

Chapter 1

The System

This Chapter describes the flywheel energy storage system. It includes a general description of the system as well as the top level control requirements.

A survey of the literature about this plant and of the main components is also presented. In particular, typical modes of operation, the widely used dq-transformation, and the dynamical equations of the doubly-fed induction machine are introduced. A similar study is developed for the back-to-back converter.

Part of the results of this Chapter can also be found in [11][12].

1.1 The System

The system studied in this Thesis is an autonomous energy-switching system that regulates the energy flow between a local prime mover (a flywheel) and the electrical power network, in order to satisfy the demand of a time-varying electrical load. This system, used in the CERN (Centre Européen pour la Recherche Nucléaire) to store electrical energy for a particle accelerator or in the Okinawa Electric Power Company, has been also studied in [3]. The main goal of the system is, basically, to store kinetic energy into a flywheel and deliver it when an external load requires a high energy flow.

The system (see Figure 1.1) is composed by a doubly-fed induction machine (DFIM) coupled to a flywheel and controlled through the rotor windings by a back-to-back converter (B2B). This is the most common control architecture of the DFIM [3][44][68][69][70][71][85][89]. If the AC source of the B2B is connected to the 3-phase power grid, this architecture is also known as Scherbius drive [68], *i.e.* the power converter is in a closed-loop with the DFIM. In practice, due to the fact that the power flowing through the power converter is much smaller than the power flowing to the DFIM stator side, it is common to neglect this *feedback* connection.

The DFIM is controlled through the rotor windings port $(V_r, I_r \in \mathbb{R}^3$, where V and I are the three-phase voltage¹ and current variables, and subindex r refers to the rotor). It is coupled to an energy-storing flywheel with port variables $(\tau_e$ electrical torque, ω mechanical speed). An electrical network modelled by an ideal AC voltage source with port variables $(V_n, I_n \in \mathbb{R}^3$, subindex n refers to the network variables), and a generic electrical three-phase load, represented by its impedance Z_l , is connected to the stator port variables $(V_s, I_s \in \mathbb{R}^3)$.

¹In this work all the three-phase voltages are line-to neutral voltages. It is assumed that the neutral references of all three-phase system are common.

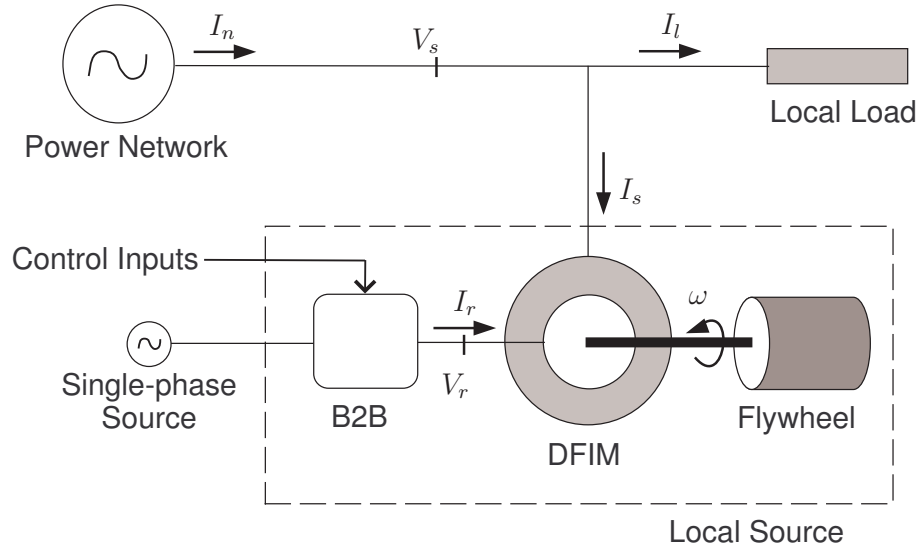


Figure 1.1: Doubly-fed induction machine coupled to a flywheel, controlled by a back-to-back converter and connected to a power network and a load.

In this scheme the network equations are given by Kirchhoff laws

$$I_l = I_n - I_s, \quad V_n = V_s.$$

In this work only equilibrated three-phase variables are considered. However most of the considerations taken in the modelling section (see Section 1.2) can be assumed in a more general setting.

As mentioned above, the main objective of the system is to supply the required power to the load with a high network power factor. Depending on the load demands, the DFIM acts as an energy-switching device between the flywheel and the electrical power network. The control problem is to optimally regulate the power flow.

The performance objective, assuming a maximal active power of the network P_n^{MAX} , can be summarized as follows:

- **To supply** the extra energy required by the load. Notice that this objective concerns the active power, and considering a constant grid voltage, $V_n = ct$, this requirement is achieved by the stator currents.
- **To store** kinetic energy in the flywheel while the load does not require all the grid power.
- **To compensate** the power factor ($\cos \phi$), i.e., the whole system (load and local source) acts as a pure resistor. That is $\cos \phi \sim 0$, or, $Q_n \sim 0$.

These requirements can be achieved by commuting between different steady-state regimes. The switching strategy is studied in Section 1.4.

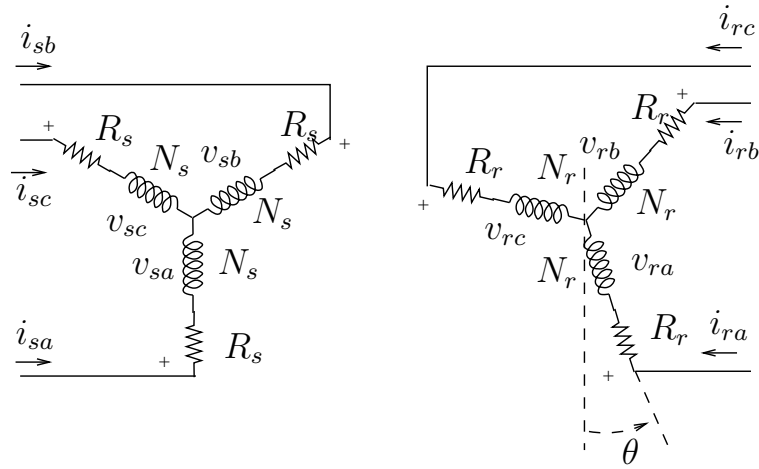


Figure 1.2: Basic electrical scheme of the doubly fed induction machine

1.2 The doubly-fed induction machine

Doubly-fed induction machines (DFIM) form a class of induction machines which have become very popular for renewable energy applications. They have been proposed in the literature, among other applications, for wind-turbine generators [68][82], hybrid engines [22] or high performance storage systems [3]. The attractiveness of the DFIM stems primarily from its ability to handle large speed variations around the synchronous speed (see [71] for an extended literature survey and discussion). Another advantage is that the power electronic equipment to control the machine only has to handle a fraction (maximum 20 – 30%) of the total power [72]. Therefore, the losses in the power electronic converter can be reduced, compared to a system where the converter has to handle the total power. In addition, the cost of the converter becomes lower.

It is usual to consider [23][50] that the machine is symmetric (all windings are equal) and the stator-rotor cross inductances are smooth, sinusoidal functions of θ (the rotor angle) with just the fundamental term. Figure 1.2 shows an electrical scheme of a doubly-fed, three-phase induction machine. It contains 6 energy storage elements with their associated dissipations and 6 ports (the 3 stator and the 3 rotor voltages and currents).

The parameter machines N_s , N_r are number of turns of the coils, where R_s , R_r represent the losses (for the stator and rotor windings respectively), and the electrical variables are the three-phase (abc) stator and rotor voltages and currents

$$\begin{aligned} V_s^T &= [v_{sa}, v_{sb}, v_{sc}] \\ V_r^T &= [v_{ra}, v_{rb}, v_{rc}] \\ I_s^T &= [i_{sa}, i_{sb}, i_{sc}] \\ I_r^T &= [i_{ra}, i_{rb}, i_{rc}] \end{aligned}$$

where three phase variables are defined as

$$f_{abc} = F \left[\cos(\omega_s t), \cos\left(\omega_s t - \frac{2}{3}\pi\right), \cos\left(\omega_s t + \frac{2}{3}\pi\right) \right]^T.$$

The angle θ is the rotor position relative to the system.

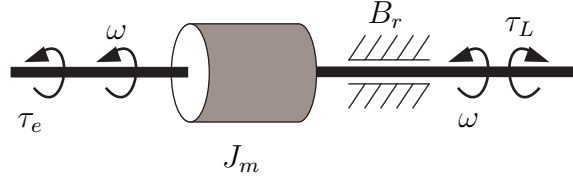


Figure 1.3: Basic mechanical scheme of a rotating machine

The mechanical part (see Figure 1.3) is basically composed by the rotor mass and its corresponding inertia J_m , turning at the mechanical speed ω ($\omega = \dot{\theta}$), with a certain loss represented by the damping coefficient B_r , and subjected to the action of the electrical torque τ_e and of an external torque (or load torque, τ_L).

1.2.1 Dynamical equations of a DFIM

The doubly-fed induction machine is an electromechanical system. The two domains are related by the magnetic field which produces forces and induces currents. Our presentation starts from the well-known electrical and mechanical dynamical equations; see [23] for an extended explanation on how AC machines operate.

For the electrical part the dynamical equations are given by

$$\dot{\Lambda}_s = R_s I_3 I_s + V_s \quad (1.1)$$

$$\dot{\Lambda}_r = R_r I_3 I_r + V_r \quad (1.2)$$

where Λ_s, Λ_r are the inductor fluxes,

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \in \mathbb{R}^{3 \times 3}$$

and the linear magnetic relationship of the fluxes and current is given, neglecting the saturation effects, by the following θ -dependent matrix (in \mathbb{R}^6)

$$\begin{bmatrix} \Lambda_s \\ \Lambda_r \end{bmatrix} = \tilde{L}(\theta) \begin{bmatrix} I_s \\ I_r \end{bmatrix}$$

where

$$\tilde{L}(\theta) = \begin{bmatrix} \tilde{L}_s & \tilde{L}_{sr}(\theta) \\ \tilde{L}_{sr}^T(\theta) & \tilde{L}_r \end{bmatrix} \in \mathbb{R}^{6 \times 6},$$

$$\tilde{L}_{sr}(\theta) = L_{sr} \begin{bmatrix} \cos \theta & \cos(\theta + \frac{2}{3}\pi) & \cos(\theta - \frac{2}{3}\pi) \\ \cos(\theta - \frac{2}{3}\pi) & \cos \theta & \cos(\theta + \frac{2}{3}\pi) \\ \cos(\theta + \frac{2}{3}\pi) & \cos(\theta - \frac{2}{3}\pi) & \cos \theta \end{bmatrix} \in \mathbb{R}^{3 \times 3},$$

$$\tilde{L}_s(\theta) = L_s \begin{bmatrix} 1 & \cos(\frac{2}{3}\pi) & \cos(-\frac{2}{3}\pi) \\ \cos(\frac{2}{3}\pi) & 1 & \cos(\frac{2}{3}\pi) \\ \cos(-\frac{2}{3}\pi) & \cos(\frac{2}{3}\pi) & 1 \end{bmatrix} \in \mathbb{R}^{3 \times 3},$$

$$\tilde{L}_r(\theta) = L_r \begin{bmatrix} 1 & \cos\left(\frac{2}{3}\pi\right) & \cos\left(-\frac{2}{3}\pi\right) \\ \cos\left(\frac{2}{3}\pi\right) & 1 & \cos\left(\frac{2}{3}\pi\right) \\ \cos\left(-\frac{2}{3}\pi\right) & \cos\left(\frac{2}{3}\pi\right) & 1 \end{bmatrix} \in \mathbb{R}^{3 \times 3},$$

where L_s , L_r are the so-called, respectively, stator and rotor inductances and L_{sr} the self-inductance²

The mechanical equation comes from Newton laws. Considering a 2-pole machine it takes the form

$$J_m \dot{\omega} = \tau_e - B_r \omega - \tau_L$$

where

$$\tau_e = I_s^T \tilde{\mathcal{L}}_{sr}(\theta) I_r. \quad (1.3)$$

$$\tilde{\mathcal{L}}_{sr}(\theta) = L_{sr} \begin{bmatrix} -\sin(\theta) & \cos\left(\theta - \frac{\pi}{6}\right) & -\cos\left(\theta + \frac{\pi}{6}\right) \\ -\cos\left(\theta + \frac{\pi}{6}\right) & -\sin(\theta) & \sin\left(\theta + \frac{\pi}{3}\right) \\ \cos\left(\theta - \frac{\pi}{6}\right) & \sin\left(\theta - \frac{\pi}{3}\right) & -\sin(\theta) \end{bmatrix} \in \mathbb{R}^{3 \times 3}$$

Notice that the electrical equations are highly nonlinear, due the dependence of the rotor position θ .

1.2.2 The dq-transformation

The dq-transformation (also known as Blondel-Park Transformation) is widely used in the study of power systems [50]. This mathematical transformation is used to decouple variables, to facilitate the solution of difficult equations with time-varying coefficients, or to refer all variables to a common reference frame. In addition, the θ -depending model of the machine (as is explained in the previous subsection), is simplified using the dq-model.

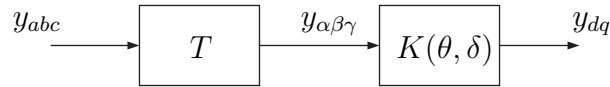


Figure 1.4: Basic scheme of the dq-transformation.

²In the literature a $\frac{2}{3}$ factor is added in the expressions of \tilde{L}_s , \tilde{L}_r and \tilde{L}_{sr} . In this Thesis the coefficients are defined as

$$L_s = \frac{\pi \mu_0 l_1 l_2 N_s^2}{8g}$$

$$L_r = \frac{\pi \mu_0 l_1 l_2 N_r^2}{8g}$$

$$L_{sr} = \frac{\pi \mu_0 l_1 l_2 N_s N_r}{8g}$$

where μ_0 is the magnetic permeability of the air, l_1 is the length of the rotor, l_2 is the rotor diameter, g is the air gap length and N_s , N_r are the number of turns of stator and rotor windings respectively. See [23] for further details.

The dq-transformation can be split in two steps (see Figure 1.4). First the original three-phase variables y_{abc} (currents, voltages or magnetic fluxes) are transformed to the $\alpha\beta\gamma$ static reference by means of

$$y_{\alpha\beta\gamma} = T y_{abc} \quad (1.4)$$

where

$$T = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \in \mathbb{R}^{3 \times 3}.$$

Remark 1.1. Notice that, since $T^T = T^{-1}$, this is a power-preserving transformation:

$$\langle i_{\alpha\beta\gamma}, v_{\alpha\beta\gamma} \rangle = \langle i_{abc}, v_{abc} \rangle.$$

△

For three-phase equilibrated systems, *i.e.* $y_a + y_b + y_c = 0$, this transformation allows to work only with the two first transformed components ($\alpha\beta$) and neglect the third one (the homopolar, γ) which is zero for any balanced set and which, in any case, is decoupled from the remaining dynamical equations.

Secondly, if the output variables are periodic orbits (sinusoidal functions), they can be transformed into equilibrium points rotating in a reference framework. This procedure also eliminates the θ -dependence of the dynamical equations of the DFIM. Let us define the dq-variables as

$$\begin{bmatrix} y_{s\alpha\beta} \\ y_{r\alpha\beta} \end{bmatrix} = K(\theta, \delta) \begin{bmatrix} y_{sdq} \\ y_{rdq} \end{bmatrix} \quad (1.5)$$

with

$$K(\theta, \delta) = \begin{bmatrix} e^{J_2\delta} & O_2 \\ O_2 & e^{J_2(\delta-\theta)} \end{bmatrix} \in \mathbb{R}^{4 \times 4}$$

where δ is an arbitrary function of time (usually δ is the stator frequency, ω_s), and

$$e^{J_2\eta} = \begin{bmatrix} \cos(\eta) & -\sin(\eta) \\ \sin(\eta) & \cos(\eta) \end{bmatrix} \in \mathbb{R}^{2 \times 2},$$

$$O_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{2 \times 2},$$

$$J_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \in \mathbb{R}^{2 \times 2}.$$

1.2.3 The dq-model of the DFIM

From the dynamical equations of the DFIM described in subsection 1.2.1, and using the dq-transformation, the dq-model of the DFIM can be obtained.

Applying the T transformation (equation (1.4)) to the electrical variables, equations (1.1) and (1.2) yields

$$\dot{\lambda}_{s\alpha\beta} + R_s I_2 i_{s\alpha\beta} = v_{s\alpha\beta} \quad (1.6)$$

$$\dot{\lambda}_{r\alpha\beta} + R_r I_2 i_{r\alpha\beta} = v_{r\alpha\beta} \quad (1.7)$$

where $\lambda_{s\alpha\beta}, \lambda_{r\alpha\beta}, i_{s\alpha\beta}, i_{r\alpha\beta} \in \mathbb{R}^2$ are fluxes and currents in the $\alpha\beta$ framework (neglecting the homopolar component). The electrical torque (1.3) is transformed into

$$\tau_e = L_{sr} i_{s\alpha\beta}^T J_2 i_{r\alpha\beta} \quad (1.8)$$

Linking fluxes and currents are now related by

$$\lambda_{\alpha\beta} = L_{\alpha\beta}(\theta) i_{\alpha\beta} \quad (1.9)$$

where

$$\lambda_{\alpha\beta} = \begin{bmatrix} \lambda_{s\alpha\beta} \\ \lambda_{r\alpha\beta} \end{bmatrix} \in \mathbb{R}^4, \quad i_{\alpha\beta} = \begin{bmatrix} i_{s\alpha\beta} \\ i_{r\alpha\beta} \end{bmatrix} \in \mathbb{R}^4, \quad L_{\alpha\beta}(\theta) = \begin{bmatrix} L_s I_2 & L_{sr} e^{J_2 \theta} \\ L_{sr} e^{-J_2 \theta} & L_r I_2 \end{bmatrix} \in \mathbb{R}^{4 \times 4}.$$

Putting together (1.6) and (1.7)

$$\dot{\lambda}_{\alpha\beta} + R i_{\alpha\beta} = v_{\alpha\beta} \quad (1.10)$$

where

$$v_{\alpha\beta} = \begin{bmatrix} v_{s\alpha\beta} \\ v_{r\alpha\beta} \end{bmatrix} \in \mathbb{R}^4, \quad R = \begin{bmatrix} R_s I_2 & O_2 \\ O_2 & R_r I_2 \end{bmatrix} \in \mathbb{R}^{4 \times 4}, \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{2 \times 2}$$

The steady-state for the equations above are periodic orbits that can be transformed into equilibrium points by means of the (1.5), where δ is, for convenience, selected as

$$\dot{\delta} = \omega_s,$$

with ω_s the line frequency, which is assumed constant³. From the electrical equation (1.10) and using (1.5)

$$\dot{K}(\theta, \delta) \lambda_{dq} + K(\theta, \delta) \dot{\lambda}_{dq} + R K(\theta, \delta) i_{dq} = K(\theta, \delta) v_{dq}$$

where

$$\lambda_{dq} = \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} \in \mathbb{R}^4, \quad i_{dq} = \begin{bmatrix} i_s \\ i_r \end{bmatrix} \in \mathbb{R}^4, \quad v_{dq} = \begin{bmatrix} v_s \\ v_r \end{bmatrix} \in \mathbb{R}^4,$$

or

$$\dot{\lambda}_{dq} = -K(\theta, \delta)^{-1} \dot{K}(\theta, \delta) \lambda_{dq} - K(\theta, \delta)^{-1} R K(\theta, \delta) i_{dq} + K(\theta, \delta)^{-1} K(\theta, \delta) v_{dq}.$$

Notice that defining

$$\Omega(\omega) = K(\theta, \delta)^{-1} \dot{K}(\theta, \delta) = \begin{bmatrix} \dot{\delta} J_2 & O_{2 \times 2} \\ O_{2 \times 2} & (\dot{\delta} - \dot{\theta}) J_2 \end{bmatrix} \in \mathbb{R}^{4 \times 4}$$

³This is the so-called synchronous reference frame.

where $\dot{\delta} = \omega_s$ and $\dot{\theta} = \omega$, the dynamical equations of the electrical variables can be written as

$$\dot{\lambda}_{dq} = -\Omega\lambda_{dq} - R i_{dq} + v_{dq}, \quad (1.11)$$

The relationship between fluxes and currents (1.9) can be written in the dq-frame as

$$K(\theta, \delta)\lambda_{dq} = L_{\alpha\beta}(\theta)K(\theta, \delta)i_{dq}$$

or

$$\lambda_{dq} = \mathcal{L}i_{dq}, \quad (1.12)$$

where

$$\mathcal{L} = K(\theta, \delta)^{-1}L_{\alpha\beta}(\theta)K(\theta, \delta) = \begin{bmatrix} L_s I_2 & L_{sr} I_2 \\ L_{sr} I_2 & L_r I_2 \end{bmatrix} \in \mathbb{R}^{4 \times 4}. \quad (1.13)$$

The T transformation (1.5) also modifies the electrical torque (1.8) as follows

$$\begin{aligned} \tau_e &= L_{sr} i_{s\alpha\beta}^T J_2 e^{J_2\theta} i_{r\alpha\beta} = \\ &= L_{sr} i_s^T (e^{J_2\delta})^T J_2 e^{J_2\theta} e^{J_2(\delta-\theta)} i_r = \\ &= L_{sr} i_s^T J_2 i_r. \end{aligned} \quad (1.14)$$

and finally the mechanical equation can be written as

$$J_m \dot{\omega} = L_{sr} i_s^T J_2 i_r - B_r \omega - \tau_L. \quad (1.15)$$

The overall system consists of the fourth-order electrical dynamics (1.11) together with the scalar mechanical dynamics (1.15). The electrical variables are either fluxes or currents (related by (1.12)). Usually, in the study of the classical induction machine, the variables are the stator currents and the rotor fluxes, due to the fact that the rotor currents are not measurable. For the control of the DFIM, the ability to obtain directly the rotor currents allows to use the four currents as electrical variables⁴. The control input is the two-dimensional rotor voltage v_r , and the stator voltage v_s is viewed as a constant disturbance.

1.3 The back-to-back converter

Electronic power converters [32] are devices able to deliver electrical energy in a suitable way for the applications, *i.e.*, with prescribed frequency, voltage amplitude or any other specification. They do the trick by periodically storing the energy in inductors and capacitors before releasing it in the desired form; in a given period the converter goes through a series of topological circuit changes by means of controlled switches (for instance IGBT switches).

The back-to-back converter consists of two converters, a machine-side converter and a grid-side converter, that are connected "back-to-back". Between the two converters a dc-link capacitor is placed, as energy storage, in order to keep the voltage variations (or ripple) in the dc-link voltage small. With the machine-side converter it is possible to control the torque or the speed of the DFIM and also the power factor at the stator terminals, while the main objective for the grid-side converter is to keep the dc-link voltage constant.

⁴However in Chapter 2, using the Port-controlled Hamiltonian framework, the electrical variables are the Hamiltonian variables, *i.e.* the fluxes.

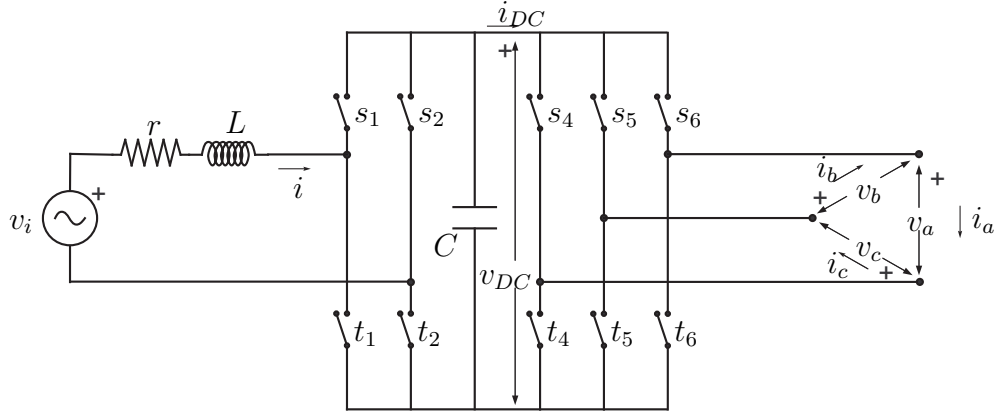


Figure 1.5: Back-to-back converter.

Figure 5.19 shows the back-to-back converter selected for this system. It differs from the typical topology [68][69] in the grid-side converter; in this case the dc-link voltage is controlled by a single-phase boost rectifier instead of a three-phase rectifier (the main reason is to follow as close as possible the available experimental setup). The machine-side converter is a three-phase dc/ac inverter. The whole converter has an ac single *input* and its *outputs* are three-phase PWM (pulse width modulation) voltages which feed the rotor windings of the electrical machine. This system can be split into two parts: a dynamical subsystem (the full bridge rectifier, containing the storage elements) and an static subsystem (the inverter, which, from the energy point of view, acts like a transformer).

A single-phase ac voltage source v_i provides the energy in the direct operation mode. L is the inductance, C is the capacitor of the dc-link, r takes into account all the resistance losses (inductor, source and switches), s_k and t_k ($k = 1, 2, 3, 5, 6$). Switch states take values in $\{-1, 1\}$ and t -switches are complementary to s -switches: $t_k = \bar{s}_k = -s_k$. Additionally, $s_2 = \bar{s}_1 = -s_1$.

One of the principal requirements is that the B2B converter has to allow a bidirectional power flow. This is due to the fact that, in some stationary regimes and, obviously, in the transient, the DFIM can extract energy through the rotor. This feature is achieved using IGBT switches instead of the cheaper, but less versatile, diodes and thyristors option implemented in [45].

1.3.1 Dynamical equations of a full bridge rectifier

As explained above, only the full-bridge rectifier has dynamics. The dynamical equations of a full-bridge rectifier have the form

$$\begin{aligned}\dot{\lambda} &= -Sv_{DC} - ri + v_i \\ \dot{q} &= Si - i_{DC}\end{aligned}$$

where the discrete variable S takes value $+1$ when s_1 is closed ($v_{s1} = 0$), and -1 when s_1 is open ($i_{s1} = 0$). λ and q are the inductor flux and the capacitor charge respectively. Considering the inductor L and the capacitor C as ideal elements, the relationship between the flux/charge and current/voltage is

$$\begin{aligned}\lambda &= Li \\ q &= Cv_{DC},\end{aligned}$$

which yields

$$\begin{aligned}L \frac{di}{dt} &= -Sv_{DC} - ri + v_i \\ C \frac{dv_{DC}}{dt} &= Si - i_{DC}.\end{aligned}\tag{1.16}$$

Usually this discontinuous model is approximated by an averaged system, and S will take values in a continuum set; the discrete implementation of the switch is recovered then by means of a suitable sampling procedure, such as a pulse width modulation (PWM) scheme.

The control objectives of this part are:

- the DC value of v_{DC} voltage should be equal to a desired constant v_{DC}^d , and
- the power factor of the converter should be equal to one.

1.3.2 Equations of a three-phase inverter

The three-phase inverter can be described by means of a set of static equations,

$$v_{abc} = f v_{DC}$$

where

$$f = \frac{1}{2} \begin{bmatrix} s_6 - s_4 \\ s_5 - s_6 \\ s_4 - s_5 \end{bmatrix}.$$

1.4 Power management

In this Section the power flow management of the flywheel energy storage system is discussed. From the dq-power definitions an exhaustive study of the stator and rotor power flows, in a steady-state, allows to define an optimal speed, which will be used, finally, to determine the optimal management of the system. See Appendix A for electrical power definitions.

It is important to note that in the flywheel energy storage system no-external torque τ_L is applied in the mechanical equation (1.15). The only torque from the mechanical domain is the one due to the linear damping $B_r\omega$ term.

1.4.1 Steady-state power study of a DFIM

To formulate mathematically the power flow strategy described above we need to express the various modes in terms of equilibrium points. In this way, the control policy will be implemented transferring the system from one equilibrium point to another. Towards this

end, first the fixed points of the system (1.11) and (1.15) are computed, *i.e.* the values i_{dq}^* , ω^* , v_r^* such that

$$\begin{aligned} 0 &= -\Omega^* \lambda_{dq}^* - R i_{dq}^* + v_{dq}^* \\ 0 &= L_{sr} i_s^{*T} J_2 i_r^* - B_r \omega^* \end{aligned}$$

where

$$\Omega(\omega)^* = \begin{bmatrix} \omega_s J_2 & O_{2 \times 2} \\ O_{2 \times 2} & (\omega_s - \omega^*) J_2 \end{bmatrix} \in \mathbb{R}^{4 \times 4}.$$

Explicit separation of the rows corresponding to the stator, rotor, network and mechanical equations yields the following system of equations:

$$\omega_s L_s J_2 i_s^* + \omega_s L_{sr} J_2 i_r^* + R_s I_2 i_s^* - v_s = 0 \quad (1.17)$$

$$(\omega_s - \omega^*) [L_{sr} J_2 i_s^* + L_r J_2 i_r^*] + R_r I_2 i_r^* - v_r^* = 0 \quad (1.18)$$

$$L_{sr} i_s^{*T} J_2 i_r^* - B_r \omega^* = 0. \quad (1.19)$$

It is clear that—assuming no constraint on v_r —the key equations to be solved are (1.17) and (1.19).

Using the power definitions it can be seen that the DFIM has an optimal mechanical speed for which there is minimal power injection through the rotor (These calculus are detailed in Appendix B). Indeed, from (1.18) one immediately gets

$$P_r^* \triangleq i_r^{*T} v_r^* = (\omega_s - \omega^*) L_{sr} i_r^{*T} J_2 i_s^* + R_r |i_r^*|^2,$$

where $|\cdot|$ is the Euclidean norm. Further, using (1.19), we get

$$P_r^* = B_r \omega^* (\omega^* - \omega_s) + R_r |i_r^*|^2. \quad (1.20)$$

Although the ohmic term in (1.20) does depend also on ω , its contribution is small for the usual range of parameter values, so $|P_r|$ is small near $\omega^* = \omega_s$. Another consideration that is made to justify the choice of "optimal" rotor speed, ω^* , concerns the reactive power supplied to the rotor—that we would like to minimize. It can be shown that

$$Q_r^* \triangleq i_r^{*T} J_2 v_r^* = (\omega^* - \omega_s) f(Q_n, \omega^*),$$

where $f(\cdot, \cdot)$ is a bounded function of its arguments. Consequently, $Q_r^* = 0$ for $\omega^* = \omega_s$. Taking this into account, we will set the reference of the mechanical speed as $\omega^* = \omega_s$.

1.4.2 Power strategy

The power flow strategy of the flywheel energy storage system has been proposed in [12]. The power management schedule is determined according to the following considerations. The general goal (described in Section 1.1) is to supply the required power to the load with a high network power factor, *i.e.*, $Q_n \sim 0$. On the other hand, as has been shown above, the DFIM has an optimal mechanical speed for which there is minimal power injection through the rotor. Combining these two factors suggests to consider the following three modes of operation:

- **Generator mode.** When the real power required by the local load is larger than the maximum network power P_n^{MAX} the DFIM is used as a generator. In this case the references for the network real and reactive powers are fixed as $P_n^* = P_n^{MAX}$ and $Q_n^* = 0$.
- **Storage (or motor) mode.** When the local load does not need all the network power and the mechanical speed is far from the optimal value the "unused" power network is employed to accelerate the flywheel. From the control point of view, this operation mode coincides with the *generator mode*, and thus the same references are fixed—but now the goal is to extract the maximum power from the network in order to transfer it to the flywheel.
- **Stand-by mode.** Finally, when the local load does not need all the power network and the mechanical speed is near the optimal one, we just compensate for the flywheel friction losses by regulating the speed and the reactive power. Hence, the reference for the mechanical speed is fixed at its minimum rotor losses value (to be defined below) and $Q_n^* = 0$.

The operation modes boil down to two kinds of control actions as expressed in Table 1.1, where P_l is the load power and $\epsilon > 0$ is some small parameter.

$P_n^* < P_l$	$ \omega - \omega_s \leq \epsilon$	Mode	References
True	True	Generator	$P_n^* = P_n^{MAX}$ and $Q_n^* = 0$
True	False	Generator	$P_n^* = P_n^{MAX}$ and $Q_n^* = 0$
False	True	Stand-by	$Q_n^* = 0$ and $\omega^* = \omega_s$
False	False	Storage	$P_n^* = P_n^{MAX}$ and $Q_n^* = 0$

Table 1.1: Control action table.

The power references P_n^* and Q_n^* will be achieved by means of the stator current control. The value of i_s , expressed in appropriate coordinates and considering a constant voltage amplitude of the power grid, translates directly into the value of the power flowing through the stator. This indirect control of P_s and Q_s , which will be detailed in Chapter 4, will achieve maximum efficiency from the grid, both in terms of active power and power factor compensation.