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in

AGENT - BASED SIMULATION OF MACROECONOMIC SYSTEMS

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Dedicated to my family

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The teacher who walks in the shadow of the temple, among his followers, gives not of his wisdom but rather of his faith and his lovingness.

If he is indeed wise he does not bid you enter the house of his wisdom, but rather leads you to the threshold of your own mind.

Kahlil Gibran

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Introduction

Purpose and aim of the Research

In the last 30 years economists have developed new economic theories in which the micro enviroment (economic and behaviural microfundation) interacts with the macro enviroment (macroeconomic variables). To understand the real world, economists have created a new way to observe the reality, that is far away from the view of static and homogeneous agents in a complicated (but not complex) economic system. In the real world, and in this new perspective, agents are different from each other, they have different behavior, different rationality and capability and different endowements. The new line of research is focused on the complexity of the world, on the heterogenousity of agents and on the interaction of agents among them and with the enviroment. From these interactions *emerges* proprieties at macroeconomic level, a world (complex system), that is not foreseable only observing (and aggregating) the average agent that acts in the world (the *representative agent*). What emerges at macroeconomic level from the interaction of heterogenous agents affects the agent (at microeconomic level) and his (inter)actions, and this generates a perpetual cycle.

Economists started to analyze the world through a new kind of model: the model based on *agents behavior* (agent - based model) that is interacting and heterogeneous (HHI).

Economists use informatic tools (computer science) and concept of physics to implement and carry out their theories [1].

In this framework economists have taken advantage from computer scientist to create a simulated complex economic system inhabitated by heterogenous interacting agents [Tesfatsion, (2003)]. The explosive computer power, over the past several decades, offers new tools and opportunity for economists as computational methods to solve standard economic models (general equilibrium models) and also method as Agent - Based Comutational Economics (ACE) to develop end explore new kinds of economic models.

In this Doctoral thesis we use an Agent- Based Model (ABM) to analyze variables and dynamics of the economy , building a complex system that is very close to the real world behavior.

Concretely, the main aims of the research are organized in four chapters. In the first chapter (i), we focus on the evolution of the macroeconomic theory over time; in the second chapter (ii) it is realized an analytic study of an Agent-Based Model *(ABM)* in a economy populated by firms and a bank system [2]; in the third chapter (iii) we focus our analysis on the distribution of growth rate and profit rate with an ABM and in last chapter (iv) we analyze a model that is an evolution of the [Delli Gatti et. al, (2005)] in a system

[1] Advances in the tools of modelling have greatly expanded the possibility set for economists [Arthur et al., (1997)], [Day & Chen, (1993)], [Epstein & Axtell, (1996)], [Holland, (1992)], [Krugman, (1996)], [Sargent, (1993)], [Young, (1998)].

[2] In our case, we analyze the model proposed by [Delli Gatti et. al, (2005)].

populated by heterogenous firms in a multibanks system.

More precisely, the specific aims of the present thesis are the following:

- to examine an ABM model studing analytically its dynamics using a deterministic version of the model (we use [Delli Gatti et. al, (2005)]) to understand the behavior and the features inherent to the model. We use a deterministic model to understand and explain the dynamics behind an ABM, examining , throught the analytic study, the fluctuations and the structure of the fluctuations of GDP;

- to analyze the GDP growth dynamics and fluctuations, using some accounting index (Roa, Roe, Leverage) and industrial index (Herfindhal-Hirshman Index), comparing the result of the fluctuations to what was found in the real world by [Gabaix, (2011)];

- to prove that an ABM is able to replicate the empirical distribution of the growth rate and profit rate that [Alfarano et al., (2008), (2012)], [Mundt et al., (2014), (2016)] have found studing the real world data. We use an ABM to prove that this kind of model is able to show the same empirical distributions (in our case, of growth rate and prodit rate) that arise from the analysis of the real world data;

- to show that an ABM is able to replicate and explain the relationship between bank and firms and describing, throught the analysis of the distribution of profit, debt, firms' size, the emergence property that arise in a heterogenous interacting world [Delli Gatti et. al, (2007)]. We analyze the financial fragility and the business fluctuation [Gallegati et. al, (2003)] , [Delli Gatti et. al, (2003), (2004), (2005)] in a system with firms and multibanks in an ABM [Grilli et. al, (2014), (2016)], focusing on the empirical findings that the model is able to replicate.

In order to address the aims that were set, in the thesis we simulate three different real world and we examinate the result.

Methodology

The specific aspects of the firms and bank behaviors, their interactions and features, and the evolution of aggregate behavior, are analyzed using a methodology that moves away from the vision of the traditional neoclassical theory and uses a new approach based on macroeconomic approach based on microeconomic foundation (microfoundation).

This analysis is based on the Agent-Based Heterogeneous Model approach which is far from the traditional Mainstream theory of general equilibrium. This approach considers heterogenous interacting agents. We know the structure of the model, the agents interact and we can observe aggregate dynamics different from the sum of the individual behavior. Aggregate dynamics are due to the interactions and from them arise emergent properties. Through a feedback mechanism, the aggregate behavior affect the microlevel (agent reactions), and, in turn, it affect again the macrolevel. The Agent-Based Model has been developed to study the interaction of many heterogeneous agents. These kind of models are based on micro-foundations, but the new concept of equilibrium doesn't require that any element is in equilibrium: a state of macroeconomic equilibrium is manteined by a large number of interactions, [Feller, (1957)].

In the models we propose, we also use, to improve our analysis, the OLS estimation to evaluate the scaling coefficient of the firms' size distribution, that confirms us that the estimation of the tail is close to one (Zipf plot), that is consistent with real data [Gaffeo et. al, (2003)]. Moreover, research presented by physicists [Amaral et al., (1998)] and [Marsili and Zhang, (1998)] has shown that the heterogeneity and the direct or indirect interaction among units bring naturally out power laws distributions.

We can note that a power law distribution emerges for firms size distribution, profit distribution [Fujiwara, (2003)] and debt distribution [Fujiwara (2003)]. In terms of a power law distribution, it means that firms are located along a curve whose coefficient is stable and the intercept changes very slowly over time.

Moreover, in *chapter 3*, to evaluate the growth rate distribution for the empirical distribution analysis, to eliminate possible trends in firm size, we consider the normalized (logarithmic) size, so we 'de - trend' the growth rate. For emipirical distribution analysis, both growth rate and profit rate, as in [Bottazzi and Secchi, (2006)] , we test , throught a *maximum likelihood estimation*, the Laplacian hypothesis with a more general distributional class, the *Subbotin distribution* (1923) , also known as the generalized exponential-power distribution.

Contribution

We can show, as follow, the contribution that this Doctoral thesis can be made at research level:

- with the deterministic analysis we computed the interest rate that grant the growth of the economy ($\bar{r}$) .We found that there is a time invariant growth rate for the level of all variables (equity, capital, bank equity, supply and demand of credit), that is equal to the *roe*.

- we found that all variables (bank and firm assets, demand and supply of credit, capital,..) and their level, grow exponencially, because they are all proportional to the level of (bank or firm) equity. To compute the deterministic model, we used some accounting indeces as *roa*, *roe* and *leverage*, through the analysis of the parameters that characterize the firm and bank behaviors.

- we discovered that the *bank roe* is equal to the *firms roe*, although, the parameters from which they depend on are apparently differents.

- Both the *roes* of bank and firm depend on the interest rate, but in opposite sense: when the interest rate increases the roe of bank increases while the roe of the firm decreases.

- we found out that if the interest rate is equal to $\bar{r}$, the system bank-firms reaches a state characterized by an exponential growth of the size of the bank and of the firm and a balanced situation in demand and supply of credit.

- recent empirical research has shown that firms entering and exiting markets mechanism contributes as much to macroeconomic fluctuations as firms continuing their activity [Davis et al., (1996)]. Hence, any theory of business fluctuations (GDP [3] fluctuations), as we do, should pay particular attention to the way in which entrances and exits of firms are modeled.

- we discovered that, considering the firms entrances/exit dynamics, the GDP fluctuations are due to the *roa* standard deviation times the herfindhal - hirshman index [4] . Analyzing the *roa* in our models, we can assert that the standard deviation of GDP is equal to $\frac{\phi}{\sqrt{3}} * H$. We also analyzed the H herfindhal- hirshman index. As in [Gabaix, (2011)] , we show and confirm that the fluctuation of the GDP definitively depends on the H fluctuation. We also found that the model [Delli Gatti et. al, (2005)] tends to generate a giant firm that is able to get all the amount of loans that the bank can allow. In our model, when the H explodes, a firm dominates the system.We can argue, that the model tends to finance a firm (sector) more than another and the fluctuation of this firm (sector) affects the GDP fluctuation.

- we found that in the model we analyze there is dependency between the aggregate growth rate (GDP fluctuation) and the the profit rate of large firms.

- throught the MLE (maximun likelihood estimation) we showed that

[3] The GDP is the gross domestic product.

[4] ss argued by [Gabaix,(2011)] , the fluctuations of GDP depend on Herfindhal Index.

the ABM model is able to replicate a laplace Distribution for profit rate and a more leptokurtik laplace distribution for growth rate.

- we found out that even in a multibanks - firms system the deterministic assumptions hold true. Moreover, we found out the model that reproduces a wide range of stylized facts that characterize a complex dynamic system (world).

Introducción (Spanish)

Propósito y objetivo de la investigación

En los últimos 30 años los economistas han desarrollado nuevas teorías económicas en las que el microambiente (microfundación económica y de comportamineto) interactúa con el macroambiente (variables macroeconómicas). Para comprender el mundo real, el economista ha creado una nueva forma de observar la realidad, que está lejos de la visión de agentes estáticos y homogéneos en un sistema económico complicado (pero no complejo). En el mundo real, y en esta nueva perspectiva, los agentes son diferentes unos de otros, tienen comportamientos diferentes, racionalidad y capacidad diferentes y dotaciones diferentes. La nueva línea de investigación se centra en la complejidad del mundo, en la heterogeneidad de los agentes y en la interacción del agente entre ellos y con el ambiente. A partir de estas interacciones *emergen* las propiedades a nivel macroeconómico, un mundo (sistema complejo), que no son previsibles sólo observando (y agregando) el agente promedio que actúa en el mundo (el *agente representativo*). Lo que surge a nivel macroeconómico a partir de la interacción de agentes heterogéneos afecta al agente (a nivel microeconómico) y sus (inter) acciones, y esto genera un ciclo perpetuo.

El economista comenzó a analizar el mundo a través de un nuevo tipo de modelo: el modelo basado en el *comportamiento de los agentes* (modelo basado en agentes) que interactúan y que son heterogéneos (HHI).

Los economistas utilizan herramientas informáticas (*computer science*) y conceptos de física para implementar y llevar a cabo sus teorías. [5]. En este marco el economista se ha aprovechado de la informática para crear un sistema económico complejo simulado habitado por agentes heterogéneos interactuantes [Tesfatsion, (2003)]. El poder explosivo de las computadoras en las últimas décadas ofrece nuevas herramientas y oportunidades para los economistas como métodos computacionales para resolver modelos económicos estándar (modelos de equilibrio general) y también como método de Economía Computacional Basada en Agentes (ACE) para explorar nuevos tipos de modelos económicos.

En esta tesis doctoral utilizamos un modelo basado en agentes (ABM) para analizar variables y dinámicas de la economía, construyendo un sistema complejo que está muy cerca del comportamiento real.

Concretamente, los objetivos principales de la investigación se organizan en cuatro capítulos. En el primero Capítulo (i), nos centramos en la evolución de la teoría macroeconómica a lo largo del tiempo; en el segundo capítulo (ii) se realiza un estudio analítico de un modelo basado en agentes *(ABM)* en una

[5] Los avances en los instrumentos de modelismo enormemente han ampliado el juego de posibilidad para economistas [Arthur y col., (1997)], [Day & Chen, (1993)], [Epstein & Axtell, (1996)], [Holland, (1992)], [Krugman, (1996)], [Sargent, (1993)], [Young, (1998)].

economía poblada por empresas y un sistema bancario [6]; en el tercer capítulo (iii) enfocamos nuestro análisis en la distribución de la tasa de crecimiento y la tasa de beneficio con un modelo ABM y en el último capítulo (iv) analizamos un modelo que es una evolución del [Delli Gatti y col., (2005)] en un sistema poblado por empresas heterogéneas en un sistema donde actuan múltiples bancos.

Más concretamente, los objetivos específicos de la presente tesis son los siguientes:

- examinar un ABM que estudia analíticamente su dinámica usando una versión determinista del modelo (usamos [Delli Gatti y col., 2005)] para entender el comportamiento y las características inherentes el modelo. Utilizamos un modelo determinista para comprender y explicar la dinámica del ABM, examinando, a través del estudio analítico, las fluctuaciones y la estructura de las fluctuaciones del PIB;
- analizar la dinámica de crecimiento del PIB y la fluctuación, utilizando índices contables (Roa, Roe, Leverage) e el índice industrial (Índice Herfindhal-Hirshman), comparando el resultado de las fluctuaciones a lo que encontramos en el mundo real mediante el estudio hecho por [Gabaix, (2011)];
- demuestre que un ABM es capaz de replicar la distribución empírica de la tasa de crecimiento y la tasa de beneficio que [Alfarano y col., (2008), (2012)], [Mundt y col., (2014), (2016)] han estudiado los datos del mundo real. Utilizamos un modelo ABM para demostrar que este

[6] En nuestro caso, analizamos el modelo propuesto por [Delli Gattiy col., (2005)]

tipo de modelo es capaz de mostrar las mismas distribuciones empíricas (en nuestro caso, de tasa de crecimiento y tasa de beneficio) que surgen del análisis de los datos del mundo real;

- un ABM es capaz de replicar y explicar la relación entre el banco y las empresas y describir, a través del análisis de la distribución del beneficio, la deuda, el tamaño de las empresas, la propiedad emergente que surge en un mundo interactivo heterogéneo [Delli Gatti y col., (2007)]. Analizamos la fragilidad financiera y la fluctuación de la economia [Gallegati et. al, (2003)] , [Delli Gatti y col., (2003), (2004), (2005)] en un sistema con empresas y multibancos en un ABM [Grillii y col., (2014), (2015)], enfocándonos en los hallazgos empíricos que el modelo es capaz de replicar.

Con el fin de abordar los objetivos que se establecieron, en la tesis simulamos tres mundo real diferente y examinamos los resultados.

Metodología

Se analizan los aspectos específicos de las empresas y el comportamiento bancarios, sus interacciones y características y la evolución del comportamiento agregado, utilizando una metodología que se aleja de la visión de la teoría neoclásica tradicional y utiliza un nuevo enfoque macroeconómicos basado en la fundación microeconomía (microfundación).

Este análisis se basa en el modelo heterogéneo basado en agentes, que está lejos de la teoría tradicional del equilibrio general. Este enfoque considera agentes heterogéneos interactuantes. Conocemos la estructura del modelo, los agentes interactúan y podemos observar dinámica agregada diferente de la suma del comportamiento induvidual. La dinámica agregada se debe a las interacciones y de ellas surgen propiedades emergentes. A través de un mecanismo de retroalimentación, el comportamiento agregado afecta al micro nivel (reacciones del agente), y, a su vez, afecta de nuevo al nivel macro. El modelo basado en agentes se ha desarrollado para estudiar la interacción de una moltitud de individuos heterogéneos. Este tipo de modelos se basan en micro−fundaciones, pero el nuevo concepto de equilibrio no requiere que cualquier elemento esté en equilibrio: un estado de equilibrio macroeconómico es manteined por un gran número de interacción [Feller, (1957)].

En los modelos que proponemos, utilizamos también, para mejorar nuestro análisis, la estimación *OLS* para estimar el coeficiente de escala de la distribución del tamaños de las empresas, lo que nos confirma que la estimación de la cola es cercana a los resultados obtenido con el anaísis con datos reales [Gaffeo y col., (2003)]. Por otra parte, las investigaciones presentadas por los físicos han demostrado que la heterogeneidad y la influencia directa o indirecta de la interacción entre las unidades trae naturalmente hacia fuera distribuciones de ley de potencia.

Podemos notar que una distribución de ley de potencia emerge para la distribución de tamaño de las empresas, la distribución de beneficios [Fujiwara, (2003)] y la distribución de la deuda [Fujiwara (2003)]. En términos de una distribución de ley de potencia, significa que las empresas están situadas a lo largo de una curva cuyo coeficiente es estable y la intercepción cambia muy lentamente en el tiempo.

Además, en el capítulo 3, para evaluar la distribución de la tasa de crecimiento para el análisis de la distribución empírica, para eliminar posibles tendencias en el tamaño de la empresa, consideramos el tamaño normalizado (logarítmico), por lo que 'de - tendencia' la tasa de crecimiento. Para el análisis de distribucińn emipírica, tanto la tasa de crecimiento como la tasa de beneficio, como en [Bottazzi y Secchi, (2006)], probamos, a través de una estimación de máxima verosimilitud (EMV), la hipótesis laplaciana con una clase distributiva más general, la distribución de *Subbotin (1923)*, también conocida como la distribución de potencia exponencial generalizada.

Contribución

Podemos mostrar, como sigue, la contribución que esta tesis doctoral puede hacerse a nivel de investigación:

- con el análisis determinista hemos encontrado la tasa de interés que garantiza un crecimiento del economia ($\bar{r}$). Hemos encontrado que hay una tasa de crecimiento invariable en el tiempo para el nivel de todas las variables (patrimonio, capital, patrimonio bancario, oferta y demanda de crédito), que es igual al *roe*.

- hemos encontrado que todas las variables (activos bancarios y firmes, demanda y oferta de crédito, capital, ...) y su nivel, crecen exponencialmente, porque son todos proporcionales a el nivel de capital (bancario o de empresa). Para calcular el modelo determinístico, hemos utilizado algunos índices contables como *roa*, *roe* y *apalancamiento*, mediante el análisis de los parámetros que caracterizan la empresa y los comportamientos bancarios.

- hemos descubierto que el *roe del banco* es igual a el *roe de las empresas*, aunque, los parámetros de los que dependen aparentemente son diferentes;

- los *roes* del banco y de la empresa dependen de la tasa de interés, pero en sentido contrario: cuando la tasa de interés aumenta, también aumenta el roe bancario mientras que el roe de la empresa disminuye.

- hemos descubierto que si la tasa de interés es igual a $\bar{r}$, el sistema banco-empresas alcanzan un estado caracterizado por un crecimiento exponencial del tamaño del banco y de la empresa y una situación equilibrada en la demanda y oferta de crédito.

- investigaciones empíricas recientes han demostrado que el mecaniscmo en que las empresas entran y salen del mercado contribuyen tanto a las fluctuaciones macroeconómicas como las empresas continuan sus actividades [Davis y col., (1996)]. Por lo tanto, cualquier teoría de las fluctuaciones de la economía (fluctuaciones del PIB [7]), como lo hacemos, deberían prestar especial atención a la forma en que en el modelo entran y salen las empresas;

- hemos descubierto que, considerando la dinámica de entradas / salidas de las empresas, las fluctuaciones del PIB se deben a la desviación es-

[7] El PIB es el producto interno bruto.

tándar *roa* y al índice herfindhal - hirshman [8]. Analizando el *roa* en nuestros modelos, podemos afirmar que la desviación estándar del PIB es igual a $\frac{\phi}{\sqrt{3}} * H$. También hemos analizado el índice de H (herfindhal - hirshman index) . Como en [Gabaix, (2011)], mostramos y confirmamos que la fluctuación del PIB depende definitivamente de la fluctuación H. También hemos encontrado que el modelo [Delli Gatti y col., (2005)] tiende a generar empresas gigantes que pueden obtener todo la cantitad de préstamos que el banco se puede permitir. En nuestro modelo, cuando el H explote, una empresa domina el sistema. Podemos argumentar, que el modelo tiende a financiar una empresa más que la otros y la fluctuación de esta empresa afectan a la fluctuación del PIB;

- hemos encontrado que en el modelo que analizamos hay dependencia entre la tasa de crecimiento agregado (fluctuación del PIB) y la tasa del beneficio de las grandes empresas.

- a través de la EMS (estimación de máxima verosimilitud) hemos demostrado que el modelo ABM puede replicar una distribución laplace para la tasa de beneficio y una distribución más leptokurtika de una laplace para la tasa de crecimiento;

- hemos descubierto que incluso en un sistema multibancos - empresas, la suposición determinista es verdadera. Por otra parte, hemos descubierto que el modelo reproduce una amplia gama de hechos estilizados

[8]Como argumentado por [Gabaix, (2011)], las fluctuaciones del PIB dependen del índice Herfindhal .

que caracterizan a un sistema dinámico complejo (mundo).

Chapter 1

The Evolution of Macroeconomics Theory

1.1 Introduction

Based on its historical path, economic theory is divided into two major periods: the first one that goes from the eighteenth century to the second half of the nineteenth century, the classical economic theory (whose proponents were Adam Smith, David Ricardo and Carl Marx), and the second one that goes from the mid-nineteenth century to the present, where the Neoclassical Economic Theory is born.

The neoclassical economic theory includes all canonical theories that count keynesian and monetary theory and the theory derived from them.

The neoclassical theory has as research object the detemination of price through a *general equilibrium (GE) approach*. This equilibrium is get by the match of demand and supply curves. These two curves are obtained as result of maximization of firms and consumers behaviours. Demand and supply curves are always in equilibrium, the equilibrium does not depend on the numbers of agents in the market or on the number of the markets in the economy (*Walras Law*).

In *General Equilibrium Theory* humans are considered rational (homo economicus) [1] and motivated by individual interest: in this theory the individual is considered the best judge of himself. The economy operates under competitive conditions and absence of frictions that may affect the market (there is perfect information).
In Neoclassical Economics there is always equilibrium, there is always a perfect match between supply and demand, there is no economy growth assumptions, and is always possible to determinate the price. The only important thing is how optimally allocate goods and resources.
The theory of the general equilibrium explains as a decentralized economy, composed by numerous independent agents that act according to their interest, is compatible with an equilibrium on all the markets. The assumption of neoclassical theory is about the microeconomic world: the agents makes rational and optimal choice, with scarcity of recourses, and based on their budget constraint. The major exponents of this theory are Leon Warlas and Vilfredo Pareto. In this theory the focus is on macroeconomics dynamics (markets dynamics) without consideing the microlevel.

The optimal choices are represented in the market equilibrium (in com-

[1] The Homo oeconomicus is a prudent man who acts with rationality after having well considered the situation: he lives in a state where he/she never takes risks. It is a median social individual that is conscious that prudence is fundamental to develop the human consortium.

plete market) in absece of asymmetric information and in presence of perfect competition. This lead to a optimal allocation of recurses (as established by Walras Theory). The market equilibrium is always Pareto efficient, that is, there is not possibility to increase the wealth of a unit without decrease the wealth of other units. Every Pareto efficient allocation can be supported as an equilibrium by some set of price. It is required to reach some particular Pareto efficient result of a redistribution of initial endowements of agents. It is possible to reach this outcome only if the market is left alone to do its work.

The general equilibrium theory attempts to explain the demand, supply and price behavior in the whole economy (in several interacting market) by seeking to prove that interaction of demand and supply will always result in an overall equilibrium. Thus the equilibrium exists and it is efficient. The GE contrasts to the theory of partial equilibrium, which only analyzes the single market.

Leon Walras is considered the "father" of the general equilibrium theory. He proposes and describes the price formation mechanism with an equation system that explain the match between demand and supply in an economic system considered as whole (top-down approach) [Walras, (1874)].

The Walras theory implies budget constraint, that is the value of excess of demand (across all markets) must sum to zero:

$$\sum_{j=1}^{k} p_j \ D_j - \sum_{j=1}^{k} p_j \ S_j = 0 \tag{1.1}$$

where p_j is the price good and D_j and S_j are respectively the total demand

of the economy and the total supply of the economy.
According to Walras Theory, considering any particular market, if all markets in the economy are in equilibrium, then the specific market we analyze must also be in equilibrium. Any market is efficient. When arise an equilibrium that is not efficient, there is a sort of *market failure.*

One of the most important contribution of Walras in neoclassical economic theory is the *Walras Auction.* Walras proposes a dynamic process by which general equilibrium may be reached (groping process). It is a type of simultaneous auctions where each agent calculates its demand for the good at every possible price and submits this to an auctioneer. The price is then set, thus the total demand across agent is equal to the total amount of goods in the market. A *Walras Auction* perfectly macth the supply and the demand of goods.
Walras suggests that the equilibrium will be achived through a process of *tâtonement* (trial and error) a form of hill climbing. The Walras auctioneer is an auctioneer that match supply and demand in a perfect competition market. There is perfect information and no transiction costs. The process *tâtonement* or groping finds the market clearing price for all commodities and from this arise the general equilibrium. The clear market price is set when total demand across agents is equal to total amount of goods. The auctioneer announces the price and the agent decides how of their goods they offer (supply side) or purchase (demand side). There is no transiction or disequilibrium price.

Neoclassical theory formulates its theories with assumptions that the quantitative change of macro-economic variables over time and / or in relation to other variables is got by a view of all phenomena tend to equilibrium and not to the change.

Neoclassical theory studies the properties of the economic system based on general equilibrium paradigm. Through the assumption of neoclassical theory are been realized economic models in which macroeconomic and microeconomic approach are intended to be two separated elements of the same system. In this framework, the theoretical constructions do not reflect the relations between the individual (micro level) and the aggregate (macro level) existing in the real world.

The first criticism moved to the neoclassical theory comes from Lucas [Lucas Critique, (1976)], that explains as the neoclassical models do not consider the microfundations as base of economic analysis and he considers impossible to explain the macroeconomic dynamics without having as reference the behavior in the microeconomic level. In fact, if we analyze a model in *GE*, if the economic policies change, we can see that the GE is constant. The model, for Lucas, have to include microfundation for the explanation of macromodel.
Economists have to study the economic decision-maker as operating through time in a complex probabilistic enviroment. Also, they have to incorporate probabilistic elements and dynamics into economic theory. The central node for Lucas includes the the dynamics in economic theory.

The *Lucas Critique* is a critique of pure macroeconomics model: model that does not present microfundation analysis to explain a macroeconomic behavior. Lucas does not get distance from the general equilibrium, but he endows it in a macroeconomics model based on microfundations.

He focused his works on economic theory that studies behaviors of optimizing agents. The law of motion of the economy is the aggregation of these optimizing agents behaviors to predict and understand the macrolevel dynamics.

Lucas critique proposes a revolution in the way to make economic theory and built economic models that envolve the economy as whole.
The model proposed by Lucas does not refuse the imperfect competition, typical object of keynesian analysis [2], although it is difficult to find an analytic solution in presence of imperfect competition.
In Neoclassical theory models are "static" , the system is in a perpetue "state of rest" and for Lucas this state of rest is an anachronism. Lucas propose a new practical way to view price and quantity path following complicated stochastic processes. Lucas wanted to change the way the economists thought.

To maximaze the agent choices in an interpolate contest, the individual choice depend on what the agent think about the future (expectations). The new approach considers *rational expectations* of the agent in a contest of the *dynamic model of general equilibrium.* Thus, the new economic model have to explain the dynamics of the system with a rule of formation of expectations. But expectations change, and what agents expect for the future affects the choice of today. In this new approach changes in economic policies are analyzed in a dynamic contest.

[2] The perfect competition was elaborated by the keynesian school that started to pay attention to markets not perfectly competitive.

Lucas talked about a new economic dynamic and microfunded theory to evaluate the effect in policy changes.
With this point of view the economic system is a result of interpolated optimization process in a contest of uncertainly of any single economic agents.
Lucas also focused on the economic cycle, he view the economic cycle not as a point (of equilibrium), but as a path, that is an equilibrium path. This mechanism is inherent in a process of dynamic optimization of agent in an uncertainty conditions.
Althought Lucas uses imperfect information (but perfect competition) in his works he is not able to reproduce a cyclical realistic fluctuations.

From the new point of view proposed by Lucas, it is born the idea to build models that start their analysis from choices and preferences of units at microeconomic level. The problem, in this new theory, is the problem of aggregation: how to aggregate the microbehavior to analyze the macrodinamics.
To solve this problem, the new theory to build a macroeconomic model gets back the *Marshal* idea of representative firms. From this concept is born the figure of an agent that is able to represent the maximization of any agent prensent in the market.

Thus, with the [Lucas Critique, (1976)], the concept of *representative agent* was used, and now too, to understand and explicate the problem related to optimization and to choice. Lucas introduces also the possibility to do not have perfect information in the system, although he assumes that each individual agent is able to choose with racionality.
The macrodynamics are evaluated as simple sum of the individual behaviors

represented by the *representative agent* (*RA*). From *Lucas Critique* on the figure of the *representative agent* , that is un individual agent that sums up the expectation, the preference and the choices of a class of individual units, is caught in the economic research. The *representative agent* , called *RA*, has rational expectation and takes decision s that are optimal for the welfare of the whole aggregate.The behavior of the economy in its whole is reduced to the study of the simple sum of microeconomic units behavior: they isolate and analyze the microeconic units of the economy (the individual and his/her behavior as single part) to understand the macroeconomics level behavior. The *RA* is an hypotetic agent that has all the wealth of the economy. It is used as semplification instrument to solve the problem of the optimal allocation and the problem of complexity in economy, due, by definition, to the exclusion of direct interaction.
This new approach, called *reductionust approach*, of the economic theory, it is based on phyisic assumption. In the reductionistic approch from the microeconomic investigation we can perfectly reproduce the behaviours at macroeconomic level.
In this new framework, using microfundation to understand macrodynamics, the result get by induvidual behaviors (micro level) is able to predict the dynamics at macro level: there is no difference between micro and macro, because the dynamics at macro level are the simple sum of the individuals behaviours.

Today the neo-classical economic theory, or mainstream theory, is the base of most of the major research work.
This framework is characterized by specific assumptions about human be-

havior: every human being is identical to the other one [Parisi, (2006)] and any person own all the information that need to make rational decisions. Any humans being takes decision as if it were equipped with perfect ability to reasoning and evaluation.

The traditional application of microfundation to analyze the macro level, as sum of single units, has shown the failure of the *RA* and the new needs for the economist to find a new better way to analyze the macroeconomics dynamics.

In the years economists have tryed to leave the neoclassical assumption in which the fully rational agent take optimal decision with perfect information, and started to consider other aspect as imperfect and incompleate information (asymetric information), the competitiveness and the analysis of the relations among agents that have local interactions.

1.2 From RA to Agent - Based Model

The model based on RA has been intoducted in 70s with the new classical macroeconomic model. There are two reason for which RA are used in modern macroeconomy theory: (1) Lucas' critique and (2) the Warlasian's tradition of equilibrium model. The new macroeconomic has three important charatestics: (1) agents choices are based on real quatities, (2) agents are in equilibrium and (3) agents have rational expectations [Hoover, (1988)].

The construction of RA models is a consequence of Lucas' critique. The RA is born to assure the best way to construct macroeconomic models with microfundation, to avoid the problem caused to solve the complex analytic

structure of the models (think about the introduction of imperfect information or imperfect competion in the model and the difficulty to solve the anlytic formulations with these new assumptions).
With RA economists regain the total control of mathematical method to solve macroeconomic models in a contest of welfare economy. The RA seems to be an instrument that allows to develop economic Walrasian models.
The RA is an answer for the problem of microfundation in macroeconomic models.
In this new contest economists attempt to create a relation, a consistency, between macroeconomic model and microeconomic theory.
New macroeconomic models are based on the choice, rational choice, of an agent that is able to take decision equivalent to what the whole set of agents do.
Also, in years in which computer thecnics were not so developed, the RA was the best way to get microfunded macroeconomics models.
The RA endows the tipic agent behavior in a given time and "freeze" his features. The first step is the maximization of the RA utility. The parameters of the object function (reducionism optimization approach) are constant over time and there is not any relationship between evironment variations and agents preference. In the RA model all agent in the system are supposed to be identical.

This simplified assumption used in macroeconomic model to get models with microfundations is obviously unreal. In the RA model aggregate is a mery sum of the RA behavior. If the behavior agents function is not linear and the features of agents are not identical to each other, there is always an error in aggregation of function behavior. It is unreal to think that the macrodynamics are well approximated or that the economy behavior is well represented by a typical and unique type of agent.

These models, with RA, are microfunded models, but they are not able to represent the macroeconomics dynamics and the complexity of macroeconomic real world.

With the imposition of RA hypotesis , economists delate an essential factor for the determination of economic enviroment: *the interactions among agents.*

The very simple thing, but important, is that the economic system is formed by millions of individual decisions and any agent persues its goals with direct interaction wirh other agents and with the environment, creating the "emergence" [3] of a collective behaviour. The RA simplify the analysis, but it creates a lots of problems at the aggregate level.

Based on [Kirman,(1992)] there are moltiple reasons that explan the failure of RA and its inefficiency to describe an economic system as a complex system.

First of all, the maximization done at the individual level do not represent the aggregate rationality because the behavior of a person (unit, agent) may be different from an aggregate behaviors. Second, even if we are able to accept the rational individual choice as representative of aggregare behaviours, it will be some problems: the reaction of RA related to the change of sit-

[3] With the word emergence we refer to the arising of stable and ordenarly aggregate structure from simple adaptative individual behavior rules [Epstein, Axtell, (1996)].

uations could be different from the reaction of the aggregate of individuals (units, agents) he represents. Third, the maximization made by RA related to good A with respect to good B could be different from the choice took by the aggregate, that could prefer good B instead of good A. Fourth, the aggregate behaviour that emerges from a mery sum or the RA behaviours leads to macroeconomics behaviours that conducts to unnatural behaviors of any single agent.

This process leads to a big problem about the model based on RA: *the problem of aggregation.* To get strict aggregation of individual preferences, RA models have to impose stricted hypotesis on the function that represent the agent's behaviours and on the function of wealth distribution.
Any kind of aggregation of preferences is chosen it affects the way the RA reacts, and he/she can make mistakes or/and they can reveal inconsistent with the sum of the all agent's reactions. Furthermore, the way to aggregate the preference affect the reaction of RA and this reaction may be contrary compared to the sum of the reaction of all other agents [Keller, (1980)].
Moreover, RA model often requires that the equilibrium is unique and stable.

There is a *fallacy of composition* [Delli Gatti et al., (2006)], that is, to justify RA hypotesis the individual should own determinated properties. Only with determinated individual properties the aggregate is well represented by RA.

It is impossible make behavior hypotesis of the agent property (as preferences) that can ensure the uniqueness and stable equilibrium at aggregate level [Fagiolo and Roventini, (2008)].
Moreover, the way choosen for aggregation of preferences affects also the be-

haviour of the economy, that is, the presence or the absence of unique and stable equilibrium.
Even if the agents are all idential to each other (same preferences, same initial endowement) the uniqueness equilibrium and the stability can not be assured [Kirman and Koch, (1986)] .

Moreover, the function of the RA is assumed as invariant to policies, and this is not true in the reality [Geweke, (1985)]. If we use the RA and we consider the change in policy, the aggregate function that we get will be incorrect assesment of the effect of the policy change.

The assumptions related to a RA model only make sense if the agent take their decision on the bases of macroeconomic variables and only if all individual have the same marginal reactions [Stoker, (1993)].
Thus, if we use the RA, bigger is the class of individuals we consider, higher will be the magnitude of arbitrage that we can observe in the aggregate level .Economists break away from the RA approach to attempt to analyze not complicated system, but *complex system*.

The analysis at macroeconomics level of a system has to be done as an analysis of a complex system where interacting heterogenous agents make optinal choice based on the information they own (there is asymetric information) and based on the starting situation they had. The bases of the new point of view of the economic system are the microfundation and the microinteraction and not individual isolate choices.
Analyze the system not as a system inhabited by isolated individuals and choices but as a microfundated system that evolves and in which heterogenous individuals (agents) interact (heterogenous interacting agents, HIA)

leads to usefull information to study the aggregate activity and dynamics of a complex system [Kirman, (1992)].
As we said, the RA ignores the presence of micro level interactions and direct/indirect interactions among agents, beliving that the individual behaviours lead, anyway, to a good analysis of the evolution of the aggregate. The RA analysis of a complex system leads to wrong and confusing conclusions. If the economic agents are heterogenous, if the interactions among agents have some effects on the economic dynamics and if the agent behaviors are not formulable by typical function, if the agents do not act as unique representative maximizing agent, then the empirical evidences and results become unreal and not significant. The RA hides and delete the direct interactions among agents [Delli Gatti et al., (2006)], although we know that in the political economy has always been considered the "interaction in the market" (microlevel interaction) without economic interest.

[Lippi, (1988)] shown as the aggregate behaviour can be derived not by a complicate maximization of RA, but it can be replicated by aggregation of heterogenous agents.

From the 70s economists point of view has changed. Economists try to explain the economic dynamics using natural law to justify that the economic behavior does not require the adoption of *reductionism* paradigm. Physicists have shown that scaling laws are generated by HIA [Marsili, Zhang, (1998)], [Amaral et al., (1998)] and this is not compatible with reducionism approach. Moreover, scaling laws are not compatible with the mainstream economics. Interactions among agents exist, we can see them every day. According to the *holistic approach*, the aggregate is different from the sum of

its component, cause the interaction of units. In this new vision economists pass from the reductionist approach to holistic approach. In the *holistic approach* the models are focused on the complex system dynamics. A system is analyzed defining its aggregate poperty only after studying the individual units that compose the system and the feeedback mechanism that elapses from single units and the system. The principal and most important point of the holistic approach is that there is a substancial difference between the macro-level dynamics and the mery sum of each single part. These models (models with heterogenous interactng agents) show the lack of robustness of microeconomic fundament used as sum to generate macro dynamics. RA analysis show also limit related to macroeconomics relations that they got throught the observation of rational behaviours of single agent.
In the *holistic approach* the equilibrium of the system does not require that any element is in equilibrium, the aggregate is quasi-stable: a large number of transictions in opposite directions generate and maintain a state of microeconomic equilibrium. It is reached an *static equilibrium* in which the aggregate equilibrium is compatible with the individual disequilibrium [Feller, (1957)]. The economic system is formed by agents, heterogenous agents, that take decisions, that interact, that have different preferences. Thus, it is very important to consider the interaction among agents, because in this interactions emerge the macroeconomic dynamics.

The normal interactions that affect the agents are indirect interactions. The agent decisions determine demand and supply, that determine the prices which affect the whole set of agent choices.
The interaction on preferences happens when the preference related to agent

alternative situations depend on the action of the other agents.
The disagreement between the Solow theory of growth, that describe the convergence of economies in the long-run, and the empirical observation, that present consistent divergences between poor and rich countries, have highlighted the needs of theory based on agent interactions.

In this contest, in this empty space from macroeconomic behavior and microfundation, it is born the new theory of complex economic system with heterogenous interacting agents: the Agent - Based Model (ABM) overlook on the academic space.

1.3 Complex System and Simulation: ABM Methodology

The lack in the dominant economic theory pushed to the development of a new economic approach, theories and methods able to represent the economics system reality in more satisfactory way.
The mainstream macroeconomic theory remains firmly rooted on *general equilibrium with microfundations* [Colander, (2006)]. This theory put emphasis on the isolated optimal choice behaviours of agents and on profit maximization. The equilibrium state is got by external imposition of specific conditions that required fullfilled expectaction and market with perfect competition .
There was no space for important real world factors, incompleate market and

imperfect informations. Up to the recent crisis, nowadays, there are *DSGE* models approach that study the economy with financial frictions (new Keynesian model) [Eggertsson & Krugman, (2012)], [Guerrieri & Lorenzoni, (2016)].

It has shown clear the need to create models throught the representation of heterogenous agents that interact, creating no-linear interaction.
From the mid of 1980s, several economists have attempted to develop agent-based computational economy to capture the complexity of the real world.

The models with interaction among agents have to answer to two important questions: (i) how the individual behaviors are affected by the aggregate behavior and (ii) how the group are formed.

The needs of macroeconomic microfundated models to get significant macroeconomic theory is clashed to the difficults of the mathematical analysis of the economy.
The economy is a complex system [Tesfatsion, (2005)] where choices of an agent affects choices of the other agents and affect also the environment in which agents live.
Relize and formalize mathematical assumption of the economic complex system is very difficult, from this problem arise the need to build economic model with a "bottom-up" prospective. In this prospective the analysis is formed on the micro features of economic system with strong contrast with the "top-down" prespective of the tradictional macroeconomics theory [Fagiolo and Roventini, (2008)].
These new models are models based on agents, a million of agents, and any

agent has his properties and his wills. The behavior of these different agents become the behaviour of the aggregate and often it may be very different from we are able to deduce from the mery features of the single components [Terna, (2006)].

The emergent properties that arise from different agent interactions can be considered as the *invisible hand* described by Adam Smith: if the equilibrium exist, it arise from the individual choices of heterogeneous interacting economic agents [Tesfatsion, (2005)], [Axtell, (2005)].

The need of appropriate microfundations, the improvement in science and in computer tools helped to develop a new computational approch to the economy. This approach analyzes and studies the economy as a complex dynamic system [Gilbert, (2004)].

The world is a complex system. The complex system is made of many highly interconnected part of many scale, the interaction of which, results in a complex behavior requires separate interpretations of each level (micro, meso, macro). This realization leads us to undestand that new features emerge as one moves from a scale to another. Thus, complexity is about revealing the principles that govern the way in which these new properties appear. Each level is charaterized by new emergent law that rule it. These systems are self-organized with self-adaption and scaling (for example Power Law dependence). In a complex system, we accept that many process occur at same time in differt levels and that each process is important to generate the intricate behaviour of the system. The behavior of the system depens on its units considered together. In a complex system randomness and determinism are both relevant for the system behavior. These systems (complex systems) exist at the edge of caos: over time the system can change stochastically (and dramatically) as result of small change [Vicsek, (2002)]

Over the years economists have attempted to use the microfundations assumption in a way more wide and interacting. The analysis has started from heterogenous individuals and from their local interaction to create models of agents that are able to replicate complex situation comparable to the real world and arise a new concept of the *emergent stucture* [Terna, (1999)].

The Agent-Based Model is a methodology to represent simulation model based on object-orienting programming. In Agent-Based model we use the informatic simulation instead of mathematical analysis. The thecnique to redact an informatic simulation needs to be carefully studied. We have to pay more attention to simulation thecnique because we can incur in some mistake in the code.

The agent-based model is necessary to explain what *emerges* from the evolution of complex system. With word *emersion* we mean spontaneous formations of *auto-organized* structure at different levels of a hierarchical system configuration.
In the complex system is not possible to focus only on the average behaviour of its units, but we need to know the behaviour of any single agent to understand the dynamic of the system and its complexity.

Any individuals behaviours and the interactions among individuals generate a process that creates a entity visible as whole. The aggregration process

is favoured by catalizators. A catalizator meets one or more molecules (agents in economic analysis) and generates the autocatalitic process of aggregation. The *autocatalitic process* grants that the behaviour of the whole system is dominated by element with autocatalitic growth rate higher than the same recorded by average element. [Solomon, (2007)]. The autocatalitic processes are dynamic processes. The autocatalitic process implies that looking at average or most probability behavior is not representative of the dynamic of the system. The autocatalitic dynamic is the way to understand the emergence of free-scale distribution (as Power Law) at aggregate level.
At any level of organization of the system emerge properties that are not defineble *a priori* by the simple sum of lower level.

The Agent-Based Model use mathematical formulation to explain the relation that exist among aggregate variables. The work method is based on the use of computer simulation that is able to replicate the real world, agents that live in and the relationship that they have [Parisi, (2006)].
Thus, the Agent- Based Model of complex adaptive system is a system inhabited by a multitude of heterogenous objets (agents) that interacts with each other and with the environment.

The agent based models include also the ACE (Agent based Computational Economics) and MABM (Macroeconomics agent based models). The first one is a computational study of economic process modelled as dynamics of interacting agents [Tesfatsion, (2002)]. There are studies of [Epstein and Axtell, (1996)] and [Tesfation and Judd, (2006)] on Agent- Based computational economics (ACE) that is the computational study of economic policies modelled as dynamic system, of interacting agents. The second one describes and explains a complex adaptive system in which a multitude of heterogenous agents (firms, banks,..) interact with each other. The heterogenity can come from various factors: productivity, price, size, technology and so on.

The aggregated variables, as, for example, GDP, are computed, as we said before, "from bottom up" . In the bottom-up approach, the individual behaviors are modeled according to simple behavior rules. Agents can have local interaction and change the individual rule through adaptation. Through aggregation arise some statistical regularities that is not possible to predict from the observation of the individual behavior (*self-emerging regularities*). Thus emergent behavior feedback to the individual level and this mechanism generate a macrofundation of micro [Colander, (1996)]. The ABM is charaterized by: heterogenous interacting agents, explicit space, local interaction (rich interaction structure), bounded rationality (information are private and limited and agents are endowed with finite computing capacity) and non-equilibrium dynamics. The model has a recursive dynamic system in which state at $t+1$ is computed starting from a state at time t. In this way, it is possible to investigate what happen between start and end route.

With the ABM we are able to explain and show that the major part of the real world is controlled more by the *tail* of the distribution than by the average. It is controlled more by the expectional events than by mean events. It is controlled by catastrophes, not by normal events, by very rich calss and not by medium class..

The pourpose of Agent-Based Model is also to free the economist from the "average thinking" [Anderson, (1997)].

To build (simulate) an Agent - Based Model we have to identify the object of study and, then, structurate a model through abstraction process that are theoretically motivated. Afterward, we can observe the behaviors of the model comparing them with the *stylized facts* [Gilbert, (2008)] [Terna, (2006)]. For example, the stylized fact of the GDP distribution is a Power Law distribution and the economy growth rate distribution is well approximated by a Laplace distribution (see [Delli Gatti et al, (2005)]. The power law distribution means that firms are located along a curve whose coefficient is stable and the intercept changes very slowly over time. The transiction from one state to another is affected by stochastical change and agent actions [Bottazzi and Secchi, (2003)]. [Stanley, (1996), Amaral (1997)] obtain a laplace distribution of growth rate, just relaxing the assumpion of the independence of firms' growth rate.

In Agent- Based Model, as a simulated model, the relation among agents is not symplified and the analysis assume complex traits. We dispose of all statistic method that we are able to use when we analyze the real data, but the advantage is that we know pefectly the structure of the model.
Models are the methodology of the artificial life (ALife): they are the synthesis of the actual life in the computer.
With the Agent-Based Model the analysis move towards a more systemic investigation of the performance of learning algorithms [Tesfatsion, (2002)] to explain economic dynamics and individual behaviors.

Chapter 2

Analytic Study of ABM Model

2.1 Introduction

The recent economic crisis has led economists to consider the financial factors responsible about the trend of the aggregate assets in different countries. In general, the economic system consists of a multitude of firms with different structures and financial needs. In this perspective, as in [Delli Gatti et al., 2005] the primary need is the analysis of firms' financial fragility, that is the firms' ability to pay their debts over time. In the modern capitalist economy market − with sophisticated financial institutions − the financial fragility is described by [Misky, (1982)] as a state in which any event, also common event, can generate a default chain and financial troubles. The degree of financial fragility is determined by the average financial situation of economic units, the level of their liquidity and the share of borrowing that firms need to finance during their investment. We consider the financial position as the ability to generate profits to pay off the obligations. In this framework could not be overlooked what – in recent years – has been the core of international economic discussions, that is the analysis of large firms which affect the behavior of aggregated economic variables. From this point, we focus on aggregate level and on firms behaviours, that express the economic dynamics.

The dynamics that envolves technological and economic processes may be conceived as the result of complex patterns generated by the interaction of heterogeneous adaptive agents [Krugman, (1996)], [Delli Gatti et al., (2005) (2008)].

We present the analysis of firms financial fragility , the long-run dynamics of the aggregate and of the firms size distribution. This analysis is based on the Agent-Based Heterogeneous Model approach which is far from the traditional *Mainstream* theory of general equilibrium.

This approach considers heterogenous interacting agents. We know the structure of the model, the agents interact and we can observe aggregate dynamics different from the sum of the induvidual behavior.

The aggregate dynamics are due to the interactions among agents and from them arise emergent properties. Through a feedback mechanism, the aggregate behavior affects the microlevel (agent reactions), and, in turn, it affects again the macrolevel.

The Agent-Based Model has been developed to study the interaction of many heterogeneous agents. Hence, these kind of models are based on micro-foundations. The new concept of equilibrium does not required that any element is in equilibrium: a state of macroeconomic equilibrium is manteined by a large number of interactions.

The model we present is based on asymmetric information, heterogeneous agent and no directs interactions among agents. In the model, the heterogeneity of the agent is due to the randomness of the price of the homogeneous goods.

The model is based on *heterogenous interacting agents* (HIA) and the model simulation process is a random process. The transiction from a state to another is influenced by agents systematic actions and by chance.

We observe how firms with different level of financial robustness interact and compete in the market to get the best interest rate, the higher amount of credit and get the optimal level of production. The financial fragility imposes the conditions of firms failure. Higher leveraged firms (fragile from the financial side) are exposed to higher risk of default. In fact, the bankruptcies are due to negative value of equity. The financial fragility affect the firms growth through the decreasing access to bank credit.

The aim of our analysis is to understand the economic behavior of a system inhabited by *HIA* using the [Delli Gatti et al., (2005)] model and analyze its features building and studying a *deterministic version of the model.*

Our pourpose is also the analysis of long− run dynamics of the aggregate to study the analytic growth, through the *deterministic model analysis.* Other pourpose is to show that the deterministic model is able to replicate the dynamic of growth of the stochastic model.

The chapter is organized as follow. In the *section 2.4* we explain the model as in [Delli Gatti et al., (2005)]. We refer to this model as the stochastic version of the model, called *the stochastic model*, where firms heterogeneity is based on price.

In *section 2.3* , to understand the stochastic model, we try to study analitically a deterministic version of the model , called *the deterministic model*, composed by *1* firm and *1 bank* with price equal to the expected price and it is constant over time.

The deterministic model is not able to replicate the heteogeneity of the model, so, to keep the heterogeneity of agent in the model, we study, in the *section 2.3.3* a variant of the deterministic model. We keep the price equal to the expected price: it is constant over time and among firms. Also in this case, there is only a bank that interacts with the firm. So, we set the model with *100 firms* with imposed initial financial condition (random entry capital values). What we want to show is that the deterministic model (with imposed heterogeneity) is able to generate the trends of the stochastic model dynamics as economy growth rate, equilibrium interest rate, ROE and ROA. In the *section 2.4* we analyze the long$-$ run dynamics of the model, paying special attention to the law of motion of the aggregate of equity. In the *section 2.5* we analyze, explain and compare the results of the simulations of the different models, studying their features. In The *section 2.6* we conduct an analytic study focused on the GDP fluctuations. In the last section we propose a conclusion of our analysis.

In the Appendix we show the analysis of the ratios equity-capital and loan-capital and also parameters and variables settings.

2.2 The Model: Delli Gatti et al. 2005

In the model it is assumed that: (i) agents are heterogeneous, (ii) there are *no direct interactions* between them, (iii) there are market frictions such as asymmetric information and (iv) agents are able to sell all goods (homogeneus goods) that they believe are an optimal level of production, the goods market condition depends only on supply side, as in [Greenwald and Stiglitz, (1990), (1993)]. Thus, our model is *fully supply determined.* [1]

2.2.1 Firms Behavior

At any time t, with $t = 1, .., T$,the economy is populated by a constant number of firms N, whose indexes are $i = 1, .., N$. At each time t firms produce homogeneous goods with only an input, capital K and with a constant level of technology ϕ [2] ,hence the level of production of $i - th$ firm is:

$$Y_{it} = K_{it}\phi. \tag{2.1}$$

At any time t, firms sell their production and the individual selling price is, , as in [Greenwald and Stiglitz, (1990), (1993)], the random outcome of

[1]Greenwald and Stiglitz (1990, 1993a) assumptions are: (i) firms face price uncertainty in the form of probability distribution of the individual firm's sale price around the market price of output; (ii) full firms equity rationing, that is, the demand for input (Kapital) is got by internal (equity base) and external (bank credit) funds; (iii) standard debts contract between firms and bank.

[2]In the model the level of technology ϕ is equal among firms and constant along time and the capital stock never depreciates.

a market process around the average market price of output P_t, according to the law $P_{it} = u_{it}P_t$, with expected value $[E(u_{it})] = 1$ and finite variance. The price of firms' output is a stochastic variable. As a consequence, the normalized price is:

$$u_{it} = \frac{P_{it}}{P_t} \tag{2.2}$$

The u_{it} is a random variable uniformly distributed on the support $[0, 2]$.

By assumption, firms are fully rationed on the equity market and the only external source they dispose is bank credit. The $i - th$ firm finances its capital using both internal net worth (or equity) E^f_{it} and external sources (bank loan) L_{it}, according to the balance sheet identity $K_{it} = E^f_{it} + L_{it}$. Under the assumption that firms and bank hold a long-term contractual relationship, the debt commitment in real term is $r_{it}L_{it}$, where r_{it} is the interest rate and it is also the return of net worth. Thus, each firm face financial cost equal to $g\ r_{it}(L_{it} + E^f_{it})$, that is $gr_{it}K_{it}$ and we assume that g is another financial costs related to the capital $(g > 1)$.

The equity at time t, E^f_{it} is given by the equity at previous time and the profit π^f_{it} at time t, where π^f_{it} is the firm's profit (see below). Furthermore, in this model, firms go to bankrupt when their net worth becomes negative, that is $E^f_{it} < 0$.

$$E^f_{it} = E^f_{it-1} + \pi^f_{it} \tag{2.3}$$

At the end of each period t, the profit depends on the choices made by the $i-th$ firm about capital, level of production and level of credit loans. Thus the profit is given by:

$$\pi_{it}^{f} = u_{it}Y_{it} - g\ r_{it}K_{it}. \tag{2.4}$$

Where the *expected profit* is given by the equation below :

$[E(\pi_{it}^{f})] = [\phi - gr_{it}]\ K_{it}$. Make use of equation (2.3) and equation (2.4), the

firms bankruptcy state occurres when:

$$u_{it} = \frac{1}{\phi}(gr_{it} - \frac{E_{it-1}^{f}}{K_{it}}) \equiv u^{*}{}_{it} \tag{2.5}$$

where $u^{*}{}_{it}$ is the normalized price of the output of firm i. If the i-th firms sell

its output below $u^{*}{}_{it}$, it will go to bankruptcy. In the otherway, the failure will occur whenever $E_{it}^{f} < 0$.

Thus, the probability of i-th firm bankruptcy $[PrB_{it}]$ can be written as follows:

$$\begin{aligned} PrB_{it} &= P\ [E_{it}^{f}\ <\ 0] \\ &= P\ [E_{it-1}^{f} + \pi_{it-1}^{f}\ <\ 0] \end{aligned}$$

$$= P\ [E^f_{it-1} + u_{it}\ \phi\ K_{it} - g\ r_{it}\ K_{it}] < 0$$
$$= P\ [u_{it} < u^*{}_{it} = \frac{g\ r_{it}}{\phi} - \frac{E^f_{it-1}}{\phi\ K_{it}}]$$

We want to remeber that :

− the price is uniformly distribuited, so $u_{it}\ \ U\ [0, 2]$,

− the price expected value is $E(u_{it}) = 1$,

− and $u{*}_{it}$ is the critical value of the selling price. To get the value of $[PrB_i t]$

we have to solve the integral below:

$PrB_{it} = \int_0^{u^*_{it}} p(u)du = \frac{1}{2}u^*_{it}$,

Since we assumed prices to follow a uniform distribution *p(u)* in [0, 2], it holds, for instance, $PrB_{it} = 0.5$, if $u^*_i = 1$. Furthermore, PrB_{it} increases when: (i) the net worth at $t-1$ decreases; (ii) the capital and the total debt increase, and (iii) the variable costs increase.

Given that the equity base of the previous period depends on the past profits, the history of the firm profits affects the current probability of bankruptcy, i.e., there is *path dependence* as in [Greenwald and Stiglitz (1993)] [3].
We focus on a model with bankruptcy in which as firms produce more, the probability of bankruptcy increases: higher is the production, higher is the probability of bankruptcy. Bankruptcy is a cost, and firms take these costs in to account in their production decision.

Thus, the probability of bankruptcy, PrB, is incorporated directly into the firm's profit function because bankrupt is a cost, and this cost increases

[3]They argue that as firms produce more, they must bear more risk.

with the firm's output. That *function of bankrupt cost* is quadratic, that is $C^f_{it} = cY^2_{it}$, with $c > 0$.

The objective function, Γ *function*, is the difference between the expected profit, $[E(\pi^f_{it}])$ and the bunkruptcy costs times the bankruptc probability, $(C^f * PrB)$, that is:

$$\Gamma = (\phi - g\ r_{it})\ K_{it} - \frac{c\ \phi}{2}(g\ r_{it}\ K^2_{it} - E^f_{it-1}\ K_{it}) \tag{2.6}$$

Then, we maximize the function Γ with respect to K, finding K^*_{it}, that is the i-th firm's *desired capital*. Thus, we get:

$$K^*_{it} = \frac{(\phi - g\ r_{it})}{c\ \phi\ g\ r_{it}} + \frac{E^f_{it-1}}{2\ g\ r_{it}} \tag{2.7}$$

At any time the $i - th$ firms have a target level of K_{it} equal to the *Desired Capital* K^*_{it}, the difference between K^*_{it} and K_{it} is the *Investment* ,i.e., I_{it}. That is : $I_{it} = K^*_{it} - K_{it-1}$

To finance the *investment*, the $i - th$ firm recurs to profit and, if it needs, to new debt, so

$$I_{it} = \pi^f_{it} + \Delta L_{it} \tag{2.8}$$

where $\Delta L_{it} = L_{it} - L_{it-1}$.

The $i-th$ firm *firms' credit demand* L^d_{it} - that is their *liabilities* - at time t depends on the level L^d_{it-1} at previous time, on the level of investment, and on the amount of previous profit. [4] Hence:

$L^d_{it} = L^d_{it-1} + I_{it} - \pi^f_{it-1}$. We can see this *i-th firm credit demand* as the

difference between K^*_{it} (desired capital), the profit at time $t-1$, π^f_{it}, and the equity at $t-1$, E^f_{it}. Thus, we can also write the i-th firm credit demand as follow:

$$L^d_{it} = \frac{(\phi - \ g \ r_{it})}{c \ \phi \ g \ r_{it}} + \frac{E_{it-1}}{2 \ g \ r_{it}} - \pi^f_{it-1} + E^f_{it-1}. \tag{2.9}$$

2.2.2 Banking Sector

Firms and the Bank interact in the credit market: firms finance their production by the internal source (equity) and by external credit. the Bank provides the credit to firms.

The interaction between the firms and the bank it is a long-term relationship.

The level of credit that the bank can loan is functional of its equity and

[4] The profit is the firms self-financing. Thus, a positive profit reduces the need of external credit.

the level of deposit [5] That is:

$$L_t^s = (E_t^B + D_t^B). \tag{2.10}$$

To determine the aggregate level of credit supply we assume that banks are subject to a prudential rule such that $L_{it}^s = \frac{E_{t-1}^B}{\nu}$. The bank manages the risk of outstanding debts by reducing the level of supply of credit. This process is possible applying a risk coefficient. So, the Total Loan Supply , that the bank could lend , is the ratio between the E_{t-1}^B and the risk coefficient ν [6] :

$$L_{it}^s = \frac{E_{t-1}^B}{\nu} \tag{2.11}$$

where ν is a the risk coefficient : the healthier is the bank from a financial point of view, the higher is the aggregate of credit supply [Hubbard et al., (2002)].

The credit is allotted to each individual firm i on the basis of the mortgage it offers, which is proportional to its size, and to the amount of cash available to serve debt. Thus, the real credit that the Bank allows to the $i-th$ firm is related to the ratio of $i-th$ firm equity and aggregate equity and the ratio of $i-th$ firm capital and the aggregate capital.

$$L_{it}^s = \lambda \ L_t^s \ \ k_{it} + (1-\lambda) \ L_t^s \ \ e_{it}. \tag{2.12}$$

[5] The value of bank deposit, D_t, in this model is $D_t = \sum L_{it} - E_{t-1}^B$.

[6] The coefficient is constant along time and it is in accord to the Basilea II agreement

where e_{it} is the level of firm financial fragility [7] and k_{it} [8] is the size of the firm business capability. The coefficient λ is the parameter to allot credit according to capital and equity [9].

The bank plays a prudential role; it sets the level of *loan* in the credit market. The equilibrium in the credit market is got by the *equilibrium interest rate*. The interest rate of the i-th firm is determinated when the demand of credit equals the supply of credit.

$$r_{it} = \frac{2 + E^f_{it-1}}{2\ g\ c\ [(\frac{1}{\phi}) + E^f_{it-1} + \pi^f_{it-1}] + 2\ c\ g\ L^s_t\ [\lambda\ k_{it} + (1-\lambda)e_{it}]}. \tag{2.13}$$

A rise (decrease) in profit and in equity base, decreases (rise) the interest rate.

If we assume that the return on bank's equity is given by the average of lending interest rate $\overline{r_t}$ and the deposit is remunerated with the borrowing rate r^a_{it} , that is $r^a_{it} = \overline{r_t}$, the bank profit, π^B_t, is:

$$\pi^B_t = \sum r_{it} L^s_{it} - \overline{r_t}[(1-\omega)D_{t-1} + E^B_{t-1}]. \tag{2.14}$$

where $\frac{1}{(1-\omega)}$ is the spread between lending and borrower interest rate and ω is the degree of competition in the banking sector: the higher is ω, the

[7] e_{it} is $E^f_{it-1}\ /\ \sum_{i=1}^{N_{t-1}} E^f_{it-1}$.
[8] k_{it} is $K_{it-1}\ /\ \sum_{i=1}^{N_{t-1}} K_{it-1}$
[9] The value of λ is $0 < \lambda < 1$.

higher is the interest spread . The interest spread increases with a higher monopolistic power of banks, [Delli Gatti ed al., (2003)].

The bank equity E_t^B is given by:

$$E_t^B = \pi_t^B + E_{t-1}^B - B_t^{debt}. \tag{2.15}$$

where B_t^{debt} is the sum of bad debts, that is the sum of the equity of the bankrupted firms, $\sum E_{it}^f < 0$.

An increase of *bad debts* changes the aggregate of credit supply: it turns down. Consequently, the financial costs increases beacause of the higher interest rate. The firms equity distribution affects the average of lending interest rate, that , in turn, influences the bank profit and, hence, the credit supply.

The firms dynamics affect each other through indirect interactions.

Thus, when *i−th firms* goes to bankruptcy, the aggregate of credit can go down, the interest rate can rise up and the risk of bankruptcy of survivor firms can increase. The *domino effect* could be a consequence of firms' failure.

2.3 Deterministic Model

The first step we do is to study the dynamical system *firm-bank* without price shocks, in order to characterize this very simple system. We consider the *firm-bank* deterministic version as a benchmark to describe afterward the original version with stochastic shocks and N firms.

All this part of the thesis is original and, we can see, will be very usefull for a full description of the original model.

The deterministic version of the original model is constituted by a firm and a bank, with the price equal to the expected price of the stochastic version of the model ($E[P_t] = 1$). In this way the firm is not subject to the shocks on the price

The firm's *profit* is equal to the expected value of the stochastic version, that is:

$$\pi_t^f = (\phi - g\ r_t)\ K_t \tag{2.16}$$

With the deterministic analysis we want to compute the value of equilibrium of the interest rate. We will find a time invariant growth rate for all the level variables (equity, capital, bank equity, supply and demand of credit).

Now, we consider the equity law of motion of the firm and bank. We assume that bad debt is zero, which is equivalent to assume that the single firm does not fail.

$$E_t^f = E_{t-1}^f + \pi_t^f \tag{2.17}$$

$$E_t^b = E_{t-1}^b + \pi_t^b \tag{2.18}$$

where π_t^f is the firm profit and π_t^b is a bank profit.

Lets start our analysis , focusing on firm and bank *roe*, *roa* and *leverege*

2.3.1 Roa, Roe and Leverage

In this section we compute the return on assets of the firm. The *roa* is the return on assets ρ^f, that is what a firm gains through the use of its resources in its entrepreneurial activity.
The ρ^f is the ratio between the firm profit π_t (in our case it is net profit) and the total assets. In this model the total assets are well approximated by capital K_t; thus we have:

$$ROA_t^f = \rho_t^f = \frac{(\phi - g\ r_t)\ K_t}{K_t} = (\phi - g\ r_t) \tag{2.19}$$

The *roe* is the return on equity χ, generally it is what the firm gains in its activity instead to direct its resources to alternative investment (bond, shares..).
In the model the *roe* (return on equity) is essetially the growth rate of the equity (both firm and bank), respectively,$g_t^E = roe^f$, $g_t^{E^b} = roe^b$.

Using the equations (2.17) and (2.18) we can demonstrate that the *growth rate of the equity* is equal to the *roe*. So, we have:

$\frac{{E^f}_t - {E^f}_{t-1}}{{E^f}_{t-1}} = \frac{\pi_t^f}{{E^f}_{t-1}}$ => $g_t^E = roe^f$ firm growth rate

$\frac{{E^b}_t - {E^b}_{t-1}}{{E^b}_{t-1}} = \frac{\pi_t^b}{{E^b}_{t-1}}$ => $g_t^{E^b} = roe^b$ bank growth rate

The χ is the ratio between the net profit at current time π_t and the equity at previus time E_{t-1}. Now, we try to explain this relationship.

Lets' compute the return on asset of the firm. The value of capital, total assets, comes from the optimization of the object function Γ: it is the firm *desired capital* .

$$K^*{}_{it} = \frac{(\phi - g\ r_{it})}{c\ \phi\ g\ r_{it}} + \frac{E_{it-1}}{2\ g\ r_{it}} \tag{2.20}$$

Current and desired capital coincide because the bank always satisfies the credit demand of the firm by adjusting the credit interest rate to the firm financial conditions. Anticipating the final result, meaning that the equity of the firm increases exponentially, the first term in (2.20) is negligible and we can approximate the total assets formula as follows:

$$K_t \simeq \frac{E^f_{t-1}}{2\ g\ r_t} \tag{2.21}$$

Thus, if we want to compute the *roe*, return on equity, we can re-write the equity as function of capital:

$$K_t = \frac{E^f_{t-1}}{2\ g\ r_t} ==> E^f_{t-1} = K_t\ 2\ g\ r_t \tag{2.22}$$

We also re-write the ratio profit $-$ equity (roe) , considering the profit equation in (2.16) and the equity as we have computed in (2.20), we have:

$$ROE^f{}_t = \chi_t^f = \frac{\pi_t}{K_t \; 2 \; g \; r_t} = \frac{(\phi - g \; r_t)}{2 \; g \; r_t} \tag{2.23}$$

in other way, we can see the ROE as ($\chi^f{}_t = \rho_t^f * l_t^f$), where ρ^f is the firm roa and l_t^f is the firm *leverage*.

The leverage is the ratio between the capital and the equity. If we use the equation (2.21) for the capital, we have:

$$l^f{}_t = \frac{K_{t-1}}{E_{t-1}^f} = \frac{E_{t-1}^f}{2 \; g \; r_t} \frac{1}{E_{t-1}^f} = \frac{1}{2 \; g \; r_t} \tag{2.24}$$

Thus, at the end, the firm *roe* is:

$$ROE^f{}_t = \chi^f{}_t = \rho^f{}_t * l^f{}_t = \frac{\Pi_t}{E_{t-1}^f} * \frac{1}{2 \; g \; r_t} = \frac{(\phi - g \; r_t)}{2 \; g \; r_t} \tag{2.25}$$

We can observe that firm ROE depends non-linearly on the interest rate. In fact, it decays nonlinearly when interest rate increases, and it vanishes when $r_t = \phi/g$. Assuming the [Delli Gatti et al., (2005)] model setting, the value of ϕ/g is equal to 0.0909.

Now, we start our analysis on the bank side: *roe*, *roa* and leverage. As in equation (2.10), the bank deposits are calculated as residuals, by the difference between total credit supply of the bank and the bank equity. From the equation (2.11), we can compute the deposit and the total supply expressed as function of equity. Considering that, $L_t^s = E_t^b/\nu$, thus, the deposits, will be:

$$D_t = L^s{}_t - E_t^b \tag{2.26}$$

Where $L^s_t = \frac{E^b_{t-1}}{\nu}$. So, D_{t-1} can be approximated by: $\frac{E^b_{t-1}}{\nu} - E^b_{t-1}$.

Recaling the bank profit of the stochastic model, we have that the bank profit is:

$$\begin{aligned}
{\pi^b}_t &= r_t\ L^s_t - r_t \left[(1-\omega)\ \ D_{t-1} + E^b_{t-1}\right] \\
&= r_t\ \tfrac{E^b_{t-1}}{\nu} - r_t\ \left[(1-\omega)\ \left(\tfrac{E^b_{t-1}}{\nu} - E^b_{t-1}\right) + \ E^b_{t-1}\right]. \\
&= r_t\ \tfrac{E^b_{t-1}}{(\nu)} - r_t\ \left[\left(\tfrac{E^b_{t-1}}{\nu} - E^b_{t-1}\ - \tfrac{E^b_{t-1}}{\nu}\ \omega + E^b_{t-1}\ \omega\right) + E^b_{t-1}\right] \\
&= r_t\ \tfrac{E^b_{t-1}}{(\nu)} - r_t\ \tfrac{E^b_{t-1}}{\nu} + r_t\ \tfrac{E^b_{t-1}}{\nu}\ \omega - r_t\ E^b_{t-1}\ \omega \\
&= r_t\ \omega E^b_{t-1}\ \left(\tfrac{1}{\nu} - 1\right).
\end{aligned}$$

From this point on, we use the parameter β to express $\left(\frac{1}{\nu} - 1\right)$.

Thus, the equation of *deterministic bank profit* is the following one:

$$\pi^b_t = r_t\ \omega\ \beta\ E^b_{t-1} \tag{2.27}$$

Given the equation of bank profit, we can compute bank roe.

$$\chi^b_t = \frac{{\pi^b}_t}{E^b_{t-1}} = \frac{r_t\ \omega\ \beta\ E^b_{t-1}}{E^b_{t-1}} = r_t\ \omega\ \beta \tag{2.28}$$

Now, we can calculate the *bank roa*, that is the ratio between the bank profit and its total assets; in this case the total assets are well approximated by the *total supply loans*, and we have:

$$\rho^b_t = \frac{\pi^b_t}{L^s_t} = \frac{r_t\ \omega\ \beta\ E^b_{t-1}}{L^s_t} = \frac{r_t\ \omega\ \beta\ E^b_{t-1}}{\frac{E^b_{t-1}}{\nu}} = r_t\ \omega\ \beta\ \nu = r_t\ \omega\ (1\ -\ \nu) \tag{2.29}$$

The previous formula essentially says that the *roa* of the bank is about approximatevely $r_t\ \omega$, i.e. the spread between the interest rate paid to the bank deposit and the interest paid to the bank by the firm. The leverage is about $l^b = (\beta + 1)$

As we made for firm ROE, even in this case, the bank ROE is the bank ROA times the bank leverage. Thus, we have:

$$\chi_t^b = \frac{\pi_t^b}{E_t^b} = \frac{r_t\ \omega\ \beta\ E_t^b}{E_t^b} = r_t\ \omega\ \beta \tag{2.30}$$

The *roe* of the bank increases linearly with the interest rate. Both the ROEs of bank and firm depend on the interest rate, but in the opposite direction: when the interest rate increases the *roe of bank* increases while the *roe of the firm* decreases.

The basic idea to obtain the equilibrium interest rate $\bar{r}$ is to equalize the firm and bank *roe* :

$$\chi_t^b = \chi_t^f \qquad \Longrightarrow \qquad r_t = \bar{r} \tag{2.31}$$

Thus, we found the situation in which the *interest rate* r_t is equal to the *interest rate* $\bar{r}$, that is $r_t = \bar{r}$. The r_t is the interest rate that the firm get by the bank at time t and $\bar{r}$ is the interest rate that equalizes the two ROEs. If $r_t = \bar{r}$, the bank and the firm grow at constant rate equal to χ and we have an exponential growth of the equity of the firm E^f and bank E^b.

We can find different situations in which the interest rate r_t is different from $\bar{r}$. If $0 < r_t < \bar{r}$, the demand for credit grows at rate higher than the supply of credit, the firm grows more than the bank; if $\bar{r} < r_t < \phi/g$, the supply of credit grows at a rate higher than the demand of credit, the bank grows more than the firm; if $r_t > \phi/g$, the growth rate of the firm equity will be negative, with a corresponding shrinking of size of the firm.

At the interest rate $\bar{r}$, the bank grows exponentially in its equity and it can provide a higher and higher supply of credit to the firm. On the other side, the firm has always a positive profit, althought often close to zero. In this way, it therefore can expand its production capacity because of the availability of the credit. The system bank-firm reaches a *stationary state* characterized by an exponential growth of the size of the bank and of the firm and a balanced situation in demand and supply of credit. On the contrary, if the interest rate would take a different value, either there would be enough credit to satisfy the demand of the firm, or there would be an excess of credit supply. Both scenarios create a pressure on the interest rate to adjust, with opposite effects, the growth of credit demand and supply. Moreover, if the interest rate is too high, the firm will have negative profits and might go, after some time, bankrupt.

2.3.2 Equilibrium Interest Rate: Calculation

We can evaluate the bank and firm *roe* to identify the *equilibrium interest rate*. First, we equalize the two expression of the *ROEs*, obtained as implicit equation of $\bar{r}$:

$$\frac{(\phi - \ g \ \bar{r})}{2 \ g \ \bar{r}} = \omega \ \beta \ \bar{r} \tag{2.32}$$

We can derive $\bar{r}$ from the equation (2.27) , and we get:

$$\bar{r} = \frac{-1 \ + \ \sqrt{1 \ + \ \frac{8\omega\beta\phi}{g}}}{4\omega\beta} \tag{2.33}$$

Based on the parameters of the model [Delli Gatti et al., (2005)], the second term in the squared root is much smaller than 1, thus we can employ the following *Taylor s expansion*:

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{1}{8}\, x^2 \ + o(x) \tag{2.34}$$

From the second term we can get:

$$\bar{r} = \frac{\phi}{g} - 2\ \omega\ \beta\ \frac{\phi^2}{g^2} \tag{2.35}$$

From now, we use the $\bar{r}$, the interest rate computes with Taylor expansion (2.34), instead of r.

We have to consider the expantion equation (2.32) up to the second order since limiting the expansion of the first order; the profit of the firm will be always zero, thus, the ROA and ROE will vanish and subsequently, the equity growth rate will be also zero ($g_e = ROE$).

The second term in the expansion give the crucial contribution to the growth of the firm and bank system and to the entire economy.

Using $\bar{r}$ and equalizing the firm and bank *roe* we have:

$$\chi^b = \chi^f = \omega\ \beta\ \frac{\phi}{g} \tag{2.36}$$

This value represents the *growth rate of the whole economy*; the roe is the growth rate of the equity (bank and firm equity) and all variables depend on it.

We found that all variables (bank and firm assets, demand and supply of credit, capital,..) and their levels, grow *exponentially*, because they are all proportional to the level of (bank or firm) equity.

We said that if the roe of the bank and the roe of the firm is equal and so the interest rate is equal to $\bar{r}$, the economy grows exponentially.

Now, we focus on the *firm side*, we know that the capital is : $K_t = E^f_{t-1} + \pi_t + L_t$, but also we know that the level of *loans* depends on equity: from the desired capital equation, we found that *loans* are:

$L_t = \frac{(\phi - gr_t)}{c\phi gr_t} + E^f_{t-1}(\frac{(1-2gr_t)}{2gr_t})$.

Thus, loans depend on equity, and the *capital* depends on equity directly and indirectly through the loans.

Now, we focus on the *bank side*, we know that the total supply loans is: $L^s_t = \frac{E^b_{t-1}}{\nu}$, and that the deposit, that is residual, is computed also in relation to the bank equity $D_t = E^b_t(\frac{1}{\nu} - 1)$.

Thus, even in this case, the level of deposit and supply loans depend on the level of bank equity.

We found that $\chi^f = \chi^b$ is equal to firm and bank growth, thus their growth are also, obviously, equal betwen them. Therefore, all the system grows at the same rate of *roe*.

The growth rate of the economy depends on four parameters of the model: (i) two bank parameters (β and ω, that are, respectively, leverage and spread) and (ii) two firm's parameters (ϕ the technological level of the firm and g the capital cost).The growth rate of the economy will be zero if the productivity of capital is zero (there is no firm essentially) and if the spread ω is zero. But the vanishing profit of the firm would prevent the system from growing. The second term in the expansion is responsible for the non vanishing profits of the firm and the growing of the system bank-firm.

Moreover, the model implies that the demand for goods grows exponentially with the same rate of the equation (2.36), which means an implict validation of Say's law.
Considering the lending term in the expantion of the $\bar{r}$ in the equation (2.35), the interest rate r^* reflects the perfect competition rule, the competition equilibrium condition, thai is:

$$\pi^f = 0 \tag{2.37}$$

It means that r^*, has to be equal to ϕ/g . If this condition holds true, there is no growth of the system.

The interest rate that reflect the perfect competition condicion is expressed by r^*. Its value is equal to ϕ / g. So, the interest rate of the firm has to be lower to the r^* to get positive profit. The interest rate which tends the firm to get positive profit is expressed by $\bar{r}$. Its value is equal to $\frac{\phi}{g} - 2\,\omega\,\beta\,\frac{\phi^2}{g^2}$.
It is the second term of the expansion that allows the growth of all system This small deviation from perfect competition, allows the firm to growth, because $\bar{r}$ is slightly smaller than r^*, and this leads to small, but positive, profits.
So, this leads to a sistematic positive profit of firm; we can express it in a

growing demand for credit, supplied by the bank, that also makes profit and therefore its equity to grow expoentially.

From the equation (2.35) we know the parameters that enter in the formula for $\bar{r}$, the interest that equilibrates the supply and the demand of credit.

If we sobstitute the $\bar{r}$ in the equation (2.19), we can compute other accounting ratios:

$$ROA_t^f = \rho_t^f = 2\ \omega\ \beta\ \frac{\phi^2}{g} \tag{2.38}$$

As we saw before, the ROE is the ROA times the leverage, thus, we can write the bank leverage, as the ratio between the ROE and the ROA, as we compute in the following formula.

$$l_t^b = \frac{\chi_t^b}{\rho_t^b} = \frac{\phi\omega\ \beta}{g}\ \frac{g}{\phi\ \omega\ (1-\nu)} = \frac{\beta}{(1-\nu)} = \beta + 1 \tag{2.39}$$

Another interesting relationship is that:

$$\rho_t^f\ *\ l_t^f = \rho_t^b\ *\ l_t^b \tag{2.40}$$

If the leverage of the bank is higher than the leverage of the firm, the ROA of the firm will be higher than that of the bank.

2.3.3 Deterministic Model with Heterogeneity in the Initial Condition of the Financial Structure of Firms

In order to study the dynamical properties of the deterministic system, we generalize the firm-bank deterministic system to a system single firm-bank with N heterogenous firms, which is still deterministic. We still keep the deterministic skeleton, with N firms endowed with different credit structure. We want to study what happend to the deterministic system in the case of N heterogenous firms. Do they grow at the same rate asymptotically of the single firm system?

We try to introduce a heterogeneity in the initial condition of the financial structure of firms (capital and, consequently, equity and liabilities).

What we expect? We assume that the deterministic model is able to explain the sthocastic model growth. We also expect, that the model with heterogeneity in initial condition tends to the deterministic value, showing that , even with initial hetrogeneity, the model reachs, over time, the benchmarks of interest rate, roe and roa we computed in the equations (2.35), (2.36), (2.38).

In this part of the thesis,to create the heterogeneity we use the random numbers generated by: (i) *uniform distribution* (case 1) or (ii) *power law distribution* (case 2) .
We want to remind that in the deterministic model firms have a constant price $u = 1$. In this way we loose the heteogeneity on the price that we have in the stochastic model, but we acquire heterogeneity on initial conditions for each of the N firms.

We expect to find that the deterministic model with heterogeneity converges, in the long run, to values predicted by the analytical analysis [10], the model have to reach some target values:

− $\bar{r} = 0.0905$, $roe = 0.0021$ and $roa = 0.00041$

− size of firms distributed by power law;

− aggregate growth equal to analytic growth.

In the next paragraph we analyze the two way to generate heteorgeneity.

2.3.4 Case 1: Random Numbers Uniform Distributed

We start our analysis by heterogeneity in the initial conditions of the financial structure of the firms : *initial capital*. We assigne the heterogeneity on the capital becuase it is the proxy more used for the dimension.
We geretated the initial values as random numbers uniformly distributed in the interval $[2, 10]$.

We maintain the same proportion we have in stochastic model between capital and equity: the initial value of the equity is the 20% of the initial value of the capital.

The interest rate, in median, evolves close to the $\bar{r}$, but is not equal.
We find, analyzing the simulation results, that over the years the interest rate will be equal to $\bar{r}$, see equation (2.35).

The convergence of the interest rate to $\bar{r}$ depends on firms' equity.

We observe that higher is the firm equity, faster the firm converge to $\bar{r}$.

[10] The analytic values are the value we have found in the deterministic model with 1 firms and 1 bank.

This is becasuse, the value of the equity affects the interest rate charged by the bank to the firm.

The heterogeneity in the interest rate among firms, and the difference convergence to the $\bar{r}$ is coused by the heterogeneity of firms' equity, see the section (2.6) form more details about this result.

2.3.5 Case 2: Random Numbers Power Law Distributed

Our second analysis is focused to the study of the system with heterogeneity in the initial conditions of the financial structure of the firms: *initial capital*. In this case, we generated the initial values as random numbers power law distributed, with the shape parameter of Pareto Distribution $\alpha = 1.1$ and location parameter of Pareto distribution $b = 2$, meaning that the minimun value of the capital is 2.
We maintain the same proportion we have in stochastic model between capital and equity: the initial value of the equity is the 20% of the initial value of the capital. We elected this distribution because in the stochastic model the firms hetrogeneity leads to a power law distribution.

Also, in this case, the interest rate will converges $\bar{r}$.

The amount of the equity (financial rubustness) strongly affect the interest rate and the latter convergences to $\bar{r}$, see the section (2.6) form more details about this result.

2.4 Long-Run Dynamics

The interest rate is assumed constant, equals to the value of the interest rate that allows to the economy to grow, equivalent to $\bar{r}$ (see equation (2.35)).

From the equation (2.4), (2.5) and (2.9) we can compute the law of motion of the equity, and with some aproximation, we get:

$E_t^f = (1 \ + \ \frac{\phi \ - \ g \ r}{2 \ g \ r}) \ E_{t-1}^f \ + \ \frac{(\phi \ - \ g \ r)^2}{c \ \phi \ g \ r}$

Given $c = 1$, $\phi = 0.1$, $g = 1.1$ and $r = 0.0905$ we got that $\frac{(\phi \ - \ g \ r)^2}{c \ \phi \ g \ r}$ is equal to 0.00002. So, we can approximate $\frac{(\phi \ - \ g \ r)^2}{c \ \phi \ g \ \bar{r}}$ to zero and we have

$$E_t^f = (1 \ + \ \frac{\phi \ - \ g \ r}{2 \ g \ r}) \ E_{t-1}^f$$

As we saw, $\frac{\phi \ - \ g \ r}{2 \ g \ r}$ is exactly the firms' roe, χ^f.

We know that firms' roe χ^f is equal to bank roe χ^b.

Thus our χ is equal to $\omega \ (\frac{1}{\nu} - 1) \ \frac{\phi}{g}$.

Moreover, if we take the starting value of equity E_0 and its evolution over the time, we get:

$$E_t^f = E_0^f \cdot (1 \ + \ \chi)^t \tag{2.41}$$

To evaluate the long-run equity dynamics, that is the rate of growth of the equity, we generate the logaritmic of the last equation:

$$\log(E_t^f) = \log(E_0^f) + t \log(1 \ + \ \chi) \tag{2.42}$$

2.5 Comparison of the Two Versions : Analysis and Results

In this section we compare the results of the 2 two versions of the model: stochastic and deterministic.

We start the analysis with the aggregate of equity and its growth of stochastic and deterministic version of the model (see Figure (2.1)). Under the analysis we made about the long-rum dynamics of the equity in the stochastic version of the model, we expect that the aggregate of equity grows at the same speed as the deterministic model growth (long - run theoretical growth) . This means that the deterministic version of the model is able to replicate the growth, in long run, of the stochastic model. The growth of the aggregate equity is pervaded by fluctuations: they are probably caused by the decreasing/increasing of large firms.
The decreasing/increasing phases of large firm can affect also the evolution of firms' interst rate (average interest rate).

The model generates a system where the bank is stimulated to lend credit to large firms. Large firm (under our analysis) enters , in a certain period of time, in a virtuous cicle where it gets very small interest rate, much smaller than $\bar{r}$, and than grows and grows for some periods (growing persist in a considerable number of time).
Then, the large firm face a downturn trend and decreases, but, in the same time, another large firm gets small interest rate and grows, and the process continues over time.

The system generates granularity (few large firms and a multitude of small firms) and a very large firm that own almost all the economy wealth.

The model follows closly the analytical prediction , i.e. the growth is equal to the return on equity (*roe*), see equation (2.35).
The fluctuation in aggregate equity and production (stochastic model) is due to a decreasing of large firms that have also remarkable impact on the bussiness cycle through the financial system [Gabaix,(2003)].

As say [Gabaix,(2011)], the idiosyncratic firm-level shocks can explain an important part of aggregate fluctuations. The *granular hypothesis* suggests that movement of large firms can explain one third of variation in output growth. So, the macroeconomic behaviour can be clarified by looking at the behavior of large firm(s).

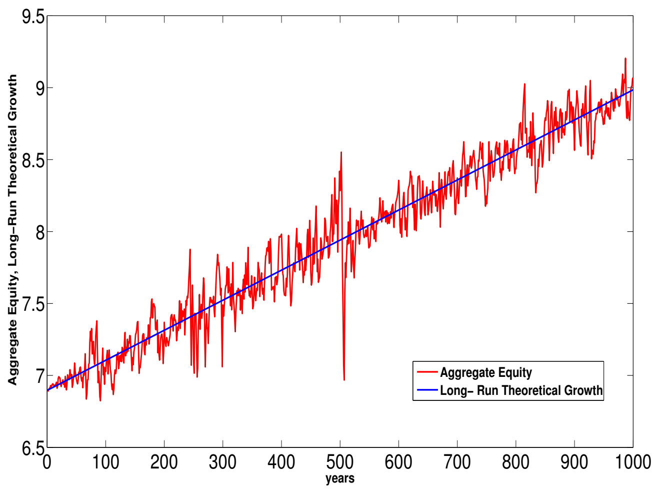

Figure 2.1: **Aggregate Equity Growth (red line) and Long-Run Analytic Growth**

From the Figures (2.2) we can observe that the aggregate equity is very close to the long-run analytic growth (see equation (2.42)).

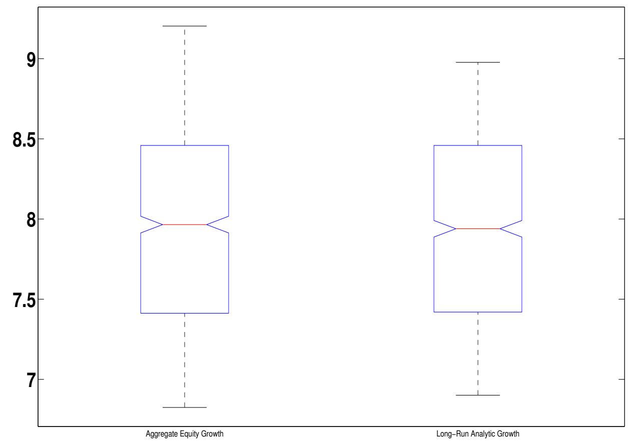

Figure 2.2: **Boxplot of the Aggregate Equity (left side) and Long-Run Analytic Growth (rigth side): we computed the OLS of the aggregate growth**

Now, we try to analyze the interest rate, roe and roa in stochstic model.

The figures (2.3, 2.4 and 2.5) represent the interest rate, the firms' roe and the firms' roa in a stochastic model and their respective theorethical benchmarks, from the equations (2.35), (2.36), (2.38).

The interest rate evolves (and fluctuate) between the 2 red lines. The highest red line is the interest rate based on perfect competitive equilibrium in goods market r^* and the lowest red line is the interest rate that equalizes the supply and the demand of credit ($\bar{r}$), where $r^* = \phi/g$ and $\bar{r} = \phi/g - 2 * \omega * \beta * (\phi^2/g^2)$. The interest rate evolves inside the range $[\bar{r}, r^*]$. The average's average interest rate (over 100 simulations), in the stochastic model, is equal to 0.0906, that is very close to the deterministic interest rate 0.0905, (see Figures 2.3).

We can observe that the ROE evolves close to its theoretical benchmark (equation (2.36)). The roe benchmark is 0.0021, this is the firms and bank

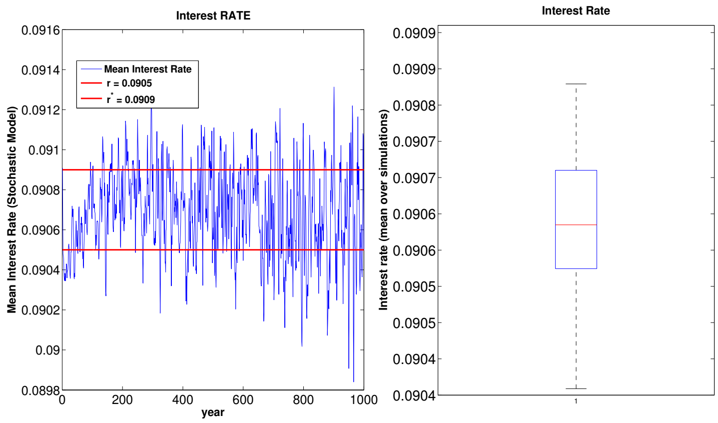

Figure 2.3: **Mean Interest Rate over time**: the blue line is the mean interest rate, the red lines are the analytic interest rate $\bar{r}$ and r^*. In the rigth side of the figure we can obserse the boxplot of the interest rate evaluated over 100 simulations. The median value of the boxplot is about 0.0906 that is close to the deterministic value of interest rate 0.0906.

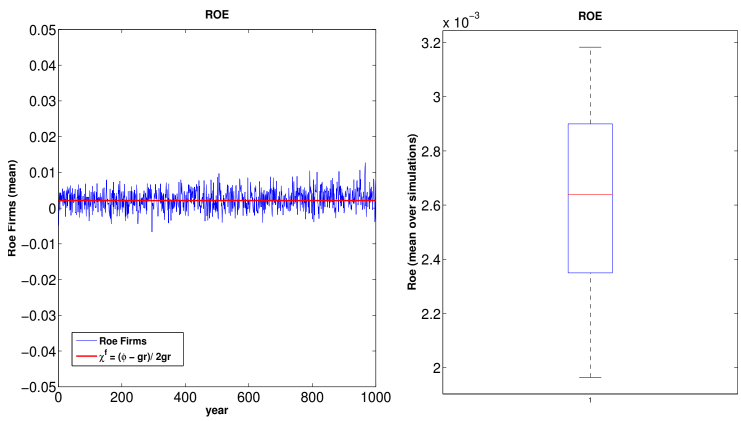

Figure 2.4: **Return on Equity (ROE) over time**: the blue line is the mean roe and the red line is the analytic firms roe χ..In the rigth side of the figure we can obserse the boxplot of the interest rate evaluated over 100 simulations. The median value of the boxplot is about 0.0026 that is close to the roe deterministic value 0.0021.

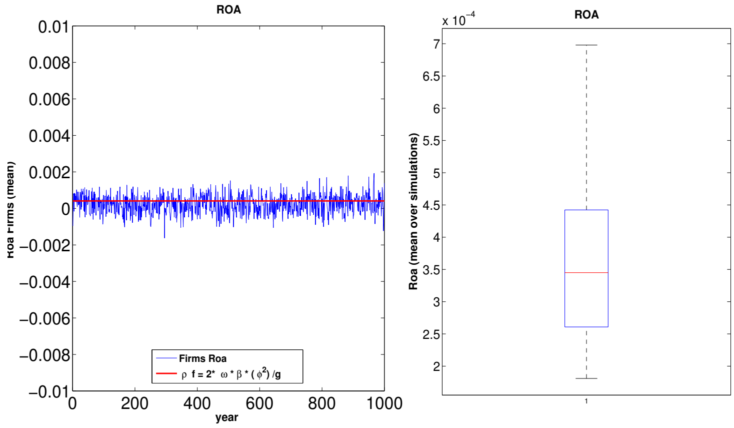

Figure 2.5: **Return on Assets (ROA) over time**:the blue line is the mean roe and the red line is the analytic firms roa ρ. In the rigth side of the figure we can obserse the boxplot of the interest rate evaluated over 100 simulations. The median value of the boxplot is about 0.00036 that is close to the roa deterministic value 0.00041.

roe. In analytic terms, the roe is $\omega * \beta * (\phi/g)$. We found that in the stochastic model , the average's average roe (over 100 simulations) is equal to 0.00026, that is close to its theoretical benchmark, see Figures (2.4).

Even the *ROA* evolves around the value we found studying the deterministic model. The theoretical roa is $2 * \omega * \beta * (\phi^2/g)$ (equation (2.38)), that is equal to 0.00041. The average's average roa (over 100 simulations), in the stochastic model, is equal to 0.00035, that is very close to its theoretical benchmark, (see Figures 2.5).

This positive value of roa can explain the growth of firms and the growth of the model. In fact, we know that in perfect competive good market the interest rate is ϕ/g, there is no profit and no firms can grow, and this returns a zero roa. If we decuct from the roa from the interest rate ϕ/g, we get a lower interest rate that grants a positive profit: the roa is the distance between the zero profit and the growth of firms (economy). Moreover the ROA gives us an idea of how the use of assets generates earnings.

Moreover, we simulated a model with imposed heterogeneity (case 1: random intial capital uniform distributed; case 2 random intial capital power law distributed), to proove that, the models, tends to move around the theoretical benchmarks. We can obserse, even if we impose heterogeneity, and although in long run, firms' interest rate reach the theoretical benchmark of interest rate (0.0905), the firms' ROE reach the value $\omega * \beta * (\phi/g)$ (0.0021) and the firms' ROA reach the value equal to $2 * \omega * \beta * (\phi^2/g)$ (0.00041). Whatever the case chosen, (case 1 or case 2), all firms reach the theoretical benchmarks, but firms get these values with different speeds. In any case the firm that reach slower the theoretical benchmarks is the firms with lowest equity. So, lower the equity, slower the interest rate, ROE and ROA converge to the anlytic values. This result is due to the fact that the equity has a great impact on the interest rate that firms get and the amount of interest rate affects the roa and the roe. Through the interest rate the supply credit market and the demand credit market are in equilibrium. We know from equation (2.12) that the credit that any firm gets depends on the total credit supply (on bank side) and on weighted firm equity and capital (on firm side). As we know from the equation (2.21), the capital depends on firm equity. Thus, essentially, the credit, that any firm gets, depends only on its equity.

Higher the firm equity, closer the interest rate to $\bar{r}$. Thus, it is the amount of firm equity that determines the speed in which any firm converges to the theoretical benchmarks.

After some period of time (years) all firms grow at the same rate. The system

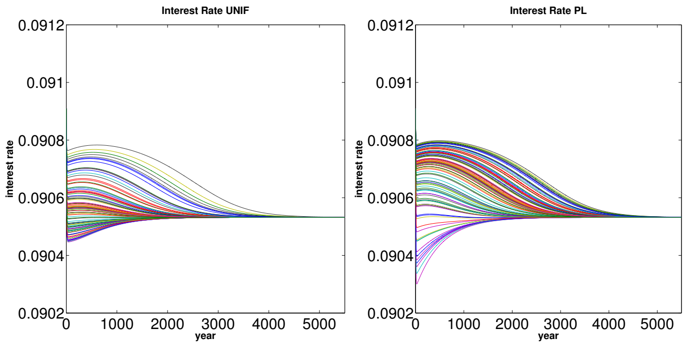

Figure 2.6: **Interest Rate (with heterogeneity in the initial condition of the financial structure of firms) over time** : the right side is the CASE 1 (random intial capital uniform distributed) and the left side is the CASE 2 (random initial capital power law distributed).

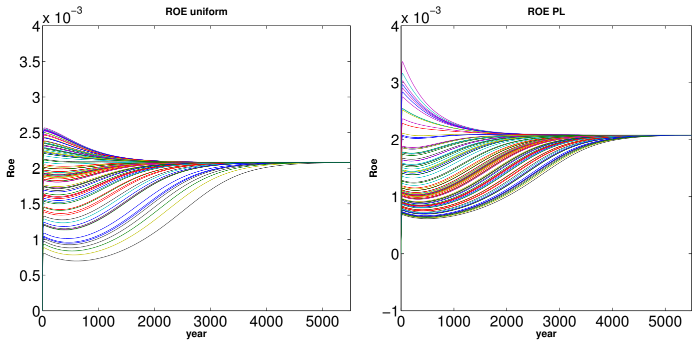

Figure 2.7: **ROE (with heterogeneity in the initial condition of the financial structure of firms) over time**:the right side is the CASE 1 (random intial capital uniform distributed) and the left side is the CASE 2 (random initial capital power law distributed).

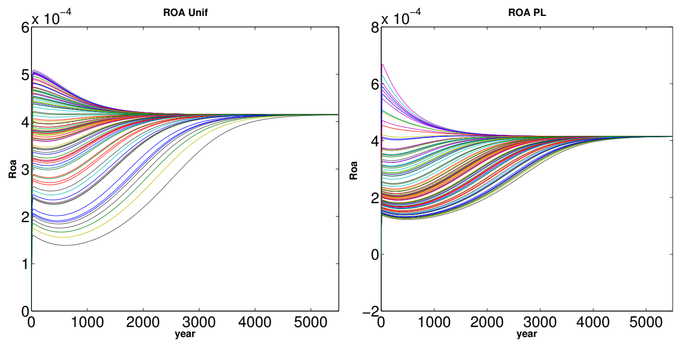

Figure 2.8: **ROA (with heterogeneity in the initial condition of the financial structure of firms) over time** :the right side is the CASE 1 (random intial capital uniform distributed) and the left side is the CASE 2 (random initial capital power law distributed).

tends to theoretical growth rate. The largest firm converges to theoretical benchmarks faster than the smallest one, that, of course, is the last one that converges [11], see Figures (2.5 , 2.6, 2.7).

As we found that the stochastic roe and the stochastic roa are close to their theoretical benchmarks, equation (2.36) (2.38), we can argue that the stochastic leverage is also close to its theroretical value, see equation (2.24).

The high level of the *firms leverage* is explained by the *euity-capital* and *loan-capital* ratios: in the model, the system tends to ratio , respectively, equal to 0.20 and 0.80 (see the Appendix).
It means that the model produces a system with financial fragility and high level of loans.

As we said, If we compare the average's average of the roa, roe and leverage of firms we find that their value are very close to the theoretical one.
The values of average's average of firms roa and bank roa are not equal, the first one is more of the doble value of the bank roa. Obviously, their value (in mean) are positive: the model tends to share the profit between firms (in special way large firms) and bank. In this way we have perpetual and mutual positive growth in the aggragate of firms equity and in the bank equity.

The firms size distribution is power law distributed, also confirmed by an ols estimation of the tail of the distribution. The firms' size distribution in *stochastic version of the model* (Figure 2.9) shows a power law estimation

[11] In case 1 the last firm converges to the anlytic interest rate, ROE and ROA at, respectively, $T = 3819$, $T = 4396$, $T = 5448$. In case 2 the last firm converges to the anlytic interest rate, ROE and ROA at, respectively, $T = 3990$, $T = 4565$, $T = 4795$.

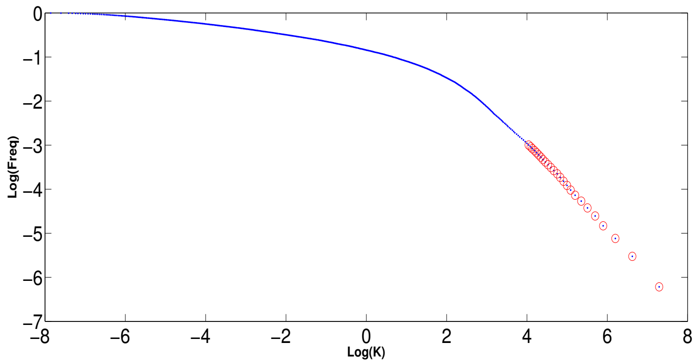

Figure 2.9: **Firms Size Distribution Stochstic model**: the size distribution is made as mean of size distribution in 100 different simulaitons.

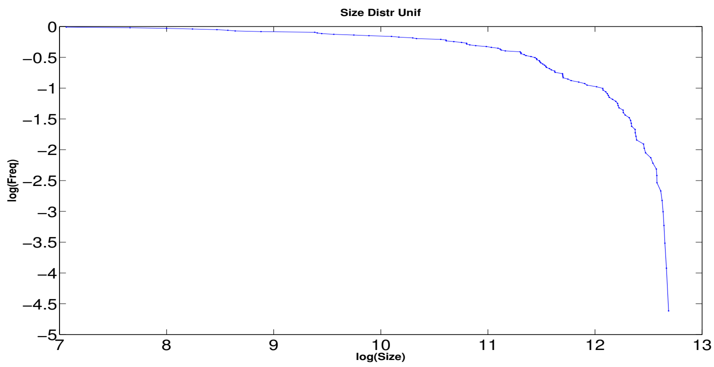

Figure 2.10: **Firms Size Distribution Case 1**

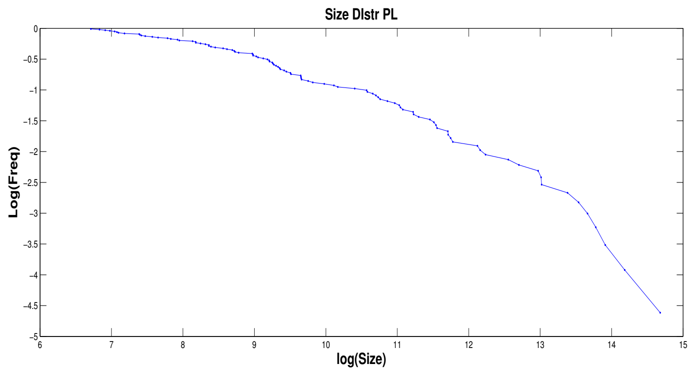

Figure 2.11: **Firms Size Distribution Case 2**

and the scaling exponent recorded ($\alpha = -0.9928$) [12] is consistent with what [Gaffeo et al., (2003)] found in real data. [13] . The estimation is made considering the 25 largest firms.

We also fit the firms size distribution with imposed heterogeneity generated by random initial capital power law distributed. The OLS estimation reveals that the scale parameter in the tail analysis is -0.9759, very close to 1 [14].

Essentially, the results of the Figures (2.10), (2.11) is that depending on the heregeneity in the initial value, we can observe this heterogeneity in the firms' size distribution.

Although in both cases (case 1 and case 2) all firms converge to benchmark, only in the case 2 we have a good fit for power law distribution of firms' size.

We compute the firms' size distribution analysis of the stochastic version of the model in different GDP phases. In growing phases (when the GDP standard deviation is higher) the size distribution tends to shift to the right ($T = 508$ cyan line in Figure (2.12)), while during the recessions the firms' size distribution shift to the left ($T = 504$ pink line in Figure (2.12)). During the expansion phases, larger firms tend to grow faster than smaller firms and this generate an *higher slope of the interpolate line.* The decreasing

[12] We analyzed the firms' size distribution in the last year of simulation.

[13] The ols estimation of the tail of the firms size distribution generates the follwing resuts: the scaling exponent α is equal to -0.9928 with s.e. 0.0033 and p-value 0.

[14] With the OLS analysis we estimate the tail of the distribution: we get an equation $Y = \beta_0 + \beta_1 * X$, where the scale parameter β_1 is -0.9759 with s.e. 0.0767 and p-vale $1.04 * 10^{-}08$ and the β_0 is 10.1221, with s.e. 1.0145 and p-value $1.85 * 10^{-}07$.

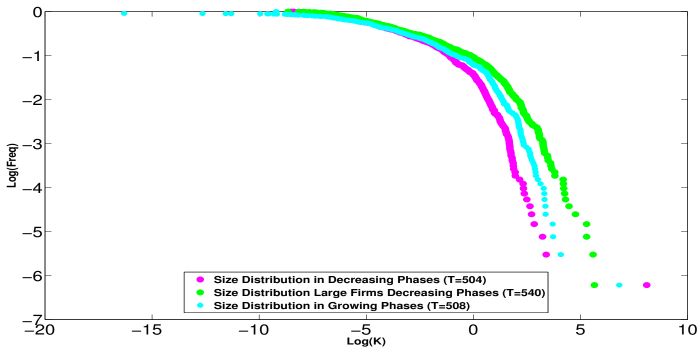

Figure 2.12: **Firms Size Distribution over different phases of GDP**

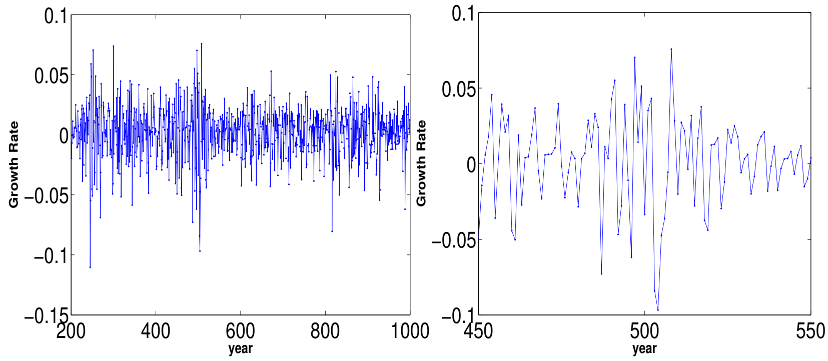

Figure 2.13: **GDP growth rate**: the left side of the figure is the GDP growth rate over the time simulation in the right side there is a GDP growth rate between $T = 450$ and $T = 550$.

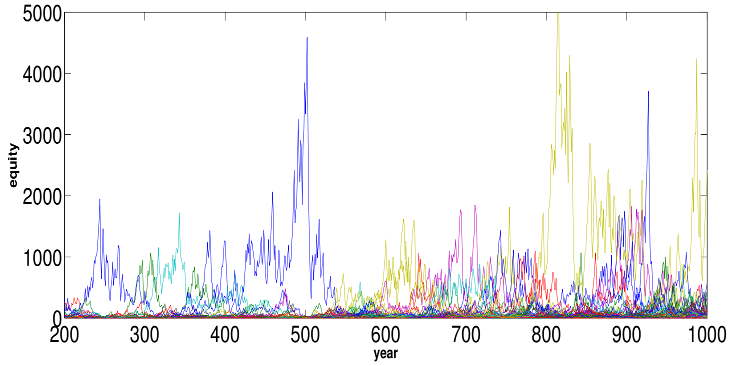

Figure 2.14: **The Evolution of Firms Equity**

of large firms during downturns cause a more equal firms' size distribution ($T = 540$ green line in Figure (2.12)). This is what was observed in real data [Gaffeo et al., (2003)].

As we have said, growing phases $T = 508$ (see the Figure (2.12 cyan line) of *gdp growth rate*) the scaling exponent is equal to -0.8540 [15], large firms tend to grow more. During the recession phases, there is a great difference between the large firms and the other firms: large firms dominate the economy. In this case the scale exponent is equal to -0.6614, lower than the growing phases [16]. When the large firms decrease ($T = 540$) we can observe a more equal firms' size distribution [17]

2.6 GDP Fluctuations

What we want to show in this paragraph is how the GDP evolve over time and what elements affect its fluctuation over time. Before to analyze the GDP structure, we focus on the *herfindhal - hirshman index* that represent the industrial concentration of firms and we can help us to understand how large firm affects the economy.

"*The idiosyncratic firm-level shocks can explain an important part of aggregate movements and provide a microfoundation for aggregate shocks. Existing research has focused on using aggregate shocks to explain business cy-*

[15]The ols estimation the scale exponent in $T = 508$ is equal to -0.8540, with s.e 0.0838 and p-value $5.45 * 10^-10$.

[16]The ols estimation the scale exponent in $T = 504$ is equal to -0.6614, with s.e 0.0797 and p-value $8.87 * 10^-08$.

[17]The ols estimation the scale exponent in $T = 540$ is equal to -1.0703, with s.e 0.0457 and p-value $1.54 * 10^-17$. All the estimations are made considering the 25 largest firms.

cles, arguing that individual firm shocks average out in the aggregate. I show that this argument breaks down if the distribution of firm sizes is fat-tailed, as documented empirically. The idiosyncratic movements of the largest 100 firms in the United States appear to explain about one-third of variations in output growth. This granular hypothesis suggests new directions for macroeconomic research, in particular that macroeconomic questions can be clarified by looking at the behavior of large firms. This idea and analytical results may also be useful for thinking about the fluctuations of other economic aggregates, such as exports or the trade balance. " Gabaix, (2011)

We try to analyze the GDP behaviour and how the large firm actually affect the aggregate. What we expect to find is that the fluctuation of the large firm strongly affect the aggregate behavior.

2.6.1 Herfindhal-Hirschman Index

In this paragraph, we compute a benchmark value for the *Herfindhal - Hirshman Index*,H, assuming that the largest 125 firms follows a Zipf relation, i.e. $s_{it} = s_1/i$ where $2 <= i <= 125$. We try to analyze the H to get its analytic value to better understand the behavior of the model.

The standard deviation of GDP depends on the standar deviation of *roa* and on the Herfindhal-Hirschman Index, as we will show later on. Now, we try to explain the analytic value of H. The squared H is equal to the sum of the all i-th squared size s_i (where s_i is the measure of the size of the firms over the total size S. As total size we use the capital of firms , so, we get:

$$H^2 = \sum \left(\frac{S_{it}}{S_t} \right)^2 \tag{2.43}$$

where $S_t = \sum_{i=1}^{N} \; S_{it}$ and $S_{it} = \frac{s_{1t}}{i}$.

We want to demostrate that the largest firm affects the GDP fluctuation. We take in to account the 125 largest firms and we can re-write the squared total size S_t as *harmonic mean*, where any firm smaller than the largest one is a fraction of the largest one. So, we have that the total size (S_t) is the sum of the fraction of the 125 largest firms.

$$S_t = s_{1t} \sum_{i=1}^{N} \frac{1}{i} \simeq \; s_{1t} \;\; = \;\; s_{1t} \;\; (ln(N) + 1) \tag{2.44}$$

where $N = 125$ and $s_1 = \frac{s_i}{i}$ and $2 \leq i \leq 125$

We can re-write the sum of the squared summed i-th size $\sum {s_{it}}^2$ as function of *Riemann Zeta Function* . The approximation is valid as the number N tends to infinity. So, we have:

$$\sum {S_{it}}^2 \;\; = \;\; {s_{1t}}^2 \sum_{i=1}^{N} \frac{1}{i^2} \;\; \simeq \;\; s_{1t}^2 \; \frac{\pi^2}{6} \tag{2.45}$$

Thus, we can re-write the squared H, using the firm capital as size:

$$H^2 = \frac{K_{1t}^2 \; \frac{\pi^2}{6}}{{K_1}^2 \;\; (ln(N) + 1)^2} \tag{2.46}$$

At the end the **analytic herfindhal-hirschman index** [18] is :

$$\bar{H} \simeq \frac{\frac{\pi}{6}}{(lnN+1)} \tag{2.47}$$

As in [Gabaix, (2011)] the fluctuation of the GDP are related to the herfindal-hirshman index, i.e. to a measure of dispersions of the granular fluctuation.

2.6.2 Figures: HHI, Firms Size Distributions, OLS estimator

We set the value of the *exit equity*, i.e. the threshold for which firms exit form the market, as E_{exit} at time $t = 1$ equal to 0.001. The evolution of the exit equity is described by the following equation:

$$E_t^{exit} = E_{t-1}^{exit} * (1 + (\beta * \omega * \frac{\phi}{g})) \tag{2.48}$$

Where $(\beta * \omega * (\phi/g))$ is the *growth factor* and it is equal to 0.0021, that is exactly the average of the aggregate growth rate.

As we can see in Figure (2.15), the H evolve around its analytic value, with some strong fluctuations. The fluctuation of H represents the fluctuations of the large firm. The fluctuations of H are strongly influenced by the presence of the largest firm. In fact , as we can observe in Figure (2.16),

[18] If N is equai to 125, the $\bar{h}$ is equal to 0.2201.

cutting off the large firm from the analysis of H, the H evolves in the range between its mean value and the its analytic value.

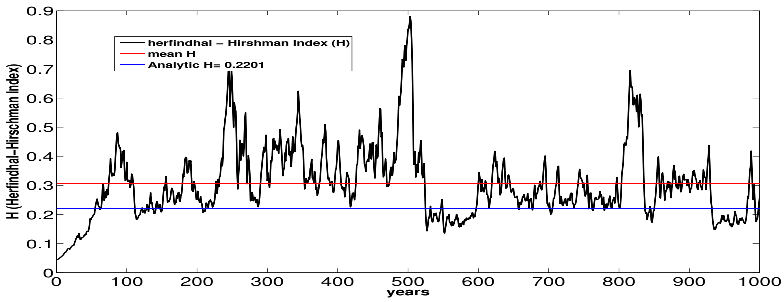

Figure 2.15: **Herfindal - Hirshman Index (all firms)**
The Herfindhal-Hirshman Index over time (black line) , the mean of the H and the Analytic H ($\frac{\frac{\pi}{\sqrt{6}}}{(ln\ N+1)}$). The *analytic* H is a good proxy of the H in normal trend of GDP (no lower or upper peak).

2.6.3 GDP Fluctuations Analysis

What we want to do in this section is to realte the aggregate fluctuations of the GDP of our economy to the fluctuations of the firms. Recent empirical work has shown that firms' entering and exiting mechanism contributes almost as much to employment and macroeconomic fluctuations as firms continuing their activity [Davis et al., (1996)]. Hence, any theory of business fluctuations should pay particular attention to the way entries/exits of firms are modeled.

Exits are endogenously determined as financially fragile firms that go bankrupt, that is as their net worth becomes negative. Besides making the total output to shrink, exits cause the decreasing of equity of the banking sector and, in turn, aggregate credit to go down. As discussed above, this

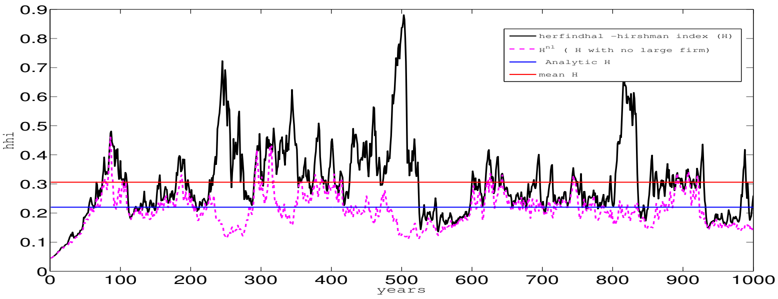

Figure 2.16: **Herfindhal - Hirshman Index: H of all firms vs H with no large firm (H^{nl}).**
The Herfindhal-Hirshman Index of all firms over time (black line) , the mean of the H (red line), the Analytic H ($\frac{\frac{\pi}{\sqrt{6}}}{(ln\ N+1)}$) (blue line). The *analytic H* is a good proxy of the H in normal trend of GDP (no lower or upper peak). The H^{nl} shows us that if the *large firm* is cut off from the analysis, the H evolve in the range between its mean value and the its analytic value.

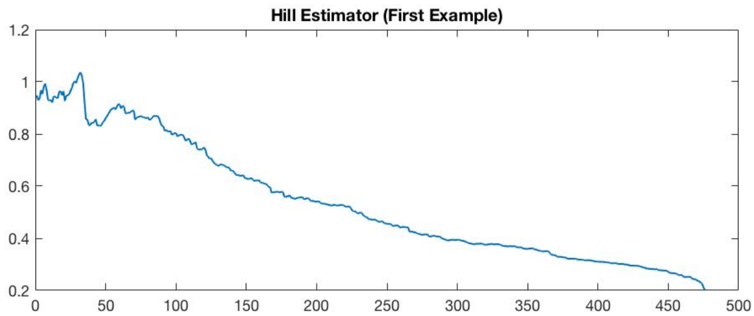

Figure 2.17: **Hill Estimator**.
In probability theory, heavy-tailed distributions are probability distributions whose tails are not exponentially bounded: that is, they have heavier tails than the exponential distribution. We compute the *hill estimator* of the tail index.

mean field interaction in terms of a bank effect , [Hubbard et al., (2002)], amplifies and propagates idiosyncratic shocks all over the economy.

We start our analysis taking into account the balance sheet equation of the entire economy from [Delli Gatti et al.,(2005)]. Thus, at aggregate level, we have:

$$K_t = L_t + E_t - \Pi_{t+1} \tag{2.49}$$

where the aggregate capital is the sum of the N firms capital ($K_{it} = \sum_{i=1}^{N} k_{it}$), the aggregate loan is the sum of the loan of N firms ($L_{it} = \sum_{i=1}^{N} l_{it}$), the sum of the N firms equity is the aggregate equity ($E_{it} = \sum_{i=1}^{N} e_{it}$) and the aggregate profit is the sum of the profit of N firms $\Pi_{it} = \sum_{i=1}^{N} \pi_{it}$.

If we consider the variation of the quantities above, we have:

$$\Delta K_t = \Delta L_t + \Delta E_t - \Pi_{t+1} + \Pi_t \tag{2.50}$$

in the model [Delli Gatti et al., 2005] there is a turn-over process on firms side: firms fail and exit from the system at time t and new firms enter in the system (the same number of the exiting firms) at time $t + 1$.

If we consider that there is no firms' bankruptcy, we can consider the following equation:

$$\Delta E_t - \Pi_{t+1} = 0 \tag{2.51}$$

i.e., the generated profit is all accumulated in the aggregate equity.

If we have some firms bankruptcy, we can aproximate the equation above with the following equation:

$$\Delta E_t - \Pi_{t+1} \simeq F_t^e < E^{en-ex}{}_{t-1} > \tag{2.52}$$

where F_t^e is the number of entering firms and $< E^{en-ex}{}_{t-1} >$ is the average value of the flow of new equity which is the difference of the equity of the new firms and the equity of the exting firms.

Now we wanto to compute the growth rate of aggregate capital:

$$\frac{\Delta K_t}{K_t} = \frac{\Delta L_t}{K_t} + F_t^e \frac{< E_{t-1} >}{K_t} + \frac{\Pi_t}{K_t} \tag{2.53}$$

We know that the ratio between the variation of aggregate capital and the aggregate capital is the growth rate of the aggregate capital g_k. The GDP is the aggregate of production at time t, Y_t. Since $Y_t = \phi K_t$, and we can approximate g_k as the growth rate of GDP. So, we define $\Delta K_t / K_t = g_k$.

We try to compute the the growth rate of GDP:

$$g_k = \frac{\Delta L_t}{L_t} \frac{L_t}{K_t} + \frac{F_t^e \; E_{t-1}}{N \; K_t} + \frac{\sum \pi_{it}}{\sum K_{it}} \tag{2.54}$$

where N is the total number of firms.

Now, we can calcolate the expected value of the growth rate of GDP :

$$E\left[g_k\right] = E\left[g_l\right] * \; E\left[\frac{L_t}{K_t}\right] + E\left[\frac{F_t^e}{N}\right] * \; E\left[\frac{E_{t-1}}{K_t}\right] + \; E\left[\sum roa_{it} \; \lambda_{it}\right] \tag{2.55}$$

where roa is the return on assets and we can approximate it with the profit rate. The λ_{it} is the weigth that each firm has in the economy, acconrding to it, the weighted roa is: $\lambda_{it} \frac{K_{it}}{\sum K_{it}}$.

After some simulations, we found that the expected value of the growth size measured as capital is equal to the expected value of the growth rate of loans, thus we have $E[g_k] = E[g_l]$. Moreover, from the analysis of the deterministic model, we know that the firms' leverage, that is K_t/E_t, is equal to $1/2\phi$. Thus, we know that the ratio E_t/K_t is equal to 2ϕ. From this formulas, we expected that the value of ratio between loans and capital is equal to $1 - 2\phi$, that is $E[L_t/K_t] = (1 - 2\phi)$.
From above, we can get the following equation:

$$
\begin{aligned}
E\left[g_t^k\right] &= E\left[g_t^l\right]\ (1-2\phi) + \ E\left[\frac{F_t^e}{N}\right]\ 2\phi + \ E\left[roa_{it} *\ \lambda_{it}\right] \\
&=> E\left[g_t^k\right]\ 2\phi = \ E\left[\frac{F_t^e}{N}\right] *\ 2\phi + \ E\left[roa_{it} *\ \lambda_{it}\right] \\
&=> E\left[g_t^k\right]\ = (\ E\left[\frac{F_t^e}{N}\right] + \ E\left[roa_{it} *\ \lambda_{it}\right]\ * \frac{1}{2\phi})
\end{aligned}
$$

where g_t^l is the growth rate of the aggregate loans.

Moreover, from the analysis of the *long-run dynamics* , see section (2.5), we know that the economy grows at the rate $\omega * \beta * \frac{\phi}{g}$.

The right hand of the equation is equal to 0.0023 (in any simulations we made), that is close to the value of the growth rate of the economy :

$$
(\ E\left[\frac{F_t^e}{N}\right] + \ E\left[roa_{it} *\ \lambda_{it}\right]\ * \frac{1}{2\phi}) \quad \simeq \quad \omega\ * \beta\ * \frac{\phi}{g}.
$$

So, we have:

$$
E\left[g_t^k\right] = \omega\ * \beta\ * \frac{\phi}{g}.
$$

Now, what we try to do it is to calculate the standard deviation of GDP, to try to understand its fluctuations. We have:

$$\frac{\Delta K_t}{K_t} = \frac{\Delta L_t}{L_t} + \frac{F_t^e \; E_{t-1}}{N \; K_t} + \frac{\Pi_t}{K_t} \tag{2.56}$$

The first term on the rigth hand of the equation, that is the variation of loans, is very small, it depends on the variation of the bank equity, because what the bank can offer to firms L_t^s, the total supply loan, depends on the bank equity, $L_t^s = E_{t-1}^b/\nu$. The variation of bank equity E_t^b/E_{t-1}^b is essentially deterministic, because we have almost no firms' bankruptcies. Therefore we can *neglect* this first term in the rigth hand of the equation.
The fluctuations of the second term on the right hand of the equation are very small, it depends on the flow of entering/exiting firms. Its standard deviation is equal to

$\frac{\sigma_{fe}}{N} \; 2\phi = \sigma_{fe} \; 0.0004$

and it is close to zero.

The number of firms bankruptcies (and the number of new firm) [19] is very low (see Figure 2.18).

So, we can argue that the standard deviation of the growth rate is affected and dominated by the last term of the equation, that is Π_t/K_{it} and we can

[19] The number of firms in the system is constant, so, the number of entry firms is equal to the number of exit firms. What change is the threshold of entry/exit equity.

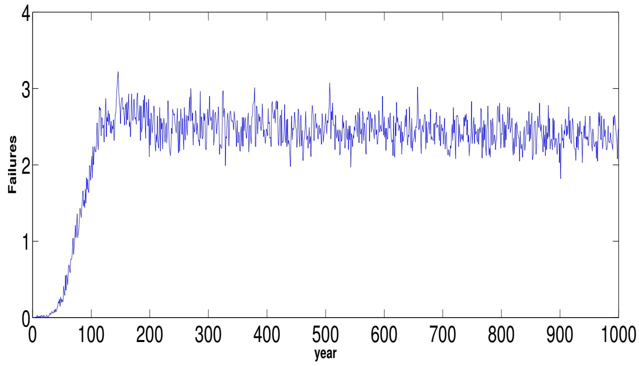

Figure 2.18: **Firms Failures/Entries over time:**mean over 100 simulations

also write it as $\sum roa_{it}\ \lambda_{it}$.

Then, we have :

$$Var\ [g_k] = Var\ [\sum roa_{it}\ \lambda_{it}]$$

We can write the last equation as follow:

$$Var\ [\sum roa_{it}\ \lambda_{it}] = \sigma^2_{ROA}\ \ (\lambda_{it})^2$$

Now, we try to demostrate the equation above:

$$Var\ [\sum roa_{it}\ \lambda_{it}] =$$
$$Var(\sum roa_{it})\ Var(\sum \lambda_{it}) + (E[\sum \lambda_{it}])^2\ Var(\sum roa_{it})$$
$$+(E[\sum roa_{it}])^2\ Var(\sum \lambda_{it})$$

The first term and the last term of the rigth hand of the equation are zero, so, we have that $Var(\sum roa_{it}\ \ \lambda_{it}) = \ \sigma^2_{ROA}\ \ (\lambda_{it})^2$.

The parameter λ expresses the weight that any firm have in the market, we can define it as industrial concentration that is well known as the *herfindal-hirshman index*
So, we have

$$Var[g_k] = \sigma^2_{ROA} \quad H^2 \tag{2.57}$$

where H is the *Herfindhal Index* and assuming that the variation of the roa is indipendend of the variation of the firms size.
At the end the standard deviation of growth rate, that is the squared root of the equation above, is:

$$\sigma_t^{gk} = \sigma_{ROA} \quad H \tag{2.58}$$

Moreover, we can solve and get the analytic value of σ^2_{ROA} trought the following integral because the roa_{it} depend on the price that is uniformly distributed:

$$\sigma^2_{ROA} = \frac{1}{2\phi} \int_{-\phi}^{\phi} x^2 \, dx = \frac{\phi^2}{3} \tag{2.59}$$

Thus, we get that the standard deviation of growth rate is :

$$\sigma_t^k = \frac{\phi}{\sqrt{3}} \quad H \tag{2.60}$$

At the end, we can argue that the fluctuations of GDP depend on *Herfindhal Index*, as in [Gabaix,(2011)].

Below there is a Figure (2.19) that compares the *GDP standard devia-*

tion calculated over 15 time steps and the *analytic GDP standard deviation* computing using (2.60).

The equation tell us the connection between the idiosincratic shocks at firm level and the aggregate fluctuation of GDP.

Given that the distribution of firms size is *power law* with a tail index very close to 1, and given N, we have a high value of H. The value of H decrease very slowly with the number of firms.

Thus, in order to have aggregate fluctuactions in this model, we do not need to have aggregate shocks, but just heterogeneous (power law distributed) interacting (they are all financed by the same bank) firms.

This argument is very similar to [Gabaix, (2011)]. If all firms would have been of the same size, than H would have been proportional to $1/\sqrt{N}$.

We can suppose that in these two different trends appear some difference in the *size distribution*: we suppose that when there is higher value of H, there is a presence of some very large firm that strongly affects the GDP fluctuations.

In the figure 2.20 , there is shown the linear regression to compare the standard deviation of GDP (over 15 time steps) and its analytic value (that include squared Herfindhal Index).

We found that there are a reletions between the values of standard deviation GDP and its analytic values.

The mean of GDP , as we can see using the equation (2.54) is equal to $\omega\ \beta\ \phi/g$, that is the ROE. More precisely, it depends on growth of loans, on the expected value of the ratio loans-capital and on the sum of weighted roa.

To get the equality between g_k, that we obtain in the simulation results, and the right side of the equation, the analysis suggest us that we have to get a negative value of sum weighted roa.
Why we have a negative summed weigthed roa? We have found that in some periods of time, when large firm start decreasing, their profit are strongly negative. Thus, as its profit has a greater weight on the economy, and its size also has a great impact on the economy, the sum of the weighted ROA is negative (see the figures 2.21 and 2.22).

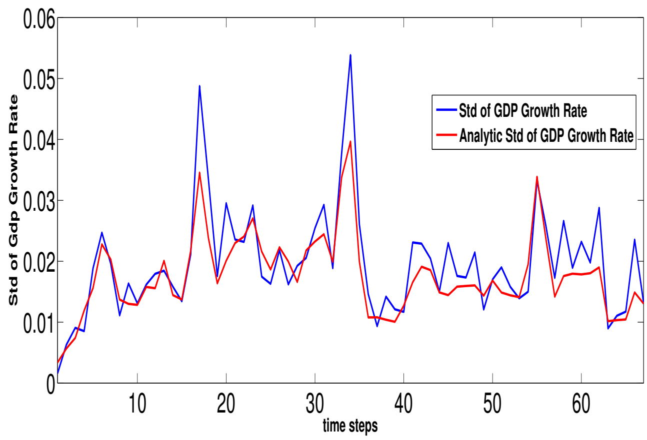

Figure 2.19: **GDP Standard Deviation**: The blu line is the GDP STD in simulated model , the red line is a Analytic GDP STD and the black line is the equilibrium.

The evolution of firms' equity /Figure (2.14)) shows that in some periods the largest firm record a fall in growth and maybe this can affects negatively the ratio of aggregate of profit and aggregate of capital.

We try to analyze what is the percengate of equity (over whole economy) own by firms that record the lowest profit for any year (see the figure 2.22). We have noted that in some time simulation $T = 245$ and $T = 504$ the percentage of equity own by the firm that records the lowest profit rate is very

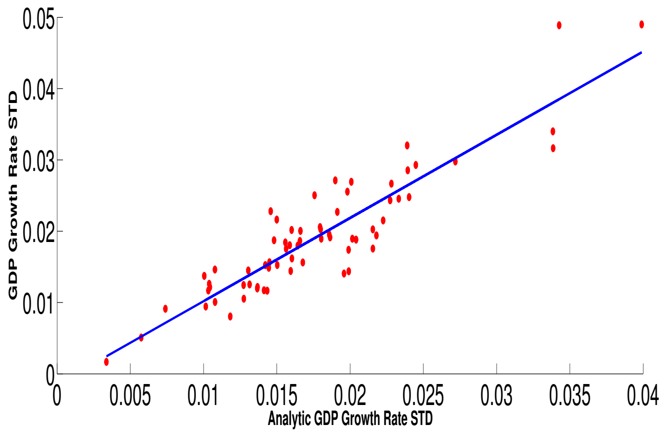

Figure 2.20: **The Evaluation of GDP Standard deviation**: The GDP analysis is based on GDP STD in simulated model and GDP as analytic result. The line $Y = b{+}ax{+}e$, where $a = -0.0015$ and $b = 1.1668$, with s.e 0.0663 and p-value 0.

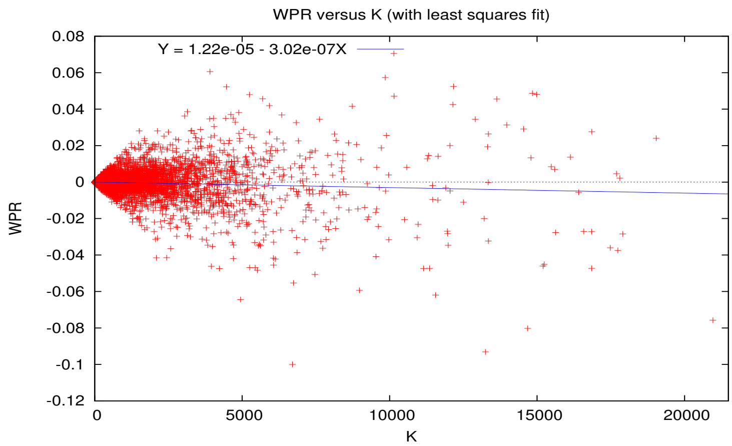

Figure 2.21: **Regression**: We have estimated the relation between the capital and the weighted profit rate of the 80% largest firms. Where the capital is the independent variable and the weighted profit rate the dependent variable. higher the capital, more negatively is the slope of the regression line. We can suppose that the largest firms in some periods record higer negative profit that affect all the aggregate. $Y = \beta_0 + \beta_1 X$, where β_0 is $1.21 * 10^{-5}$, with s.e. $1.59 * 10^{-6}$ and p-value 0, and β_1 is $3.02 * 10^{-7}$ with s.e. $5.21 * 10^{-9}$ and p-value 0.

high.

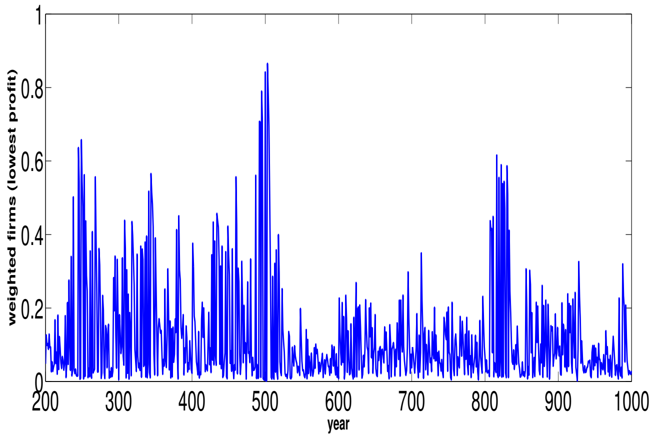

Figure 2.22: **The Evolution Share of Equity of firm that records the lowest (negative) profit**

We can suppose that the high negative profit related to high weight of the firm (based on its size) do affect the whole aggregate and for this reason we have a negative ratio aggregate profit - aggregate capital ($\sum \pi / \sum K$).

We analyze firms with lowest profit at time $T = 245$ and $T = 504$.
In $T = 245$ the lowest profit is $-906,81$. Before the big negative profit, the share of equity, that the large firm owns, was almost 63% of total equity. After the loss, it was about 54%. We have analyzed why this large firm records a big negative profit: the price it sell the goods is 0.0261 and the interest rate, that is the cost of the loans, is 0.1253.
As we can analyze, the large firms get high amount of loans, thus the cost of loans is high. Instead, the gain it had for selling its good is low, due to the low price. This explain the negative profit and this high negative profit

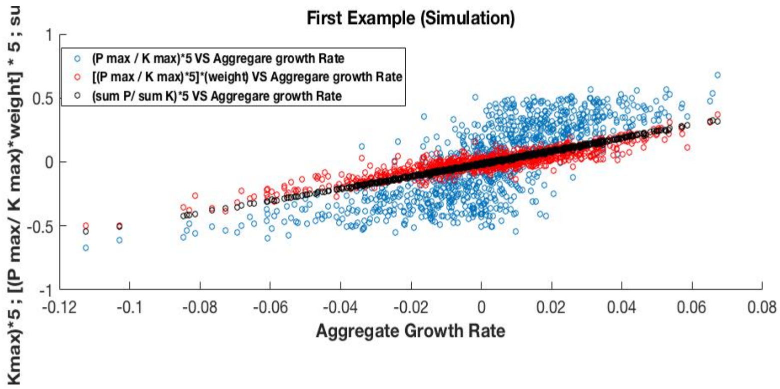

Figure 2.23: **Aggregate Growth Rate and Aggregate Roa Analysis** . We try to compare the relation between the Aggregate Growth Rate and the Aggregate Roa or Roa of the largest firm (at any year) or the weighted Roa of largest firm, over the time span of the simulation. Al the Roa are multiplied by $1/(2 * \phi)$.

related to one larger firms lead to a negative ratio aggregate profit-aggregate capital. In $T = 245$ this ratio $\sum \pi / \sum K$ is -0.1072.

In the time $T = 504$ we have the same situation. The firm that records the lowest (negative) profit is a large firm. It possessed the 78% of the whole firms equity before the loss and then it was 69%. The profit is -1432.31 and its price and interest rate are respectively 0.2824 and 0.1245. The ratio aggregate profit - aggregate capital ($\sum \pi / \sum K$) is negative and it is -0.0950.

We can observe that there is a relation between the aggregare growth rate and the aggregate roa and the largest firm weighted roa. This result confirm us, that there is a large firm, for any year, that drive the economy and it is able to affect the whole growth of the economic system, see Figure (2.23).

We can conclude that higher is the percentage of equity own by the firm that records the lowest (negative) profit, much more it affect the ratio of aggregate profit-aggregate capital. This is the reason why the average of the ratio of aggregate profit- aggregate capital (along all time simulation) is a negative value. The negative profit of large firms deeply affects the aggregate.

2.7 Conclusion

The *deterministic model* describes the interaction between the real sector (the firm) and the financial sector (the bank). The bank has an advantage in paying a lower interest rate to the deposits, while can charge a higher interest rate to the firm. We equalize the firms and bank ROEs and we analyze the interest rate and we consider the two interest rate: r^*, the interest rate based on the perfect competitive equilibrium in the goods' market, and $\bar{r}$, the interest rate equilibrating the two ROEs. thus, we have the following cases:

- $0 < r_t < \bar{r}$: in this case the demand for credit grows at a rate higher than the supply of credit. This happens because the demand and supply of credit are essentially proportional to the equity of firm and bank, respectively. The corresponding disequilibrium tends to adjust the interest rate upward, reducing the growth rate of the demand for credit and contemporaneously increasing the growth of the supply of credit.

- $\bar{r} < r \leq r^*$: in this case the supply of credit grows at a rate higher than the demand of credit. The corresponding disequilibrium tends to adjust the interest rate downward, reducing the growth rate of the supply of credit and contemporaneously increasing the growth of the demand for credit.

- $r_t > r^*$ the growth rate of the firm equity will be negative, with a corresponding shrinking of the size of the firm. The scenario might end up with the bankruptcy of the firm or the decreasing the interest rate and therefore to a switch to positive profits of the firm.

Said all that, the only value of the interest rate compatible with a stationary condition is $r = \bar{r}$, meaning an equalization over time of an exponentially increasing demand and supply of credit.

Interestingly, the model gives some insights on the determinants of the growth of the system bank-firm-economy. To increase the overall growth rate of the economy, the bank can charge a higher spread to the deposits, increasing ω, but it might determine a reduction in deposits. An alternative strategy would be to increase the leverage parameter β. In the deterministic version of the model, such strategy increases the growth of the economy, since the bank can exploit more its oligopolistic position further leveraging ω and, at the same time, charging to the firm a lower interest rate, which, in turns, determines higher profits both for the bank and the firm. On the contrary, in the stochastic environment with heterogeneous firms, a too high leverage might destabilize the system bank-firms, creating a fragile growth. A trade-off leverage-growth-fragility might emerge out of the model. Alternative ways to increase the growth of the system are related to increasing ϕ and/or decreasing g, both parameters linked to a quite standard concept of technological progress.

The aim of the analysis of the stochastic model is, studying the deterministic model, to find some theoretical predictions of the model and to solve some aspect of the dynamics of the model (for example GDP dynamics). In fact, through the analytic study we found that the GDP fluctuations are due to the *roa* standard deviation times the herfindhal - hirshman index. As in [Gabaix, (2011)], we show and confirm that the fluctuation of the GDP definitively depends on the H fluctuation.

The result of our analysis, maybe, could be also applied to other similar model present in the literature.

2.8 Appendix

2.8.1 Parameter Setting and Simulation Procedure

All simulations refer to a benchmark parameter setup. We considered the economy populated by $N^f = 500$ (stochastic model) and 100 (deterministic model with imposed heterogeneity) firms and $N^b = 1$ bank. In total, there are 6 parameters, which are calibrated as follows: $\phi = 0.1$, $\nu = 0.08$, $\lambda = 0.3$, $\omega = 0.002$, $g = 1.1$, $c = 1$.

The exit of firms is by the size of the equity: at first time the threshold is equal to 0.001From the second time this threshold increase by the rule $E_{t-1}^{exit} * (1 + (\beta * \omega * \frac{\phi}{g}))$ at any time.

2.8.2 Variables and Parameters

Variables

- u = random price;
- K = capital;
- L = firms' Liabilities;
- E = firms' Equity;
- Π = firms' Profit;
- I = firms' Investment;
- K^* = firms' Desired Capital;

- $I^l f$= firms' Investment in Labour Force;
- PrB = firms' bankruptcy probability;
- ρ = firm roa;
- χ = firm roe;
- r = interest rate;
- ${L^s}_t$ = bank total credit Supply;
- ${L^s}_{it}$ = bank credit Supply for each firm;
- E^B = bank Equity;
- Π^B = bank Profit;
- B^{debt} = bad debts;
- D = bank deposit.
- ρ^b = bank roa;
- χ^b = bank roe.

Parameters

- t = year;
- ϕ = capital productivity;
- g = total variable costs;
- λ = parameter to allot credit according to Basilea II;

- ν = bank risk coefficient;
- ω = the degree of competition in the banking sector;
- c = parameter for bankruptcy costs;

2.8.3 Simulation Settings

Initial Settings

- T = 1000 ;
- Firms Number = 1, 100, 500;
- $K^{entry} = 10$;
- $E^{entry} = 2$;
- $L^{entry} = Kentry - Eentry$;
- $E_t^{exit} = E_{t-1}^{exit} * (1 + (\beta * \omega * \phi/g))$.

Parameters Settings

- u = [0 ; 2]
- $\phi = 0.10$;
- g = 1.1 ;
- $\lambda = 0.3$
- $\nu = 0.08$;

- ω =0.002;
- c = 1;

2.8.4 Equity−Capital and Loans−Capital Ratio

in this section we show some figure related to the equity − capital and loan − capital ratios in the deterministic /case 1 and case 2) and stochastic models.

We take the ratios value of the last time simulation. We have:

− Stochastic Model: $\frac{E^f}{K} \simeq 0.20$ and $\frac{L}{K} \simeq 0.80$;

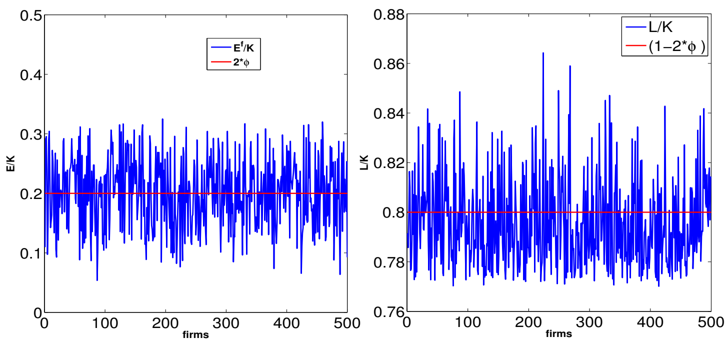

Figure 2.24: E^f/K **and** L/K **in the 500 firms in the last time simulation**

Chapter 3

Profit Rate and Growth Rate Analysis in Agent-Based Model

3.1 Introduction

In this chapter, we will study the profit rate and growth rate statistical properties of the model of [Delli Gatti et al., (2005)]. Including the analysis of long− lived firms over a time span of thirty years, we find that profit rates and their volatilities are independent of size. We find that the empirical densities of both profitability and growth can be described by exponential power (or Subbotin) distribution, but there are pronounced differences in their parameterizations. Instead, in our case, autocorrelation structures are quite similar in growth rate and profit rate. On the base of their empirical densities, the model is able to replicate, via simulation, the growth rate and profit rate dynamic in a new perspective, as studied by [Alfarano et. al (2008), (2012)] and [Mundt et al. (2014), (2016)]. In this new perspective we use the Subbotin density in presence of complex interactions among competitive heterogenous firms.

The empirical regularities, as a relatively stable skewed firm size distribution [Axtell, (2001)] and [Gaffeo et al., (2003)], and other important type of stylized facts are well reproduced by agent-based models.

Following the agent based model approach, in this chapter we develop a model able to replicate jointly empirical regularities in industrial dynamics (e.g., firm size distributions) and macro statistical properties (mean and standard deviation of profit and growth rate, empirical distribution densities analysis and autocorrelation). This work is based on an existing agent−based model [Delli Gatti et al., (2005), (2008)] which, simulating the behavior of interacting heterogeneous firms and of the banking system, is able to generate a large number of stylized facts, but we built the model to address our analysis on other new vision: the growth rate and profit rate analysis. As we can see in the [Delli Gatti et al., (2005), (2008)] the growth rate distribution follows a double exponential distribution (Laplace distribution) [1]. We show that the growth rate distriburtion does not follow a Laplace distribution, as in the [Stanley et al. (1996)] and [Bottazzi et al., (2001)] analysis, but it follows a more leptokurtik distribution than Laplacian. The profit rate distribution, instead, is well approximated by a Laplace distribution. These results are robust with the analysis of US non−banking firms analyzed by [Mundt, (2014), (2016)], [Livan et al, (2015)]. The profit rate analysis and its empirical distribution has previously been considered by [Alfarano et al. (2008), (2012)] and [Mundt et al., (2014), (2016)] who pro-

[1] The growth rate distribtion in Delli Gatti et al, (2005), is a right skewed distribution

posed a diffusion model to account for the Laplace distribution of profit rates. Their model will guide our present investigation and analysis.

The chapter is structured as follow. In the section 3.2 we present the model. In section 3.3 we describe firms dynamics, the growth rate and profit rate structures. In the section 3.4 we analyze the growth rate and profit rate distributions. In the section 3.5 we study the autocorrelation of the rates (growth and profit) and in the section 3.6 we show the result (figures) of simulations. In the last section we propose our conclusions.

In the Appendix we can find the variables and parameters settings.

3.2 Dynamics of the Model

As in the model we studied in *chapter 2*, [Delli Gatti et al., (2005)], we propose the same agent-based model, with differences in the price whose firms sell their homogeneous good.

3.2.1 Firms Behavior

At any time t, with $t = 1, .., T$,the economy is populated by a constant number of firms N, whose indexes are $i = 1, .., N$. At each time t firms produce homogeneous goods with only an input, capital K and with the level of technology ϕ [2] ,hence the level of production of $i - th$ firm is:

$$Y_{it} = K_{it}\phi. \tag{3.1}$$

[2] In the model the level of technology ϕ is equal among the firms and constant along the time and the capital stock never depreciates.

We set two different processes to generate the price dynamics in the model.

In the first case, we set the price as a random walk, with upper and lower bound which coincides with 0 and 2, the extremes of the interval of the price in the previous version of the model. In this way the expected value of the price is the same that in the case the price is uniformly distributed with uncorrelated realizations at any instant of time. This implies that we can solve the bankrupt probability in the same way of the model [Delli Gatti et al., (2005)] . In the problem of optimization, we consider the unconditional expected value of the price when computing the optimization problem for Γ.

From time $t+1$ firms sell their goods at price p_{t-1} plus a random variable normal distributed; from the second period of time, the price follows a **random walk**.

$$p_{it} = p_{it-1} + z_{it}. \tag{3.2}$$

The variable z is normal distributed with $\mu = 0.00020$ and $\sigma = 0.016$ [3]. The variable z_{it} changes every year and among firms [4].

In the second case, we use, as in [Delli Gatti et al., (2005)], the price is

[3] $z \sim$ N $(0.00020, 0.016)$.

[4] We have introduced two bounds: the lower bound is 0 and the upper bound is 2. In this way we maintain the expected price close to 1. We define the j-th firm, the firm that, at time t, get the price out the lower and upper boundaries (where $0 < j < N^f$). In the case the j-th price is lower than 0, we use the $|p_{jt}|$; when the i- th price goes beyond the value 2, we set the price as 2 - $|z_{jt}|$.

a random price uniformlly and unconditionally distributed in the range $[0, 2]$. This price is an *iid* variable.

Firms sell their production and market price of output is P_{it}, according to the law

$$P_{it} = u_{it}. \tag{3.3}$$

The u_{it} is a uniform random variable in the range $[0, 2]$, that changes among firms.

3.3 Firms Dynamics, Profit Rate and Growth Rate

3.3.1 Firms Dynamics

In this section, we focus the analysis on some properties of the baseline of the model. Our main goal consists in showing that the system is able to reproduce a number of macroeconomic stylized facts, which characterize the most industrialized countries under normal economic conditions. Frist of all, we analyze the evolution of the aggregate of firms equity. The aggregate of firms equity grows along the time. Related to analysis of the equity, we found that the model dynamic generates a fat tail distribution of size (we use the equity as proxy of firms size) [5]. As in real industrialized economies,

[5] The aggregate equity and the size distribution are evaluated considering all firms in the system (long-lived firms and new entry firms).

our model well reproduces an important stylized fact: firms' size distribution is power law distributed . Figures (3.10) and (3.11) show this evidence and that the distribution is well fitted in the tail by a power law distribution.

3.3.2 Growth Rate and Profit Rate

We focus our analysis on the study of long−lived firms, that are the firms able to operate in the market in the whole time span of 30 years [6] . We have chosen the period of 31 time because our aim is to compare our simulated data to the empirical data in [Mundt et al., (2014), (2016)]. The long-lived firms represent the 70% of total number of firms. They represent more than 70% of the total assets, aggregate equity and production.The importance of such a *granular* view of the economy has recently been argued by [Gabaix, (2011)], who finds that about one third of variations in GDP growth [7] can be attributed to the idiosyncratic destinies of the largest firms.

Lets explain how we compute the *profit rate* and *growth rate*.

We evaluate the annual *profit rate* (or return on assets) , pr_{it}, as the ratio between the i-th firm *profit*, Π_{it} and the i-th firm total assets, TA_{it}. In this case, the total assets are well approximated by K_{it}.

$$pr_{it} = \frac{\Pi_{it}}{K_{it}}. \tag{3.4}$$

[6]In our model, we consider the long-lived firms, in any simulation, in the period from 980 to 1010. It means that firms that the long-lived firms we analyze are firms that do not fail in the time of 30 years (from T=980 to T=1010).

[7]Gabaix has carried out his analysis on USA non−banking firms.

The annual growth rate gr_t is the logaritmic difference in size (in this case the size is the total assets, that is the capital) in two period of time:

$$gr_{it} = \log(K_{it}) - \log(K_{it-1}). \tag{3.5}$$

To better understand the distribution dynamics of growth rate and profit rate, we analyze the evolution of profit rate and growth rate; more exactly we study their median and standard deviation. It is importat to focus on these quantities (mean and standar deviation) : we find that the profit rate of firm exibits less fluctuations than the growth rate of the capital of the firm, (see Figure 3.2). We show that the evolution of firms' profit rate is very stable, even compared to the firms' growth rate.

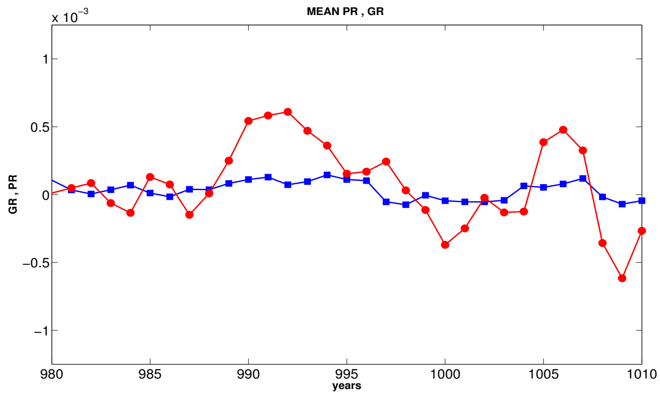

Figure 3.1: **Evolution of the mean of Profit Rate and Growth Rate of long-lived firms during the 31 periods of time. We have chosen the period of 31 time because our aim is to compare our simulated data to the empirical data in [Mundt et al., (2014), (2016)].**

3.4 Empirical Densities

In this section, we want to study the fit of the empirical distribution related to growth and profit rates that our model have generated.

Moreover, we want to compare and analyze the growth rate and the profit rate empirical densities in 2 cases: (i) when the price follows a random walk process and (ii) when the price is iid uniformly distributed variable.

In the recent literature about growth rate distributions, it is common practice, to evaluate the growth rate distribution, to eliminate possible trends in firm size by considering the normalized (logarithmic) size, so we "de−trend" the growth rate. The normalized logaritminc size is expressed in the following equation:

$$ns_{it} = log(S_{it}) - \ N^{-1} \sum_{i=1}^{N} log(S_{it}). \tag{3.6}$$

Thus, we obtain the *normalized logaritmic size* subtracting the average of log−size of all firms at time t to the log−size of the i-th firm. Then the normalized growth rate is defined as :

$$s_{it} = ns_{it} - ns_{it-1}. \tag{3.7}$$

The **normalized growth rate** is the difference of two consecutive periods of times of normalized logaritmic size.
For the *profit rate* we use the raw form, that is $pr_{it} = \Pi_{it}/K_{it}$.

Furthermore, in the literature, we find several models that reproduce the

Laplacian distribution for firms growth rates, [Stanley,(1996)] [Bottazzi and Secchi, (2006)].

As in [Bottazzi and Secchi, (2006)] we try to test the Laplacian hypothesis with a more general distributional class, the *Subbotin distribution*, also known as the generalized exponential-power distribution. The Subbotin is characterized by three parameters: a location parameter m, a scale parameter σ , and a shape parameter α . The last parameter is responsible for qualitative differences in the distribution. To fit the empirical distributions of growth and profit rates, we employ the exponential power distribution first suggested by [Subbotin (1923)]. The form of this function is:

$$f(x|m,\sigma,\alpha) = \frac{1}{2\sigma\alpha^{\frac{1}{\alpha}}\Gamma(1+\frac{1}{\alpha})} \exp\left(-\frac{1}{\alpha}\,|\frac{x-m}{\sigma}|^{\alpha}\right) \tag{3.8}$$

where α and σ are $\in \mathbb{R}^+$, $m \in \mathbb{R}^+$ and $\Gamma(.)$ indicates the *gamma function*.

More precisely, when the shape parameter α is equal to one $\alpha = 1$, the distribution fits to the Laplacian; instead if α is equal to two ($\alpha = 2$) the distribution fits to Gaussian. [8]

Lets' us analyze the case in which the price is a *random walk variable*. In this case the α shape parameter assumes value equal to 1, in the *profit rate* and less than 1 in the *growth rate*.

In our analysis we study the profit rate and growth rate distribution considering the annual pooled empirical densities.

[8] in this model, we compute m as the median value of growth rate (profit rate).

Especially, if we consider the **growth rate** distribution, the estimation of α shape parameter is 0.8648, the confidence interval is $[0.8467, 0.8828]$. This estimation shows, and confirms, that the growth rate is not Laplacian (or double exponential distribution), but it is more leptokurtic than a laplacian distribution [9] (see Figure (3.2 left side).

On the other hand, if we consider the *profit rate distribution*, we can observe the estimated α shape parameter is 1.0276 with the confidence interval is $[1.0042, 1.0510]$, see Figure 3.2 right side. [10]. This confirms us that the profit rate distribution follows a Laplace distribution.

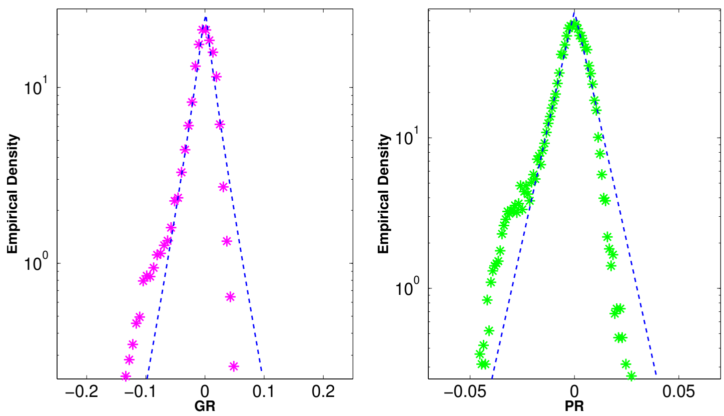

Figure 3.2: **Profit Rate (rigth side) and Growth Rate (left side) of pooled data (random walk price variable): the empirical analysis of the distribution gives us some intersting results.**
The estimated shape parameter of profit rate of long− lived firms is equal to $\hat{\alpha}_{pr} = 1.0276$, very close to the laplace shape parameter distribution (1), and the estimate scale parameter is equal to $\hat{\sigma}_{pr} = 0.0072$.
The mean shape parameter of growth rate of long− lived firms is $\hat{\alpha}_{gr} = 0.8648$, leptokurtik distribution, and the estimate scale parameter is equal to $\hat{\sigma}_{gr} = 0.0185$.

Besides, we have studied the year−by−year analysis, on the long−lived

[9] Similar estimates of Subbotin shape parameter are reported by [Bottazzi et al., (2011)].

[10] All the figures and all the results are evaluated as the mean value of the estimated paramenters over 100 different simulations (seeds).

time range [980; 1010], to understand the behaviour of the distributions and analyze if the results we have got analyzing pooled data are representative of the year-by-year analysis. Hence, we focused on the shape and scale parameters of the growth and profit rate distribution to verify towards which distributions they tend, see Figure 3.3 for the evolution of *shape parameter* and and Figure 3.4 for the evolution of the *scale parameter*) [11].

The shape parameter of the year-by-year analysis confirms us that the *growth rate shape parameter* is always lower than 1 (Figure 3.3 left side) ; this analysis is robust with the estimation of the shape parameter of pooled data. The *profit rate shape parameter* is very close to 1 (see Figure 3.3 right side). This analysis shows the consistence of the estimation .

As we said, the *Figure (3.4)* shows us also the evolution over time of the scale parameter of growth rate (left side) and profit rate (right side). The estimation of the scale parameter of growth rate, $\hat{\sigma}_{gr}$, is higher in value and variations that the parameter of profit rate, $\hat{\sigma}_{pr}$.

The estimated scale parameter of growth rate is:

(1.) $\hat{\sigma}_{gr}$ is 0.0185 (with confidential interval $[0.0182, 0.0188]$).

and the estimated scale parameter of profit rate is:

(2.) $\hat{\sigma}_{pr}$ is 0.0072 (with confidential interval $[0.0071, 0.0073]$),

[11] In specific way, we use the error bar figures to study and to understand the evolution of the distributions

Thus, as we can better see in Figure 3.3 , the variation of the width of the distribution is more evident for growth rate.

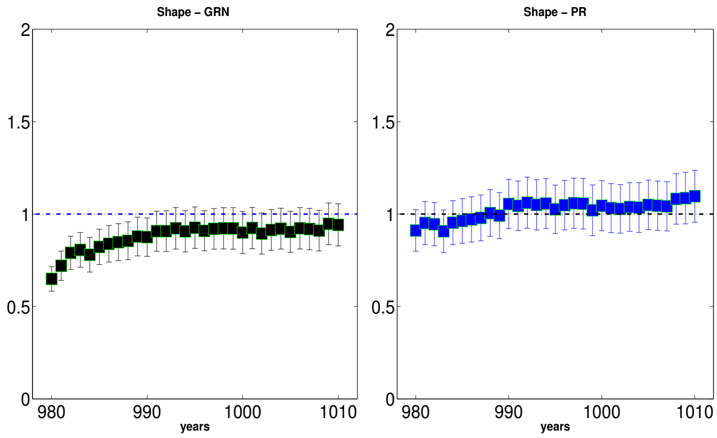

Figure 3.3: **Profit Rate (left side) and Growth Rate (right side) - random walk price variable: the empirical analysis of the distribution gives us some intersting results.**
The mean shape parameter of profit rate of long−lived firms in the 31 periods of time is equal to $\bar{\alpha}_{pr} = 1.0276 \pm 0.0.02$**, very close to the laplace shape parameter distribution (1).**
The mean shape parameter of growth rate of long−lived firms in the 31 periods of time is equal to $\bar{\alpha}_{gr} = 0.8648 \pm 0.018$**, a leptokurtik distribution. The blue - dotted line is the Laplace shape parameter benchmark (**$\alpha = 1$**).**

Lets' us analyze the case in which the price is a *iid variable.*

In this case the α shape parameter assume value bigger than 2, both in the *growth rate* and in the *profit rate*. Especially, if we consider the **growth rate** α parameter estimation, it is 2.2288, with the confidence interval is $[2.1718, 2.2859]$, see Figure 3.2 right side.

The *growth rate distribution* do not follow a Laplacian distribution (see Figure 3.5 (left side)). We can observe an irregular distribution which shape parameter suggest that it is closer to Gaussian distribution.

On the other hand, if we consider the *profit rate distribution*, we can

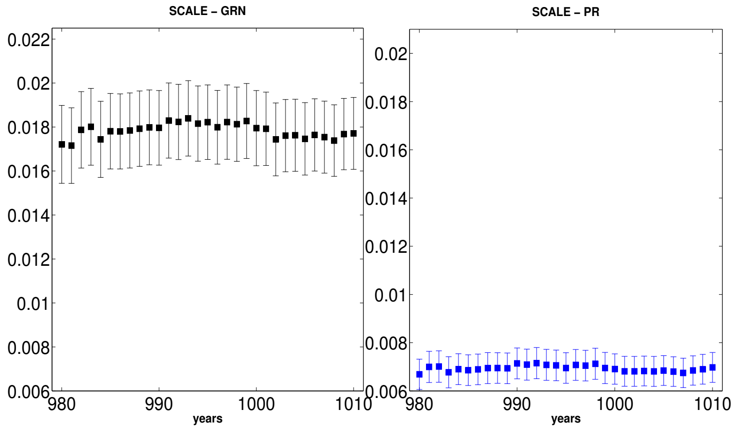

Figure 3.4: **Profit Rate (left side) and Growth Rate (right side) - random walk price variable: the empirical analysis of the distribution gives us some intersting results.**
The mean scale parameter of profit rate of long− lived firms in the 31 periods of time is equal to $\bar{\sigma}_{pr} = 0.0072 \pm 0.0001$**.**
The mean scale parameter of growth rate of long− lived firms in the 31 periods of time is equal to $\bar{\sigma}_{gr} = 0.0181 \pm 0.0003$**.**
The estimation shows us that the variation of the growth rate is higher that the profit rate.

observe that the estimated α shape parameter is equalt to 15.3689, with the confidence interval is $[14.7315, 16.006]$.

As we have analyzed in the previous section, we know that the profit rate is expressed as the following equation: $(\phi\ u_{it}\ -\ g\ r_{it})$. Thus, the distribution is strongly affected by, or better, it is strongly dominated the price distribution, that we know to be a uniform distribution, If we look at the Figure 3.5 (right side) we can observe that the *Subbotin* curve suggest us that the *profit rate distribution* follows a uniform distribution. In fact, if the parameter α tends to infinite, $\alpha -> \infty$, the distribution will be an *uniform distribution*, as discussed in Chiodi, 2000.

In this case, both for growth rate distribution and for profit rate distribution a maximum likelihood test rejects the Laplace in favor of the Subbotin

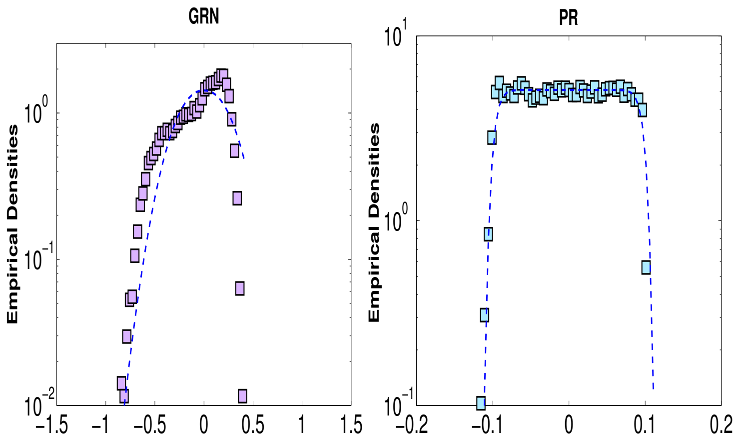

Figure 3.5: **Profit Rate (rigth side) and Growth Rate (left side) of pooled data (iid price variable): the empirical analysis of the distribution gives us some intersting results.**
The estimated shape parameter of profit rate of long− lived firms is equal to $\hat{\alpha}_{pr} = 15.3689$ **and the estimate scale parameter is equal to** $\hat{\sigma}_{pr} = 0.0851$**.**
The mean shape parameter of growth rate of long− lived firms is $\hat{\alpha}_{gr} = 2.2288$ **and the estimate scale parameter is equal to** $\hat{\sigma}_{gr} = 0.2759$**.**

distribution (it is shown in the Figure 3.5).

As we said, the *Figure (3.7)* shows us also the evolution over time of the scale parameter of growth rate (left side) and profit rate (right side). The estimation of the scale parameter of growth rate, $\hat{\sigma}_{gr}$, is higher in value and variations that the parameter of profit rate, $\hat{\sigma}_{pr}$.

The estimated scale parameter of growth rate is: $\hat{\sigma}_{gr}$ is 0.2759 (with confidential interval $[0.2726, 0.2793]$) and the estimated scale parameter of profit rate is $\hat{\sigma}_{pr}$ is 0.0851 (with confidential interval $[0.0845, 0.0856]$),

All the estimation of *scale* and *shape* parameter and the confidence intervals are estimated by the *maximum likelihood estimation*.

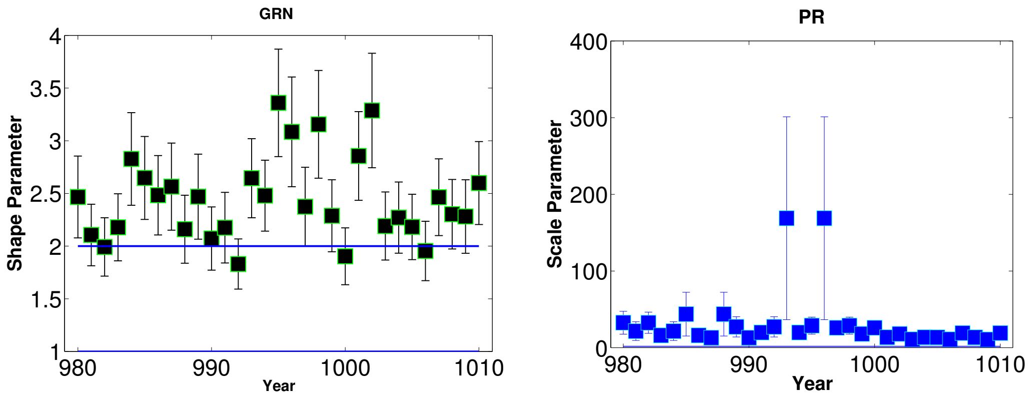

Figure 3.6: **Profit Rate (left side) and Growth Rate (right side) - iid price variable:** the empirical analysis of the distribution gives us some intersting results. The scale parameter estimation confirm us that the growth rate distribution is far from the Laplace distribution and its particular distribution is closer to the Gaussian distribution. In the case of profit rate distribution, it is represent the price distribution: it is a uniform distribution with very high value of shape paramenter.

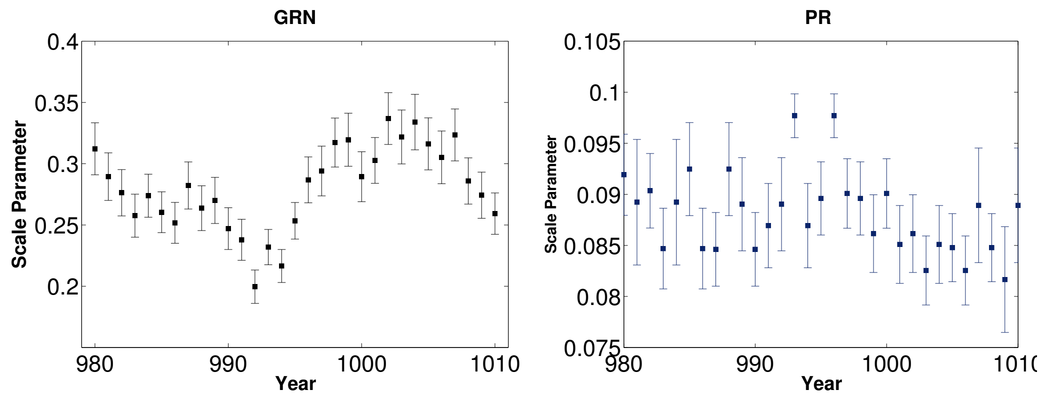

Figure 3.7: **Profit Rate (left side) and Growth Rate (right side) - iid price variable:** the empirical analysis of the distribution gives us some intersting results. The scale parameter estimation confirm us that the variation of the width of the distribution is more pronounced for growth rate.

3.5 Autocorrelation

Lets' us analyze the case in which the price is a *random walk variable*. The random walk price of p_{it} strong affects the *profit rate* and *growth rate*, autocorrelation. We can observe that the **profit rate** has a significant autocorrelation in time, more precisely in the first 4 lags, (Figure 3.6 right side). Also the **growth rate** has a positive autocorrelation in the first 4 lags, (Figure 3.8 left side).

In [Mundt et al., (2014), (2016)], [Alfarano et al., (2008), (2012)] we can observe a positive autocorrelation in **profit rate**, the same that we found in our simulated model. Instead for **growth rate** in [Mundt et al., (2014), (2016)], [Alfarano et al., (2008), (2012)] we can observe that there is no autocorrelation, this result is very different from what we have got in our model.

This different result is due to the fact that the price follows a random walk that creates a dependency (in our model) of the profit over time [12]. The dependency of the profit over time explains also the positive autocorrelation of the growth rate. In our model, the growth rate, as we explained before, is a logaritmic difference of the capital (our firms total assets). The capital is the result of the sum of the firm liabilities and firm equity (see balance sheet identity in *section 3.2*) and equity depends on the profit got by firms at any period of time (see equation 3.4). Thus, even the capital depends on the profit, and this dependency generates a positive autocorrelation also in growth rate.

[12]In fact, as we have seen, the profit rate is autocorrelated.

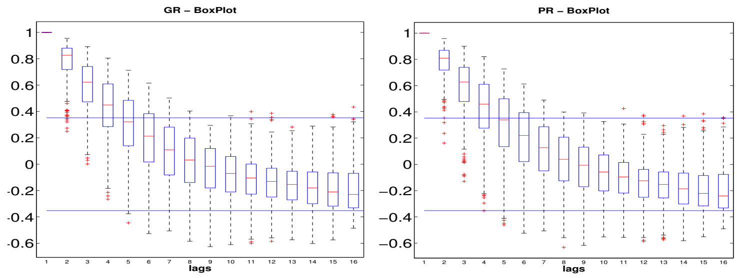

Figure 3.8: **Box−Plot of Autocorrelation of Growth Rate (right side) and Box−Plot of Autocorrelation of Profit Rate(left side). The price is a random walk variable:** box−plot show us autocorrelation in the first lags. This results is given by the random walk of the price that affects the profit rate and the growth rate. The variation (increasing or decreasing) of the price depends on the random variable.
The box−whiskers plot for *growth rate* autocorrelation. Boxes include the 25 percent quantile, the median, and the 75 percent quantile. The blue streight lines show the 95 percent confidence interval under the null hypothesis of zero autocorrelations. The interval has been computed as $\pm\frac{1.96}{\sqrt{T}}$ where $T = 31$, that is the length of the profit rate timeseries.

Lets' us analyze the case in which the price is a *iid variable*.

If the price is an iid variable, we can observe that the **profit rate** doesn't have autocorrelation over time. This results is consistent with the structure of the price, that change over time. The profit rate is , as we said, strongly affected by the price. The variation of the price strong affects the *profit rate*. If u_{it} is always different between u_{it-1}, there is no autocorrelation along the time series of profit rate.

Also, the **growth rate** does not has a significant positive autocorrelation. This result is consistent with what [Mundt et al., (2014), (2016)], [Alfarano et al., (2008), (2012)] have found in their research.

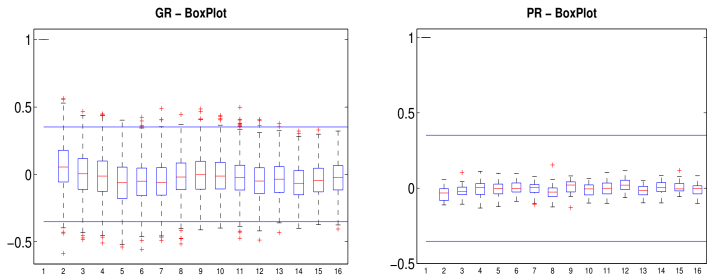

Figure 3.9: **Box−Plot of Autocorrelation of Growth Rate (right side) and Box−Plot of Autocorrelation of Profit Rate(left side). The price is an iid variable):** box−plot show us autocorrelation in the first lags. This results is given by iid price that affects the profit rate and the growth rate. The variation (increasing or decreasing) of the price depends on iid price variable.
The box−whiskers plot for *growth rate* autocorrelation. Boxes include the 25 percent quantile, the median, and the 75 percent quantile. The blue streight lines show the 95 percent confidence interval under the null hypothesis of zero autocorrelations. The interval has been computed as $\pm\frac{1.96}{\sqrt{T}}$ where $T = 31$, that is the length of the profit rate timeseries.

3.6 Other Figures and Results

We also found that the model generate a power law firms' size distribution for both cases we analyze:

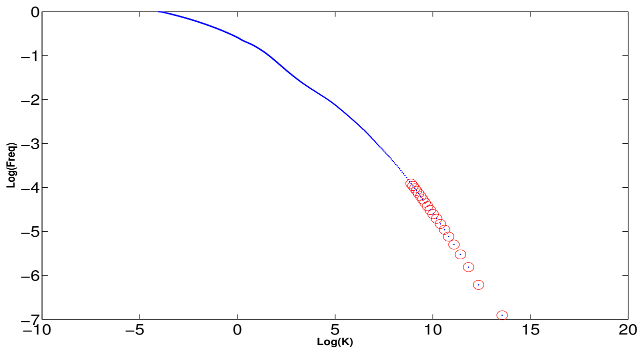

Figure 3.10: **Size distribution (Equity): price as random walk variable** The OLS estimation confirms us that the distribution is well fitted in the tail by a power law distribution $y = Ax^{\alpha}$, the scaling exponent α is equal to -0.6530 with s.e. 0.0048 and p-value 0.

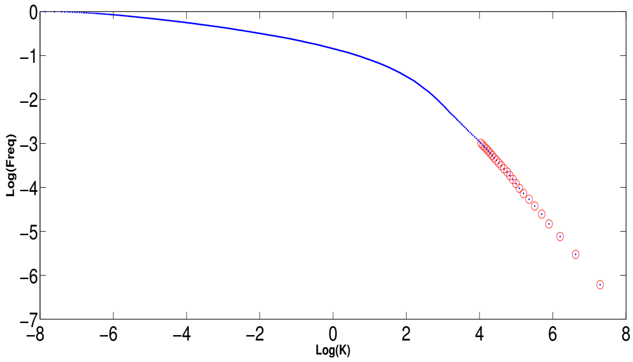

Figure 3.11: **Size distribution (Equity): price as iid variable** The OLS estimation confirms us that the distribution is well fitted in the tail by a power law distribution $y = Ax^{\alpha}$: the scaling exponent α is equal to -0.9928 with s.e. 0.0033 and p-value 0.

3.7 Conclusions

The model shows us some evidence of the dynamics of the firms: the firms' size follows a right skewed distribution and it is a power law distribution. The empirical analysis of the growth rate has shown different result form the [Stanley et al., (1996)], [Bottazzi et al., (2001)], but, in the case the price ia a random walk variable, the analysis is able to confirm the analysis conduct by [Mundt et al., (2014), (2016)], [Alfarano et al., (2008), (2012)] that represent the analysis of the real data in the US market. The growth does not follow a Laplace distribution, but the analysis shows that it follows a more leptokurtic distribution than Laplace. The empirical analysis of the profit rate, as in [Mundt et al., (2014), (2016)], [Alfarano et al., (2008), (2012)] shows that it follows a laplace distribution. This results are confirmed also by the estimation of shape and scale parameter in the year-by-year analysis.

In the case the price is a iid variable, the growth rate distribution follow a distribution which estimated shape parameter suggests that it is more close to a Gaussian Distribution than a Laplacian distribution, although the growth rate distribution has a particual form. The profit rate distribution confirm us that the profit rate is strongly affected by the price. In fact, its distribution follow a uniform distribution.

Moreover, in the first case, we find that both growth rate and profit rate are autocorrelated in the first lags. This result depends on the fact that both of them are affected by the evolution of profit and, in turn, it is affected by the price. The fact that the profit rate is autocorrelated confirms the analysis of the profit rate autocorrelation of profit rate US firms [Mundt et al., (2014), (2016)].

Otherwise, in the second case, the growth rate is autocorrelated only in the first lag, as in [Mundt et al., (2014), (2016)], and the profit rate shows no autocorrelation. We have this result because the price is a iid variable that affect the evolution of the growth rate and profit rate.

3.8 Appendix

In this section, we want to describe assumptions and procedures we followed to simulate the model. A simulation is completely described by the parameter values, and the initial conditions. Firstly, we set the parameter values and the initial conditions for state variables needed to start the simulation. These parameters are relative to the firm, bank and the entry process.

3.8.1 Variables and Parameters Setting

3.8.2 Variables

- P_{it} = iid random price variable;
- p_{it} = random walk price variable;
- K = capital;
- L = firms' Liabilities;
- E = firms' Equity;
- Π = firms' Profit;
- I = firms' Investment;
- K^* = firms' Desired Capital;
- $I^l f$= firms' Investment in Labour Force;
- PrB = firms' bankruptcy probability;
- r = interest rate;

- ${L^s}_t$ = bank total credit Supply;
- ${L^s}_{it}$ = bank credit Supply for each firm;
- E^B = bank Equity;
- Π^B = bank Profit;
- B^{debt} = bad debts;
- D = bank deposit.

3.8.3 Parameters

- t = year;
- ϕ = capital productivity;
- g = total variable costs;
- λ = parameter for allotting credit according to Basilea II;
- ν = bank risk coefficient;
- ω = the degree of competition in the banking sector;
- c = parameter for bankruptcy costs;
- τ = interval time of Long-Lived Firms.

3.8.4 Simulation Settings

Initial Settings

- T = 1500 ;
- Firms Number = 1000;
- $K^{entry} = 10$;
- $E^{entry} = 2$;
- $L^{entry} = K^{entry} - E^{entry}$;
- $E^{exit}_{t0} = 0.001$;
- $E^{exit}_{t} = E^{exit}_{t-1} * (1 + (\beta * \omega * \phi/g))$.

Parameters Settings

- u = [0 ; 2];
- z ∼ N (μ ; σ);
- $\mu = 0.00020$;
- $\sigma = 0.016$;
- $\phi = 0.10$;
- g = 1.1 ;
- $\lambda = 0.3$
- $\nu = 0.08$;

- ω =0.002;
- τ =30;
- c = 1;

Chapter 4

Financial Fragility and Business Fluctuaction in Agent-Based Model: A Multibanks - Firms Model

4.1 Introduction

The role of the banks is to support the credit needs of firms (large, medium and small firms). Their function is to be the intermediary for firms that need credit to invest in their business and firms that need credit for lack of internal founds.

The function of banks, their activity as intermediary, is very important (expectially at current time) since it strongly affects the entire economy [Grilli et., al (2014), (2015)]. In time of crisis, when productions and economic activity slowdown, the relation between banks and firms once again becomes the focal point of many economics theories. In fact, the bank-firm relationship play a crucial role in exposure of the risk of the avalanches and domino effects [Battiston et al., (2007)], [Delli Gatti et al., (2006)], [Stiglitz and Greenwald, (2003)].

What we want to show is that in the market (any market) can survive different categories of banks (some larger than others) and that the evolution of the GDP, in our model, does not depend on the way firms choose the bank to link to.

The rationality of the agents (firms) is limited, *bounded rationality*, they can choice the largest bank to link to following different approaches: (i) firms choice to link to the largest bank (with higher equity) in the system in a direct way (*direct approach*) or (ii) firms can select the largest bank by a probability (*probabilistic approach*).

We want to focus our attention on the evolution of the system if firms (new firm) link to the largest bank.

Our analysis aims to prove that the analytic results that we got studing the deteministic model *chapter 2*, analytic study of the model [Delli Gatti et al., (2005)], are robust even in the case in which there is the presence of a multitude of banks.

We want to point out that this model generate a system in which a multiple banks can survive simultaneously in the market and (all) grow over time. As we found in the analysis of the model [Delli Gatti et al., (2005)] in *chapter 2*, a bank is able to stay and grow in the economy even if it is linked to a firm. In fact, in the latter case, the bank supports the firm over time, lending more and more credit and the firm repays its financial costs. In this way, the firm becomes large and the bank increases its size (equity).

What we want to show is that no matters the way firms link to the bank, the system, and the banks equity, grows over time at the same rate we found in the model in the *chapter 2*.

Moreover, the interection of heterogeneous agent is one important implication. In fact, the structure of aggregate behaviour (macro) actually emerges from the interaction between agents (micro).Thus, statistical regularities emerge as a self-organised process at the aggregate level. Complex patterns of interacting individual behaviour may generate certain regularity at the aggregate level [Delli Gatti et al., (2005)].

As well, at the same time, even in this model, the main facts of firm demography (like power law distribution of size firms) and economics dynamics (the fluctuation of business cycle) emerge endogenously (and are more resistant to external shocks).

We want to prove that the model is able to replicate some empirical findings that characterize the ABM models, in relation to industrial dynamics and financial facts.

The chapter is divided as follows. In the section 4.2 we explain the model. In the section 4.3 we expose some simulation results and stilyzed facts that the model is able to generate. In the section 4.3 we also show the figures of the outcome of the simulations. In the last section we draw some conclusions.

In the Appendix we show the variables and the parameters setting that we use to costruct the model.

4.2 Dynamics of The Model

Consider a sequential economy over time ($t = 1, 2, ..., T$) populated by a multitude of heterogeneous agents belonging to two different sectors: market sector populated by N^f firms, with $i = 1, 2, ..N^f$, and a banking sector composed by N^b, with $j = 1, 2, ..N^f$. Firms produce and sell all final goods (homogeneous goods) at a stochastic price [1]. The production of the final goods requires only a productive inputs: *capital*. The level of technology is ϕ [2] ,hence the level of production of $i-th$ firm is: $Y_{it} = K_{it}\phi$. We assume that there is no barriers to entry. Firms demand and obtain credit from banks to finance their investment, at the level of interest rate r_t.

At any time t, firms sell their production and the individual selling price is, as in [Greenwald and Stiglitz, (1990), (1993)], the random outcome of a market process around the average market price of output P_t, according to the law $P_{it} = u_{it}P_t$, with expected value $[E(u_{it})] = 1$ and finite variance. The price of firms' output is a stochastic variable. As a consequence, the normalized price is $u_{it} = \frac{P_{it}}{P_t}$.

The variable u_{it} is an uniform random variable support on $[0, 2]$.

By assumption, firms are fully rationed on the equity market (they can not acces to the market share to get the money they need to finance new investments) and the only external source they dispose is bank credit. The $i-th$ firm finances its capital using both internal net worth (or equity) E^f_{it} and external sources (bank loans) L_{it}, according to the balance sheet identity

[1]Firms are like an island. Each firm has its own market, thus it can sell all the quantity it considers optimal. They are *price−taker*.

[2]In the model the level of technology ϕ is equal among firms and constant along the time and the capital stock never depreciates.

$K_{it} = E^f_{it} + L_{it}$.

Firms and bank hold a long-term contractual relationship, the debt commitment in real term is $r_{it}L_{it}$, where r_{it} is the interest rate and it is also the return of net worth. Thus, each firm face financial cost equal to $r_{it}(L_{it} + E^f_{it})$, that is $gr_{it}K_{it}$, where g as other financial cost related to the capital $(g > 1)$ [3].

The equity at time t, E^f_{it} is:

$$E^f_{it} = E^f_{it-1} + \pi_{it} \tag{4.1}$$

Firms go to bankrupt when their net worth becomes negative, that is $E^f_{it} < 0$.

At the end of each period t, the profit is given by $\pi_{it} = u_{it}Y_{it} - g\ r_{it}K_{it}$.

Where the *expected profit* is given by $E(\pi_{it}) = [\phi - gr_{it}]\ K_{it}$.

The probability of i-th firm bankruptcy $[PrB_{it}]$ is

$$PrB_{it} = P\ [u_{it} < u^*{}_{it} = \frac{g\ r_{it}}{\phi} - \frac{E^f_{it-1}}{\phi\ K_{it}}] \tag{4.2}$$

As in [Greenwald and Stiglitz (1993)] , higher is the production, higher is the probability of bankruptcy. Bankruptcy is a cost, and firms take these costs in to account in their production decision. In fact, the probability of bankruptcy, PrB, is part of the firm's profit function. Bankrupt is a cost that increases with the firm's output. That *function of bankrupt cost* is quadratic, that is $C^f = cY^2_{it}$, with $c > 0$.

The objective function, Γ *function*, is the difference between the expected profit, $E(\pi_{it})$ and the bunkruptcy costs times the bankrupt proba-

[3] The constant g should be such that the condition $E^f_{it} < gr_{it}K_{it}$ holds.

bility, $(C^f \; PrB)$, that is:

$$\Gamma = (\phi - g\ r_{it})\ K_{it} - \frac{c\ \phi}{2}(g\ r_{it}\ K_{it}^2 - E_{it-1}^f\ K_{it}) \tag{4.3}$$

Maximizing the function Γ with respect to K , we got K_{it}^*, (*desired capital*):

$$K_{it}^* = \frac{(\phi - g\ r_{it})}{c\ \phi\ g\ r_{it}} + \frac{E_{it-1}^f}{2\ g\ r_{it}} \tag{4.4}$$

We can describe the *investment* as the difference between the i-th firm capital and the capital (derired capital) that the i-th firm desires to have, and we have: $I_{it} = K_{it}^* - K_{it-1}$

Thus using (4.8) eqution we got that the demand of credit of the i-th firm is:

$$L_{it}^d = \frac{(\phi\ g\ r_{it})}{c\ \phi\ g\ r_{it}} + \frac{E_{it-1}^f}{2\ g\ r_{it}} - \pi_{it-1} + E_{it-1}. \tag{4.5}$$

4.2.1 Banking Sector

Firms and the banks interact in the credit market and the interactions between the firms and the banks generate the equilibium interest rate for i-th firm. The relation between the *j-th* bank and the *i-th* firm is a long-term relationship.

Banks enter in the economy with the same initial financial conditions.

The first link (attachment) bank-firm is set up following a radom variable uniformly distributed [4].

We computed two different versions of model. In the first version we study the system populated by 2 banks, in the second version of the model we analyze the economic dynamics of a system populated by ten banks. For any wersion, we analyze two different way to link the new firm to the largest bank in the system: the *direct approach* and the *probabilistic approach*. In real world we can observe that the firm prefers to attach it to largest banks to get better conditions.

Now, we explain the two different approaches we use.

In the *direct approach*, whatever is the number of banks, any new firm links directly to the largest bank in the system.

In the *probabilistic approach*, any $j-th$ bank links to a new firm through a *probability* that depends on the size of each bank. At any period of time, we weight the size of any bank over the total equity of banks present in the system, *preferential attachment mechanism* [Barabási & Albert, (1999)].

$$Pr_{jt}^{b} = \frac{E_{jt-1}^{B}}{\sum E_{jt-1}^{B}} \tag{4.6}$$

Pr_{jt}^{b} is the *probability* that any bank has to get a link with new firm.

The level of credit that the bank can loan is functional to its equity and

[4]The random attachment of new entry firm operates until t=200.

the level of deposit [5]. That is:

$$S_{jt} = (E^B_{jt} + D^B_{jt}) \tag{4.7}$$

To determine the aggregate level of credit supply, we assume that banks are subjected to a prudential rule, thus:

$$S_{jt} = \frac{E^B_{jt-1}}{\nu} \tag{4.8}$$

where ν is a the risk coefficient [6]. The healthier is the bank from a financial point of view, the higher is the aggregate of credit supply.

Credit is allotted to each individual firm i on the basis of the mortgage it offers, which is proportional to its size, and to the amount of cash available to serve debt.

$$L^s_{it} = \lambda \ S_{jt} \ k_{it} + (1 - \lambda_j) \ S_{jt} \ e_{it}. \tag{4.9}$$

where e_{it} is the level of firm financial fragility [7] and k_{it} [8] is the size of the firm business capability. The coefficient λ_j is the parameter to allot credit according to capital and equity. It is constant over time, but differnt among

[5]The value of bank deposit, D_{jt}, in this model is $D_{jt} = \sum L_{it} - E^B_{jt-1}$.

[6]The coefficient is constant along the time and it is in accord to the Basilea II agreement

[7]e_{it} is $E^f_{it-1} \ / \ \sum_{i=1}^{N_{t-1}} E^f_{it-1}$.

[8]k_{it} is $K_{it-1} \ / \ \sum_{i=1}^{N_{t-1}} K_{it-1}$

banks. [9]

We have equilibrium in the credit market through the interest rate (r_{it}) that determine when the demand of credit equals the supply of credit.

$$r_{it} = \frac{2 + E^f_{it-1}}{2\ g\ c\ [(\frac{1}{\phi}) + E^f_{it-1} + \pi_{it-1}] + 2\ c\ g\ S_{jt}\ [\lambda_j\ k_{it} + (1-\lambda j)e_{it}]}. \quad (4.10)$$

A rise (decrease) in profit and in equity base, decreases (rise) the interest rate.

If we assume that the return on bank's equity is the average of lending interest rate $\overline{r_t}$ and the deposit is remunerated with the borrowing rate ${r_{it}}^a$.

The bank profit, π^B_t, is:

$$\Pi^B{}_{jt} = \sum r_{it} L^s{}_{it} - \overline{r_t}, (1-\omega) D_{jt-1} + E^B_{jt-1}. \quad (4.11)$$

where $\frac{1}{(1-\omega)}$ is the spread between lending and borrower interest rate: the higher is ω, the higher is the interest and the monopolistic power of banks. The value of omega is tha same for any banks.

The bank equity E^B_t is given by:

$$E^B_{jt} = \pi^B_{jt} + E^B_{jt-1} - B^{debt}_{jt}. \quad (4.12)$$

[9]The value of λ_j is $0 < \lambda_j < 1$.

where B^{debt}_{jt} are the sum of bad debts, that is all the negative equity of firms, $\sum E^{f}_{it} < 0$.

At the end on any period of time, the *j-th* bank bankrupts if its equity is lower than zero: $E^{b}_{jt} < 0$.

4.3 Empirical Findings, Stylized Facts and Results

We simulated a model with $N^{f} = 500$ (number of firms) and $N^{b} = 2$ and $N^{b} = 10$ (number of banks).

The model we propose is able to represent very important results: whatever is the way firms choose a bank, the model is able to replicate the analytic dynamics we found studyng the model [Delli Gatti et al., (2005)] in the *chapter 2*. In the *direct approach*, the bank with the smallest number of firm links is able to grow over time and increase its equity, even if it is linked to one firm. There is no differences in the banks equity.
We can observe in Figures (4.1) - (4.4) that whatever is the way the new firm is linked to bank, the banks equity grows over time, even when a bank has a few (or only one) links. This result undelines what we found analyzing the deterministic version of the model [Delli Gatti et al., (2005)]. There is a dependency in growth between the bank and the firm(s): bank lends credit to the firm that become larger and larger and this growth allows to the bank to also grow. In this model we propose, what change is not the growth of the banks equity or the growth of the GDP, but it is the *topology* of the

bank-firms links that changes. In the *direct approach* a bank , at the end of the simulation, is linked to almost all firms and the other banks are linked to few (one) firm(s); in the *probabilistic approach* the number of firms linked to the *j-th* bank , over time, fluctuates around the ratio N^f/N^b (see Figures (4.1) and (4.3)).

Moreover, in any version of the model we can observe (Figures (4.5) - (4.4)) that the aggregate of firms equity (and consequently the GDP) grow over time, at the rate of *roe* [10], but with some fluctuations. The growth rate we observed follows the *long-run dynamics* that we found in the deterministic version of the model (chapter 2), which coincides with the analytical formula developed in chapter 2.

Other important result, that the model is able to generate, is the value that we studied in the *deterministic model* analysis (chapter 2). We found the two following results that we are goning to explain.

1. The standard deviation of GDP [11] is well described by the following equation:

$$\sigma_{gdp} = \frac{\phi}{\sqrt{3}} H \tag{4.13}$$

where $\frac{\phi}{\sqrt{3}}$ is the is the analytical expression of the standard deviation *roa* (return on assets) and H is the *Herfindal - Hirshman Index*, similarly to [Gabaix, (2009), (2011)].

[10] As we find in the deterministic model analysis, (chapter 2).
[11] The aggregate capital is our proxy of GDP

We can observe that the right hand of the equation (4.13) is a good proxy of the fluctuations of GDP, see Figures (4. 9) - (4.10) [12]. So, the model populated by a multitude of banks is able to generate, no matter which approach we use to banks-firms link, the same GDP dynamics of the model with only one bank (see the model in chapter 2).

2. The *expected value* of the GDP ($E[g_k]$) is close to the *roe* (return on equity) and it is equal to the following equation:

$$E[g_k] = E[g_t^l]\ (1 - 2\phi) + \ E[\frac{F_t^e}{N}]\ 2\phi + \ E[roa_{it}\ \lambda_{it}] \tag{4.14}$$

Where $E[g_t^l]$ is the growth rate of the aggregate of loans, the $E[\frac{F_t^e}{N}]$ is the expected value of the weighted new entries and $E[roa_{it}\ \lambda_{it}]$ is the aggregate weighted (by size) roa. From this point on, we call the right hand in the equation (4.14) as $\bar{g}_k$.

As we said, the *expected GDP* is equal to roe. Thus we can express the expected GDP as $E[g_k] = \omega\ \beta\ \frac{\phi}{g}$.

In the two version of the model the $E[g_k]$ and the $\bar{g}_k$ are the following:

	$N^b = 2$	$N^b = 10$
(direct approach)	$E[g_k] = 0.00212$	$E[g_k] = 0.00228$
(probabilitistic approach)	$E[g_k] = 0.00216$	$E[g_k] = 0.00225$
(direct approach)	$\bar{g}_k = 0.0024$	$\bar{g}_k = 0.0023$
(probabilistic approach)	$\bar{g}_k = 0.0023$	$\bar{g}_k = 0.00238$

[12] In the figures we have evaluated the standar deviation of GDP and $\frac{\phi}{\sqrt{3}}\ H$ over a time step equal to 15.

The result shows that the mean growth rate of GDP is equal (or very close) to the value of the analytic study, $E[g_k] = \bar{g}_k$.

What we also want to prove is that the model we propose is able to generate some important stylized facts. The stilyzed facts are the result of microeconomic and macroeconomic dynamics. These dynamycs emerge from the interaction of heterogenous interacting agents at micro level (firms and banks) and these interactions affect the macrolevel and the macro also affect the micro level, generating a feedback mechanism. Thus, we want to analyze how our models are able to replicate some empirical findindgs: industrial dynamics, financial facts and business cycle [Delli Gatti et al., (2007)].

On the industrial dynamics, we can affirm that our models are able to show that the firms size is a *power law* and it is right skewed [Axtell et al., (2001)] [Gaffeo et al., (2003)] (Figure (4.8)) and the growth rate of surviving firms decrease as size increase [13].

Furthemore, the probability of being in the market is correlated with firms size and age and there is lower variance in the aggregate than in individual agents level [Amaral et al., (1997)] [Gabaix, (2002)] (for example the variance of the aggregate of producion). Moreover, the growth rate of the large firm decreases as firm size increases, see Figures (4. 11) - (4.14).

More over, the interest rate is *a-cyclical* and the relation (ratio) between the firms capital and the banks capital is constant (approximately) over time

[13] The firms fail when the equity is lower than zero, thus, higher the size, lower the probability of bankrupt.

[Gallegati et al., (2003)], Figures (4.15) and (4.16) .

It is important to condidar that also in this model the loan distribution (Figures (4. 17) and (4. 18)) follows a pawer law distribution [Fujiwara, (2003)] and also the firms profit distribution follows a power law distribution (Figures (4.19) and (4.20)) [Fujiwara, (2003)] [14]. Other important financial facst is that the equity ratio decreases almost monotonically as the time of firms failure approaches, this means that financial ratio is good predictor of firms bankruptcy.

Completely different from what happened to the interest rate is *the business cycle*, that is *procyclical*, see Figures (4.21) and (4.22). That is any economic quantity that is positively correlated with the overall state of the economy. If there is any quantity that tends to increase in expansion and tends to decrease in recession, it is classified as procyclical. Gross Domestic Product (GDP) is an example of a procyclical economic indicator. Moreover, firms size shift over the business cycle. During the espansion phases of the cycle firms become larger and the distribution becomes less steep [Gaffeo et al., (2003)]. Firms size distribution tends to shift to the right during growing phases, while during recessions the estimated parameter α decreases. Deacreasing of great firms during downturns cause a more equal firms size distribution.

[14] To compute the firms profit distribution, we have ignored the negative data.

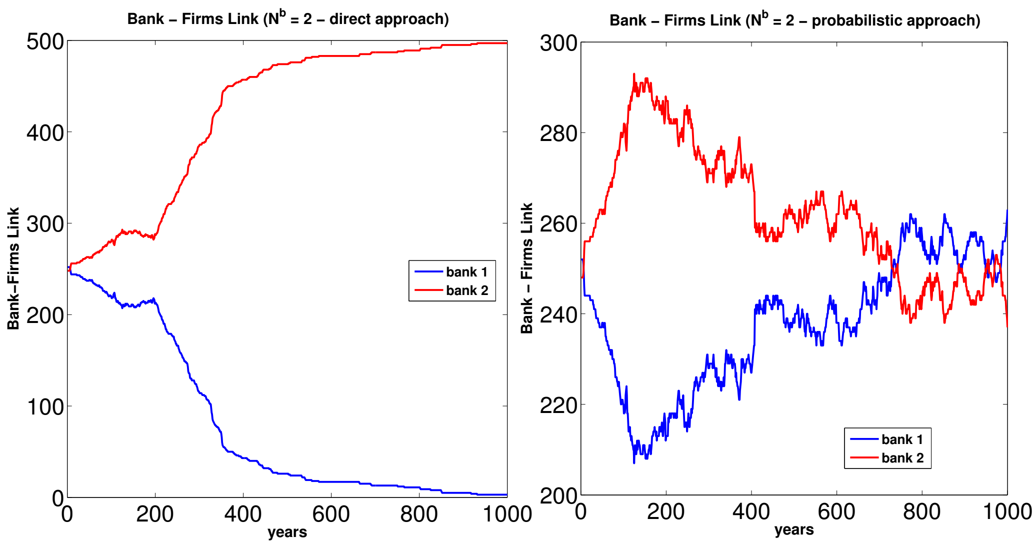

Figure 4.1: The Evolution of Banks- Firms Links over time $N^b = 2:$ the direct approach (left side) and the probabilistic approach (right side).

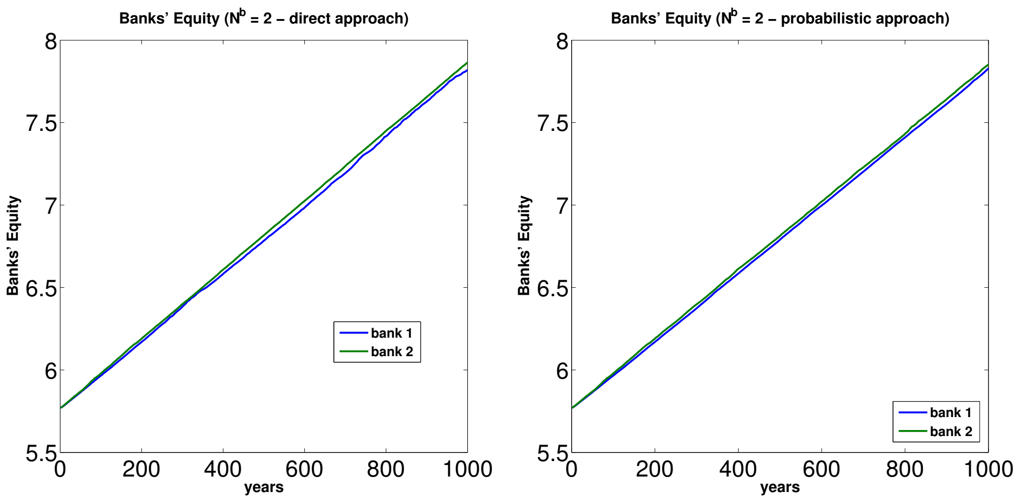

Figure 4.2: The Evolution of Bank Equity over time $N^b = 2:$ the direct approach (left side) and the probabilistic approach (right side).

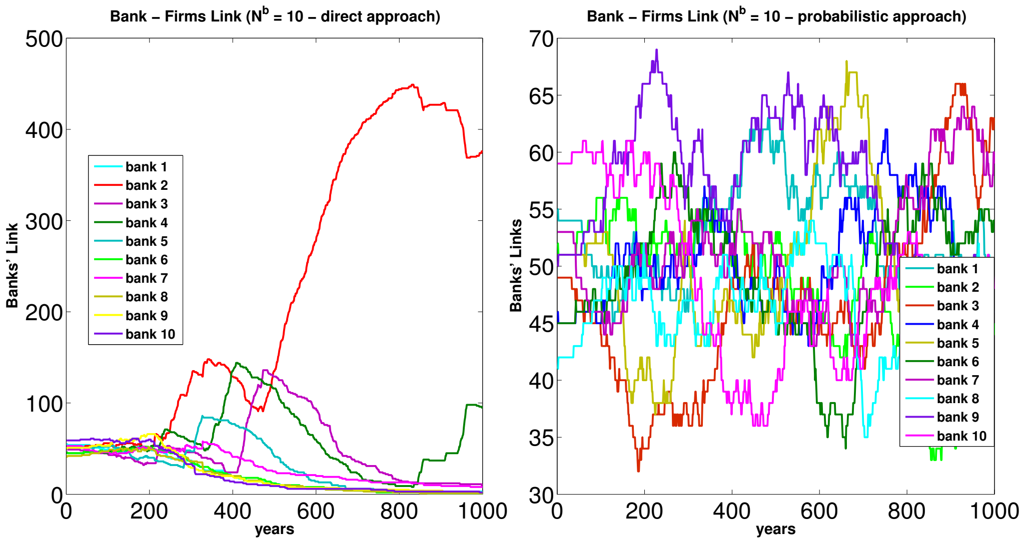

Figure 4.3: The Evolution of Banks- Firms Links over time $N^b = 10$: tthe direct approach (left side) and the probabilistic approach (right side).

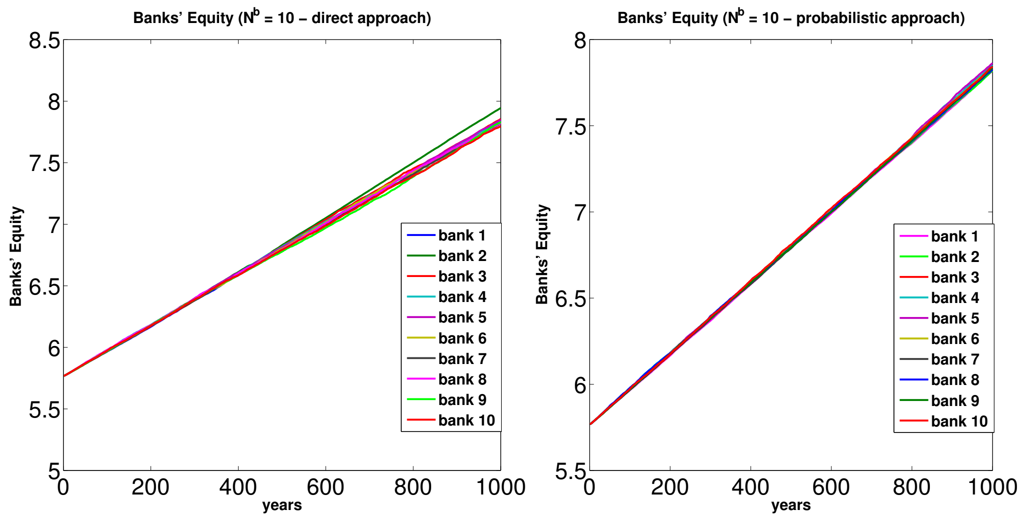

Figure 4.4: The Evolution of Bank Equity over time $N^b = 10$: the direct approach (left side) and the probabilistic approach (right side).

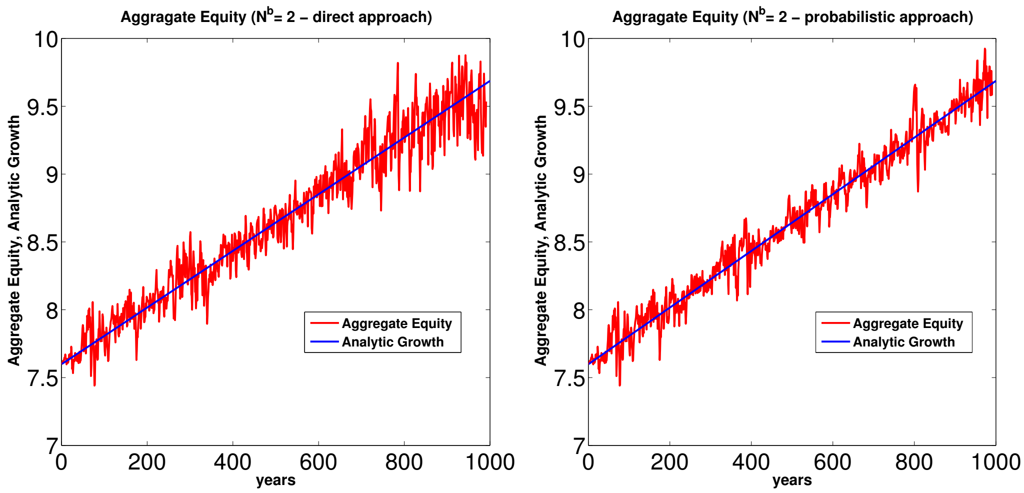

Figure 4.5: The Aggregate Equity Growth over time $N^b = 2$: the direct approach (left side) and the probabilistic approach (right side).

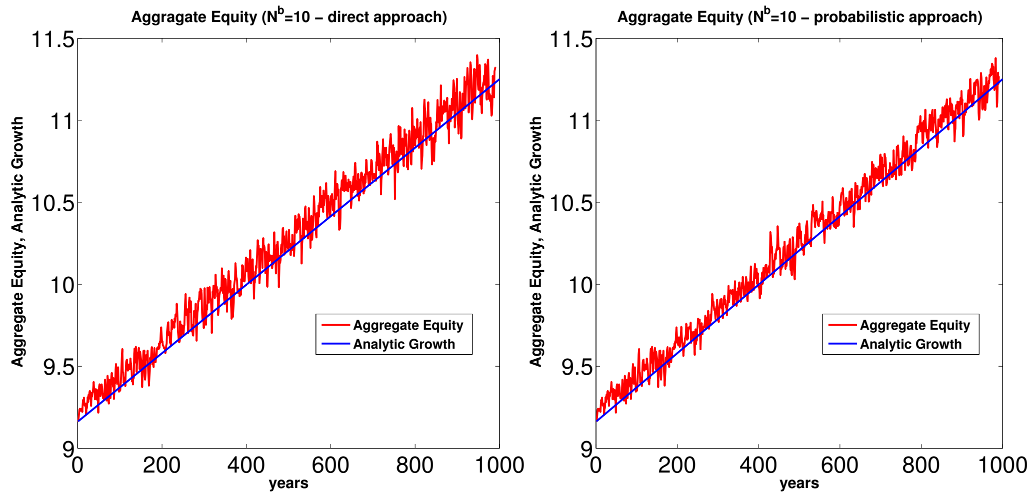

Figure 4.6: The Aggregate Equity Growth over time $N^b = 10$: the direct approach (left side) and the probabilistic approach (right side).

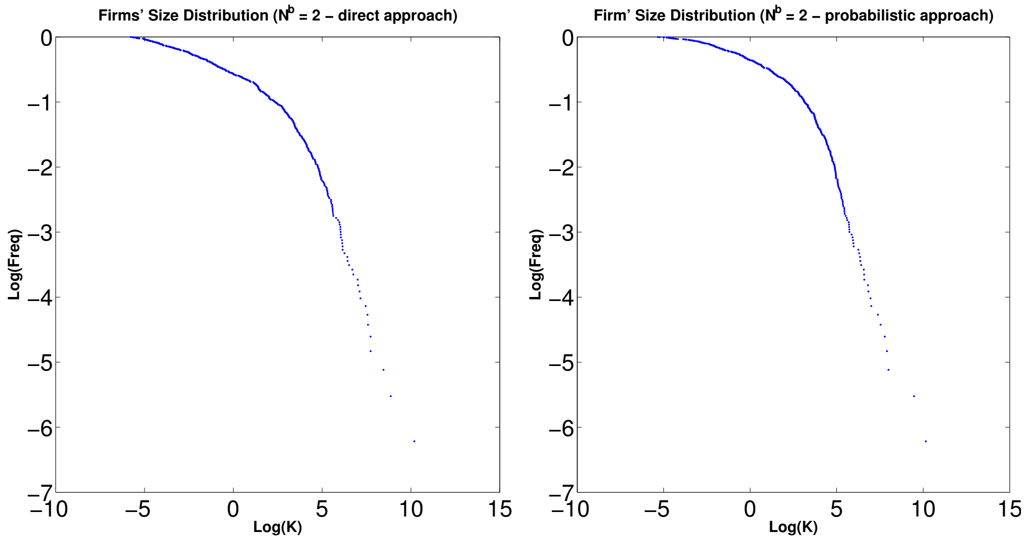

Figure 4.7: Firms Size Distribution $N^b = 2$: the direct approach (left side) and the probabilistic approach (right side).

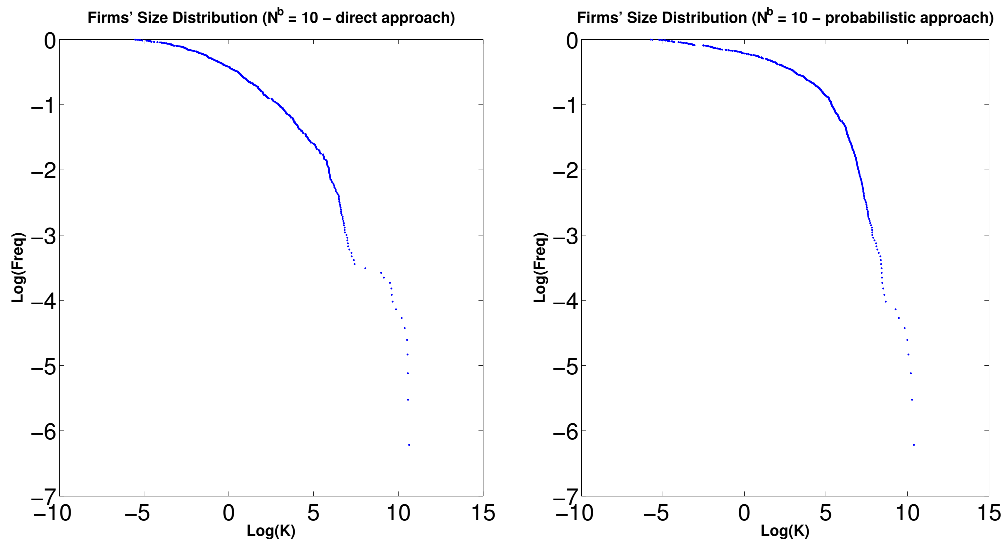

Figure 4.8: Firms Size Distribution $N^b = 10$: the direct approach (left side) and the probabilistic approach (right side).

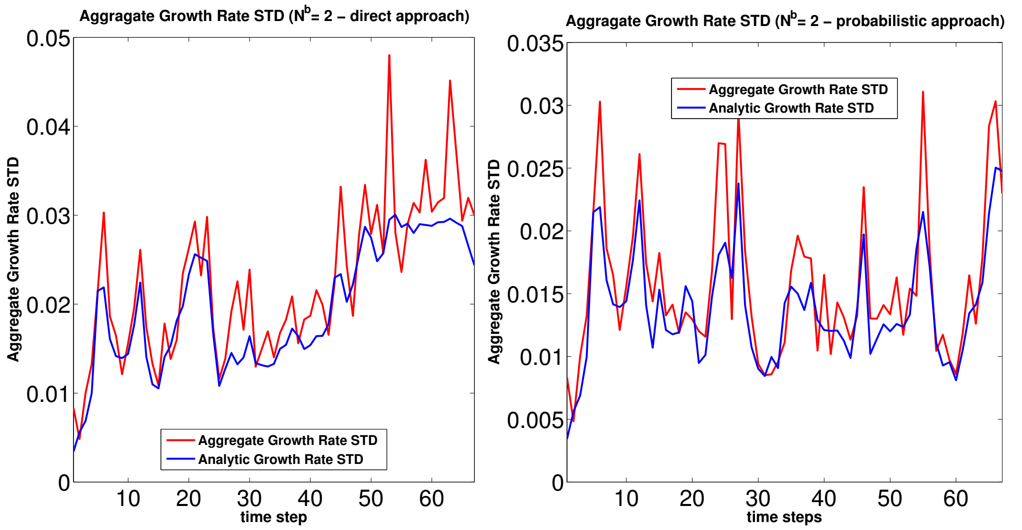

Figure 4.9: The Aggregate Growth Rate Standard Deviation over 15 time steps and the Analytic Standard Deviation of Aggregate of Growth Rate $N^b = 2$:the direct approach (left side) and the probabilistic approach (right side).

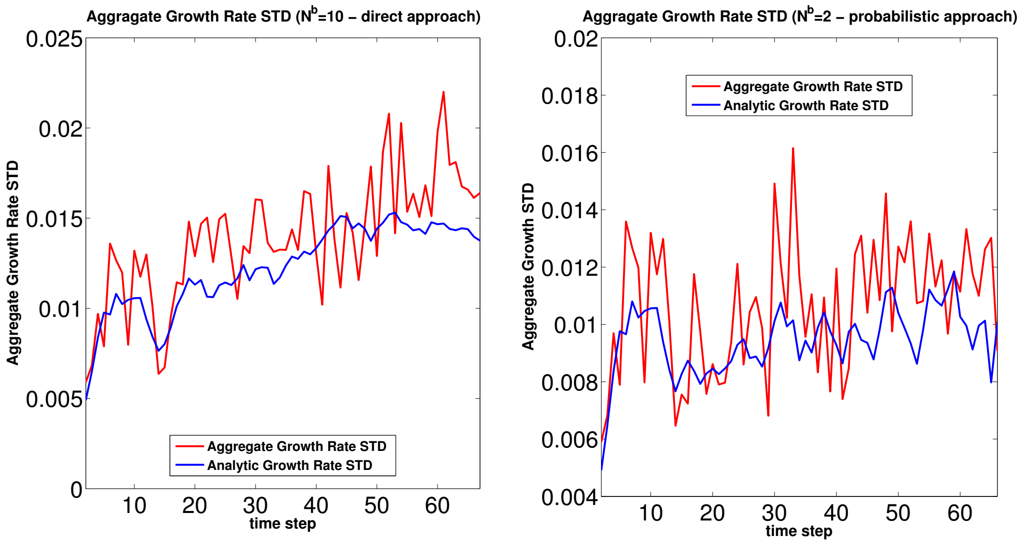

Figure 4.10: The Aggregate Growth Rate Standard Deviation over 15 time steps and the Analytic Standard Deviation of Aggregate of Growth Rate $N^b = 10$: the direct approach (left side) and the probabilistic approach (right side).

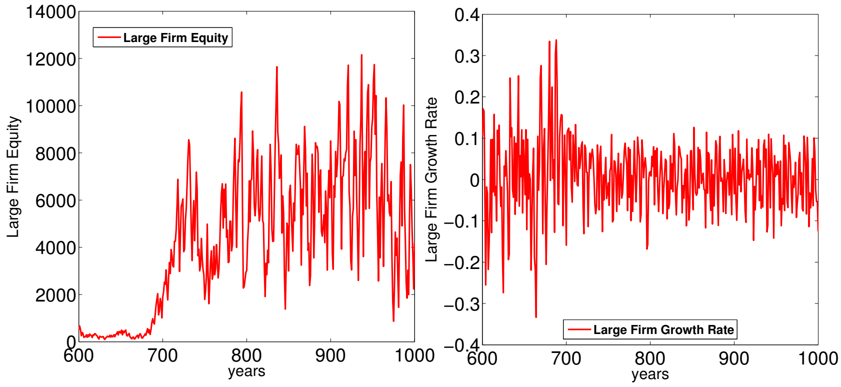

Figure 4.11: The Evolution of Equity of Large Firm and its Growth Rate over time $N^b = 2:$ direct approach.

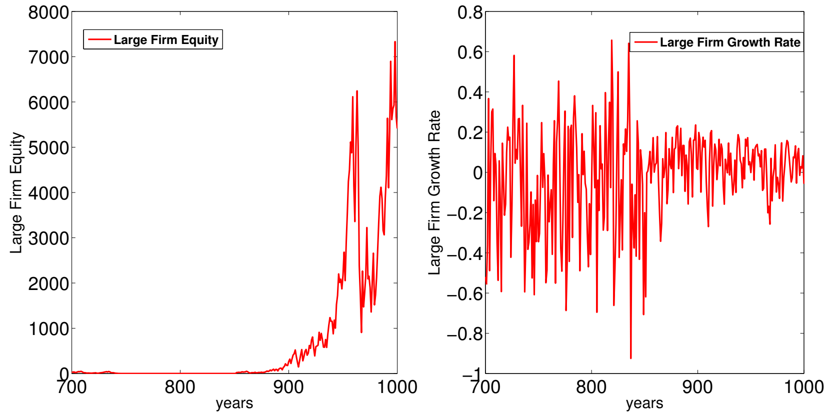

Figure 4.12: The Evolution of Equity of Large Firm and its Growth Rate over time $N^b = 2:$ probabilistic approcah.

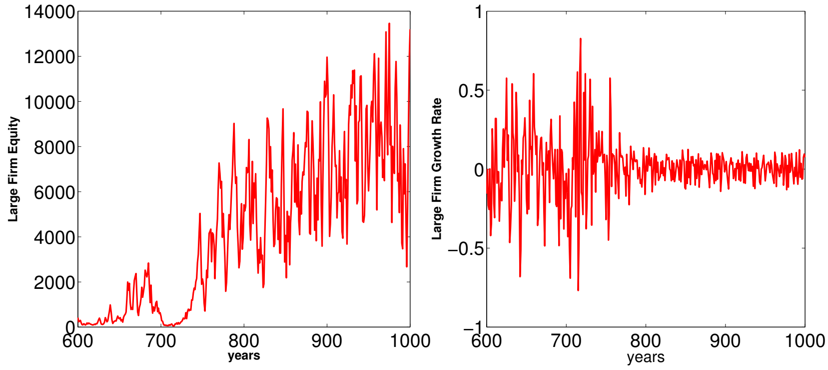

Figure 4.13: The Evolution of Equity of Large Firm and its Growth Rate over time $N^b = 10$: direct approach

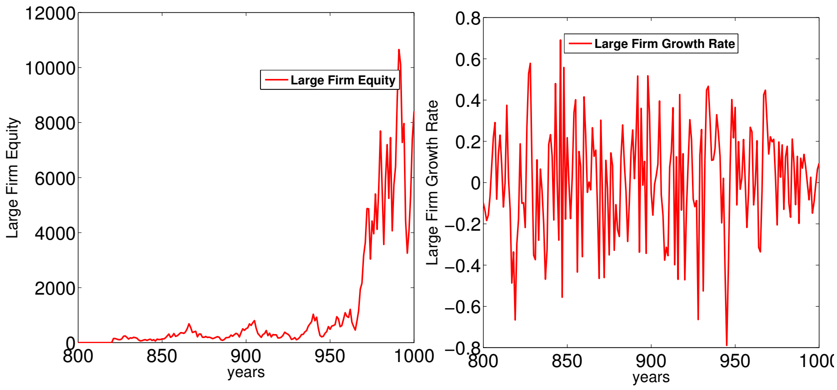

Figure 4.14: The Evolution of Equity of Large Firm and its Growth Rate over time $N^b = 10$: probabilistic approach

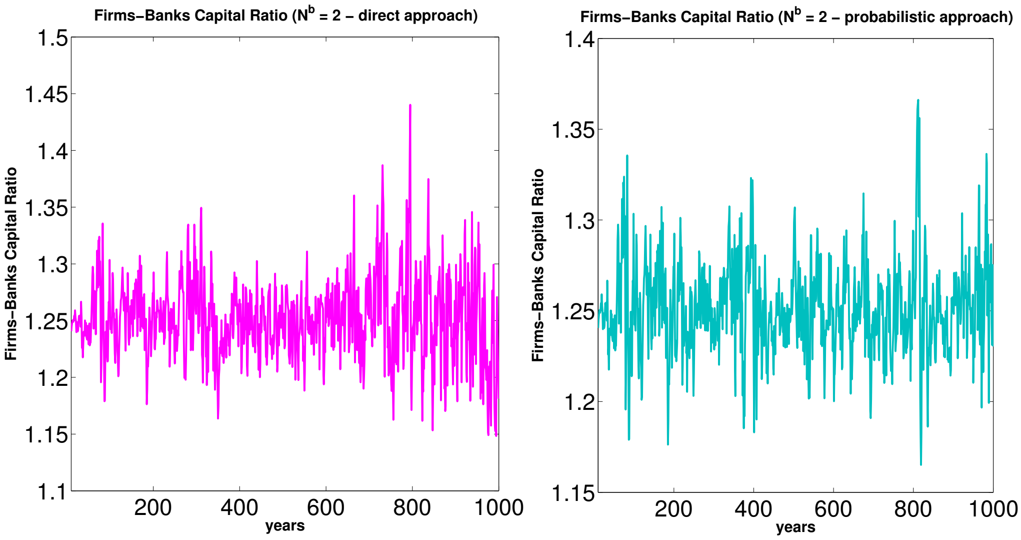

Figure 4.15: The Evolution of the Firms Capital and Banks Capital Ratio $N^b = 2$: the direct approach (left side) and the probabilistic approach (right side).

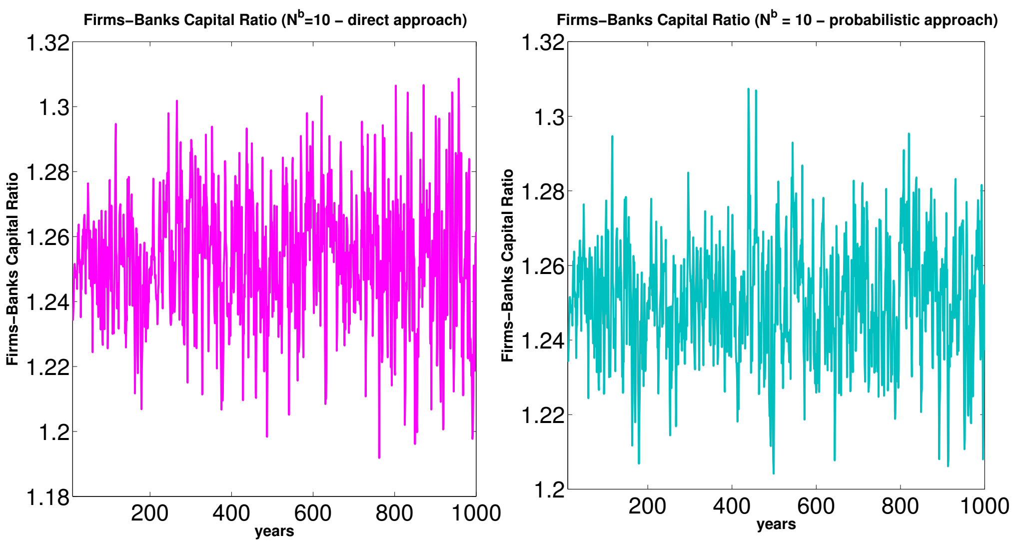

Figure 4.16: The Evolution of the Firms Capital and Banks Capital Ratio $N^b = 10$: the direct approach (left side) and the probabilistic approach (right side).

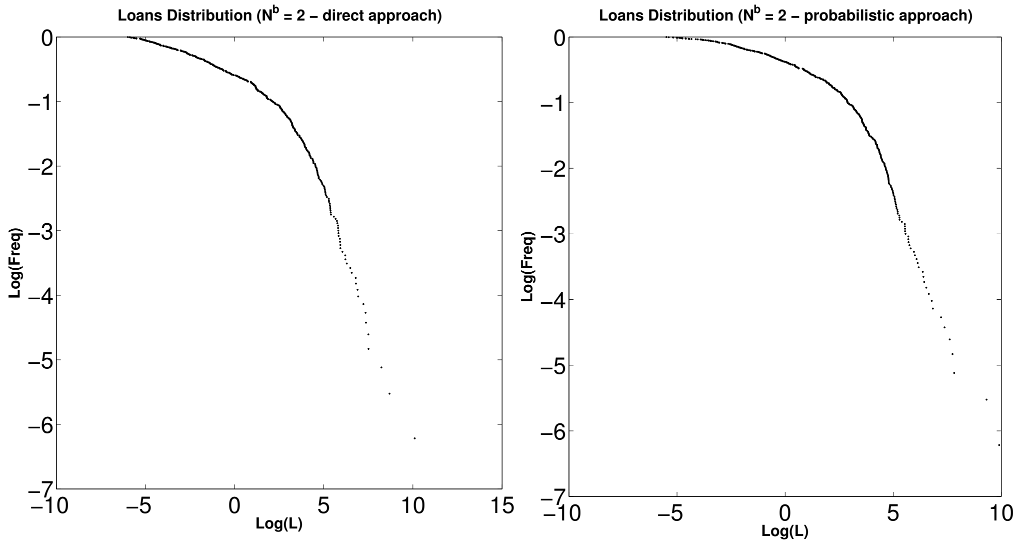

Figure 4.17: Loans Distribution $N^b = 2$: the direct approach (left side) and the probabilistic approach (right side).

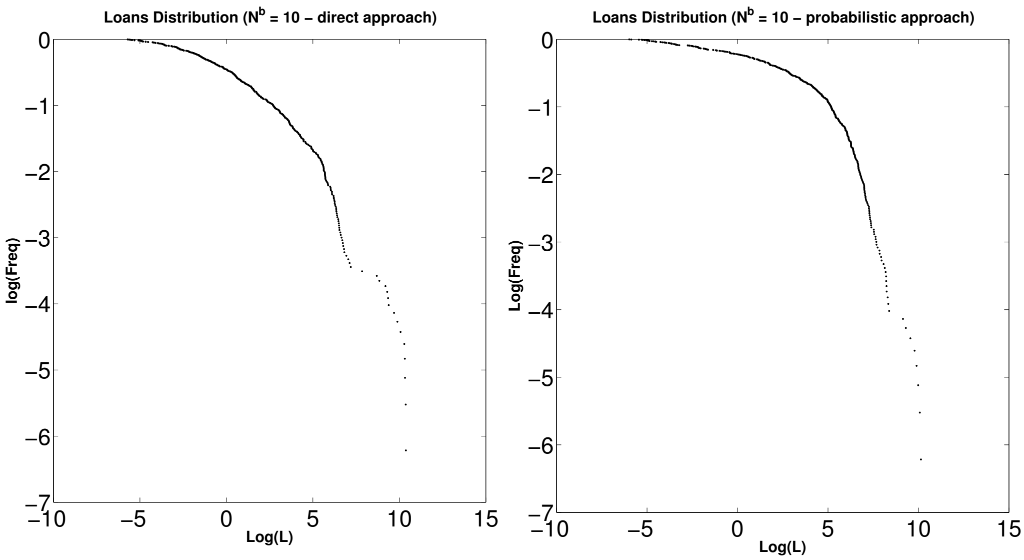

Figure 4.18: Loans Distribution $N^b = 10$: the direct approach (left side) and the probabilistic approach (right side).

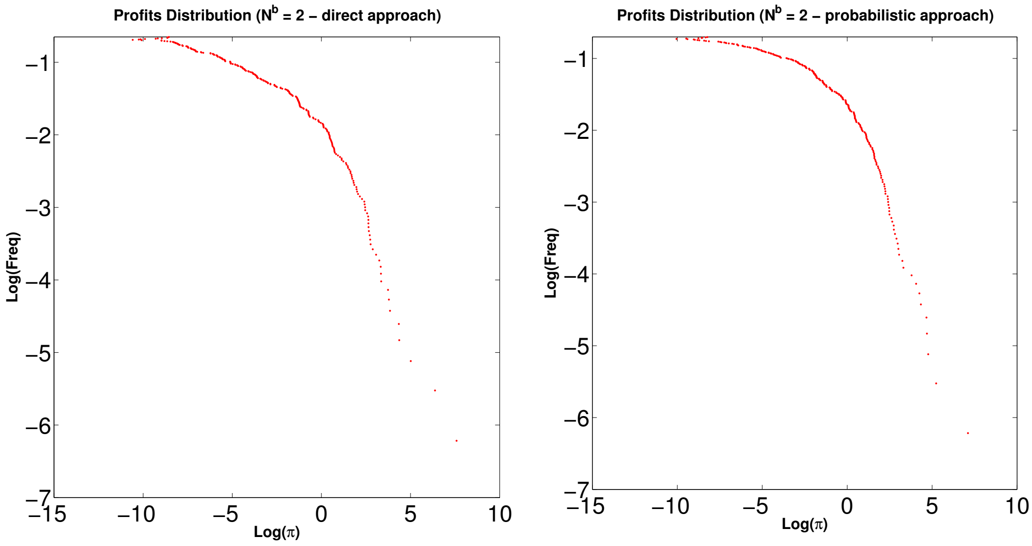

Figure 4.19: Profits Distribution $N^b = 2$: the direct approach (left side) and the probabilistic approach (right side).

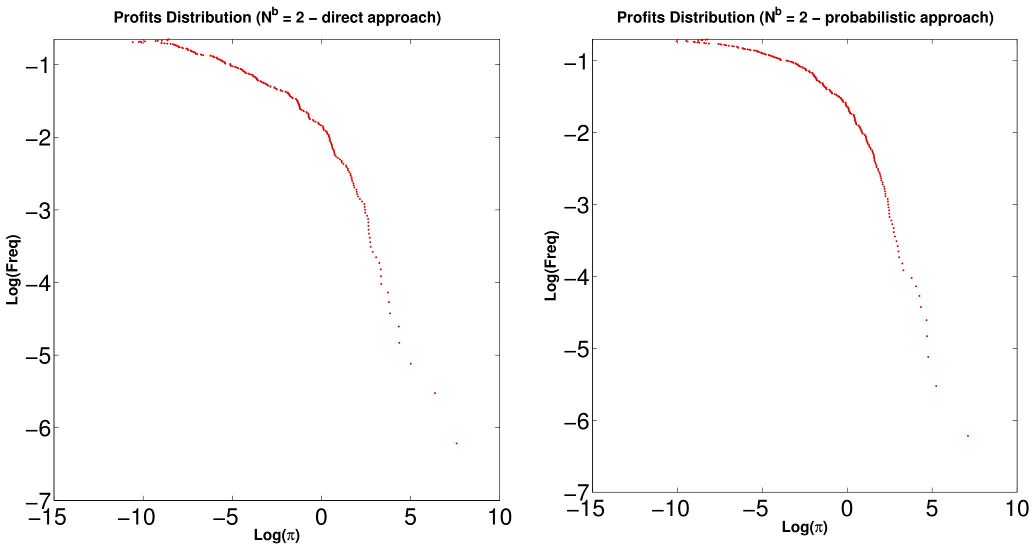

Figure 4.20: Profits Distribution $N^b = 10$: the direct approach (left side) and the probabilistic approach (right side).

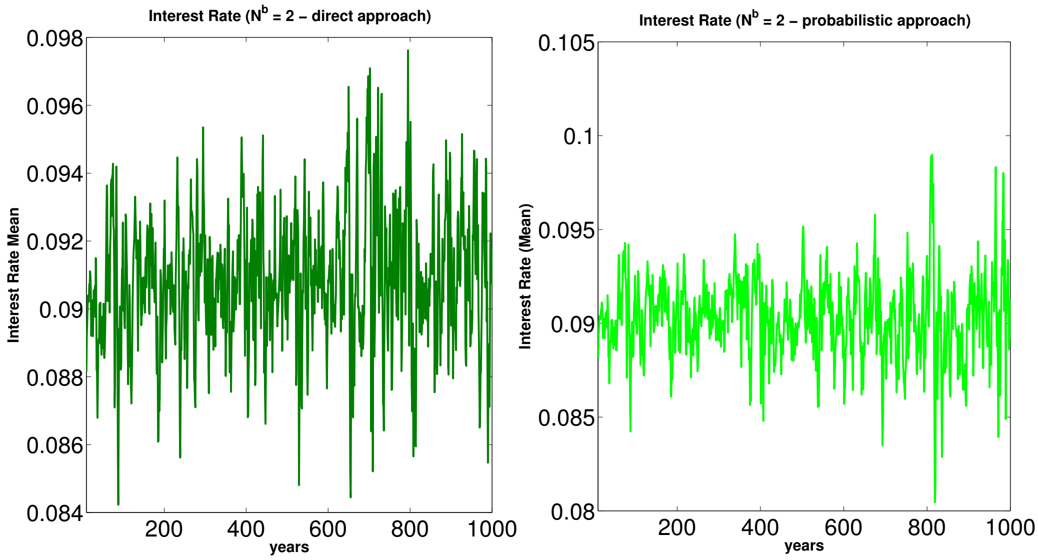

Figure 4.21: The Interest Rate (Mean) over time $N^b = 2$: the direct approach (left side) and the probabilistic approach (right side).

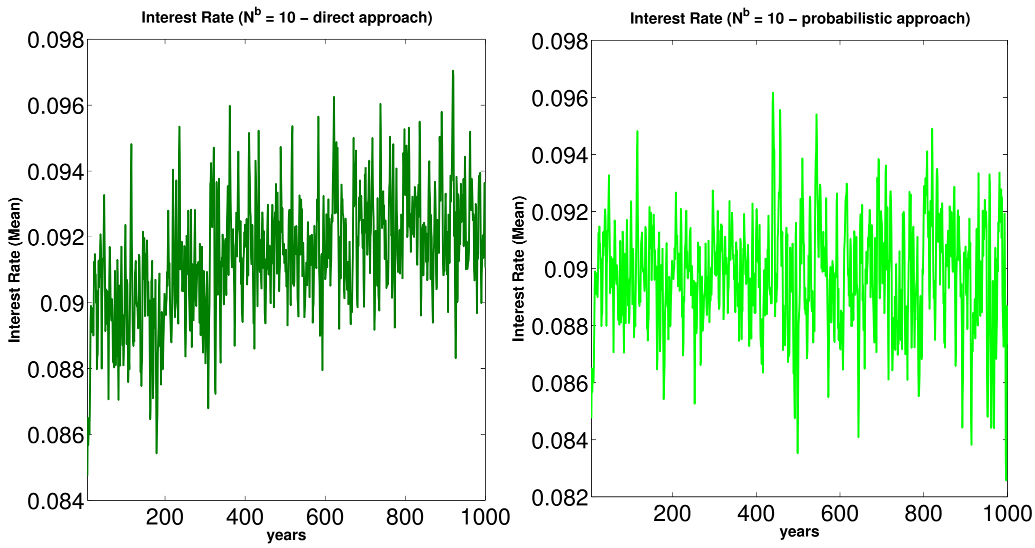

Figure 4.22: The The Interest Rate (Mean) over time $N^b = 10$: the direct approach (left side) and the probabilistic approach (right side).

4.4 Conclusions

The model generate a wider picture of what we found trough the analysis of the model of the *chapter 2*. If we extend the analysis in a model populated by a multiple banks we can observe the same dynamics we found studying the model [Delli Gatti et al., (2005) with only a bank.

So, the model is also able to replicate the result we found in the *deterministic model analysis*. This great results give to our analysis strong robustness. The analytic study of the *deterministic model* is based on the interaction of a multitude of firms and banks. In this framework we found the same result in a different contest, comparing them with model populated by two and ten banks with differents approaches to link to the firms. This means that the analytic findings are able to represent a wide range of similar model, independently from the number of banks that act in the market and independently of the way firms choose to link to the bank.

In conclusion, the model we propose is able to describe a wide range of dynamics that we can observe in real data analysis. This result is due to the *agent based approch*, where micro and macro level interact and generate emergencies that are not possible to predict only with the analysis of the micro level. The analysis of the *representative agent* is not able to generate a real world dynamics. The real world is a complex world, thus, we have to study the dinamics analyzing complex system generated by the interaction of a large number of heterogenous agents.

We simulated the model that is able to reproduce a wide range of stilyzed facts of different aspect of complex world: (i) industrial dynamics, (ii) financial facts and (iii) business cycle. These results are possible only using a methodological approach based on agent-based simulations of a system with *HIAs*.

4.5 Appendix

In this section, we want to describe the assumptions and procedures we followed to simulate the model. A simulation is completely described by the parameter values and the initial conditions. Firstly, we set the parameter values and the initial conditions of that variables we need to start the simulation. These parameters are relative to the firm, bank and the entry process.

4.5.1 Variables and Parameters Setting

4.5.2 Variables

- u = random price;
- K = capital;
- L = firms' Liabilities;
- E = firms' Equity;
- Π = firms' Profit;
- I = firms' Investment;
- K^* = firms' Desired Capital;
- PrB = firms' bankruptcy probability;
- r = interest rate;
- ${S^{jt}}_t$ = j-th bank total credit Supply;
- ${L^s}_{it}$ = bank credit Supply for each firm;

- E^B = bank Equity;
- Π^B = bank Profit;
- B^{debt} = bad debts;
- D = bank deposit;
- Pr^b_{jt} = probability of new firms to link to the largest bank.

4.5.3 Parameters

- t = year;
- ϕ = capital productivity;
- g = total variable costs;
- λ = parameter to allot credit according to Basilea II;
- ν = bank risk coefficient;
- ω = the degree of competition in the banking sector;
- c = parameter for bankruptcy costs;

4.5.4 Simulation Settings

Initial Settings

- T = 1000 ;
- Firms Number = 500;

- Banks Number= 2, 10;
- $K^{entry} = 10$;
- $E^{entry} = 2$;
- $L^{entry} = Kentry - Eentry$;
- $E_t^{exit} = E_{t-1}^{exit} * (1 + (\beta * \omega * \phi/g))$;
- $E_{exit}^b = 0.1$.

Parameters Settings

- u = [0 ; 2]
- ϕ = 0.10;
- g = 1.1 ;
- λ = [0.1 ; 0.7] ;
- ν = 0.08;
- ω =0.002;
- c = 1.

Conclusions

This Doctoral thesis is based on the evaluation and analysis of variables and dymamics of a complex system through a construction of an Agent - Based Model (ABM) , where agents are heterogenous and interact among themselves and with the environment.

We also want to show that the ABM are able to generate and enphasize the empirical results that we can get analyzing real data. The second chapter of the present Doctoral thesis examined the [Delli Gatti et al., (2005)] model, constructing a deterministic model that helped us to do an analytic study of the ABM. Our result corroborate that the model generate an exponentially growing system where bank and firms share each other the market profit (*roa*) to persist in their growth. The model generate, as well, a system where the bank is estimulated to lend credit to large firms.

Large firm (under our analysis) enters , in a certain period of time, in a virtuous cicle where it get very small interest rate (smaller than $\bar{r}$ [15]). Due to this extraordinary interest rate (so much smaller than $\bar{r}$), 'lucky' firm grows and grows for some period and we can observe that the growth of this firm (become now large firm) persist in a considerable period of time. Then, large

[15] $\bar{r}$ is the interest rate that equilizes the bank roe and the firm roe

firm faces a downturn trend and decreasess, but, in the same time, another firm gets small interest rate and grows, so the process continues over and over time. We found out that the system generate granularity (few large firms and a moltitude of small firms) and a very large firm that can own almost all the economy wealth. Besides, in *chapter 2* we saw that large firm can hold even the 70% of all richness (equity).

Moreover, we found out that the deterministic model describes the interaction between the real sector (the firm) and the financial sector (the bank). In our deterministic analysis, we equalize the firms and bank ROEs and we analyze the interest rate. We have found out 2 levels: the interest rate based in the perfect competition in the goods market (r^*) and the interest rate that equalize the two ROEs $\bar{r}$.

Thus, if the demand for credit grows at a rate higher than the supply of credit, the interest rate r_t [16] will be between zero and $\bar{r}$, $0 < r_t < \bar{r}$. This happens because the demand and supply of credit are essentially proportional to the equity of firm and bank, respectively. The corresponding disequilibrium tends to adjust the interest rate upward, reducing the growth rate of the demand for credit and contemporaneously increasing the growth of the supply of credit.

Otherwise, if $r_t < \bar{r} < r^*$, the supply of credit grows at rate higher than the demand of credit. The corresponding disequilibrium tends to adjust the interest rate downward, reducing the growth rate of the supply for credit and, at the same time, increasing the growth of the demand of credit.

[16]The interest rate r_t is the interest rate that get any i-th firm and that equilibrate the supply and the demand of credit of i-th firm.

Moreover, if the growth rate of the firm equity will be negative ($r > r*$), with a corresponding shrinking of the size of the firm, we can see the bankruptcy of the firm or the decreasing interest rate and therefore to a switch to positive profits of the firm.

Interestingly, the model gives some insights on the determinants of the growth of the system bank-firm-economy. To increase the overall growth rate of the economy, the bank can charge a higher spread to the deposits, increasing ω, but it might determine a reduction in deposits. An alternative strategy would be to increase the leverage parameter β. In the deterministic version of the model, such strategy increases the growth of the economy, since the bank can exploit more its oligopolistic position further leveraging ω and, at the same time, charging to the firm a lower interest rate, which, in turns, determines higher profits both for the bank and the firm. On the contrary, in the stochastic environment with heterogeneous firms, a too high leverage might destabilize the system bank-firms, creating a fragile growth. A trade-off leverage-growth-fragility might emerge out of the model. Alternative ways to increase the growth of the system are related to increasing ϕ and/or decreasing g, both parameters linked to a quite standard concept of technological progress. The aim of the analysis of the stochastic model, studying the deterministic model, was to find some statistical equilibrium of the model and to solve some aspect of the model dynamics (for example GDP dynamics). In fact, through the deterministic analysis we found that the GDP fluctuations are due to the *roa* standard deviation times the herfindhal - hirshman index [17] (H) .

[17] As argued by [Gabaix,(2011)] , the fluctuations of GDP depend on Herfindhal

As in [Gabaix, (2011)] , we show and confirm that the fluctuation of the GDP definitively depends on the H fluctuation. We also found that the model [Delli Gatti et. al, (2005)] tends to generate a giant firms that are able to get all the amount of loans that the bank can allow. In our model, when the H explodes, a firm dominates the system. We can argue, that the model tend to finance one firm more than another and the fluctuation of this firm affects the GDP fluctuation.

Instead in the *chapter 3*, we examinate the **profit rate** and the **growth rate** structure and fluctuations. Our result confirm the empirical findind in the profit rate and growth rate that [Mundt et al., (2014), (2016)] , [Alfarano et al., (2008), (2012)] found in their analysis of real data (US 500 largest firms).

The model shows us some firms dymanics evidence: the firms size follows a right skewed distribution and it is a power law distribution. The empirical analysis of the growth rate showed different results from the [Stanley et al., (1996)], [Bottazzi et al., (2001)] , but is able to confirm the analysis conduct by [Mundt et al., (2014,2016)] , [Alfarano et al., (2008), (2012)] that represent the analysis of the real data. The growth rate does not follow a Laplace distribution, but the analysis shows that it follows a more leptokurtic distribution than Laplace, the estimation of the shape parameter α confirms us that it il lower than one ($\alpha < 1$). The empirical analysis of the profit rate, as in [Mundt et al., (2014), (2016)] , [Alfarano et al., (2008), (2012)] shows that it follows a laplace distribution with α close to 1. This result is confirmed also by the estimation of shape and scale parameter in the year-by-year

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analysis. Moreover, we find that both growth rate and profit rate are autocorrelated in the first lags. So, the profit rate is autocorrelate as in the analysis of real data in [Mundt et al., (2014), (2016)] . This result depends on the fact that both of them are affected by the evolution of profit and, in turn, it is affected by the price, that follows a *random walk*.
When the price is a *iid variable* the growth rate distribution follow a different distribution from the Laplace and the estimation of the shape parameter suggests us that is close to Gaussian distribution ($\alpha \simeq 2$).
The profit rate distribution follow a uniform distribution ($\alpha \longrightarrow \infty$). This because the profit rate is strongly affected by the price distribution. When the price is a *iid variable* , the autocorrelation of the growth rate reflects what [Mundt et al., (2014), (2016)] found in the analysis or real data of US firms. The growth rate does not show autocorrelation and also the profit rate shows no autocorrelations (caused by the price).

Finally, in the last chapter (*chapter 4*), in order to observe which are the effects of a system populated by more than a bank, we simulated a model where heterogenous firms interact in the market where operate a moltitude of banks.

We simulated a model in which new firm can link to the largest bank (higher equity) in 2 different way, through a (i) *direct approach* in which new firm directly links to the largest bank, and through a (ii) *probabilistic approach* in which the new firm links to the largest bank according to a probability.

The model we propose generates a system in which banks equity grows over time, even if a bank is linked to a firm. The fact that a bank with lower numbers of links is able to grow over time, confirms us that the results (deterministic analysis) we found in *chapter 2* hold true also for this model.

The growth analysis of the model confirm us that also with a multi banks-firms system the economy grows following the *long-run dynamics* we observed studying the model in chapter 2. Whatever is the way we use to link new firm to the bank, the model grows at the same rate of the model populated by one bank.

Analyzing different approaches, what changes in this multi banks-firms model is not the way the model grows, but the *topology* of the bank-firms links. In fact, in the *direct approach* a bank , at the end of the simulation, is linked to almost all firms and the other banks are linked to few (one) firm(s); in the *probabilistic approach* the number of firms linked to the *j-th* bank , over time, fluctuates around the ratio N^f/N^b.

So, the model is also able to replicate the result we found in the *deterministic model analysis.* This great result give to our analysis strong robustness.

This means that the analytic findings are able to represent a wide range of similar model, independently from the number of banks that act in the market.

In conclusion, the models we propose and analyze are able to describe a wide range of dynamics that we are able to observe in real data analysis. This results is due to the *agent based approch*, where micro and macro level interact and generate emergencies that are not possible to predict only with the analysis of the micro level. The analysis of the *representative agent* is not able to generate a real world dynamics. The real world is a complex world, thus, we have to study its dinamics analyzing complex system generating by the interaction of a large number of heterogenous agents.

We simulated models that we propose in this thesis and they are able to reproduce a wide range of stylized facts of different aspects of complex world: from the GDP dynamics to the firms size distribution, from the empirical distribution of profit and growth rate to the fluctuations (and equilibrium) of the intetest rate, from the dynamics of firms to the distribution of profit and debt, from the aggregate growth rate to the behavior of large firms.

Future Line of Research

The future lines of research that we propose, starting with this Doctoral thesis, can be summarized as follows:

- extend our determinist analysis, analysis of GDP fluctuation (aggregate growth rate fluctuation) to other ABM where the equilibrium between bank supply loan and firms demand of credit is given by the interest rate. Indeed, we think that the result we propose in our deterministic analysis of the model [Delli Gatti et. al, (2005)] can be the base for the analysis of other ABM that are caracterized by the same mechanism of equilibrium for lending credit.

- Create a ABM model that is able to replicate the other empirical findings (not only growth rate and profit rate distribution) that hold true for the analysis of real data.

- Our further analysis take towards the construction of ABM populated by a large number of firms and banks that interact and compete in the market to get the best amount of credit at the lowest cost (firm side) and to link a higher number of large firms (bank side), studing the netwok that these interactions can create.

Conclusiones (Spanish)

Conclusiones y comentarios finales

Esta tesis doctoral se basa en la evaluación y análisis de variables y dinámicas de un sistema complejo a través de la construcción de un Modelo basado en Agente (ABM), donde los agentes son heterogéneos e interactúan entre sí y con el entorno.

También queremos mostrar que los ABM son capaces de generar y enfatizar los resultados empíricos que podemos obtener analizando datos reales.

El segundo capítulo de la presente tesis doctoral ha examinado el modelo de [Delli Gatti y col., (2005)], construyendo un modelo determinista que nos ha ayudado a realizar un estudio analítico de la ABM. Nuestro resultado corrobora que el modelo genera un sistema de crecimiento exponencial donde el banco y las empresas comparten el beneficio de mercado (*roa*) para persistir en su crecimiento. El modelo genera, además, un sistema en el que se estima que el banco presta crédito a las grandes empresas.

La empresa grande (bajo nuestro análisis) entra, en un cierto período de tiempo, En un círculo virtuoso donde obtienen una tasa de interés muy muy pequeña (menor que $\bar{r}$ [18]). Debido a este tipo de interés extraordinario

[18] $\bar{r}$ es la tasa de interes que iguala el roe de la empresa y el roe del banco

(mucho más pequeño que $\bar{r}$), la empresa 'afortunada' crece y crece durante algún tiempo y podemos observar que el crecimiento de la empresa (que ahora se hace grande) persiste en un número considerable de tiempo. Entonces, la empresa grande se enfrenta a una tendencia a la baja y disminuye, pero, al mismo tiempo, otra empresa obtiene una tasa de interés pequeña y crece, y el proceso continúa y en el tiempo. Hemos descubierto que el sistema genera granularidad (pocas empresas grandes y una moltitud de pequeñas empresas), con una empresa muy grande que posee casi toda la riqueza de la economá. Ademś, en textit capítulo 2 hemos visto que la gran empresa puede mantener incluso los 70% de toda la riqueza (equidad).

Por otra parte, hemos descubierto que el modelo determinista describe la interacción entre el sector real (la empresa) y el sector financiero (el banco).

En nuestro análisis determinístico, igualamos las empresas y los ROEs de los bancos y analizamos la tasa de interés. Hemos encontrado dos niveles: r^* la tasa de interés en caso de competición perfecta en el mercado de los bienes y $\bar{r}$ la tasa de interés que equilibra el oferta y demanda de crédito. Así, si la demanda de crédito crece má que la tasa de la oferta de crédito, la tasa de interés r_t [19]. estará entre zero y $\bar{r}$ ($0 < r < \bar{r}$). Esto ocurre porque la demanda y la oferta de crédito Son esencialmente proporcionales al patrimonio de la empresa y del banco, respectivamente. El correspondiente desequilibrio tiende a ajustar la tasa de interés hacia arriba, reduciendo la tasa de crecimiento de la demanda de crédito y simultáneamente aumentando el crecimiento de la oferta de crédito.

[19] La tasa de interés r_t es la tasa de interés que pone en equilibrio demanda y oferta de credito.

De lo contrario, si $r_t < \bar{r} < r^*$, la oferta de crédito crece a una tasa superior a la demanda de crédito. El correspondiente desequilibrio tiende a ajustar la tasa de interés a la baja, reduciendo la tasa de crecimiento de la oferta de crédito y, al mismo tiempo, aumentando el crecimiento de la demanda de crédito.

Por otra parte, si la tasa de crecimiento del capital de la empresa será negativa ($r_t > r*$), con una contración del tamaño de la empresa, podemos ver la bancarrota de la empresa o la disminución de la tasa de interés y por lo tanto a un cambiar a beneficios positivos de la empresa.

Curiosamente, el modelo da algunas ideas sobre los determinantes del crecimiento del sistema economico banco-empresa. Al aumentar la tasa de crecimiento de la economá global, el banco puede cobrar un *spread* más alto a los depósitos, aumentando ω, pero eso podría determinar una reducción en los depósitos. Una estrategia alternativa sería aumente el parámetro de apalancamiento β. En la versión determinista del modelo, la estrategia es de aumentar el crecimiento de la economía, ya que el banco puede explotar más su posición oligopolística aprovechando aún más ω y, al mismo tiempo, cobrando una tasa de interés más baja, lo que, a su vez, determina mayores beneficios tanto para el banco que para la empresa. Por el contrario, en el entorno estocástico con heterogeneidad de las empresas, un apalancamiento demasiado alto podría desestabilizar el sistema banco- empresas, creando un frágil crecimiento. Podrá surgir un *trade-off* entre apalancamiento-crecimiento-fragilidad del modelo. Las formas alternativas de aumentar el crecimiento del sistema están relacionadas con el ϕ creciente y / o decreciente y con el g, ambos parámetros relacionados con un estándar bastante basico de concepto de progreso tecnológico. El objetivo del análisis del modelo estocástico, estudiando el modelo determinista, fue encontrar algún equilibrio estadístico del modelo y resolver algúnos aspectos de la dinámica del modelo (por ejemplo, la dinámica del PIB). De hecho, a travś del anĺisis determinista hemos encontrado que las fluctuaciones del PIB se deben a la desviacińn estándar del *roa* moltiplicado por el índice herfindhal - hirshman [20] (H) .

Como en [Gabaix, (2011)], mostramos y confirmamos que la fluctuacińn del PIB depende definitivamente de la fluctuación del H (Índice Herfindhal - Hirshman).

También hemos encontrado que el modelo [Delli Gatti y col, (2005)] tiende a generar empresas gigantes que pueden obtener todo la cantidad de préstamos que el banco se puede permitir. En nuestro modelo, cuando el H explote, una empresa domina el sistema. Podemos argumentar, que el modelo tiende a financiar una empresa más que la otra(o)s y la fluctuación de esta empresa afecta a la fluctuación del PIB.

En cambio, en el *capítulo 3*, Examinamos la estructura **tasa de beneficio** y la estructura **tasa de crecimiento** y las fluctuaciones. Nuestro resultado confirma el hallazgo empírico en la tasa de beneficios y la tasa de crecimiento que [Mundt y col., (2014), (2016)] , [Alfarano y col., (2008), (2012)] han encontrado en sus analíisis de datos reales (500 empresas más grandes de EE.UU.).

El modelo nos muestra algunas dinámica empírica de las empresas: el

[20]Como argumentado por [Gabaix, (2011)], las fluctuaciones del PIB dependen del Índice Herfindhal.

tamaño de las empresas siguen un distribución asimétrica derecha y una distribución de ley de potencia. El análisis empírico de la tasa de crecimiento ha mostrado resultados de forma diferentes de lo propuesto por [Stanley y col., (1996)]; pero es capaz de confirmar la conducta de análisis de [Mundt y col., (2014), (2016)] and [Alfarano y col., (2008) ,(2012)] que representan el análisis de los datos reales. La tasa de crecimiento no sigue una distribución de Laplace: el análisis muestra que sigue una distribución más leptocúrtica que una Laplace. El análisis empírico de los beneficios, como en [Mundt y col., (2014), (2016)], [Alfarano y col., (2008), (2012)] muestra que sigue una distribución laplace. Este resultado es confirmado también por la estimación del parámetro de forma y escala en el análisis año por año. Además, encontramos que tanto en la tasa de crecimiento como en la tasa de beneficio se ves autocorrelacción, pero sólo en los primeros retrasos. Este resultado depende del hecho de que ambos se ven afectados por la evolución del beneficio y, a su vez, se ve afectada por el precio, que sigue una *random walk*.

En el caso el precio siguera una variable aleatoria 'iid' (indipendent identically distributed random variable) la distribución de la tasa de crecimiento sigue una distribución cuya estimacíon de su parametro de forma (α) nos dice que se acerca más a una distribución Gaussiana, $\alpha \simeq 2$. La distribución de la tasa de retabilidad sigue una distribucíon uniforme ($\alpha \longrightarrow \infty$) . Eso porque la tasa de rentabilidad está altamente afectada por la distribución del precio.

En caso que el precio fuera una variable aleatoria 'iid', la autocorrelación de la tasa de crecimiento refleja lo que [Mundt y col., (2014), (2016)] han encontrado analizando los datos de las empresa de EE.UU. . La tasa de crescimiento no muestra autocorrelacón. Tambien en la tasa de rentabilidad no podemos observar autocorrelación, esto porque la tasa de rentabilidad está afectada del precio.

Finalmente, en el último capítulo (*capítulo 4*), para observar cómo son los efectos de un sistema poblado por más de un banco, hemos simulado un modelo en el que las empresas heterogéneas interactúan en el mercado donde operan una moliedad de bancos.

Hemos simulado un modelo en el cual la nueva empresa puede vincularse al banco más grande (con más equity) de dos manieras diferentes: a través de un (i) *enfoque directo* en el cual la nueva empresa conecta directamente con el banco más grande y a través de un (ii) *enfoque probabilístico* en el cual la nueva empresa se vincula al banco más grande con una probabilidad.

El modelo que proponemos genera un sistema en el cual la equidad (equity) de los bancos crece en el tiempo, incluso si el banco está vinculado con una (pocas) empresa(s). El banco con el menor numero de empresas conectada es capaz de crecer con el tiempo. Esto nos confirma que los resultados (análisis determinista) que encontramos en el *capitulo 2* también son válidos para este modelo.

El análisis de crecimiento del modelo nos confirma que también con un sistema multi bancos - empresas la economía crece siguiendo la dinámica de largo prazo que observamos estudiando el modelo en el *capitulo 2.* Cualquiera que sea la forma que usamos para vincular la nueva empresa al banco, el modelo crece a la misma tasa del modelo poblado por un solo banco.

Analizando diferentes enfoques lo que cambia esn este modelo multi bancos-empresas no es la forma en que el modelo crece, sino la *topologiá* de la coneccíon banco-empresas. De hecho, en el *enfoque directo* un banco, al final de la simulación, está vinculado a casi todas las empresa y el (los) otro(s) banco(s) están vinculado con una (pocas) empresa(s); en el *enfoque probabilístico* el numero de empresas vinculadas al *j-esimo* banco, con el tiempo, fluctúa alrededor de la proporción N^f/N^b.

El modelo también es capaz de replicar el resultado que hemos encontrado en el *análisis de modelo determinístico*. Estos excelentes resultados dan a nuestro análisis robustez. Esto significa que los hallazgos analíticos son capaces de representar una amplia gama de modelos similares, independientemente del número de bancos que actúan en el mercado.

En conclusión, los modelos que proponemos y analizamos son capaces de describir una amplia gama de dinámicas que podemos observar en el análisis de datos reales. Este resultado se debe al *enfoque basado en agentes*, que interactúan niveles micro y macro y generan emergencias que no son posibles predecir sólo con el análisis del nivel micro. El análisis del *agente representativo* no es capaz de generar una dinámica del mundo real. El mundo real es un mundo complejo, por lo tanto, tenemos que estudiar sus dinámicas analizando un sistema complejo generado por la interacción de un gran número de agentes heterogéneos.

Hemos simulado los modelos que proponemos en esta tesis y son capaces de reproducir una amplia gama de hechos estilizados con diferentes aspectos del mundo complejo: desde la dinámica del PIB hasta la distribución del tamaño de las empresas, desde la distribución empírica del beneficio y la tasa de crecimiento hasta las fluctuaciones (y el equilibrio) de la tasa de interés, de la dinámica de las empresas a la distribución del beneficio y de la deuda, de la tasa agregada de crecimiento a el comportamiento de las grandes empresas.

Futura Línea de Investigación

Las futuras líneas de investigación que proponemos, a partir de esta tesis doctoral, pueden resumirse de la siguiente manera:

- ampliar nuestro análisis determinista, el análisis de la fluctuación del PIB (fluctuación de la tasa de crecimiento agregado) a otro ABM donde el equilibrio entre el préstamo de suministro bancario y la demanda de crédito de las empresas es dado por la tasa de interés. De hecho, pensamos que el resultado que proponemos en nuestro análisis determinista del modelo [Delli Gatti et. Al, (2005)] puede ser la base para el análisis de otras ABM que se caracterizan por el mismo mecanismo de equilibrio para el crédito;

- crear un modelo ABM que sea capaz de replicar los otros hallazgos empíricos (no sólo la tasa de crecimiento y la distribución de la tasa de ganancia) que son válidos para el análisis de datos reales;

- cuestro análisis posterior toma hacia la construcción de ABM poblado por un gran número de empresas y bancos que interactúan y compiten en el mercado para obtener la mejor cantidad de crédito al menor costo (lado de la empresa) y vincular a un mayor número de grandes empresas (lado del banco), estudiando la red que estas interacciones pueden crear.

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