvirta (1996), this test be generalized to have power not only against an omited additive VSTR component but also against omission of important lags from the estimated VSTR model.

well-known Mean Square Error

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(y_t - \widehat{y_t})^2, \tag{14}$$

based upon the distance between actual (y) and estimated ($\widehat{y}$) GDP growth (T is the sample size). The latter may be investigated with the loss function Turning Points Error

$$TPE = \frac{1}{T}\sum_{t=1}^{T}(d_t - \widehat{d_t})^2, \tag{15}$$

where d_t is an indicator variable taking value 1 at the official NBER recessions. Recall that logistic transition functions (in models with baseline parameters greater than zero) could be interpreted as probabilities of being in expansion. This leads to define $\widehat{d_t} = 1 - F_y(z_{ty})$.

We want not only to order the different models, but would also like to test whether one of them is significantly superior to the others. Diebold and Mariano (1995) provide a statistic (DM henceforth) for testing the hypothesis of no difference in the accuracy of any two competing forecasts that follows a $N(0,1)$ distribution under the null.[6]

4 Empirical results

In this section we examine the nonlinear relations between real GDP, and the Composite Index of Leading Indicators (CLI) using both in-sample and real-time analysis.[7]

[6] See Camacho and Perez-Quiros (2000) for a specific definition of the DM test applied to forecasting GDP growth and business-cycles.

[7] The composite index of leading indicators is a weighted average of ten macroeconomic leading variables which are expected to turn before the aggregate economy. This series, issued by the Conference Board, is subject to statistical revisions (due to revisions in the components), and definitional revisions (index's components are reselected and reweighted).

In-sample, we analyze the 151 quarterly observations of GDP running from June 1960.2 to 1997.4. We use the monthly CLI series released in January 1998 (the first which contains figures for December 1997), which is transformed into quarterly series by selecting the data corresponding to the last month of each quarter.

In real-time, we predict values from 1972.2 to 1998.1. At any quarter t we use the CLI series published two months after the beginning of such quarter (these substitute the approximate figures available one month after the beginning of each quarter) converting the quarterly CLI observations into monthly series as in the in-sample analysis. [8]

Following the specification strategy outlined in Figure 1, we need to specify an appropriate linear VAR which is the base for the nonlinear models. In a preliminary analysis of data, the augmented Dickey-Fuller tests detect unit root in the log of both variables. This suggests the use of the stationary rate of growth of output and CLI, hereafter y and x, for estimating a VAR model of order one (the Schwarz selection criterion was minimized at one) in first differences (the Johansen's ratio test of the null of no cointegration is 14.48). This contradicts previous results in the literature (Granger et al., 1993, and Hamilton and Perez-Quiros, 1996), confirming that the absence of cointegration with GDP is a characteristic of the latest CLI revisions.

Linearity and model selection tests require the specification of a set of variables z and a set of values of q. For the former we use lagged values of x and y within a year. For

[8] For example, to obtain the first real-time prediction we estimate the models with GDP available in 1972.1 and the CLI series issued in February (representing the information available until 1971.4). We use these estimates to forecast GDP for the second quarter of 1972 (figures available in August) with the CLI series available in May (representing the information available up to the first quarter). The procedure is updating until the last forecast.

the latter we use (square) weighted averages of the one to four lagged deviations from the linear path. Model selection tests are applied for each candidate rejecting linearity. Table 6 displays that one logistic ($z_t = y_{t-2}$), three exponential ($z_t = x_{t-1}$, x_{t-2}, and x_{t-3}), and one logistic-deviated ($q = 1$) models are the candidates for representing the nonlinearities of output and CLI together.

We estimate the parameters of the models by maximum likelihood, checking whether the size of the model can be reduced. First, the nonlinear autoregressive parameters are examined to be significant. Second, autoregressive parameters corresponding to the least significant (if nonsignificant) estimates are repetitively removing and reestimating the reduced model. This procedure leads to final specifications which agree with Hamilton and Perez-Quiros (1996) results: lagged growth does not help in forecasting either current growth or current CLI in any bivariate model. Additionally, the intercept is proved to be statistically insignificant both on the second equation of EVSTR(x_{t-1}), EVSTR(x_{t-3}), and LVSTR-D(1), and on the first equation of LVSTR(y_{t-2}).[9] Finally, $\widetilde{b}$ and $\widetilde{d}$ are jointly insignificant in each nonlinear specification. Table 7 shows estimates of the significant parameters.[10]

In illustrating how these nonlinear models work, let us analyze the output equation of LVSTR(y_{t-2}) and EVSTR(x_{t-2}). The logistic model presents a smoothness parameter 1.85, indicating that the transition between the two extreme regimes (characterized by $F = 0$ and $F = 1$) is smooth. The estimated threshold is 0.13 and marks the

[9] Results leading to these choices have been omitted, but are available from the author upon request.

[10] Note that, as Teräsvirta (1994) has emphasized, a precise joint estimation of the smoothness parameter and the threshold is a problem when the former is large.

halfway point between regimes. On the other hand, the exponential model shows a much higher smoothness parameter (11.20) which implies sharper transitions between the middle ground ($F = 0$), marked by values of x_{t-2} near to 0.83, and the other extreme regime ($F = 1$). Figure 2 shows the transition function values as a function of the observed switching expressions. These pictures allow the reader to readily see some characteristics of the transition function: its shape and its more frequent values.

The first approximation to the estimated models is the study of the adequacy of these specifications to the data. For this purpose, Table 8 reports the results of the tests SI for values of r from 1 to 4, the test NRN, and the tests PC for values of k from 1 to 3.[11] These show that there is evidence of serially uncorrelated errors and parameter constancy of any model at any lag, but EVSTR(x_{t-2}). The p-values of the test NRN show that there is no strong evidence against these models.[12] Thus, we conclude that the proposed models pass the tests of accuracy fairly well.

Tables 9 and 10 describe the performance of the models' predictions at representing both the US GDP growth and the US business-cycles phases.[13] As measures of the accuracy of the models at forecasting, Table 9 reports the Certain Negative Rates (CNR) and Certain Positive Rates (CPR). The former (latter) signals the percentage of quarters when the models correctly anticipate GDP falls and recessions (GDP rises and expansions).[14]

[11] Note that monotonic as well as nonmonotonic changing parameters are special cases of the test PC with $k = 3$.

[12] The unique exception is the p-value of 0.01 of EVSTR(x_{t-3}), but this has not been followed up since the number of parameters which would appear make the model intractable.

[13] In line with Stock and Watson (1992), we interpret an estimated probability of recession above 0.75 (below 0.25) as a signal of recession (expansion).

[14] Let Nl (Ng) be the number of quarters when the rate of growth of GDP is actually less (greater) than

On the other hand, Table 10 shows the False Negative Signal Rates (FNR) and False Positive Signal Rates (FPR) which examine the percentage of mistakes when the models forecast GDP rises and recessions (FNR) and GDP failures and recessions (FPR).[15] These Tables point out that LVSTR(y_{t-2}) always presents one of the best accuracy and that the false signals rate of LVSTR(y_{t-2}) is similar (if not lower) to the other nonlinear models' rates at estimating growth, but much lower than the rate of LVSTR-D(1) at replicating the NBER schedule.

Finally, we formally test the in-sample and out-of-sample accuracy of the selected nonlinear models at identifying both actual GDP growth and the NBER chronology. At fitting GDP growth in-sample, Table 11 reveals that LVSTR(y_{t-2}) is the model with lowest MSE, especially during recessions. Additionally, Table 12 confirms the within-recessions improvement in the accuracy of such model against any other nonlinear specification, but EVSTR(x_{t-3}). On the other hand, LVSTR(y_{t-2}) is both numerically (TPE of 0.16 vs. 0.51) and statistically (DM equals to 8.52) superior to LVSTR-D(1) at identifying in-sample the US business-cycles .

zero, and among them, let $\widehat{Nl}_i$ ($\widehat{Ng}_i$) be the number of these quarters for which model i estimates that GDP falls (rises). In addition, let Nr (Ne) be the number of quarters of NBER recessions (expansions), and let $\widehat{Nr}_i$ ($\widehat{Ne}_i$) be the number of these periods for which the estimated probability of recession from model i is above 0.75 (below 0.25). Thus, $\widehat{Nl}_i/Nl$ and $\widehat{Nr}_i/Nr$ ($\widehat{Ng}_i/Ng$ and $\widehat{Ne}_i/Ne$) are defined as the CNR (CPR) of model i.

[15]Let $\widehat{Nl}_i$ ($\widehat{Ng}_i$) be the number of quarters for which model i estimates that GDP falls (rises), and let Ng (Nl) be the number of these quarters for which GDP actually rises (falls). Also let $\widehat{Nr}_i$ ($\widehat{Ne}_i$) be the number of quarters for which the estimated probability of recession from model i is above 0.75 (below 0.25), and let Ne (Nr) be the number of these quarters classified by the NBER as expansions (recessions). Hence, $Ng/\widehat{Nl}_i$ and $Ne/\widehat{Nr}_i$ ($Nl/\widehat{Ng}_i$ and $Nr/\widehat{Ne}_i$) are the FPR (FNR)of model i.

Let us move to the out-of-sample results. The last two columns of Table 11 show that LVSTR(y_{t-2}) is again the model with lowest MSE. However DM tests do not find that such model was statistically superior in real-time.[16] At forecasting the NBER schedule, LVSTR(y_{t-2}) presents numerical (TPE of 0.17 vs. 0.39) and statistical (DM of 4.77) gains with respect to LVSTR-D(1).

Summarizing, LVSTR(y_{t-2}) is the selected nonlinear model for three reasons. First, Table 8 highlights the poorer adequacy of exponential models to the data. Second, Table 11 concludes that LVSTR(y_{t-2}) shows numerically better accuracy than any other nonlinear specification at representing both the US GDP growth and the US business cycles phases. Additionally, Table 12 presents evidence in favor of its statistical superiority, especially during recessions. These facts were anticipated by the analysis of Tables 9 and 10, where LVSTR(y_{t-2}) presented the best certain and false positive and negative rates. Third, logistic models allow for a richer economic interpretation when applied to output growth since they can be used for forecasting the US business-cycles behavior.

One of the major contributions of this paper is to analyze the ability of STAR models in forecasting the business-cycles sequence. Figure 3 assesses the degree to which LVSTR(y_{t-2}) characterizes the US business-cycle fluctuations in real-time. Specifically, its transition function may classify the data into two subsamples based on $1 - F$ being above and below 0.5.[17] With this criterion, we detect correlation between low values of the transition function (high values of $1 - F$) and the NBER recessions. Thus, the transition

[16] Note that, when we restrict the analysis to recessionary periods, LVSTR(y_{t-2}) is better than LVSTR-D(1) and EVSTR(x_{t-1}), and there exists some evidence in favor of using this model instead of EVSTR(x_{t-2}).

[17] Among others, this criterion is used by Filardo (1994), and Etrella and Mishkin (1998).

function may be considered as a filter which infers the probability of being in recession.

5 Conclusion

We provide a vector autoregressive extension of Smooth Transition Regression models. Following maximum likelihood estimation, we have adapted linearity, model selection and model adequacy tests, as long as several model evaluation techniques to analyze these models.

We have found empirical evidence in favor of nonlinear behavior of US output and CLI. Specifically, we select a logistic nonlinear model whose transition variable is output growth with two periods of lag. The estimated nonlinear specification describes the US output growth features fairly well, especially at the more turbulent recessionary periods. This is confirmed by both the in-sample and the real-time analysis. Interestingly, this nonlinear model is also able to anticipate the US official NBER business-cycles phases.

Appendix. LM test of serial independence of errors in VSTR models

It is well known that the test statistic (11) follows a limiting χ^2 distribution with as many degrees of freedom as the number of parameters which are assumed to be zero under the null. Thus, our target is to find an explicit definition for expressions appearing in this test.

Following the notation used Section 3.4.1, let the $(1 \times r)$ vector $\Phi'_{ij} = \left(\Phi^1_{ij}, \cdots, \Phi^r_{ij}\right)$ be the block ij of the matrix Φ' such that $U_t = \Phi' V_t$, with $i,\ j = y,\ x$. Let us collect the $4r$ elements of Φ in the column vector $\overline{\Phi} = (\Phi'_{yy}, \Phi'_{yx}, \Phi'_{xy}, \Phi'_{xx})'$, and let us define the vector $\vartheta = (\overline{\Phi}', \Psi')'$.Hence, the null of serially uncorrelated errors may be expressed as $H_0 : \overline{\Phi} = 0$. To derive the test, it is useful to left-multiply the model (9) by $I - \Phi(L)$, which leads to the likelihood function:

$$l_t = C - \frac{1}{2} \ln |\Gamma| - \frac{1}{2} \left(\varsigma^2_{yt} \Gamma^{yy} + 2\varsigma_{yt}\varsigma_{xt}\Gamma^{yx} + \varsigma^2_{xt}\Gamma^{xx} \right), \tag{A1}$$

where Γ^{ij} is the block ij of the symmetric matrix Γ^{-1}, with $i,\ j = y,\ x$.

To derive the estimate of score under the null, it is useful to note that $\partial\widetilde{l}_t/\partial\Phi_{yj} = \left(\Gamma^{yy}\widetilde{\varsigma}_{yt} + \Gamma^{xy}\widetilde{\varsigma}_{xt}\right)\widetilde{v}_{jt}$ and $\partial\widetilde{l}_t/\partial\Phi_{xj} = \left(\Gamma^{yx}\widetilde{\varsigma}_{yt} + \Gamma^{xx}\widetilde{\varsigma}_{xt}\right)\widetilde{v}_{jt}$, with $j = y,\ x$. This leads to the $(4r \times 1)$ vector

$$m_\Phi = \sum \left(\partial\widetilde{l}_t/\partial\Phi'_{yy}, \partial\widetilde{l}_t/\partial\Phi'_{yx}, \partial\widetilde{l}_t/\partial\Phi'_{xy}, \partial\widetilde{l}_t/\partial\Phi'_{xx} \right)', \tag{A2}$$

which corresponds to the expression $\sum \left(\widetilde{\Gamma}^{-1}\widetilde{\varsigma}_t \otimes \widetilde{V}_t \right)$. Similarly, we can derive that

$$m_\Psi = \sum \left(\partial\widetilde{l}_t/\partial\Psi \right) = \sum \left[\left(\widetilde{\varsigma}_t\widetilde{\Gamma}^{-1} \otimes I_\varrho \right) \underline{\widetilde{Z}}_t \right] \tag{A3}$$

corresponds to the matrix $\sum \left[\left(\widetilde{\varsigma}_t\widetilde{\Gamma}^{-1} \otimes I_\varrho \right) \underline{\widetilde{Z}}_t \right]$, where $\underline{\widetilde{Z}}_t$ is the $(\varrho \times 1)$ vector $\left(\widetilde{z}'_{yt}, \widetilde{z}'_{xt}\right)'$.

To estimate the expressions related to the Hessian matrix, let us decompose the matrix

$$M = \frac{1}{T} \sum \partial^2 \widetilde{l}_t / \partial\vartheta\partial\vartheta' \tag{A4}$$

into four blocks $[M_{ij}]$, with $i, j = \Phi, \Psi$. The upper left block $M_{\overline{\Phi\Phi}}$ is similarly composed of 16 blocks, $M(\Phi_{ij}, \Phi_{hk}) = \frac{1}{T} \sum \partial^2 \widetilde{l}_t / \partial\Phi_{ij}\partial\Phi'_{hk}$ which correspond to $M(\Phi_{ij}, \Phi_{hk}) = \widetilde{\Gamma}^{jk} \widetilde{v}_{it} \widetilde{v}'_{ht}$, for $i, j, h, k = x, y$. This implies that $M_{\overline{\Phi\Phi}} = \frac{1}{T} \sum \left(\widetilde{\Gamma}^{-1} \otimes \widetilde{V}_t \widetilde{V}'_t \right)$. The upper right block $M_{\overline{\Phi}\Psi} = \frac{1}{T} \sum \partial^2 \widetilde{l}_t / \partial\overline{\Phi}\partial\Psi'$ is $\frac{1}{T} \sum \left(\widetilde{\Gamma}^{-1} \widetilde{Z}'_t \otimes \widetilde{V}_t \right)$, whereas the lower left block is $M'_{\overline{\Phi}\Psi}$. Finally, the lower right block $M_{\Psi\Psi} = \frac{1}{T} \sum \partial^2 \widetilde{l}_t / \partial\Psi\partial\Psi'$ is $\frac{1}{T} \sum \left(\widetilde{Z}_t \widetilde{\Gamma}^{-1} \widetilde{Z}'_t \right)$.

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Table 1. Linear approximation of VSTR models

z belongs to X_t	
Logitic models	Exponential models
$y_t = \varepsilon_{y0} + \sum_{h=0}^{3} \xi'_{yh} X_t w^h + v_{yt}$ $x_t = \varepsilon_{x0} + \sum_{h=0}^{3} \xi'_{xh} X_t w^h + v_{xt}$	$y_t = \varepsilon_{y0} + \sum_{h=0}^{2} \xi'_{yh} X_t w^h + v_{yt}$ $x_t = \varepsilon_{x0} + \sum_{h=0}^{2} \xi'_{xh} X_t w^h + +v_{xt}$
z does not belong to X_t and deviated models	
Logitic models	Exponential models
$y_t = \sum_{h=0}^{1} (\varepsilon_{yh} w^h + \xi'_{yh} X_t w^h) + v_{yt}$ $x_t = \sum_{h=0}^{1} (\varepsilon_{xh} w^h + \xi'_{xh} X_t w^h) + v_{xt}$	$y_t = \sum_{h=0}^{2} (\varepsilon_{yh} w^h + \xi'_{yh} X_t w^h) + v_{yt}$ $x_t = \sum_{h=0}^{2} (\varepsilon_{xh} w^h + \xi'_{xh} X_t w^h) + v_{xt}$

Note. This Table applies to the case $z_y = z_x = z$ and $q_y = q_x = q$. Variable w is the (square) weighted deviation from the linear path in deviated models whereas it is the transition variable in other VSTR models. In the case of logistic models including transition variables within the set of explanatory variables, the identification problem is avoided by means of a third order Taylor series approximation. In deviated models, a second order approximation is applied for a posterior discrimination between logistic and exponential models. The remaining VSTR models are approximated with a first order linear expansion.

Table 2. Linearity tests.

Auxiliary regressions	Null of linearity
z belongs to X_t	
$y_t = \varepsilon_{y0} + \sum_{h=0}^{3} \xi'_{yh} X_t w^h + v_{yt}$ $x_t = \varepsilon_{x0} + \sum_{h=0}^{3} \xi'_{xh} X_t w^h + v_{xt}$	$\xi_{i1} = \xi_{i2} = \xi_{i3} = 0$
z does not belongs to X_t and deviated models	
$y_t = \sum_{h=0}^{2} (\varepsilon_{yh} w^h + \xi'_{yh} X_t w^h) + v_{yt}$ $x_t = \sum_{h=0}^{2} (\varepsilon_{xh} w^h + \xi'_{xh} X_t w^h) + v_{xt}$	$\varepsilon_{i1} = \varepsilon_{i2} = 0$ $\xi_{i1} = \xi_{i2} = 0$

Note. This Table applies to the case $z_y = z_x = z$ and $q_y = q_x = q$. Variable w is the (square) weighted deviation from the linear path in deviated models whereas it is the transition variable in other VSTR models. Parameter i refers to y and x.

Table 3. Model selection tests

z_t belongs to X_t				Choice
Hypoth.	Test 1	Test 2	Test 3	
H_0	$\xi_{i3}=0$	$\xi_{ij}=0$, $j=2,\ 3$	$\xi_{ij}=0$, $j=1,2,\ 3$	
H_a	$\xi_{i3}\neq 0$	$\xi_{i2}\neq 0$ $\xi_{i3}=0$	$\xi_{i1}\neq 0$ $\xi_{ij}=0$ $j=2,\ 3$	
	Reject	...	...	Logistic
	Accept	Reject	Accept	Exponential
	Accept	Accept	Reject	Logistic
	Accept	Reject	Reject	No decision
z_t does not belongs to X_t and deviated models				Choice
Hypoth.	Test 1			
H_0	$\varepsilon_{i2}=0$, $\xi_{i2}=0$			
H_a	$\varepsilon_{i2}\neq 0$, $\xi_{i2}\neq 0$			
	Accept			Logistic
	Reject			Exponential

Note. This Table applies to the case $z_{ty}=z_{tx}=z_t$ and $q_y=q_x=q$. Variable w is the (square) weighted deviation from the linear path in deviated models whereas it is the transition variable in other VSTR models. Parameters ε_i and ξ_i refer to the auxiliary regressions in Table 2. Parameter i refers to y and x. As an example, let z_t be any variable belonging to X_t. If Test 1 and Test 2 cannot be rejected, but Test 3 is rejected, then a logistic transition function should be used.

Table 4. Test of no remaining nonlinearity.

Auxiliary regressions	Null no remaining nonlinearity
z belongs to X_t	
$y_t = \beta_y' A_{yt} + \widetilde{\beta}_y' A_t F_y^1 + \sum_{h=1}^{3} \xi_{yh}' X_t w^h + v_{yt}$ $x_t = \beta_x' A_{xt} + \widetilde{\beta}_x' A_t F_x^1 + \sum_{h=1}^{3} \xi_{xh}' X_t w^h + v_{xt}$	$\xi_{i1} = \xi_{i2} = \xi_{i3} = 0$
z does not belongs to X_t and deviated models	
$y_t = \beta_y' A_{yt} + \widetilde{\beta}_y' A_t F_y^1 + \sum_{h=1}^{2} (\varepsilon_{yh} w^h + \xi_{yh}' X_t w^h) + v_{yt}$ $x_t = \beta_x' A_{xt} + \widetilde{\beta}_x' A_t F_x^1 + \sum_{h=1}^{2} (\varepsilon_{xh} w^h + \xi_{xh}' X_t w^h) + v_{xt}$	$\varepsilon_{i1} = \varepsilon_{i2} = 0$ $\xi_{i1} = \xi_{i2} = 0$

Note. This Table applies to the case $z_y = z_x = z$ and $q_y = q_x = q$. Variable w is described in Table 1. Parameter i refers to y and x.

Table 5. Test of parameter constancy

Auxiliary regressions	Null of constant parameters
$y_t = \theta_{y0}' \beta_{yt} + \theta_{y1}' \beta_{yt} t + \cdots + \theta_{yk}' \beta_{yt} t^k$ $\left[\widetilde{\theta}_{y0}' \widetilde{\beta}_{yt} + \widetilde{\theta}_{y1}' \widetilde{\beta}_{yt} t + \cdots + \widetilde{\theta}_{yk}' \widetilde{\beta}_{yt} t^k \right] F_y + v_{yt}$ $x_t = \theta_{x0}' \beta_{xt} + \theta_{x1}' \beta_{xt} t + \cdots + \theta_{xk}' \beta_{xt} t^k$ $\left[\widetilde{\theta}_{x0}' \widetilde{\beta}_{xt} + \widetilde{\theta}_{x1}' \widetilde{\beta}_{xt} t + \cdots + \widetilde{\theta}_{xk}' \widetilde{\beta}_{xt} t^k \right] F_x + v_{xt}$	$\theta_{i1}' = \cdots = \theta_{ik}' = 0$ $\widetilde{\theta}_{i1}' = \cdots = \widetilde{\theta}_{ik}' = 0$

Note. Variable w is described in Table 1. Parameter i refers to y and x.

Table 6. Results of linearity and model selection tests

	Lin. test	Test 1	Test 2	Test 3	Decision
$z = y_{t-2}$	26.47 (R)	11.73 (A)	$\cdots$	$\cdots$	LVSTR(y_{t-2})
$q = 1$	41.55 (R)	12.54 (A)	$\cdots$	$\cdots$	LVSTR-D(1)
$z = x_{t-1}$	30.94 (R)	7.25 (A)	16.54 (R)	7.14 (A)	EVSTR(x_{t-1})
$z = x_{t-2}$	41.65 (R)	20.98 (R)	$\cdots$	$\cdots$	EVSTR(x_{t-2})
$z = x_{t-3}$	36.94 (R)	21.35 (R)	$\cdots$	$\cdots$	EVSTR(x_{t-3})

Note. Tests are developed as Tables 2 and 3 describe. Statistics are displayed only for models which reject linearity. Second column shows the results for linearity tests whereas third to fifth columns present the results for model selection tests. Only x_{t-1} belongs to the set of explanatory variables (a three-stage testing procedure applies) since these tests refer to a vector autoregressive specification with lag length one. Results of the tests at 1% are in parentheses (A: non-rejection, R: rejection).

Table 8. Testing the adequacy of VSTR models

	Test SI				Test NRN	Test PC		
	$r = 1$	$r = 2$	$r = 3$	$r = 4$		$k = 1$	$k = 2$	$k = 3$
EVSTR(x_{t-1})	0.09	0.08	0.11	0.08	0.69	0.64	0.16	0.69
EVSTR(x_{t-2})	0.02	0.01	0.01	0.01	0.04	0.02	0.18	0.01
EVSTR(x_{t-3})	0.44	0.09	0.19	0.04	0.01	0.76	0.29	0.95
LVSTR(y_{t-2})	0.08	0.08	0.20	0.12	0.05	0.10	0.98	0.97
LVSTR-D(1)	0.20	0.13	0.20	0.23	0.06	0.40	0.24	0.11

Note. As Section 3.1 describes, tests SI, test NRN, and tests PC refer to tests of serial independence of the errors, test of no remaining nonlinearity and tests of parameter constancy respectively. Each entry shows the p-values of these tests. Tests NRN and PC have also power against omission of important lags from the estimated model.

Table 7. Maximum likelihood estimates of parameters.

	Model estimation	VARCOV
LVSTR(y_{t-2})	$\widehat{y}_t = \underset{(0.12)}{0.91}\widehat{F}_y + \underset{(0.05)}{0.60}x_{t-1}$ $\widehat{x}_t = \underset{(0.18)}{0.42} - \underset{(0.13)}{0.31}\widehat{F}_x + \underset{(0.05)}{0.43}x_{t-1}$ $\widehat{F}_y = \left[1 + \exp\left(-\underset{(0.26)}{1.85}(y_{t-2} - \underset{(0.01)}{0.13})\right)\right]^{-1}$ $\widehat{F}_x = \left[1 + \exp\left(-\underset{(15.66)}{87.57}(y_{t-2} + \underset{(047)}{0.33})\right)\right]^{-1}$	$\widehat{\sigma}_{11} = \underset{(0.02)}{0.55}$ $\widehat{\sigma}_{22} = \underset{(0.02)}{0.51}$ $\widehat{\sigma}_{12} = \underset{(0.03)}{0.11}$
LVSTR-D(1)	$\widehat{y}_t = \underset{(0.07)}{0.72} - \underset{(0.10)}{0.18}\widehat{F}_y + \underset{(0.06)}{0.61}x_{t-1}$ $\widehat{x}_t = \underset{(0.09)}{0.30}\widehat{F}_x + \underset{(0.07)}{0.35}x_{t-1}$ $\widehat{F}_y = \left[1 + \exp\left(-\underset{(13.64)}{14.85}(y_{t-1} - \underset{(0.08)}{0.56} - \underset{(0.07)}{0.11}y_{t-2} - \underset{(0.08)}{0.57}x_{t-2})\right)\right]^{-1}$ $\widehat{F}_x = \left[1 + \exp\left(-\underset{(1.93)}{1.93}(x_{t-1} - \underset{(0.07)}{0.23} - \underset{(0.06)}{0.15}y_{t-2} - \underset{(0.07)}{0.50}x_{t-2})\right)\right]^{-1}$	$\widehat{\sigma}_{11} = \underset{(0.02)}{0.59}$ $\widehat{\sigma}_{22} = \underset{(0.02)}{0.53}$ $\widehat{\sigma}_{12} = \underset{(0.03)}{0.09}$
EVSTR(x_{t-1})	$\widehat{y}_t = \underset{(0.36)}{1.20} - \underset{(0.36)}{0.59}\widehat{F}_y + \underset{(0.06)}{0.58}x_{t-1}$ $\widehat{x}_t = \underset{(0.04)}{0.14}\widehat{F}_x + \underset{(0.05)}{0.45}x_{t-1}$ $\widehat{F}_y = 1 - \exp\left(-\underset{(2.48)}{185.54}\left(x_{t-1} - \underset{(0.03)}{0.83}\right)^2\right)$ $\widehat{F}_x = 1 - \exp\left(-\underset{(3.29)}{145.85}\left(x_{t-1} - \underset{(0.78)}{2.31}\right)^2\right)$	$\widehat{\sigma}_{11} = \underset{(0.02)}{0.59}$ $\widehat{\sigma}_{22} = \underset{(0.02)}{0.52}$ $\widehat{\sigma}_{12} = \underset{(0.03)}{0.09}$
EVSTR(x_{t-2})	$\widehat{y}_t = \underset{(0.12)}{0.83} - \underset{(0.13)}{0.29}\widehat{F}_y + \underset{(0.05)}{0.59}x_{t-1}$ $\widehat{x}_t = \underset{(0.49)}{1.02} - \underset{(0.48)}{0.90}\widehat{F}_x + \underset{(0.05)}{0.42}x_{t-1}$ $\widehat{F}_y = 1 - \exp\left(-\underset{(0.14)}{11.20}\left(x_{t-2} - \underset{(0.12)}{0.31}\right)^2\right)$ $\widehat{F}_x = 1 - \exp\left(-\underset{(35.89)}{1558.55}\left(x_{t-2} - \underset{(0.02)}{0.25}\right)^2\right)$	$\widehat{\sigma}_{11} = \underset{(0.02)}{0.59}$ $\widehat{\sigma}_{22} = \underset{(0.02)}{0.51}$ $\widehat{\sigma}_{12} = \underset{(0.03)}{0.07}$
EVSTR(x_{t-3})	$\widehat{y}_t = \underset{(0.11)}{0.97} - \underset{(0.36)}{0.94}\widehat{F}_y + \underset{(0.06)}{0.54}x_{t-1}$ $\widehat{x}_t = \underset{(0.36)}{0.67}\widehat{F}_x + \underset{(0.05)}{0.39}x_{t-1}$ $\widehat{F}_y = 1 - \exp\left(-\underset{(0.19)}{0.26}\left(x_{t-3} - \underset{(0.36)}{1.41}\right)^2\right)$ $\widehat{F}_x = 1 - \exp\left(-\underset{(0.37)}{0.42}\left(x_{t-3} + \underset{(0.17)}{0.18}\right)^2\right)$	$\widehat{\sigma}_{11} = \underset{(0.02)}{0.55}$ $\widehat{\sigma}_{22} = \underset{(0.02)}{0.50}$ $\widehat{\sigma}_{12} = \underset{(0.03)}{0.08}$

Note. Standard errors are in parentheses.

Table 9. Certain negative and positive detection rates (CNR and CPR).

	In-sample				Out-of-sample			
	y_t		Business cycles		y_t		Business cycles	
	y_t <0	$y_t \geqslant 0$	NBER recessions	NBER expansions	y_t <0	$y_t \geqslant 0$	NBER recessions	NBER expansions
	CNR	CPR	CNR	CPR	CNR	CPR	CNR	CPR
EVSTR(x_{t-1})	25	98	...	...	33	100	...	...
EVSTR(x_{t-2})	35	98	...	...	13	97	...	...
EVSTR(x_{t-3})	35	97	...	...	20	97	...	...
LVSTR(y_{t-2})	50	97	29	56	26	98	22	72
LVSTR-D(1)	30	98	37	40	33	100	5	17

Note. "In-sample" refers to 1960.2-1997.4. "Out-of-sample" refers to 1972.2-1998.1. Variables y_t and x_t denote rate of growth of GDP and CLI at time t. In both in and out-of-sample analysis, last (first) two columns present the percentage of quarters when each model correctly anticipates the NBER schedule (y_t sign). Notice that, in line with Stock and Watson (1992), an estimated probability of recession above 0.75 (below 0.25) is interpreted as a signal of recession (expansion).

For instance, of the 20 (131) in-sample falls (rises) of GDP, the LVSTR-D(1) model detects negative (positive) growth 6 times (128 times), which implies 30% of CNR (98% of CPR). On the other hand, this model predicts 10 times probability of recession above 0.75 within the 27 quarters of NBER recessions (37 % of CNR) and estimates 120 times probability of recession below 0.25 within the 124 quarters of NBER expansions (97% of CPR).

Table 10. False negative and positive signal rates (FNR and FPR)

	In-sample				Out-of-sample			
	GDP		Business cycles		GDP		Business cycles	
	$\widehat{y} \geqslant 0$	$\widehat{y} < 0$	$\widehat{p} \leqslant 0.25$	$\widehat{p} \geqslant 0.75$	$\widehat{y} \geqslant 0$	$\widehat{y} < 0$	$\widehat{p} \leqslant 0.25$	$\widehat{p} \geqslant 0.75$
	FNR	FPR	FNR	FPR	FNR	FPR	FNR	FPR
EVSTR(x_{t-1})	10	29	...	...	10	0	...	...
EVSTR(x_{t-2})	9	22	...	...	13	60	...	...
EVSTR(x_{t-3})	9	36	...	...	12	50	...	...
LVSTR(y_{t-2})	7	28	7	50	11	33	9	50
LVSTR-D(1)	10	25	25	86	10	0	31	94

Note. "In-sample" refers to 1960.2-1997.4. "Out-of-sample" refers to 1972.2-1998.1. Symbols $\widehat{y}$ and $\widehat{p}$ denote estimated rate of growth of GDP and estimated probability of recession. In both in and out-of-sample analysis the first (second) column presents the percentage of negative (positive) actual rate of growth of GDP within the periods of positive (negative) estimated growth rate. In addition, the third (fourth) column shows the percentage of NBER recessions (expansions) within the quarters of estimated expansions (recessions). Notice that in line with Stock and Watson (1992), an estimated probability of recession above 0.75 (below 0.25) is interpreted as a signal of recession (expansion).

For instance, of the 143 (8) in-sample positive (negative) estimated rates of growth of GDP with the LVSTR-D(1) model, the GDP actually falls (rises) 14 (2) times, which corresponds to a FNR (FPR) of 10% (25%). On the other hand, of the 67(72) times within sample that the LVSTR-D(1) model predicts probability of recession less than 0.25 (higher than 0.75), 17(62) of those turned out to be actual NBER recessionary (expansionary) quarters, which correspond to a FNR of 25% (FPR of 86%).

Table 11. In-sample and out-of-sample MSE and TPE

	In-sample		Out-of-sample	
	MSE	TPE	MSE	TPE
EVSTR(x_{t-1})	0.59 1.08 0.49	...	0.72 1.65 0.53	...
EVSTR(x_{t-2})	0.59 1.08 0.48	...	0.76 1.53 0.60	...
EVSTR(x_{t-3})	0.55 0.90 0.48	...	0.72 1.49 0.56	...
LVSTR(y_{t-2})	0.55 0.88 0.48	0.16 0.30 0.13	0.70 1.46 0.54	0.17 0.55 0.09
LVSTR-D(1)	0.59 1.08 0.48	0.51 0.61 0.49	0.70 1.67 0.49	0.39 0.50 0.37

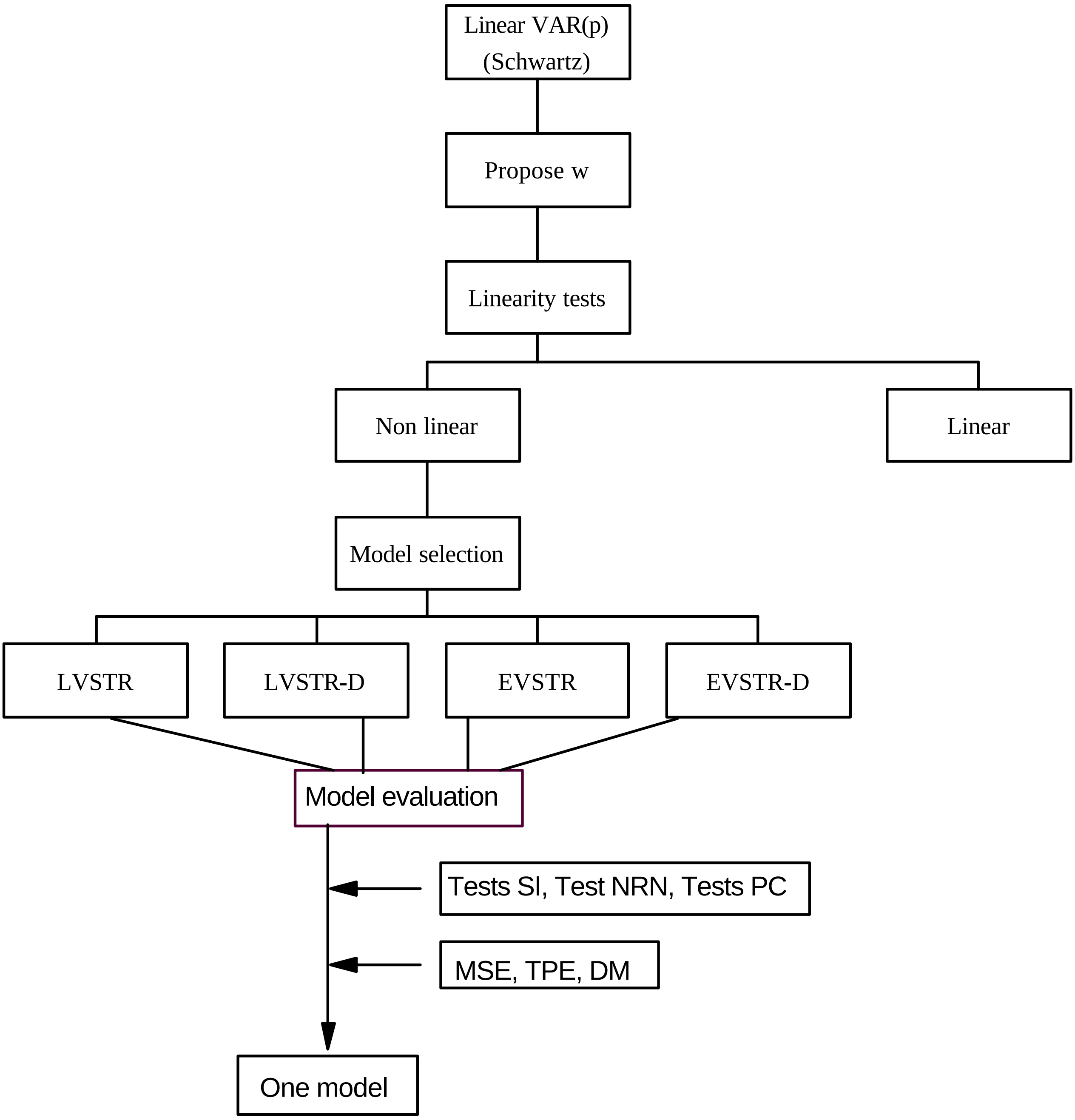
Note. For each model, first entry refers to the entire sample. Following the NBER chronology, second (third) entry refers to recessionary (expansionary) periods. "In-sample" refers to 1960.2-1997.4. "Out-of-sample" refers to 1972.2-1998.1. MSE and TPE, defined in (14) and (15), are loss functions that evaluate the model's accuracy at anticipating GDP growth and the US business-cycle phases.

Table 12. Diebold and Mariano (DM) tests

	In-sample		Out-of-sample	
	MSE	TPE	MSE	TPE
LVSTR(y_{t-2}) EVSTR(x_{t-1})	1.43 2.96 0.11	...	0.86 1.93 0.19	...
LVSTR(y_{t-2}) EVSTR(x_{t-2})	1.27 2.51 0.01	...	1.04 1.01 0.70	...
LVSTR(y_{t-2}) EVSTR(x_{t-3})	0.15 0.30 0.40	...	0.64 0.63 0.37	...
LVSTR(y_{t-2}) LVSTR-D(1)	1.28 2.61 0.13	8.52 4.16 7.71	0.30 2.14 1.02	4.77 0.24 5.41

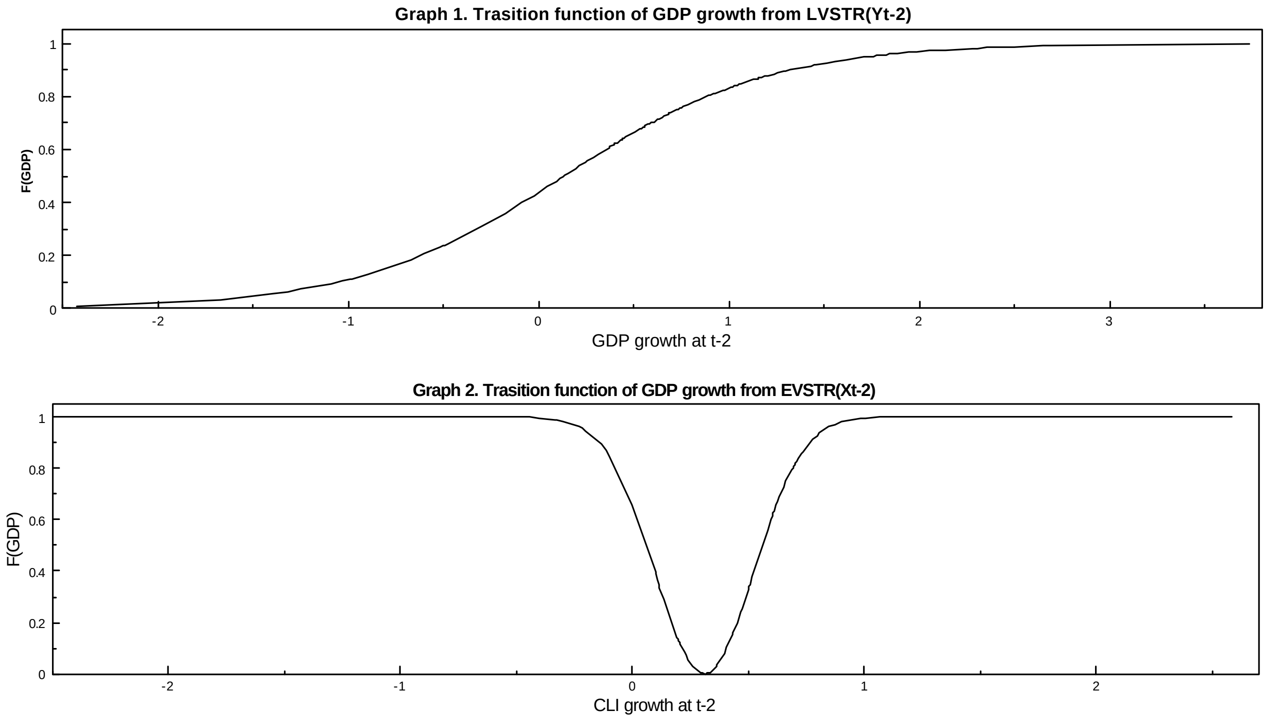
Note. This Table presents the absolute values of the DM test statistic. For each model, first entry refers to the entire sample. Following the NBER chronology, second (third) entry refers to recessionary (expansionary) periods. "In-sample" refers to 1960.2-1997.4. "Out-of-sample" refers to 1972.2-1998.1 MSE and TPE are loss functions defined in (14) and (15). For example, 1.28 (8.52) is the absolute value of the DM statistic under the hypothesis of no difference in the accuracy of models LVSTR(y_{t-2}) and LVSTR-D(1) at anticipating in-sample growth (recessions), calculated with the entire sample.

Figure 1. Description of VSTR selection



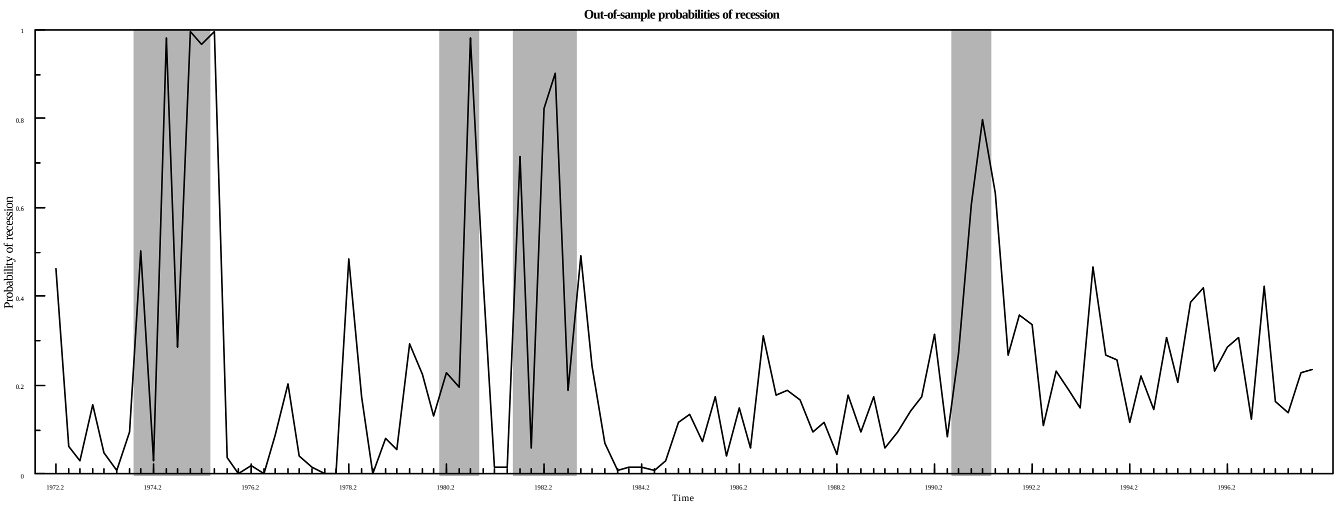
Note. This figure describes the specification of VSTR models in four steps. First, a linear VAR and its maximum lag length is specified. Second, linearity tests are applied for w (either the -square- weighted deviation from the linear path in deviated models whereas or the transition variable in other VSTR models) the researcher proposes. Third, for each w for which linearity was rejected, model selection tests are carried out. Finally, the validity of these models is evaluated by testing their adequacy (tests of serialy independence of errors, test of no remaining nonlinearity and tests of parameters constancy), and by checking the accuracy of the resulting models at anticipating both output growth (MSE) and turning points (TPE), formally tested through Diebold and Mariano (DM) tests. The procedure concludes with the selection of one nonlinear specification.

Figure 2: Estimated transition function vs the transition variable



Note: Figure 2 plots the transition function of GDP growth equation against Yt-2 (Graph1) and Xt-2 (Graph2).

Figure 3: Out-of-sample probabilities of recession



Note: Figure 3 shows the out-of-sample (period 1972.2-1998.1) probabilities of recessions from the LVSTR(Yt-2) model. Shaded areas correspond to the official NBER recessions.