UNIVERSITAT DE BARCELONA

Essays in Economic Theory

Ausias Ribó Argemí

de Barcelona

PhD in Economics | Ausias Ribó Argemí

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PhD student:

Ausias Ribó Argemí

Advisor:

Antonio Manresa Sánchez

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Contents

1 Overview

This thesis is composed of 3 independent essays on economic theory. Each essay is meant to be read separately, including footnotes and appendices. In particular, essays 2 and 3 include specific bibliography. The general bibliography is included at the end of the thesis.

The first essay reviews some well known conceptual and empirical problems that appear when economic theorists deal with preferences and choice theory, in general. While assessing those problems, the essay lays the ground for a detailed discussion of the possibility of preference learning, formation and change. The essay concludes proposing a theoretical framework to study these phenomena.

The second essay, although independent from the first, is also devoted to the issue of preference change. In particular, it studies the possibility that cultural preferences evolve as a result of the combination of technological innovation and cultural transmission mechanisms. At the same time, it allows for the possibility that those cultural preferences determine the short term outcome of economic variables. In addition, it builds a framework where the combination of technological innovation, cultural transmission and economic structure lead to a process of endogenous preference heterogeneity and clustering. Hence it provides a model to understand how culture and the economic structure interact and coevolve.

The third essay presents some theoretical problems that arise when using the concept of a matching function as a modelling device for the labor market. In particular, necessary conditions for the ratio of the number of matches per job searcher to be interpreted as the average job finding probability are established.

2 Acknowledgements

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This thesis is devoted to Isa, with love.

Part I

Preference and choice theory in economics: some conceptual and empirical problems

3 Economics and Philosophy of Science.

The goal of the present essay is to review some of the problems that arise when economists attempt to study and model preferences and choices. In particular, we are interested in how those modelling concerns and empirical challenges impact the current methodological stance towards preference learning, formation and change. To that end, we start by the reviewing the relation between theoretical and applied economics and philosophy of science.

The two questions we briefly review are about the object of economics (economic ontology) and about the process by which economic knowledge is acquired (economic epistemology).

3.1 Economic ontology

The first level to consider is that of ontology, which tries to answer the question of what are the basic elements of that part of the reality that is to be studied, namely economic phenomena. There have been several approaches to this problem. Without being exhaustive, we characterize the most relevant for the present work. Depending on which is deemed to be the basic unit of economic analysis, we can distinguish between the *individualistic approach*, that takes the individual to be the main center of action, decision and movement of the economic reality, the *holistic or globalistic approach* adapted to economics, which deems the individual to be totally determined by its social or historical environment, leading to the existence of classes or groups as the basic building blocks of the economic ontology (within this class, we can include the Marxist approach and the social constructivism approach), and all kinds of *middleground approaches*, that take the basic constituent of economic reality to be a mix of individuals and the society as a whole. One of such strands is the *systemic approach*, which we characterize briefly here. A system can be represented by its components (in the economic case, those would be individuals), the environment within which those components are embedded (for economics, this would include those elements of the physical environment that interact with the individuals) and the structure (which is the collection of relations between the individuals themselves and between the individuals and the objects of the environment). The relations that constitute the structure are defined by some properties, and those can be, in turn, quantifiable or not. Crucially, systems can have emergent properties, that is properties that are not shared by its components. And finally, different systems interact with each other (the most prominent example in economics being the interaction between the economic and the physical system within which it is embedded).

The previous differences between approaches do not necessarily coincide with the difference between macro and microeconomics: the individual, the holistic and the systemic approach can accomodate such distinction, but they differ on the importance attached to each one and, most crucially, on the direction of causal relations between both levels. This has led some authors to argue that a meso-economic ontological level is a necessary complement to the micro and macro levels of analysis (see [Kirman, 1989], [Dopfer, 2004]).

The choice of the ontological scenario will have strong consequences on the epistemological and methodological stance that is to be taken with regard to the study of economic reality, and particularly on the kind of theory that is to be built. Most notably, the neoclassical tradition has been built around individualism, while other traditions (institutionalism, post-keynesianism, marxism) have adopted differing ontologies not exclusively based on the individual. This variety in postulating an ontology is especially relevant for 3 problems that will be discussed later:

- Who is the subject of preferences: are individual preferences well defined independently of the social group and environment where individuals find themselves? or are they derived from those?. Here, the roles of institutions, context dependence and interpersonal dependence will be central in trying to explain phenomena such as preference clustering.

- How does aggregation interact with preferences: are individual preferences stable to interaction, and if so, are their properties predicable of the aggregate? do new behavioural properties arise when we consider aggregation? .This is relevant when we consider the role of preferences in social choice contexts, consumer theory and general equilibrium, and the problem of the empirical content of such theories. It is this question that mostly accounts for the differing frameworks between a large part of neoclassical economics (mostly based on an axiomatic foundation, assuming stability and looking for conditions under which aggregation preserves the individual properties of preferences) and behavioural economics, evolutionary game theory and institutionalism in general (usually starting from the individualist ontology to depart from it by assuming that interaction generates new properties, for example in Agent Based Models, henceforth, ABM).

- Do preferences change over time: this is independent from the other two in that even if individual preferences are well defined independently of the enviroment and we do not consider aggregation problems, there can be variation in individual preferences along the life of individuals and from one generation to another. In short, this question adresses the problem of how preferences are formed and vary. Here, we will be interested in looking at the interaction between preferences and technology[1].

3.2 Economic epistemology

We start by reviewing, from a historical point of view, some economic epistemologists that are particularly related to the issues we want to discuss, and then highlight several aspects of economic methodology that we deem particularly important for our subject[2],[3].

3.2.1 Historical review

John Stuart Mill. In economics, the problems highlighted above can be traced back at least to the work of John Stuart Mill ([Mill, 1994]), the most influential methodologist in economics during the nineteenth century, who maintains that direct inductive methods cannot be used to study phenomena in which many

[1] See Chapter 2.

[2] We are well aware that probably the most important authors in economic methodology are being left out of this review.

Among those, David Ricardo, William Jevons, Karl Marx, Alfred Marshall, Joseph Schumpeter, Wassily Leontieff, Piero Sraffa. However, the aim of this section is not to compile a history of economics methodology but to highlight some very specific problems in this field, that are related to the issue of preference and choice modelling, and by association, to the issue of preference formation and change.

[3] This section is largely and loosely based on the Stanford encyclopedia of philosophy, entry for *Philosophy of Economics.*

causal factors are in play. Mill argues in favor of an indirect inductive method, which he calls *a priori*: in the first stage, Mill proposes to use direct inductive methods to determine the laws governing individual causal factors in domains well suited for the use of such methods, while in the second stage, the combined consequences of the individual causal factors are studied deductively. In this method, verification of the combined consequences is only used to check whether the deductions were correctly performed and whether important factors were not accounted for ([Hausman, 1992].

Because economic theory includes only the most important causes and necessarily ignores minor causes, its claims are inexact. Its predictions will be imprecise, and sometimes far off. However, according to Mill, it is still possible to develop and confirm economic theory by using simpler domains to study the laws governing the major causal factors and then deducing their consequences in more complicated circumstances. This is important insofar as it reflects current economics methodological practice[4] , despite the fact that few economists explicitly defend it.

Milton Friedman. During the twentieth century, Mill's view that the basic principles of economics should be empirically established was put to test and doubts were cast on some of the basic principles of economics, particularly the view that firms attempt to maximize profits ([Hall and Hitch, 1939]). Some authors who were more in line with Popperian approaches to philosophy of science denounced pure theory in economics as unscientific ([Hutchison, 2000]). At the same time, several attempts were made to replace the Millian view with a more modern methodology that would serve as a benchmark to justify much of current practice ([Machlup, 1955], [Koopmans, 1957]). The most influential of these was Milton Friedman's contribution ([Friedman, 1953]).

[4] See [Hausman, 1992]

Friedman differentiates in a conventional way positive from normative economics and conjectures that disagreements about policies are due to disagreements about the consequences of alternatives. This attempt to reduce normative disputes to the need of a more exhaustive knowledge of positive consequences is coupled with the assertion that correct prediction concerning phenomena not yet observed is the ultimate goal of all positive sciences. He holds a practical view of science and looks to science for predictions that will guide policy.

Since it is difficult and often impossible to carry out experiments and since the uncontrolled phenomena economists observe are difficult to interpret (owing to the same causal complexity involved), it is hard to judge whether a particular theory is a good basis for predictions or not. This problem is what misled economists, according to Friedman, to believe that they could test theories by the realism of their assumptions rather than by the accuracy of their predictions. He presents the case of testing whether firms maximize profits as the clearest example. Friedman argues against this, by stating that the realism of a theory's assumptions is irrelevant to its predictive value, which is the only scientific criterion.

Friedman does not clearly define what he means by assumptions and unrealistic. If those are taken to mean central economic generalizations and false, then Friedman is arguing at least that it is a mistake to appraise theories by investigating whether their central generalizations are true or false. However, in Friedman's view, economists are interested in only some of the implications of economic theories. Other predictions, such as those concerning the results of surveys of managers, are irrelevant to policy. Summarizing, Friedman believes that economic theories should be evaluated only by their predictions concerning prices and quantities exchanged on markets. In his view, what matters is "narrow predictive success" , not overall predictive adequacy. So economists can simply ignore the disquieting findings of surveys. This provides cover for the practice of ignoring whether one's assumptions hold, as long as the model used makes correct predictions. But in addition, since anomalous market outcomes could be due to any number of uncontrolled causal factors, while experiments are difficult to carry out, it turns out that economists need not worry about ever encountering evidence that would disconfirm fundamental theory. Detailed models may be confirmed or disconfirmed, but fundamental theory is safe. In this way one can understand how Friedman's methodology, which appears to justify the eclectic and pragmatic view that economists should use any model that appears to "work" regardless of how absurd or unreasonable its assumptions might appear, has been deployed in service of a rigid theoretical orthodoxy.

Contemporary approaches to methodology in economics. Karl Popper's falsificationist methodology ([Popper, 1968], [Popper, 1969]) has had important influence in contemporary economics methodology. His view that theories should be "logically falsifiable" (that is, inconsistent with some possible observation reports) is based on the idea that unfalsifiable claims that rule out no observations are uninformative. In his view, scientists should extensively test their theories and reject them when they fail the tests, while any unrejected theory should be thought of as a conjecture at best. The idea is that no test can confirm a theory. With regards to economics, Popper defends what he calls *situational logic* as the correct method for the social sciences ([Popper, 1985]). However, specific economic theories are rarely logically falsifiable. When they are, the widespread acceptance of Friedman's methodological views ensures that they are not subjected to serious test. When they apparently fail tests, they are rarely rejected. Economic theories, which have not been well tested, are taken to be well-established guides to policy, rather than merely conjectures ([Eichner, 1983]).

The two most important Popperian methodologist in economics are Mark Blaug ([Blaug, 1992]) and Terence Hutchison ([Hutchison, 1977]) both of whom call for more testing and a more critical attitude, but transforming Popper's demarcation criteria into little more than a warning that scientists should be critical and test their theories. Blaug's and Hutchison's criticisms have sometimes been challenged on the grounds that economic theories cannot be tested, because of their ceteris paribus clauses and the many subsidiary assumptions required to derive testable implications[5]. But this response ignores Popper's insistence that testing requires methodological decisions not to attribute failures of predictions to mistakes in subsidiary assumptions or to interferences.

As is well known, applying Popper's views on falsification literally would leave almost no elements in the set of scientific theories. Not only neoclassical economics, but all known economic theories would be condemned as unscientific, and there would be no way to discriminate among economic theories. One major problem general problem with a naive reading of Popper's views is that one cannot derive testable implications from theories by themselves. Subsidiary assumptions concerning distributions, measurement devices, proxies for unmeasured variables and the absence of various interferences are needed. This problem was first posed by [van Orman Quine, 1976]. and is known as the Duhem-Quine problem. Popper argues that this can be solved by a methodological decision from the scientist to regard a failure of the deduced testable implication to be a failure of the theory. But in economics the subsidiary assumptions are dubious and in many cases known to be false. Making the methodological decision that Popper requires is unreasonable and would lead one to reject all economic theories.

Imre Lakatos ([Lakatos and Musgrave, 1970]) has also been influential in economics. He argues that any modification of a preexistent theory that adds testable implications is a reason to employ new theories, highighting that the ultimate goal of scientific research is to be *emprically progressive*[6]. This appears to solve the

[5] See [Caldwell, 1984]

[6] In Lakatos terminology.

problem of how to evaluate mainstream economic theory by arguing that what matters is empirical progress or retrogression rather than empirical success or failure. Also, drawing on Kuhn's notion of a *paradigm* ([Kuhn, 1962]), Lakatos emphasized that there is a *hard core* of basic theoretical propositions that define a scientific research programme and are not to be questioned within the research programme. In addition members of a research programme accept a common body of heuristics that guide them in the articulation and modification of specific theories. These views have also been attractive to mainstream economic methodologists, since theory development in economics faces seemingly rigid rules and the theory of rational action, joined with the general equilibrium approach did seem to play the role of a hard core in mainstream economics, at least until the last two decades. The fact that economists do not give up basic theoretical postulates that appear to be false might be explained and justified by regarding them as part of the *hard core* of the neoclassical research programme. Yet Lakatos views do not provide a satisfactory account of how economics can be a reputable science despite its reliance on extreme simplifications. Furthermore, science is for Lakatos more empirically driven than is contemporary economics ([Hands, 1992]). Apart from that, several authors have questioned whether economics research has a hard core ([Hausman, 1992]).

Also influential have been Deirdre McCloskey's views ([McCloskey, 1983]) that the only relevant and significant criteria for assessing the practices and products of a discipline are those accepted by the practitioners. According to those, economists can dismiss arrogant pretensions of philosophers to judge economic discourse.

From another perspective, there have also been substantial efforts to apply structuralist views of scientific theories ([Sneed, 2012]) to economics[7].

[7] See [Stegmüller et al., 2012] and [Balzer, 1982], for example.

3.2.2 Some aspects of economic methodology

Positive versus normative economics. One of the central tenets in economics methodology is the distinction between positive and normative economics. Setting aside the implicit problems in defining what are scientific facts in general, the tenet holds that it is reasonably feasible to distinguish between economic facts and values. In that sense, it adheres to the view of economics as a mostly positive science that helps policy makers choose means to accomplish their ends, without taking a stance on the choice of ends itself.

Several authors have raised questions about this view ([Mongin, 2006]) arguing that: 1) economists do affect the choice of ends itself because the constraints and goals of the policy makers are often incomplete in nature, and so must be fully interpreted and articulated before being subjects of study ([Machlup, 1969]); 2) the central role of normative theory of rational action in economics infuses with its own intrinsic values the construction of models and the assesment of alternative policies; 3) values and beliefs are coevolving, as for example, the larger the exposure to the idea of individuals as self-interested, the larger the likelihood to hold self-interest as an important value ([Marwell and Ames, 1981]); 4) the incentives faced by researchers play a role in their judgment ([Marx, 1867]).

We are interested in particular in wether preferences and choice structures have to be considered a normative or positive part of economics. In general, as noted in 2), the notion of rational action plays the role of a governing principle in modelling preferences and choice structures. However, the particular axioms or properties through which this principle is included in the modelling (be it, for example, any of the variations on the Weak Axiom of Revealed Preference, WARP, or the notions of completenness and transitivity of preferences) have been defended both as normative mechanisms of the governing principle (it is against the intuition of rational action that someone could have intransitive preferences and be repeatedly tricked by a money pump type of situation and not change her actions) and as positive features (individuals do, in general, choose some of their available and affordable options when faced with alternatives, which can be argued as a reason to assume completenness).

But defending them as positive given that they are false as empirical regularities (see below), is only possible if they are either simplifications (so that they would represent what preferences would be in absence of distorting factors) or insofar as they help to capture a more empirically accurate description of human action, meaning that the structure formed by axiomatizing preferences, setting budget constraints and maximization delivers predictions about what individuals would do in a particular situation that are consistent with what they actually do. The first possibility begs the question about which external factors could be at work to account for the differences between the simplifications and the observed behaviour. The second possibility implies, however, adhering to the view that only narrow predictive success matters, without regard for the realism of the hyopthesis, à la Friedman, which is problematic in itself (see below).

If instead, we take them to be normative devices, it is necessary to show that no alternative devices (incompletenness, intransitivity) are consistent with our intuition of the rational action that is guiding our construction.

Yet, the previous discussion does not take into account the question of whether preferences and choice functions are static or dynamic, stabe or unstable. That is, preference change or stability plays an important role when considering preferences as normative or positive in nature. In order to tackle transitivity, for instance, it is necessary to define first what it means when preferences are changing. Is it a property of the old, the new or the joint set of preferences? if it is predicable only of the static set of current preferences, then in order to check whether it holds empirically one must define the set of factors, including time period, that can affect them. This is also true of completenness, and specially relevant of axioms on choice functions (for example, the Generalized Axiom of Revealed Preference, GARP, [Varian, 1982],[Varian, 1983], assumes stability of preferences across choice observations, which implies time stability). In addition to problems of definition, assuming change of preferences forces a redefinition of the choice-based approach in order to keep its claim to be the observable counterpart to the preference approach. This is particularly clear in the expanding literature on change of tastes, temptation, self-control, be it from a perspective of set-dependent choice or from a multiple selves approach ,which will be discussed later.

It would be interesting to study from an evolutionary point of view if transitivity-intransitivity and completenness-incompletenness can coexist in the long run in a population with changing preferences. It is easy to imagine scenarios where one property entails an evolutinary advantatge : for example, groups with intransitive preferences might be more prone to dissapear or endogenously change them in the face of repeated money pump type situations (it is easy to imagine a population with transitive and intransitive preferenced individuals where they randomly meet to bargain. If the transitive group is non zero measure, there is some probability for the intransitive individuals to successively meet transitive individuals and after sequentially trading, they could end up worse than in the beggining, whilst the same outcome is not likely for the transitive group. This would trigger an increase in transitive individuals), and groups with incomplete preferences could be at a disadvantage when faced with bargaining situations (imagine that whenever two individuals meet and bargain, no trade takes place if one individual's offer falls into the incomplete part of the preferences of the other individual. That would make a trade agreement between complete individuals more likely). However, the reverse could be also defended: intransitivity could be an evolutionary advantage if for example, one of the items traded becomes necessary for survival (imagine there is food, f, housing, h and transportation, t as bargaining options. Suppose an initial population where everybody prefers f>h, h>t and being transitive, f>t. Now, some natural disaster decimates the food and clothing supplies. Then a mutated individual who has f>h, h>t, but t>f would be more likely to survive) and incompletenness can arise as a precaution against possible threats (imagine that a new type of individual who bargain more aggressively, is introduced in the population, increasing the number of trades that end up in a much worse situation for the initial population. Then incompletenness would imply a lower likelihood of falling in that trade and a higher probability of at least mantaining the status quo).

Reasons versus causes The second methodological problem we want to discuss is the role of preferences and choice functions as reasons or causes: in the examples given in 1 above, preferences in particular are taken to be reasons for action. Reasons because the individual that knows her preferences has a reason to choose one option instead of another. However, they are not taken to be causes insofar as liking one bundle more than another does not per se trigger the act of choosing it, for example, if it is not affordable. That is, while preferences are reasons per se, they are causes only in conjunction with the constraints faced.

If we take the choice approach instead, the problem is turned upside down in the sense that a choice function provides a cause for action (in the sense that it provides the causal relation between a given choice set and the choice made. In that sense, the choice function is a causal factor), but without giving an explicit reason for it. The reason is only provided in this case by the assumption that choices reveal a consistent ranking, which again invokes the stability assumption. The difference is that preferences compare choices in the abstract while a choice function gives behaviour in the concrete case. It is precisely the need to deal with observable, concrete actions what drives the construction of choice functions as fundamentals for action.

Moreover, if we accept that explanation involves a certain assimetry between explanandum and explanans ([Hempel and Oppenheim, 1953]), the same argument that concludes that preferences explain actions implies that choices cannot explain preferences. This would have consequences for the presumed equivalence between the preference approach and the choice-based approach.

Since rationality plays a central role in economics, it is natural that the concept of reason for an action acquires a relevance that does not have in the natural sciences.

Explaining by reasons differs from explaining by causes in that reasons justify the actions, can be evaluated and critized and must be understood by the agent of the actions. These are all features associated with rational action, and have led some authors to raise doubts about the possibility of causal explanations in the social sciences ([von Wright, 1971]).

In particular, some authors have argued that the appeal of rational action as a guiding principle is that rational actions are self-explanatory, that is, the fact that an action is consistent with the system of preferences and beliefs of an agent is sufficient reason to explain the action if the agent is rational. This would be a more satisfying procedure than invoking causal factors (sociological, psychological, etc) that rely on black boxes type of explanations ([Becker, 1978],[Boudon, 1998]). However, this presumes a certain theory of what constitutes scientific explanation. Moreover, in economics in particular, a very important difference between the two types of explanations is that reasons are constructed on the basis of preferences and beliefs, which might be erroneous or incomplete ([Knight, 1925]).

Social scientific naturalism This leads us directly to the third methodological problem, which is the difference between natural and social sciences. This difference can be characterized as an answer to the following questions:

1) Are theories and explanations in natural and social sciences different in structure or concept?

2) Are there fundamental differences in goals? some authors have argued that social sciences should, in addition to explanations, provide understanding of the actions ([Weber, 1949], [Knight, 1925])?

3) Are laws and theories well suited to capture social phenomena? this has to do with the perception that many social phenomena might be too irregular to be captured by lawful statements?

In economics in particular, a basic tool to deal with this observed irregularity and multiplicity of causal factors has been the use of simplifications, abstractions and ceteris paribus clauses. This leads us to the next methodological problem.

Abstraction and idealization in economics One of the most important methodological issues concerning economics involves the very considerable simplification, idealization, and abstraction that characterizes economic theory and the consequent doubts these features of economics raise concerning whether economics is well supported. Mainstream economic models often assume that everyone is perfectly rational and has perfect information or that commodities are infinitely divisible. This highlights the question about how much simplification, idealization, abstraction is legitimate, and how to interpret simplifications because if they are interpreted as universal generalizations, they are false. Can a science rest on false generalizations? If these claims are not universal generalizations, then what is their logical form? And how can claims that appear in this way to be false or approximate be tested and confirmed or disconfirmed?

Economics and ethics Economists typically evaluate outcomes exclusively in terms of welfare. In order to justify this procedure, some attempts have been made to find conditions under which questions regarding welfare can be separated from questions about equality, freedom, or justice. In particular, this is one of the interpretations of the Second Welfare Theorem in general equilibrium theory. Here, welfare is taken to mean material well-being. However, there is nothing a priori that precludes introducing concerns about equality, freedom and justice in the preference structure, so that raises the question about whether welfare is equivalent to maximizing one's preferences.

Welfare One central question of moral philosophy has been to determine what things are intrinsically good for human beings. This is a central question, because all plausible moral views assign an important place to individual welfare or well-being. This is obviously true of utilitarianism (which hold that what is right maximizes total or average welfare), but even non-utilitarian views must be concerned with welfare, if they recognize the virtue of benevolence, or if they are concerned with the interests of individuals or with avoiding harm to individuals.

There are many ways to think about well-being, and the prevailing view among economists themselves has shifted from hedonism (which takes the good to be a mental state such as pleasure or happiness) to the view that welfare can be measured by the satisfaction of preferences. A number of prominent economists are currently arguing for a return to hedonism, but they remain a minority. ([Frey and Stutzer, 2010], [Kahneman and Thaler, 2006]) Unlike hedonism, taking welfare to match the satisfaction of preference specifies how to find out what is good for a person rather than committing itself to any substantive view of a person's good. Note that equating welfare with the satisfaction of preferences is not equating welfare with any feeling of satisfaction. If welfare can be measured by the satisfaction of preferences, then a person is better off if what he or she prefers comes to pass, regardless of whether that occurrence makes the agent feel satisfied.

Since mainstream economics attributes a consistent preference ordering to all agents, and since more specific models typically take agents to be well-informed and self-interested, it is easy for economists to accept the view that an individual agent A will prefer X to Y if and only if X is in fact better for A than Y is. This is one place where positive theory bleeds into normative theory. In addition the identification of welfare with the satisfaction of preferences is attractive to economists, because it prevents questions about the justification of paternalism (to which most economists are strongly opposed) from even arising.

Welfare and the satisfaction of preferences may coincide because the satisfaction of preferences constitutes welfare or because people are self-interested and good judges of their own interests and hence prefer what is good for them. There are many obvious objections to the view that well-being is the satisfaction of preferences. Preferences may be based on mistaken beliefs. People may prefer to sacrifice their own well-being for some purpose they value more highly. Preferences may reflect past manipulation or distorting psychological influences ([Elster, 1983]). In addition, if preference satisfaction constitutes welfare, then policy makers can make people better off by molding their wants rather than by improving conditions. Furthermore, it seems unreasonable that social policy should attend to extravagant preferences. Rather than responding to these objections and attempting to defend the view that preference satisfaction constitutes well-being, economists can blunt these objections by taking preferences in circumstances where people are self-interested and good judges of their interests to be merely good evidence of what will promote welfare ([Hausman and McPherson, 2009]). There are some exceptions, most notably Amartya Sen ([Sen, 1985]), but most economists take welfare to coincide more or less with the satisfaction of preferences.

3.3 Rationality in economics

Since the different notions of rationality are a central part of the disagreements noted above, it is useful to take a look at the role that they have played in economic analysis. As has been pointed out[8], it is not necessary for an economic theory to be built on a technical notion of rationality, even if one adopts the individualistic ontological view for economics, since a model of individual economic behaviour can be constructed on the concept of habits or some other rule, and in fact, such elements play a role in many models even if rationality is assumed[9]. With this in mind, we distinguish two different concepts of individual rationality in mainstream economics: in a *broad sense*, it is equivalent to assume that agents optimize some defined objective function when choosing their actions. This should be contrasted with other hypothesis about individual behaviour that have been proposed, such as satisfaction (by [Simon, 1982]), rules of thumb (behavioral economics), random behavior, etc. This broad sense is not the same as the more restrictive notion of selfishness, since it is possible to build a model based on non-maximizing self-interested individuals and also one of maximizing altruistic agents. In a more *narrow and specific sense*, rationality is defined partially, but not exclusively, by the properties of individual preferences: transitivity and completeness, or the equivalent conditions on choice functions. As will be discussed later, both conditions have been challenged from a normative and positive perspective[10]. Nonetheless, a large part of the literature on preferences and choice has

[8] See [Arrow, 1986], [Simon, 1982], among others.

[9] Perhaps the clearest formulation is in [Arrow, 1986], p.21: "Let me dismiss a point of view that is perhaps not always articulated but seems implicit in many writings. It seems to be asserted that a theory of the economy must be based on rationality, as a matter of principle. Otherwise, there can be no theory. This position has even been maintained by some who accept that economic behavior is not completely rational."

[10] To capture the disputed nature of this definition, we can look at the early attempts to overcome it as a justification for aggregate behaviour properties: [Becker, 1962], for example,

been centered on defining this more specific notion, studying its properties and constructing axiomatic frameworks that provide equivalent testable conditions, the main example being revealed preference theory. However, rationality in both senses also imposes some structure on social behaviour because it owes much of its explanatory power to its combination with the concepts of equilibrium, perfect competition and complete markets, so that if some of those assumptions are not met, rationality per se can become self-contradictory. But even with perfect competition and complete markets, the fact that in most theoretical approaches (for example, general equilibrium theory), the existence of an equilibrium does not imply anything regarding out of equilibrium trading implies that such a potentially self-contradictory concept of rationality is pervasive ([Arrow, 1986]).

With regards to cognitive complexity, this narrow sense of rationality implies a computational and memoristic capacity for individuals that is at odds with theoretical and empirical data on the functioning of the brain[11]. That is especially true when rationality is paired with an informational assumption about the economic fundamentals, as for example in perfect knowledge or rational expectations. This is in stark contrast with the notion of the invisible hand acting on minimal informational requirements that was typical of the classical economists vision, as exemplified by the role of prices: in the classical vision of prices as coordinating devices that encode information, there is a trade-off between knowledge of prices and the amount of additional information an individual needs to gather in order to make choices. However, this trade-off dissappears when knowledge of all future

builds a rationale for a downward sloping market demand curve that is compatible with purely random or habit individual behaviour.

[11]As an example, according to the usual theory on consumer behaviour, the number of calculations an individual needs to perform to maximize utility subject to budget constraint, if there is a continuum of options imply that the consumer problem is, in computer science terminology, NP-hard.

prices is built directly into the individual. In addition, as Arrow pointed, an empirical set of complete equilibrium prices will not exist as long as there is some search cost associated with knowledge of prices. In the case of incomplete markets, the informational requirements imply that it is necessary for each individual to form costly expectations about the behaviour of other agents.

This is not to say that cognitive or informational complexity per se is to be rejected as a feature of a model or as a description of behaviour. But the merits of introducing complexity have to be assessed through comparison with competing models and tested through data. Moreover, the mathematical notion of complexity is not equivalent to the behavioural or psicological one, since the former relies heavily on an assumed philosophy of the mind (e.g. the computer analogy).

But perhaps it is aggregation that poses the most difficult question for the narrow concept of rationality. This is due to the problems it presents to general equilibrium theory, which was assumed to be the organizing paradigm of neoclassical economics, as it ensured existence, local uniquenness, stability and other properties of equilibria that are crucial for comparative statics, a fundamental tool for many other branches of economics. General equilibrium theory was also charged with the goal of ensuring that macroeconomic models, constructed to match qualitative empirical features (such as the law of demand) coud be obtained from rigorous microfoundations. The main problem with aggregation and rationality is that only certain properties of individual excess demand functions obtained under the narrow individual rationality assumption jointly with price-taking behaviour and perfect information are inherited by aggregate excess demand functions.[12] This represented a challenge in a doble way: first, it undermined the program

[12] According to the Debreu-Mantel-Sonenscheim theorem, 1973, that has been extended for the case of incomplete preferences by [Mas-Colell, 1986] and incomplete markets by [Momi, 2003]. See [Rizvi, 2006] for an historical account.

to build microfoundations for macroeconomic models, and in particular, the attempt to model macroeconomic behaviour as equivalent to a representative agent maximizing preferences that inherit those properties assumed at the individual level. Second, it suggested that general equilibrium theory does not generate restrictions refutable from aggregate data alone. Hence, the empirical content of general equilibrium theory is somewhat dubious. Even if some attempts have been made to establish that general equilibrium could, in principle, generate testable restrictions when individual data are known ([Brown and Matzkin, 1996]), it remains clear that those restrictions do not concern the basic properties of local uniquenness, stability and comparative statics in general. Hence, it cannot say much about these features of the economy.

In response to those challenges, different branches of economics developed different concepts of rationality and attempted to bypass the microfoundations problem by defining new equilibrium concepts. In particular, developments in game theory in its evolutionary version, behavioral economics, experimental economics, and recursive computable macroeconomic models with heterogeneous agents were in part a response to them.

An important question related to those problems is that of heterogeneity of preferences. Once it was clear that the defining properties, as rationality in the narrow sense is concerned, of individual preferences are not inherited by the functions defining aggregate behaviour, the need to assume definite mechanisms and rules of aggregation became evident. In macroeconomic models, this led to the emergence of new equilibrium concepts, based on the informational structure of the agents. It also led to the prominence of approaches focussed on the possibility to compute simulated behaviour and study their properties, since it was not possible to rely on theoretical foundations to ensure those properties. However, in many macro models, the strategy was simply to rely on simple aggregation mechanisms coupled with the assumption that individual preferences were homogeneous. This begs the question about the role and importance of the homogeneity assumption in this procedure: it has been argued that homogeneity can be defended on the grounds of simplicity in modelling. However, several critics see it as a backward engineering technique in order to obtain a representative agent, and have raised different concerns: it contradicts empirical test on heterogeneity of tastes and preferences (to the point where such things can be measured), it is specially relevant as ad hoc assumption in macro models where individual preferences are backward-engineered in order to get representability, and it can be contrasted with models of evolution of population tastes (see [Heifetz et al., 2007], [Bisin and Verdier, 2001]) where heterogeneity arises endogenously as an equilibrium.

4 Preference formation and change.

It is well known that within economic theory, and in particular within the theory of individual decision making, the debate about representability of preferences, as well as its manipulability has generated a large body of literature. We will give specific reasons for focussing on this aspect of social phenomena but it is worth stressing that many macro models make extensive use of preference displaying either a recursive structure or a dependence of current preferences over consumption on past decisions, be them individual or aggregate. This is specially clear when we consider the literature on habits and aspirations in macroeconomics, and also the literature on cultural transmission of preferences[13].

There are two main different but complementary approaches to individual decision making, namely the preference based and the choice based approach. Here, we attempt to discuss some aspects of each of them, and its relevance for

[13] See Chapter 2.

the issue at hand.

4.1 The preference based and the choice based approaches

For the preference based approach, the primitive object given to the theory is the preference relation that encapsulates the tastes and propensions of the individual. There are several aspects of the object that should be taken into account.

- what qualifies as a preference relation, including the nature of the objects that can be thought of as being in it (physical goods, services, other individuals choices, social orderings)?
- what is its relation with time (both logical and historical), that is, how does it evolve?
- how does it relate to notions such as uncertainty and risk?
- what generates or influences a preference relation?
- how can its existence be ascertained from data, and if so, how can it be measured (in particular, what is the relation between preferences and choice structures)?

However, all of the previous aspects are intertwined with the properties and uses of the choice based approach to individual decision making, so we present both approaches briefly and then discuss their relations with respect to each of the aspects cited above..

4.1.1 Definition and properties of preference relations

For the moment, let us consider the simplest setting, abstracting from uncertainty and risk issues, as well as from the process of formation and change of preferences.

The first thing will be to define what has been understood as a preference relation in the literature. To do this, we will abstract from many streams and focus on some deemed more central with regard to the goal of our work.

A preference relation is usually defined in terms of a bynary relation over a set X (whose cardinality and topological properties will be discussed later). It can either be given in terms of a strict preference relation (hereafter labeled $\succ$) or in terms of a weak preference relation ($\succeq$). If we take as a primitive the weak preference relation, the usual definition is the following[14].

Definition 1 (rational preference relation) *A binary relation $A \subseteq X \times X$ is a weak preference relation on X if it is a complete preorder. Equivalently, if $(x \succeq y)$ denotes that $(x, y) \in A$, then A is a weak preference relation $\succeq$ on a set X if*

1. $\forall x, y \in X$, $(x \succeq y) \vee (y \succeq x)$ *(completeness)*
2. $\forall x \in X$, $(x \succeq x)$ *(reflexivity)*
3. $\forall x, y, z \in X$, $(x \succeq y) \wedge (y \succeq z) \Rightarrow (x \succeq z)$ *(transitivity)*[15]

In terms of the strict preference relation, the definition is as follows:

Definition 2 (Strict preference relation) *Let $(x \succ y)$ denote that $(x, y) \in A \subseteq X \times X$. Then A is a strict preference relation on X if*

[14] We will draw heavily on [Mas-Colell et al., 1995]., and on [Kreps, 1988], both for definitions and notation.

[15] Conditions 1 and 2 jointly constitute the definition of rational preferences. The term rational in this definition is somewhat technical and not related to a more broad-based meaning of the word. In particular, see [Dietrich and List, 2010] for some comments on substantive and procedural rationality.

1. $\forall x, y \in X,\ x \succ y \Rightarrow \neg(y \succ x)$ *(assymetry)*

2. $\forall x, y, z \in X,\ x \succ y \Rightarrow (x \succ z) \vee (z \succ y)$ *(negative transitivity)*

The technical notion of rationality that this definition carries will be discussed below. Note that by appropiately defining one object in terms of the other, it is clear that the properties of one of the objects $(\succ, \sim, \succsim)$ are implied by those of the others. Hence, which is taken as primitive is a matter of convenience and tastes. Moreover, in both cases, an indiference relation $\sim$ can be defined, which is reflexive and symmetric[16].

One important question concerning preference relations (be them rational or not) is whether they can be represented by a utility function, $u \in \mathbb{R}^X$, in the sense that:

i) any normative or descriptive implication that can be obtained taking the preference relation as the primitive is preserved when taking the utility function as its surrogate.

ii) the knowledge of the utility function is sufficient to characterize the type of preferences that it represents.

In order to asses this questions, we give some definitions first:

Definition 3 (Representation of a preference relation) *A complete preorder $\succsim$ defined on X is said to be represented by a utility function $u \in \mathbb{R}^X$ if , for all $x, y \in X, x \succsim y \iff u(x) \geq u(y)$*

Definition 4 (Strict upper and lower contour) *Let $\succsim$ be a relation defined on X. For any $x \in X$, the sets $U_{\succ}(x) := \{y \in X : y \succ x\}$,$L_{\succ}(x) := \{y \in X : x \succ y\}$ are called the strict upper and lower contours of x.*

[16] Cfr[Mas-Colell et al., 1995], p.6

Definition 5 (Upper and Lower semicontinous relation) *A binary relation* $\succsim$ *on a separable metric space* X *is upper (lower) semicontinous if for all* $x \in X$*,* $L_{\succ}(x)(U_{\succ}(x))$ *is open.*

Definition 6 (continous relation) *A binary relation* $\succsim$ *on a separable metric space* X *is continous if it is upper and lower semicontinous.*

There are many results concerning representability of preferences, but we choose to list only some of them, deemed especially important for our purposes.

Proposition 1 *Let* X *be an arbitrary set and* $\succ$ *a binary relation. If* $\succsim$ *is a complete, transitive and reflexive relation and there exists a countable,* $\succsim$*-order dense* $Z \subseteq Y$*, then* $\succsim$ *can be represented by a utility function* $u \in \mathbb{R}^X$*.*

Proposition 2 (Rader representation theorem) *Let* X *be a separable metric space, and* $\succsim$ *a complete, transitive and reflexive relation. If* $\succsim$ *is upper semicontinous, then it can be represented by an upper semicontinous utility function* $u \in [0,1]^X$*.*

Proposition 3 (Debreu representation theorem) *Let* X *be a separable metric space, and* $\succsim$ *a complete, transitive and reflexive relation. If* $\succsim$ *is continous, then it can be represented by a continous utility function* $u \in \mathbb{R}^X$*.*

4.1.2 Definitions and properties of choice functions

Let us define now the fundamentals of the choice approach. This takes as a primitive the observed choice of the agents, instead of the underlying preferences.

Let $\mathcal{F}$ be a family of non-empty subsets of X. and $c : \mathcal{F} \to P(X) \setminus \varnothing$ be a function[17].

[17] $c()$ can be thought of as a function that assigns sets to sets or as a correspondence that assigns subsets of sets to each set in $\mathcal{F}$. Also, recall that $P(X)$ stands for the power set of X.

Definition 7 (Choice structure) *A choice structure is a pair* $(\mathcal{F}, c(\cdot))$ *such that for all* $A \in \mathcal{F}, c(A) \subseteq A$.

The idea of a choice structure is that it gives information about the possible elements within a set that are chosen by the individual in each situation that can be presented to the individual. Note that, by the same definition, not all possible situations must be presented to the individual. Hence, this formulation allows for external restrictions on choice situations, based for example on social, institutional or other basis. It also allows for a way of defining uncertainty by redefining the capability of individuals to imagine situations as being given by the family $\mathcal{F}$. However, the specification of this restrictions is left to the researcher, guided by the problem faced. Hence, it would be useful to design some instrument in order to asses the ex-ante effect of such restrictions on the possible preferences to be represented. Moreover, as was discussed above, the nature of the elements in X is not specified. Note that the choice need not be unique. The requirement that it be non empty is imposed in order to ensure some predictive power when starting from this primitive of the theory.

Usually, some axioms are imposed on the primitive of this approach (that is, the choice structure). Let us first reference some of them in order to discuss their role.

Axiom 1 (Weak axiom of revealed preference (WARP)) *A choice structure* $(\mathcal{F}, c(\cdot))$ *sastisfies the weak axiom of revealed preference if* $\{x, y\} \subseteq A, B$ *and* $x \in c(A), y \in c(B)$*, then* $x \in c(B)$[18]

Basically, it says that if x is chosen from set A when y was available, then, whenever y is chosen from another set B where x is available, x should also be chosen. This axiom seems like a minimal consistency requirement for choices.

[18]It should be noted that this axiom was first mentioned by Paul Samuelson.

However, as seen when discussing temptation, from a descriptive point of view it might be highly restrictive. When time and choice for menus are taken into consideration, this requirement seems too strong in order to account for the effective behaviour of individuals. The same axiom has also been stated slightly differently by [Sen, 1997]:

Axiom 2 (Sen's Property α) *A choice structure $(\mathcal{F}, c(\cdot))$ satisfies Sen's property α if, for all $A, B \in \mathcal{F}$, if $x \in B \subseteq A$ and $x \in c(A)$,then $x \in c(B)$*

Axiom 3 (Sen's property β) *A choice structure $(\mathcal{F}, c(\cdot))$ satisfies Sen's property β if, for all $A, B \in \mathcal{F}$, if $x, y \in c(A)$, $A \subseteq B$, and $y \in c(B)$, then $x \in c(B)$*

Sen's axioms are jointly equivalent to the WARP. We are interested in the relation between $\succsim$ and $(\mathcal{F}, c(\cdot))$. To study that, we first give a couple of definitions.

Definition 8 (Revealed preference relation) *Given a choice structure $(\mathcal{F}, c(\cdot))$, x is revealed weakly prefered to y (denoted $x \succsim^* y$) if there exists $B \in \mathcal{F}$, such that $\{x, y\} \subseteq B$ and $x \in c(B)$.*

Definition 9 (Induced choice set) *Given a rational preference relation, $\succsim$, then for any $B \in \mathcal{F}$, the choice set induced by $\succsim$ is $c^*(B, \succsim) := \{x \in B : \nexists y \in B$ such that $y \succ x\}$.*

We note that this family of choice sets forms a choice structure when X is finite and $\succsim$ is rational[19].

[19] Alternatively, if the strict preference $\succ$ is taken to be the primitive, then the following holds.

Acyclicity: Let $x_i, i = 1, .., n$ be a set of choices. We say that $\succ$ is acyclic if there is no $n \in \mathbb{N}$ such that $x_i \succ x_{i-1}, i = 2, .., n$ and $x_n = x_1$.

Then, if X is finite, $\succ$ is acyclic iff there exists a choice function $c(\cdot) = c^*(\cdot, \succ)$.

Now, 2 questions are posed.

1) Given an individual with a rational preference relation $\succsim$, do her induced choice sets when facing all $B \in \mathcal{F}$ generate a choice structure satisfying the weak axiom of revealed preference?

2) Given a choice structure $(\mathcal{F}, c(\cdot))$, satisfying the weak axiom of revealed preference, is there a rational preference relation $\succsim$ that rationalizes this choice structure in the sense that $c(B) = c^*(B, \succsim)$ for all $B \in \mathcal{F}$?

The first question aims at the normative role of the WARP. From a methodological point of view, what is the role of such axioms with regards to answering those 2 questions? The rationale given for the axioms is that of ensuring some kind of "rationality" of the choice behaviour of the individual[20]. Here, rationality should be taken to mean consistency. However, for the same reasons pointed out in the case of the preference approach, and as [Arrow, 1986] pointed out, this is neither a necessary nor a sufficient condition in order to have a sound theory of choice behaviour based on observability. It is important to note that the specific meaning given here to rationality is not the most common among non economists[21].

Now, as it is well known, the answer for the first question is yes[22], while in general, the second question is answered in the negative. As shown by [Arrow, 1959], a sufficient condition to answer question 2 in the positive is to restrict $\mathcal{F}$ so that it includes at least all elements of $P(X)$ with 3 or less elements.

One of the major areas where this comes into play is that of consumer de-

[20] Cfr [Mas-Colell et al., 1995], page 10.

[21] For example, Arrow gives the following account of rationality in everyday usage."*It is noteworthy that the everyday usage of the term "rationality" does not correspond to the economist's definition as transitivity and completeness, that is, maximization of something. The common understanding is instead the complete exploitation of information, sound reasoning, and so forth*". [Arrow, 1986], p. 390

[22] Cfr. [Mas-colell, 1995], p.14

mand, especially considering that many results in partial and general equilibrium theory draw on the properties of individual demand and aggregate excess demand functions. In this regard, stronger conditions have been developed (such as the Strong axiom of revealed preference, SARP, [Houthakker, 1950] and the Generalized axiom of revealed preference, [Afriat, 1967], [Varian, 1982]) to ensure that the implied behaviour of the individual complies with what would be obtained from maximization of a rational preference relation subject to the individual budget set. In particular, conditions on the definitenness of the Slutsky matrix are obtained. However, as noted in the introduction, this takes into consideration only one (very important) aspect of preferences while disregarding its other components (be them social, institutional or computational). This could be defended again on the basis of economic relevance and simplicity, but nonetheless it would be helpful to design an instrument to assess the impact of such restrictions and approach on the other economic and non-economic aspects of preferences. Hence, the possibility of comparing this approach with other competing research programs seems desirable.

4.1.3 The preference domain.

Preference over what? The previous results regarding representability and properties of preferences are strictly related to the mathematical assumptions about the nature of the preference domain, X. However, some of the most important questions about preferences and change are related to the conceptual nature of the objects in that domain. It is also worth noting that the assumptions on the nature of X and their interpretation have far reaching implications on the role of preference relations in proving existence theorems in general equilibrium and in representation theorems. We discuss these questions briefly.

Actual versus potential goods: imagination and memory constraints and the dimensionality problem Which objects can be elements of X?. It seems natural to start by assuming that any existing and traded commodity or service that the individual can conceivably think of should be susceptible of being included in $\dot{X}$. However, this does not exclude other currently non-existing or non-traded commodities and services also entering this set. This leads to the question of what the agent can imagine, as limited by his present state knowledge and capabilities. If a good or commodity is potentially conceivable (say, a computer program that performs certain tasks but is not developed as of yet), should this be a candidate for membership of X?. This is somewhat of a dual question to the problem that will be discussed below when considering uncertainty and specifically targetting unexpected events (in the sense of states of the world that were not taken into account because of limitations of knowledge or for other reasons, when making the decision ex-ante). One argument would have it that, since there is (as of yet) no market for the computer program, then it should not enter the maximization program of the individual, and hence, it does not matter if we include it or not in X. However, this argument is bound to meet some problems if one wants to take into account product innovation on the one hand, and the empirical data on market research that firms undergo in order to ensure that there is some future market demand for the products they are about to produce. This suggest that potential commodities, at least to some extent, should be included in a preference relation if we are to make sense of both of these well documented facts.

One possible answer is to leave some of the elements of X unspecified (for example, X could have infinite dimensionality to account not only for present but also for future potential commodities). A problem with this strategy is that it needs to identify from which sources and by which procedure does an individual learn about the properties of those unspecified objects. Furthermore, learning can take the form of new facts, the discovery of regularities among known facts, or the discovery of more efficient rules for organizing information in general (see [Aragones et al., 2005] for an account of the possibility of the second and third types). The most commonly assumed sources of learning are new information external to the individual and introspection (or both). For new information, the gathering process has to be taken into account; for example, an information set can be defined, jointly with a law of motion for its change. For introspection, a system of beliefs can be defined. However, the modification of the system of beliefs can be modeled in different ways, according to the model of learning behaviour that is deemed more plausible: imitation, adaptative learning, bayesian learning, among others have been used extensively in economics. What is important to note is that in this context, learning, be it about new objects or new rules to organize known objects, implies either uncovering a part of one's preference relation that was previously unknown (or inexistent) or a new form for our preference orderings, so that a process of learning one's preferences is necessary. This is one of the strongest arguments to consider preferences as a process rather than as a state, and in this sense, as a partially endogenous object of the theory: both from a positive and a normative point of view, it seems plausible that preferences are developed sequentially. But this leads to a second question: from which temporal perspective should we judge rationality of preferences? we might assume that it was rational only relative to the initial information set and beliefs of the individual. However, this is compatible with choice behaviour that violates rationality sequentially. This is ruled out by some standard axiomatizations of choice behaviour, most notably the GARP ([Afriat, 1967],[Varian, 1982]). Hence, to mantain the hypothesis of rationality in the narrow sense along the preference development process would imply a restriction of the class of admisible learning processess to be assumed in order to avoid any sequential incompleteness or intransitivity of the sequential order being uncovered. The cost in modelling flexibility of these stringent conditions might be excessive, especially if we take into account the empirical evidence on the violation of GARP and the intrapersonal development of sequential rationality of choice (see [Harbaugh et al., 2001]).

As an alternative, one can assume that all present and future or potential (in the sense of conceivable) commodities are already known to the individual, and thus belong to X. This makes the assumption of completeness even more heroic than it is usually admitted[23], since usually, the completeness is predicated of currently existing and trade commodities, and the associated limitation (that of not being able to gather information about all current existing commodities) could be thought of as steeming from memory capacity constraints which could be potentially suplemented by information storing technologies, while the other (not being able to imagine potential commodities) can be most closely related with imagination limitations, and so a projective technology devising new potential goods should be postulated. It is not obvious that both assumptions are equally plausible. But it is relevant to note that, in the case of natural cognitive limitations (such as memory thresholds, imaging capacity, etc), the completeness assumption (relative to the sinchronic information set) is closely related to the technological state of the economy ([Lancaster, 1966]). A case in point, often omitted is trade: a precondition for trade is the knowledge of the existence of other products, which

[23] The Arrow-Debreu solution of date-event indexed goods is a solution only insofar as all future events are taken into account when making the decision. If we were to admit that there is no complete state space of future events that the agent can rely on (for example, if we want to leave room for unexpected events, not just probabilistic), then the rational expectations desideratum that the agent has in mind the same model of the economy as the theoricist (following the Lucas criticque) implies that the modeller has to build the same limitatjon into her theoretical construction. One attempt at doing so can be found in [Dekel et al., 1998].

is only made possible by the development of transportation technologies and the dissemination of information about new products.

Even if we abstract from the problem of potential commodities, the dependence of the narrow rationality assumption (completeness on X) on the technological state of the economy is even more clear when one considers the analysis of consumption. As shown by the large literature on limited rationality, there is an implicit philosophy of the mind behind the idea of a rational agent (in the narrow sense of having complete, reflexive and transitive preferences) that is able to gather all this information and extrapolate into the future[24]. A problematic implication of this philosophy for some versions of consumption theory is that, according to the completeness axiom and assuming that individuals as consumers behave maximizing, the order of magnitude of simple operations (say, comparing rations of marginal utilities to ratios of prices for each pair of goods available) that an individual should perform just to go shopping is implausibly high. This complexity problem arises when one assumes that the consumer has preferences over at least all objects that lie in her consumption possibility set[25] and that her decision is made by exhaustively comparing each pair of objects that are affordable to her with respect to their preference ordering and their prices. Obviously, the narrow rationality assumption is silent about the specific procedure that consumers use to compare affordable alternatives, as long as the choices made respect the underlying preference and constraints. But again, for normative and descriptive reasons, the class of procedures (algorithms) able to implement narrow rational behaviour, and compatible with a reasonable degree of complexity is very restricted. Hence, indirectly, the rationality assumption implies an important cost in terms of flexibility of modelling: it leaves aside the theoretical question about why a procedure should be preferred to another when

[24] See for example, [Hollis and Nell, 1975] and [Bunge, 1993]

[25] The same is true if we restrict the consumer to compare affordable goods.

comparing alternatives with respect to price, and the empirical question about how individuals develop such a procedure and perform those operations. This cost is usually weighted against the parsimony gained and the discipline imposed by the narrow rationality assumption. However, as the large literature on psychology of consumption, buying habits and marketing can attest, it is difficult to defend this stance. The complexity problem belongs to a larger class that includes the question of substantive versus procedural rationality, the emerging properties of group behaviour in consumption (imitation effects, endogenous diffusion) versus the embedded nature of individual preferences in utility functions or choice axioms, and the sociology of consumption in general. And it is strongly related to the question of emerging properties of behaviour, not reducible to a set of preestablished fundamentals. There are several ways to deal with this problem by modelling the decision making process in a more detailed, specific way. As an example, a usual assumption is that individuals choose consumption through a two stage decision problem: first they choose expenditures share for each type of good and then decide on how much of each individual good to consume, where individual goods provide utility according to an aggregator function or utility index. This provides a reduction in the complexity of the calculus involved. Moreover, the first stage decision can be engrained in the form of the utility aggregator itself, by fixing certain parameters that determine the expenditure share of each good when price-taking maximization behaviour takes place[26]. But that begs the question of what gave rise to the functional form assumed or the particular parameter values. This is especially relevant for the empirical estimation of demand systems for consumers, given information about incomes, prices, and quantities. Some econometric models have been designed to adress this problem, such as the Rotterdam demand systems ([Theil, 1975]) and the Almost Ideal Demand System

[26] This is the case for the cobb-douglass specification and some of the Stone-Leary variants.

([Deaton and Muellbauer, 1980], henceforth, AIDS) and its modifications (including quadratic AIDS, among others), which provide ample flexibility in terms of functional forms, satisfy the basic assumptions of classical consumption theory (such as homogeneity of degree zero of demand, symmetry of the substitution effect) and that bypass the problem of aggregation of demand across consumers, without having to restrict to parallel linear Engel curves[27]. However, since those approaches are built on the local non-satiation assumption (which entails consuming on the budget line), once the number of goods for which the demand should be estimated increases, the curse of dimensionality appears, both in the number of parameters to be used and in the problem of unbounded utility obtained by individuals[28]. As an alternative, other authors have tried to study under what conditions demand systems are negatively sloped in prices purely on the properties of the budget set, without resorting to specify preferences (see [Hildenbrand, 2014]). Another field where the question of the dimension and type of goods over which preferences are defined and the estimation of budget shares is important is in models of imperfect competition and product differentiation, where the use of utility aggregators over varieties is widespread (see, for example, [Dixit and Stiglitz, 1977] and [Boldrin and Levine, 2009]). In this case, the problem is deeply embedded in the nature of the models, since competition and growth are tightly related to the development of new products, which hinges upon the presence of taste for varieties. Hence, the development of tastes for such products and the ability to compare them with previous ones is crucial. It can be argued that varieties are to be considered groups of goods, so that preferences are defined

[27] In the AIDS case, by assuming a particular class of preferences, PIGLOG, that satisfy that perfect aggregation property. However, see [Gorman, 1987], for the plausibility of this strategy.

[28] See [Nevo, 2003] and [Berry and Pakes, 2007b], for a summary of the problems related to demand estimation whithin the commodity approach, when the number of commodities grows or there are new products.

over groups only, and thus the role for aggregators. But in this case, even more that in the demand problem, the preference for variety can only be predicated on the basis of the consumer being able to distinguish them according to some criterion. And that leads to the question of what distinguishes them. Even if we consider that the permanent increase in the number of varieties is a technical shortcut, and that the models can be redefined with a finite non-increasing number of varieties (for example, due to obsolescence), the problem remains in that new products constantly appear and appeal to consumers. As stated, aggregators reduce the need for ordering them. But the nature and restrictions that such aggregators pose on the underlying preferences, and the testability of such devices remain open questions. The role of an aggregator used to deal with varieties is deeply connected with the idea of a technology of consumption, that is, that consumption is a complex activity involving the interaction of different physical goods in many cases. In that sense, aggregators are a dual solution to the kind of procedures that deal with the increasing dimension of the consumption set that we will explore next: the characteristics approach. The aggregator solution proposes to recognize the existence of activities of consumption as aggregation of certain goods, while the characteristics approach proposes to recognize that each good is a package of different characteristics that define the consumption experience.

The characteristics approach is an alternative strategy to deal at once with the dimensionality problem and the question of what distinguishes new commodities and varieties from existing ones. This consists in taking as objects of preference the characteristics provided by commodities instead of the goods per se. Hence, two basket of goods are not distinguished by the physical quantities of each good they contain, but by the characteristics bundle they represent. This was pioneered in [Lancaster, 1966], and was introduced in econometrics through several models of discrete choice estimation (see [Manski and McFadden, 1981] and [McFadden, 1984]). There are two different versions of this approach, the pure characteristics model, where the dimension of characteristics space is assumed constant, and the "taste for new products" approach, where it is assumed that new characteristics can appear (see [Berry and Pakes, 2007a]). The theoretical implications of the two versions are quite different. However, since the latter version does not solve the problem of dimensionality, here we will just consider the former. If the space of characteristics is assumed finite, discrete and invariant, the dimensionality problem can be reduced since new commoditties are simply different combinations of the (underlying) invariant set of characteristics and the computation problem becomes tractable (note that this is not the case with goods, since even if every good is present only in discrete finite quantities, new goods can appear). In addition, this approach helps explaining how an individual can choose different quantities of very similar goods, even when their prices are the same (non-price competition) or why two almost equal varieties can support a different price. This explains the pervasive use of this model in the literature (especially in marketing) to explain the demand for new goods and varieties, the differences in quality between products and the impact of advertising on consumer behaviour. The simplest setting to do that is to assume a (possibly random) utility model where the consumer chooses one of several different mutually exclusive varieties, since in those cases, one can obtain data on some characteristics shared to varying degrees by the variants and try to discriminate which one will be chosen based on price competition and characteristics differences. Moreover, if a random utility model with discrete choice is assumed, the presence of differences in characteristics can explain a large part of the choice behaviour previously attributed to randomness (see [Manski, 1977]). However, when using this approach, two further questions arise: First, which is the relation between utility functions in terms of characteristics and in terms of goods consumed? the answer depends on the relation assumed between characteristics production and commodittiess consumption. In Lancaster, a linear analysis for the production of characteristics was assumed, mostly because of simplicity, and the relation depended on the dimensions of the characteristics and the goods space. Complex industrial societies are associated by Lancaster with situations in which the number of goods exceeds that of characteristics, due to product innovation and technological change. In this case, Lancaster shows that assuming a quasi-concave utility in characteristics and linear activities for production of characteristics from goods, the derived utility function over goods is also quasi-concave. On the other hand, [Rustichini and Siconolfi, 2008] shows that if the production function for characteristics satisfies typical neoclassical conditions, and the underlying utility from characteristics is continous and quasi-concave, any continous utility function on goods can be generated. That is, the use of the characteristics framework does not limit the form of admisible utility functions for goods that is generated. In principle, this would present a problem if only data from commodity choices were available, in the sense that the underlying structure of preferences over characteristics would not be testable from individual behaviour. However, as Rustichini points out, Lancaster does not assume that there should be a derived preference relation over goods. His approach is best suited to work directly with data on both choice of goods and characteristics of each good in order to sort out the primitive preference relation between characteristics. A more problematic feature of this solution is that the reduction in complexity depends explicitly on the assumption of finite, discrete characteristics and the nature of the technology relating goods and characteristics, both extremely difficult to test. Second, since the approach is used in estimating demands for varieties or differentiated products, its results in the aggregate should be consistent with the theory of demand. Usually, aggregate demand systems of varieties of the same good are assumed to satisfy the gross substitution property (the idea being that varieties satisfy a similar need). The question is, what are the conditions that the distribution of preferences for characteristics among the population has to satisfy in order to be compatible with an aggregated demand system for varieties that satisfies the gross substitution property? [Anderson et al., 1989] answers this question in a very simple framework. Assuming that each individual has a best combination in the space of characteristics, he concludes that the dimension of the characteristics space has to be of the same order than that of the goods space. This implies that in his setting, the dimensionality problem in the utility function and the compatibility with an aggregate demand system for varieties cannot be solved jointly by using the characteristics approach. This is true for a CES aggregator over varieties, which is one of the most commonly used functional forms for utility in the product differentiation literature.

Endogenous versus exogenous objects of preference A second question is whether we allow for the individual to have preferences over the objects that usually the model wants to explain, that is, on prices of goods, income levels, etc. Usual models of general equilibrium preclude preferences to depend on prices, drawing on a two-fold argument that prices are taken as given by consumers and that the period considered is short enough for preferences to be considered stable, that is unchanging with respect to prices (we will discuss this assumption later on) but this can be taken to mean either that rational individuals do not have definite preferences over the value of prices or that their preferences are not state dependent with respect to prices (e.g., prices do not constitute a parameter that affects preferences over the other goods). Note that stability in the second sense does not preclude a rational decision maker to have definite preferences over the value of particular prices (or their distribution). However, there is by now a large body of empirical evidence in the fields of behavioral and experimental economics that challenges the assumption of stability of preferences with respect to prices in both senses[29]. First, the literature has identified clear cases where price-dependent preferences are related to quality perception and to status quo perception[30]. Second, in a more general setting concerning the interrelation between psychology and economics, [Kahneman et al., 1982] initiated the so called heuristics and biases approach (focussed on studying the judgement process under uncertainty) with the goal of introducing availability, representativeness, anchoring and adjustment as heuristics usually employed by individuals when judging and to uncover the errors to which those individuals are prone in uncertain judgement environments. As a result, the neoclassical assumption that preferences are invariant with respect to the framing of the problem faced by the consumer, changes in contextual conditions of the choice or the procedure used to elicit preferences has been severely challenged[31]. For price dependence in particular, a large literature in marketing and consumer research has studied the dependence of reservation price valuation for a product on the fact of being provided with previous information about current or past prices of that product, the distribution of prices for that product or simply a (unknown to the subjects in the experiment) randomly generated price for the product[32] (see [Monroe and Hoseason, 2003] for a review).

With regards to the price-taking hypothesis, the factual exposition to many prices for the same or equivalent goods by individuals when making choices is generally ruled out by the assumption of the "law of one price". However, the degree of accuracy of such an hypothesis depends on the market and good considered.

[29] See [Kahneman and Tversky, 1979];[Chapman and Johnson, 1999], [Mazar et al., 2010]; [Grewal et al., 1998]; [Simonson and Drolet, 2004]; [Ariely et al., 2003], among others.

[30] In particular, the snob effects and the preference for status models are some of the explanations offered for those findings. See [Veblen, 1965],[Scitovsky, 1992], [Pollak, 1977].

[31] Some of the problems identified are related to the so called loss aversion, endowment effect and anchor effect. See [Kahneman et al., 1982] for definitions.

[32] This is the so called anchor effect.

Note that the Arrow-Debreu strategy of differentiating goods by date-event can be successful in defending this law of one price, since it allows to condition prices on very specific situations, modelized as date-events, but only on the newer assumption that the data collector is able to distinguish such date-events (which in the case of unexpected events the modeller cannot do, if it is to satisfy the criterium that information is the same for the agent than for the modeller, as Lucas pointed out). This is closely related to the difficulty of including unawareness in the standard neoclassical model as usually stated (see [Dekel et al., 1998]).

In addition, from the theoretical point of view, a reason to study price dependence is that the model of temporary financial equilibrium has a reduced form that is mathematically equivalent to an economy with price-dependent preferences.

In view of the above, there have been some attempts to find necessary and sufficient conditions to extend the competitive equilibrium properties that hold for price-independent preferences to the case where there is price-dependency[33]. These properties include existence, local uniquenness and genericity among others. They are dubbed minimal from the perspective of comparative statics, but they stem from the assumption of continuity and rationality of preferences. However, even under those conditions, it is easy to construct examples of economies with price-dependent preferences where the usual properties of the demand functions do not hold. In particular, negative semi-definiteness of the Slutzky matrix may not obtain. Furthermore, with price-dependent preferences, the WARP and other theoretical constructs cease to rationalize choice. As an example, consider the following situation, where preferences are defined over two bundles of goods/prices, $y_1 = (x_1, p_1)$, $y_2 = (x_2, p_2)$, $x_i \in \mathbb{R}^2$ where i is the time period where the bundle was chosen, according to the following rule (that could be thought to rationalize the preferences of a consumer that wants to have as much income as possible and

[33] See [Arrow and Hahn, 1971],[Greenberg, 1977],[Balasko, 2003].

at the same time values the second good according to its relative price, as a sign of quality[34]): $y \succ y'$ if either $px > p'x'$ or $px = p'x'$ and $\frac{p^{2'}}{p^{1'}} < \frac{p^2}{p^1}$[35]. Now assume that $x_1 = (1.2, .1.2)$, $x_2 = (0, 2.39)$, $p_1 = (1, 1)$, $p_2 = (1, 1.041841)$. At time 1 we have that $p_1 x_2 < p_1 x_1$ so that x_2 was affordable, and at time 2 $p_2 x_2 = p_2 x_1$ so that x_1 was affordable. If observed, those situations would lead to the violation of WARP. However, they could be rationalized by a preference relation over goods that satisfies the usual properties. Hence, the link between observation and normative assumptions about preferences breaks down. Obviously we have abstracted from the problem of defining what it means in this context to have an induced preference over goods. But the example is enough to show that in those cases, the observational requirements have to be substantially changed, if maintained at all. This is somewhat a case of set dependence when choosing, since the same baskets of goods can be included in different choice sets, depending on the relative prices faced by the agent. This also could be said of wealth or income, especially if we acknowledge that preferences are formed by transmission (education, cultural environment, etc) as well as by genetic (individual) traits.

More importantly, if we are interested in the formation process by which preferences come to be or develop, then it is only natural to ask wether the exposition to certain levels of income could impact such process.

A third question would be if the elements of X can themselves be domains from which to choose. For example, an individual can have defined preferences over menus of bundles, where for each menu, a preference relation is defined and would

[34]This is even more relevant if the consumer has to discover its own preferences by trial and error, which seems plausible for many kinds of goods, such as food, music, certain types of durables, etc. Another line of justification for this kind of preferences comes from uncertainty about the goods quality.

[35]This represents the combination of a higher income-preference effect with a pure price-preference effect for higher relative prices of the second good as a sign of quality.

guide her action once that particular domain is considered. This has been explored at length in the literature on temptation and regret. The most relevant feature of this possibility is that it allows for preferences that represent set-dependent choice behaviour. Obviously related with the previous question is wether we allow preferences on preferences (this is tightly related to the general literature on higher order beliefs in the context of uncertainty). Several psychological and ethnographical studies have provided ample evidence that such aspects play a role on individual decision making. It seems unclear whether the current models of individual decision making can accomodate such objects (in particular, they are tackled within uncertain or risky environments, by allowing regret, ambiguity and temptation to be formalized in the decision making process). We argue that such objects should be taken into consideration, and embedded within a general theory of preference formation and change (see for instance, [Dietrich and List, 2010], [Dekel et al., 2007]). Moreover, the abundant literature on habits, temptation preferences[36], recursive preferences and other instances point out to the progressive inclusion of such objects into X. But then again, the relation of this objects with physical units (independent of time), and with testable counterparts is problematic. This points out again to the problem of the relation between a preference relation formulation and its observable counterparts.

In the literature, the nature of X depends, as should be expected, on the goals to be achieved by the particular work. In particular, to prove results of existence and smoothness of demand functions, and of existence and regularity of equilibriums in general equilibrium theory, usually it is assumed to be a topological vector space, or a topological space, be it totally or partially ordered, and conditions are stated for the existence of a continous utility function representing a partial order or preorder.

[36] See [Gul and Pesendorfer, 2001],[Dekel et al., 2004] and [Noor, 2011].

In order to assess the implications of the properties assumed for X, it is important to relate the definition of preferences to the choice-based approach to individual decision theory, where the choices of the agents are taken to be the primitives. The relation between both approaches is constructed by representation theorems that ensure the existence of a utility function that rationalizes the choices of the individual. The problem of recovering a preference relation from the actual choices is known as the recoverability problem and much work has been done to link the structure of preferences to the possibility of uniquely recovering them from empirical knowledge about choices[37].

There is also abundant literature on both the existence of equilibrium and a representation of a preference relation by a continous function when either completenness and /or transitivity are missing. This is connected to both a theoretical goal to enlarge the class of binary relations that can be accounted for by different economic models and also by the emergence of empirical evidence suggesting that both completeness and transitivity are violated by the choices of agents.

4.1.4 The completenness and transitivity axioms: a brief history

As we have seen, in the preference based approach to individual choice theory, rationality is defined as completeness and transitivity. Since both are axioms imposed on preferences, it is natural to ask whether there is a rationale for imposing them. As noted by [Mas colell, 1995], both assumptions entail a different kind of restriction. While completenness is related with the underlying view of human capabilities (memory, imagination), aided by present state technological devices, transitivity has to do with consistency in behaviour and/or in tastes. The main question here is to inquire if these differences in restrictions arise from the dif-

[37] see [Al-Najjar, 1993] in particular, for a discussion of the recoverability problem with non transitive preferences.

ferent role played by both axioms in the general structure of a certain strand of economic theory, namely neoclassical economics. A working hypothesis we would like to explore is that while completeness arises from tractability issues, transitivity is accepted as a desideratum of the theory, as translating the everyday usage of rationality into economics. Two related questions would be the possibility of alternative translations of such a common notion, and whether a theory of choice should be mainly based on the assumption of rationality of agents. Moreover, drawing on the history of the notion of rationality in economics, we would like to unveil the dependence of such an axiom on the goal of explaining individual choice as a result of maximizing behaviour of some kind (see below).

On a normative basis, transitivity has been defended on the grounds of excluding behaviours such as that observed in the so called *money pumps*, where some agent takes advantage of the non transitivity of preference of some other to start a cycle of trade that leaves the inducing agent strictly better off and the induced agent strictly worse. However, in order for a money pump to be present, several more restrictive assumptions about different preferences, non-satiability and informational constraints have to be made, thus leaving room to categorize these arguments as ad hoc. From a descriptive perspective, there are well documented cases where transitivity seems to be violated, namely when there are perception threshold effects or when there are framing effects (as defined by [Kahnemann, 1979]). Some of these cases can be avoided by using several more primitive rational relations that interact to give rise to the a priori posed preference relation of the individual. Regarding completeness, several authors ([Aumann, 1967], [Bewley, 1986]) have suggested that decidability might not always be an empirically accurate description of behavior. However, it has been argued that completeness has the same nature of the frictionless assumptions in phisical models of movement. This is taken to mean that completeness imposes an idealization of the individual capabilities, abstracting from elements that might be present and can be measured but are not crucial for the object of study. We contend that this is not the case. First, the assumption that the disrupting forces that are not being taken into consideration (in our case, those that would block completeness) are measurable is difficult to sustain. Second, given the dual nature of the assumption (normative/descriptive), no account can be given of it without explicit reference to what the theory regards as rational behaviour. Thus, any observed departure from completeness can either be disregarded on normative grounds of irrational behaviour (by assumption) or attributed to a restriction that is not measurable. If we were to take seriously the stance that the empirical content of a theory consists of all possible situations in which it can be tested, then we should conclude that completeness is not testable. This is closely related with the methodological stance that is commonly held in economics. As has been noted in the introduction, there is a certain strand of methodology in economics, mostly dating back to Friedman, that define the goodness of a theory almost exclusively by its narrow predictive power, and consider that the realism of the hypothesis should play no role in assessing how good a theory is, at least in the stage of its empirical validation. However, Friedman itself has in mind a research process that starts with data, abstracts a set of stylized facts to explain, and then constructs a theory to make sense of them. Only after that, the theory is developed in order to extract predictions that are testable against new or old data. And he clarifies that it is not in the process of choosing the best hypothesis to represent the data known where the realism of such hypothesis is a hindrance, but only in the subsequent developing of testable predictions[38]. Hence, it would be natural to ask what are the a priori restrictions that the choice of certain assumptions places on the size and shape of potential stylized facts to be explained. In particular, it would be

[38] See [Hammond and Hammond, 2006].

desirable to separate by nature, extension and implications the restrictions placed by hypothesis made on behalf of tractability from those explicitly related to the stylized content to be accounted for. Moreover, note that as is usual in comparing models, different assumptions aiming to capturate the same piece of evidence should be compared in order to establish advantages and disadvantages. Here, it should be emphasized that, since economic activity does not happen separately from other aspects of social life, other disciplines are clearly involved in trying to explain social behaviour. In particular, psychology, sociology, history, biology are all components of the aggregate human behaviour, from which the economic part is just an aspect. Thus, one research program could posit as a desideratum of the theory that the restrictions on the set of potentially explained stylized facts brought about by the choice of hypothesis should be checked not only against thoses deemed important in economics, but also versus those that steem from the other disciplines. A counterargument to this view could ask which is the particular element in economics that makes necessary such a cross-discipline stylized fact checking and why does it not apply to other fields. Moreover, this objection could be raised on the basis of the difference in nature between economics and other social fields, especially regarding formalization and or the very nature of some of the objects to be explained (as the movements in prices, demands, etc) which are observable. However, it is debatable that such analogy is valid because this common line of defence takes preferences as an external given primitive, hence begging the question of how to justify such study when recoverability breaks down. For instance, it is known that if preferences are not transitive, then unique recoverability is not guaranted ([Al-Najjar, 1993]). Moreover, when there are expectations involved in the determination of choices, it can be impossible to disentangle preferences from expectations on the basis of observed behaviour ([Polemarchakis, 1983]).

Incompleteness: an introduction Incompletenness has been argued as a sensible assumption due to the possibility of indecisivenness on the part of the individual (see [Aumann, 1962], [Bewley, 1986], [Mandler, 2005] among others). From an empirical point of view, indecisivenness can stem from limits on human capabilities (imagination, memory), from weakness of will, or simply from lack of interest in the psychological comparison. This prompted a strand of literature to develope a somewhat different approach to the representation of preferences, focussing on the sufficient conditions that a function should satisfy in order to see it as expressing a possibly incomplete preorder. This is known as the Richter-Peleg approach (see [Richter, 1966]) and is based on the following concept.

Definition 10 (Richter-Peleg utility function) *A Richter-Peleg utility function for an incomplete preorder $\succsim$ defined on X is $u \in \mathbb{R}^X$ sucht that for all $x, y \in X$, $x \succ y \implies u(x) > u(y)$ and $x \sim y \implies u(x) = u(y)$*

It can be shown that with this definition, if X contains a countable $\succsim$-dense subset, then there exists a Richter-Peleg utility function for the incomplete preorder $\succsim$. However, the problem with such representation is that it is not possible to recover all the information about $\succsim$ starting from u. The typical case is that if $u(x) > u(y)$, then we cannot tell apart the situations where the individual prefers x at least as much as y and the one where she cannot compare them. Hence, we cannot recover the region of potential indecisivennes and some information about $\succsim$ is lost. To overcome this fact, another approach has been proposed (for example, in a certainty setting, by [Ok, 2002], and in an expected utility setting, by [Dubra et al., 2004]). It basically amounts to represent $\succsim$ by a vector valued function, in the following sense.

Definition 11 (Vector valued representation of a preference relation) *Let $\succsim$ be a preorder defined on X. A set $\mathcal{U} \subseteq \mathbb{R}^X$ represents $\succsim$ if $x \succsim y \iff u(x) \geq$*

$u(y)$ *for all* $u \in \mathcal{U}$ *, for all* $x, y \in X$*.*

With this definition, it turns out that the same condition that we stated for the Richter-Peleg approach ensures the existence of such a representation for a preference relation $\succsim$ on X. The difference is that such a representation conveys all the information in $\succsim$, including the indecisivenness parts. Is there an intuitive interpretation that can be given for such a representation? One that is usually found in the literature[39] is that it could be thought of as if each u was representing a different dimension of the objects in X in which the individual is interested, bringing this approach closer to multi-objective maximization.

In the choice-based approach, the problem of incompleteness appears related to the question of separating cases where there is indifference between choosable alternatives and cases where there is indecisiveness. Consider the following example, borrowed from [Nehring, 1997]. An individual ranks alternatives x, y, z according to a combination of 2 criteria, c, q. Now, suppose that the individual additively aggregates the criteria through some weight w. The options are stated in table 1:

$$\begin{bmatrix} \text{alternative/criteria} & c & q \\ x & 10 & 0 \\ y & s & s \\ z & 0 & 10 \end{bmatrix} \tag{1}$$

Assume $s < 5$. Now, faced with the heterogeneity of criteria, the individual takes as relevant to his decision every possible weight $w \in [0, 1]$. Hence, from a choice-based perspective, when faced with pairwise comparisons of elements of $A = \{x, y, z\}$, any alternative is acceptable in the sense that a choice function

[39] see, for example, [Ok, 2002].

$C(\cdot)$ describing her judgement (not necessarily her behaviour) would have

$$\begin{bmatrix} C(\{x,y\}) = \{x,y\} \\ C(\{x,z\}) = \{x,z\} \\ C(\{z,y\}) = \{z,y\} \end{bmatrix}$$

Now, if Sen's condition β is imposed as an axiom, this would imply that $C(\{x,y,z\}) = \{x,y,z\}$. However, from the individual perspective, no matter what the particular value of the weight used in the aggregation, if both x and z are within the choice set, y will never be acceptable. This would suggest that what is precluding the WARP axiom to represent a situation where there is indecisivenness is precisely condition β. This is strictly linked with the interpretation given to the choice function by the revealed preference relation. $C(\{x,y,z\}) = \{x,y\}$ is interpreted as saying that x,y are revealed at least as prefered as z but at the same time, they are revealed indifferent. However, another possible interpretation would be that x,y are revealed not inferior to z and also not inferior when mutually compared. The advantatge of this formulation is that it allows for the case where there is indecisiveness. It is notewhorty to point out that the problem with condition β is not specifically linked to incompleteness, as some often used choice functions, such as the Pareto correspondence, do not satisfy property β. Moreover, in uncertainty environments, decision makers guided by stochastic dominance would be deemed irrational if we were to measure their actions according to WARP. As it is well known, similar considerations play a leading role in the extense literature on relaxations of the WARP axiom. However, we will focus here on the specific strand that relates to incompleteness.

Among others, [Nehring, 1997], [Eliaz and Ok, 2006] and [Sen, 1997] have studied the relation between WARP and incompletenness. However, the solutions proposed are very different in scope. Nehring attempts to incorporate non-binariness in the main findings of the choice-based approach, by allowing preferences that compare sets of choices with particular choices. This is closely related with the problem of defining X. Eliaz et al. relax the WARP axiom to a Weak Axiom of Revealed Non Inferiority (WARNI) that allows them to obtain a choice-based characterization of transitive but possibly incomplete preference, whithin a certain class, that they label *regular* preferences. They obtain a characterization of the choice situations that allow the observer to infer (under the above restrictions) when a particular individual is indifferent or indecisive between alternatives. Both Nehring and Eliaz highlight the difficulties of defining what is observationally equivalent to $C(\{x, y, z\}) = \{x, y\}$. That is, when eliciting choice from an individual, what counts as an instance of the choice function being multivalued? A possible answer is to distinguish between psychological (as implied by desires, etc) and behavioral preferences (as implied by actual choices). Whith that distinction, when multiple choices are deemed acceptable from a psychological preferences point of view, there is an assumed device (possibly random) that will decide which of the acceptable bundles is actually consumed. This is important when studying consumer demand. However, when elicited by actual observation, multiple choices are related with repeated appearances of the same choice sets. This fits better into a dynamic choice theory than in the static framework within which WARP is usually stated. Taking this into account, [Mandler, 2005] incorporated a sequential choice framework to make sense of the possibility of incomplete psychological preferences and intransitive behavioral preferences coexisting, but not viceversa. To do so, Mandler adopts a different concept of rationality, namely *outcome rationality*, based on the idea of non-dominated outcomes, which does not satisfy WARP. He applies this framework to explain the Status Quo Maintenance (SQM) phenomenon, a common pattern of nontraditional decision-making that underlies the endowment effect, loss aversion, and the willingness to pay-willingness to accept disparity. SQM is modelled as the situation where an individual facing sequential choice sets is only willing to choose a bundle if it dominates its previous round choice. Note that this includes the possibility that two non-comparable bundles, say x and y, were part of the previous round choice but the individual is not willing to exchange one of them for the other, on all occasions. It could be the case that a new bundle, z (z'), is available in the next choice set (that also includes x and y) which strictly dominates x but not y (y but not x). Hence, depending on wether z or z' shows up in the next round choice set, then either x or y will dissapear from the set of acceptable choices but not necessarily both at the same time. This choice pattern induces a behavioural preference that is intransitive. More importantly, Mandler shows that apart from the SQM phenomenon, the combination of intransitive behaviour with the assumed incompleteness of psychological preferences allows agents to be outcome rational and not be subject to the usual money pump arguments when intransitive preferences are present. Again, this weakens the case for defending both tenets.

Moreover, Mandler introduces a question that will be of central importance for this work, and that lately has received a good deal of attention both in the preference based and the choice based approach. That is, the existence of set dependence when considering choices, and the possibility that preferences may be dependent on the part of the domain specified. As [Koszegi and Rabin, 2007] has pointed out, this is of utmost importance for the study of welfare economics, and has far reaching implications on what can be inferred from observations and in which sense a preference relation can be recovered from a sequence of choices. We will go back to this when we introduce time.

Also, it is important to distinguish the approaches highligthed above from the previously mentioned strand of the literature, pionereed by K.Lancaster[40], that focusses on the preference for certain attributes that the objects in X possess in

[40] [Lancaster, 1966].

different degrees. Such an approach also hinges upon multi-valued maximization criteria to predict the behaviour of the individual. But the Lancaster approach directly defines $\succsim$ over attributes and, thus considers the possible objects of choice as bundles of different amounts of such attributes. Hence, the implications of both approaches are different from a theoretical and empirical point of view. This begs the question of which approach should be followed, if it is possible either to use the new concept of representation or to redefine the choice set X. Especially if indecisivenness can be viewed either as genuine or a result of such mispecification of the choice set X. Again, an instrument to compare the a priori constraints that each approach poses on the explanative power of the theory and on the possibility of comparing its axioms with other comparable sets of axioms in different disciplines would be useful.

4.1.5 Intransitivity and economic rationality: problems and approaches

As pointed out before, transitivity has been defended using money pump type of arguments. This type of arguments appeal to a certain normative property that rational behaviour ought to have: outcome rationality, in the sense that somebody with intransitive preferences could be induced into an exchange that leads to a worse outcome than the one she would obtain by not participating in such an exchange. On the other hand, in general, transitivity is used as a condition to build well defined choice functions from which to recover the preferences. But, as noted above, the assumption has been questioned both on empirical an theoretical grounds. Some authors have argued that the problem can be bypassed by assuming that preferences are separable since it can enhance the possibility of recovering preferences from a given set of data and can ensure that the decisions of the individual comply with what would be the case if she were maximizing a rational preference relation, even if the relation is not de facto transitive (see

[Epstein and Zhang, 2003]). That begs the question of which strategy is less restrictive and has more explanatory power: assuming transitivity of the preference relation or assuming separability without transitivity. Again, some measure of the implicit tradeoff seems desirable.

4.1.6 Time

When introducing time in relation to preferences and choice, we have to distinguish two different aspects. First, time appears as a dimension that the bundles of goods in X, the domain, can have, or as a change in preferences through time. In the choice-based approach, time is present whenever we choose from bundles to be consumed at a latter moment, or when the structure of choice sets faced by the individual is sequential.

Starting with the preference-based approach, and with respect to the first question, goods can be indexed by the time moment they will be available. As a benchmark, many dynamic models impose the idea of stability of preferences through time. That is, whenever the individual considers bundles of future goods, she can imagine perfectly how they will be in the future. Hence, in this approach, we could think of X as composed of many dimensions, each of which corresponding to a good and a time period. However, this begs the question of when is the individual made aware of having such preferences. That is, should there be a zero moment where the individual elicits all her preferences in order to draw guidance for behaviour. If that is so, then it is not trivial to defend that such a moment is at the beggining of the period studied (which would be the case, for example, in Arrow-Debreu models of general equilibrium). Normatively, to require somebody not to discover anything about her preferences during her lifetime is akin to requiring her to live in a state of absolute self-consciousness about her future desires and tastes. This is a sense of temporal completenness that affects not only the memory and imagination of the individual but that requires it to be conscient of all her future developments (in some sense, it is akin to the idea of a monad[41]) However useful from an analitical point of view, a normative dynamic theory of preference should require the agent to evolve (here, the sociological and psychological studies on tastes are of great help, and also, the study of body and brain evolution can shed some light). A clear case would be how to model the change in tastes that a kid is likely to experience whenever growing up, or of an adult entering her old age. We note that the constant tastes solution implies that each individual is born fully equipped with a fundamental knowledge of those traits, which again is bound to be problematic if a certain trait is state-dependent and there is no complete knowledge of possible future states. A common variation on this would be to pose that what is constant is, for example, genes, and it is genes that determine our preferences, through darwinian selection mechanisms (this would be consistent with the classical thesis of Dawkins), so that the basic characteristics of preference are indeed preexisting and invariant (potentially invariant when considering all the potential combinatorics of genetic basis, without precluding genetic mutations or innovations which in this view would be a case of uncertainty about when it could occurr). This also has been defended within the cognitive science framework. The basic motto would be that adaptation has shaped our behaviour resulting in a maximizing, if not conscious, set of procedures to ensure the replication of our genes. This would provide, in a single stroke, a basis for defending maximizing behaviour without the need of a conscious individual and also a basis for posing that the fundamental preference relation is constant (what would change in this approach is the maximizing strategy adopted, depending on genes inherited, environmental pressure, genetic innovation, and comparative advantatges). However, we would be back to a sort of the Lan-

[41] See [Leibniz, 1989].

caster approach, and then any claim that tastes for observed or consumed goods are constant would be rendered doubtful (diet studies are a good example of the problems of the above mentioned approach) since the same good could represent an advantatge for many different combinations of genes, leaving room for context dependence. And no explanation would be given of the process by which we discover and acquire the preference for such goods . Moreover, any conscious decision (risk taking, political action, etc) would have to be deemed as the result of such forces. Indeed, many models of altruism versus non altruism have taken this form. In evolutionary game theory, this led to developing models in order to know if it is possible for non-maximizing strategies (individual payoff-dominated strategies) to survive (in fitness models, for example). However, if genes, or any other constant fundamental were solely responsible for our taste structure, we should be able to track them down. That is not the case both because studying the particular work of genes on tastes is operationally complex and because even if caused by an underlying fundamental, the particular expression of that fundamental is associated with an emergent dimension for the individual: that of recognizing what she likes and how she likes it. Not only the intensity is the problem here, but also the non transferability and intrinsic non-measurability of such dimensions (in analytical philosophy of mind, this problem is well known). Obviously, models can be defended as simplifications, and processes of empathy (to whatever degree) can be alleged to give us information on others tastes related to our own. But again, these models cannot explain how tastes are acquired or changed through life. Nor is the social impact incorporated in the model.

Also, if constant tastes are to be defended (be it in its classical version of time zero complete elicitation of future tastes, or in its genetic version of fitness models), then this assumption should bring more explanatory power regarding the type of stylized facts that are to be rationalized. In short, is this constant assumption sufficiently fruitful from a predictive or explanatory point of view in rationalizing observed changes of preferences, adictions, and other phenomena of the same type to justify not considering other more complicated but possibly also more fertile hypothesis?.

Note that even the simplest fitness models incorporate strategic behaviour. Problems of maximization in time would have then to be understood as a solution to life organization processes, enthropic sistems. The equivalent solution of asserting that when reevaluating her preferences each period, the individual agrees with her past selves is a particular case of a more general model that seems better equipped to accomodate the above cited phenomena ranging from change in tastes to addictions, temptation and others: a model of continous reevaluation of tastes, every period. But can this more general model be more easily achieved by redefining X while keeping the posited preference relation unchanged or by time-varying the preference relation over a fixed set of objects? These two different strategies correspond to representation hypothesis by a single utility function over the entire domain of all possible future goods or by multiple utility functions, each defined over a temporarily indexed set of goods. This is especially relevant for OLG models, where some sort of separability of utility functions with respect to time (be it additive, multiplicative or of some other sort) is often assumed. It is also crucial when trying to model dynamically inconsistent behaviour, as in the model of the battle of the selves (see, for example, [Diamond & Kosegi, 2003]), where the individual has to take into account the different perspective of her future selves, each of which using a different discount factor, when making her optimal decision. This model can be embedded as a particular case of the modelization of temptation ([Krussell, 2009]; [Noor, 2011]). To give another well known example, the representative consumer in the Ramsey-Cass-Koopmans model is sometimes assumed to have preferences over a sequence of time-indexed goods, $\{x_t\}_{t=0}^{\infty}$, defined by $U(\{x_t\}) = \sum_0^\infty \beta^t u(x_t)$. This formulation assumes that any future reevaluation of tastes will coincide with the fundamental one made at time 0. In addition, it introduces the notion of a rate of time preference (β) which in this case is constant. This rate serves two different purposes: it implies that future consumptions are less valuable to the individual (as long as β is strictly between 0 and 1) than present consumption, and it allows for an intergenerational interpretation of the infinite horizon model, where each generation cares about its descendants, β being the degree of intergenerational altruism ([Barro, 1988]).

Note that separability is a problematic assumption that has been often discussed in economics. In particular, since the works of [Hicks, 1939], [Leontieff, 1977], and others, some limitations of this assumption have been highlighted. Separability is important both in the theoretical literature on intertemporal equilibrium, on the partial equilibrium approach to defend that marginal utility does not depend on the consumption of all goods, and in applied work especially when trying to construct micro-econometric devices to capture individual demand and its relation to aggregate data.

More generally, in many dynamic settings, it is usual to assume time-stationary preferences (implying, as was noted before, that the age of the individual when deciding cannot change her preference for a given bundle of goods that is the same from the point of view of the time moment when the decision is made), and some kind of separability. The recursive utility model, due to Koopmans (see [Mas-Colell, 1995]) leaves room for time varying discount factors and at the same time preserves the condition that ranking of different bundles is independent of what they offered in the past. This is in stark contrast with recent empirical evidence showing that individuals, at any point in their lifetime, consider tastes and preferences as already determined (coinciding with what they experience to be their preferences and tastes at that point in time) even if their tastes and preferences are shown to vary with their age (see [Quoidbach et al., 2013]).

An even more general framework is used in models with habits. Typically, habits can be built in the utility function directly by letting $u_t(.) = u(x_{t-\tau}, ..., x_t)$ for any value of τ. Moreover, it can be assumed that there is a given household technology that produces the habit at time t as a joint product with the consumption vector at $t-1$ (for example, if $z_t = f(x_{t-\tau}, ..., x_{t-1})$). This introduces the possibility of writing this simply as $u(x_{t-\tau}, ..., x_t) = u(z_t, x_t)$ that depends only on time t variables. Here, a question is how do habits come to be formed? There are two strands of the literature, the first one relying on the idea that habit formation depends on aggregate past consumption and so, the habit stock at each moment in time is taken as given by the individual (external habits, see [Pollak, 1977] and [Abel, 1990]) and the second one focussing on the influence that individual past consumption has on the present stock of habits (internal habits). Moreover, the dependence on the stock of habits can be cast, among others, in the form of quasi-difference (absolute habits) or in the form of a ratio (relative habits). Finally, the habit formation can be modelled as taking place just for the aggregate level of consumption of all goods or as being formed differently for each type (or group) of goods (this is the deep habits literature, see [Ravn et al., 2006]). The habits focus has been used to tackle a wide variety of issues, ranging from the so called equity-premium puzzle ([Abel, 1990]) to replication of hump-shaped response of consumption to expansionary shocks, which has proven to be a difficult feature of the data to replicate by general equilibrium real bussiness cycle models, since in the absence of habit formation, the response in consumption in those models tends to be an inmediate jump to their peak and a posterior descent into the previous level, while on the data, the peak is achieved several periods after the shock ([Christiano et al., 2005]). For the present work, several questions arise: which relation do habit persistence models bear to the choice-based view of preferences?

what are the underlying preference relations that give rise to habits models? .

We will discuss models where tastes do change when we review the temptation literature.

4.1.7 Heterogeneity

Up to this point, we have mostly pointed out some salient features of the individual preferences, relating them to the maximizing approach to individual decision making, and trying to establish their dependence on what is usually postulated (at least in neoclassical framework) as their empirical counterpart, the observable choice behaviour. However, the possibility of heterogeneous preference across consumers has been left out. This issue is crucial in applied work, since the estimation of individual demand and the construction of representative aggregates rely on the assumptions about heterogeneity of behaviour with respect to income on the part of individual consumers. That is, the stability assumption through different income groups when it comes to budget shares of expenditures in different categories of goods. The classical problem of obtaining Engel curves by imposing seemingly commonsensical restrictions on behaviour (homogeneity , symmetry of compensated price effects) has gathered a lot of attention[42]. On the other hand, some methods that do not rely on assumptions on the behaviour of individual consumers have been developed[43]. Heterogeneity here concerns tastes and attributes of consumers. In turn, heterogeneity of preferences is an issue for macro models based on aggregation. The existence of a representative consumer depends heavily on the heterogeneity structure in the economy. This issue is related to preference formation through the specification of processes by which individuals may end up

[42] See Gorman (1950) Dale W. Jorgenson, Lawrence J. Lau, and Stoker (1980, 1982) , the AIDS models of Angus S. Deaton and John Muellbauer (1980a, 1980b) and the QUAIDS system of James Banks, Blundell, and Lewbel (1997).

[43] See [Hildenbrand, 2014].

with different preferences due to their exposition to different social and economic environments (social group, political system, etc) and family environments (values transmitted by the parents, etc). Moreover, change in preferences can account for some of the problems highlighted in the above noted literature.

In macroeconomics, the problem of heterogeneity led to the development of conditions under which the aggregate magnitudes of an economy could be treated as if arising from the presence of a representative consumer even in the presence of heterogeneity in endowments, skills and tastes[44].

4.2 Uncertainty

There are 2 main approaches that have been proposed to capture the relation between uncertainty and preferences, and also with choice. Their difference has to do with different concepts of uncertain prospects. The first one is the Von Newmann-Morgenstern Expected Utility approach, that deals with settings where uncertainty is viewed as objective, in the sense that there is a given set of possible future states, Z (usually identified with prizes or consequences) and a set P of known probability measures or probability distributions on Z. The second one is the Savage Subjective Expected Utility approach, that deals with settings where the probability distributions are unknown. This section will only review some aspects of the Expected Utility approach.

Definition 12 (simple (discrete) probability measure) *A simple (discrete) probability measure on* Z *is a function* $p : Z \longrightarrow [0, 1]$ *such that supp(p):=*$\{z \in Z : p(z) > 0\}$ *is a finite (countable) set and* $\sum_{z \in \sup p(p)} p(z) = 1$. *Let the set of simple (discrete) probability measures on* Z *be denoted by* P_s (P_d)

Definition 13 (mixture space) *A mixture space is a set of objects,* Π *with typ-*

[44] See Caselli & Ventura, 2000; Sorger, 2000; García-Peñalosa & Turnovsky, 2006.

ical elements π, ρ, μ *and* ν, *and a family of functions* $h_a : \Pi \times \Pi \longrightarrow \Pi$ *for* $a \in [0,1]$ *such that:*

i) $h_1(\pi, \rho) = \pi$

ii) $h_a(\pi, \rho) = h_{1-a}(\rho, \pi)$

iii)$h_a(h_b(\pi, \rho), \rho) = h_{ab}(\pi, \rho)$

Examples of mixture spaces are the set of probability measures defined on countable sets with addition and multiplication by scalars defined componentwise. Another one is the set of all probability measures with finite support defined on an arbitrary set.

The previous definition is useful in stating some versions of the expected utility representation theorem:

Theorem 4 (VNM representation theorem) *Suppose* Z *is finite,* $\pi, \rho, \mu \in P_s$*. Then*

Proposition 5 ***Theorem 6*** *(a)* $\succ$ *is a preference relation (strict) on* P_s

(b) $\pi \succ \rho$ *and* $a \in (0,1]$ *implies* $(a\pi + (1-a)\mu \succ a\rho + (1-a)\mu$ *for all* $\mu \in P_s$

(c) $\pi \succ \rho \succ \mu$ *implies that there exists* $a, b \in (0,1)$ *such that* $a\pi + (1-a)\mu \succ \rho \succ b\pi + (1-b)\mu$

if and only if

(d) there exists a function $u : Z \longrightarrow \mathbb{R}$ *such that* $\pi \succ \rho$ *iff* $E_Z[ud\pi] > E_Z[ud\rho]$*. Moreover, if* u *represents* $\succ$ *in the sense of (d), it is unique up to positive affine transformations.*

Theorem 7 (Affine representation) *Suppose* Π *is a mixture space and* $\succ$ *is a binary relation on* Π*. Then*

(a) $\succ$ *is a preference relation (strict)*

(b) $\pi \succ \rho$ *and* $a \in (0,1]$ *implies* $h_a(\pi, \mu) \succ h_a(\rho, \mu)$ *for all* $\mu \in \Pi$,

(c) $\pi \succ \rho \succ \mu$ *implies that there exists* $a, b \in (0,1)$ *such that* $h_a(\pi,\mu) \succ \rho \succ h_b(\pi,\mu)$

if and only if

(d) there exists a function $F : \Pi \longrightarrow \mathbb{R}$ *such that* $\pi \succ \rho$ *iff* $F(\pi) > F(\rho)$ *and* $F(h_a(\pi,\rho) = aF(\pi) + (1-a)F(\rho)$*. Moreover, if* F *represents* $\succ$ *in the sense of (d), it is unique up to affine transformations.*

This framework deals with a situation where there is an apriori known set of possible future states of nature, and where probabilities are defined over those. This type of theorem focusses on conditions to impose on preferences in order to allow the possibility of representing the effect of lotteries over prizes, with known probabilities, directly as an expected utility representation, where there is an index function that gives the values of prizes and each lottery is evaluated according to the expected value of such function when restricted to the particular probability distribution representing the lottery attribution of likelihoods for events on the prize set. Moreover, the theorem exactly characterizes the class of preferences over lotteries that are representable by such an expected utility form. We note that the mixture space representation theorem falls short of an expected utility representation, since it only ensures an affine form for F. It is well known that either by imposing a sure-thing axiom or some form of continuity (for example, continuity in the weak topology), an expected utility representation can be obtained.

What are the central elements in this approach?

First, the state space (set of prizes or consequences) Z is known in advance. This is difficult to defend either on a normative or descriptive basis for the same reasons given above in the case of completeness of preferences, but even more if we are to make any sense of the concept of unexpected events in its most intuitive sense. Obviously one can always resort to defining the potential surprise or unexpectedness of an event as its likelihood, and include unconceived events as events that are assigned zero probability in every possible prospect. However, if such an event has objective zero probability in every prospect that the agent examines, then the only way in which it could happen is if there is incompleteness in the space of lotteries. Moreover, when considering conditional probabilities of such an event, they could not be calculated according to Bayes rule.

To face this, some models have been proposed that do not rely on an ex-ante known state space for consequences .

The second element is the fact that probabilities for events are objective, in the sense that the individual takes them as externally given. In the static setting we are considering, it is akin to impose that agents are Bayesian updaters who somehow have arrived at the same posterior on any event. Here, the question is how do they arrive to this situation?. To find some justification for this result in a dynamic setting, see [Dekel et al., 1998], where a dynamic process of choices with no previously defined objective state space can induce, through modification of strategies, an expected utility maximization behaviour on the part of individuals.

The third element is that the utility function on prizes is not dependent on states.

Now, in addition to being a preference relation, the additional axioms imposed on preferences (namely, conditions (b) and (c) in the VNM and mixture space representation theorem) are known as the substitution axiom and archimedean axiom respectively. In order to discuss (b), let us use an example from [Kreps, 1988]. Let p be the set of dishes to choose from in a restaurant, and identify a single dish with a degenerate lottery giving it with probability one, and a full course with a lottery giving 2 different dishes, each with probability 0.5. Consider the choice between two menus, each one composed of first and second course, one consisting of smoked salmon and steak, the other consisting of smoked salmon and grilled salmon. Suppose that the individual prefers grilled salmon to steak. Now, the substitution axiom implies that she would prefer the second menu to the first, because it assumes that she can compare "parts" of the menu interchangeably. But intuitively, she could prefer the first menu, even if it has steak, on the grounds of having more diversity. That highlights the fact that intuitively, when comparing prizes, we evaluate them globally. This makes the assumption normatively dubious, and descriptively false in many applications. On the other hand, the archimedean axiom is equivalent to a continuity assumption on preferences. There are many versions of such theorems, and some of them imply continuity and/or boundedness of u in the VNM expected utility representation.

The typical case where this kind of representation fails is when it is intuitive that preference are state-dependent. For example, suppose that the goods to be had are bundles of equippment, one including one umbrella and one including another item instead, when the possible states are rain or no rain. Then it seems clear that the preference relation representation will have to include the state in the utility function. A case of this class of representation is a state separable expected utility function representation for preferences. This kind of representation is used in models of preference for variety, change of tastes, temptation, etc. As an example, [Dekel et al., 2004] find conditions under which there is a subjective state space that can be extracted from the information about her preferences.

4.3 Temptation and Set dependence.

As has been said, one of the main criticisms of choice theory is related to the possibility of set-dependent choice behaviour. A case that has been studied is that of temptation and preference for committment. Although there are several models that tackle with this issue, we will focus on the work of [Gul and Pesendorfer, 2001] (hereon, G&P) and [Noor, 2011],[Noor and Ren, 2011], with occasional mention of [Kreps, 1988],[Dekel et al., 1998], [Krussell, 2009] and [Laibson, 1997]. This brief review will serve as background against which to present our discussion of learning about one's preferences.

In G&P, temptation is identified with a situation where the individual can choose from a set of menus, each menu consisting of a set of bundles of goods, and then when the menu is realized, the agent can choose a bundle from it. The element of delay in the second step decision allows for temptation in the following sense: It is related to a difference between the choice of menus from which to choose at a later date and the actual choice within the menu. The intuition is that, when presented with the possibility to choose between menus of goods that will be available at a future date, there is a trade-off between the committment preference, that is the preference as taken from the point of view of the individual if it could committ to posterior choices, and the preference once the choice set has been fixed. In some cases, this trade-off can be taken to denote that the individual is tempted by some element of the choice set she is considering. The prototypical example has a consumer who likes salads (s) and hamburguers (h). She has to choose whether to go to a restaurant where only salads are available, to one where both are available or to one where only hamburguers are available. So its choice sets are $A = \{s\}, B = \{s, h\}, C = \{h\}$. Suppose that she could express preferences over sets and that its preference reveals that $A \succ B \succsim C$. This can be taken to mean that, when presented with menu B, she is tempted by the presence of h in that menu, and would preferr, if she could committ to a choice, to stick with s. For example, she believes her appetite for burguers will be difficult to resist if she has that option, eventhough from a health point of view she would like to choose salad. On the other hand, the fact that she preferrs B to C expresses the fact that at least there is some gain in having the possibility of choosing salad, making it clear that the new item on the menu could improve her final outcome, potentially. Here, the second part of the preference could be interpreted as a preference for variety, such as in the original model by Kreps, but it has been used mostly to show that there is temptation in the sense suppose there are two menus, $A = \{a, b, c\}, B = \{a, b\}$. Then, temptation arises if $B \succ A$ when this is interpreted as showing that the presence of c in the first menu makes the individual worse off, since from a normative point of view she would prefer not to choose c, but once given the option to choose from set A, she will likely feel tempted to choose c. This setting is mostly a static one (even if it involves two stages) and identifies temptation as a preference for committment, revealed by the fact that the agent prefers B to A in the first stage. But the shortcomming is that, as a basis for dynamic choice behaviour, it can only identify temptation in the next period and not in the current one, that is, menus themselves do not tempt, but only some options included in them. By contrast, Noor adresses the issue by first inferring a normative preference for the agent as the limit of a sequence of delayed preference relations. Each element of the sequence is indexed by t , which denotes that the comparisons are made today for bundles of goods to be traded and consumed after t periods. In the special case where there is a limit (for t) of this sequence, that limit is identified with the normative preference of the agent. This encapsulates the idea that (temporally) distancing oneself from the objects of choice allows the individual to ascertain its normative preference relation. Then, temptation is defined as deviation from normative preference. While the extensions of G&P to a dynamic setting are based on a stationarity assumption (namely, that delaying the choice between two alternatives for any t periods will not change the preference involved), the Noor model is based precisely on violating that axiom, since imposing this assumption would entail that the limit preference is equal to any given one, thus precluding temptation in the sense adopted by Noor. However, the Noor model relies on another sense of stationarity, namely that involving current, non-delayed choices between bundles. That is, it assumes that for each period t, the preference relation comparing bundles from the perspective of that same period to be consumed in that same period does not change, since if it does, it probably would also affect the limiting preference, and preclude to infer the presence of temptation from an observed deviation in past periods with respect to current ones. This assumption is present in both setting, and in this sense, temptation in this models is not an instance of preference change. This feature is also shared by [Laibson, 1997] when modelling dinamically inconsistent behaviour as an intrapersonal game played by the agent against its future selves. Moreover, even in [Krussell, 2009], where a normative preference is obtained for some values of the parameter for temptation, the underlying preference is unchanged.

From the point of view of sophistication of the agent (meaning that she correctly anticipates her future behaviour), G&P and much of the literature take it as an assumption, while Noor, by working with the dynamic preference as its primitive, leaves room to check and refute the hypothesis of sophistication. In that sense, it would seem that the Noor model is better equipped to deal with tests of choice behaviour.

From the operational point of view, G&P and Noor share the goal to define preferences over (sets of) menus, instead of over bundles of goods. The mechanism is to take a compact metric space X, understood as the space of bundles, and consider the set of probability measures over X on the Borel $\sigma-$algebra of X, $\Delta(X)$ endowed with the weak convergence topology[45] . This set is understood as the set of lotteries over bundles of goods that the agent can envision. However, here the problem of defining uncertainty comes into play again, as it would be naive to concede only for this part that the agent has defined beliefs over what

[45] $\Delta(X)$ is compact and metrizable.

she can or cannot consume, and beliefs that are compatible whith expected utility maximization. From $\Delta(X)$, the set $K(X)$ of all nonempty compact subsets of $\Delta(X)$ endowed with the Hausdorff topology is a compact metric space and serves as the space of menus.

In the Noor model, the primitive is a non-empty closed-valued choice correspondence $C \rightsquigarrow Z$ that for any menu x gives the set of most preferred consumptions tomorrow and the set of most preferred menus from which to choose tomorrow.Hence, the agent choose at each point in time next period consumption and next period menu from which she will choose again both future consumption and menu.

The representation result that comes from G&P is the following:

Definition 14 *The preference relation $\succeq$ over $\Delta(X)$ admits a G&P representation if there exists linear functions $U, V : \Delta(X) \longrightarrow \mathbb{R}$ such that $\succeq$ over $\Delta(X)$ can be represented by $W(x) := \max_{\mu\in x}\{U(\mu) + V(\mu) - \max_{\eta\in x} V(\eta)\}$*

Noor's representation is given by:

Definition 15 *The choice correspondence C over $\Delta(X)$ admits a Noor representation if there exists functions $U, V : \Delta(X) \longrightarrow \mathbb{R}$ such that $C(x) = \arg\max_{\mu\in x}\{U(\mu)+ V(\mu)\}$, for all $x \in Z$ and $U(\mu) = \int_{C\times Z}(u(c)+\delta W(x))d\mu$ and $V(\mu) = \int_{C\times Z}(v(c)+ [\beta W(x) + \gamma \max_{\eta\in x} V(\eta)])d\mu$ for all $\mu \in \Delta(X)$, where $W : Z \longrightarrow \mathbb{R}$ is defined in the G&P representation and u, v are continous functions and δ, γ, β are scalars satisfying $\delta \in (0, 1), \gamma \in [0, \delta], \beta > \gamma - \delta$.*

Z is a constructed space of infinite horizon menus, where each menu is a compact set of lotteries.

What is important to note from the previous definition is that the agent ends up maximizing the sum of a normative preference (U) and a temptation preference

(V). However, the normative preference is itself a discounted sum of future consumptions and future normative value of menus. This makes the Noor approach vulnerable to a similar argument to the one expressed before against the substitution axiom. Suppose your choices are smoked salmon, steak, apple, versus smoked salmon, grilled salmon, apple. You may prefer grilled salmon to steak but in order to avoid repetition prefer the first menu to the second, but that violates the substitution axiom. This can be translated into a relation between current consumption and continuation value of temptation in the model of dynamic choice, which may lead to the violation of some the axioms, due to the presence of future temptation.

4.4 Learning

Learning is important for various reasons in economics. First, it is important to distinguish the concept of learning new information without changing the underlying preferences, be it through a bayesian updating process or through some other means, which do not necessarily imply learning new facts (see, for example [Aragonés 2005], which studies the possibility of fact-free learning in the sense of different strategies that the individual can follow to obtain statistical inferences from the set of known data at her disposal), from learning how the preferences are (as has been noted above, this second sense is important for many goods whose preference can only be learned through experience). We will be mainly focussed on this second sense of learning.

With regard to general equilibrium theory, learning has been argued as a mechanism that could justify how the economy arrives to a competitive equilibrium (be it with rational expectations or not). Existence theorems for general equilibrium have a long tradition in economic theory. However, one of the oldest arguments against this strand of the literature (mostly developed by Arrow and Debreu) is that it cannot explain how the economy arrives at such an equilibrium (in some models, to justify the existence of an equilibrium, it is necessary to impose that consumers do not trade at any prices different from the equilibrium ones, which seems a bit puzzling and absolutely counterintuitive). Learning could be used to provide a part of the missing link in that an economy that develops over time and whose agents learn from past values, can have a dynamic path that converges to the competitive equilibrium. Learning in a rational expectations setting has been extensively studied within new keynesian models, where the individual expectation about the other agents expectation enters the determination of equilibrium.In that context, the idea of learnability of an equilibrium has been developed.

4.4.1 Learning about one's preferences.

For the present work, learning is important as a means to understand the uncovering of the agents preferences as this agent evolves. First, there is the question of an apriori knowledge of ones preferences (implicit in the assumption of complete preferences over the entire possible choice set). It can be argued that as a simplfying assumption and an implementation device, this hypothesis is too strong, and as noted above, does not leave room for surprises, unexpected changes in preferences or common sense processes like growing, etc. Moreover, its empirical counterpart is far from obvious, since the fact that a consumer always chooses when faced with alternatives does not lend the hypothesis any strenght (recall that it is only on certain assumptions about the interpretation of completeness and transitivity that a sequence of observed choices can be rationalized as maximizing some kind of preference order, and that the question of set-dependence in choice can interfere with the extraction of information from actual choices). An additional problem in the case of sequential decision problems that can be reduced to one shot decisions (for example, an Arrow-Debreu general equilibrium) where the agents preference over future time and states is assumed complete is that assuming complete a priori knowledge of one's preferences entails assuming a special place for period 0 in the life of the agent, as this is the period from which all her decisions can be modelled. In particular, in models where time indexes goods, this special period should be identified with some moment where the agent is capable of taking all her decisions responsibly, not being determined by others influence (parents, etc). For this, note that consumption at certain ages (childhood) or in certain situations (school, prison, old people residences) is determined in large part by guidance of an external agent (mentee consumption), so observed behaviour in those cases need not correspond to revealing preferences, unless you construct a preference on situations (menus) where the agent assesses how good it is for her to join certain institutions, taking into account that in those institutions her consumption choices will be severely restricted by external guidance. This way out would need a reformulation of the choice set in the same vein as it has been done in the temptation literature. But even with a priori perfect knowledge of pairs of comparisons, there are complexity issues that can arise when trying to pass judgment on different alternatives. This can trigger different responses by identical individuals as per what concerns their preferences and their perfect knowledge of them. The alternative is to assume sequential decision making with every decision dependent on the new set of preferences that are realized every time period (or state). Devices like shocks to the marginal utility of consumption or leisure can somewhat generalize the previous models and can be taken into account. Moreover, one of the driving forces behind the some parts of the habits literature can be interpreted as designing a process where learning one's tastes is highly dependent on the persistent effect of our own past consumption, maybe because the individual partially discovers the marginal valuation of consumption or leisure after consumption or leisure takes places. This approach is taken in the model presented in part 2.

Axiomatic characterization of learning behaviour. We will, from now on, restrict to studying learning about our preferences in an environment where the agent cannot assign any probability to future events. To give some intuition about how learning in this sense can take place, let us start with an example. Suppose that there are 2 time periods, 0 and 1, and that at time zero, the agent must make two decisions: choose a bundle for time zero between x, y and z, and choose a choice set for time 1, between B={x,y} and B'={x,y,z}. At time 1, the agent will have to choose an option from the choice set specified at time 0. The agent has a complete and transitive ranking of the 3 choices, but at time 0, she only knows that x>y. We assume that if the agent chooses choices x or y, nothing new is revealed from z, whilst if the agent chooses z, the relationship with x and y is revealed at the moment of choice. So, there are only four relevant actions to consider, namely (x,B), (x,B'), (z,B) and (z,B'). Let us say that the agent displays a learning behaviour if she chooses (z,B'), so that the choice of something unknown in the first period can help the agent make the choice in the second period. If we assume that the initial preference over x,y,z is stable through time, that the consumption order does not matter, that the agent can make conjectures about all possible extensions about her preferences, and that she can only rationalize a choice if that choice is the best under all possible conjectures, then this behaviour is not rationalizable. However, this behaviour can be rationalized by a preference on the ordered pairs of the form $\langle x, z\rangle$, taken to mean the preference for ordered consumption, independently of the unknown (to the agent) relationship between x and z in the initial preference relation[46]. This ranking can also mean that the agent has a preference for learning beforehand in order to be able to choose from

[46] The ranking of alternatives would only have to satisfy that $< z, x > \sim < z, z >$ are preferred to any other alternative.

a wider set.

Let us define more precisely learning behaviour in this framework. Suppose that an agent has to choose at time $t = 0$, a bundle from $B_0 = \{x_i\}_{i=1}^N$, the set of bundles available for the agent at time $t = 0$ and a choice set from which to choose at $t = 1$. The agent's preferences are given by a linear order, $\succsim$ over the bundles to choose at $t = 0$ but she only knows an incomplete but transitive and reflexive part of it, $\succsim_0$ on B_0, at $t = 0$. Moreover, the choice sets from which to choose at $t = 1$ must be chosen among the elements of the power set of B_0. We assume that preferences are complete when restricted to a certain non-empty subset of B_0. Now, let $\triangle = \{x \in B_0 : \nexists x' \neq x$ such that $< x, x' > \in \succsim_0$or $< x', x > \in \succsim_0\}$, the set of initial bundles not related with any other bundle. We assume that, whenever an element of $\triangle$ is chosen, then all the relations between that bundle and the bundles previously related are revealed at time $t = 1$, before the agent has to choose from her choice set. Also, let $\mathcal{P}(B_0)$ be the power set of B_0.and $\overline{\succsim}$ be a partial order defined over $B_0 \times \{\mathcal{P}(B_0) \backslash \emptyset\}$, with tipical element $< x, B >$.

Remark 1 *Note that, while we use a similar setting than that in the temptation literature, we have 2 additional problems: 1) we do not impose the usual axioms for representability, since even if we can extend any partial order to a total order, preserving the initial order*[47]*, and (owing to the finitenness of* B_0*) represent this extension by a utility function, this function will not characterize completely the partial order (as discussed in the section concerning completenness); 2) we are not interested in an expected utility representation, since the goal is to rationalize learning behaviour and characterize the incomplete preference relations that can give rise to it, without appealing to probabilistic considerations, be them objective or subjective probabilities.*

Which conditions could we impose to the agent behaviour to ensure that it is

[47] This is the content of Szpilrajn's theorem.

interesting in the learning sense specified, and at the same time non-trivial?

We propose several intuitive properties that the choice function consistent with learning should satisfy:

Definition 16 *The agent displays choice for flexibility if* $\forall B, B' \in \{\mathcal{P}(B_0) \setminus \emptyset\}, B \subseteq B'$ *,*$\forall x \in B_0$*, the agent chooses* (x, B') *over* (x, B)*. That is, the agent chooses more options than less.*

Definition 17 *The agent displays pure incompletenness aversion if* $\forall B, B' \in \{\mathcal{P}(B_0) \setminus \emptyset\}$ *such that* $M(B) = \{x \in B : x \text{ is maximal for } \succsim_0 \text{on } B\} = M(B')$ *,then* $\forall x \in B_0 \setminus \triangle$*,* $x' \in \triangle \setminus B'$*, ,the agent chooses* (x, B) *over* (x', B') *. That is, if the first period choice carries no information on the second period choice, and the sets are equivalent without that information, she will choose the known bundle.*

Definition 18 *The agent displays pure learning behaviour if* $\forall B, B' \in \{\mathcal{P}(B_0) \setminus \emptyset\}$ *such that* $M(B) = \{x \in B : x \text{ is maximal for } \succsim_0 \text{on } B\} = M(B')$ *, then* $\forall x \in \triangle \cap B$*,* $x' \in \triangle \setminus B'$*, the agent chooses* (x, B) *over* (x, B')*. That is, a new choice that carries informational increase in the second period is chosen over one that does not, when choice sets are equivalent from the period* 0 *point of view.*

Definition 19 *The agent displays learning behaviour if* $\exists x \in \triangle, y \in B_0 \setminus \triangle$*,* $B \in \{\mathcal{P}(B_0) \setminus \emptyset\}$*, with* $x, y \in B$ *such that the agent chooses* x *as the time* 0 *bundle and* B *as the time* 1 *choice set.*

We would like to know which incomplete preference relations over B_0 induce a preference relation over $B_0 \times \{\mathcal{P}(B_0) \setminus \emptyset\}$ whose associated choice function satisfies choice for flexibility, pure incompletenness aversion, pure learning behaviour and learning behaviour at the same time.

However, at this point, we can only conjecture that such incomplete preference relations can be completely characterized. This is still part of an ongoing research program.

Learning behaviour and outcome rationality: correct knowledge as a learning incentive Given the previous framework, if the agent displays learning behaviour, it seems that a natural way to think about its learning process is by extending a non complete partial order. However, none of the previous behavioral properties is a sufficient condition for the agent to acquire new knowledge about her preferences and at the same time preserve outcome rationality (as defined in the incompletenness section). It could well be that all of her choices correspond to unknown objects, but that she is wrong in the sense of ending in a worse situation than by choosing the known objects. In that case, the agent will learn but only by deviating from a minimal requirement of rationality. So we want to include the knowledge dimension into our concept of learning, in order to make this concept compatible with outcome rationality. To that end, we must consider how the agent's conjectures about her preference's missing parts are formed, and under which conditions will this process of forming conjectures lead to a situation of acquiring knowledge, that is, learning in a more specific way.

Let us first define this particular type of learning process about our unchanging (through time) preferences that are only partially known. So we focus on the possibility of sequentially completing our preference ordering, assuming for the moment that preferences are transitive. To this end, we follow [Mandler, 2005], and distinguish between psicological preferences, which provide reasons to make choices, and behavioral preferences, which are derived from the choices taken. We take as fundamentals a non-complete partial order over a set Y of alternatives (this represents the agent knowledge about her preference at the beggining of the period), jointly with the possible conjectures about how the missing parts of our ordering turn out to be. A conjecture is a set of partial orders over Y, interpreted as imagination and computational efforts we have made, but between which we are in doubt (non probabilistic). On the other side, if a conjecture does not include a given partial order, this is interpreted as steeming from some hidden heuristic process we follow to try to uncover our preferences, coupled with a limitation of our cognitive abilities. Crucially, we assume that the agent does not know whether her preferences are changing or not. Note that in this setting, from the point of view of the agent, there can be change in preferences from date t to date t+1, even if there is a well defined (but unknown) final preference ordering.

At each point in time, the choice of the agent includes a bundle and a choice set, C_t . Note that this, jointly with the assumption of learning behavior, implies that each period, the set from which to choose has at least one new element. For each combination of initial knowledge and conjectures, and each choice set C_t , there will be a unique ex-post realization of choices. These choices, jointly with the knowledge we already have, will determine a unique behavioral preference, that might or might not belong to the set of conjectures[48]. We assume that the agent displays learning behaviour.

There are two different senses in which the agent learns in this setting. The first one occurs whenever the choices imply information that was not included in the given knowledge. This forces the agent to update her conjectures. We call this **conjectural learning**, since what the agents obtains is a richer set of

[48] As discussed in Mandler, 2005, although it is reasonable to expect that psicological preferences are transitive (to avoid money pump arguments) and that behavioral preferences are complete (to avoid situations where the agent does not choose), it is consistent with the notion of outcome rationality to admit that psicological preferences can be incomplete and behavioral preferences have intransitive elements. To avoid this problem, we assume that the realized behavioural preferences of the agent are transformed into a preorder, by applying the transitive closure operator on them.

future conjectures. However, in the present setting, learning in this sense happens trivially, by construction, every time a new choice is made. But more important, this learning only provides guidance towards our future choices if we decide to limit our options to choices that we already did in the past.

The second sense in which learning occurs in the present setting is whenever the realized behavioral preference happens to be part of our conjectures. We call this **factual learning**, since the agent partially unveils her true preference by hypotesizing some structure for her preferences, which will presumably influence her choices, and finding out that what she can inferr from the choices is consistent and exhausts what she hypothesized. This kind of learning does not occur trivially in this model. More importantly, it is the sense in which learning can help explain why the agent makes new choices. This is why we will restrict to study factual learning.

Remark 2 *Note that both types of learning admit the possibility of being wrong.*

Given that the agent displays learning behaviour, which conditions about conjectures would ensure that factual learning will happen?

The simplest condition to ensure learning every period would be the rule to include all one-point extensions from the current knowledge in every possible conjecture, jointly with the base knowledge. However, this would make factual knowledge trivial.

A weaker condition to ensure learning would be to follow the simple rule of starting with the base knowledge as a conjecture, and for every period where learning did not happen, add the realized ex-post preference to the previous conjecture. It turns out that, with a slight addition, this is enough to ensure learning.

Proposition 8 *(**Completion Learning 1**) Suppose that i) for a given initial knowledge, all the elements in a conjecture preserve our present knowledge and* *the only thing all conjectures have in common is the possibility that our present knowledge is all there is to know about our preferences, ii) that any realized ex-post preference is included as part of the next period conjecture, and that conjectures can be generalized in the sense that, if we have any set of conjectures that are totally related by inclusion pairwise, then we can also envision a conjecture that is the union of them all. Under this conditions, we can ensure that, for any given initial knowledge, factual learning will take place in finite time.*[49]*.*

Remark 3 *However, this says nothing about the plausibility of completing our preferences. Even if we can put an upper bound on the number of simple operations (comparing two alternatives, for example) to be performed in order to form every potential conjecture, we do not know the ratio of operations per amount of time unit that we can perform, and with a period of given lenght, we cannot assure that completenness will be reached. That is, our definition serves as a basis for studying improvements in knowledge, but does not ensure that those improvements will end in completenness within the time specified*[50]*.*

Remark 4 *Note that, although we could extract some information from the current conjectures about our probabilistical beliefs regarding which is our true preference relation, the present setting is not probabilistic, in the sense that different potential partial orders could be weighted differently or equal and still be part of the same conjecture.*

Remark 5 *The present concept of factual learning implies that, once it is obtained, the individual updates her initial knowledge with the correct conjecture. However, since the agent believes that her preferences might be changing, that*

[49] See the appendix for proofs.

[50] This would be bypassed if we allowed for infinitely lived individuals and finite consumption sets.

need not be the end of the unveiling process (since it can happen for a partial order). We would like to study which additional conditions ensure that the agent completely unveils her final preference relation. Moreover, the next natural step would be to generalize this framework to the case where the choice sets from the agent combine with an outside restriction, say a budget set, and see if the result holds. Note that choosing the same point might not lead to the same ex-post behavioral preference, since the budget set could constrain the elements with which we can compare the choice with.

Now consider the possibility of a technology that enhances our imaginational and computational capabilities, or equally, reduces the cost of each simple operation. In the simpler case, this would clearly improve the chance of learning, and of sequentially completing our preferences. However, technical change need not reduce the time to perform every comparison in the same way. For example, an internet search engine might improve our ability to compare potential holidays destinations by giving us an idea of their characteristics, including pictures, etc, but it will probably be less useful when comparing different kinds of wine. Hence, some technological improvements might drive our process of unveilling preferences in a particular direction, and thus affect the kind of choices we make, and the kind of conjectures we adopt[51]

Note that in the present setting, nothing is said about what motivates the interim choices and in particular, each one of them is made from a situation of incomplete preferences, but those choices have to be consistent with what we know about our preferences. This is a consistency requirement that can be critized on the basis of being both too strong and too weak. But in this context, we

[51] As an example, consider the impact of TV and internet on the leisure choices of young adults in the US in the last decade. http://www.ara.cat/premium/que-shan-acabat-festes_0_1435656436.html

are assuming that the underlying preferences do not change and that even with incomplete preferences, agents will make a choice.

To conclude, the future goals of this research program would be:

1) to completely characterize (establish sufficient and necessary conditions) those incomplete preference relations over B_0 which induce a preference relation over $B_0 \times \mathcal{P}(B_0)$ whose associated choice function satisfies choice for flexibility, pure incompletenness aversion, pure learning behaviour and learning behaviour at the same time.

2) The present concept of factual learning implies that, once it is obtained, the individual updates her initial knowledge with the correct conjecture. However, since the agent believes that her preferences might be changing, that need not be the end of the unveiling process (since it can happen for a partial order). We would like to study which additional conditions ensure that the agent completely unveils her final preference relation.

3) We would like to study if the results hold if we include an external constraint each period in the form of a budget set.

5 APPENDIX

Definition 20 *Y is the set of alternatives among which to choose*

Definition 21 *$\mathcal{F}$ is the set of all partial orders on Y,that is, the set of elements that can become part of a conjecture about our preferences.*

Definition 22 *$\succsim_0$is the initial partial order on Y, that is, the initial knowledge.*

Definition 23 *$\mathcal{F}(\succsim_0)$ is the set of all partial orders on Y that preserve $\succsim_0$.*

Definition 24 *$\mathcal{P}(\mathcal{F})$ is the set of all subsets of $\mathcal{F}$, with tipical element P that is, the set of all possible conjectures.*

Definition 25 *$\mathcal{P}(\mathcal{F}(\succsim_0))$ is the set of all subsets of $\mathcal{F}(\succsim_0)$, with tipical element P that is, the set of all possible conjectures that preserve the initial knowledge.*

Definition 26 *$\mathcal{A} \subset \mathcal{P}(\mathcal{F}(\succsim_0))$is the set of actual conjectures the agent forms.*

Definition 27 *$\mathcal{A}_J = \{A_j : A_j \in \mathcal{A}; j \in J\}$ is a subset of $\mathcal{A}$.*

Definition 28 *$\sigma : \mathcal{P}(\mathcal{F}) \longrightarrow \mathcal{F}$ is a single-valued function that assigns a behavioural, ex-post, preference relation to any possible conjecture the agent holds.*

Definition 29 *We say that there is learning in the sense of partial completion if there is $P \in \mathcal{P}(\mathcal{F}(\succsim_0))$ such that $\sigma(P) \in P$. That is a situation where the agent behavioural ex-post preference relation coincides with one of the conjectures she held initially.*

Proposition 9 *(**Completion Learning 1**)Suppose that $\succsim_0$ is any initial incomplete partial order over Y and $\mathcal{A}$ is such that i) all the elements in a conjecture preserve our present knowledge and the only thing all conjectures have in common is the possibility that our present knowledge is all there is to know about our preferences, ii) that any realized ex-post preference is included as part of the next period conjecture if this was not correct, and iii) that conjectures can be generalized in the sense that, if we have any set of conjectures that are totally related by inclusion pairwise, then we can also envision a conjecture that is the union of them all. Under this conditions, we can ensure that, for any given initial knowledge, factual learning will take place in finite time.*

Proof. Define recursively the conjectures associated with each time period by $\{A_0\} \in \mathcal{P}(\mathcal{F}(\succsim_0))$, $\succsim_0 \in A_0$, $A_{t+1} = G(A_t, \sigma(A_t))$ and suppose that:

i) $\forall A_t, [\sigma(A_t) \notin A_t \Rightarrow G(A_t, \sigma(A_t) = A_t \cup \sigma(A_t)]$. Then, i) $\cap \mathcal{A} = \{A_0\} \in \mathcal{A}$,

ii) $\forall A_t \in \mathcal{A}$, $\sigma(A_t) \cup A_t \in \mathcal{A}$,[52]

[52] This can be proved by induction.

iii) $\forall \mathcal{A}_J \subseteq \mathcal{A}$ such that ($\forall j, k \in J$, $A_j \subseteq A_k$ or $A_k \subseteq A_j$), then $\cup_{j \in J} A_j \in \mathcal{A}$.

Then, according to the Fixed Point Theorem in Berge, *Topological Spaces*, p.40, there exists $\exists t_0 < \infty$, $A_{t_0} \in \mathcal{A}$ such that $\sigma(A_{t_0}) \in A_{t_0}$.

Part II

Culture and Technology: a model of technologically induced preference change.

6 Introduction

Why fewer women than men do market work? Why Koreans or Americans work longer hours than Europeans? Ask a sociologist and she will likely point to cultural heterogeneity as the main explanation of these differences. While recognizing the importance of culture, economists tend to understand better explanations based on opportunity costs or incentives and try to avoid those involving differences in preferences (of which we tend to be suspicious at least since [Stigler and Becker, 1977]. One problem with differences in preferences is that, although tastes are clearly heterogeneous across individuals, it is hard to understand why a specific preference should be more frequent among individuals living in a particular country or belonging to a particular group. However, [Fernández, 2010] surveys a number of papers that provide ample cross-sectional, historical and experimental evidence that this is indeed the case.[53] As a consequence, it is increasingly clear

[53] Many of these works suffer from an identification problem: as both the environment and the behaviour are changing simultaneously, it is hard to ascribe unambiguosly the different behaviors to different preferences. Nevertheless, [Fernandez, 2007], [Eugster et al. ,2011] and [Hoff et al.

that we need to account for the role that culture (broadly defined) plays in the determination of preferences, see e.g., [Fehr and Hoff ,2011].

Our understanding is that culture shapes individual preferences (and, therefore, it should not be surprising to find group or geographical clustering of preferences), but this begets the question of what gave rise to that particular culture in the first place. We propose an explanation in which it is economic factors (more specifically, technology) that generate a culture that, in turn, shapes the individual preferences. Culture, thus, determines through preferences the short run equilibrium values of economic variables. In the long run, however, it is culture what changes in response to the underlying economic fundamentals.

Previous efforts to understand the evolution of preferences have stressed the central role that exposure and experience play in the development of tastes.[54] Of course, the notion that preferences are shaped by habit and custom is not new in Economics, as it can be traced back at least to [Marshall ,1920]. We share the view of [Bowles ,1998] that some basic preferences might be innate, possibly genetically transmitted, while the rest would be learned. Those innate would be very general in nature, such as the preference for sweet tastes or the dislike of pain, and probably are the result of the gene selection process in the evolution of the species.[55] The real problem lies in understanding why a particular culture

,2011], among others, are able to overcome this difficulty by examining the case of culturally differentiated groups that live under an homogeneous institutional environment.

[54]See [Bowles, 1998] for evidence taken from other disciplines on the relevance of exposure in the development of tastes. [Bowles, 1998], hence, takes an ontogenetic perspective. For a phylogenetic approach see [Robson, 2001], who stresses the reciprocal influence between human evolution and the development of preferences.[Ostrom, 2000] takes both an ontogenetic and a phylogenetic approach as it ascribes changes in preferences to exposure and changes in the distribution of preferences to the natural selection mechanism.

[55]Some tastes, e.g., bitter taste, are asociated with natural poisons, e.g., hemlock, so it is natural that avoiding those tastes became favored by natural selection. However, the learning

arises in which we learn to appreciate specific qualities of the goods.

In the evolution-of-preferences literature, two strands can be identified, that of "internal" and that of "external habits" depending on whether this 'exposure and experience' is intended or not by the individual herself.[56] "Internal habits," thus, denotes the idea that it is the current choices of the individual what shape her future preferences, a fact of which she is fully aware. This literature originated in [Stigler and Becker ,1977] and was later developed by [Becker and Murphy ,1988]. Clearly, this approach sheds light on the 'causes' of preferences in the later stages of the life of the individual, but is silent on what determines them in earlier stages, e.g., we could say 'granpa likes classical music because he's been listening to it for the past 50 years' but we would not know what led him to start listening to it in the first place (as opposed to going to the movies).

By "external habits" we refer to the idea that it is unintended exposure to a given good or service what helps the individual to develop a taste (or aversion) to it. It is conventional to cite [Duesenberry ,1949] as the precursor of this habit formation theory that adopted its canonical form under [Constantinides ,1990] and [Abel ,1990], and was latter extended to allow for intergenerational interactions by [De la Croix ,1996]. In our view, most if not all papers in this strand are not particularly concerned by what the evolution of preferences is and take them as if they were given. However, it is clear that in these models the steady state value (or equivalent equilibrium notion) of preferences is determined within the model.

We posit that this evolution is governed by endowments and technology, or,

process may well trump that innate taste and help develop an apreciation for tonic water or, at least, for gin and tonic.

[56] A related literature is that of the evolution of the *distribution* of preferencences, in which, although preferences themselves are given, their distribution is endogenously determined. Within this strand, changes in the distribution can be due to the socialization process (à la [Bisin and Verdier, 2001] or to some "fitness" criterion (as in [Dekel, et al., 2007]. We will briefly address this literature below.

more precisely, by their interaction. For example, if we look at fish and seafood consumption per capita, we observe that it is highest in tiny Pacific island nations and lowest in some landlocked countries. Using country data, the correlation coefficient between fish and seafood consumption per year per capita and the ratio of coastline length to total land area is of 0,66 with a t-statistic of 8,64. Even excluding Maldives, whose exceptionally large per capita consumption may be driven by tourism, the correlation coefficient remains substantial with a value of 0,47 (6,09).[57] A plausible rationale is that, where fish was readily available, individuals developed a taste for fish, something that did not happen where it was not, and this "cultural preference" is driving consumption decisions today, when improved transportation methods have increased dramatically fish accessibility everywhere. If our intuition is correct, it would help understand why we observe that people living in a given country or belonging to a given group tend to have similar preferences.

It should be noted that this "availability" could be the result of either endowments or technology. In our earlier example, fish could be easily accessible in a particular area because fish stocks are extremely abundant and even a rudimentary fishing technology yields a large production. In this case, fish availability for consumption would be due to the endowment. Alternatively, a more refined fishing technology could yield the same production from far less plentiful stocks. In this event fish availability would be the result of technology. In either case, the product of labor could be the same and, to the extent that this is the case, the two alternatives would be indistinguishable from our perspective. Hence, it is the interaction of endowments and technology what, in our view, shapes preferences.

As far as technology is concerned, there is a budding empirical literature deal-

[57] Fish and seafood consumtion data are from 2007, retrieved from FAOSTAT. Coastline length and land area by country were obtained from The CIA Factbook.

ing with the technology-to-preferences line of causation. [Alesina et al. ,2010] provide a nice example: they uncover a link between historic plough use and current self expressed attitudes about the role of women in society. Their argument is that plough use increased the productivity of males relative to that of females and that induced men to specialize in 'market production' while women specialized in 'home production.' In turn, this specialization gave rise to beliefs and attitudes that have persisted until the present and manifest themselves in, for example, lower female labor force participation among first and second generation immigrants to the US from countries that historically used the plough.[58] [Ross ,2008] describes the case of oil producing countries and suggests that it is the fact that the oil producing technology crowds out women from the labor market what reinforces the prevalence of patriarchal norms and values. These two papers underscore the role of technology in shaping individual preferences, which are then shown to exhibit a significant correlation with, among other things, the female labor market participation rate.[59]

As we have already mentioned, to some extent, the fact that technology shapes preferences is implicit in most papers of what we have called the "external habits" literature. In those papers, steady state values of parameters affecting individual preferences are ultimately driven by technology (perhaps among other factors), although this dependence is not usually underscored. A few recent papers, how-

[58] [Boserup, 1970] is credited by [Alesina et al. , 2010] as the origin of the idea that it is the historic mode of agricultural production what shaped, at least in part, gender role attitudes.

[59] A related literature is that of papers dealing with the impact of external events on preferences. Examples are, [Nunn and Wantchekon, 2011], [Alesina and Fuchs-Schündeln, 2007] or [Guiso et al., 2008]. [Nunn andWantchekon, 2011] trace back mistrust to the likelihood of ancestors being subject to slave trade. [Alesina and Fuchs-Schündeln, 2007] document the impact of Comunism on East Germans preferences. [Guiso et al, 2007] relate differences in social capital between the North and South of Italy to the free city states experience in the North of Italy at the turn of the first millennium.

ever, have made this relationship explicit, e.g., [Fernández, et al. ,2004], [Lindbeck and Nyberg ,2006], or [Doepke and Zillibotti ,2008].[60] [Fernández et al. ,2004] underscore the role of received (dis)utility to men of having a working wife in the determination of the proportion of working wives and show that it depends critically on the productivity of women. [Lindbeck and Nyberg ,2006] model the impact of subsidies on effort to achieve a high productivity, that, in turn, shapes the influence of social norms on preferences. [Doepke and Zillibotti ,2008] is perhaps the closest to our paper in that they focus on preferences on leisure (and on the time discount parameter); however, they do so in a manner that enables them to explain the intensive margin whereas we focus on the extensive margin. In addition, they concentrate on the role of direct transmission of preferences while, in our case, it is the oblique transmission mechanism the one that operates the most.[61] In this sense, we view our approach as complementary to that of Doepke and Zillibotti ,2008].

Our paper focuses on the determination of labor market participation rate in a model in which preferences are shaped by a culture whose evolution is driven by technology.[62] We build a simple OLG model that captures the evolving nature of culture. Every (two period lived) generation will be identical except for the culture they inherit from the previous generation. For a given generation, the current state of culture and the economic decisions of the individuals given prevailing wages will determine their labor market participation rate in the second period of their lives.

[60] [Palacios-Huerta and Santos, 2002] and [Fershtman and Heifetz, 2006] also feature explicitly endogenous preferences, but their focus is more on the institutional framework than on the role of technology.

[61] In fact, we can accommodate both oblique and direct transmission of preferences, as can be seen in Section 5.

[62] We understand that culture need not be neutral with respect to technological change, and that, to some extent, changes in economic fundamentals may be driven by culture itself. However, we will concentrate in the economics to culture direction of causality.

In the steady state, the labor market participation rate will be determined by the marginal product of labor (our technological parameter).

For each generation, as the labor market participation rate depends on the quit rate, the received culture will affect the wage schedule offered by firms (see e.g., [Oi, 1962]. Hence, we will have reciprocal causation between labor market participation rate and expected present value of wages. Note that we refrain from talking about wages and use instead the more cumbersome ‘expected present value of wages’ expression. This is because, although perfect competition among firms guarantees that the expected present value of labor costs must equal the expected present value of the marginal product of the workers, it has no direct implication on the particular value of the wages paid at every point in time. Therefore, some additional assumption regarding how each period wage is fixed is needed to close the model. Contract theory provides the natural option: we impose that firms offer whichever wage schedule is preferred by individuals.[63]

Our model, thus, integrates both contract theory and the role of culture in an OLG model to shed some light on the labor market participation decision. In this sense, our paper is related to [Hauk and Saez-Marti ,2002], [Escriche et al. ,2004] and [Escriche ,2007]. In these papers, authors embed cultural transmission of values à la [Bisin and Verdier ,2001] and asymmetric information in an OLG framework. The main difference between our approach and theirs is that they all take as given the existence of several subcultures, each characterized by specific preferences, and focus on determining their equilibrium distribution, i.e., the proportion of individuals having each of those preferences. These papers, thus, feature an endogenous distribution of preferences but specific preferences themselves are exogenous and, as such, unaffected by economic factors. In contrast, our

[63]Some papers have indeed pursued this line of thought (e.g., [Lazear and Rosen, 1990], but they have always confined themselves to the static (or one generation) case.

agents are ex ante identical and it is only changes in the economic fundamentals what could give rise to the appearance of these differentiated groups.

To underscore that our model provides a mechanism for cultural convergence distinct from the oblique socialization of [Bisin and Verdier ,2001], consider the following situation. Imagine that there is immigration of a non zero mass of individuals from a low to a high productivity country. If individuals from the low productivity country had indeed lower market attachment, immigration would result in two cultures coexisting in the destination country: one with higher and one with lower labor market attachment, corresponding to natives and immigrants respectively. Under most (if not all) mechanisms of cultural transmission, these two cultures would perpetuate themselves with no impact on each other. In contrast, our model implies that the higher productivity of labor in the new country would set off a process of cultural convergence of the immigrants towards the behaviour of the natives. Once the new steady state is attained, all individuals belong to the same group, that of “high” labor market attachment, irrespective of the country of ancestry of each of them.

In some sense, hence, it could be argued that our model rationalizes heterogeneous preferences: to the extent that individual preferences can be the product of other economic fundamentals, heterogeneity in preferences would simply reflect variation in the relevant fundamentals. For example, one could account for the male-female labor market participation gap as the result of past differences in productivity, as described in [Alesina et al. ,2010]. In fact, heterogeneous preferences can arise from any factor that impacted the relevant technological parameter. For example, we would expect individuals living in areas in which property rights enforcement is limited to work shorter hours. This, in turn, might lead to the development of a negative attitude towards work within those areas. Also in this vein, the difference in hours worked across countries could be understood as taxation originating the preference for leisure that some (e.g. [Blanchard, 2004] suggest as an explanation of the fact that individuals work longer hours in some countries.

The paper is organized as follows. Section 2 presents the model for consumers and firms. Section 3 describes the equilibrium conditions. Section 4 analyzes some implications of the model. Section 5 considers the robustness of the results to alternative preference transmission mechanisms and Section 6 concludes.

7 The Model

7.1 Consumers

Each generation is formed by a continuum of individuals (indexed by i) of measure 1 who live for two periods, say t and $t+1$, and are endowed with one indivisible unit of labor time per period. Individuals derive utility from current and future consumption and from future leisure according to a time separable utility function

$$U_i\left(c_t, c_{t+1}, L_{t+1}\right) = u\left(c_t\right) + \beta u\left(c_{t+1}\right) + \beta \lambda_i \left(1 - L_{t+1}\right)$$

where $c_t, c_{t+1} \in \mathbb{R}_+$ denote time t and time $t+1$ consumption levels, $L_{t+1} \in \{0, 1\}$ indicates labor time, β is a subjective discount factor and λ_i is the marginal valuation of time. Since individuals do not value current leisure, they inelastically supply their endowment time to the labor market when young. At times it will be convenient to indicate whether we are referring to the generation an individual belongs to; in those cases, the superscripts t and $t+1$ will be used, but, otherwise, they will be dropped to enhance readability. We assume further that $u(\cdot)$ is differentiable, strictly increasing and strictly concave, that all derivatives have constant sign over the entire domain and that the usual Inada conditions apply. We also assume that the absolute risk aversion, as measured by the Arrow-Pratt coefficient, is non increasing in income for $u(\cdot)$. Notice that, as L_{t+1} is assumed to be a binary variable, the fact that the utility function is linear in labor time implies no loss of generality.

Individuals do value leisure in the second period of their lives, but only learn about their marginal valuation of leisure at the onset of that second period. The idea behind this specification is that, as the evidence cited by [Bowles ,1998] indicates, preferences are commonly formed as a result of the previous consumption experience. In our case, for preferences for leisure to develop, the individual must first experience work, which she does when young. This working experience allows her to develop a taste for leisure (or a distaste for work), upon which she will subsequently choose whether to work or not when old. Before working, individuals are only aware of the common (to all individuals) distribution function of leisure valuation. In particular the known distribution of λ_i is as follows, let q_{t+1} be the probability with which an individual born at time t receives $\lambda_i = 0$ (we will refer to q_{t+1} as the unconditional probability of working), and $F(\lambda)$ the conditional cdf of λ for $\lambda_i > 0$. We assume that $F(\lambda)$ is twice continuously differentiable and its density, $f(\lambda)$, is strictly positive and non-increasing. The actual realization of λ_i is information private to the individual and is unverifiable in the asymmetric information sense, i.e., the outcome of contracts can not be dependent on its value, as it is impossible to verify it independently (e.g., by a judge). Obviously, this implies that markets are incomplete.

The unconditional probability of working can be understood as reflecting the prevailing attitudes towards work and leisure, and, in this sense, as reflecting the impact of culture on labor market participation. From the point of view of each generation, this unconditional probability of working is given because the culture in which they are bred is heavily influenced by that of the previous generation.

Hence, consumers born at time t only learn about their particular marginal valuation of leisure before time $t+1$ consumption and leisure or work takes place. This specification of uncertainty implies that all individuals are *ex-ante* (as of time t) homogeneous and *ex-post* (as of time $t+1$) heterogeneous and is akin to the one used in [Lazear and Rosen,1990], whose workers only receive information about their reservation wage just before deciding whether to work or not. At the time of hiring, therefore, an individual has no more information about her characteristics than her potential employers and this makes the problem one of genuine uncertainty rather than one of asymmetric information. However, as there is no "law of large numbers" applicable in the context of a continuum of agents (see [Feldman and Gilles, 1985], this individual uncertainty translates into aggregate uncertainty which we would like to avoid. We, thus, take the approach in [Alós Ferrer, 2002] whereby individual uncertainty is specified so that it disappears in the course of aggregation. This specification removes aggregate uncertainty at the cost of losing independence between individuals, but this is immaterial in our model, because it only implies that the index i is correlated with the realization λ_i. In so far as the index itself is not observable, this is irrelevant.

Consider an individual born at time t. On the first period of her life, the individual is assumed to work, and will receive a wage of w_t that will be devoted to current consumption, c_t, and to savings, s_t, which will earn interest R_{t+1}. The decision on whether to work in the next period hinges upon the actual realization of the λ_i parameter she receives. At time $t+1$ she will choose between staying employed, which allows consumption $\overline{c}_{t+1} = s_t R_{t+1} + w_{t+1}$, or quitting her job, in which case her consumption will be of only $\underline{c}_{t+1} = s_t R_{t+1}$ but she will receive $\beta\lambda_i$ in additional utility from leisure. It is clear that she will remain employed if and only if $\lambda_i < \widehat{\lambda}_{t+1} = u\left(\overline{c}_{t+1}\right) - u\left(\underline{c}_{t+1}\right)$, a critical value that is decreasing in savings and increasing in the wage received when old. Therefore, the probability with which an individual born at time t will work at time $t+1$, will be, thus,

$$p_{t+1} = q_{t+1} + (1 - q_{t+1}) F\left(\widehat{\lambda}_{t+1}\right). \tag{2}$$

It is important to note that our previous assumptions on the distribution of λ_i have a number of implications. First, the probability of working will be always strictly below one. Second, the Inada conditions together with $0 < p_{t+1} < 1$ imply non-negative savings. And third, p_{t+1} coincides with the proportion of old individuals that will work at time $t+1$. Culture, thus, determines a 'base labor market participation rate,' q_{t+1}, whose influence is combined with that of economic factors to yield the actual labor market participation rate, p_{t+1}.[64]

The problem for the consumer is, thus, to choose s_t to maximize her expected utility of current and future consumption and of (contingent) future leisure, subject to her budget constraints, i.e., $c_t + s_t = w_t$ and $c_{t+1} = s_t R_{t+1} + w_{t+1} L_{t+1}$, and taking into account that the individual chooses $L_{t+1} = 1$ if and only if $\lambda_i < \widehat{\lambda}_{t+1}$. The objective function of the individual can be written as

$$\begin{aligned}
& E_{f(\lambda_i)}\left[U_i\left(c_t, c_{t+1}, L_{t+1}\right)\right] \\
= \;& q_{t+1}\left[u\left(c_t\right) + \beta u\left(\bar{c}_{t+1}\right)\right] \\
& + (1 - q_{t+1}) \int_0^\infty \left\{u\left(c_t\right) + \beta\left[u\left(c_{t+1}\right) + \lambda_i\left(1 - L_{t+1}\right)\right]\right\} f\left(\lambda_i\right) d\lambda_i \\
= \;& u\left(c_t\right) + E_{p_{t+1}}\left[\beta u\left(c_{t+1}\right)\right] + \beta\left(1 - q_{t+1}\right) \int_{\widehat{\lambda}_{t+1}}^\infty \lambda f\left(\lambda_i\right) d\lambda_i \qquad (3) \\
= \;& u\left(c_t\right) + E_{p_{t+1}}\left[\beta u\left(c_{t+1}\right)\right] + \left(1 - p_{t+1}\right) \beta E_{f(\lambda_i)}\left[\lambda_i \middle| \lambda_i > \widehat{\lambda}_{t+1}\right], \qquad (4)
\end{aligned}$$

where $E[\cdot]$ and $E[\cdot|\cdot]$ are the expectation and the conditional expectation operators respectively and the subindex indicates the probability under which the expectation is taken. The FOC can be shown to be the usual

$$-u'\left(c_t\right) + E_{p_{t+1}}\left[u'\left(c_{t+1}\right) \beta R_{t+1}\right] = 0. \tag{5}$$

[64] The time $t+1$ labor market participation rate is actually $\frac{1+p_{t+1}}{2}$.

To understand the disappearance of terms connected to leisure, observe that as the probability of working increases the individual gives up utility of leisure in exchange for utility from consumption. At the margin, the added probability of working, reduces the utility from leisure in exactly its critical value, i.e., $\widehat{\lambda}_{t+1}$; also at the margin, the gain in utility from consumption is $u\left(\bar{c}_{t+1}\right) - u\left(\underline{c}_{t+1}\right)$. This two values are identical, and, hence, it is as if savings had no effect on the utility of leisure.[65]

Note that the Inada conditions imply that the LHS of (5) is (large) positive for values of s_t close to 0 and (large) negative for values close to w_t, which together with its continuity guarantees existence of optimal savings. However, the effect on the LHS of (5) of an increase in savings is unclear because, in addition to the usual negative effect on the expected marginal utility of consumption, there is also a positive indirect effect that operates through the increase in the probability of working, as can be seen in

$$\begin{aligned}\frac{\partial^2 E_{f(\lambda_i)}\left[U_i\left(c_t, c_{t+1}, L_{t+1}\right)\right]}{\left(\partial s_t\right)^2} &= u''\left(c_t\right) + E_{p_{t+1}}\left[\beta u''\left(c_{t+1}\right) R_{t+1}^2\right] \\ &\quad + \frac{\partial p_{t+1}}{\partial \widehat{\lambda}_{t+1}} \left(\frac{\partial \widehat{\lambda}_{t+1}}{\partial s_t}\right)^2.\end{aligned}$$

To guarantee that a unique solution to this problem exists, we provide a natural extension of the traditional definition of 'normal good' to our uncertain environment: we characterize a good as 'generalized normal' if an increase in current wealth leads to an increase in its expected consumption.[66]

Proposition 1 If time $t+1$ leisure is a generalized normal good as defined, optimal savings are uniquely determined. In addition, they are non decreasing

[65] From an analytical standpoint, this is more easily seen taking the derivative of (3) with respect to s_t, rather than that of (4).

[66] The following proposition can be viewed as an extension of the classical result in Diamond (1965).

in w_t and non increasing in w_{t+1}.

Proof The expected time $t+1$ leisure is $E\left[1-L_{t+1}\right]=1-p_{t+1}$, and the above definition of a generalized of normal good implies that

$$\frac{\partial E\left[1-L_{t+1}\right]}{\partial w_t}=-\frac{\partial p_{t+1}}{\partial s_t}\frac{\partial s_t}{\partial w_t}>0.$$

As the probability of working has been shown to decrease with savings, it must be the case that $\frac{\partial s_t}{\partial w_t}>0$. The only way an individual can increase the expected amount of leisure she will experience is through the reduction of the probability of working, and this is accomplished by increasing the savings level. Note that this assumption does not imply necessarily that time $t+1$ consumption is also a normal good because, although the increase in savings will lead to an increase in c_{t+1} irrespective of whether the individual works or not at $t+1$, it will also shift the probability towards the low consumption state and, hence, the net effect on the expected future consumption is unclear.

Implicit differentiation of the FOC yields

$$\frac{ds_t}{dw_t}=\frac{u''\left(c_t\right)}{\frac{\partial^2 E_{f(\lambda_i)}\left[U_i\left(c_t,c_{t+1},L_{t+1}\right)\right]}{\left(\partial s_t\right)^2}},$$

which has been shown to be positive. As the numerator is negative, this implies a negative denominator. This ensures that a unique strictly positive solution to (5) exists, and optimal savings $\widehat{s}_t^j=S\left(w_t,w_{t+1},r_{t+1},q_{t+1}\right)$ are well defined. The amount saved depends, thus, on the wages paid at t and $t+1$ and on the prevailing interest rate between these two periods. We have already shown $\frac{dS(w_t,w_{t+1},R_{t+1})}{dw_t}>0$, and to establish the sign of $\frac{dS(w_t,w_{t+1},R_{t+1})}{dw_{t+1}}$, we calculate

$$\frac{\partial^2 E_{f(\lambda_i)}\left[U_i\left(c_t,c_{t+1},L_{t+1}\right)\right]}{\partial s_t \partial w_{t+1}}=E_{p_{t+1}}\left[\beta u''\left(c_{t+1}\right)R_{t+1}\right]+\frac{\partial p_{t+1}}{\partial w_{t+1}}\frac{\partial \widehat{\lambda}_{t+1}}{\partial s_t}<0$$

i.e., consumers react to a higher future wage by reducing savings and, thus, increasing the probability of working.

7.2 Firms

We assume that there is perfect competition among a continuum of firms that share a common technology displaying constant returns to scale, that the marginal product of both capital and labor are positive and decreasing everywhere, and that, in addition to paying for the factors used, firms face a one-time cost when hiring a new worker. It will be seen below that workers have no incentive to make a job-to-job transition and, hence, this cost will only be borne when hiring young workers. The production of a representative firm at t is $Y_t = Y(K_t, L_t) = y(k_t) L_t$ where, as usual, K_t and L_t are the capital and labor used respectively, k_t stands for the capital labor ratio and $y(k_t)$ denotes output per worker. In addition, we impose that $L_t = L_t^{t-1} + L_t^t$, i.e., all workers are equally productive irrespective of their age. We assume further that firms are able to hire as much capital as they want at a constant cost r. In addition, financial arbitrage implies that the rate of return on savings, R_{t+1}, must coincide with the cost of capital r. The problem of the firm is, thus, to maximize the present value of profits, that, at each t, are defined by

$$Y_t - \left[\left(w_t^t + \kappa\right) L_t^t + w_t^{t-1} L_t^{t-1} + K_t r\right].$$

Notice that the wage rate paid to old and young workers need not be the same.

The FOC of this problem with respect to K_t is the usual and can be written as

$$y'(k_t) = r, \tag{6}$$

implying that firms choose a fixed capital per worker ratio (denoted k), which, in turn, determines that output per worker is constant.

We assume that firms can credibly commit to any wage schedule, i.e., they can post jobs offering a wage pair $w = \left\{w_t^t, w_t^{t+1}\right\}$ that will be the actual values of wages paid. On the other hand, workers cannot commit to working for more than one period, because of the possibility of receiving a large λ_i. Denote P_{t+1} the proportion of old individuals willing to work at that wage, w_t^{t+1}; as already noted, our previous assumptions on the distribution of λ_i imply that $P_{t+1} = p_{t+1}$,[67] and, hence, $P_{t+1} = P\left(s_t, w_{t+1}\right)$. We do not want firms to face uncertainty with respect to the measure of workers that will choose to stay on the job when old. To guarantee this, we impose that each firm hires a countably infinite number of workers. This assumption warrants that the ex-ante probability of working that each individual faces coincides with the ex-post proportion of old-age agents that choose to work not only for the overall economy but also at the firm level. In addition, for each young worker the firm employs at time t, it will also be able to employ P_{t+1} old workers at time $t+1$.[68] This allows us to rewrite the time t present value of profits as

$$\Pi_t = \sum_{\tau=0}^{\infty} \frac{1}{r^\tau} \left\{Y_{t+\tau} - \left[\left(w_{t+\tau}^{t+\tau} + \kappa\right) L_{t+\tau}^{t+\tau} + w_{t+\tau}^{t+\tau-1} P_{t+\tau} L_{t+\tau-1}^{t+\tau-1} + K_{t+\tau} r\right]\right\},$$

an expression that underscores the fact that the choice variable for the firm is the number of young workers it hires. Furthermore, that hiring decision has implications on profits at times t and $t+1$. The FOC with respect to the number of young workers hired from a given generation (e.g., the one born at time t) implies

$$w_t^t + \kappa + w_{t+1}^t \frac{P_{t+1}}{r} = \left[y\left(k\right) - y'\left(k\right) k\right] \left(1 + \frac{P_{t+1}}{r}\right), \tag{7}$$

[67] This is tantamount to assuming that firms compute P_{t+1} under the assumption that the continuum of individuals behave as would a representative (measure one) individual.

[68] For this statement to be accurate, we need that those individuals willing to work when old have no incentives to accept offers from other employers. We will see below that the conditions under which this is true are implied by *competition among firms*.

and we will denote $\overline{w} = y(k) - y'(k)k$ the one period marginal product of labor. This condition simply means that the expected present value of the marginal product of labor must equal the expected present cost of that worker. It is worth stressing that, as firms know the number of old age workers they will be able to employ and adjust wages paid accordingly, the fact that a proportion P_{t+1} of workers quit is immaterial from the firm point of view, as the hiring cost incurred in their hiring was fully covered by the reduced present value of wages paid to all workers.

If we write the present (as of time t) value of profits as

$$\begin{aligned}\Pi_t &= \sum_{\tau=0}^{\infty} \frac{1}{r^{\tau}} \left\{Y_{t+\tau} - \left[\left(w_{t+\tau}^{t+\tau} + \kappa\right) L_{t+\tau}^{t+\tau} + w_{t+\tau}^{t+\tau-1} P_{t+\tau} L_{t+\tau-1}^{t+\tau-1} + K_{t+\tau} r\right]\right\} \\ &= \sum_{\tau=0}^{\infty} \frac{y(k)}{r^{\tau}} \left(1 + \frac{P_{t+\tau+1}}{r}\right) L_{t+\tau}^{t+\tau} - \\ &\sum_{\tau=0}^{\infty} \frac{1}{r^{\tau}} \left[w_{t+\tau}^{t+\tau} + \kappa + w_{t+\tau+1}^{t+\tau} \frac{P_{t+\tau+1}}{r} + rk\left(1 + \frac{P_{t+\tau+1}}{r}\right)\right] L_{t+\tau}^{t+\tau},\end{aligned}$$

simple substitution of (6) and (7) into this expression shows that firms make indeed zero profits.

As workers cannot commit to staying in their jobs when old, firms face an additional restriction when choosing the wage schedule they offer: they need to make sure that no firm can hire only old workers and make a profit. It is clear that such a firm would have profits described by

$$\begin{aligned}\Pi_t &= Y_t - \left[\left(w_t^{t-1} + \kappa\right) L_t^{t-1} + K_t r\right] \\ &= \left[y(k) - \left(w_t^{t-1} + \kappa\right) - rk\right] L_t^{t-1} \\ &= \left[y(k) - \left(w_t^{t-1} + \kappa\right) - y'(k)k\right] L_t^{t-1} \\ &= \left[\overline{w} - \left(w_t^{t-1} + \kappa\right)\right] L_t^{t-1},\end{aligned}$$

and, hence, w_t^{t+1} is bounded below by

$$w_t^{t+1} \geq \overline{w} - \kappa, \quad \forall t. \tag{8}$$

The fact that old workers always receive at least $\overline{w} - \kappa$, i.e., the most they could make at any other firm, is what warrants that they have no incentive to change firms when old and justifies our previous claim that firms face no uncertainty about the number of old workers it can employ. To summarize, then, the contract space we are considering is the set of all wage pairs that satisfy (7) and (8) simultaneously. Clearly, condition (7) only places a restriction on the present value of the wages paid to a worker, but not on the time distribution of this amount, i.e., not on the particular values of w_t and w_{t+1}. The lower bound for old age wage in (8) reduces further the set of admissible wage pairs. We argue that competition among firms leads them to offer whichever distribution is most desirable for workers, i.e., competition forces firms to choose w_t, w_{t+1} so that individuals maximize their utility over those wage pairs for which profits are zero and are above the lower bound, i.e.,

$$\{w_t, w_{t+1}\} \in \arg\max \{(4) \;\; s.t. (7), \; (8)\} \tag{9}$$

This condition is akin to the incentive compatibility constraints commonly found in the asymmetric information literature. However, it is worth stressing that in our setting it arises as a consequence of competition among firms, and not as a restriction on the problem of the firm.

There is an interesting specific case: when $w = \{\overline{w} - \kappa, \overline{w}\}$, i.e., every worker is paid her current marginal product net of current hiring costs. We label this contract the 'age contract.' Under this contract, firms obtain the same profits from hiring any worker, irrespective of her age. Furthermore, wages do not depend on the quit rate. In all other contracts, however, wages and quit rates are jointly determined.

In all, competition among firms has three effects. First, firms pay workers the full amount of their marginal product net of hiring costs (7). Second, the old age wage must be above a given threshold (8). And third, within the limits imposed by the previous two conditions, the time distribution of wages is the one preferred by workers (9).

8 Equilibrium

8.1 Equilibrium within a generation

Consider the generation born at time t and recall that members of this cohort take q_{t+1} as given. We study first the case where $q_{t+1} = q, \forall t$. For this generation, an equilibrium is a manifold that includes the savings decision, the probability of working and the wage rates, i.e., equations (5), (2), and (9), must be satisfied simultaneously. Let $R_A(c_t)$ denote the Arrow-Pratt measure of absolute risk aversion, and $\lambda_m = u(\overline{w}(1+r) - \kappa) - u(r\overline{w})$. Proposition 2 establishes the existence of the equilibrium for such an economy.

Proposition 2 If

(2.i) $f(\lambda_m) \leq M$,[69] and

(2.ii) time $t+1$ leisure is a generalized normal good

then there exists a unique, stable equilibrium for the economy.

Condition (2.i) imposes a limit on the probability mass in the right tail (beyond some point λ_m) of the distribution of λ_i.

Proof We give here a sketch of the proof and defer the formal proof to Appendix A.1[70]. We treat the problem in two separate parts. First, by substituting

[69] M is a positive real number whose precise definition is given in Appendix A.1.

[70] In the Appendix, we ommit most technical details for the sake of brevity. Detailed proof is available from the authors.

(2) in (5), (7) and (9), we construct the following value function, which is well defined given s_t :

$$\begin{aligned} V(s_t) &= \max_{\{w_t, w_{t+1}\}} u(c_t) + E_{p_{t+1}}[\beta u(c_{t+1})] + (1 - p_{t+1}) \beta E_{f(\lambda)} \left[\lambda_i \middle| \lambda_i > \widehat{\lambda}_{t+1} \right] \\ & \quad s.t. (7), (8), \text{ and } w_t - s_t \geq 0 \end{aligned}$$

Under the assumptions, both the objective function and the restrictions are quasi-concave, and given that a solution to this problem exists and that the constraint qualification is met everywhere, the Kuhn-Tucker conditions are necessary and sufficient to characterize the unique solution to the problem, $\{w(s_t), p_{t+1}(s_t)\}$, with $w(s_t) = \{w_t, w_{t+1}\}$. Note that the inverse function theorem ensures that this solution is differentiable with respect to s_t. Second, we construct an auxiliary function $g(s_t, w(s_t))$ that is simply the first order condition of the problem of the individual when wages are such that solve the problem of the firm, and show that it has at least one zero. Let $s^*(w_t, w_{t+1})$ be such that $g(s^*, w(s^*)) = 0$. Then, s^* defines an equilibrium for the economy. Note that, in principle, any feasible contract can be an equilibrium contract, i.e., equilibrium considerations do not limit further available contracts. This implies that the equilibrium contract can be, but need not be the age contract. Finally, we ensure that irrespective of the equilibrium contract, this equilibrium is unique.

8.1.1 Equilibrium wage schedules

The presence of the seniority payments in the labor market is pervasive. However, the age contract is the maximally back loaded wage scheme i.e., the one that implies the largest difference between young and adult wages (or the highest seniority payments). We are interested in determining under what conditions individuals prefer wage schedules involving a less steep wage schedule. Therefore, we find conditions on the primitives of our model that guarantee that the contract prevailing in equilibrium is not the age contract. If this is the case, as we have already noted, wage rates will be dependent on the quit rate.

Consider a wage schedule consisting in equal payments in both periods, i.e., $w_t^t = w_{t+1}^t$, and label it 'constant wage schedule.' We will prove that, under some conditions, it is strictly preferred to the age contract and that this preference is robust in the space of parameters for which proposition 2 holds.[71] Obviously, if a constant wage schedule is preferred to the age contract, this constant wage schedule may or may not be the equilibrium contract but we are certain that the equilibrium contract will be distinct from the age contract. Let $\underline{\beta}, \underline{W}, \underline{r} > 0$ denote the minimum values $\beta, \overline{w}$, and r can take, also let $\underline{s} = S(\overline{w} - \kappa, \overline{w})$, and observe that $\underline{s}$ will be the minimum savings for given parameter values. Our initial parameter space is, then $\widehat{\Theta} \subset [\underline{\beta}, 1) \times [0,1] \times (0,1] \times \left[\overline{W}, \infty\right) \times [\underline{r}, \infty) \times \mathbb{R}_{++}$, with typical element $\widehat{\theta} = (\beta, q, \kappa, \overline{w}, r, f(\lambda_m))$,where $\lambda_m = u(\overline{w}(1+r) - \kappa) - u(r\overline{w})$. Also note that, given the definition of the parameter space and the consumer problem, we can obtain $\inf_{\widehat{\theta} \in \widehat{\Theta}} \underline{s} = \underline{\underline{s}} > 0$, which can be taken as a parameter, as it is independent of the specific values of the other parameters. Hence, we take our enlarged parameter space as $\Theta = \widehat{\Theta} \times \mathbb{R}_{++}$, with typical element $\theta = \left(\beta, q, \kappa, \overline{w}, r, f(\lambda_m), R_A\left(r\underline{\underline{s}}\right)\right)$.

Proposition 3 Let $\mu(\cdot)$ be the Lebesgue measure on Θ and, given $\theta \in \Theta$, $w_t^*(\theta) = \left\{w_t^*, w_{t+1}^*\right\}$ be the unique equilibrium wage schedule for the generation born at time t. Then, there exists $I \subset \Theta$ such that $\mu(I) > 0$ and for all $\theta \in I$, $w_t^*(\theta) \neq \{\overline{w} - \kappa, \overline{w}\}$.

Proof We offer a formal proof in Appendix A.2 that proceeds along the following

[71] This preference will, hence, be shown to be robust with respect to a particular $u(c)$. We take this to be given from the outset, since the range of the cdf, the density and the elasticities are dependent on its functional form, as well as on $\overline{w}$

lines: We first construct a 'constant wage' contract, i.e., one in which $w_t^t = w_{t+1}^t$ that satisfies (7) and (8), and show that, for small values of κ, it is preferred to the age contract. As (9) has to be satisfied in equilibrium, this implies that the prevailing wage scheme will be different from the age contract.

In words, this implies that there is an open set of parameters for which the equilibrium of the economy will imply a wage schedule other than the age contract. An economic rationale for this is that, in presence of hiring costs, the constant wage contract provides some insurance against the eventuality of not working when old, because consumers receive higher income (compared to the age contract) when young (i.e., when they are sure to work), and, hence, are better able to smooth their consumption levels.

8.2 Steady state equilibrium

We have already mentioned that the unconditional probability of working reflects the impact of culture on labor market participation. In particular, we posit that individuals observe the quit behavior of the previous generation and derive from it some assessment of the 'acceptability' of quitting. On the other hand, it is already clear that the received culture is not the only relevant factor, because the economic environment is also crucial in the labor market participation rate determination. The inherited unconditional probability of working, together with the economic environment the individuals face, translates into their actual behavior regarding quits, i.e., the actual probability of working is determined according to (2). The fact that individuals do not simply work (or quit) with their inherited probability, but change it to account for economic factors is key, because it causes culture to evolve through time in response to these economic factors. We have, thus, that the current value of economic variables is heavily influenced by culture, but that, at the same time, it is economic fundamentals that shape culture in the long run. We introduce, thus, a process that governs the evolution of q_t through time. Let

$$q_{t+1} = \pi(q_t, p_t) : [0,1]^2 \rightarrow [0,1]$$

be the law of motion (which we assume to be common knowledge) for the unconditional probability of working. Therefore, q_{t+1} can be viewed as the result of cultural influences received by the individual. Knowledge of past values of the unconditional probability of working (the past acceptability of quitting) and of the actual proportion of old age workers helps determining how acceptable it is to quit. The higher these values, the least acceptable should be this behavior and, hence, the higher the base labor market participation rate, q_{t+1}. As a consequence, the law of motion should be increasing in both q_t and p_t. In addition, and to guarantee that the steady state equilibrium is unique and stable, we will assume that $\pi(q_t, p_t)$ is continuously differentiable and that its partial derivatives are bounded above by ψ, a positive real number which will be defined in Appendix A.2. Finally, as q_{t+1} is a probability, we impose that

$$1 > \pi(q_t, p_t) > 0, \forall (q_t, p_t) \in [0,1]^2 \tag{10}$$

The assumed law of motion for q_t embodies the idea that preferences are partly transmitted across generations. However, this transmission can take on several forms and steams from different sources, from which three are especially relevant. First, there is transmission of cultural values within the family, and it can be explicit (i.e., intended by the individual parents) or implicit (i.e., derived from the repeated exposition of the individual as a child to certain situations, from which endogenous modifications of tastes arise). Moreover, both types of intra family transmission interact, as the individual can observe as a child their parents actual behavior and contrast it to the values intended to be transmitted. Second, individuals are embedded in a broader social framework, e.g. the neighborhood, the school or the institutional and legal framework of the country they live in. This represents a constraint on their actions and an external (to the family) source of interaction, and sets another channel for preference formation. Finally, the individual as a child can observe directly the actual economic behavior of the previous generation as a group and this observation can also impact the preference generation mechanism.

We show in the robustness section that our modelization of the process is flexible enough to accommodate as particular cases some of the usual theoretical approaches on preference transmission, including the cultural transmission model of [Bisin and Verdier, 2001], the asymmetric information model of [Fernández , 2007], and preference transmission induced by aspirations à la [De la Croix, 1999].

Irrespective of what the particular preference transmission mechanism is, we want to ensure that a unique steady state exists in this economy, and that is the purpose of the next proposition.

Proposition 4 If

(4.i) $\frac{\partial R_A(c_t)}{\partial c_t} \leq 0, \forall c_t,$

(4.ii) $f(\lambda_m) \leq M^*$.[72]

(4.iii) time $t+1$ leisure is a generalized normal good

(4.iv) $q_{t+1} = \pi(q_t, p_t) : [0,1]^2 \rightarrow [0,1]$ is continuously differentiable, with positive partial derivatives bounded above by ψ and (10),

then there exists a unique steady state equilibrium for the economy, with $q_t = q \in (0,1)$, $\forall t$. This equilibrium is globally stable, in the sense that, for any initial value of (q_0, p_0) the economy will converge to that unique steady state. Moreover, the steady state value of the unconditional probability of working is increasing in $\overline{w}$.

[72] M^* is a positive real number whose specific definition is given in Appendix A.3.

Proof As before, we give a sketch of the proof and refer the reader to Appendix A.3. First, given $q_t \in [0,1]$, we can apply the proof of proposition 2 to obtain a unique solution to the problem of the generation born at time $t-1$. Note that part of this solution is $p_t = q_t + (1-q_t) F\left(\widehat{\lambda}_t\right)$, which allows us to write it as $p_t = p_t(q_t)$. Hence, for generation born at time t, it is clear that q_t is the only relevant state variable, and we can write

$$q_{t+1} = \pi(q_t, p_t(q_t)) = \pi(q_t)$$

and note that our previous arguments imply that the solution is differentiable with respect to the state variable q_t. Thus, we can obtain an expression for the differential

$$\frac{dq_{t+1}}{dq_t} = \frac{d\pi(q_t)}{dq_t} = \frac{\partial\pi(q_t, p_t)}{\partial q_t} + \frac{\partial\pi(q_t, p_t)}{\partial p_t}\frac{dp_t}{dq_t}.$$

This, together with the definition of $\frac{dp_t}{dq_t}$, which can be obtained from the solution to the problem of the generation born at time t, and assumption (4.iv) above, ensure that the total effect on q_{t+1} is less than proportional to the change in q_t so that for all $(q_{t-1}, p_{t-1}) \in [0,1]^2$:

$$\sup_{q_t, \in [0,1]} \left|\frac{dq_{t+1}}{dq_t}\right| = \sup_{q_t, \in [0,1]} \left|\frac{d\pi(q_t)}{dq_t}\right| = \sup_{q_t, \in [0,1]} \left|\frac{\partial\pi(q_t, p_t)}{\partial q_t} + \frac{\partial\pi(q_t, p_t)}{\partial p_t}\frac{dp_t}{dq_t}\right| < 1.$$

Hence $\pi(q_t)$ is a contraction on a complete metric space, and there exists a unique fixed point $q_{t+1} = \pi(q_{t+1}) \in (0,1)$.

This proposition shows that the steady state equilibrium of the unconditional probability of working, q, is an increasing function of the marginal product of labor and constitutes the main result of the paper. The mechanism through which $\overline{w}$ affects q should be apparent: $\overline{w}$ is among the determinants of the wage schedule. In turn, the wage schedule drives the work vs. leisure decision. Observe that this decision rests on the comparison of the consumption levels associated with each state. Obviously, if markets were complete, individuals would insure themselves and, provided the price of such insurance was fair, there would be no difference between states in their consumption level. Therefore, market incompleteness plays a crucial role in our model.

8.3 Complete vs. incomplete markets

To underscore the relevance of market incompleteness, consider what would happen if markets were complete. To complete markets, we introduce an insurance contract with premium ϕ paid when young that returns the amount φ in the event of the worker not being employed when old.[73] Clearly, for the contract to be actuarially fair, it should happen that

$$\varphi_{t+1} = \frac{\phi_t r}{1 - P_{t+1}}. \tag{11}$$

The problem for an individual born at t is again (4) but with somewhat redefined variables: current consumption is $c_t = w_t - s_t - \phi_t$, while future consumption will be either $\underline{c} = \frac{r\phi_t}{1-P_{t+1}} + s_t r$ or $\overline{c} = w_{t+1} + s_t r$. The individual must now choose both her savings and the premium she wants to pay, to fulfill the FOC for savings given by (5), and the FOC for the premium given by

$$-u'(c_t) + (1 - p_{t+1})\,\beta u'(\underline{c})\,\frac{r}{1 - P_{t+1}} = 0.$$

If insurers are rational and compute P_t under the assumption that the continuum of individuals behave as would a representative (measure one) individual, we have that $p_{t+1} = P_{t+1}$ and, hence

$$-u'(c_t) + \beta u'(\underline{c})\,r = 0.$$

[73] *This is similar in spirit to the unemployment accounts of [Brown et al., 2008]*

Using this in (5) results in $u'(\overline{c}) = u'(\underline{c})$ and, hence $\overline{c} = \underline{c}$, i.e., the individual chooses to fully insure herself against the possibility of not working when old, $\varphi_{t+1} = w_{t+1}$. Of course this is nothing but the classic result of a risk averse individual choosing full insurance when offered a fair premium.[74]

If the price for insuring the non employment state is either actuarially fair or lower, we would have $\overline{c} \leq \underline{c}$ and at most a proportion q_{t+1} of individuals would choose to work. This would sever the connection between the marginal product of labor and the unconditional probability of working. From the moment such insurance was available onwards, $\overline{w}$ would play no role in the determination of the unconditional probability of working and it would drift towards wherever $\pi(q_{t+1}, q_t)$ leads it. Note that when the price is higher than the actuarially fair, individuals choose less than full insurance, and all our results hold. As an example, imagine that the government implemented a program of intergenerational transfers from the young to the non working old. The balanced budget constraint implies that

$$\phi_t = (1 - P_t)\,\varphi_t. \tag{12}$$

where now ϕ_t denotes the tax paid by the young and φ_t the subsidy received by the old. From the point of view of the generation born at time t, it will pay taxes amounting to ϕ_t and receive a subsidy of $\frac{\phi_{t+1}}{1-P_{t+1}}$ in the event of not working. The comparison of this last value and the RHS of (11) reveals that, unless $\phi_{t+1} \geq \phi_t r$, either the tax is too high or the subsidy too low relative to the actuarially fair values. Given that w_t^t is bounded above, a policy of ever increasing taxes levied on the young is unfeasible. Hence, at some point we would have $\phi_{t+1} < \phi_t r$. As a consequence, the tax-subsidy pair will not be actuarially fair and $\overline{c} > \underline{c}$ and, therefore, $p_{t+1} > q_{t+1}$. Most importantly, the probability of working would still be dependent on the marginal product of labor and the steady state value of q would

[74]Note that in this case $s_t > 0$ is no longer guaranteed.

remain an increasing function of $\overline{w}$.

In other words, only those instruments (or policies) that lead to $\overline{c} = \underline{c}$ would be able to sever the link between $\overline{w}$ and q. In particular, any program of intergenerational transfers (either voluntary, e.g., through 'gifts' to parents, or compulsory, e.g., through taxes and subsidies) that resulted in $\overline{c} \neq \underline{c}$ would leave the connection between $\overline{w}$ and q untouched.

9 Implications

How can our model help explain the observed differences in labor market participation rates across groups defined e.g., by gender or by nationality? To the extent that p_t is the product not only of current economic conditions but also of the inherited culture, q_t, all factors determining q_t are relevant in the explanation of the present labor market participation rate. Notably, past values of the marginal product of labor will be shaping current attitudes towards work provided that the economy is not in a steady state: Imagine a world in which, individuals belonged to one of two distinguishable groups, call them 'Bees' and 'Drones.' Suppose that, initially, both the marginal product of labor and the distribution of the marginal valuation of leisure was identical across the two groups. Assume that an exogenous technological innovation causes the marginal product of Bees to increase while that of Drones remains constant, $\overline{w}^B > \overline{w}^D$. This could be what happened with the introduction of the plough, as [Alesina et al. ,2010] suggest: the superior strength needed to handle the plough increased male productivity, but left female productivity unchanged. Assume also that the prevailing wage scheme is not the one associated with the age contract. As shown in Proposition 3 above, this will be the case at least for small values of κ. In such a world, the unconditional probability of working of each group would converge to different values, $q^B > q^D$. Hence, economic factors would give rise a "cultural" artifact that associates group membership with labor market attachment. In turn, this larger labor market attachment would imply a larger expected present value of wages for Bees than for Drones.

From this standpoint $\left(\overline{w}^B > \overline{w}^D,\ q^B > q^D\right)$ suppose now that some exogenous innovation caused the marginal product of Drones to increase and equal that of Bees, à la [Galor and Weil, 1996]. Clearly, we would observe a transition phase during which both the expected present value of wages and the unconditional probability of working of Drones converge to the same values Bees experience. During this transition the expected present value of wages would be lower for Drones than for Bees even though they are equally productive, and we would observe a wage gap between them[75]. This wage gap would arise between groups of groups differing only in the culture they have inherited from the previous generation, and bears no relation whatsoever to education, productivity or any of the usual determinants of wage.

Frequently any difference between wages of distinguishable groups (e.g., male vs. female, or black or Mexican vs. white) that can not be ascribed to these 'usual determinants' is attributed to pure discrimination. Our model suggests that this need not be the case: differences in wages may arise as a result of pure cultural (as opposed to economic) factors, and, to the extent that they are not included in the analysis, discrimination estimates might be biased upwards. In this vein, [Erosa, et al. , 2005] show that the better part of the unexplained wage differential between men and women vanishes when one considers only women with no children. In our context, this would merely imply that women without

[75] Actually, we would observe a lifetime expected wage gap because it is the expected present value of wages what would be lower for drones than for bees. As the equilibrium wage schedule has not been determined, we make no claims about the existence of wage gaps between wages at each point in time.

children face cultural restrictions with respect to quitting similar to those men do.

Our model can also be brought to bear on the issue of cross country differences in labor market hours worked. There is broad consensus in this literature about the crucial role taxes play in explaining these differences (see [Prescott, 2004], but explanations stressing differences in preferences over consumption/leisure choices are not uncommon (e.g. [Blanchard, 2004]. We suggest what could be viewed as a consensual alternative between the two previous explanations. Imagine two identical countries, both in steady state equilibrium. Being identical, all variables, e.g., wages or the unconditional probability of working, would take the exact same values. Now suppose one country introduces some form of taxation and spends the proceeds in a manner that those taxed also reap the benefits of spending. There is a wedge, however, between taxes paid and services received, perhaps due to collection costs. This wedge is what will be crucial. As a consequence of the wedge, individuals will work less, and this will generate a departure in the culture of the high tax country from that of the low tax country: on average, leisure will be more highly valued in the high tax country. Thus, we would observe that individuals work less in countries with higher taxes (or more precisely, with higher inefficiency in their tax collection mechanisms). This lower number of individuals working would be a consequence of both, taxes and the pro leisure culture that taxes would induce.

In addition, the model can also be use for policy evaluation: For example, [Alesina et al. , 2007] raised the issue of gender based taxation, essentially on efficient taxation grounds. They contend that, given the different labor supply elasticities of male and female workers, marginal tax rates should be different for this two groups in order to satisfy the Ramsey criterion. They argue that as a by product of achieving efficiency, gender based taxation would help close the gender income gap, because it would induce women (men) to work longer (shorter) hours. Obviously, the gender wage gap would also reduce on an after tax base, but the gross (pre-tax) gender wage gap would not be affected.

Our model suggests that such gender based taxation will have implications on quit rates which, in turn, will impact not only net, but gross wages. These 'induced' changes will reinforce the effect of the differential tax rates, provided the taxed and subsidized populations are chosen carefully. In particular, a small tax on old working males would, in addition to reducing their net wages, induce an increase in their quit rate, which, in turn, would further reduce their pre-tax expected present value of wages. If the proceeds of the tax were distributed among old working women, their net income would increase; this would cause their quit rate to decrease, and this lower quit rate would lead to an increase in their pre-tax expected present value of wages. Note that, although in [Alesina et al. (2007] wages themselves are unaffected, in our model wages change in response to changes in quit rates. What does change in [Alesina et al. (2007] are the labor supply elasticities of male and female workers and they suggest that in the event of those elasticities converging, there would disappear the rationale for a differential tax treatment of men and women. In a similar vein, our results indicate that gender based taxation can be used to speed the convergence of quit rates. A formal treatment of this problem can be found in Appendix A.4.

10 Robustness to alternative preference transmission mechanisms

We wish to show that, with slight modifications, our model can accommodate most of the transmission mechanisms developed in the cultural transmission literature, and that it can encompass several preference dynamics. To that end, we distinguish two different approaches to preference transmission.

The first approach is centered around the idea that individuals have some cultural traits that they intend to transmit to their offspring, even though transmission is costly. A prominent example is the cultural transmission mechanism formalized in [Bisin and Verdier ,2001], where parents have some cultural trait that can be transmitted, and care about their children welfare but have imperfect empathy.[76] This mechanism can be embedded in our setting by identifying the cultural trait as the marginal value of leisure. Let an individual born at time t know q_{t+1}. The individual chooses s_t in order to solve her problem, and at the beginning of period $t+1$, her marginal valuation of labor is revealed. At each moment in time, there will be two cultural groups of adults (indexed by $v \in \{0,1\}$), one having marginal value of leisure $\lambda^i_{t+1} = 0$ (a proportion $q^0_{t+1} = q_{t+1}$ of the adults) and one having a marginal value of leisure given by $\lambda^i_{t+1} \in (0, \Lambda)$ (a proportion $q^1_{t+1} = 1 - q_{t+1}$ of adults). Thus, the cultural trait corresponds to whether individuals value leisure positively or not.[77] Suppose that each individual born at time t has a child at the beginning of period $t+1$. The child is socialized as follows: with probability $\xi^v = \xi(e^v_{t+1}, q^v_{t+1})$ depending on effort e^v_{t+1} made by

[76] This means that parents evaluate their kids welfare with their own utility function, evaluated at their kids expected choice. The relaxation of the rationality hypothesis in this case consists of both not being able to fully account for the children utility and of not taking into account the cost that children will have to bear in order to socialize their own children.

[77] To simplify, we assume that parents with strictly positive marginal valuation of leisure just care about their children also having strictly positive marginal valuation of leisure, and not about the magnitude of such valuation. An interpretation of this is that society is composed of two groups: one that has a strong attachment to the labor market (committed to work under any circumstances related to health, childs, independent of the wage offered) and one that is is willing to consider the trade-off between wage income and personal circumstances when deciding whether to work or not, and thus takes into account the level of wages to make a decision. It is possible to include in the model a continuum of cultural traits, each of which is a particular magnitude of the marginal valuation of leisure, but this won't be pursued here.

her parent with trait v and on the current share of population with the parents traits, the child inherits the same trait (vertical transmission), and with probability $(1-\xi^v)$ the child is randomly matched with some individual of the society at large (oblique transmission) and inherits the trait of that individual. In this case, the transition probability of the kid receiving the same trait of her parent, for group v, is given by:

$$\Xi^{v,v} = \xi^v + (1-\xi^v)q_{t+1}^v, \tag{13}$$

while the transition probability of the kid receiving a trait different from that of her parent is:

$$\Xi^{v,-v} = (1-\xi^v)q_{t+1}^{-v} \tag{14}$$

Analogous expressions hold for parents with trait $v=1$. As in [Bisin and Verdier ,2001], we assume that effort for direct socialization is costly in terms of utility, and that parents evaluate their daughters future choices through their own utility function. Let the value for a parent with cultural trait v of having a child with the same trait be V^{vv} and the value of having a child with different trait be $V^{v,-v}$. Under the previous assumptions, parents display a preference for socializing to their own cultural trait ($V^{v,v} > V^{v,-v}$). Assume that the effort can be exerted irrespective of whether the parent works or not in period $t+1$ and let $C(e_{t+1}^v)$ be the effort cost of direct transmission, which is differentiable, strictly increasing, strictly quasi-convex in effort, and satisfies $C(0)=0$, $\left.\frac{\partial C(e_{t+1}^v)}{\partial e_{t+1}^v}\right|_0 = 0$. Moreover, the direct socialization probability, $\xi^v = \xi(e_{t+1}^v, q_{t+1}^v)$ is assumed to be differentiable, strictly increasing and strictly quasi-concave in effort, with $\xi(0, q_{t+1}^v) = 0$. Under our assumptions, it is clear that the effort choice does not interact with the choice of savings and, hence, the individual can solve (4) in the same manner as before. What is new is that, at time $t+1$, once her type is revealed[78], the individual will

[78] Recall that the socialization effort does not interact with the savings decision. Hence, it it

have to choose the socialization effort according to

$$\max\left[\Xi^{vv}V^{v,v}+\Xi^{v,-v}V^{v,-v}-C(e_{t+1}^{v})\right]$$
$$s.t.\ 13,14.$$

In turn, e_{t+1}^{v} can be used to obtain $\xi^{v}(e_{t+1}^{v},q_{t+1}^{v})$. Given this value, and aggregating over all individuals with type v, we obtain the law of motion for q_{t+1}:

$$q_{t+2}=\pi(q_{t+1},p_{t+1})=q_{t+1}+(1-q_{t+1})(\xi^{0}-\xi^{1})$$

Once q_{t+2} is known, individuals born at time $t+1$ make their own choice regarding s_{t+1}. It can be shown that this law of motion satisfies the conditions for the existence of a steady state of proposition 4, without having to impose that direct and oblique socialization are substitutes in order to ensure that the steady state will display cultural heterogeneity.

Moreover, although [Bisin and Verdier , 2001] do not specify the specific means by which a child is influenced either by her parent or by a member of society at large, it seems clear that, in their model, socialization is the result of particular matching with some individual. In addition to this matching, our framework can also accommodate the influence of the actual behaviour of the society at large. It is clear that all parents who receive $\lambda_{t+1}^{i}=0$ (a measure q_{t+1}) whish to transmit a strong work ethic to their children. The remaining $1-q_{t+1}$ are trying to convey to theirs the idea that leisure is a gratifying activity. However, not all of these are in fact enjoying leisure, as some of them (a measure $(p_{t+1}-q_{t+1})$) have chosen to work instead because of the economic conditions.

The second approach to preference transmission is based on the idea that the parent trait or predisposition to certain behaviour is inherited by her child, but no explicit effort is made by parents to transmit it, and the ensuing dynamics are

clear that we could inlcude the decision regarding e_{t+1}^{v} in the consumer problem in (4). However, so doing would only result in a change in the timing of the decision.

an outcome of the aggregation of the individual decisions made by the members of each group. The most prominent class of models in this approach is that of evolutionary game theory, where fitness is the guiding principle for the population dynamics.[79] This could be translated to our setting by introducing sexual reproduction through a random matching process and allowing parents with different traits to choose the number of offspring they wish to have, while assuming that they make no explicit effort to transmit a particular trait. It differs from the cultural transmission strand in that parents do not necessarily derive utility from their offspring sharing their values.[80] Since all agents survive for the same number of periods, we choose our fitness criterion to be the maximization of the fraction of the population sharing a particular trait. Let parents born at t and with revealed trait v choose the number of children they wish to have (n_{t+1}^{v}) after their type is revealed. Suppose that the cost (in utility) of raising offspring is given by $C(n_{t+1}^{v})$, a function sharing the same properties as the cost of effort. In order to to concentrate on the features of transmission related only with fitness, we assume that there is no impact of socialization and that individuals are matched randomly with an individual of the population at large in order to have children. If the individuals share the same trait, their children will inherit the same trait for sure. If individuals have different traits, their children will inherit each trait with probability $\frac{1}{2}$, and the decision with respect to the number of children while be made by the wife in half the cases. Note that now the population can grow and the law of motion for q_t should reflect that. Defining the ratio of descendants across groups $N_{t+1} = \frac{n_{t+1}^{0}}{n_{t+1}^{1}}$ and using the same notation as before for the value of a child for a parent, the problem solved by parents of type v is then:

[79] See [Vega-Redondo, 1996], [Weibull, 1995] for an introduction to evolutionary game theory.

[80] That is, we do not assume imperfect empathy, and so, the ranking of V^{vv} and $V^{v,-v}$ from the point of view of the parent depends on the economic conditions expected during their children life.

$$\max_{\{n^v_{t+1}\}} n^v_{t+1}\left[\frac{1}{2}V^{v,v}+\frac{1}{2}V^{v,-v}\right]-C(n^v_{t+1}) \tag{15}$$

Under the previous assumptions, this is a convex problem that can be solved for $n^v_{t+1}(q^v_{t+1})$. From this, we can obtain the law of motion for q_{t+2}:

$$q_{t+2}=\pi(q_{t+1},p_{t+1})=\frac{q_{t+1}\left[(N_{t+1}+1)-q_{t+1}N_{t+1}\right]}{\left[q_{t+1}(1-N_{t+1})+N_{t+1}\right]}$$

Note that this is indeed a probability. However, it does not satisfy the conditions for proposition 4. This is not surprising since, as discussed in [Bisin and Verdier, 2001], this class of mechanisms might not be able to generate an heterogeneous stable long run distribution of cultural traits, which is at odds with evidence. However, even if we cannot ensure its convergence or independence from its initial value, the evolution of q_{t+2} is still linked to economic factors (namely, productivity), preserving our main result, i.e., that culture is affected by economic fundamentals in the long run.

In addition to the evolutionary class of models, the second approach to preference transmission includes two other types of models which we are able to embed in our setting. First, those in the vein of [Fernández , 2007], where individuals receive from the previous generation a prior with respect to the disutility of work and update that prior based on some private signal. This can be incorporated in our model by taking q_t to be the inherited prior, while the fact that her parent did work or not could constitute the private signal. Second, those based on cultural transmission by aspirations, à la [De la Croix, 1996], can be incorporated rather simply in our model by choosing $\pi\left(q_{t-1},p_{t-1}\right)=\alpha q_{t-1}+(1-\alpha)\,\beta p_{t-1}$ where α describes the degree of aspiration with respect to the past generation explicit norms and β stands for the degree of aspiration with respect to the past generation actual economic behavior.[81] It is worth noting that, in models of aspirations,

[81] It should be clear that we need $\alpha, \beta < 1$.

the utility the younger generation obtains from consumption in the first period of their lives usually depends on the consumption experience of the previous generation when young. In our model, it is the distribution of the marginal valuation of leisure what is endogenous and determines the probability of working and the expected consumption of leisure. Of course, to the extent that this probability is influenced by the behavior of the previous generation, aspirations are reflected in it. And, because the probability of working is involved, these aspirations also have an impact on the utility derived from the consumption of other goods.

11 Summary and conclusions

In recent years, we have witnessed a rapid and steady growth in papers dealing with the relationship between culture and economics, and in particular, with the influence of culture on preferences. Several strands of this literature can be identified, each of dealing with a particular aspect of the problem. There are papers that document (but do not attempt to model) changes in preferences stemming from changes in either the cultural environment or the economic fundamentals. There are also papers that emphasize the importance of the cultural background in explaining differences in behavior, but ignore what caused those diverse preferences. Finally, and somewhat independently from the previous literature, there is a number of papers dealing with the mechanism of intergenerational transmission of preferences. We build a model that integrates all these (to some extent partial) analysis. Individual preferences are shaped by culture, which, in turn, evolves through time in response to changes in the economic fundamentals.

In particular, and in keeping with a large part of this literature, we analyze the labor market participation rate, and we do so through the quit to non participation decision. Individuals may exit the labor force in the second period of their lives (thus determining the labor market participation rate) and, in taking this decision, they weight their marginal valuation of leisure, whose distribution reflects the inherited culture, against the economic incentives of working. The comparison is not trivial: as numerous empirical studies have shown, quit rates and wage rates depend on each other, and the model reflects it. In addition, it must be noted that even if the present value of the product of labor was fixed, its time distribution (the wages paid to young and old members of the same generation) would be not, and we impose that competition among firms leads them to offer whatever wage schedule workers prefer. Hence, at each time, the labor participation rate, and the wage schedule need to be determined simultaneously. The following generation observes the behavior with respect to labor market participation of its predecessors and updates its beliefs about the acceptability of quitting, thus modifying the distribution of the marginal valuation of leisure, i.e., the culture. This updating is done in a manner compatible with the mechanisms of intergenerational transmission of preferences most frequent in the literature. Culture, therefore, evolves through the generations and ends up reflecting the current state of economic fundamentals. In the transition, however, both past and current fundamentals are relevant in the determination of equilibrium values. This mechanism helps explain why it is possible to observe differences in the present expected value of wages of workers even when they are equally productive, or why individuals in countries with higher taxes seem to develop a preference for leisure.

12 Specific bibliography for essay 2.

References

[Abel, 1990] Abel, A., "Asset Prices under Habit Formation and Catching up with the Joneses," *The American Economic Review* 80 Papers and Proceedings of the Hundred and Second Annual Meeting of the American Economic Association (May 1990), 38-42

[Alesina and Fuchs-Schünden, 2007] Alesina, A., and N. Fuchs-Schündeln "Goodbye Lenin (or Not?): The Effect of Communism on People's Preferences," *The American Economic Review* 97 (September 2007) 1507–1528.

[Alesina et al., 2010] Alesina, A., P. Giuliano, and N. Nunn, "*The Origins of Gender Roles: Women and the Plough*," mimeo, Harvard University, 2010.

[Alesina et al., 2007,b] Alesina, A., A. Ichino and L. Karabarbounis, "*Gender Based Taxation and the Division of Family Chores*," NBER Working Paper No. 13638, November 2007.

[Alós-Ferrer, 2002] Alós-Ferrer, C., "*Individual Randomness in Economic Models with a Continuum of Agents,"* mimeo, Universität Konstanz, 2002

[Becker and Murphy, 1988] Becker, G. S. and K. M. Murphy, "A Theory of Rational Addiction," *Journal of Political Economy*, 96 (August 1988), 675-700.

[Bisin and Verdier, 2001] Bisin, A. and T. Verdier, "The economics of cultural transmission and the dynamics of preferences," *Journal of Economic Theory* 97 (April 2001) 298–319.

[Blanchard, 2004] Blanchard, O., "The Economic Future of Europe," *Journal of Economic Perspectives* 18 (Fall 2004), 3-26.

[Boserup, 1970] Boserup, E., *Woman's Role in Economic Development*. (London: George Allen and Unwin, 1970).

[Bowles, 1998] Bowles, S., "Endogenous Preferences: The Cultural Consequences of Markets and Other Economic Institutions," *Journal of Economic Literature* 36 (March 1998), 75-111.

[Brown et al., 2008] Brown, A.J.G., J.M. Orszag and D.J. Snower, "Unemployment Accounts and Employment Incentives," *European Journal of Political Economy* 24, (September 2008), 587-604.

[Constantinides, 1990] Constantinides, G. M., "Habit Formation: A Resolution of the Equity Premium Puzzle," *The Journal of Political Economy*, 98 (Jun., 1990), 519-543.

[De la Croix, 1996] de la Croix, D., "The dynamics of bequeathed tastes," *Economics Letters* 51 (October 1996), 89-96.

[Dekel, 2007] Dekel, E., J.C. Ely and O.Yilankaya, "Evolution of Preferences," *Review of Economic Studies* 74, (July, 2007), 685-704.

[Diamond, 1965] Diamond, P., "National Debt in a Neoclassical Growth Model," *American Economic Review* 55 (December 1965), 1126-1150.

[Doepke and Zilibotti, 2008] Doepke, M. and Zilibotti, F. "Occupational Choice and the Spirit of Capitalism," *The Quarterly Journal of Economics*, 123 (May, 2008), 747-793.

[Duesenberry, 1949] Duesenberry, J. S., *Income, Saving, and the Theory of Consumer Behavior.* (Cambridge, Mass.: Harvard Univ. Press, 1949).

[Erosa et al., 2005] Erosa, A., L. Fuster and D. Restuccia, "A Quantitative Theory of the Gender Gap in Wages," University of Toronto Working Papers tecipa-199, September 2005.

[Escriche, 2007] Escriche, L., "Persistence of occupational segregation: The role of intergenerational transmission of preferences," *The Economic Journal* 117 (April 2007), 837–857.

[Escriche et al., 2004] Escriche, L. G. Olcina and R. Sánchez, "Gender discrimination and intergenerational transmission of preferences," *Oxford Economic Papers* 56 (July 2004), 485-511.

[Eugster et al., 2011] Eugster, B., Lalive, R., A. Steinhauer and J. Zweimüller, "The Demand for Social Insurance: Does Culture Matter?," *The Economic Journal,* 121 (November 2011), F413-F447.

[Feldman and Gilles, 1985] Feldman, M. and C. Gilles, "An Expository Note on Individual Risk without Aggregate Uncertainty," *Journal of Economic Theory* 35 (February 1985), 26-32.

[Fehr and Hoff, 2011] Fehr E. and K. Hoff, "Introduction: Tastes, Castes and Culture: The Influence of Society on Preferences," *The Economic Journal*, 121 (November 2011), F396-F411.

[Fernández, 2007] Fernández, R., "Culture as Learning: The Evolution of Female Labor Force Participation over a Century," NBER Working Paper No. 13373, September 2007.

[Fernández, 2010] Fernández, R., "Does culture matter?," CEPR, Discussion Paper No. 7965, (August 2010).

[Fernández, 2004] Fernández, R., Fogli, A. and C. Olivetti, "Mothers and Sons: Preference Formation and Female Labor Force Dynamics," *The Quarterly Journal of Economics* 119 (November, 2004), 1249-1299

[Fershtman and Heifetz, 2006] Fershtman C. and A. Heifetz, "Read My Lips, Watch for Leaps: Preference Equilibrium and Political Instability," *The Economic Journal*, 116 (January, 2006), 246-265.

[Galor, 1996] Galor O. and D.N. Weil, "The Gender Gap, Fertility, and Growth," *The American Economic Review* 86 (June 1996), 374-87.

[Guiso et al., 2008] Guiso, L., P. Sapienza and L. Zingales, "Long Term Persistence," NBER working paper 14278, August 2008.

[Hauk, 2002] Hauk, E. and M. Saez-Marti, "On the Cultural Transmission of Corruption," *Journal of Economic Theory* 107 (December 2002), 311–335.

[Lazear and Rosen, 1990] Lazear, E.P. and S. Rosen, "Male-Female Wage Differentials in Job Ladders," *Journal of Labor Economics* 8 (January 1990), S106-S123.

[Lindbeck and Nyberg, 2006] Lindbeck, A. and S. Nyberg, "Raising Children to Work Hard: Altruism, Work Norms and Social Insurance," *The Quarterly Journal of Economics*, 121, (November 2006), 1473-1503.

[Marshall, 1920] Marshall, A., *Principles of Economics: An Introductory Volume*. 8th ed. (London: Macmillan, 1920).

[Nunn and Wantchekon, 2011] Nunn, N. and L. Wantchekon, "The Slave Trade and the Origins of Mistrust in Africa," *The American Economic Review* 101, (December 2011), 3221-3252.

[Oi, 1962] Oi, W.Y., "Labor as a Quasi-Fixed Factor," *The Journal of Political Economy* 70 (December 1962), 538-555.

[Ostrom, 2000] Ostrom, E., "Collective Action and the Evolution of Social Norms," The Journal of Economic Perspectives 14 (Summer, 2000), 137-158.

[Palacios, 2004] Palacios Huerta, I. and T.J: Santos, "A theory of markets, institutions, and endogenous preferences," *Journal of Public Economics,* 88 (March 2004), 601– 627.

[Prescott, 2004] Prescott, E.C., "Why Do Americans Work So Much More Than Europeans?," *Federal*

Reserve Bank of Minneapolis Quarterly Review 28 (July 2004), 2–13.

[Robson, 2001] Robson, A.J., “The Biological Basis of Economic Behavior,” The Journal of Economic Literature 39 (March 2001), 11-33.

[Ross, 2008] Ross, M.L., “Oil, Islam, and Women,” *American Political Science Review* 102 (February 2008), 107-23.

[Stigler and Becker, 1977] Stigler, G.J. and G.S. Becker, “De gustibus non est disputandum,” *The American Economic Review* 67 (March 1977), 76-90.

[Stokey, 1988] Stokey, N. and R. Lucas, *Recursive methods in Economic dynamics*, With Edward C. Prescott (Cambridge, Mass. and London: Harvard University Press, 1989)

13 Appendix

13.1 Proof of proposition 2. Existence and uniqueness of the equilibrium within a generation.

Let $q_t = q \in [0, 1]\,, \forall t$ be given. As we are considering each generation separately, we will only keep the time subscript to distinguish wages earned when young from those received when old. In particular, c will denote young age consumption while old age consumption will be either $\overline{c}$ or $\underline{c}$ in the event of working or not working when old respectively, while p will denote the probability of working when old. Let $w_t^{\max}$ be the maximum wage the firm can offer to young workers when w_{t+1} is set equal to its minimum value and let $w_{t+1}^{\max}$ be the maximum wage the firm can offer to old workers when w_t is set equal to its minimum value, $w_t^{\min}$[82]. As optimal savings have been shown to depend positively on the current wage and negatively on the (expected) future wage, the maximum amount saved will be $\overline{s} = S\left(w_t^{\max}, \overline{w} - \kappa\right)$, whereas minimum savings are positive and given by $\underline{s} = S\left(w_t^{\min}, w_{t+1}^{\max}\right)$.

Recall that we defined $\underline{\beta}, \underline{W}, \underline{r} > 0$ as the minimum values $\beta, \overline{w}$, and r can take. Our initial parameter space is, then $\widehat{\Theta} \subset \left[\underline{\beta}, 1\right) \times [0, 1] \times (0, 1] \times \left[\overline{W}, \infty\right) \times [\underline{r}, \infty) \times \mathbb{R}_{++}$, with typical element $\widehat{\theta} = (\beta, q, \kappa, \overline{w}, r, f(\lambda_m))\,,$ where $\lambda_m = u\left(\overline{w}(1 + r) - \kappa\right) - u\left(r\overline{w}\right)$[83]. Let M denote the normalized value of the upper bound of the density function at λ_m, whose precise value will be given later. Also note that, given the definition of the parameter space and the consumer problem, we can

[82]As will be seen below, firms will never offer $w_t^{\min} = 0$, since no worker would choose that offer.

[83]Since $\widehat{\lambda} = u(\overline{c}) - u(\underline{c})$ is increasing in w_{t+1} and decreasing in s_t , the minimum value it can achieve in the set $\Omega(s)$ defined below is λ_M. Moreover, since $\frac{\partial f(\lambda)}{\partial \lambda} < 0$, by assumption, any upper bound for $f(\lambda_M)$ is also an upper bound for $f(\lambda)$ in the set $\Omega(s)$

obtain $\inf_{\theta\in\Theta} \underline{s} = \underline{\underline{s}}$[84], which can be taken as a parameter, as it can always be computed and is independent of the values of the other parameters.[85] Also, we can define maximum probability $\overline{p} = q + (1-q)\, F\left(u\left(\overline{w} + r\underline{\underline{s}}\right) - u\left(r\underline{\underline{s}}\right)\right)$ Hence, we take our enlarged parameter space as $\Theta = \widehat{\Theta} \times \mathbb{R}_{++}$, with typical element $\theta = \left(\beta, q, \kappa, \overline{w}, r, f(\lambda_m), R_A\left(r\underline{\underline{s}}\right)\right)$. Finally, let $\Lambda = u\left(\overline{w} + r\underline{\underline{s}}\right) - u\left(r\underline{\underline{s}}\right)$ be the maximum value that $\widehat{\lambda}$ can attain (that this is indeed a maximum can be seen from the definition of $\underline{\underline{s}}$ given above and the fact that $\widehat{\lambda}$ is decreasing in s). Before stating the proof, let us highlight that condition (ii) holds true, for example, for any rescaled exponential density, say $f(\lambda) = J_1 e^{-J_2\lambda}$, $J_1, J_2 > 0$, where both parameters are chosen appropriately.

The strategy of the proof is based on solving first a problem that determines the firm's optimal wage offer under equilibrium conditions, for every given possible individual choice of savings. Then, we proceed to find that choice of savings that is a best response to itself in the sense that it is the optimal choice for the individual when presented with wages given precisely by the optimal wage offer of the firm for that savings level. That savings level determines an equilibrium.

The proof is organized in several steps, that are numbered to enhance readability:

1. We determine the optimal solution for the firm in equilibrium

    (a) Since the equilibrium level savings should allow firms to make non-negative profits, pay wages above the given thresholds and individuals to obtain strictly positive consumption when young, we compute the maximum savings $\widehat{s}_0$ for which this is possible.

[84] *This includes the case for* $\underline{\underline{s}} = 0$

[85] This can be seen by noting that for the case $\beta = \underline{\beta}, \overline{w} = \overline{W}, r = \underline{r}, q = 0, f(\lambda_M) = 0$, $\underline{s}$ is still defined by the FOC of the consumer and is strictly positive.

(b) For each possible $0 < s < \widehat{s}_0$, we define an ancillary problem, $V(s)$, that characterizes the firms optimizing behaviour in any equilibrium. This problem consists in taking as given the individual savings and choosing the optimal wage schedule that maximizes the individual utility subject to obtaining non-negative profits and satisfying the lower bounds for both wages.

2. We define $g(s)$ as the FOC of the individual when wages are given by $w(s)$. Observe that a zero of $g(s)$ completely characterizes optimizing behaviour by the individual when firms optimize in equilibrium. Therefore, we define an equilibrium as that level of savings, s^*, such that $g(s^*) = 0$,. We prove that s^* exists and that it must satisfy $\underline{s} \leq s^* \leq \overline{s}$.

3. To guarantee uniqueness, we ensure that $g(s)$ is decreasing around s^* for any $s^* \in (\underline{s}, \overline{s})$.

   (a) We write the condition $\frac{dg(s^*)}{ds} < 0$ as an inequality of the form $A > f\left(\widehat{\lambda}\right) B$.

   (b) We specify an upper bound on $f(\lambda_m)$ involving only primitives that guarantees that $g(s)$ is decreasing around s^*. Since the assumptions guarantee that the primitives satisfy such a uniform upper bound, this completes the proof.

13.1.1 Firms optimal behaviour in equilibrium

1.a It is clear that if firms take savings as exogenous, they cannot achieve non negative profits for all savings levels, because the non negativity of consumption in the first period requires that $w_t \geq s$. We need to determine that upper bound. Imagine firms offered $w_{t+1} = \overline{w} - \kappa$, the lowest possible amount. This would enable them to pay $w_t^{\max}$, the largest possible amount in t (because s is given) that produces zero profits. Obviously, the largest savings compatible with this would be $\widehat{s}_0 = w_t^{\max}$ and this value is determined by :

$$a\left(\widehat{s}_0\right) = \overline{w}\left[\kappa\left(\frac{q+(1-q)\,F\left(u\left(\overline{w}-\kappa+\widehat{s}_0 r\right)-u\left(\widehat{s}_0 r\right)\right)}{r}\right)\right]-\left(\widehat{s}_0+\kappa\right)=0.$$

Note that the existence of $\widehat{s}_0$ is guaranteed by the fact that the above function is monotonic in s. Moreover, note that such $\widehat{s}_0$ could never be optimal for the individual, since her first period marginal utility of consumption would be infinite. Hence, since $\overline{s} = S\left(w_t^{\max}, \overline{w}-\kappa\right)$ is the optimal response of the individual when facing the wage schedule $\left(w_t^{\max}, \overline{w}-\kappa\right)$, it follows that $\overline{s} < \widehat{s}_0$.

1.b Given $s \in (0, \widehat{s}_0)$, let

$$\begin{aligned} V\left(s\right) \;=\; & \max_{\{w_t, w_{t+1}\}\in A} E_{f(\lambda_i)}\left[U_i\left(c_t, c_{t+1}, L_{t+1}\right)\right] \\ & s.t. \\ & \left(\overline{w}\left(1+\tfrac{p}{r}\right)-w_t-\kappa-w_{t+1}\tfrac{p}{r}\geq 0\right) \quad (\nu_1) && (16)\\ & \left(w_{t+1}-\overline{w}+\kappa\geq 0\right) \quad (\nu_3) && (17)\\ & \left(w_t - s\geq 0\right) \quad (\nu_4) && (18)\end{aligned}$$

be the problem whose solution gives the optimal behaviour of the firm in equilibrium, where $A = (0, +\infty) \times (0, \overline{w}+1)$ is open and ν_i denote the corresponding Kuhn-Tucker multipliers. It can be shown that $V\left(s\right)$ is well defined, has a unique solution completely characterized by the Kuhn-Tucker conditions provided

$$\beta\frac{F\left(\lambda_m\right)}{\Lambda}\left(\overline{w}-\kappa\right)R_A\left(\overline{w}\left(1+r\right)\right)=M_1>f\left(\lambda_m\right). \tag{19}$$

13.1.2 Equilibrium savings: existence

Now, in order to ensure that the individual chooses s through maximizing behavior, we construct the following function

$$g\left(s\right)=-u'\left(w_t\left(s\right)-s\right)+\beta r\left[pu'\left(w_{t+1}\left(s\right)+sr\right)+\left(1-p\right)u'\left(sr\right)\right].$$

which is simply the FOC of the consumer evaluated at s when wages are given by $w(s)$ Under the assumptions, $\lim_{s\to 0^+} g(s) = +\infty$, $\lim_{s\to \widehat{s}_0} g(s) = -\infty$, (since $\lim_{s\to \widehat{s}_0} w_t(s) = s$) and the continuity of $g(s)$ ensures that there exists $s^* \in (0, \widehat{s}_0)$ such that $g(s^*) = 0$. Note that such s^* determines an equilibrium for this economy, as defined in the proposition. Moreover, $s^* \in [\underline{s}, \overline{s}]$. To see it, recall that s^* being an equilibrium implies that it is an optimal response of the consumer to $w(s^*)$, that is $s^* = S(w_t(s^*), w_{t+1}(s^*))$ and $w(s^*)$ being a solution to $V(s)$ implies that $w_t^{\min} \leq w_t(s^*) \leq w_t^{\max}$, $(\overline{w} - \kappa) \leq w_{t+1}(s^*) \leq w_{t+1}^{\max}$. Then, as $S(w_t, w_{t+1})$ has been shown to be increasing in w_t. and decreasing in w_{t+1}, we have that $\underline{s} = S\left(w_t^{\min}, w_{t+1}^{\max}\right) \leq s^* = S(w_t(s^*), w_{t+1}(s^*)) \leq \overline{s} = S(w_t^{\max}, (\overline{w} - \kappa))$.

13.1.3 Equilibrium savings: uniqueness

3.a In order to ensure the uniqueness of this equilibrium, it is enough to show that for any s^* in the open interval $(\underline{s}, \overline{s})$, $\frac{dg(s^*)}{ds} < 0$. Note that for any such s^*, (17) and (18) must not be binding (so that $\nu_3 = \nu_4 = 0$) while (16) must hold with equality. This can be seen as follows: as shown above, (16) must hold with equality and (18) is an strict inequality at any solution of the $V(s)$ problem. For (17), note that if $w_{t+1} = (\overline{w} - \kappa)$, (16) implies that $w_t = w_t^{\max}$, which is precluded by the fact that we are considering $s^* < \overline{s} = S(w_t^{\max}, (\overline{w} - \kappa))$. Hence, (17) must not be binding at any $s^* < \overline{s}$. It can be shown that $w(s)$ is a continuously differentiable function around any equilibrium savings, s^*.

Let $c^* = w_t - s^*$, $\overline{c}^* = w_{t+1} + s^* r$, $\underline{c}^* = s^* r$ and write

$$\begin{aligned}
\frac{dg(s^*)}{ds} &= -u''(c^*)\left[\frac{dw_t(s^*)}{ds} - 1\right] \qquad (20)\\
&\quad +\beta r(1-q) f\left(\widehat{\lambda}\right)\left[\frac{\partial \widehat{\lambda}}{\partial s} + u'(\overline{c}^*)\frac{dw_{t+1}(s^*)}{ds}\right]\frac{\partial \widehat{\lambda}}{\partial s}\frac{1}{r}\\
&\quad +\beta r\left[pu''(\overline{c}^*)\left(\frac{dw_{t+1}(s^*)}{ds} + r\right) + (1-p) u''(\underline{c}^*) r\right].
\end{aligned}$$

We want to show that $g(s)$ is decreasing around s^*, and we will prove the following stronger condition[86]

$$\frac{dg(s^*)}{ds} - \beta r^2 (1-p) u''(\underline{c}^*) < 0. \tag{21}$$

Changes in savings can be shown to affect optimal wages as follows

$$\begin{aligned}
\left.\frac{dw_{t+1}}{ds}\right|_{s=s^*} &= \frac{u''(c^*) b(1+rb)}{C_1 + b^2 u''(c^*)} - \frac{rC_2 + u''(c^*) rb^2}{C_1 + b^2 u''(c^*)} \\
&\quad - \frac{u''(c^*) \frac{b}{r} (\overline{w} - w_{t+1})(1-q)\left[u'(\overline{c}^*) - u'(\underline{c}^*)\right]}{C_1 + b^2 u''(c^*)} f\left(\widehat{\lambda}\right) \\
&= W_1 + W_2 + W_3 f\left(\widehat{\lambda}\right),
\end{aligned}$$

where $b = \frac{1}{r}\left[(\overline{w} - w_{t+1})(1-q) f\left(\widehat{\lambda}\right) u'(\overline{c}^*) - p\right]$, C_1 and C_2 are given by

$$\begin{aligned}
C_1 &= \beta p u''(\overline{c}^*) + (1-q)\beta f\left(\widehat{\lambda}\right)\left(u'(\overline{c}^*)\right)^2 \\
&\quad + (1-q)\frac{\nu_1}{r}\left\{(\overline{w} - w_{t+1})\left[f'\left(\widehat{\lambda}\right) u'(\overline{c}^*)^2 + f\left(\widehat{\lambda}\right) u''(\overline{c}^*)\right] - 2f\left(\widehat{\lambda}\right) u'(\overline{c}^*)\right\} \\
C_2 &= \beta p u''(\overline{c}^*) + (1-q)\beta f\left(\widehat{\lambda}\right)\frac{\partial\widehat{\lambda}}{\partial s}\frac{1}{r} u'(\overline{c}^*) \\
&\quad + (1-q)\frac{\nu_1}{r}\left\{(\overline{w} - w_{t+1})\left[f'\left(\widehat{\lambda}\right)\frac{\partial\widehat{\lambda}}{\partial s}\frac{1}{r} u'(\overline{c}^*) + f\left(\widehat{\lambda}\right) u''(\overline{c}^*)\right] - f\left(\widehat{\lambda}\right)\frac{\partial\widehat{\lambda}}{\partial s}\frac{1}{r}\right\},
\end{aligned}$$

and the definitions of W_i are obvious from the expression above. Note that $W_1, W_3 < 0$ and that both are bounded[87]. Let $\widehat{W_1} = \inf_{\theta\in\Theta} W_1$, and $\widehat{W_3} = \inf_{\theta\in\Theta} W_3$. Also

$$\begin{aligned}
\left.\frac{dw_t}{ds}\right|_{s=s^*} &= -\frac{brC_2 - u''(c^*) b^2}{C_1 + b^2 u''(c^*)} + \frac{C_1 (\overline{w} - w_{t+1})(1-q)\frac{\partial\widehat{\lambda}}{\partial s}\frac{1}{r}}{C_1 + b^2 u''(c^*)} f\left(\widehat{\lambda}\right) \\
&= A_1 + A_2 f\left(\widehat{\lambda}\right)
\end{aligned}$$

where, again, the definitions of A_i are obvious. Note that $A_2 < 0$. Using these

[86] This condition will be useful in Appendix A.3

[87] It is easily seen that these bounds involve only parameters.

values, we can now write $\frac{dg(s^*)}{ds}$ as

$$\begin{aligned}\frac{dg\left(s^*\right)}{ds} &= u^{''}\left(c^*\right)\left[1-A_1\right]+\beta r^2\left[pu^{''}\left(\overline{c}^*\right)+\left(1-p\right)u^{''}\left(\underline{c}^*\right)\right]+\beta r\left[W_1+W_2\right]pu^{''}\left(\overline{c}^*\right)\\ &+f\left(\widehat{\lambda}\right)\left\{\left(1-q\right)\beta\left(\frac{\partial\widehat{\lambda}}{\partial s}\right)^2+\beta r\left[W_1+W_2\right]\left(1-q\right)\frac{\partial\widehat{\lambda}}{\partial s}u^{'}\left(\overline{c}^*\right)\right.\\ &\left.+W_3\beta r\left[pu^{''}\left(\overline{c}^*\right)+\left(1-q\right)\frac{\partial\widehat{\lambda}}{\partial s}u^{'}\left(\overline{c}^*\right)\right]-u^{''}\left(c^*\right)A_2\right\}.\end{aligned} \tag{22}$$

Expression (21) can be written, using (22), as

$$\begin{aligned}&-u^{''}\left(c^*\right)\left[1-A_1\right]-\beta r^2pu^{''}\left(\overline{c}^*\right)-\beta r\left[W_1+W_2\right]pu^{''}\left(\overline{c}^*\right)>\\ &\quad f\left(\widehat{\lambda}\right)\left\{\left(1-q\right)\beta r^2\left(\frac{\partial\widehat{\lambda}}{\partial s}\right)^2+\beta r\left[W_1+W_2\right]\left(1-q\right)\frac{\partial\widehat{\lambda}}{\partial s}u^{'}\left(\overline{c}^*\right)\right.\\ &\qquad\left.+\beta rW_3\left[pu^{''}\left(\overline{c}^*\right)+\left(1-q\right)\frac{\partial\widehat{\lambda}}{\partial s}u^{'}\left(\overline{c}^*\right)\right]-u^{''}\left(c^*\right)A_2\right\}.\end{aligned} \tag{23}$$

3.b It can be shown that the LHS of (23) is positive and

$$-u^{''}\left(c^*\right)\left[1-A_1\right]-\beta r^2pu^{''}\left(\overline{c}^*\right)-\beta r\left[W_1+W_2\right]pu^{''}\left(\overline{c}^*\right)>\frac{-u^{''}\left(c^*\right)\left[1-\overline{p}\right]^2}{1+\frac{1}{\beta r}\frac{R_A\left(r\underline{\underline{s}}\right)}{R_A\left(\overline{w}(1+r)\right)}}.$$

Therefore, (23) would follow if either the RHS is negative or if it is positive and

$$\begin{aligned}\frac{-u^{''}\left(c^*\right)\left[1-\overline{p}\right]^2}{1+\frac{1}{\beta r}\frac{R_A\left(r\underline{\underline{s}}\right)}{R_A\left(\overline{w}(1+r)\right)}} &> f\left(\widehat{\lambda}\right)\beta r\left(1-q\right)\left\{r\left(\frac{\partial\widehat{\lambda}}{\partial s}\right)^2+\left[W_1+W_2\right]\frac{\partial\widehat{\lambda}}{\partial s}u^{'}\left(\overline{c}^*\right)\right.\\ &\left.+W_3\left[\frac{pu^{''}\left(\overline{c}^*\right)}{\left(1-q\right)}+\frac{\partial\widehat{\lambda}}{\partial s}u^{'}\left(\overline{c}^*\right)\right]-\frac{u^{''}\left(c^*\right)A_2}{\beta r\left(1-q\right)}\right\}\end{aligned} \tag{24}$$

Now, dividing both sides of (24) by $u^{'}\left(c^*\right)$, dividing and multiplying $\frac{\partial\widehat{\lambda}}{\partial s}$ by $\widehat{\lambda}$ and using the definition of absolute risk aversion, we can rewrite it as

$$\left[\frac{1}{1+\frac{1}{\beta r}\frac{R_A\left(r\underline{\underline{s}}\right)}{R_A\left(\overline{w}(1+r)\right)}}\right]\frac{R_A\left(c^*\right)\left[1-\overline{p}\right]^2}{C_3}>f\left(\widehat{\lambda}\right),$$

where

$$
\begin{aligned}
C_3 &= (1-q)\,\beta r^2 \left\{ \frac{\widehat{\lambda}^2 R_A\left(\widetilde{w}+s^* r\right)^2}{u'\left(c^*\right)} - \left[W_1+W_2\right] \frac{u'\left(\overline{c}^*\right)}{u'\left(c^*\right)} \widehat{\lambda} R_A\left(\widetilde{w}+s^* r\right) \right. \\
&\left. -W_3 \frac{u'\left(\overline{c}^*\right)}{u'\left(c^*\right)} \left[\frac{p}{(1-q)\,r} R_A\left(\overline{c}^*\right) + \widehat{\lambda} R_A\left(\widetilde{w}+s^* r\right) \right] + \frac{R_A\left(c^*\right) A_2}{\beta r^2 (1-q)} \right\}
\end{aligned}
$$

Now, note that:

i) since $c^* < \overline{c}^* < \overline{w}\,(1+r)$, we have that $\frac{u'(\overline{c}^*)}{u'(c^*)} < 1$ and $\frac{u'(\overline{w}(1+r))}{u'(c^*)} < 1$.

ii) It can be shown that $-\beta r\left[W_1+W_2\right] < -\beta r\left[W_1-r\right] < -\beta r\left[\widehat{W}_1-r\right]$ and, by the definitions given above, we have $-W_3 < -\widehat{W}_3$

iii) The term $R_A\left(c^*\right) A_2$ is negative.

iv) λ_m is a lower bound for any equilibrium $\widehat{\lambda}$.

v) Λ is the maximum value that $\widehat{\lambda}$ can attain.

vi) By the definition of $\underline{\underline{s}}$ given above, for any $\widetilde{w} \in (0, w_{t+1})$, s^* such that $g\left(s^*\right)=0$, we have $R_A\left(\widetilde{w}+rs^*\right) < R_A\left(rs^*\right) < R_A\left(r\underline{\underline{s}}\right) < \infty$.

Thus, using the appropriate (either the largest or smallest) values for all these variables in C_3, results in

$$
\overline{C}_3 = R_A\left(r\underline{\underline{s}}\right)(1-q)\,\beta r^2 \left\{ \frac{\Lambda^2}{u'\left(\overline{w}\,(1+r)\right)} R_A\left(r\underline{\underline{s}}\right) - \left[\widehat{W}_1 - r\right]\Lambda - \widehat{W}_3 \left[\frac{1}{(1-q)\,r}+1\right] \right\}
$$

being the largest possible value C_3 can take. Hence, a sufficient, but not necessary, condition for (24) to hold is

$$
\left[\frac{1}{1+\left[\frac{1}{\beta r}\right] \frac{R_A\left(r\underline{\underline{s}}\right)}{R_A\left(\overline{w}(1+r)\right)}} \right] \left[\frac{R_A\left(c^*\right)\left[1-\overline{p}\right]^2}{\overline{C}_3} \right] = M_2 > f\left(\lambda_m\right). \tag{25}
$$

Note that M_2 does not depend on any endogenous object. Now we can define $M = \min\left[M_1, M_2\right]$. Since assumption *(ii)* guarantees that (19) and (25) hold, this completes the proof.[88]

[88] Note that for the case $q=1$, we obtain $p_t=1$. In that case the problem is reduced

13.2 Proof of proposition 3: Equilibrium wages

We will prove the proposition for any given $(q_0, p_0) \in [0,1]^2$. First, let us consider a slightly enlarged parameter space, allowing $\kappa = 0$. Also, let $\Theta = \{\theta \in \mathbb{R}^7_+ : \theta$ is such that Proposition 2 holds$\}$, with typical element $\theta = \left(\beta, q, \kappa, \overline{w}, r, f(\lambda_m), R_A\left(r\underline{\underline{s}}\right)\right)$. Also, let Θ_{-z} denote the projection of Θ in all of the coordinates except for parameter z, with typical element θ_{-z} We will be interested in the projection $\Theta_{-\kappa}$. In order to prove the proposition, we will show that for an open set of parameters, there is always a contract and a corresponding savings level, satisfying the consumer FOC, the zero profit condition, as well as the lower bound for both period wages, such that the individual prefers it to the age contract.

Consider the contract paying the same wage in both periods (and call it the "constant contract"), denoted by $w^*(\theta)$. Let $s(w^*(\theta))$ be the corresponding savings level chosen by the individual, and hence satisfying FOC. Note that both depend on the set of parameters chosen, θ. It can be shown that, for $\kappa \to 0$ there is a unique value $w^*(\theta) > \overline{w} - \kappa$ that implies zero profits when consumers choose their savings level optimally.

Now, let us denote $\triangle U(\kappa, \theta_{-\kappa})$ the difference between the utility level attained with the constant contract and that obtained under the age contract when the individual chooses savings optimally. Note that, since $w^*(\theta)$, and $s(w^*(\theta))$ are continuously differentiable with respect to κ for all $\theta_{-\kappa} \in \Theta_{-\kappa}$, $\triangle U(\kappa, \theta_{-\kappa})$ is continuously differentiable in κ. We will be done if we can show that there exists $0 < \widetilde{\kappa}$ such that, for all $\kappa \in I_{\widetilde{\kappa}} = (0, \widetilde{\kappa})$, $\triangle U(\kappa, \theta_{-\kappa}) > 0$. This would imply that the age contract is not optimal because deviating towards the constant contract would improve individual welfare without violating any of the requirements, and therefore, the age contract would not be chosen for any $\theta \in I = I_{\widetilde{\kappa}} \times \Theta_{-\kappa}$.

to one without uncertainty, and thus our basic assumptions on the utility function ensure the uniqueness of the result.

First, it is clear that

$$\kappa = 0 \Longrightarrow \begin{cases} \triangle U\left(\kappa,\theta_{-\kappa}\right) = 0 \\ w^{*}\left(\theta\right) = \overline{w} \\ s\left(w^{*}\left(\theta\right)\right) = s\left(\overline{w},\overline{w}\right) = s \\ p\left(w^{*}\left(\theta\right), s\left(w^{*}\left(\theta\right)\right)\right) = p\left(\overline{w},s\right) = p \end{cases}.$$

Also, since $s > 0$ solves the consumer problem, it also satisfies the first order condition

$$u'\left(\overline{w}-s\right) = p\beta r u'\left(\overline{w}+rs\right) + \left(1-p\right)\beta r u'\left(rs\right)$$

and this implies that

$$u'\left(\overline{w}-s\right) > \beta r u'\left(\overline{w}+rs\right).$$

Taking the derivative of $\triangle U\left(\kappa,\theta_{-\kappa}\right)$ with respect to κ, using the first order conditions from the consumer problem as well as the implicit differentiation of the equilibrium savings with respect to κ and evaluating at $\kappa = 0$ it can be shown that:

$$\frac{\partial \triangle U\left(\kappa = 0,\theta_{-\kappa}\right)}{\partial\kappa} = u'\left(\overline{w}-s\right) > 0 \qquad \forall\theta_{-\kappa}\in\Theta_{-\kappa}.$$

Since $\frac{\partial\triangle U(\kappa=0,\theta_{-\kappa})}{\partial\kappa}$ is continuous, there exists $\widetilde{\kappa} > 0$ and $I_{\widetilde{\kappa}} = \left(0,\widetilde{\kappa}\right)$ such that:

$$\frac{\partial \triangle U\left(\widetilde{\kappa},\theta_{-\kappa}\right)}{\partial\kappa} > 0, \qquad \forall\left(\kappa,\theta_{-\kappa}\right)\in I = I_{\widetilde{\kappa}}\times\Theta_{-\kappa}, \text{ with } \mu\left(I\right) > 0.$$

This implies that, by doing a Taylor expansion at $\left(\kappa = 0,\theta_{-\kappa}\right)$:

$$\forall\left(\kappa,\theta_{-\kappa}\right)\in I, \qquad \triangle U\left(\kappa,\theta_{-\kappa}\right) = \triangle U\left(0,\theta_{-\kappa}\right) + \frac{\partial \triangle U\left(\overline{\kappa},\theta_{-\kappa}\right)}{\partial\kappa}\kappa > 0$$

for some $\overline{\kappa}\in\left(0,\kappa\right)$. This completes the proof.

The economic rationale for this result is that, given $\overline{w}$, and no hiring costs, an increase in the hiring costs will only decrease the present wage in the age contract while diminishing both the present and expected future wage in the constant wage contract. Hence, savings will decrease in the age contract. In the constant contract, however, there are two opposite effects whose total sign is unclear. Thus, given a sufficiently high risk aversion and variance of the density function of the marginal valuation of leisure, savings in the constant wage contract will react less, allowing a smother consumption path.

Analogously, it can be shown that the proposition is non trivial. The economic reason for this result is that, given relatively high hiring costs, if $\overline{w}$ increases starting from zero, the net increase in the second period wage with the age contract more than compensates for the loss of wage in the first period, and, at the same time, increases the probability of working in the second period. Here, the fact that savings cannot react too much because of the non negativity constraint plays a crucial role.

13.3 Proof of proposition 4. Existence and stability of the steady state equilibrium.

First, note that, given $q_t \in [0,1]$ proposition 2 ensures that there exists a set of parameter values, dependent on the value of q_t such that there is a unique solution for the problem of generation t. Now, we want to strengthen that condition so that the parameter values do not depend on the particular value taken by q_t. Since q_t does not appear in the definition of the other parameters, from the proof of proposition 2, such restriction can be accomplished by defining dynamic restrictions as follows. Recall that (19) and (25) define the values of the upper bounds for $f\left(\widehat{\lambda}\right)$. First note that q_t does not appear in the definition of M_1. Let:

$$
\begin{aligned}
M_{1,}^{*} &= \inf_{q_t \in [0,1]} M_1 = M_1 > 0 \\
M_{2,}^{*} &= \inf_{q_t \in [0,1]} M_2 > 0 \\
M^{*} &= \min\left[M_1^{*}, M_2^{*}\right] > 0.
\end{aligned}
$$

The strict positivity of M_2^* is ensured by the definition of $\pi(q_t, p_t)$ together with the definition of W_1, W_2 and $1 - A_1$ (see Appendix A.1). Also in Appendix A.1 the conditions for $g(s_t)$ to be decreasing around s_t^* were established.

Hence, the definition of M^* together with assumptions $(4.i)$, $(4.ii)$ and $(4.iii)$ ensure that, given $q_t \in [0,1]$, we can apply the proof of proposition 2 to obtain a unique solution to the problem of the generation born at time $t-1$. Note that part of this solution is $p_t = q_t + (1 - q_t) F\left(\widehat{\lambda}_t\right)$, which allows us to write it as $p_t = p_t(q_t)$. Hence, for generation born at time t, it is clear that q_t is the only relevant state variable, and we can write

$$q_{t+1} = \pi(q_t, p_t(q_t)) = \pi(q_t)$$

and note that our previous arguments imply that the solution is differentiable with respect to the state variable q_t. Thus, we can obtain an expression for the differential

$$\frac{dq_{t+1}}{dq_t} = \frac{d\pi\left(q_t\right)}{dq_t} = \frac{\partial\pi\left(q_t, p_t\right)}{\partial q_t} + \frac{\partial\pi\left(q_t, p_t\right)}{\partial p_t}\frac{dp_t}{dq_t}.$$

If we can prove that, for all $(q_{t-1}, p_{t-1}) \in [0,1]^2$, $\frac{dp_t}{dq_t}, \frac{\partial\pi(q_t,p_t)}{\partial q_t}, \frac{\partial\pi(q_t,p_t)}{\partial p_t}$ are such that

$$\sup_{q_t\in[0,1]}\left|\frac{dq_{t+1}}{dq_t}\right| = \sup_{q_t\in[0,1]}\left|\frac{d\pi\left(q_t\right)}{dq_t}\right| = \sup_{q_t\in[0,1]}\left|\frac{\partial\pi\left(q_t, p_t\right)}{\partial q_t} + \frac{\partial\pi\left(q_t, p_t\right)}{\partial p_t}\frac{dp_t}{dq_t}\right| < 1$$

then $\pi\left(q_t\right)$ is a contraction, and by the contraction mapping theorem, it has a unique fixed point $q_t = \pi\left(q_{t+1}\right) = q$.[89] Moreover, assumption (iv) guarantees that such fixed point is in $(0,1)$ and we will be done.

Given $\theta = \left(q_0, \kappa, f\left(\lambda_m\right), \zeta, \overline{w}, \beta, r, R_A\left(0\right)\right) \in \Theta = \left\{\theta \in \mathbb{R}^8_+\right\}$, let $q_t = \pi\left(q_{-1}, p_{-1}\right)$ be known, and $y_t = \left(w_t, w_{t+1}, s_t, \nu_{1,t}, \nu_{2,t}, \nu_{3,t}, \nu_{4,t}\right)$ be the set of endogenous variables. In order to obtain an expression for $\frac{dp_t}{dq_t}$, let us recall the definition of p_{t+1}:

$$p_{t+1} = q_{t+1} + \left(1 - q_{t+1}\right) F\left(u\left(w_{t+1} + rs_t\right) - u\left(rs_t\right)\right).$$

Hence,

$$\frac{dp_{t+1}}{dq_{t+1}} = 1 - F\left(\widehat{\lambda}_{t+1}\right) + \left(1 - q_{t+1}\right) f\left(\widehat{\lambda}_{t+1}\right)\left[u'\left(\overline{c}_{t+1}\right)\frac{dw_{t+1}}{dq_{t+1}} + \frac{\partial\widehat{\lambda}_{t+1}}{\partial s_t}\frac{ds_t}{dq_{t+1}}\right].$$

This value can be shown to be continuous in $q_{t+1} \in [0,1]$. By the extreme value theorem, there exists

$$M_3 = \sup_{q_{t+1}\in[0,1]}\left|\frac{dp_{t+1}}{dq_{t+1}}\right| > 0.$$

Finally, let $M_4 = \max\left[M_3, 1\right]$, define

$$\psi = \frac{1}{3M_4},$$

and note that assumption (iv), allows us to place bounds on the derivatives of π

$$\sup_{q_t\in[0,1]}\left|\frac{\partial\pi\left(q_t\right)}{\partial q_t}\right|, \sup_{q_t\in[0,1]}\left|\frac{\partial\pi\left(q_t\right)}{\partial p_t}\right| \leq \frac{1}{3M_4}.$$

[89] See Stokey and Lucas (1989), p. 50.

This implies that

$$\sup_{q_t \in [0,1]} \left| \frac{d\pi(q_t)}{dq_t} \right| \leq \sup_{q_t \in [0,1]} \left| \frac{\partial \pi(q_t)}{\partial q_t} \right| + \sup_{q_t \in [0,1]} \left| \frac{\partial \pi(q_t)}{\partial p_t} \right| \left| \frac{dp_t}{dq_t} \right| \leq \frac{1}{3} + \frac{1}{3M_4} \left| \frac{dp_t}{dq_t} \right| < 1,$$

which completes the proof.

13.4 Policy assessment: gender based taxation

Let B stand for 'man' and D for 'woman.' Define the 'gross wage gap' as the difference in their expected present value of pre-tax wages and the 'net wage gap' as the difference in the expected present value of their net labor income. Assume that the economy is in a interior stationary steady state characterized by men having a larger marginal product of labor and, as a consequence, a larger unconditional probability of working $\left(\overline{w}^B > \overline{w}^D,\ q^B > q^D\right)$. As mentioned earlier, a technological change that increases the marginal product of women so that it equals that of men would induce a transition phase during which both the expected present value of wages and the unconditional probability of working of women converge to men values. During this transition, the expected present value of wages would be lower for women than for men even though they are equally productive.

Should the government want to speed the transition, it could engage in a budget balanced policy consisting in a lump sum payroll tax on old working men and distributing the proceeds among old working women. Note that, as the tax revenue depends on the market participation rate of old men, the subsidy each women receives is also dependent on it. Therefore, such a tax scheme links two markets that initially were independent of each other, or more precisely, makes the outcomes in the market for female workers to be dependent on the equilibrium values in the market for male workers. It should also be underscored that, as this policy modifies the stationary value of q^j, it needs to be dynamically adjusted, and eventually eliminated. Otherwise it would lead to a new stationary steady state in which the unconditional probability of working of women was larger than that of men. An indicator of the gap between the two groups is simply $q_t^B - q_t^D$. We argue that this difference decreases faster with the aforementioned policy.

As we are assuming that the government maintains its budget balanced at every time t, the amount of the subsidy an individual receives is determined at each time by the tax revenue and the participation rates of both groups. Let $T^t = \{T_{t'}\}_0^\infty$ denote a tax scheme where $T_{t'+1}$ is the tax faced by the generation born at time t', for all $t' \leq t$ but is 0 for all generations born at time $t' > t$. Let $q_{t+1}^j (T^t)$ denote the unconditional probability of working of type j individuals born at time t given that the previous generations have paid the taxes implied by tax scheme T^t. Recall that $q_t^B (0) > q_t^D (0), \forall t \in \mathbb{N}$, i.e., there exists a positive gap between the unconditional probability of working of both groups when there is no tax. Finally, let $q^j (T)$ denote the unique steady state value of type j individuals unconditional probability of working if every generation had faced a fixed tax T.

Proposition 5 If:

(5.i) the conditions for the existence of a steady state (Proposition 4) hold

(5.ii) κ is such that the equilibrium wages depend on the unconditional probability of working for all t

then there exists $\overline{T} > 0$ such that a budget balanced government policy consisting of a flat rate tax scheme (T^t) on type B working old and a subsidy of $\frac{T_{t+1} w_{t+1}^B P_{t+1}^B}{P_{t+1}^D}$ to type D working old, will cause both the lifetime gross and net wage gap to shrink. In addition, unconditional probabilities of working would converge faster in the sense that

$$\left. \frac{\partial \left(q_{t+1}^B (T^t) - q_{t+1}^D (T^t) \right)}{\partial T^t} \right|_{T^t = 0} < 0,$$

for all t and $0 < T_t < \overline{T}$

Proof First, let κ be small but strictly positive. In order to prove the claim, we first construct the problem of type B and D individuals, as well as the problem of the firm. Since we impose a payroll tax T_{t+1} on type-B individuals that decide to work when old, and use the revenues to subsidize type-D individuals that work when old, the time t constraint is the same as in the original problem for both individuals while time $t+1$ constraints are now given by $c^B_{t+1} = s^B_t r + \left(w^B_{t+1} - T_{t+1}\right) L^B_{t+1}$ and $c^D_{t+1} = s^D_t r + \left(w^D_{t+1} + T_{t+1} \frac{P^B_{t+1}}{P^D_{t+1}}\right) L^D_{t+1}$ where $P^j_{t+1} = E_j \left[p^j_{t+1}\right]$ stands for the expectation across type j agents born at time t. Hence, the previous definitions of $c^j_t, \overline{c}^j_t, \underline{c}^j_t$ are modified accordingly. When making their decisions, both individual agents and firms take T_{t+1} and q^j_{t+1} as given with respect to their actions.

As in Appendix A.1, let us define the problem of the firm and note that in this case, firms face an additional constraint, given when facing type B individuals:

$$w^B_{t+1} + s^B_t r - T_{t+1} \geq 0 \quad \left(\nu^B_{5,t}\right).$$

To ensure that under our assumptions, the new problem of the firm is well defined, and has a unique solution, we restrict $T_t \in [0, \overline{w} - \kappa]$, for all t, so that the new constraint is never binding and we can eliminate it from the problem. Denote by $V^j\left(s^j_t, q^j_{t+1}, T^{t+1}\right)$ the problem of the firm when facing individual of type j and by $\Omega^j\left(s^j_t, q^j_{t+1}, T^{t+1}\right)$ its constraint set, $j = B, D$ As in Appendix A.1 and A.2, our assumptions ensure that the firm problem is well defined and has a solution. Moreover, by a reasoning analogous to that of previous appendices, the individual utility is strictly quasiconcave in wages, it has non-zero gradient, and the problem satisfies the constraint qualification at any solution. Hence, for a given triplet $\left(s^j_t, q^j_{t+1}, T^{t+1}\right)$, the unique solution is totally characterized by the Kuhn-Tucker conditions. On the other hand, our assumptions also guarantee that given $\left(T^{t+1}, q^j_{t+1}\right)$ the FOC of the individual agent problem has a unique solution. Note that a change in T^{t+1} will have two effects: first it will affect the transition for each generation t and, second it will affect the steady state value of both quit rates. Define now the lifetime gross wage gap,

$$w_t^B + w_{t+1}\frac{p_{t+1}^B}{r} - w_t^D - w_{t+1}^D\frac{p_{t+1}^D}{r} = \overline{w}\left(\frac{p_{t+1}^B}{r} - \frac{p_{t+1}^D}{r}\right),$$

by the zero profits condition. Also, recall that $q^j(T)$ denote the stationary value of q^j when all generations face a tax of amount T.

By differentiating the system of equations that defines the equilibrium for a given generation, and evaluating at the equilibrium, it can be shown that the following conditions hold

$$\begin{array}{ll} i) & \left.\frac{dp_{t+1}^B}{dT^{t+1}}\right|_{T^{t+1}=\{0\}_0^\infty} < 0, \qquad \left.\frac{dp_{t+1}^D}{dT^{t+1}}\right|_{T^{t+1}=\{0\}_0^\infty} > 0 \\ ii) & \left.\frac{dq_{t+2}^B}{dT^{t+1}}\right|_{T^{t+1}=\{0\}_0^\infty} < 0, \qquad \left.\frac{dq_{t+2}^D}{dT^{t+1}}\right|_{T^{t+1}=\{0\}_0^\infty} > 0 \\ iii) & \left.\frac{dp_{t+1}^j}{dq_{t+1}^j}\right|_{T^{t+1}=\{0\}_0^\infty} > 0 \\ iv) & \left.\frac{dq^B(T)}{dT}\right|_{T=0} < 0, \qquad \left.\frac{dq^D(T)}{dT}\right|_{T=0} > 0. \end{array}$$

Taken together, these inequalities imply the result.

Although the intuition behind this result is straightforward, it is worth noting that the proof is not trivial, because of the endogenous nature of contracts. Take the contract of men: anticipating the tax men will face when old, firms will adjust the wage scheme offered to diminish its impact on the worker welfare. In particular firms will offer a lower wage to young men and a higher one to old workers, because this lessens the impact of the tax, as it shifts income from the 'low' to the 'high' marginal valuation of consumption period of the life of the worker. This could lead men to increase their likelihood of working when old. Proposition 5 shows that this is not the case, and men end up with a reduced probability of working

Part III

The matching function as a modelling device: theoretical restrictions.

14 Introduction

The use of matching functions in economic models allows for the introduction of market frictions in a tractable fashion. The matching function describes how the number of job searchers and the number of open vacancies relate to the number of new job matches that occur within a period. It is common in this literature to interpret the number of matches per job searcher as the average job finding probability. This approach has proven important when studying unemployment and its relationship to other phenomena. The extensive body of literature on matching models shows its many applications (see [Petrongolo and Pissarides, 2001] and [Rogerson et al., 2005] for surveys).

Our contribution to the literature consists on investigating the functional form of the matching function. We find the restrictions that different functional forms of the matching function must satisfy to ensure that the number of matches per job searcher can be interpreted as a job finding probability. These restrictions are useful in those models where the matching function is simply an instrument to introduce frictions in the labor market without explicitly modelling their microfoundations ([Blanchard and Galí, 2010]). The role of these frictions are then to allow for the existence of unemployment in equilibrium. In these cases, it is natural to interpret the number of matches per job searcher as a probability, i.e., the probability to find a job by a worker[90].

15 Basic framework

Let us denote by M the number of new matches created in one period, which is assumed to be a function of the number of job seekers (S) and the number of open vacancies (V) at the beginning of the period, $M = m(S, V)$.[91] We assume, as it is common in this literature, that the matching function is increasing in its arguments and concave. Moreover, we consider that there is no time aggregation bias nor measurement error of the number of job searchers and matches. In this set-up, the average probability of finding a job by a worker is the number of matches per job searcher, $p = m(S, V)/S$, and the average probability of filling in a vacancy by a firm is the number of matches per vacancy, $q = m(S, V)/V$.

We assume free entry in vacancies, with a positive cost of opening a vacancy. The number of posted vacancies is determined by:

$$qB = \kappa, \tag{23}$$

where q is the probability to fill in a vacancy, B are the expected future profits of a filled vacancy for the firm and $\kappa > 0$ is the cost of posting a vacancy. Note that B is endogenous to the model and it includes any effects of the separation rate on the expected future profits. It is however independent of the number of job searchers. This condition applies to a large variety of random matching models, from those with exogenous or endogenous separation rate to those with match-specific productivity. These models will differ in their specification of B

[90] The author whishes to warmly thank Montse Vilalta.

[91] Time subscripts have been omitted to enhance readability.

(see [Rogerson et al., 2005]). For the purpose of this paper, though, we do not need to specify any particular form.

Definition 1 *An **interior equilibrium** occurs when the number of matches is bounded from above by the number of job searchers and the number of vacancies in the economy ($M < \min\{S, V\}$).*

Definition 1 implies that the probability of finding a job (p) and the probability of filling in a vacancy (q) are both below unity. Notice from (23) that the condition $q < 1$ is automatically satisfied in equilibrium, since vacancies will be open if and only if $B > \kappa$. Otherwise, the expected benefits of a filled vacancy would be lower than its cost and noone would have incentives to open any vacancy.[92] This result holds in equilibrium for any functional form of the matching function. However, the matching function will be important in determining whether the probability of finding a job is lower than 1. In the following sections, we study the conditions under which several matching functions lead to an interior equilibrium. Since we proved that $q < 1$ in any equilibrium, it is only left to check whether $p < 1$ holds in equilibrium.

16 Theoretical restrictions for an interior equilibrium

In this section we derive the theoretical restrictions that must be satisfied for a matching function to be consistent with the existence of an interior equilibrium. The following theoretical restrictions should be satisfied by any matching function when the number of matches per job searcher is to be interpreted as a probability.

[92] We follow the common assumption in the literature that each vacancy is open for one individual and the cost of openning the vacancy occurs each time you recruit a new worker.

In all cases, we assume that there is no time aggregation bias neither problems with the measurement of the number of job searchers and matches. We study the restrictions for the Cobb-Douglas and two CES specifications of the matching function.

16.1 Cobb-Douglas matching function

In this section we analyze the necessary restrictions for a Cobb-Douglas matching function to have an interior equilibrium. We also distinguish the implications of these restrictions under different returns to scale on the matching technology.

Proposition 10 *In a general equilibrium random matching model with a Cobb-Douglas matching function, $m = AS^{\alpha}V^{\beta}$, where $A > 0$ is a scale parameter, $\alpha \in (0,1)$ and $\beta \in (0,1)$, an equilibrium (M, S, V, κ, B) is interior if and only if*

$$1 > \frac{\kappa}{B} > \left(\frac{A}{S^{1-\alpha-\beta}}\right)^{\frac{1}{\beta}}.$$

Proof. Let us define the matching function to be Cobb-Douglas, $m = AS^{\alpha}V^{\beta}$, where $A > 0$, $\alpha \in (0,1)$ and $\beta \in (0,1)$. Take any equilibrium (M, S, V, κ, B). By the definition, it is interior if and only if $M < V$ and $M < S$. The first inequality is satisfied as long as $B > \kappa$.

To obtain the second result, substitute q in (23) for its expression, m for its assumed functional form and solve for V. Then substitute V for this expression in the condition $M < S$, where M is defined by the Cobb-Douglas matching function. Rearranging, we obtain the second inequality in proposition 1:

$$\frac{\kappa}{B} > \left(S^{\alpha+\beta-1}A\right)^{\frac{1}{\beta}}.$$

Notice that $A < S^{1-\alpha-\beta}$ is a necessary (although not sufficient) condition to have any interior equilibrium.

Corollary 1. *Different returns to scale in the matching function imply different conditions to have an interior equilibrium:*

a) with constant returns to scale ($\alpha + \beta = 1$), there is an upper bound to the scale parameter A: $1 > \frac{\kappa}{B} > A^{\frac{1}{\beta}}$,

b) with non-constant returns to scale ($\alpha+\beta \neq 1$), there is a joint upper bound to the number of job searchers and the scale parameter A: $1 > \frac{\kappa}{B} > \left(S^{\alpha+\beta-1}A\right)^{\frac{1}{\beta}}$.

Proposition 1 states the necessary conditions for any Cobb-Douglas matching function to lead to an interior equilibrium. Notice also that in the case of CRS, a Cobb-Douglas matching function with scale parameter (A) larger or equal to 1 cannot be a good representation of the labor market frictions, since it would imply that any job searcher fills a vacancy with probability one.

16.2 CES matching function

Another matching function commonly used in the literature takes a CES specification. In this section we analyze two CES matching functions and the conditions they must satisfy to ensure an interior equilibrium.

Proposition 11 *In a general equilibrium random matching model with a matching function represented by a CES function $m = A\left(\eta S^{\sigma} + (1-\eta)V^{\sigma}\right)^{1/\sigma}$, where $A > 0$ is a scale parameter, $\eta \in (0,1)$, $\sigma < 1$ and $1/(1-\sigma)$ is the elasticity of substitution between job searchers and vacancies, an equilibrium (M, S, V, κ, B) is interior if and only if:*

a) $\sigma < 0$, $\frac{(1-\eta)^{-1/\sigma}\kappa}{A} < B < \left(\frac{A^{-\sigma}-\eta}{1-\eta}\right)^{1/\sigma}\kappa$ or,

b) $\sigma > 0$ and $B < \left(\frac{A^{-\sigma}-\eta}{1-\eta}\right)^{1/\sigma}\kappa$.

Proof. Let us define the matching function to be CES such that

$$m = A\left(\eta S^{\sigma} + (1-\eta)V^{\sigma}\right)^{\frac{1}{\sigma}},$$

where $A > 0$, $\eta \in (0,1)$ and $\sigma < 1$. By the definition, (M, S, V, κ, B) is interior if and only if $M < S$. By substituting q in (23) for its expression, m for its assumed functional form, we derive the following equation:

$$\eta\left(\frac{S}{V}\right)^{\sigma} = \left(\frac{\kappa}{AB}\right)^{\sigma} - (1-\eta). \tag{24}$$

Notice that for the market tightness (V/S) to be positive, the RHS of the previous equation must be positive. Therefore, in any equilibrium $(\kappa/AB)^{\sigma} > (1-\eta)$. This translates into $B < (1-\eta)^{-1/\sigma}(\kappa/A)$ if $\sigma > 0$ and $B > (1-\eta)^{-1/\sigma}(\kappa/A)$ if $\sigma < 0$.

Using equation (24) we solve for V and use this expression to substitute V in the condition $S > M$, where M is defined by the CES matching function above. We obtain the following inequality:

$$\left(\left(\frac{\kappa}{AB}\right)^{\sigma} - (1-\eta)\right)^{1/\sigma} > \frac{\kappa}{B}\eta^{1/\sigma}. \tag{25}$$

This inequality is satisfied if and only if $B < \left(\frac{A^{-\sigma}-\eta}{1-\eta}\right)^{1/\sigma}\kappa$ for any σ. Putting all the conditions together, we find the result in proposition 2. Notice that in the case of $\sigma > 0$, the inequality $\frac{(1-\eta)^{-1/\sigma}}{A} < \left(\frac{A^{-\sigma}-\eta}{1-\eta}\right)^{1/\sigma}$ is always satisfied as long as $A^{\sigma}\eta > 0$, which is true by assumption.

Proposition 12 *In a general equilibrium random matching model with a matching function represented by the following CES function,* $m = (S^{\sigma} + V^{\sigma})^{1/\sigma}$*, where* $\sigma < 1$ *and* $1/(1-\sigma)$ *is the elasticity of substitution between job searchers and vacancies, an equilibrium* (M, S, V, κ, B) *is interior if and only if* $B > \kappa$ *and* $\sigma < 0$*.*

Proof. Let us define the matching function to be CES such that

$$m = (S^{\sigma} + V^{\sigma})^{\frac{1}{\sigma}}$$

where $\sigma < 1$. Substituting q in (23) for its expression, m for its assumed functional form and rearranging, we obtain the following:

$$\left(\frac{\kappa}{B}\right)^{\sigma} - 1 = \left(\frac{S}{V}\right)^{\sigma}.$$

Notice that, given $B > \kappa$, the market tightness (V/S) is positive if and only if $\sigma < 0$. Therefore, we need $\sigma < 0$ in any equilibrium.

Now, solving for V and substituting it in the condition $S > M$, where M is defined as the CES matching function above, we obtain the following inequality:

$$1 > \frac{\kappa}{B} \frac{1}{\left((\kappa/B)^{\sigma} - 1\right)^{1/\sigma}},$$

which, given $B > \kappa$, is satisfied for any $\sigma < 0$.

Propositions 2 and 3 report the necessary conditions for two different CES matching functions to lead to an interior equilibrium. The CES function specified in proposition 2 requires restrictions on the expected future profits of a filled vacancy, as in the Cobb-Douglas case with CRS. Moreover, since $B > \kappa$, it can be shown that the scale parameter A cannot be larger than 1 when $\sigma < 0$, and cannot be smaller than 1 when $\sigma > 0$. In contrast, the only requirement for the CES matching function studied in proposition 3 is to have an elasticity of substitution between job searchers and vacancies below unity.

The latter especification of the matching function is equivalent to the one used by [Den Haan et al , (2000)]. They study the propagation of aggregate shocks in a dynamic general equilibrium with labor frictions represented by the following matching function where $\rho = -\sigma$:

$$m(S, V) = \frac{SV}{\left(S^{\rho} + V^{\rho}\right)^{1/\rho}}.$$

They recognize that with a Cobb-Douglas matching function "*truncation is necessary to rule out matching probabilities greater than unity*" (den Haan et al. 2000, p. 485). In section 3.1 we showed under which conditions this is true. In this section we demonstrate that the matching function they propose leads to an interior equilibrium as long as $\rho > 0$ ($\sigma < 0$). Although [Den Haan et al., (2000)] do not explicitly state this condition in their paper, their calibration is consistent with our results.

17 Empirical application

In applied work, many researchers use matching functions as a device to capture frictional labor markets without explicitly stating the underlying dynamics. In these cases, a common procedure is to assume a particular functional form for the matching function and estimate the parameters directly from data on employment, change of unemployment and vacancies. Because of tractability and economic interpretation, Cobb-Douglass and CES functions are the most used. However, as stated above, theoretical consistency imposes some bounds on endogenous variables and parameters in order to preserve the interpretation of the number of matches per job searcher as a job finding probability. It seems natural to use these restrictions to construct tests for model specification.

Let $i \in I$ index the data, which in general will denote time. The minimal data required to test the reduced form model consists of $\{M_i, V_i, S_i\}_I$. Adittionally, one can have information about k_i, B_i.

17.1 Cobb-Douglass

First, suppose we want to test wether the model specification is Cobb-Douglas, and that the only available data are $\{M_i, V_i, S_i\}$. Our goal is to construct a test that exploits the interior equilibrium conditions to test the model specification.

In this case, the model consists of one mantained assumption about the functional form of the matching function $M_i = AS_i^{\alpha}V_i^{\beta}$, with $A > 0$, $\alpha, \beta \in (0, 1)$ and two necessary conditions for interior equilibrium:

$$AS_i^{\alpha+\beta-1} < \left(\frac{\kappa_i}{B_i}\right)^{\beta} \text{ and } M_i = V_i \frac{\kappa_i}{B_i}.$$

Note that the functional form and restrictions can be loglinearized and written as:

$$\begin{aligned} \log(M_i) &= \log(A_i) + \alpha \log(S_i) + \beta \log(V_i), & (26) \\ z_i &= \log(A_i) + (\alpha+\beta-1)\log(S_i) + \beta \log(\frac{B_i}{\kappa_i}) < 0, & (27) \\ \log(M_i) &= \log(V_i) + \log(\frac{\kappa_i}{B_i}). & (28) \end{aligned}$$

Since in this case, we cannot test (28), the strategy consists in estimating using $\log(\frac{M_i}{V_i})$ as a proxy for $\log(\frac{\kappa_i}{B_i})$ and construct a test for (27). First, estimate $\widehat{\mu}_j^{OLS}$ from the following equation:

$$\log(M_i) = \mu_1 + \mu_2 \log(S_i) + \mu_3 \log(V_i) + \varepsilon_i$$

Then, evaluate model specification by

$$\begin{aligned} H_0 &: \mu_2 > 0, \mu_3 > 0, & (29) \\ H_0^{'} &: \mu_2 < 1, \mu_3 < 1. & (30) \end{aligned}$$

Note that this evaluation is independent of the degree of returns to scale. Rejection of the null would be a clear signal of model misspecification.[93] The point is, however, that even if the null is not rejected, the model might not capture adequately an interior equilibrium. To assess this, we propose to construct the new variable z:

[93] However, rejection of $H_0^{'}$ can be interpreted as a need to redefine the coeficients and A through a renormalization. The rejection can be due to $\hat{\mu}_2$ or $\hat{\mu}_3$ exceeding 1. In this case, we can renormalize the data by $\tilde{x} = x^{\frac{1}{\hat{\mu}_2+\hat{\mu}_3}}$ where $x = M, V, S$ and reestimate by OLS.

$$\widehat{z}_i = \widehat{\mu}_1 + (\widehat{\mu}_3 + \widehat{\mu}_2 - 1)\log(S_i) + \widehat{\mu}_3 \log(\frac{V_i}{M_i}).$$

Under the usual assumptions, given that it is linear in $\widehat{\mu}_j^{OLS}$ and that we assume non-stochastic regressors, it is an unbiased and consistent estimator of z_i (which is itself a function of S_i, V_i, M_i). Let $\mu_{\widehat{z}_i|\{M_i,V_i,S_i\}}$ denote the conditional expectation of $\widehat{z}_i$ for each triplet $\{M_i, V_i, S_i\}$. The second restriction is equivalent to the null hypothesis $H_0 : \mu_{\widehat{z}_i|\{M_i,V_i,S_i\}} < 0, \forall\{M_i, V_i, S_i\}$. This can be tested with the data at hand.

Suppose now that the available data are $\{M_i, V_i, S_i, B_i, \kappa_i\}$. The main difference now is that (28) becomes a new equation to be estimated and we are left with a system of two equations:

$$\begin{aligned}
\log(M_i) &= \mu_1 + \mu_2 \log(S_i) + \mu_3 \log(V_i) + \varepsilon_i, \\
\log(M_i) &= \mu_6 + \mu_4 \log(V_i) + \mu_5 \log(\frac{\kappa_i}{B_i}) + \eta_i,
\end{aligned}$$

and the restriction (27). It is clear that V is an endogenous regressor. Since $\frac{\kappa}{B}$ is likely to be correlated with V but not with the misspecification of the functional form of the matching process, this suggests estimating via instrumental variables. Once $\widehat{\mu}_i^{IV}$ have been obtained, we can check model specification by (29) and (30), and add the restriction on the values of μ_4, μ_5, namely

$$H_0^{''} : \mu_6 = 0, \mu_4 = 1, \mu_5 = 1. \tag{31}$$

Then, construct the new variable:

$$\widehat{z}_i = \widehat{\mu}_1 + (\widehat{\mu}_3 + \widehat{\mu}_2 - 1)\log(S_i) + \widehat{\mu}_3 \log(\frac{B_i}{\kappa_i}).$$

As in the previous case, and under usual assumptions, this can be tested by $H_0 : \mu_{\widehat{z}_i|\{S_i,B_i,\kappa_i\}} < 0, \forall\{S_i, B_i, \kappa_i\}$.

17.2 CES

Suppose now that the specification is a CES. Then, the functional form is assumed to be

$$M_i = A\left(\eta S_i^{\sigma} + (1-\eta)V_i^{\sigma}\right)^{\frac{1}{\sigma}} \tag{32}$$

and the restrictions are given by:

$$\begin{aligned}
B_i &> \kappa_i, \frac{M_i}{V_i} = \frac{\kappa_i}{B_i}, \\
\sigma &< 0, \frac{(1-\eta)^{-1/\sigma}}{A} < \frac{B_i}{\kappa_i} < \left(\frac{A^{-\sigma}-\eta}{1-\eta}\right)^{1/\sigma}, \\
\sigma &> 0, \frac{B_i}{\kappa_i} < \left(\frac{A^{-\sigma}-\eta}{1-\eta}\right)^{1/\sigma},
\end{aligned}$$

Assume that only $\{M_i, V_i, S_i\}$ data are available. The suggested strategy is to estimate (η, σ, A) from (32) through Nonlinear Least Squares.

Once $\widehat{\mu}^{NLS} = (\widehat{\eta}, \widehat{\sigma}, \widehat{A})$ have been obtained, use $\frac{M_i}{V_i}$ to proxy for $\frac{\kappa_i}{B_i}$ and to construct the variables:

$$\begin{aligned}
\widehat{z}_{1i} &= \log\left(\frac{M_i}{V_i}\right) - \frac{1}{\widehat{\sigma}}\log\left(\widehat{A}^{\widehat{\sigma}}(1-\widehat{\eta})\right), \\
\widehat{z}_{2i} &= \log\left(\frac{M_i}{V_i}\right) + \frac{1}{\widehat{\sigma}}\log\left(\frac{A^{-\sigma}-\eta}{1-\eta}\right).
\end{aligned}$$

Standard assumptions ensure that $\widehat{z}_i$ are consistent estimators of z_i. Let $\mu_{\widehat{z}_i}$ be the expectation of the new variables. Then the restrictions can be tested by:

$$\begin{aligned}
H_0 &: \mu_{\widehat{z}_1} < 0, \mu_{\widehat{z}_2} > 0 \text{ if } \hat{\sigma} < 0, \\
H_0 &: \mu_{\widehat{z}_2} > 0 \text{ if } \hat{\sigma} > 0.
\end{aligned}$$

Assume now that data on $\{M_i, V_i, S_i, B_i, \kappa_i\}$ are available. In this case, there are two equations to be estimated by nonlinear least squares.

$$\begin{aligned} M_i &= A\left(\eta S_i^{\sigma} + (1-\eta)V_i^{\sigma}\right)^{\frac{1}{\sigma}}, \\ \frac{M_i}{V_i} &= \frac{\kappa_i}{B_i}. \end{aligned}$$

Once $\widehat{\mu}^{NLS} = (\widehat{\eta}, \widehat{\sigma}, \widehat{A})$ have been obtained, construct the variables:

$$\begin{aligned} \widehat{z}_{1i} &= \log\left(\frac{\kappa_i}{B_i}\right) - \frac{1}{\widehat{\sigma}}\log\left(\widehat{A}^{\widehat{\sigma}}(1-\widehat{\eta})\right), \\ \widehat{z}_{2i} &= \log\left(\frac{\kappa_i}{B_i}\right) + \frac{1}{\widehat{\sigma}}\log\left(\frac{A^{-\sigma}-\eta}{1-\eta}\right). \end{aligned}$$

Standard assumptions ensure that $\widehat{z}_i$ are consistent estimators of z_i. Let $\mu_{\widehat{z}_i}$ be the expectation of the new variables. Then the restrictions can be tested by:

$$\begin{aligned} H_0 &: \mu_{\widehat{z}_1} < 0, \mu_{\widehat{z}_2} > 0 \text{ if } \hat{\sigma} < 0, \\ H_0 &: \mu_{\widehat{z}_2} > 0 \text{ if } \hat{\sigma} > 0. \end{aligned}$$

The case with functional form equal to

$$M_i = \left(S_i^{\sigma} + V_i^{\sigma}\right)^{\frac{1}{\sigma}}$$

can be tested by an analogous procedure, using nonlinear least squares, and taking into account that the only relevant restriction in this case is $\sigma < 0$.

18 Specific bibliography for essay 3.

References

[Blanchard and Galí, 2010] Blanchard and Galí. 2010. Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment. American Economic Journal: Macroeconomics, 2, 1-30.

[Den Haan, 2000] den Haan, W. J., Ramey G., Watson J., 2000. Job Destruction and Propagation of Shocks. Am. Econ. Rev. 90(3), 482-498.

[Elsby, 2013] Elsby, M.; Hobijn, B.; Sahin, A. Unemployment dynamics in the OECD. Review of Economics and Statistics, 2013, vol. 95, no 2, p. 530-548.

[Laing et al., 1995] Laing D., T. Palivos, P. Wang, 1995. Learning, matching and growth. Review of Economic Studies, 62(1), 115-129.

[Michelacci, 2007] Michelacci, C, and D. Lopez-Salido, 2007. Technology shocks and job flows. Review of Economic Studies, 74(4), 1195-1227.

[Petrongolo and Pissarides, 2001] Petrongolo, B., Pissarides C. A., 2001. Looking into the Black Box: A Survey of the Matching Function. J. Econ. Lit. 39(2), 390-431.

[Rogerson et al., 2005] Rogerson, R., Shimer R., Wright R., 2005. Search-Theoretic Models of the Labor Market: A Survey. J. Econ. Lit. 43(4) 959-988.

[Shimer, 2005] Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. American economic review, 25-49.

Shimer R. 2012. Reassessing the ins and outs of unemployment. Review of Economic Dynamics. 15, 127-148.

Part IV

Final conclusions

This thesis is composed of 3 independent essays on economic theory.

The first essay reviews some well known conceptual and empirical problems that appear when economic theorists deal with preferences and choice theory, in general. While assessing those problems, the essay lays the ground for a detailed discussion of the possibility of preference learning, formation and change. The essay concludes proposing a theoretical framework to study these phenomena.

The second essay, although independent from the first, is also devoted to the issue of preference change. In particular, it studies the possibility that cultural preferences evolve as a result of the combination of technological innovation and cultural transmission mechanisms. At the same time, it allows for the possibility that those cultural preferences determine the short term outcome of economic variables. In addition, it builds a framework where the combination of technological innovation, cultural transmission and economic structure lead to a process of endogenous preference heterogeneity and clustering. Hence it provides a model to understand how culture and the economic structure interact and coevolve.

The third essay presents some theoretical problems that arise when using the concept of a matching function as a modelling device for the labor market. In particular, necessary conditions for the ratio of the number of matches per job searcher to be interpreted as the average job finding probability are established.

Part V

General bibliography

References

[Abel, 1990] Abel, A. B. (1990). Asset prices under habit formation and catching up with the joneses. *The American Economic Review*, pages 38–42.

[Afriat, 1967] Afriat, S. (1967). The construction of utility functions from expenditure data. *International Economic Review*, 8(1):67–77.

[Al-Najjar, 1993] Al-Najjar, N. (1993). Non-transitive smooth preferences. *Journal of Economic Theory*, 60(1):14 – 41.

[Anderson et al., 1989] Anderson, S., De Palma, A., and Thisse, J. (1989). Demand for differentiated products, discrete choice models, and the CharacteristicsApproach. *The Review of Economic Studies,*, 56:21–35.

[Aragones et al., 2005] Aragones, E., Gilboa, I., Postlewaite, A., and Schmeidler, D. (2005). Fact-free learning. *The American Economic Review*, 95(5):1355–1368.

[Ariely et al., 2003] Ariely, D., Loewenstein, G., and Prelec, D. (2003). ï£¡coherent arbitrarinessï£¡: Stable demand curves without stable preferences. *The Quarterly Journal of Economics*, 118(1):73–106.

[Arrow, 1986] Arrow, K. (1986). Rationality of self and others in an economic system. *Journal of Business*, pages 385–399.

[Arrow and Hahn, 1971] Arrow, K. and Hahn, F. (1971). *General competitive analysis*. Holden-Day San Francisco.

[Arrow, 1959] Arrow, K. J. (1959). Rational choice functions and orderings. *Economica*, 26(102):121–127.

[Aumann, 1962] Aumann, R. (1962). Utility theory without the completeness axiom. *Econometrica: Journal of the Econometric Society*, pages 445–462.

[Balasko, 2003] Balasko, Y. (2003). Economies with price-dependent preferences. *Journal of economic theory*, 109(2):333–359.

[Balzer, 1982] Balzer, W. (1982). Empirical claims in exchange economics. In *Philosophy of Economics*, pages 16–40. Springer.

[Becker, 1962] Becker, G. (1962). Irrational behavior and economic theory. *The Journal of Political Economy*, pages 1–13.

[Becker, 1978] Becker, G. S. (1978). *The economic approach to human behavior*. University of Chicago press.

[Berry and Pakes, 2007a] Berry, S. and Pakes, A. (2007a). The pure characteristics demand model. *International Economic Review*, 48(4):1193–1225.

[Berry and Pakes, 2007b] Berry, S. and Pakes, A. (2007b). The pure characteristics demand model*. *International Economic Review*, 48(4):1193–1225.

[Bewley, 1986] Bewley, T. (1986). Knightian uncertainty theory: part i. *Yale University*.

[Blaug, 1992] Blaug, M. (1992). *The methodology of economics: Or, how economists explain*. Cambridge University Press.

[Boldrin and Levine, 2009] Boldrin, M. and Levine, D. (2009). A model of discovery. *The American Economic Review*, pages 337–342.

[Boudon, 1998] Boudon, R. (1998). Social mechanisms without black boxes. *Social mechanisms: An analytical approach to social theory*, 172.

[Brown and Matzkin, 1996] Brown, D. and Matzkin, R. (1996). Testable restrictions on the equilibrium manifold. *Econometrica: Journal of the Econometric Society*, pages 1249–1262.

[Bunge, 1993] Bunge, M. (1993). Realism and antirealism in social science. *Theory and Decision*, 35(3):207–235.

[Caldwell, 1984] Caldwell, B. J. (1984). Some problems with falsificationism in economics. *Philosophy of the Social Sciences*, 14(4):489–495.

[Chapman and Johnson, 1999] Chapman, G. B. and Johnson, E. J. (1999). Anchoring, activation, and the construction of values. *Organizational Behavior and Human Decision Processes*, 79(2):115 – 153.

[Christiano et al., 2005] Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1):1–45.

[Deaton and Muellbauer, 1980] Deaton, A. and Muellbauer, J. (1980). An almost ideal demand system. *The American economic review*, pages 312–326.

[Dekel et al., 2007] Dekel, E., Ely, J., and Yilankaya, O. (2007). Evolution of preferences. *The Review of Economic Studies*, 74(3):685–704.

[Dekel et al., 1998] Dekel, E., Lipman, B., and Rustichini, A. (1998). Standard state-space models preclude unawareness. *Econometrica*, 66(1):159–173.

[Dekel et al., 2004] Dekel, E., Lipman, B., and Rustichini, A. (2004). Representing preferences with a unique subjective state space. *Econometrica*, 69(4):891–934.

[Dietrich and List, 2010] Dietrich, F. and List, C. (2010). Where do preferences come from? *International Journal of Game Theory*, pages 1–25.

[Dixit and Stiglitz, 1977] Dixit, A. and Stiglitz, J. (1977). Monopolistic competition and optimum product diversity. *The American Economic Review*, pages 297–308.

[Dopfer, 2004] Dopfer, K. (2004). The economic agent as rule maker and rule user: Homo sapiens oeconomicus. *Journal of Evolutionary Economics*, 14(2):177–195.

[Dubra et al., 2004] Dubra, J., Maccheroni, F., and Ok, E. (2004). Expected utility theory without the completeness axiom. *Journal of Economic Theory*, 115(1):118–133.

[Eichner, 1983] Eichner, A. S. (1983). Why economics in mot yet a science.

[Eliaz and Ok, 2006] Eliaz, K. and Ok, E. (2006). Indifference or indecisiveness? choice-theoretic foundations of incomplete preferences. *Games and Economic Behavior*, 56(1):61–86.

[Elster, 1983] Elster, J. (1983). Exploitation, freedom, and justice. *Nomos*, 26:277–304.

[Epstein and Zhang, 2003] Epstein, L. G. and Zhang, J. (2003). Subjective probabilities on subjectively unambiguous events. *Econometrica*, 69(2):265–306.

[Frey and Stutzer, 2010] Frey, B. S. and Stutzer, A. (2010). *Happiness and economics: How the economy and institutions affect human well-being*. Princeton University Press.

[Friedman, 1953] Friedman, M. (1953). The methodology of positive economics. *Essays in positive economics*, 3(3).

[Greenberg, 1977] Greenberg, J. (1977). Quasi-equilibrium in abstract economies without ordered preferences. *Journal of Mathematical Economics*, 4(2):163–165.

[Grewal et al., 1998] Grewal, D., Monroe, K., and Krishnan, R. (1998). The effects of price-comparison advertising on buyers' perceptions of acquisition value, transaction value, and behavioral intentions. *The Journal of Marketing*, pages 46–59.

[Gul and Pesendorfer, 2001] Gul, F. and Pesendorfer, W. (2001). Dynamic inconsistency and self-control. *Econometrica*, 69(6):1403–1436.

[Hall and Hitch, 1939] Hall, R. L. and Hitch, C. J. (1939). Price theory and business behaviour. *Oxford economic papers*, (2):12–45.

[Hammond and Hammond, 2006] Hammond, D. J. and Hammond, C. H. (2006). *Making Chicago Price Theory: Friedman-Stigler Correspondence 1945-1957*. Routledge.

[Hands, 1992] Hands, D. W. (1992). *Falsification, situational analysis and scientific research programs: the Popperian tradition in economic methodology*. Springer.

[Harbaugh et al., 2001] Harbaugh, W., Krause, K., and Berry, T. (2001). Garp for kids: On the development of rational choice behavior. *American Economic Review*, pages 1539–1545.

[Hausman, 1992] Hausman, D. M. (1992). *Essays on philosophy and economic methodology*. Cambridge University Press.

[Hausman and McPherson, 2009] Hausman, D. M. and McPherson, M. S. (2009). Preference satisfaction and welfare economics. *Economics and Philosophy*, 25(01):1–25.

[Heifetz et al., 2007] Heifetz, A., Shannon, C., and Spiegel, Y. (2007). The dynamic evolution of preferences. *Economic Theory*, 32(2):251–286.

[Hempel and Oppenheim, 1953] Hempel, C. G. and Oppenheim, P. (1953). *Studies in the Logic of Explanation*. na.

[Hildenbrand, 2014] Hildenbrand, W. (2014). *Market demand: Theory and empirical evidence*. Princeton University Press.

[Hollis and Nell, 1975] Hollis, M. and Nell, E. (1975). *Rational economic man: a philosophical critique of neo-classical economics*. Cambridge University Press Cambridge.

[Houthakker, 1950] Houthakker, H. S. (1950). Revealed preference and the utility function. *Economica*, pages 159–174.

[Hutchison, 1977] Hutchison, T. (1977). On prediction and economic knowledge. *Knowledge and ignorance in economics*, pages 8–33.

[Hutchison, 2000] Hutchison, T. W. (2000). *On the methodology of economics and the formalist revolution*. Edward Elgar Publishing.

[Kahneman et al., 1982] Kahneman, D., Slovic, P., and Tversky, A. (1982). *Judgment under uncertainty: Heuristics and biases*. Cambridge University Press.

[Kahneman and Thaler, 2006] Kahneman, D. and Thaler, R. H. (2006). Anomalies: Utility maximization and experienced utility. *The Journal of Economic Perspectives*, 20(1):221–234.

[Kahneman and Tversky, 1979] Kahneman, D. and Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society*, pages 263–291.

[Kirman, 1989] Kirman, A. (1989). The intrinsic limits of modern economic theory: the emperor has no clothes. *The Economic Journal*, 99(395):126–139.

[Knight, 1925] Knight, F. H. (1925). Fact and metaphysics in economic psychology. *The American Economic Review*, 15(2):247–266.

[Koopmans, 1957] Koopmans, T. C. (1957). The construction of economic knowledge. *Three essays on the state of economic science*, pages 127–66.

[Koszegi and Rabin, 2007] Koszegi, B. and Rabin, M. (2007). Mistakes in choice-based welfare analysis. *The American economic review*, 97(2):477–481.

[Kreps, 1988] Kreps, D. (1988). *Notes On The Theory Of Choice*. Basic Books.

[Kuhn, 1962] Kuhn, T. S. (1962). The structure of scientific revolutions.

[Lakatos and Musgrave, 1970] Lakatos, I. and Musgrave, A. (1970). Criticism and the growth of knowledge.

[Lancaster, 1966] Lancaster, K. (1966). A new approach to consumer theory. *The journal of political economy*, 74(2):132–157.

[Leibniz, 1989] Leibniz, G. W. (1989). *The monadology*. Springer.

[Machlup, 1955] Machlup, F. (1955). The problem of verification in economics. *Southern Economic Journal (pre-1986)*, 22(1):1.

[Machlup, 1969] Machlup, F. (1969). Speculations on gold speculation. *The American Economic Review*, pages 332–343.

[Mandler, 2005] Mandler, M. (2005). Incomplete preferences and rational intransitivity of choice. *Games and Economic Behavior*, 50(2):255–277.

[Manski, 1977] Manski, C. (1977). The structure of random utility models. *Theory and decision*, 8(3):229–254.

[Manski and McFadden, 1981] Manski, C. and McFadden, D. (1981). *Structural analysis of discrete data with econometric applications.* MIT Press Cambridge, MA.

[Marwell and Ames, 1981] Marwell, G. and Ames, R. E. (1981). Economists free ride, does anyone else?: Experiments on the provision of public goods, iv. *Journal of public economics*, 15(3):295–310.

[Marx, 1867] Marx, K. (1867). Capital, volume i.

[Mas-Colell, 1986] Mas-Colell, A. (1986). The price equilibrium existence problem in topological vector lattices. *Econometrica: Journal of the Econometric Society*, pages 1039–1053.

[Mas-Colell et al., 1995] Mas-Colell, A., Whinston, M., Green, J., et al. (1995). *Microeconomic theory*, volume 1. Oxford university press New York.

[Mazar et al., 2010] Mazar, N., Koszegi, B., and Ariely, D. (2010). Price-sensitive preferences. *Available at SSRN 1665017.*

[McCloskey, 1983] McCloskey, D. N. (1983). The rhetoric of economics. *journal of Economic Literature*, pages 481–517.

[McFadden, 1984] McFadden, D. (1984). Econometric analysis of qualitative response models. *Handbook of econometrics*, 2:1395–1457.

[Mill, 1994] Mill, J. S. (1994). On the definition and method of political economy. *The Philosophy of Economics: An Anthology*, 2:52–68.

[Momi, 2003] Momi, T. (2003). Excess demand functions with incomplete marketsï£¡a global result. *Journal of Economic Theory*, 111(2):240–250.

[Mongin, 2006] Mongin, P. (2006). Value judgments and value neutrality in economics. *Economica*, 73(290):257–286.

[Monroe and Hoseason, 2003] Monroe, K. and Hoseason, J. (2003). Pricing: Making profitable decisions. *Journal of Revenue & Pricing Management*, 2:175–177.

[Nehring, 1997] Nehring, K. (1997). Rational choice and revealed preference without binariness. *Social Choice and Welfare*, 14(3):403–425.

[Nevo, 2003] Nevo, A. (2003). New products, quality changes, and welfare measures computed from estimated demand systems. *Review of Economics and Statistics*, 85(2):266–275.

[Noor, 2011] Noor, J. (2011). Temptation and revealed preference. *Econometrica*, 79(2):601–644.

[Noor and Ren, 2011] Noor, J. and Ren, L. (2011). Temptation and social preference. Technical report, Boston University-Department of Economics.

[Ok, 2002] Ok, E. (2002). Utility representation of an incomplete preference relation. *Journal of Economic Theory*, 104(2):429–449.

[Polemarchakis, 1983] Polemarchakis, H. (1983). Expectations, demand, and observability. *Econometrica: Journal of the Econometric Society*, pages 565–574.

[Pollak, 1977] Pollak, R. (1977). Price dependent preferences. *The American Economic Review*, pages 64–75.

[Popper, 1985] Popper, K. (1985). The rationality principle.

[Popper, 1968] Popper, K. R. (1968). Logik der forschung (1934).

[Popper, 1969] Popper, K. R. (1969). Conjections and refutations.

[Quoidbach et al., 2013] Quoidbach, J., Gilbert, D. T., and Wilson, T. D. (2013). The end of history illusion. *Science*, 339(6115):96–98.

[Ravn et al., 2006] Ravn, M., Schmitt-Grohé, S., and Uribe, M. (2006). Deep habits. *The Review of Economic Studies*, 73(1):195–218.

[Richter, 1966] Richter, M. (1966). Revealed preference theory. *Econometrica: Journal of the Econometric Society*, pages 635–645.

[Rizvi, 2006] Rizvi, S. (2006). The sonnenschein-mantel-debreu results after thirty years. *History of political economy*, 38:228.

[Rustichini and Siconolfi, 2008] Rustichini, A. and Siconolfi, P. (2008). Preferences over characteristics and utility functions over commodities. *Economic Theory*, 36(1):159–164.

[Scitovsky, 1992] Scitovsky, T. (1992). *The joyless economy: The psychology of human satisfaction*. Oxford University Press, USA.

[Sen, 1985] Sen, A. (1985). Commodities and capabilities.

[Sen, 1997] Sen, A. (1997). *Choice, welfare and measurement*. Harvard University Press.

[Simon, 1982] Simon, H. (1982). Models of bounded rationality: Behavioral economics and business organization, vol. 2.

[Simonson and Drolet, 2004] Simonson, I. and Drolet, A. (2004). Anchoring effects on consumersï£¡ willingness-to-pay and willingness-to-accept. *Journal of consumer research*, 31(3):681–690.

[Sneed, 2012] Sneed, J. D. (2012). *The logical structure of mathematical physics*. Springer Science & Business Media.

[Stegmüller et al., 2012] Stegmüller, W., Balzer, W., and Spohn, W. (2012). *Philosophy of economics: proceedings, Munich, July 1981*, volume 2. Springer Science & Business Media.

[Theil, 1975] Theil, H. (1975). The theory of rational random behavior and its application to demand analysis. *European Economic Review*, 6(3):217–226.

[van Orman Quine, 1976] van Orman Quine, W. (1976). Two dogmas of empiricism. In *Can Theories be Refuted?*, pages 41–64. Springer.

[Varian, 1982] Varian, H. (1982). The nonparametric approach to demand analysis. *Econometrica: Journal of the Econometric Society*, pages 945–973.

[Varian, 1983] Varian, H. R. (1983). Non-parametric tests of consumer behaviour. *The review of economic studies*, 50(1):99–110.

[Veblen, 1965] Veblen, T. (1965). *The Theory of the Leisure Class. 1899*. AM Kelley, bookseller.

[von Wright, 1971] von Wright, G. (1971). H.(1971):Ť explanation and understanding,Ť. *New York*.

[Weber, 1949] Weber, M. (1949). Objectivity in social science and social policy. *The methodology of the social sciences*, 78.