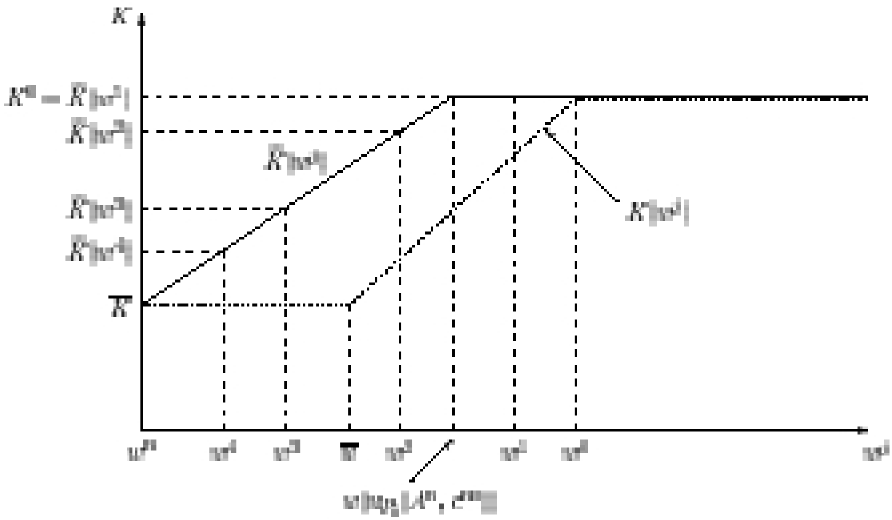
K

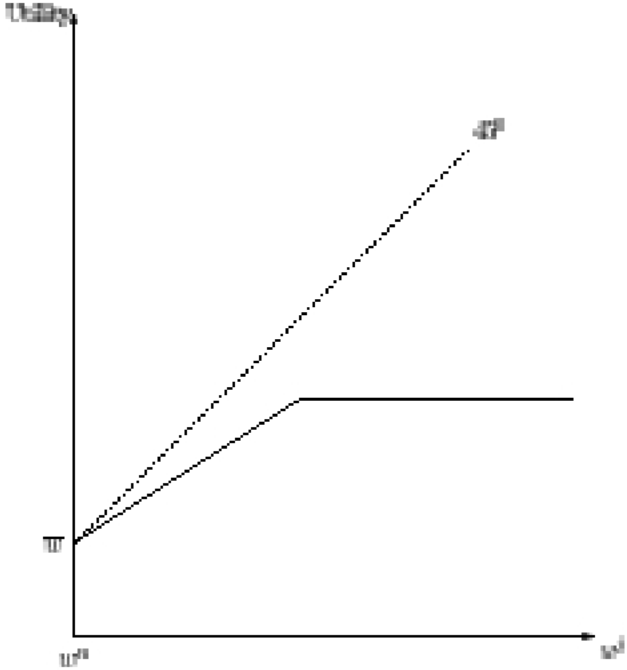


*Figure 2:* Gross and net utilities of an agent

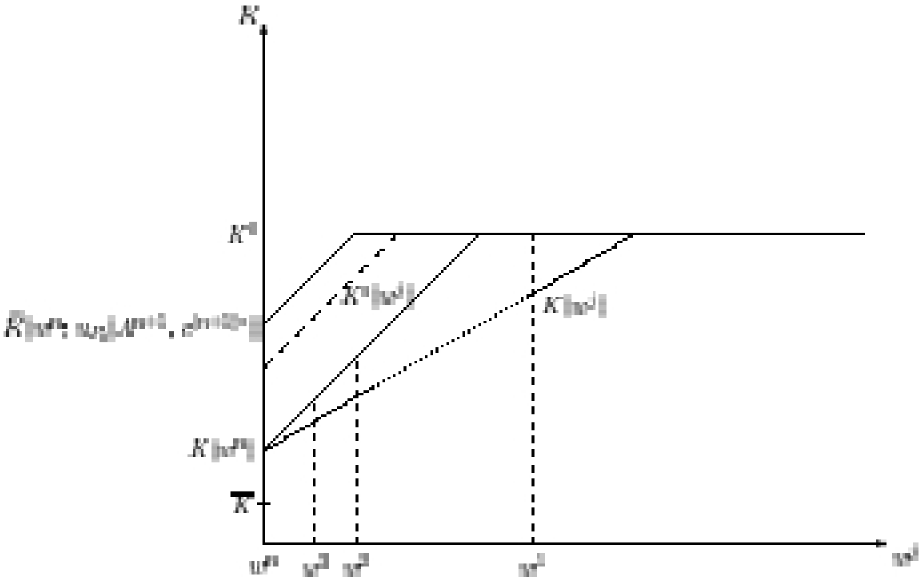

*Figure 3:* OPTIMAL INVESTMENT WHEN $u^0 > u^{m+1}$

# M

# M M M

A large set of literature on financial markets conclude that when external borrowing by firms is indispensable, capital-poor firms are denied credits by uninformed investors and they have to rely on informed capital available in the economy.[1] Informed capital owns, in general, better monitoring technology compared to less informed investors. ence, it is better able to cope with r r at the firm level that arises because of the inability to contract upon all the actions taken by the entrepreneurs in a firm seeking credit. This moral ha ard problem is more severe with the poorer firms, and hence they fail to obtain credit from the uninformed investors. oorer firms are sometimes able to invest by borrowing from informed capital which is supplied by financial intermediaries but only after being monitored more intensively.

The main goal of this chapter is to analyse a financial economy consisting of firms (run by entrepreneurs) with different levels of start-up capital, and financial intermediaries (or, investors) with different monitoring technologies. In this financial market, several firms and intermediaries interact with each other which calls for a general equilibrium framework. In this model, the payoff of each individual is determined endogenously, unlike the standard financial market models where the payoffs are determined exogenously. The framework also allows us to establish the identities of the intermediaries who become potential sources of credit to different firms.

We model the financial market as a two-sided *matching game*. If a firm convinces an intermediary to finance his project, we say that the firm and the intermediary are *matched* to form a pair. A matching is a rule that specifies all such possible pairs in the economy. An outcome of this market is an endogenous matching and a set of financial contracts, one for each firm-intermediary pair under the matching. A financial contract specifies that the intermediary finances the project and receives state-contingent claims on the project return. Each firm operates on his project after he obtains external finance and chooses a non-contractible effort level. Choice of effort influences the probability of having a high return from the project. Firm's liability is limited to his current income. Hence, differences in wealth imply differences in liabilities. We use *stability* as the equilibrium concept. An outcome is stable if there is no intermediary-firm pair that would be (strictly) better-off than under the initial outcome.

We characterize the equilibrium of the financial market. First, all contracts are Pareto optimal, i.e., given the others being equally well-off as before, no individual can strictly improve upon his/her situation in the outcome by signing a different contract. Second, there is always a subset of the firms (the poorest ones) which fail to obtain credit from any external source. If the firms form the long-side of the market, the size of this (unmatched) set is even larger. Also, capital-rich firms earn higher payoffs. On the other hand, if the intermediaries form the long-side of the market then less informed intermediaries stay out of business earning zero profit. The equilibrium payoff of each individual is endogenous in this model. We are able to establish bounds on the payoff of each firm, and these bounds depend on the other pairs formed in the economy.

Next, in this financial market with both-sided heterogeneity, we show that in a stable outcome the matching is negatively assortative, i.e., capital-poor firms rely on more informed capital, and they are monitored more intensively. In this framework, for an intermediary-firm pair, firm capital and monitoring intensities turn out to be substitutes in producing as well as transferring the surplus between each other. Negatively assortative matching patterns are consequences of this two-fold substitutability.

Our model bears resemblance with the financial intermediation models proposed by Holmstrom and Tirole [17], and Repullo and Suarez [25]. In both papers the authors consider models of bank monitoring under moral hazard. In these models, the financial economies are characterised by a continuum of firms with different wealth levels, and small number of investors. There are two types of investors: insiders and outsiders. The outside investors are not able to monitor the firms as intensively as the insiders can. In the model of Holmstrom and Tirole [17], the investors are capital constrained, and they face a moral hazard problem at the level of monitoring. They show that capital-rich firms prefer outsiders to raise finance, whereas capital-poor firms have to rely on bank (insiders) finance to invest in their projects. They also analyse the effects of different types of capital tightening in the economy. Any sort of these adverse shocks hits the poor firms more severely by taking them out of business and leads to a contraction in the bank loan which is only geared towards rich firms thus implying, what Bernanke et al. [7] call, a *flight to quality*. A change in the intermediary capital also affects the monitoring levels. Repullo and Suarez [25] consider similar kind of model where the intermediaries are not capital constrained. They draw similar conclusions as the previous paper. Moreover, they find an intermediate range of firm's wealth where the firms can choose between outside finance and bank credit. Moreover, they also study the effects of interest rate spread on the equilibrium. Besanko and Kanatas [8] also consider a theoretical model of endogenous bank monitoring and show that firms might suffer from excessive monitoring in equilibrium.

In this chapter we consider a discrete set of firms and intermediaries. This allows us to deal with small as well as large number of individuals. In the papers cited above, although a general (competitive) equilibrium framework is considered, intermediary payoff is determined by its reservation value and hence, the intermediaries of each type break-even. Since an endogenous matching market is used in the current model, the payoff each individual earns is determined endogenously. The equilibrium matching pattern, namely negatively assortative matching, conforms to the findings of Holmstrom and Tirole [17], and Repullo and Suarez [25], if we consider only two types of investors: insider with higher monitoring intensity and outsiders with lower monitoring intensity. We later consider a many-to-one market where an investor can finance more than one project only with the restriction that the project returns are uncorrelated. Another difference between the model of Holmstrom and Tirole [17] and that of ours is that they consider both the intermediaries and firms are capital constrained, whereas in the current model only firms lack capital to fund their project. In this sense, we only have demand-side considerations.

Our model is built on the theoretical model proposed by Dam and Pérez-Castrillo [12], where the authors characterise the set of stable outcomes of a principal-agent economy with identical principals and heterogeneous agents.[3] From matching theory point of view, this paper is also related to works by Becker [6], Crawford and Knoer [11], Legros and Newman [1[illegible]], and Sertel [2[illegible]], which, as Dam and Pérez-Castrillo [12] do, feature no or imperfect transferability of surplus among the agents. This non-transferability is typical to agency models characterised by provision of incentives since optimality is not implied by maximisation of total surplus. Becker [[illegible]], and Legros and Newman [1[illegible]], in a more general matching framework, also provide sufficient conditions for positively and negatively assortative matching patterns in equilibrium.

In a recent empirical paper, Ackerberg and Botticini [1], using a data set on agricultural contracts between landlords and tenants in early Renaissance Tuscany, show that, although the characteristics of a land owned by landlord is exogenous, the kind of tenant attracted to it is determined by an *endogenous matching*. They further assert

> ... [illegible].

The above suggests that strong influence of endogenous matching is not unusual in determining the terms of incentive compatible contracts among individuals.

m

We consider an economy with a ·nite set of risk neutral r s, who own a project of (fixed) si e 1 apiece. A generic firm is identified by his level of initial wealth (or, start-up capital) $w^i$. We arrange the firms according to their wealth levels in descending order as $1 > w^1$ $w^2$ $w^n$ 0. Firm's initial wealth is not sufficient to cover the entire project cost, hence each firm seeks external finance. There are risk-neutral intermediaries (or, banks) with different monitoring technologies identified by the monitoring intensities. These intermediaries are the potential investors in the market. A firm has to convince an intermediary to finance his project. Intermediary is identified by her monitoring intensity . We arrange the intermediaries with respect to their monitoring intensities in descending order as $_1$ $_2$ . Intermediaries with higher monitoring intensity are often referred to as re io r e c it . An intermediary with intensity incurs a fixed cost and commit to monitor a firm she finances. Clearly, intermediary $_1$ owns the best monitoring technology and intermediary owns the worst one. We assume that the monitoring technology does not permit an intermediary to control more than one firm.[3] Also each intermediary incurs per unit opportunity cost of fund which is equal to . Intermediaries and firms are matched in pairs. We allow for the possibility that a firm can seek for an alternative financier. ence, the matching is endogenous rather than being exogenous. Whenever matched, an intermediary-firm pair signs a financial contract and the intermediary finances the entire project.[4] Firm's wealth works as a collateral in the project even though, the firm does not invest his wealth in the project.

When an intermediary agrees to finance a project, the firm undertakes an effort [0 1] which i flue ces d o y ucces o F i ure. E oj y ld u v of u ob b l y , d 0, of f ilu om l m y ob b l y. T fi m d v fi ob o v u od o g v b fi by om g lo

F ... *Li ide* *i bi ity*

$\leq$ $w^j$ ( S)

$\leq w^j$ ( F)

T ... k ... k ... v ... , ... v ... r ... r ... fi ... v T ... v ... fi ... [0 1] S ... fi ... ( ) ... v ... , ( C) ... *rst r er c n iti n* ... fi ... x ... :

$$= ( )$$ ( C)

W ... k

**Assumption 2.** [$\frac{1}{2}$ 1] *r* .

A ... fi ... v ... W ... fi

**Definition 6.** (Feasibility)

*A c ntr ct is* **feasible for a firm** $w^j$ *i it s tis es t e restricti ns in i i u r ti n ity n i ide i bi ity.*

$\mathcal{X}^j$ ... fi $w^j$ F ... $\mathcal{Z}^j$ ... v

$$\begin{cases} x_{(\sigma,z)} & (w^j \quad ) \\ s.t. & {}^j( \quad ) \quad {}^{-j} \end{cases}$$ ( )

v ... v ... $(w^j \quad )$ ... ${}^j( \quad )$, k ... v ... v ... ( C) ... $(^{-j})$ ... v ... v x ... T

N

$$= \begin{cases} \;\;{}^{-i} & \frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i} < w^i \\ w^i & \frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i}\;\; w^i \end{cases}$$

$$= \begin{cases} \;\;{}^{-i} & \frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i} < w^i \\ w^i \;\; \frac{1}{\;}\sqrt{2\;\;(w^i \;\;\;{}^{-i})\;\;1} & \frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i} \le w^i \le \frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i} \\ \frac{\beta}{2}\;\; w^i & \frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i} > w^i \end{cases}$$

N ... j ... v ... v

F , $\frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i} < w^i$, fi

( v )

*rst best*

A

$\frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i}\;\; w^i$ W

v ... v fi ( $\frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i} > w^i$), v ... v

fi ... ff ... ${}^{-i}$ T ... v

:

$$= \begin{cases} 1 & \frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i} < w^i \\ \sqrt{2\;\;(w^i \;\;\;{}^{-i})\;\;1} & \frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i} \le w^i \le \frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i} \\ \frac{\alpha\beta}{2} & \frac{\alpha\beta}{2}\;\frac{1}{2}\;{}^{-i} > w^i \end{cases}$$

k ... ff

fi ... j ... fi $w^i$ ... $w^i$ (f)[3] S

fi ... v ... (f) T ... fi

k A

j ( )

T v q ... fi ... v

${}^{-i}$

fi ... fi

ff

[3] W ... ff ... ff = ... + ... A ... x

**Lemma 3.** *Under ... r ... i* $w^j > w^l$ ... $\leq$ ... *then* ( ... $w^j$ ... ) $>$ ( ... $w^l$ ... ) ... *r* ... *ng* ... .

*Pr* ... . S ... A ... x B

## M ... h ... g

W ... fi ... j ... fi ... , ... T ... fi ... v ... fi ... k ... fi ... $\mathcal{I} =$ ... $\mathcal{F} = w^1\ w^2$ ... $w^n$ , ... v

**Definition 7.** (Matching)
*A* ... **matching** ... *ing* $\mu : \mathcal{I} \sqcup \mathcal{F} \to \mathcal{I} \sqcup \mathcal{F}$ ... $\mu($ ... $)$ ... $\mathcal{F} \sqcup$ ... $\mathcal{I}$ ... $\mu(w^j)$ ... $\mathcal{I} \sqcup$ ... $w^j$ ... $w^j$ ... $\mathcal{F}$ ... $\mu(w^j) =$ ... $\mu($ ... $) = w^j$ ... ( ... $w^j$) ... $\mathcal{I} \times \mathcal{F}$.

T ... v ... fi ... k ... fi ... v ... k ... v ... , ... v ... W ... ( ... $w^j$) ... $\mu$ ... $\mu($ ... $) = w^j$ ( ... q ... v ... , $\mu(w^j) =$ ... )

**Definition 8.** *A* ... **C** **compatible with a matching** $\mu$ ... **C** ... $\mu$.

**Definition 9.** (Outcome)
*An* **outcome** $(\mu, C)$ ... *ing* $\mu$ ... .

T ... k ... T ... F ... , ... fi ... T ... *equilibrium* ... fi ... k ... fi ... fi ... W ... q ... fi ... ( ... v ... ) ... , ... k ... ( ... $w^j$)

**Definition 10. (Stability)**
*An outcome* $(\mu, C)$ *or the r ct is* **stable** *i t ere es n t exist mp ir ( $w^j$) m mp e m ir at $X^j$ suc t t $(w^j\ \ ) >\ \ (\mu(\ \ )\ \ ^{j})$ m $^j(\ \ ) >$ $^j(\mu(w^j)\ \ ^{j}_{\mu(w^j)})$.*

T v fi fi

b c T v

v fi k

## S f S b O m s

k W

F , B

( ) v ff v

fi k v ff T

**Lemma .** *A t e c mir als in at b e stable e re ti .*

*Pr .* S $(\mu, C)$ , $^j$ $C$ $w^j$,
$\mu(w^j) =$ , T x , ( $w^j$)
( ) $(w^j\ \ ) >\ \ (w^j\ \ ^{j})$ ( ) $^j(\ \ ) >\ ^j(\ \ ^{j})$ ( $w^j$)
k $(\mu, C)$ T $(\mu, C)$ ∎

fi

k ff ,

( $w^j$) x x k

v O

S 2.2 T

v ( )

fi $w^j$ $^{-j}$ T k

fi $w^j$, , v $^{-j}$,

then the e ntract etween $(n_h, w^d)$ is $\frac{j}{k} = \; _d(^{-j})$ an the pa ff t inter e iar $n_h$ is $(n_h, w^j, ^{-j})$

An ther pr pert sta e tc e is that a n the atche nes, a richer fir is a wa s etter ff, which is sh wn in the win e a

**Lemma 5.** *In at le stc e r t c r ot r ng $w^j$ n $w^j$ tc e $^{-j} > ^{-j}$ i $w^j > w^j$ . An r ng $w^j$ n $w^j$ un tc e $^{-j} = ^{-j} = 0$.*

*Pr* . S pp se $w^j > w^j$ are th atche an in a sta e te e $^{-j} \geq ^{-j}$ B e a 3, $(\rho(w^j), w^j, ^{-j}) > (\rho(w^j), w^j, ^{-j})$ ence, there exists $=$ $_{\rho(w^j)}(^{-j})$ c r $\varepsilon > 0$, s a en h s ch that (i) $_{\rho(w^j)}(w^j, ) = (\rho(w^j), w^j, ^{-j})$ $\varepsilon > (\rho(w^j), w^j, ^{-j})$ an $^j(\rho(w^j), ) \geq ^{-j}$ $\varepsilon > ^{-j}$ [10] ence, $(\rho(w^j), w^j)$ w ck the tc e with , which c ntra icts the sta i it the initia tc e

The sec n part ws irect r the act that i a fir is n atche then he si ns a e ntract [11] A s n te that i a fir $w^j$ is atche , then in ivi a rati na it $^{-j} \geq 0$ ∎

n the win efiniti n we intr uce the c ncept i ingness t p, which wi e se in characterisin the set sta e tc es

**Definition 11. (Willingness to Pay)**
*Gi en ng ale e $(\mu, C)$ ere fir s $w^j$ n $w^j$ at ins $^{-j}$ n $^{-j}$ res ecti e y r ng inter e i ry $n_i$ i e i ingness t y r $w^j$ g inst $w^j$ is efine s*

$$\Delta \; _i(j, j') = \; (n_h, w^j, ^{-j}) \quad (n_h, w^j, ^{-j}).$$

This expressi n eans that r iven eve s initia wea th, $w^j$ an $w^j$, an pa ffs, $^{-j}$ an $^{-j}$, i inter e iar $n_i$ is c rrent en a e with $w^j$, the a ve q antit is the axi a iti na a nt she is wi in t pa t c ntract with $w^j$ instea , r this is the axi extra a nt she wi pa t keep $w^j$ in case $n_i$ is c rrent with $w^j$ rather than an ther fir $w^j$ [11]

The a ve tw e as pr vi e n a partia characterisati n the set sta e tc es et $n_0 = n$ n e the n er fir s wh are a wa s etter ff ere it n the win the re we characterise the set sta e tc es

[10] y c c $\varepsilon - \varepsilon =$ $- \varepsilon$ $- \varepsilon$
[11] N e $\Delta n$ $j$ $j'$ $w^j$ $w^d$ $n^j$ $n^d$

**Theorem 4.** *An outcome $(\mu, C)$ for the fin nci ɑl ... iel is st ble if n ... n y if t e f ... wing c n iti ns ... :*

**(a)** *On y ... s ... ng $\gamma = \min\{m, n'\}$ p irs ... re f r ... e . If t e nu bers f fir s n inter e i ries ... re ... ifferent, t en eit er t e subset $\{w^{m+1}, w^{m+2}, \ldots, w^{n'}\}$ f fir s, r t e subset $\{z_{n'+1}, z_{n'+2}, \ldots, z_m\}$ f inter e i ries re ... ins un ... tc e .*
**(b)** *T e c ntr cts signe ... re $c_i(\overline{w}^{\mu(z_i)})$ if $z_i$ is ... tc e , n ... $c^{null}$ f r ... ny un ... tc e in ivi u .*
**(c)** *If $w^j > w^{j'}$, t en $\Delta u_{\mu(w^j)}(j, j') \geq \overline{w}^j - \overline{w}^{j'} \geq \Delta u_{\mu(w^{j'})}(j, j')$ f r ... $w^j, w^{j'}$ ... tc e . An ... $\overline{w}^j = 0$ f r ... $w^j$ un ... tc e .*

*Pr f.* We first pr ve that **(a)**-**(c)** are necessary c nditi ns f r an ... utc ... e $(\mu, C)$ t ... e sta ... e

**(a)** ... t is eas ... t ... see that a ... axi ... $\gamma$ pairs are ... r ... e , since the ... atchin ... a ... e is ne t ... ne S pp se that in a sta ... e ... tc ... e strict ... ess than $\gamma = m$ pairs are ... r e ... Then there ... st ... e at east ... ne inter e iar , sa $z_i$ an ... ne fir , sa $w^j$ are ... n ... atche , curin ... er ... pa ffs Then there exists a c ntract ... = $_i(0)$ ... c s ch that ... $_i(w^j, \ ) = \ (z_i, w^j, 0)$ ... $\varepsilon > 0$ an ... $^j(z_i, \ ) \geq \varepsilon > 0$ ... ence, $(z_i, w^j)$ ... cks the ... tc ... e with ... , which is a c ntra icti n

The a ve i p ies that i there are sa e n ... er ... fir s an ... inter e iaries, then there is n fir ... r n ... inter e iar re ains ... n atche

S pp se there are ... re fir s than inter e iaries, an ... in a sta e ... tc ... e a fir $w^j$ with $j < m$ is ... n atche ... This fir ... ets er pa ff ... $w^j$ is ... n atche , then there ... st ... e s ... e $w^k$ with $k \geq m$ ... atche ... Given ... e ... a 3, $^k(\mu(w^k), \ )$ ... st e eq a ... t ... er ... since $w^j > w^k$ Then ... win ... e ... a 1, there exists a c ntract = $_{\mu(w^k)}(0)$ ... c s ch that ... $_{\mu(w^k)}(w^j, \ ) = \ (\mu(w^k), w^j, 0)$ ... $\varepsilon >$ ... $(\mu(w^k), w^k, 0)$ an ... $^j(\mu(w^k), \ ) \geq \varepsilon > 0$ Th s, the pair $(\mu(w^k), w^j)$ ... cks the ... tc ... e with ... , which is a c ntra icti n

N w s pp se that there are ... re inter e iaries than fir s in the financia ... ar ket an ... in a sta e ... tc ... e an inter e iar ... $z_i$ with $i < n$ is ... n atche ... Then there is s ... e $z_k$ with $k \geq n$ is ... atche ... with s ... e fir , sa ... $w^j$ ... t is eas ... t check that ... $(z_i, w^j, \ ^{-j}) >$ ... $(z_k, w^j, \ ^{-j})$ since $z_i > z_k$ [12] Then there exists a c ntract

---

[illegible]

$c^j = c_j(\overline{w}^j) - \varepsilon$ with which the pair $(s_i, w^j)$ ... ks the ... te ... e

(b) F ... ws r ... e ... a 5

(c) We sh w that in a sta e ... tc ... e $(\mu, C)$, ne cann t have (i) $\Delta_{\mu(w^j)}(\hat{\lambda}, \hat{f}) >$ ... , an (ii) ... $> \Delta_{\mu(w^j)}(\hat{\lambda}, \hat{f})$ ... n (i), n tice that the ter ... n the ... e t ... si e ... the ineq a it is the wi in ness t pa ... inter e iar $\mu(w^j)$ r $w^j$ a ainst $w^j$, an the rig t ... si e is the ifference etween the ti ities ... taine ... $w^j$ an $w^j$, the extra (in ter s ... ti ities) $\mu(w^j)$ has t pa i she w ... have ffere the c ntract $_{\mu(w^j)}(\ldots)$ t $w^j$ instea ... fferin $_{\mu(w^j)}(\ldots)$ t $w^j$ This i p ies that $\mu(w^j)$ has an incentive t ... r a ... ckin pair t ether with $w^j$ An s ch a ... ckin pair is via e since there exists a c ntract ... $=$ ... $(\ldots)$ ... $\varepsilon$ s ch that (a) $_{\mu(w^j)}(w^j, \ldots) =$ ... $(\mu(w^j), w^j, \ldots)$ ... $\varepsilon >$ ... $(\mu(w^j), w^j, \ldots)$ an ( ) ... $(\mu(w^j), \ldots) \geq$ ... $\varepsilon >$ ... This c ntra icts the s pp siti n that the ... tc ... e was initia ... sta e

F r the ther part, write (ii) as ... $<$ ... $\Delta_{\mu(w^j)}(\hat{\lambda}, \hat{f}) = \Delta_{\mu(w^j)}(\hat{f}, \hat{\lambda})$ This expressi n is si i ar t (i) ... ence, it is eas t check that with the c ntract ... $=$ ... $(\ldots)$ ... $\varepsilon$ inter e iar $\mu(w^j)$ an inter e iar $w^j$ ... r a ... ckin pair, which is a c ntra icti n

The act that an ... n atche in ivi a ... ess er pa ff ... ws r ... (b)

F r s fficienc , we n ... nee t pr ve (b) an (c), which t ether with (a) i p ... sta i it The ineq a ities in (c) i p

$$(\mu(w^j), w^j, \ldots) \quad (\mu(w^j), w^j, \ldots) \geq (\mu(w^j), w^j, \ldots) \quad (\mu(w^j), w^j, \ldots)$$

t ... ws r ... the a ... ve expressi n

$$\ldots (\mu(w^j), w^j, \ldots) \ldots (\mu(w^j), w^j, \ldots) \geq \ldots (\mu(w^j), w^j, \ldots) \ldots (\mu(w^j), w^j, \ldots)$$

The ast ineq a it i p ies that the ... atchin $\mu$ is pti a in the sense that n ... ther c n i rati n s ch as $(\mu(w^j), w^j)$ can ... ake an in ivi a strict ... etter ff Th s, the pti a it ... th the c ntracts $(\ldots)$ an the ... atchin $\mu$ i p ies there ... n t

... $-$ ... Give n ... $g >$ ... b v x ... n ... c y ... v
k ... n ... n ... n ... n ... n

exist any c ntract $c^d$ different fr $g(-^d)$ an an pair wh can dc the tc e with $^1$, an hence, the tc e is sta e ∎

n the a ve the re we characterize the set sta e tc es the financia arket. First i p rtant pr pert is that a financia c ntracts in a sta e tc e are Paret pti a. This is a resu esta ishe in e a 4. C nseq ent the financia c ntract t e si ne each inter e iar fir pair is the c ntract that s ves the axi izati n pr e (F). Then we sh w that a richer fir earns hi her pa ff in a sta e tc e. This pr pert is r st irrespective the atchin pattern in a sta e tc e. O vi s , ha the fir s een eq a with respect t their initia wea th, the w have e ns e sa e pa ffs.

We are a s a e t esta ish a wer an an pper n t the pa ff a fir tains in a sta e tc e. F win e a 5, a richer fir earns hi her pa ff. Part (c) the a ve the re sh ws that the axi pa ff a fir $w^d$ can ect is the pa ff taine $w^d$, a p rer fir , p s the axi extra a nt (in ter s pa ff) that his atche partner is wi in t pa t e atche with hi rather than ein atche with $w^d$. On the ther han , the ini he can ect is $^{-d}$ p s the axi extra a nt the inter e iar he is atche with is wi in t pa r ein atche with hi rather than with $w^d$. A strai ht rwar interpretati n this is that the inter e iaries c pete r the ri ht t e atche with richer fir s. An a s an inter e iar atche with a richer fir ear t i s her c nterpart.

n Secti n 3.2.2 we have asserte that a fir ein atche with an inter e iar is eq iva ent t the fir ein t en e ere it. The inter e iaries prefer t finance an ec n ica via e pr jects rather than investin capita in the tsi e arket an earnin a risk ree interest rate. One can n tice r part (a) the a ve the re that when fir s r the n si e the arket, iven the re t re a e, n n fir s et their pr jects finance . A s , n the richest fir s an a the inter e iaries are atche an the p rest fir s sta t siness. M re interestin , when the inter e iaries r the n si e the arket, n the est nit rs sta in the arket.

n the win the re we sh w that the set sta e tc es the financia arket is n n e pt .

**Theorem 5.** *For the fin nci r el escribe b e t e set st b e tc es is* non-empty.

*Proof.* We  it a technica  pr  n the  isc ssi n that  ws the the re , we re ate the  atchin  a e  escri e  in Secti n 3 2 3 t  the  atchin  a es ana  se  in A ien an  Ga e [5], an  Craw  r  an  Kn  er [11]  Then it is i  e iate t  pr ve the existence  a sta  e  tc  e  r the financia  arket st  ie  here  □

n the c rrent  e , we  escri e a  ne t   ne  atchin  a e with in ivi  a s en a e  in tra in  ai s  c ntracts  O r  n  ew rk can  e seen as a  enera izati n  the  *assign ent g e*  etween  ers an  se ers  escri e   Shap e  an  Sh  ik [30]  n the assi n  ent  a e,  ers, wh  wn sin  e in ivisi  e  jects again, are  atche  with se ers  n this case, an  tc  e is a  atchin  an  a vect r  prices  r each  er se er pair  n er the  atchin  n  r  e , the transacti ns  cc r via c ntracts rather than prices  The ana  sis  the  pti  a c ntract si ne  an inter  e iar  fir  pair i  e iate  reveа s, as is t pica  in the incentive  e s, that the  pti a it  is n t i  p ie   the  easi i iti n  t ta  s rp  s  a partic  ar pair  n  ther w r s, the  r ntier  (  , u,  ) a pair ( , u) is n n  inear in the  ti it  space  r  the  easi i iti n pr  e   escri e  ear ier (res  tin  r  a n nc  perative  a e p a e   etween an inter  e iar  fir  pair)  ne can see that this n n inearit  i p ies i  perfect transfera i it   s rp s  etween the c ntractin  parties  The  atchin  a e  escri e  in Secti n 3 2 3 is an eq iva ent c  perative  a e p a e   etween fir s an  inter  e iaries  O r  a e,  n ike the assi n  ent  a e  Shap e  an  Sh  ik [30], is characterize   i perfect transfera i it   s rp s  Given the set  in ivi  a s, $\mathcal{I} \cup \mathcal{F}$, an  the  aret  r ntier  r a  atche  pair ( , u), the  atchin  a e  escri e  a  ve is si i ar t  a  ene ra  a e  escri e  A ien an  Ga e [5], an  the  a e  etween fir s an  w rkers  escri e  Craw  r  an  Kn  er [11]  A pr  the a  ve the re , i.e., the set  tc  es is n  e pt , is a  e as the pr  s pr vi e  in the a  ve  enti ne  papers, an  hence we  it it in  r er t  av i  technica ities  O serve that, the assi n  ent  a e  Shap e  an  Sh  ik [30] is a specia  case  rs since the  aret  r ntier in their  a e can  e written as  ( , u, ) = f( , ) , where inter  e iar  can transfer s rp s perfect  t  fir  u

Next we ana  se the  atchin  pattern in a sta  e  tc  e  We w  k at cases when the characteristic  an in ivi  a  is  n t ne in the characteristic  his/her  atche  partner, i.e., the sta  e  atchin  is  s t ne  Tw  t pes  n t ne atchin cum e c nsi ere . First, i richer fir s are atche with re efficient nit rs, then we sa that the atchin is *positive p ss rt tive* ( AM). On the ther han , i richer fir s are nit re ess intensive , then the atchin is *neg tive p ss rt tive* (NAM). The win efiniti n f r a izes these c ncepts.

**Definition 12.** *Given tc ing μ an er w ic fir s $w^1$ n $w^2$ re tic e , μ is neg tive p (p sitive p) ss rt tive if $w^1 > w^2$ i p ies $\mu(w^1) < (>) \mu(w^2)$.*

A ne ative ass rtative atchin i p ies that p rer fir s tain finance r inter e iaries with hi her nit rin intensities. On the ther han , in a p sitive ass rtative atchin richer fir s are nit re re intensive . n the win the re we sh w that in a sta e tc e, we can a wa s uarantee ne ative ass r tative atchin pattern. This kin atchin cc rs i an inter e iar with wer nit rin intensit t i s an inter e iar with hi her nit rin eve i th e pete r cin atche with a richer fir . This is the case when the wi in ness t pa r a richer fir is ecreasin in nit rin intensit .

**Theorem 6.** *In st b e ule e tc ing is neg tive p ss rt tive.*

*Pr f.* First we sh w that in a sta e tc e i the wi in ness t pa ecreases then the atchin is ne ative ass rtative. C nsi er the win *ecre sing wi ingness t p y c n iti n*

$$\Delta \;_i(j, j') \leq \Delta \;_i(j, j') \quad r \;\; n_1 > n_2 \quad an \quad w^1 > w^2. \qquad (DW\;)$$

Then the a ve an art (c) The re 4 t ether i p that in a sta c tc e we st have $\mu(w^1) = n_1$ an $\mu(w^2) = n_2$, an hence μ is ne ative ass rtative.

The n thin re ains t e sh wn is that, iven sta i it , the c n iti n (DW ) is a wa s satisfie . As we isc ss in Secti n 2 3 that the s ti n t pr ( ) is c n i ue t e pti a ver three isj int re i ns the para eter space. t is eas check that n er first est an when the fir 's in ivi a rati na it c nstraint is n t in in $\Delta \;_i(j, j') = \Delta \;_i(j, j')$ r $n_1 > n_2$ an $w^1 > w^2$.[13] S (DW ) is a t atica satisfie .

T see this in the inter e iate re i n, c nsi er the avet r rtier $(n_1, w^1, \ldots)$ t can e pr ven that $\frac{\partial}{\partial n \partial w} \leq 0$ an $\frac{\partial}{\partial^2 w} \leq 0$. Take $w^1 > w^2$ an $n_1 > n_2$. As , in

[13] A n se h h

$^{-j} < ^{-j}$ , $w^j > w^j$ T :

$$( \quad w^j \; ^{-j}) \quad ( \quad w^j \; ^{-}) \leq ( \quad w^j \; ^{-j}) \quad ( \quad w^j \; ^{-j}) \tag{3.1}$$

$$( \quad w^j \; ^{-j}) \quad ( \quad w^j \; ^{-j}) \leq ( \quad w^j \; ^{-j}) \quad ( \quad w^j \; ^{-j}) \tag{3.2}$$

T v

$$( \quad w^j \; ^{-j}) \quad ( \quad w^j \; ^{-j}) \leq ( \quad w^j \; ^{-j}) \quad ( \quad w^j \; ^{-j})$$

T (DW ), □

v v , q , fi v A v ( ), v *rat-hoc* T fi v fi j M v W , q q , v v k T v fi - T [17], R S [25] x , fi j j , v , v v q fi W v v v v , v fi q v x T v , , F , v S , T fi x v q

## 3.4 An Int rm di r in nc s Mor irms

P v u v fi fi
, , fi x q (q
) fi F , . . .
= 0 [16] A fi , > $\sum q$ =
A j N
fi j , k many-to-one
v T fi
v

**Definition 13. (Many-to-one Matching)**
*A (many-to-one)* **matching** *for the market is a mapping $\mu : \mathcal{I} \sqcup \mathcal{F} \to \mathcal{I} \sqcup 2^{\mathcal{F}}$ such that (i) $\mu( \ )$ $2^{\mathcal{F}}$ for $\mathcal{I}$, (ii) $\mu(w^j)$ $\mathcal{I} \sqcup w^j$ f r $w^j$ $\mathcal{F}$ n (iii) $\mu(w^j) =$ if n n y if $w^j$ $\mu( \ )$ f r ( $w^j$) $\mathcal{I} \times \mathcal{F}$.*

x v C
q q T q
$\mathcal{I} =$
$\mathcal{I} = \mathcal{I}_0 \sqcup \ \sqcup \mathcal{I}$ , v
fi W
= = > = = > > = = W

**Proposition 2.** *An utc e ($\mu$ C) is st b e r t e ng t ne r st i n ng i t e ing c n iti ns :*

**(a)** *E c inter c i ry fin nces q fir s n n fir s rc fin nce*
**(b)** $= (\ )$ *r ng* $\mathcal{I}$
**(d)** $\Delta$ $(j\ \mathcal{F})$ $\Delta$ $(j\ \mathcal{F})$ *r* 1 2 .

A v v , ,
fi v fi
T T fi
k

n n n n

away any c rrelati n a n the pr jects Ha the pr jects een c rrelate , then the c ntracts s vin (P) w n t a wa s e m pti a Extensi n r e t the case c rrelate pr jects w then ca r a re s phisticate c ntract esi n which is e n the sc pe the c rrent e

A specia case the e in this secti n is t c nsi er n tw inter e iaries with q tas $q_1$ an $q_2$ s ch that $q_1$ $q_2 = m$ an $a_1 > a_2$ Then a c nseq ence sta i it is that the p rer $q_1$ fir s wi e finance $a_1$ an the richer $q_2 = m$ $q_1$ fir s wi ne n $a_2$ t finance their pr ject This a s cc rs in Rep an S ares [25] The inter e iar $a_1$ is then ca e *informed* c pit an $a_2$ the *uninf r e c pit* n this e we n t a w r the p ssi i it that re than ne inter e iar can invest in ne fir " str an Tir e [17] pr vi es tw interpretati ns nit rin First, a wi r the a ve p ssi i it , nit rin serves as certifica ti n fir 's s venc that attracts nin r e capita int the fir Sec n , when n ne invest r finances a pr ject, nit rin is interprete as inter e iati n that he ps fir s, which are enie cre it nin r e invest rs, invest in their pr jects raisin in r e capita The c rrent e takes the sec n strea , an hence, thr h t the paper we re er t the invest rs as *inter e i ries*

## 5 C l s s

n this paper we e a financia ec n as tw si e atchin a e an char acterise the set sta e tc es We sh w that when fir s nee t raise externa n t finance their pr jects, in eq i i ri , the capita p r fir s have t re n re in r e capita in the arket, an the s ffer r excessive nit rin This c n r s t the fin in s " str an Tir e [17], an Rep an S are [25] Unlike these tw w rks, rs is a e with finite n er in ivi a s an we n t a w r an c rrelati n a n the pr ject ret rns B t the se atch in a es t e the financia arket a ws s en enise the pa ffs a the participatin in ivi a s We a s pr p se a ver si p e ra ew rk that is a e t s we enera (c petitive) eq i i ri es financia arkets characterise incentive pr e s The pa ffs richer fir s are t pica hi her in eq i i ri t is w rth n tin that, the res ts taine in The re s 1 are r st t an eq i i ri atchin patterns

The current e ises severa ven es r t re research. We c nsi er a ne t ne atchin a e with severa inter e iaries an severa fir s, which is a ter exten e t a an t ne e with the restricti n n c rre ati n a n the pr ject ret rns ne ap r ach n this ste re w ea r re s phisticate c ntract esi n an a an t ne atchin e , which w n t e a trivia extensi n the financia arket escri e in the c rrent paper. The eq i i ri in the case with c rre ate pr jects w then aci itate t ana se the effects ifferent kin s acr ec n ic sh cks as ne H stro an Tir e [17], an Rep an S ite [25]. An ther extensi n w et a w re than ne invest r t invest in the sa e fir n this case, as H stro an Tir e [17] interpret, i a re in r e invest r nit rs the fir then it w rks as certi in the fir 's s ency an he ps attract externa capita int the fir r ess in r e invest rs. Fina , ne can exten the e akin the inter e iaries capita c nstraine . This w ive rise t a ra hazar pr e in the eve nit rin , an it w a s e re rati na then t c nsi er c rre ate pr ject ret rns in a an t ne set p.

Chapter 4

# Does Market Concentration Preclude Risk-Taking in Banking?

[This chapter is jointly written with Santiago Sánchez Pagés]

## 4.1 Introduction

When banks are able to raise deposit to invest in assets with uncertain returns, excessive deposit i ht in uce banks to take ore risk. Involve ent in hi h risk activities is viewe as one of the principal causes of several instances of bankin crises that the worl econo has witnesse urin the last two eca es.

The ain oal of this chapter is to anal se the role of arket concentration an equit capital of banks in eter inin the risk-takin behaviour of banks in the absence of eposit insurance. The bankin sector escribe here consists of a finite nu ber of banks. Banks are i entical with respect to their equit capital. The raise eposits b offerin eposit rates. Banks choose between a *prudent* asset an a *gambling* asset to invest their total fun (equit plus eposit). The a blin asset on avera e iel s lower return than the pru ent asset, but if the a blin is successful it ives hi her return. There is a continuu of epositors. The epositors have one unit of onetar fun apiece, which the a place in a bank to earn eposit rate offere . The epositors are not insure in case the a ble fails.

We anal se the bank co petition in the context of a circular cit o el *à la* Salop [27]. Both the banks an the epositors are locate unifor l on a unit circle. The depositors incur a per unit transport cost to travel to a bank. Two types of symmetric equilibria might arise: A *prudent equilibrium*, where all banks invest in the prudent asset, and a *gambling equilibrium*, where all banks invest in the gambling asset. We use the unit transport cost relative to the number of banks as a measure of market concentration.

We completely characterise the equilibria of the banking market. We show that when concentration is low, banks compete aggressively in order to capture greater market share by offering higher deposit rate. In this case, all the depositors participate, and a competitive market is said to arise. Here, for very low concentration, all banks invest in the Gambling asset (i.e., a Competitive Gambling Equilibrium exists). As concentration increases, all banks investing in the prudent asset can also be supported in equilibrium (i.e., a Competitive Prudent Equilibrium co-exists).

For high levels of market concentration, banks never invest in the gambling asset. This is because, in a concentrated market, banks earn higher rent which induces them choose prudent asset in order to preserve that. Hence, for moderately high concentration, competition with banks investing in the prudent asset is the only equilibrium. For even higher levels of concentration, a *local monopoly* market emerges where deposit rate is so low that some depositors find it unprofitable to place their funds in banks and prefer to stay out of the deposit market.

To summarise, we show that higher market concentration is efficiency enhancing. As market concentration increases, the resulting lower competition induces banks to invest only in the prudent asset.

The equilibrium outcomes of the market also have significant implications for social welfare. For a fixed level of equity capital, higher concentration precludes banks to choose the gambling asset. Banks invest in the prudent asset and offer higher deposit rates. As a result, social welfare increases. Only when a local monopoly market arises, market concentration has adverse effects on social welfare. All this suggests a non-monotonic relation between market concentration and welfare that calls for a careful examination of competition policies. Also, when bank's equity capital increases a Competitive Gambling Equilibrium and a Monopoly Prudent Equilibrium become less likely.

Our model resembles to the literature in bank regulation. Prudential regulations of banks are viewed as instruments to prohibit banks from investing in risky projects.

To this end, several mechanisms are used by the central banking authorities. Two popular instruments are minimum capital requirements and deposit rate ceilings. Hellmann, Murdock and Stiglitz [16] show, using a dynamic model of prudential regulation, that freely determined deposit rates are inconsistent with Pareto efficiency, and that an optimal regulation policy combines minimum capital requirements and deposit rate ceiling. Repullo [24] uses a dynamic model of banking based on spatial competition à la Salop, to show that for a very low levels of market concentration all banks invest only in the gambling asset if there is no regulation. Chiappori, Pérez-Castrillo and Verdier [10] analyse the regulation of deposit rates in a circular city model of bank competition in both the deposit and loan markets.

The current setup is closest to that of Repullo [24] and Keeley [18]. The latter shows, using a static model, that increased competition leads to higher risk-taking in banking. We also use a static model assuming away any sort of deposit insurance. Standard portfolio choice theory asserts that positive risk premium (the difference between the expected return from the risky asset and the return from the safe asset) is necessary to induce (risk averse) individuals to invest positive amount in a risky asset. In the current model with negative risk premium, a gambling equilibrium might arise since a successful gamble yields more than a prudent asset. Hence, in order to ensure a prudent equilibrium, one needs an additional restriction that, given deposit rates and total volume of deposit, the profit from investing in the prudent asset must exceed that from the gambling asset (the *No Gambling Condition*). In our model, banks invest all their equity capital. This can be thought as if there is fixed capital standard posed by a regulatory authority. But, as Hellmann, Murdock and Stiglitz [16] show, capital requirements are not enough to achieve Pareto efficiency unless combined with deposit rate ceiling. In our model, unlike their work, total volume of deposit with a bank determines whether bank is going to choose a gambling asset or a prudent asset. Hence, the uninsured depositors play a major role in determining banks' risk-taking behaviour.

The main body of this chapter shows that increased market concentration ensures that all banks invest in the prudent asset. Much of the debates on bank mergers pose the view that mergers are able to enhance efficiency in face of bank risk. In the current model we also assert that if we give greater market power by allowing for bank mergers then that will ensure that banks are less likely to choose the gambling assets to invest in.