Essays on Macroeconomic Policy and Business Cycles

Shengliang Ou

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DIRECTORS DE LA TESI
Davide Debortoli i Jordi Galí

Departament d'Economia i Empresa

Universitat Pompeu Fabra Barcelona

To my parents

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Abstract

This thesis consists of three chapters on macroeconomic policy and business cycles. In chapter 1, I estimate a time-varying structural VAR to study the effects of government spending shocks on a number of U.S. macroeconomic variables. In contrast to the predictions of the standard New Keynesian models, I find no significant changes in the size of the government spending multiplier when the federal funds rate hits the Zero Lower bound (ZLB). I propose a theoretical model where the central bank, through either conventional or unconventional policies, directly controls the market interest rate, and where the policy rule parameters are subject to regime switches to capture potential changes due to the ZLB constraint. The model estimates suggest that the behavior of the market interest rate was not much affected by the ZLB constraint, and thus the government spending multiplier remained largely unaltered. In chapter 2, we provide an empirical estimate of the central bank's targeting rule that reflects the relative weight a central bank attaches to the allocation of the output gap and inflation, and of the deep parameter that characterizes a central bank's loss function, overcoming the simultaneity problem. In chapter 3, we explore the welfare implications of a reduction in the price rigidity in a New Keynesian model featuring both price rigidity and dispersed information. We find the introduction of digital price tags that may facilitate price adjustment may deteriorate the welfare. The dominant underlying mechanism is that a reduction in the price rigidity will amplify the welfare losses associated with the price dispersion within price resetting firms when agents have heterogeneous beliefs about the economy.

Resum

Aquesta tesi consta de tres capítols sobre política macroeconòmica i cicles empresarials. Al capítol 1, estimo un VAR estructural que varia amb el temps per estudiar els efectes de les xocs de la despesa pública en diverses variables macroeconòmiques dels Estats Units. A diferència de les prediccions dels nous models keynesians estàndard, no trobo canvis significatius en la grandària del multiplicador de despesa pública quan la taxa de fons federals arriba al límit zero inferior (ZLB). Proposo un model teòric on el banc central, mitjanant polítiques convencionals o no convencionals, controli directament el tipus d'interès del mercat i on els paràmetres de la regla de política estan subjectes a canvis de règim per capturar possibles canvis a causa de la restricció ZLB. Les estimacions del model suggereixen que el comportament de la taxa d'interès del mercat no es va veure molt afectat per la restricció ZLB i, per tant, el multiplicador de la despesa pública va romandre en gran part inalterat. Al capítol 2, proporcionem una estimació empírica de la regla d'orientació del banc central que reflecteix el pes relatiu que un banc central concedeix a l'assignació de la bretxa de producció i la inflació, i del paràmetre profund que caracteritza la funció de pèrdua d'un banc central, superant la simultaneïtat problema. En el capítol 3, explorem les implicacions del benestar d'una reducció de la rigidesa dels preus en un nou model keynesià amb rigidesa en preus i informació dispersa. Trobem que la introducció d'etiquetes de preus digitals que puguin facilitar l'ajust de preus podrien deteriorar el benestar. El mecanisme subjacent dominant és que una reducció de la rigidesa dels preus amplificarà les pèrdues de benestar associades a la dispersió de preus en les empreses de restabliment de preus quan els agents tenen creences heterogènies sobre l'economia.

Preface

The macroeconomic policy is widely implemented to stabilize the economy and eliminate the distortions. For instance, expansionary monetary policy and fiscal policy are usually conducted to stimulate the economy during the economic downturn. In the presence of distortions in the economy, the macroeconomic policy can be welfare improving. In this thesis, I contribute to understanding the effectiveness and welfare implications of macroeconomic policy. In particular, I investigate the effect of government spending policy during the zero lower bound period, empirically estimate the Fed's targeting rule, and analyze the welfare implications of the introduction of new technology to eliminate the nominal rigidity in the thesis.

In the first chapter, I estimate the government spending multiplier during the zero lower bound in the United States. A common prediction of many theoretical studies is that the government spending multiplier is much larger at the zero lower bound (ZLB) than in normal times, when monetary policy is not constrained. Intuitively, in normal times, the inflationary effect of a positive government spending shock can be dampened by a rise of the real interest rate, crowding out private consumption and investment, leading to a lower multiplier. In contrast, during ZLB periods, a rise of inflation causes the real interest rate to decline, further boosting aggregate demand, which in turn leads to a larger multiplier. I contribute to this literature in two ways. First, I empirically investigate whether the size of the government spending multiplier increased during the recent ZLB period in the United States and I find that there is no significant change. Second, I rationalize this fact by including unconventional monetary policy in a New Keynesian framework, which allows the central bank to target the market interest rate even when the federal funds rate hit the zero lower bound.

In the second chapter, we provide an empirical estimation of the Fed's targeting rule and loss function. Empirical estimation of this measure is challenging due to the simultaneity problem. We overcome this issue by purging out shocks that shift a central bank's targeting rule. The following results stand out. First, the Fed is willing to decrease the growth rate of the output gap by two and a half percentage points for each percentage point increase in (quarterly) inflation.

Second, the data rejects optimal discretionary policy in favor of optimal policy under commitment. Third, the Fed's targeting rule was not much changed during the pre-Volcker and post-Volcker period. And last but not least, we provide an empirical estimate of the deep parameter that characterizes a central bank's loss function. On average, the weight that the Fed attaches to the volatility of output gap relative to the volatility of (annualized) inflation is roughly 0.2.

In the third chapter, we explore the welfare implications of a reduction in the degree of nominal rigidity that could be due to the introduction of digital price tags in the economy. We address this question in a New Keynesian model, emphasizing the role of information frictions and dispersed beliefs that are previously ignored in the literature. In this model, firms have different assessments about the state of the economy due to information frictions. Therefore, in contrast to a standard model with perfect information, price dispersion emerges among those firms who can reset prices, which is inefficient because goods matter for household's utility symmetrically and firms' production technology are identical. A reduction in the price rigidity will amplify the welfare losses associated with the price dispersion within price resetting firms. We derive the conditions under which this channel will dominate and evaluate the effectiveness in a quantitative model.

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Chapter 1

THE GOVERNMENT SPENDING MULTIPLIER AT THE ZERO LOWER BOUND

1.1 Introduction

The size of the government spending multiplier—defined as the unit change of output when government spending increases by one unit—is of central concern in economics. If the government spending multiplier is larger than one, fiscal policy can stimulate economic activity without crowding out private consumption and investment. With the onset of the global financial crisis of 2007-2008, the policy rate in many major currency areas hit the theoretical zero lower bound, which contributed to an increasing relevance of fiscal policy. Importantly, a common prediction of many theoretical studies is that the government spending multiplier is much larger at the zero lower bound (ZLB) than in normal times, when monetary policy is not constrained.[1] Intuitively, in normal times, the inflationary effect of a positive government spending shock can be dampened by a rise of the real interest rate, crowding out private consumption and investment, leading to a lower multiplier. In contrast, during ZLB periods, a rise of inflation causes the real in-

[1] See, e.g. Woodford (2011b), Eggertsson (2011) and Christiano et al. (2011). For instance, in a model under some standard calibrations, the government spending multiplier at the ZLB is 5 times as large as in the normal periods.

terest rate to decline, further boosting aggregate demand, which in turn leads to a larger multiplier. Consequently, it is of great interest to measure the size of the government spending multiplier at the zero lower bound (ZLB).

I contribute to this literature in two ways. First, I empirically investigate whether the size of the government spending multiplier increased during the recent ZLB period in the United States and I find that there is no significant change.[2] Second, I rationalize this fact by including unconventional monetary policy in a New Keynesian framework, which allows the central bank to target the market interest rate even when the federal funds rate hit the zero lower bound.

Firstly, my empirical results are based on a structural vector autoregressive model with time-varying coefficients and stochastic volatility (TVC-SVAR) for the recent ZLB periods in the United States. To analyze the underlying channel, I further investigate the responses of inflation, the 10-year nominal yield rate, consumption and investment to government spending shocks during the ZLB and the pre-ZLB periods.

My identification scheme follows the literature of fiscal VAR (e.g. Blanchard and Perotti (2002)), and relies on the assumption that within-quarter government spending does not respond contemporaneously to the macroeconomic variables. One advantage of this approach, relatively to the alternative identification scheme proposed by Ramey and Zubairy (2018), is that I identify the current government spending shocks instead of the news shocks because the government spending multiplier at the ZLB is sensitive to the timing of government spending. Intuitively, if the increase in the government spending is expected to occur after the end of the zero lower bound, as would possibly be the case with military spending, then the multiplier is quantitatively small.[3]

My results can be summarized as follows. First, I find that there are no significant differences in the responses of GDP, consumption, investment, inflation and the 10-year constant maturity treasury rate to the identified government spending

[2] The size of the government spending multiplier at the ZLB is sensitive to the sample periods. For instance, Ramey and Zubairy (2018) finds that the government spending multiplier at the ZLB is sensitive to the inclusion of the WWII sample periods. However, they don't distinguish between the ZLB in the historical sample and the recent ZLB period. More details will be provided in the following section of related literature.

[3] See Christiano et al. (2011) for details.

shock. Second, using the same approach, I do find significant differences in the size of the government spending multiplier between the pre-ZLB and the ZLB period in Japan, and between the pre-Volcker and the Volcker period in the United States.[4] These results are reassuring with respect to the modeling choice of the time-varying parameter VAR, as the model is able to detect changes in the size of the multiplier.

Secondly, I provide a theoretical model to rationalize the aforementioned results, allowing for the possibility of the "substitutability" between conventional and unconventional monetary policies. I assume that the central bank directly targets the market interest rate and follows a Taylor rule. The assumption is motivated by the observation that various market interest rates were above zero and fluctuated during the period 2009Q1-2015Q4. To explore whether the behavior of the market interest rates is affected by the ZLB constraint, I allow for the policy rule parameters subject to regime switches and examine whether the Taylor coefficients for the market interest rate changed during the ZLB period.

I estimate the regime-switching DSGE model with Bayesian methods, allowing for stochastic volatility. My estimated results show that there is no structural break in the Taylor rule coefficients. This suggests that, during the recent ZLB periods, the central bank was able to adjust the market interest rate to stabilize the economy in response to government spending shocks. As a consequence, there is no significant difference in the government spending multiplier over pre-ZLB and ZLB periods, consistent with the empirical findings.

Related Literature This paper contributes to the extensive literature estimating the effect of government spending shocks on the economy. Numerous studies have investigated the size of the government spending multiplier with different identification strategies (e.g. Blanchard and Perotti (2002), Ramey and Shapiro (1998), Ramey (2011), Fisher and Peters (2010)).

The recent literature studies whether the size of the government spending multiplier can depend on the state of the economy. For example, Kirchner et al.

[4]There is an independent evidence that the monetary policy regime during the pre-Volcker period is significantly different from that implemented in the post-Volcker period (e.g., Clarida et al. (2000a)). This would predict a difference in the size of the multiplier between the pre-Volcker and the Volcker period.

(2010), Auerbach and Gorodnichenko (2012a,b), Pereira and Lopes (2014), Ramey and Zubairy (2018) examine whether the multiplier can differ when the economy is in recession. Broner et al. (2018) explore the connection between the government spending multiplier and the foreign holdings of public debt. This paper instead focuses on the size of the government spending multiplier at the ZLB.

Few papers estimate the effect of the government spending shocks at the ZLB. Crafts and Mills (2013) focus on the U.K. experience during the 1922-1938 periods. Miyamoto et al. (2018) estimate the government spending multiplier at the ZLB in Japan. Ramey and Zubairy (2018) also investigate the government spending multiplier at the ZLB using U.S. data. They cover the sample from 1889Q1 to 2015Q4. The ZLB event is defined as a union of 1932Q2-1951Q1 and 2008Q4-2015Q4. They find the size of the multiplier at the ZLB is sensitive to the inclusion of the WWII. However, they don't distinguish between the ZLB in the historical sample and the recent ZLB period. As there can be a large amount of structural change in the past 120 years, I complement their findings by focusing on recent periods and investigate the dynamics of more macroeconomic variables to analyze the underlying channel.

Further, the paper adds to the literature that estimates the government spending multiplier when monetary policy and exchange rate policy is passive. For example, Nakamura and Steinsson (2014) estimate an open economy relative multiplier considering the U.S. as a monetary union of each state assuming national monetary policy does not respond to regional variation in government spending. Ilzetzki et al. (2013) using multi-country data find the multiplier is larger in the country under the fixed exchange rate scheme than under a flexible exchange rate scheme. Dupor and Li (2015) estimate the multiplier using U.S. data from 1959-1979 when monetary policy may have been passive.[5][6] This paper differentiates from theirs by focusing the aggregate multiplier in the closed economy setting during the recent ZLB periods in the United States.

More broadly, this paper relates to the literature of empirical testing of the

[5]My findings for the pre-Volcker period (1974Q1-1979Q2) do not contradict with Dupor and Li (2015) though I use different approaches and identification strategies. I don't find a larger multiplier during the period 1959Q1-1973Q1, which is consistent with theirs.

[6]More specifically, the passive monetary policy here refers to that the Taylor coefficient on inflation during that period was less than one as documented by Clarida et al. (2000a).

theoretical predictions of the New Keynesian model under ZLB. For example, Wieland (2014), Garin et al. (2016) and Debortoli et al. (2018) estimate the impulse response of macroeconomic variables to various shocks such as supply shocks, technology shocks, demand shocks and monetary policy shocks. My result is consistent with Debortoli et al. (2018), who find that there is no structural break in the responses of a number of U.S. macroeconomic variables to the technology shocks, demand shocks, supply shocks, and monetary policy shocks focused on the same periods. I complement the literature by focusing on government spending shocks. In particular, I investigate whether the government spending multiplier is larger at the ZLB (e.g. Woodford (2011b), Eggertsson (2011) and Christiano et al. (2011)).[7]

Besides, my paper contributes to the literature that studies the effect of unconventional monetary policy. Swanson and Williams (2014), D'Amico and King (2013), Krishnamurthy and Vissing-Jorgensen (2011), Hamilton and Wu (2012) and Swanson (2017), estimate the effect of unconventional monetary policy, forward guidance or quantitative easing, on the various variables such as yield curve, exchange rate, inflation and output in reduced form. Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Del Negro et al. (2017) and Chen et al. (2012) analyze the impact of unconventional monetary policy in a quantitative DSGE model. My model is closely related to the recent work of Wu and Xia (2016) and Wu and Zhang (2016) which proposes a shadow rate to summarize both conventional and unconventional monetary policy. I instead assume the central bank targets the market interest rate at both the ZLB and non-ZLB states. Moreover, I estimate the regime-switching DSGE model allowing for a change in the market interest rate rule.

Finally, my paper relates to the literature that examines the effect of policy

[7]There is an alternative theoretical prediction on the government spending multiplier at the ZLB made by Mertens and Ravn (2014), who argue the government spending multiplier at the ZLB will be lower than in normal times. My results do not support this prediction. The difference between the two contrasting predictions lies in different equilibrium selections. The New Keynesian model suffers global indeterminacy when zero lower bound constraint is present. The analysis of Woodford (2011b), Eggertsson (2011), and Christiano et al. (2011) is based on the local determinate equilibrium around the steady state with zero (positive) targeted inflation and a positive nominal interest rate. There is another steady state, namely liquidity trap, with deflation and a zero nominal interest rate. The analysis of Mertens and Ravn (2014) is based on this liquidity trap steady state.

regimes on the economy. In a related paper, Bianchi and Melosi (2017) consider a regime-switching model and show that the uncertainty about how the fiscal policy will conduct after the end of the zero lower bound period is an important factor in explaining the dynamics of the macroeconomic variables during the recent ZLB period. Different from their work, I focus on the government spending multiplier without distinguishing the fiscal policy regime.

The remainder of the paper is organized as follows. Section 1.2 presents the implications of the government spending multiplier in a standard New Keynesian model at the ZLB. Section 1.3 describes my empirical approach. Section 1.4 presents the corresponding results. Section 1.5 provides a New Keynesian model with unconventional monetary policy to rationalize my empirical findings. Section 2.6 concludes.

1.2 The Government Spending Multiplier in a Standard New Keynesian Model

In the current section, I will show the size of the government spending multiplier in a standard New Keynesian model at the zero lower bound under some standard calibrations. The model closely follows the setup of Christiano et al. (2011).

1.2.1 Model

Consider a standard New Keynesian model with fiscal policy block. The model consists of a representative household, a continuum of monopolistically competitive firms that set the price *à la* Calvo subject to the demand constraint and production technology, a final goods producer that combines intermediate goods following a CES technology, and a government financing the spending through the lump-sum tax.[8] The linearized model is given by:

$$\frac{\hat{y}_t - s\hat{g}_t}{1-s} = E_t(\frac{\hat{y}_{t+1} - s\hat{g}_{t+1}}{1-s}) - (\hat{i}_t - E_t\pi_{t+1} - d_{\xi_t} + E_t d_{\xi_{t+1}}) \tag{1.1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t \tag{1.2}$$

$$mc_t = \frac{\alpha+\nu}{1-\alpha}\hat{y}_t + \frac{\hat{y}_t - s\hat{g}_t}{1-s} \tag{1.3}$$

$$\hat{i}_t = (1 - Z_{\xi_t})(\phi_\pi \pi_t + \phi_y(\hat{y}_t - \hat{y}_{t-1})) \tag{1.4}$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t} \tag{1.5}$$

where $\hat{y}_t$, $\hat{g}_t$, $\hat{i}_t$, π_t, mc_t is the log deviation of output, government spending, nominal interest rate, inflation, real marginal cost from their steady-state level. Equation 1.1 is the consumption Euler equation dervied from the household optimization problem that describes the relationship between consumption, inflation and the nominal interest rate. s is the steady-state level of government spending to GDP ratio. d_{ξ_t} is a discrete preference shock that can assume two values: high or low (d_h or d_l). ξ_t is a random variable following a two-state Markov chain process to control the regime. When ξ_t equals 1, d_{ξ_t} is d_h and the demand is high

[8] I assume the household period utility function is $U_t = log(C_t) - \frac{N_t^{1+\nu}}{1+\nu}$ and the production technology of intermediate goods is: $Y(i)_t = Z_t N(i)_t^{1-\alpha}$.

in the economy. When ξ_t equals 2, d_{ξ_t} is d_l and the demand is low. Equation 1.2 is the New Keynesian Phillips curve derived from the firms' optimization problem which describes the relationship between inflation, expected inflation and real marginal cost. The slope of the above Phillips curve $\kappa = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$. $1 - \alpha$ is the labor income share. β is the discount factor. Equation 1.3 is derived from the household labor supply equation and labor production function. ν is the inverse Frisch elasticity of substitution. Equation 1.4 is the monetary policy rule. More specifically, when ξ_t equals 1, d_{ξ_t} is d_h and Z_{ξ_t} is 0, which implies the economy is not constrained by the ZLB and the nominal interest rate moves according to the Taylor rule. If ξ_t equals 2, d_{ξ_t} is d_l, Z_{ξ_t} is 1, and the nominal interest rate is pegged at 0. Equation 1.5 describes the exogenous process of government spending obeying a stationary AR(1) process where $0 \leq \rho_g < 1$. $\epsilon_{g,t}$ is the exogenous government spending shock drawn from normal distribution with zero mean and σ_g standard deviation.

The model is parameterized as follows. Each period corresponds to a quarter. I set the discount factor β equal to 0.9985. The inverse Frisch elasticity ν is 1. The elasticity of substitution between good varieties $\epsilon = 6$. The frequency of price adjustment ζ_p is 0.75 which implies an average price duration of 4 quarters. α is 1/3 such that the labor income share is 2/3. The probability of remaining in the normal time regime p_h is 0.9896 while the probability of remaining in the ZLB regime p_l is 0.8 taken from Christiano et al. (2011). This implies an average duration of the normal time regime of 96 quarters, and an average duration of ZLB regime of 5 quarters. d_l is -0.3 and d_h is calculated such that the unconditional mean of the discrete shock d_{ζ_t} is zero, consistent with Bianchi and Melosi (2017).[9] The Taylor coefficient on inflation ϕ_π is 1.5 and on output growth rate ϕ_y is 0.125 during normal times. The persistence of government purchases ρ_G is 0.9. The standard deviation of a government spending shock δ_G is 0.01. These values are similar to broad business cycle literature (e.g. Gali (2015) and Christiano et al. (2011)).

Figure 1.1 displays the impulse response of output to a government spending shock and the simple cumulative multiplier in the calibrated model during the

[9] Actually, the calibration of the d_l and d_h does not affect the impulse responses of macroeconomic variables to the government spending shock.

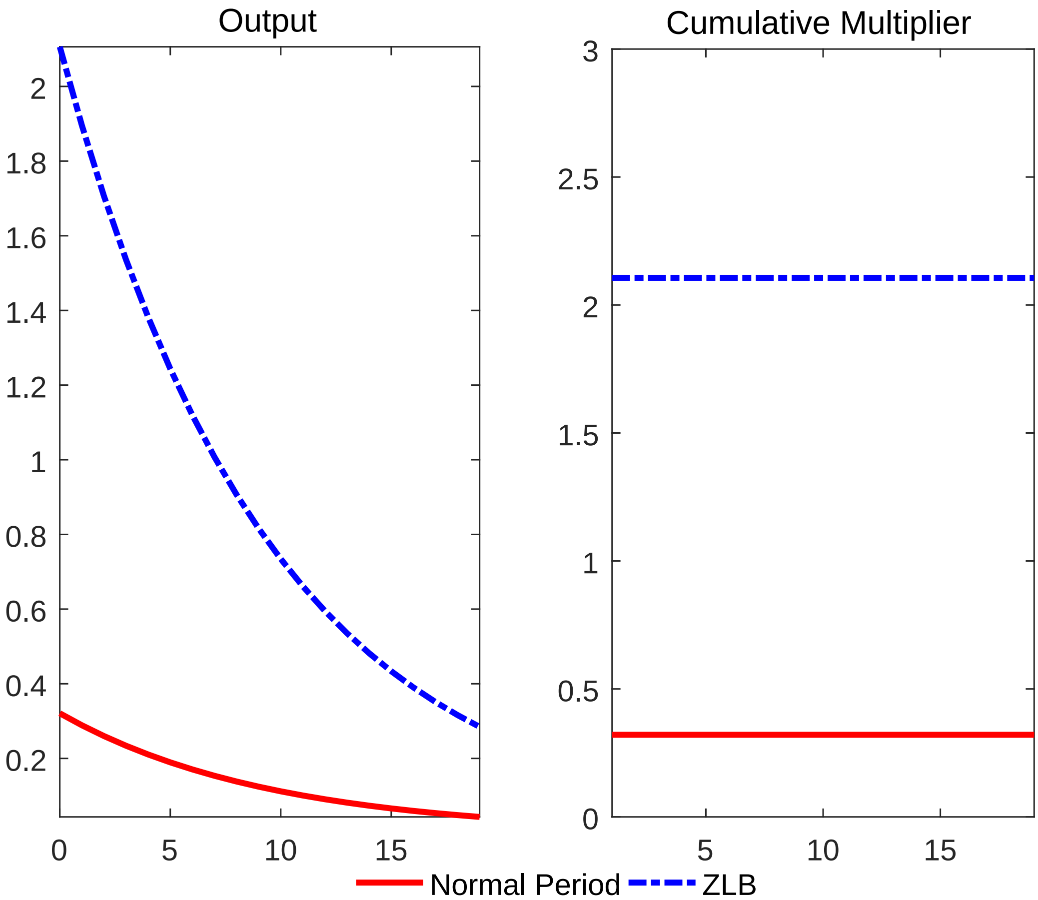
normal times and the ZLB period. The simple cumulative multiplier is defined as follows:

$$\beta_s = \frac{\sum_{k=0}^{k=s} \frac{\partial y_{t+k}}{\partial \epsilon_{g,t}}}{\sum_{k=0}^{k=s} \frac{\partial g_{t+k}}{\partial \epsilon_{g,t}}} \tag{1.6}$$

where β_s is the cumulative multiplier at horizon s, $\frac{\partial y_{t+k}}{\partial \epsilon_{g,t}}$ is the response of output at horizon $t+k$, and $\frac{\partial g_{t+k}}{\partial \epsilon_{g,t}}$ is the response of government spending at horizon $t+k$ to a government spending shock when it hits at time t. Clearly, the response of output and the cumulative multiplier are much larger at the zero lower bound. Both the difference in the response of output on impact and the cumulative multiplier between the ZLB period and normal times is around 1.8.[10] As explained in the literature (e.g. Christiano et al. (2011)), in normal times, the inflationary effects of a positive government spending shock will be dampened by the rise of the real interest rate following the Taylor principle, leading to a lower multiplier. During the ZLB period, the rise in inflation will cause the real interest rate to decline, leading to a larger multiplier.

[10] I take a conservative calibration of the persistence of the ZLB period which corresponds to 5 quarters of ZLB. If I allow for a longer duration of ZLB, the difference will be much larger.

Figure 1.1: Impulse Response to the Government Spending Shock in the Calibrated Model



Note: The figure presents the impulse response of output to a government spending shock and the simple cumulative multiplier in the calibrated model. The response of output is denoted in dollars, corresponding to a dollar change of government spending shock.

1.3 Empirical Model

This section introduces the empirical model I employ to estimate the dynamic responses of selected macroeconomic variables to the identified government spending shock. My empirical model consists of a structural vector autoregressive model with time varying coefficients and stochastic volatility (TVC-SVAR). The choice of the empirical model is motivated by two reasons. First, it allows us to assess whether the government spending multiplier is larger at the zero lower bound. Second, it imposes a flexible structure to capture other potential structural changes over time which may lead to a change in the government spending multiplier.[11]

1.3.1 Model Specification

I closely follow the model specification in Primiceri (2005). The model is given by

$$\boldsymbol{y}_t = \boldsymbol{c}_t + \boldsymbol{B}_{1,t}\boldsymbol{y}_{t-1} + ... + \boldsymbol{B}_{p,t}\boldsymbol{y}_{t-p} + \boldsymbol{u}_t,\ t = 1, ..., T, \tag{1.7}$$

where $\boldsymbol{y}_t$ is an $n \times 1$ vector of endogenous variables, $\boldsymbol{c}_t$ is an $n \times 1$ vector of time varying coefficients that multiply constant terms, $\boldsymbol{B}_{i,t}$, $i = 1, ..., p$ are respectively $n \times n$ matrices of time varying coefficients, $\boldsymbol{u}_t$ is a Gaussian white noise vector process with a covariance matrix $\boldsymbol{\Omega}_t$. The reduced-form innovations $\boldsymbol{u}_t$ is assumed to be a linear combination of underlying structual shocks $\boldsymbol{e}_t$ defined by:

$$\boldsymbol{u}_t \equiv \boldsymbol{Q}_t\boldsymbol{e}_t \tag{1.8}$$

where $\boldsymbol{E}(\boldsymbol{e_t e_t'}) = \boldsymbol{I_n}$ and $\boldsymbol{E}(\boldsymbol{e_t e_{t-k}'}) = \boldsymbol{0}$ for all t and k=1, 2, 3... $\boldsymbol{Q}_t$ is the impact matrix I need to identify. The $\boldsymbol{\Omega}_t$ is defined by $\boldsymbol{A}_t\boldsymbol{\Omega}_t\boldsymbol{A}_t' = \boldsymbol{\Sigma}_t\boldsymbol{\Sigma}_t'$ where $\boldsymbol{A}_t$ is the lower triangular matrix and $\boldsymbol{\Sigma}_t$ is a diagonal matrix.

It follows that

$$\boldsymbol{y}_t = \boldsymbol{c}_t + \boldsymbol{B}_{1,t}\boldsymbol{y}_{t-1} + ... + \boldsymbol{B}_{p,t}\boldsymbol{y}_{t-p} + \boldsymbol{A}_t^{-1}\boldsymbol{\Sigma}_t\boldsymbol{\varepsilon}_t,\ t = 1, ..., T, \tag{1.9}$$

[11] These may be related with the debt-to-GDP ratio, the condition of the financial system, the degree of openness, exchange rate regimes and level of underutilized resources.

In the recursive identification scheme, $\boldsymbol{Q}_t = \boldsymbol{A}_t^{-1}\boldsymbol{\Sigma}_t$. Let $\boldsymbol{\alpha}_t$ and $\log \boldsymbol{\sigma}_t$ be the vectors collecting respectively the non-zero elements of the matrix $\boldsymbol{A}_t$ and the diagonal elements of the matrix $\boldsymbol{\Sigma}_t$. The time varying coefficient parameters are assumed to evolve according to a random walk

$$\boldsymbol{B}_t = \boldsymbol{B}_{t-1} + \boldsymbol{\nu}_t, \tag{1.10}$$

$$\boldsymbol{\alpha}_t = \boldsymbol{\alpha}_{t-1} + \boldsymbol{\zeta}_t, \tag{1.11}$$

$$\log \boldsymbol{\sigma}_t = \log \boldsymbol{\sigma}_{t-1} + \boldsymbol{\eta}_t. \tag{1.12}$$

It is further assumed that the innovations in the model are jointly normally distributed with the following block diagonal variance-covariance matrix:

$$V = Var\left(\begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\nu}_t \\ \boldsymbol{\zeta}_t \\ \boldsymbol{\eta}_t \end{bmatrix}\right) = \begin{bmatrix} \boldsymbol{I}_n & 0 & 0 & 0 \\ 0 & \boldsymbol{Q} & 0 & 0 \\ 0 & 0 & \boldsymbol{S} & 0 \\ 0 & 0 & 0 & \boldsymbol{W} \end{bmatrix}. \tag{1.13}$$

I estimate the model following the updated MCMC algorithm in Del Negro and Primiceri (2015). See Appendix 1.7.1 for details.

1.3.2 Identification

In my baseline model, I identify the government spending shocks following Blanchard and Perotti (2002). I include real government purchase, real government current tax receipts net of transfers and real gross domestic product in a vector in the VAR denoted by [G, T, GDP]. All variables are normalized by real potential GDP.[12] The transformation is made similar to Gordon and Krenn (2010). As emphasized by Ramey and Zubairy (2018), the cyclicality of government expenditure to GDP ratio can bias the estimate of the government spending multiplier if I instead take the logarithm of the variables and convert the multiplier in percentage into dollar changes ex-post. To avoid this potential bias, I divide these variables by the real potential domestic GDP so that these variables are put in the same

[12]I first apply the GDP deflator to deflate the nominal counterpart of the government purchase, the government current tax receipts net of transfers and gross domestic product.

unit. Based on the assumption that within-quarter government spending does not contemporaneously respond to macroeconomic variables, the government spending shock is identified by the Cholesky decomposition of the variance-covariance estimates from the reduced-form VAR model. Then, the government spending shock is an unexplained component of the government spending by past government spending, output and other macroeconomic variables.

An alternative identification scheme could be the narrative approach, which the military news shocks are based on. However, it is not suitable for analyzing the government spending multiplier during the recent ZLB period in the United States as there are few shocks that could be identified through that approach during that period. In addition, the other advantage of this identification scheme, relative to the narrative approach, is that I identify the current government spending shocks instead of the news shocks because the size of the government spending multiplier at the ZLB is sensitive to the timing of the government spending. Intuitively, if the increase in the government spending is expected to occur after the zero lower bound ends, as would possibly be the case with military spending, then the multiplier is quantitatively small.[13]

1.3.3 Data

The NIPA variables are drawn from the FRED database from the period 1955Q1-2017Q4. I use variables as follows: nominal GDP, GDP deflator, Government consumption expenditures and gross investment, Federal government current tax receipts, State and local government current tax receipts, 10-year treasury constant maturity rate, CBO real potential GDP, Personal consumption expenditures, Gross private domestic investment.

1.4 Results

In the current section, I present my baseline results to investigate the size of the government spending multiplier. In my baseline model, I define pre-ZLB periods

[13] See Christiano et al. (2011).

as 2002Q1 to 2008Q4 and ZLB periods as 2009Q1 to 2015Q4. I construct the average impulse response of the two periods as a way to summarize the results.

Output and Tax Figure 1.2 presents the difference in the impulse responses of output, government spending and the tax net of transfers to a government spending shock, and the difference in the cumulative multiplier[14] between the ZLB and the pre-ZLB period. Firstly, the difference in the impulse response of output ranges from -0.25 to 0.85 on impact. Clearly, the difference is insignificant. If I ignore the uncertainty of the estimated parameters and focus on the median estimate, the magnitude is around 0.25, and much smaller than predicted by the theoretical model illustrated in section 1.2. Secondly, there is no significant difference in the cumulative multiplier. This is consistent with the result that the response of government spending to a government spending shock was also not greatly changed during the ZLB period. Finally, the difference in the impulse response of the tax net of transfers between the ZLB and the pre-ZLB period is insignificant and that implies the way of financing the government spending remained largely unaltered during the ZLB period.

Inflation and Nominal Interest Rate I expand my variables with inflation π and the ten-year constant maturity yield rate GS10 as $[G, T, GDP, \pi, GS10]$. Figure 1.3 presents the difference in the responses of inflation, the ten-year constant maturity yield rate, GDP and the tax net of transfers to a government spending shock between the ZLB and the pre-ZLB period. Firstly, the difference in the responses of output and the tax net of transfers is similar to that of my benchmark model. Secondly, the difference in the responses of inflation and the ten-year constant maturity rate is insignificant. Lastly, the median estimate of the difference in the response of inflation is around 0.1 percent that is much less than the theoretical prediction, and the median estimate of the difference in the response of the ten-year constant maturity rate is around zero. These results are consistent with the recent experience of the Fed managing long-term interest rate to stabilize the economy, and suggests the central bank follows a similar pattern to affect the long-term interest rate in both periods.

[14]See Section 1.2 for the definition.

My empirical findings stand in contrast to the above theoretical predictions from the standard New Keynesian model at the ZLB. The main difference is that in the theoretical model the nominal interest rate is fixed while I find that the nominal interest rate (the ten-year constant maturity yield rate) moves in my empirical exercise. That may suggest that unconventional monetary policy can work effectively in the recent ZLB periods and play a stabilizing role. I will build up a DSGE model allowing for unconventional monetary policy during ZLB periods to rationalize the aforementioned findings in the next section.

1.4.1 Robustness

In the current section, I briefly discuss some selected robustness exercises. The detailed analysis is contained in the Appendix 1.7.2.

Consumption and Investment The standard New Keynesian model predicts larger responses of consumption and investment to a government spending shock at the ZLB. To test this hypothesis, I expand the VAR model with consumption and investment. Similar to the results in the previous section, I find no significant change in the responses of consumption and investment.

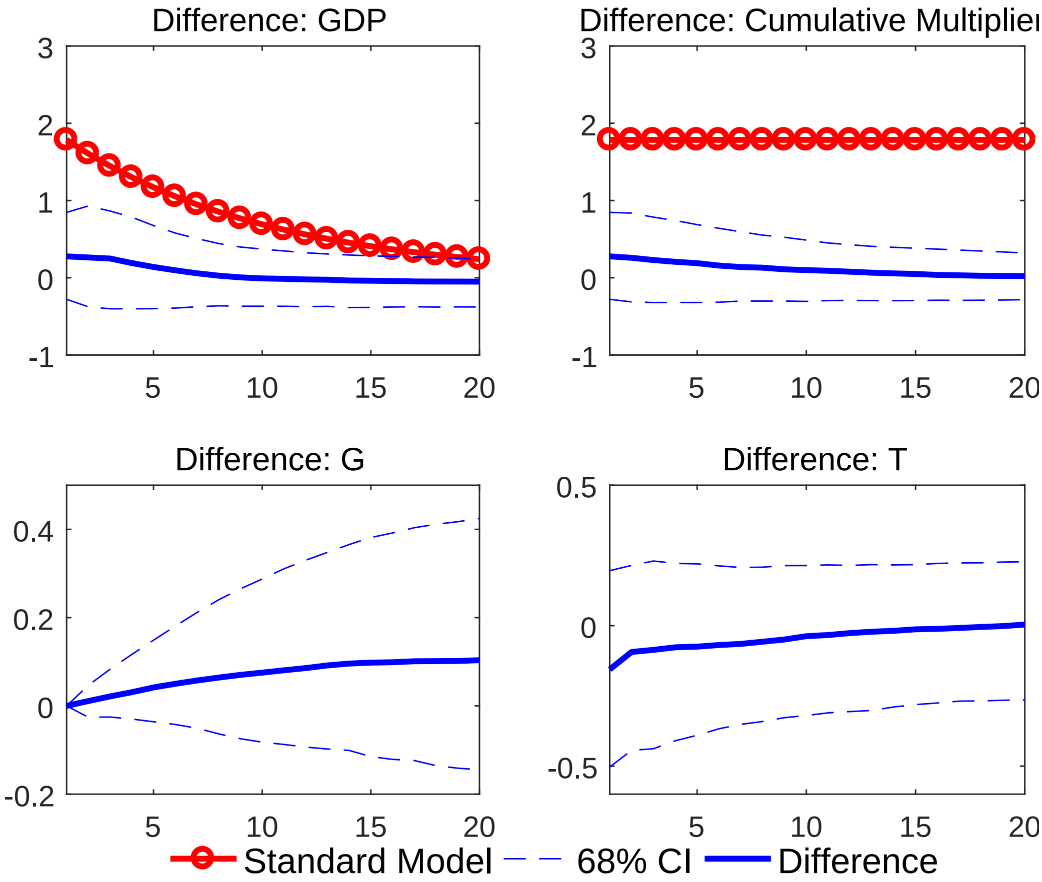
Controls To address the concern that the government spending shock in the previous VAR can be predicted, I expand the VAR with the forecast of government spending and GDP growth rate to control the timing of the government spending. The result is robust to these controls.

State of recessions To examine whether the results relate to the state of recessions, I split the sample during the ZLB period into two parts by unemployment rate. The high unemployment rate period is defined as 2009Q2 to 2011Q4 where the unemployment rate was above 8.5%. The low unemployment rate period is defined as a union of period 2009Q1 and period from 2012Q1 to 2015Q4 where the unemployment rate was below 8.5%. The size of the government spending multiplier in these two samples is similar, not significantly different from in the pre-ZLB period.

Ability of TVC-SVAR to capture a change in the government spending multiplier To address the concern that my approach would not be able to capture the change in the size of the government spending multiplier, I perform two exercise. First, I compare the government spending multiplier during the pre-Volcker period (1974Q1-1979Q2) with that of the Volcker period (1979Q3-1987Q2). There exists independent evidence that the monetary policy regime during the pre-Volcker period was significantly different from that implemented during the Volcker period which would predict a large difference in the multiplier.[15] I find a larger multiplier during the pre-Volcker period. Second, I compare the government spending multiplier during the pre-ZLB with that of the ZLB period in Japan using the TVC-SVAR approach. Miyamoto et al. (2018) find that the government spending multiplier at the ZLB in Japan is larger using the local projection method, in line with the predictions of the standard New Keynesian model at the ZLB. Consistent with their findings, I find a larger multiplier during the ZLB period in Japan. These results are reassuring with respect to the modeling choice of the time-varying parameter VAR, as the model is able to detect changes in the size of the multiplier.

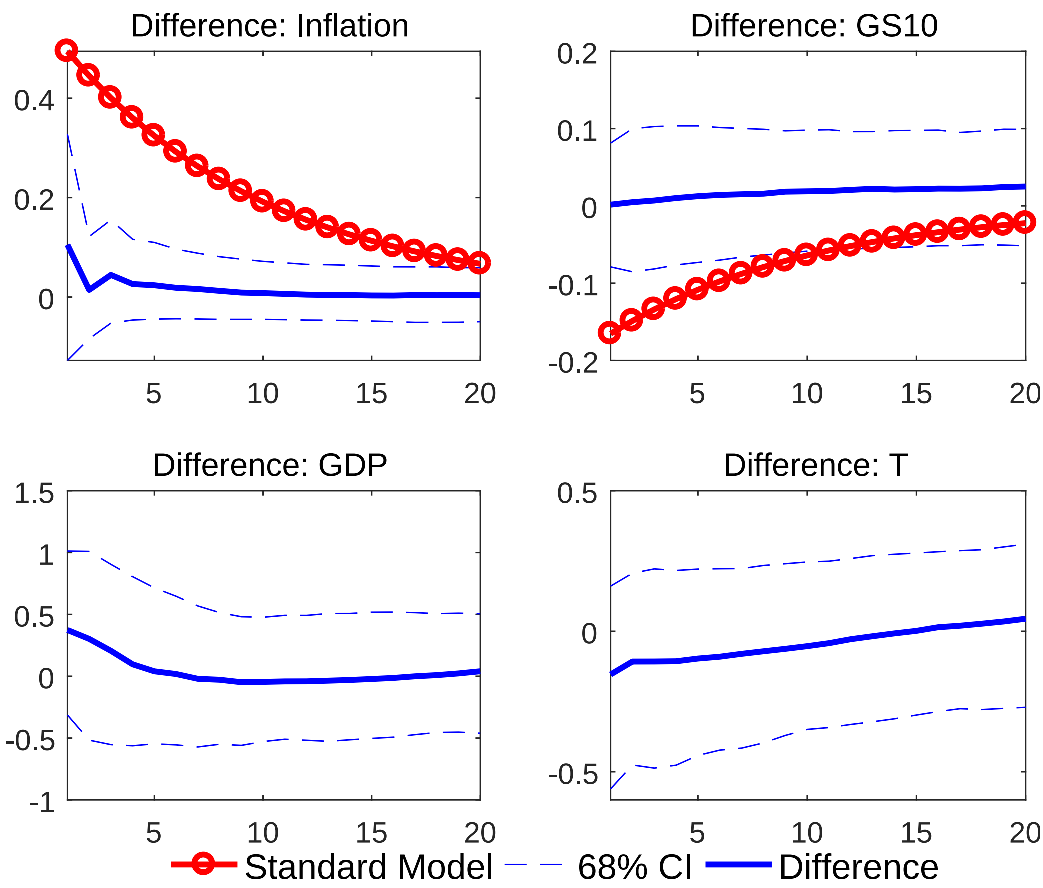
[15] See, e.g., Clarida et al. (2000a).

Figure 1.2: Difference in the Impulse Response to the Government Spending Shock: Pre-ZLB vs. ZLB Periods



Notes: The figure presents the difference in the impulse responses of GDP, government spending and the tax net of transfers to a government spending shock, and the difference in the cumulative multiplier between the ZLB and the pre-ZLB period. The blue solid line is the median estimate of the difference and the blue dashed line is the 68% confidence band. The red circle line is the theoretical prediction of the difference in the model illustrated in section 1.2.

Figure 1.3: Difference in the Impulse Response to the Government Spending Shock: Pre-ZLB vs. ZLB Periods



Notes: The figure presents the difference in responses of inflation, the 10-year constant maturity yield rate, GDP and the tax net of transfer to the government spending shock between the ZLB and the pre-ZLB period. The blue solid line is the median estimate of the difference and the blue dashed line is the 68% confidence band. The red circle line is the theoretical prediction of the difference in the model illustrated in section 1.2.

1.5 A New Keynesian Model with Unconventional Monetary Policy

In the present section, I estimate a New Keynesian model with unconventional monetary policy to rationalize the aforementioned findings. More specifically, I assume the central bank directly targets the market interest rate following a Taylor rule during both normal times and ZLB periods. This assumption is in the same spirit to Wu and Zhang (2016) who propose a shadow rate as the coherent summary of monetary policy. The underlying idea is that during normal time periods, the central bank influences both the risk-free interest rate and the premium by controlling the federal funds rate. During the zero lower bound period, the central bank directly controls the premium component of the market interest rate through unconventional monetary policy, including quantitative easing programs and forward guidance.

Household There is a representative household in the economy with the lifetime utility function:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t exp(d_t) \left\{ log(C_t - \Phi C_{t-1}) - \frac{N_t^{1+\nu}}{1+\nu} \right\} \tag{1.14}$$

subject to the budget constraint:

$$P_t C_t + B_{t+1} = B_t \underbrace{R_{m,t}}_{R_t(1+\lambda_t)} + W_t N_t + T_t \tag{1.15}$$

Here C_t is consumption, N_t is the hours, W_t is the nominal wage, T_t is the firm's profit net of lump-sum taxes paid to the government. B_{t+1} is the quantity of the one-period bond households buy at period t, $R_{m,t}$ is the interest rate of the one period bond which can be interpreted as the risky return of the financial asset with two components R_t and λ_t. R_t is the riskless rate, and λ_t can be interpreted as a premium. When the federal funds rate R_t is lowered down to zero, the central bank can still move λ_t such that the market interest rate facing the household denoted by $R_{m,t}$ tracks the evolution of the economy. The λ_t is introduced ad-hoc as

I simplify the problem by assuming the central bank directly targets this interest rate through both conventional and unconventional monetary policy. Based on the above assumptions, the market interest rate in this model corresponds to the component that the central bank can control. In the model, I keep the single interest rate $R_{m,t}$ and do not model the dynamics of R_t and λ_t. d_t is the intertemporal preference shock, following the process:

$$d_t = \rho_d d_{t-1} + \epsilon_d$$

Where ϵ_d represents an i.i.d. shock with constant variance σ_d.

Market interest rate Figure 1.4 presents the time series of several market interest rates. There are several messages outstanding. First, the federal funds rate is stuck around zero from 2009Q1 to 2015Q4. However, the commercial bank interest rate on credit card plans, the 1-Year Adjustable Rate Mortgage Average in the United States, the 2-year Finance Rate on Personal Loans at Commercial Banks, the 4-year Finance Rate on Consumer Installment Loans at Commercial Banks for New Autos were still above zero and fluctuated over time during the period 2009Q1-2015Q4. I focus on these interest rates because they are closely relevant to the household along many dimensions. Second, the 2-year constant maturity rate is constrained at the zero lower bound from period 2011Q3 to 2014Q1,[16] while the 2-year finance rate on personal loans with the same maturity still evolved over time. This provides the evidence that the market interest rate that households were faced with, may not be constrained by the zero lower bound. Third, the market interest rates above on various items are closely correlated. Based on the above observations, I assume the central bank targets the market interest rates and follows a Taylor rule.

$$\frac{R_t^m}{R} = \left(\frac{R_{t-1}^m}{R}\right)^{1-\rho_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\psi_{\pi,\zeta_t}} \left(\frac{Y_t}{Y_{t-1}}\right)^{\psi_{y,\zeta_t}}\right]^{1-\rho_r} e^{\epsilon_{R_t}} \tag{1.16}$$

[16] I define the zero lower bound as the federal funds rate is below 50 basis points, i.e. 0.5%. The highest rate during the period is 0.37%.

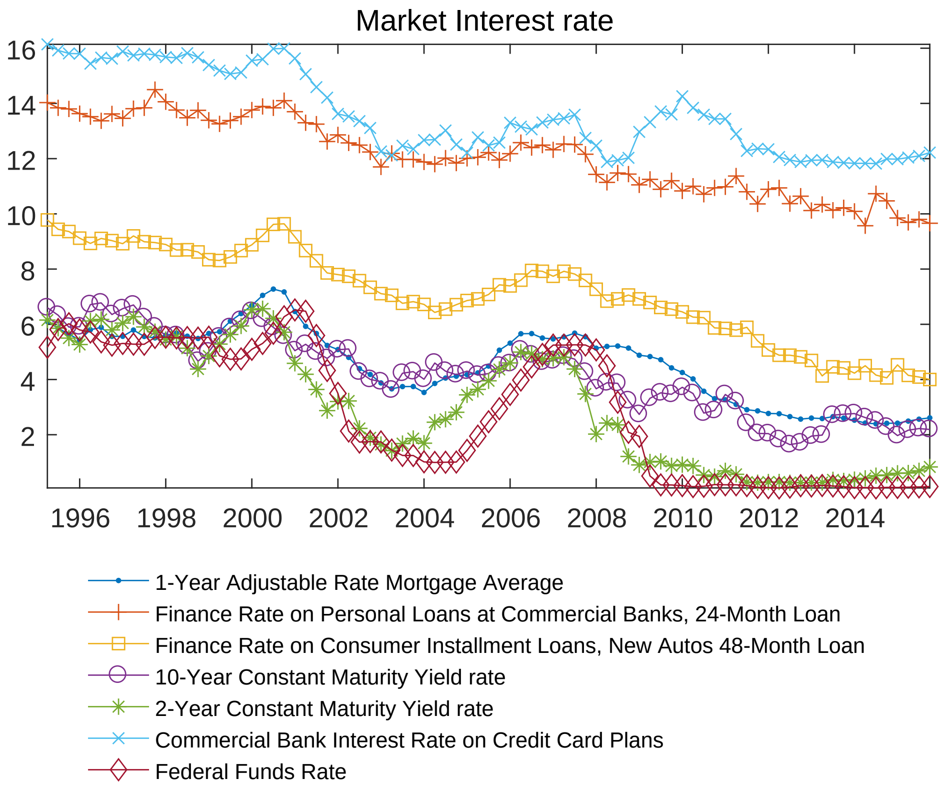


Figure 1.4: Various market interest rates

where R_t^m is the market interest rate in one quarter which is proxied by the ten-year constant maturity yield rate when estimating the model. In this specification, monetary policy shocks include both the conventional monetary policy shocks defined by the shocks to the federal funds rate and premium shocks. If the zero lower bound was an important constraint to the economy, I should observe that the Taylor coefficients changed during the ZLB periods. In the following section, I will estimate the Taylor coefficients in the regime-switching DSGE model.

Firms The firms' problem is similar to the textbook New Keynesian model.[17] There is a continuum of monopolistic firms producing the intermediate goods with production technology

$$Y_t(i) = Z_t L_t(i)^{1-\alpha} \tag{1.17}$$

[17] See, e.g. Gali (2015).

where Z_t is the technology level of the firms that produce intermediate goods. The intermediate goods producers set the price $\grave{a}$ *la* Calvo subject to the demand constraint and production technology. The final goods producers combine intermediate goods following CES technology. More details are in the appdendix 1.7.5.

Government and market clearing The government budget constraint is

$$B_t = B_{t-1}R_{m,t-1} - T_t + G_t \tag{1.18}$$

The government issues one-period bond B_t and adjusts net lump-sum taxes T_t to finance government expenditures G_t. Government purchases are assumed to evolve exogenously according to the process:

$$g_t = \rho_g g_{t-1} + \epsilon_{gt} \tag{1.19}$$

Where $g_t = log(\frac{G_t}{Z_t}) - log(\frac{G_s}{Z_{ss}})$, ϵ_{g_t} represents an i.i.d. shock with constant variance σ_g.

The market clearing condition for this economy is:

$$C_t + G_t = Y_t \tag{1.20}$$

1.5.1 Solving and Estimating the DSGE model

The model is solved with method proposed by Farmer et al. (2009). I construct the likelihood of the solution of the model using the Kalman filter and use Bayesian estimation methods to fit the model to the data. See Appendix 1.7.3 for details.

I use four series of quarterly U.S. data as observables: per capita real GDP growth, the annualized inflation rate, the market interest rate and government spending to GDP ratio. In the baseline model, the market interest rate is proxied by the ten-year constant maturity rate as the Fed managed to stabilize the economy through purchasing the long term bond. However, the ten-year constant maturity rate also reflects the future interest rate and does not correspond to the market interest rate in the model. Thus, in the robustness exercise, I use the commercial bank interest rate on credit card plans and the Wu-Xia shadow rate respectively to proxy for the market interest rate. I estimate the model by fixing the regime sequence. More specifically, I impose the period from 1985Q1 to 2008Q4 to be the pre-ZLB regime and the period from 2009Q1 to 2015Q4 to be the ZLB regime. This implies the agent in the model is faced with the possibility of regime switches while the econometrician estimating the model knows the sequence of regimes. I argue this is plausible in my application since I am trying to evaluate the average performance of unconventional monetary policy during the recent ZLB periods. Estimating the model with regime switches has some advantages over estimating the model under the fixed regime with two separate samples. First, I allow for a larger parameter region.[18] Second, in the regime-switching model, the policy function during ZLB periods also depends on the policy function during normal times when the ZLB regime ends. This is more realistic as the agent will expect the unconventional monetary policy regime to end eventually and switches back to the normal period regime.

1.5.2 Parameter Estimates

I calibrate the discount factors β to be 0.9985 to be consistent with the annualized 2% real interest rate. The habit persistence, Φ is 0.9 taken from Fernández-Villaverde et al. (2010). α is 1/3 such that the labor income share is 2/3. The government expenditure to GDP ratio in steady state is fixed at 0.2. The regimes' transition probability matrix is calibrated to be consistent with the data. The probability of conventional monetary policy regime and unconventional monetary policy regime persist are calibrated with 0.9896 and 0.8, respectively. This corresponds to 24 years of conventional monetary policy regime from 1985Q1 to 2008Q4 and 5 quarters of unconventional monetary policy regime that the agent expects. The

[18]In standard Bayesian estimation of the DSGE model, indeterminate solutions are ruled out when monetary policy is passive. In the regime-switching DSGE model, though the parameters in one regime give rise to indeterminate solutions if I assume the regime never switches, the system as a whole can still be determinate if the regime is not too persistent. Thus, I am able to allow for Taylor coefficients on inflation less than 1 in my parameter regions which allows to a greater difference to be generated in the government spending multiplier between normal times periods and ZLB periods.

rest of the parameters are estimated. I use the 10-year constant maturity yield rate summarizing both conventional and unconventional monetary policy. Table 1.1 presents the priors and posterior parameter estimates. First, the mean estimate of Taylor coefficient on inflation is 2.25 during normal times and is 1.20 during the recent ZLB periods. The change in the coefficient is quantitatively small. As I will show in the next section, the small change in the Taylor coefficient cannot generate a large difference in the government spending multiplier. Second, I set a loose prior for the Taylor coefficient on inflation during ZLB periods with the mean of 0.5 and the standard deviation of 0.5, covering a range from 0 to 1 in one standard deviation. This suggests my posterior estimate is not driven by the prior but informed from the data. In sum, the estimated results corroborate the idea that unconventional monetary policy was efficient at circumventing the constraint implied by the zero lower bound.

1.5.3 Impulse Responses

Figure 1.5 presents the impulse responses of inflation, output and the nominal interest rate to a government spending shock. During both periods, the standard New Keynesian transmission channel of government spending shocks is present which depends on the monetary policy conduct. In response to a positive government spending shock, inflation rises, and the nominal interest rate goes up more than inflation, following the Taylor principle. The rise in the real interest rate stabilizes aggregate demand. Figure 1.6 reports the difference in the response of output to a government spending shock in the TVP-VAR model, the calibrated model and the estimated MS-DSGE model with unconventional monetary policy. The difference in the response of output in the TVP-VAR model is similar to that of the estimated MS-DSGE model with unconventional monetary policy and much smaller than that of the calibrated model at the ZLB. In sum, since the behavior of the market interest rate during the ZLB periods is similar as during the pre-ZLB periods, there is no significant change in the size of government spending multiplier between the two periods.

	Posterior				Prior		
Parameter	Mode	Mean	$\%5$	$\%95$	Distr.	Mean	St. Dev.
$\rho_r(\zeta=1)$	0.8526	0.8552	0.8155	0.8908	B	0.5	0.2
$\phi_\pi(\zeta=1)$	2.2087	2.2456	1.8655	2.6296	N	1.5	0.3
$\phi_y(\zeta=1)$	0.1032	0.1301	0.0182	0.2688	N	0.25	0.1
$\rho_r(\zeta=2)$	0.6915	0.6470	0.4280	0.8257	N	0.5	0.2
$\phi_\pi(\zeta=2)$	1.0441	1.1967	0.5831	1.8471	N	0.5	0.5
$\phi_y(\zeta=2)$	0.1316	0.1440	0.0220	0.2891	N	0.15	0.1
ρ_z	0.0958	0.1784	0.0404	0.4118	B	0.5	0.2
ρ_{ζ_d}	0.7807	0.8073	0.6846	0.9322	B	0.5	0.2
ρ_g	0.9611	0.9553	0.9205	0.9835	B	0.5	0.2
$\sigma_z(\zeta=1)$	0.0117	0.0113	0.0089	0.0137	IG1	0.1	2
$\sigma_r(\zeta=1)$	0.0017	0.0018	0.0015	0.0021	IG1	0.1	2
$\sigma_{\zeta_d}(\zeta=1)$	0.0312	0.0319	0.0244	0.0411	IG1	0.1	2
$\sigma_g(\zeta=1)$	0.0167	0.0177	0.0138	0.0216	IG1	0.1	2
$\sigma_z(\zeta=2)$	0.0112	0.0114	0.0080	0.0157	IG1	0.1	2
$\sigma_r(\zeta=2)$	0.0038	0.0042	0.0030	0.0056	IG1	0.1	2
$\sigma_{\zeta_d}(\zeta=2)$	0.0233	0.0269	0.0163	0.0396	IG1	0.1	2
$\sigma_g(\zeta=2)$	0.0211	0.0226	0.0164	0.0302	IG1	0.1	2
σ_{obs4}	0.0021	0.0022	0.0017	0.0028	IG1	0.1	2
ζ_p	0.7459	0.7378	0.6745	0.7837	B	0.5	0.05
ν	1.5511	1.5525	1.1736	1.9774	G	2	0.3
400γ	2.0693	2.0690	1.5150	2.6655	G	2	0.5
400π	1.7268	1.6984	1.2705	2.1426	G	2	0.5
$400GS_{10}$	3.8881	3.7669	2.9555	4.5859	G	3	0.5

Table 1.1: Modes, Mean, 90% error bands, and prior distributions of the parameters of the Markov-switching DSGE model. $\zeta = 1$ is the normal periods. $\zeta = 2$ is the ZLB periods.

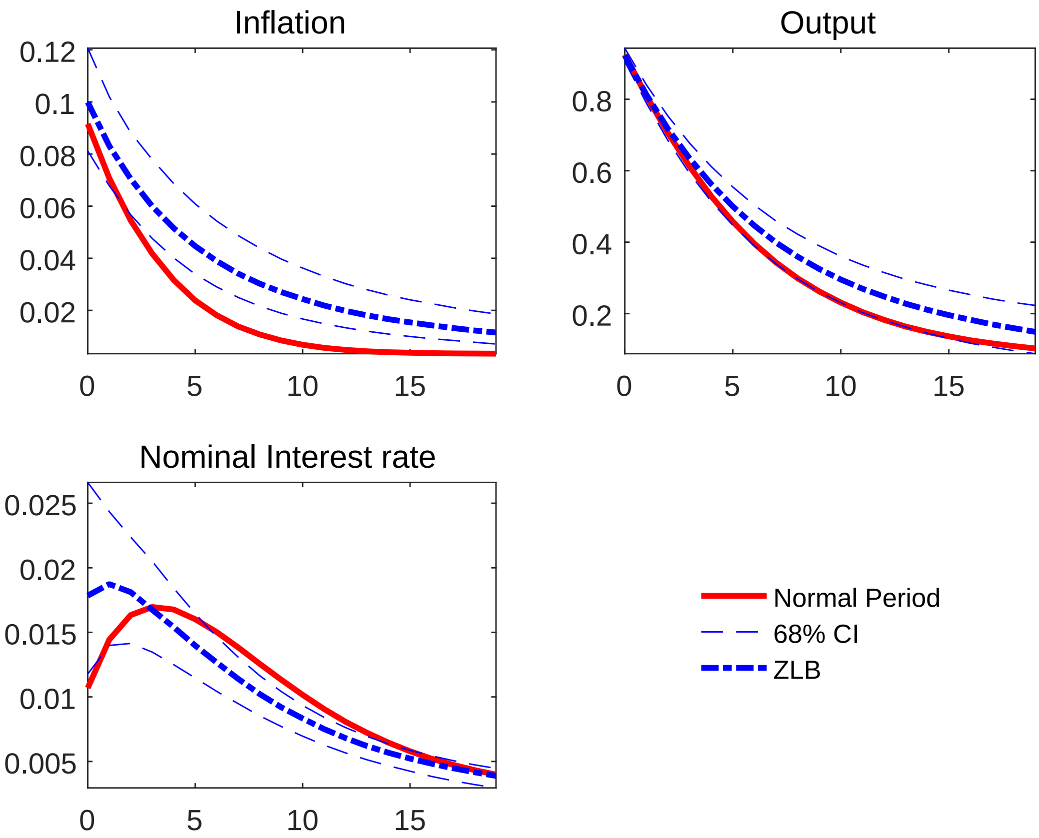
1.5.4 Discussions

In the current section, I discuss what explains the variations of the GDP growth from the estimated DSGE model with unconventional monetary policy. Figure 1.7 shows the historical contribution of each of four types of shocks (technology shock, preference shock, monetary policy shock, and government spending shock). The low growth rate during the Great Recession is mainly attributed to the negative technology shock and negative preference shock. A series of negative technology shocks hit the economy before the Great Recession consistent with the literature (e.g. Fernald (2014)). In the subsequent slow recovery periods, the negative technology shock and the negative government spending shock is an important driver of economic fluctuations. The identified negative government spending shock during the ZLB periods is consistent with the observation from the data that the government spending to GDP ratio declines during that period.

1.5.5 Robustness

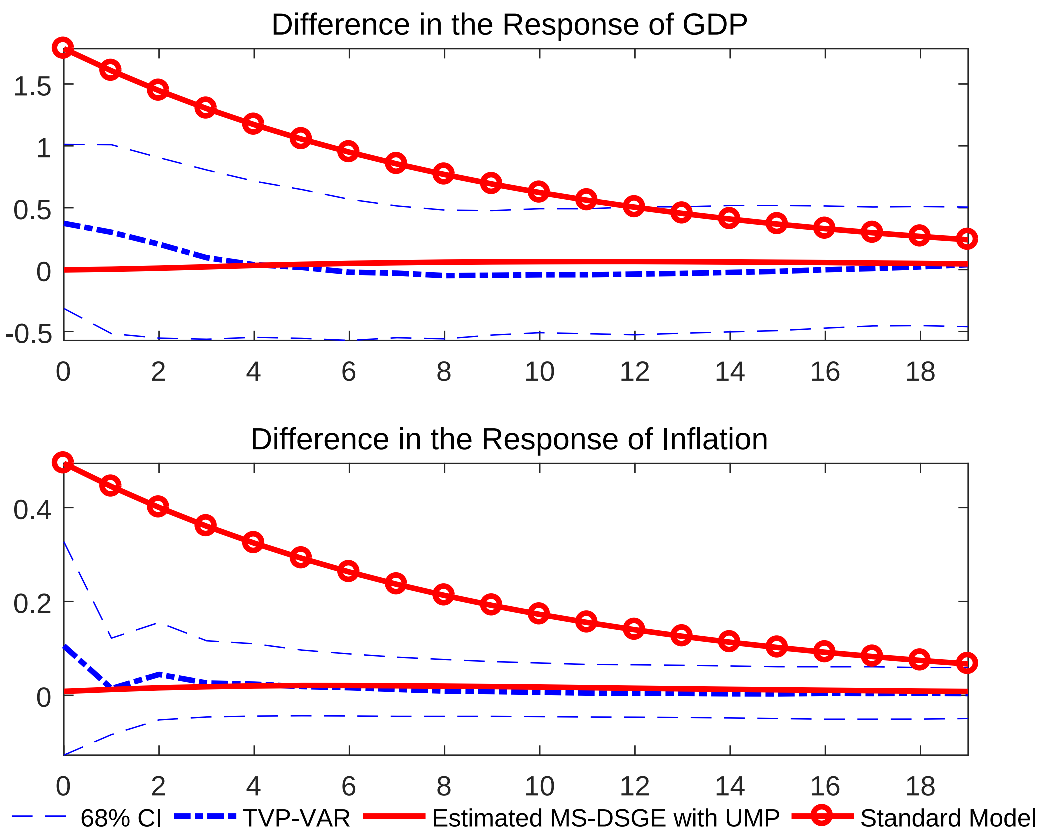
I estimate the model using the Wu-Xia shadow rate and Commercial Bank Interest Rate on Credit Plans to summarize both the conventional and unconventional monetary policy following a Taylor rule respectively. In both cases, there is no significant change in the interest rate rule between normal times and ZLB times. Following the same reasoning explained in the previous section, there is no significant difference in the size of government spending multiplier between the pre-ZLB and the ZLB periods. See Appendix 1.7.4 for detailed tables and figures.

Figure 1.5: Impulse Response to the Government Spending Shock in the Estimated MS-DSGE Model



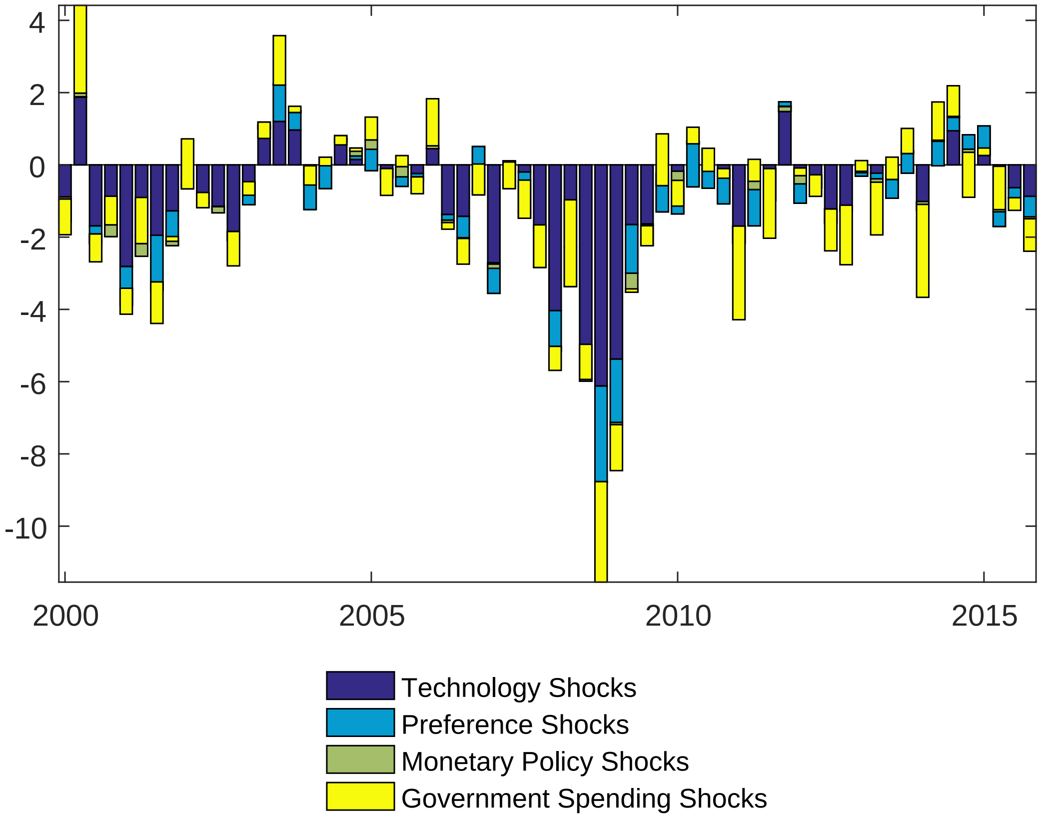
Note: The figure presents the impulse response of output, inflation and nominal interest rate to a government spending shock in the estimated model. The response of inflation and nominal interest rate is expressed as a percentage. The response of output is denoted in dollars, corresponding to one dollar-change government spending. The blue dash-dot line is the median impulse response and the blue dashed line is the 68% confidence band for the ZLB periods 2009Q1-2015Q4. The red solid line is the median impulse response for the normal periods 1985Q1-2008Q4.

Figure 1.6: Comparison of VAR results with those of the Estimated MS-DSGE Model



Note: The figure presents the difference in the impulse responses of output and inflation to a government spending shock in the TVP-VAR model, the calibrated model and the estimated MS-DSGE model with unconventional monetary policy. The blue dash-dot line is the median estimate of the difference in the impulse response and blue dashed line is the 68% confidence band. The red solid line is the median difference in the impulse response in the estimated MS-DSGE model with unconventional monetary policy. The red circle line is the difference in the impulse response in the calibrated theoretical model at the ZLB.

Figure 1.7: Historical Decomposition of Real GDP Growth rate (Annual per capita)



Note: The figure presents the historical decomposition of real GDP growth rate (Annual per capita) deviation from trend growth.

1.6 Concluding Remarks

The present paper contributes to the literature about the government spending multiplier at the zero lower bound (ZLB). I use a time-varying structural VAR to describe the dynamic responses of U.S. macroeconomic variables to the government spending shocks. In contrast to the prediction of the standard New Keynesian models, I find there are no significant changes in the responses of GDP, consumption, investment, inflation and the 10-year constant maturity treasury rate to the identified government spending shocks. One possible explanation is that unconventional monetary policy may stabilize the economy effectively during the zero lower bound periods. To test this hypothesis, I propose a theoretical model where the central bank, through either conventional or unconventional policies, directly controls the market interest rate, and where the policy rule parameters are subject to regime switches to capture potential changes due to the ZLB constraint. The model estimates indicate that there are no significant changes in the Taylor coefficients for the market interest rate between normal times and the recent ZLB periods in the United States. Therefore, even during the ZLB periods, in response to a positive government spending shock, the central bank would increase the market interest rate to stabilize the economy. As a result, the effect of government spending shocks remains largely unaltered.

My results suggest that the government spending policy was less effective than previous thought during the recent ZLB periods in the United States. This implies that in order to stimulate the economy the government should implement a larger fiscal stimulus package. Furthermore, the cost of fiscal consolidation would be small.

1.7 Appendices

1.7.1 Time-varying coefficients and stochastic volatility VAR model

In this section, I briefly explain the procedure I employ to estimate the time-varying coefficients and stochastic volatility VAR model along the lines described in Del Negro and Primiceri (2015). I simulate the posterior distribution of the model coefficients by Gibbs sampling. In each procedure, I draw the parameters from the posterior distribution conditional on the value of the remaining parameters. I describe the algorithm in each step below.

1. Draw β^T from $p(\beta^T|A^T, S^T, \Sigma^T, V, y^T)$.

2. Draw A^T from $p(A^T|\beta^T, S^T, \Sigma^T, V, y^T)$.

3. Draw S^T from $p(S^T|\beta^T, A^T, \Sigma^T, V, y^T)$.

4. Draw Σ^T from $p(\Sigma^T|\beta^T, A^T, S^T, V, y^T)$.

5. Draw V from $p(V|\Sigma^T, \beta^T, A^T, S^T, y^T)$.

Prior Specification

The prior takes the form of:

1. $B_0 \sim N(B_{OLS}, 4Var(B_{OLS)})$.

2. $A_0 \sim N(A_{OLS}, 4Var(A_{OLS)})$.

3. $log(\sigma_0) \sim N(log(\sigma_{OLS}), I_n)$.

4. $Q \sim IW(k_Q^2 * size(\tau) * Var(B_{OLS}), size(\tau))$.

5. $W \sim IW(k_w^2 * (1 + dimension(W)) * I_n, (1 + dimension(W)))$.

6. $S_i \sim IW(k_S^2 * (1 + dimension(S_i) * Var(A_{OLS}), (1 + dimension(S_i)))$.

where variable with subscript OLS corresponds to the OLS estimates in the time invariant VAR for the training sample. S_i is the variance-covariance matrix of i-th row of non-zero element of Σ. The benchmark results presented in the paper are obtained using followings values: $k_Q = 0.05, k_S = 1, K_W = 0.05$. The number of training sample, τ is set to be 80.

Gibbs sampling algorithm

Let T be the total number of observations. I draw parameters from the posterior distribution starting from $\tau/3+1$. [19] The algorithm runs as follows:

Draw β^T: Conditional on $A^T, S^T, \Sigma^T, V, y^T$, the β is drawn following the Carter and Kohn (1994) algorithm.

Draw A^T: Conditional on $\beta^T, S^T, \Sigma^T, V, y^T$, rewrite 1.9 as: $A_t(y_t - X_t' B_t) = A_t \hat{y}_t = \Sigma_t \epsilon_t$. The $i+1$-th equation of $A_t \hat{y}_t = \Sigma_t \epsilon_t$ can be written as

$$\hat{y}_{i+1,t} = -\hat{y}_{[1,\ldots i],t}\alpha_{i,t} + \sigma_{i,t}\epsilon_{i+1,t}, i = 2, 3, \ldots n$$

where $\alpha_{i,t}$ is the i-th row of non-zero element of α_t, $\hat{y}_{[1,\ldots i],t}$ is $[\hat{y}_{1,t}, \hat{y}_{2,t} \ldots \hat{y}_{i,t}]$, $\sigma_{i,t}$ is the i-th row of Σ_t, $\epsilon_{i+1,t}$ is the i-th row of ϵ_t. I can apply Carter and Kohn (1994) algorithm to the above problem equation by equation.

Draw S^T: Conditional on $\beta^T, A^T, \Sigma^T, V, y^T$, I have $A_t(y_t - X_t' B_t) = A_t y_t^\star = \Sigma_t \epsilon_t$. Taking squaring and then logarithms of every element, I obtain

$$y^{\star\star} = 2h_t + e_t \tag{1.21}$$

$$h_t = h_{t-1} + \eta_t \tag{1.22}$$

$y_{i,t}^{\star\star} = log[(y_{i,t}^\star)^2 + 1e-6]; e_{i,t} = log(\epsilon_{i,t}^2), h_{i,t} = log(\sigma_{i,t})$. I approxiate the distribution of $e_{i,t}$ by a mixture of normal distribution following Kim, Shephard and Chib (1998). Conditional on $\beta^T, A^T, \Sigma^T, V$, I sample $s_{i,t}$ from discrete density defined by

$$Pr(s_{i,t} = j | y_{i,t}^{\star\star}, h_{i,t}) \propto q_j f_N(y_{i,t}^{\star\star} | 2h_{i,t} + m_j - 1.2704, v_j^2), j = 1, \ldots, 7, i = 1, \ldots, n \tag{1.23}$$

f_N is the density function of normal distribution.

[19] I start the sample from $\tau/3+1$ instead of τ in order to keep more data points.

Draw Σ_t: Conditonal on $\beta^T, A^T, S^T, V, y^T$, I apply Carter and Kohn (1994) algorithm to draw h_t.

Draw V: Conditional on $\Sigma^T, \beta^T, A^T, S^T, y^T$, the posteriors of hyperparameters has inverse-Wishart distribution. It is easy to draw hyperparameters from inverse-Wishart distribution.

I make 45000 draws, discard the first 5000 draws and collect 1 out of 20 of the remaining 40000 draws. Parameters convergence is assessed using trace plots.

1.7.2 Empirical Results: Robustness Checks

Consumption and Investment Standard New Keynesian model predicts larger consumption and investment multiplier at the ZLB. To examine that hypothesis, I expand the VAR model with consumption and investment. The VAR model is denoted by $[G, T, Y, C, I]$. Figure 1.8 presents the differences in the impulse responses of consumption, investment, and GDP between the ZLB period and pre-ZLB period to a government spending shock. Similar to the main results, I find no significant change in the responses of consumption and investment. The median estimate of the difference is around 0.2. The difference in GDP response in this expanded VAR model is similar to that of the baseline model.

Controls To address the concern that the government spending shocks in the baseline VAR model can be predicted, I expand the VAR model with the forecast of government spending and GDP growth rate to control the timing of the government spending. The forecast of the government spending growth rate is constructed by splicing the Greenbook forecast data from period 1966Q4 to 1981Q4 and Survey of Professional Forecast Data from period 1982Q1 to 2017Q4.[20] The forecast of GDP growth rate is constructed from the Survey of Professional Forecast Data from 1969Q1 to 2017Q4.[21] The forecast of government spending and GDP is made in period t-1 for the period t value. I use the forecast of growth rate instead of levels because there have been plenty of data revisions in the National

[20] I refer the reader to Auerbach and Gorodnichenko (2012b) for details.

[21] The sample of the real GDP forecast in levels starts from 1968Q4.

Income and Product Accounts. I run two VAR models. First, I include the forecast of government spending growth rate denoted by FG in the VAR model. The VAR model is denoted by $[FG, G, T, Y]$. Second, I include both the forecast of government spending growth rate FG and the forecast of GDP growth rate $FGDP$ in the VAR model. The VAR model is denoted by $[FGDP, FG, G, T, Y]$. Figure 1.9 and Figure 1.10 display the results respectively. In both specifications, the difference in the government spending multiplier between the ZLB and the pre-ZLB period is insignificant and the median estimates are small compared with that of the theoretical prediction.

State of recessions To examine whether the results relate to the state of recessions, I split the sample during the ZLB period into two parts by unemployment rate. The high unemployment rate period is defined as 2009Q2 to 2011Q4 where the unemployment rate was above 8.5%. The low unemployment rate period is defined as a union of period 2009Q1 and period from 2012Q1 to 2015Q4 where the unemployment rate was below 8.5%. Figure 1.11 shows the difference in the response of output between the ZLB with different unemployment rates and the pre-ZLB period respectively. The size of the government spending multiplier in these two samples is similar, not significantly different from in the pre-ZLB period.

Ability of TVC-SVAR to capture a change in government spending multiplier To address the concern that my approach is not able to capture the change in the government spending multiplier, I perform two exercise. First, I compare the government spending multiplier during the pre-Volcker period (1974Q1-1979Q2) with that of the Volcker period (1979Q3-1987Q2). There exists independent evidence independent evidence that the monetary policy regime during the pre-Volcker period is significantly different from that implemented in the Volcker periods which would predict a larger multiplier during the pre-Volcker period.[22] Figure 1.12 presents the difference in the response of output between the pre-Volcker period and the Volcker period. I find a larger multiplier in the pre-Volcker period. Second, I compare the multiplier during the pre-ZLB period with that of the ZLB

[22]See, e.g., Clarida et al. (2000a).

period in Japan using the TVC-SVAR approach. Miyamoto et al. (2018) find that the multiplier at the ZLB in Japan is larger, consistent with the predictions of the standard New Keynesian model at the ZLB using local projection method. I repeat the exercise with the same data using the TVC-SVAR approach. The VAR model is [G, T, Y]. Figure 1.13 displays the difference in the response of output to a government spending shock between the ZLB and the pre-ZLB period. Similar to their findings, I find the government spending multiplier during the ZLB period (1995Q4-2014Q1) is larger than during the pre-ZLB period (1980Q1-1995Q3) in Japan. These results are reassuring with respect to the modeling choice of the time-varying parameter VAR, as the model is able to detect changes in the size of the multiplier.

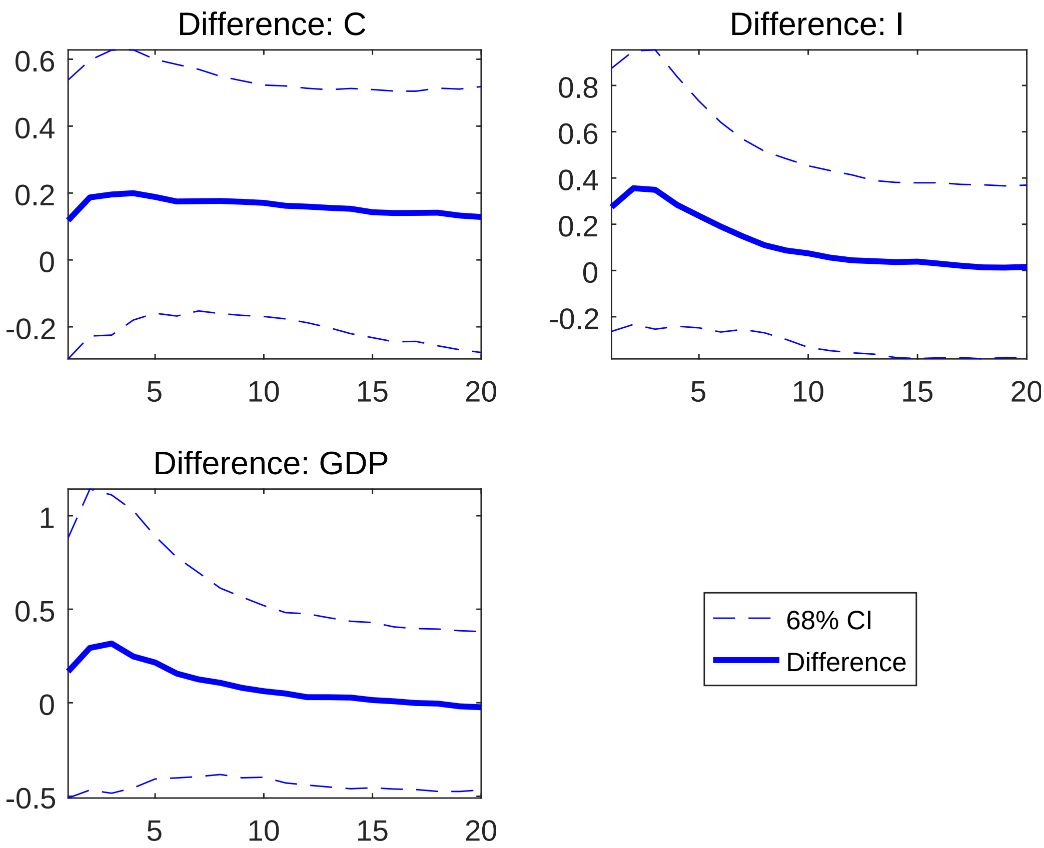
Lags To address the concern that the lags of the VAR model can affect my results, I conduct the exercise by increasing the lags of the VAR model to 4. Figure 1.15 presents the difference in the response of output. The main result is robust to the lags of the VAR model.

Identification I employ an alternative identification of government spending shocks. I include forecast error of real government purchases growth rate, real government purchases, real government current tax receipts net of transfers and real gross domestic product denoted by $[FeG, G, T, GDP]$. The variables $[G, T, Y]$ are normalized by the real potential GDP as explained in the paper. Following Ramey (2011), I order forecast error of government spending growth rate first and identify the government shocks by recursive identification scheme. I normalize the size of the government spending shock such that it increases the real government spending by one unit. Figure 1.14 presents the difference in response of output between the ZLB period and the pre-ZLB period. The difference is insignificant.

Prior of the hyperparameters To show my results is not sensitive to the priors for the hyperparameters, I estimate the model with another group of priors where $k_Q = 0.01, k_S = 0.1, K_W = 0.01$. Figure 1.16 shows that there is no significant change in the response of the output to the government spending shock.

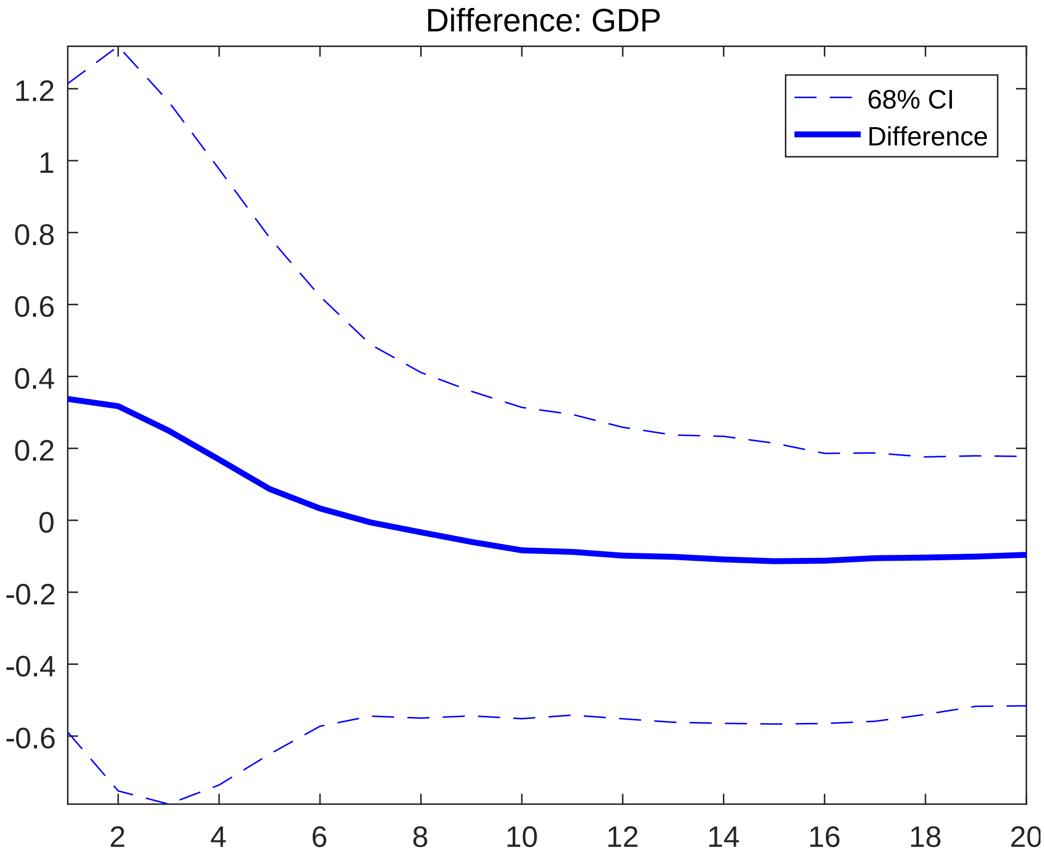
Linear VAR models with separate samples Figure 1.17 displays the impulse reponse of the real GDP to a government spending shock in a fixed-coefficient VAR model estimated over the two periods 1985Q1-2008Q4 and 2009Q1 -2015Q4. I find no significant change in the response of the output to the government spending shock.

Figure 1.8: Impulse Response to the Government Spending Shock: Pre-ZLB vs. ZLB Periods



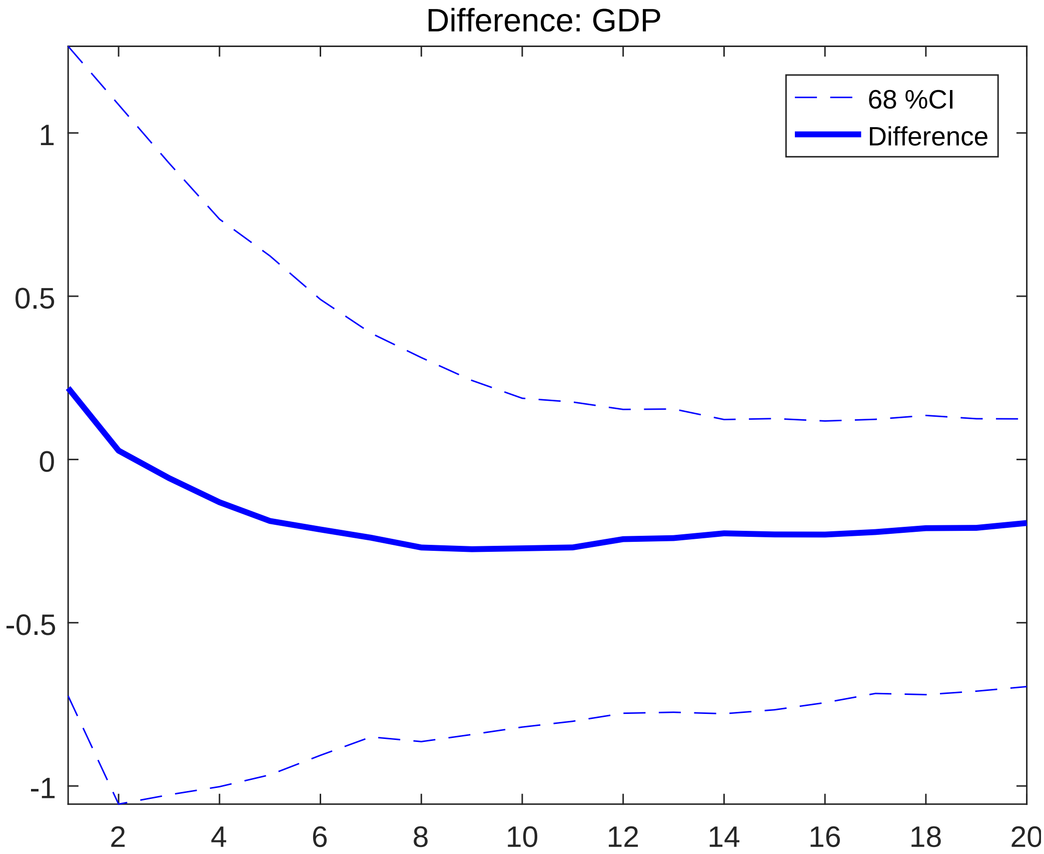
Notes: The figure presents the difference in the impulse responses of the consumption, investment and real GDP to the government spending shock between the ZLB period (2009Q1-2015Q4) and the pre-ZLB period (2002Q1-2008Q4). The blue solid line is the median estimate of the difference and the blue dashed line is the 68% confidence.

Figure 1.9: Impulse Response to the Government Spending Shock: Pre-ZLB vs. ZLB Periods



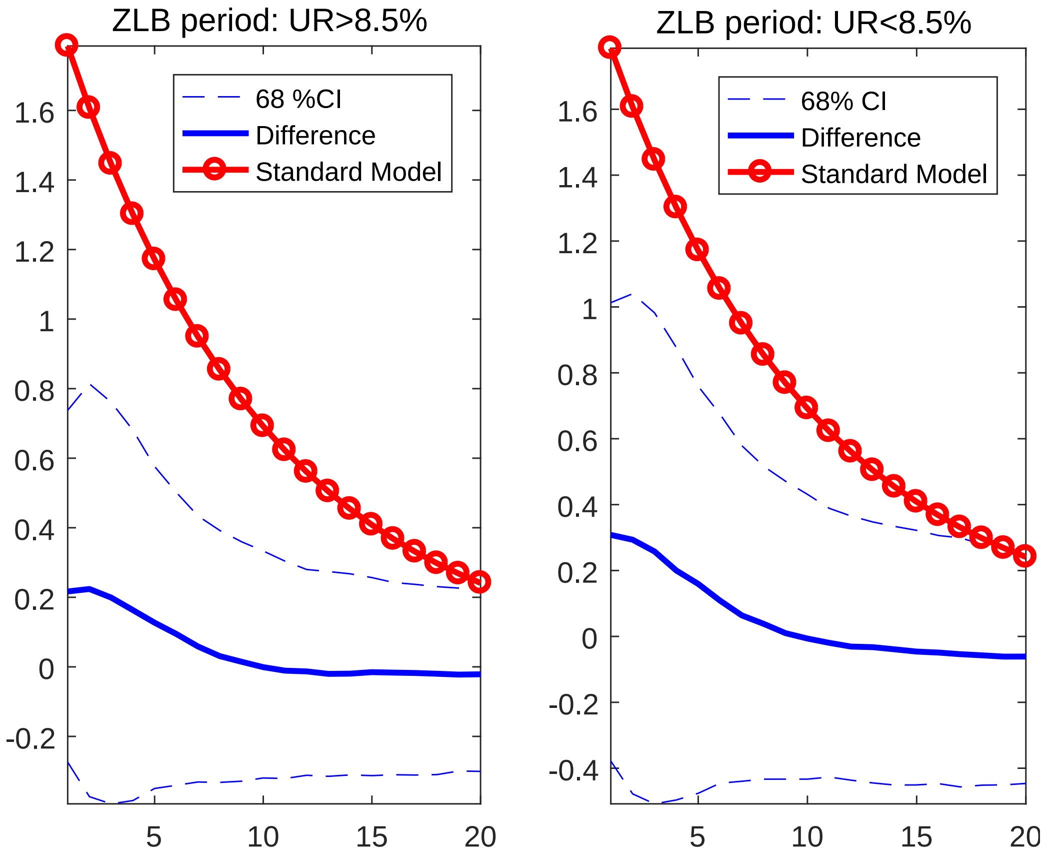
Notes: The figure presents the difference in the impulse response of the real GDP to the government spending shock between the ZLB period (2009Q1-2015Q4) and the pre-ZLB period (2002Q1-2008Q4). The blue solid line is the median estimate of the difference and the blue dashed line is the 68% confidence band.

Figure 1.10: Impulse Response to the Government Spending Shock: Pre-ZLB vs. ZLB Periods



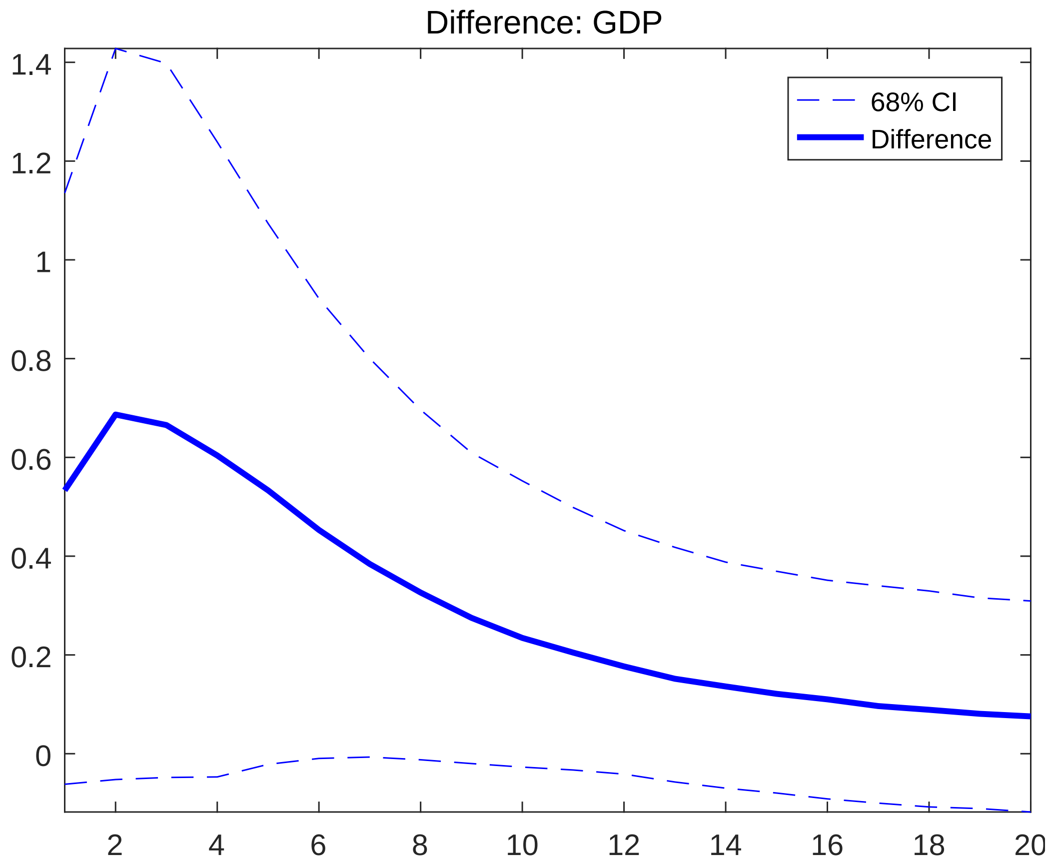
Notes: The figure presents the difference in the impulse response of the real GDP to the government spending shock between the ZLB period (2009Q1-2015Q4) and pre-ZLB period (2002Q1-2008Q4). The blue solid line is the median estimate of the difference and the blue dashed line is the 68% confidence band.

Figure 1.11: Impulse Response to the Government Spending Shock: Pre-ZLB vs. ZLB Periods



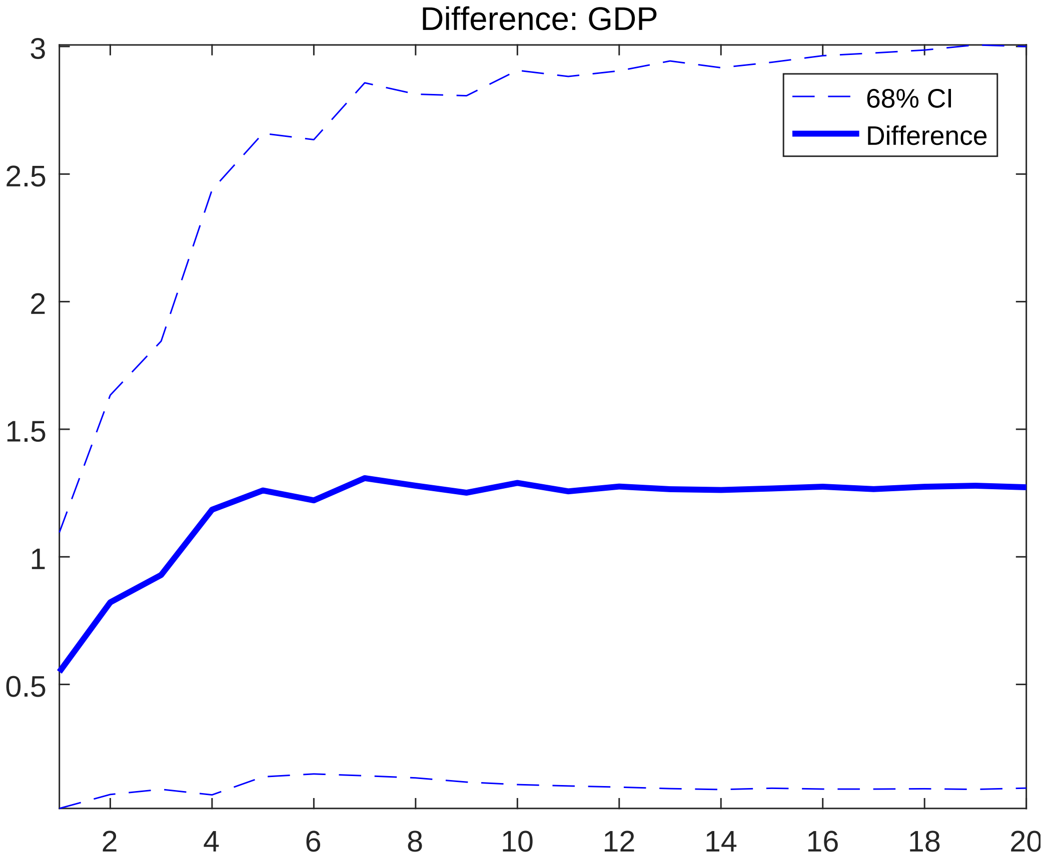
Notes: The figure presents the difference in the impulse response of the real GDP to the government spending shock between the ZLB period where either the unemployment rate is high or low and the pre-ZLB period. The high unemployment rate period at the ZLB is defined as 2009Q2 to 2011Q4 when the unemployment rate is above 8.5%. The low unemployment rate period at the ZLB is defined as a union of period 2009Q1 and period 2012Q1-2015Q4 when unemployment is below 8.5%. The blue solid line is the median estimate of the difference and the blue dashed line is the 68% confidence band. The red circle line is the theoretical prediction of the difference in the output response in the New Keynesian model at the ZLB.

Figure 1.12: Impulse Response to the Government Spending Shock: Pre-Volcker vs. Volcker Periods



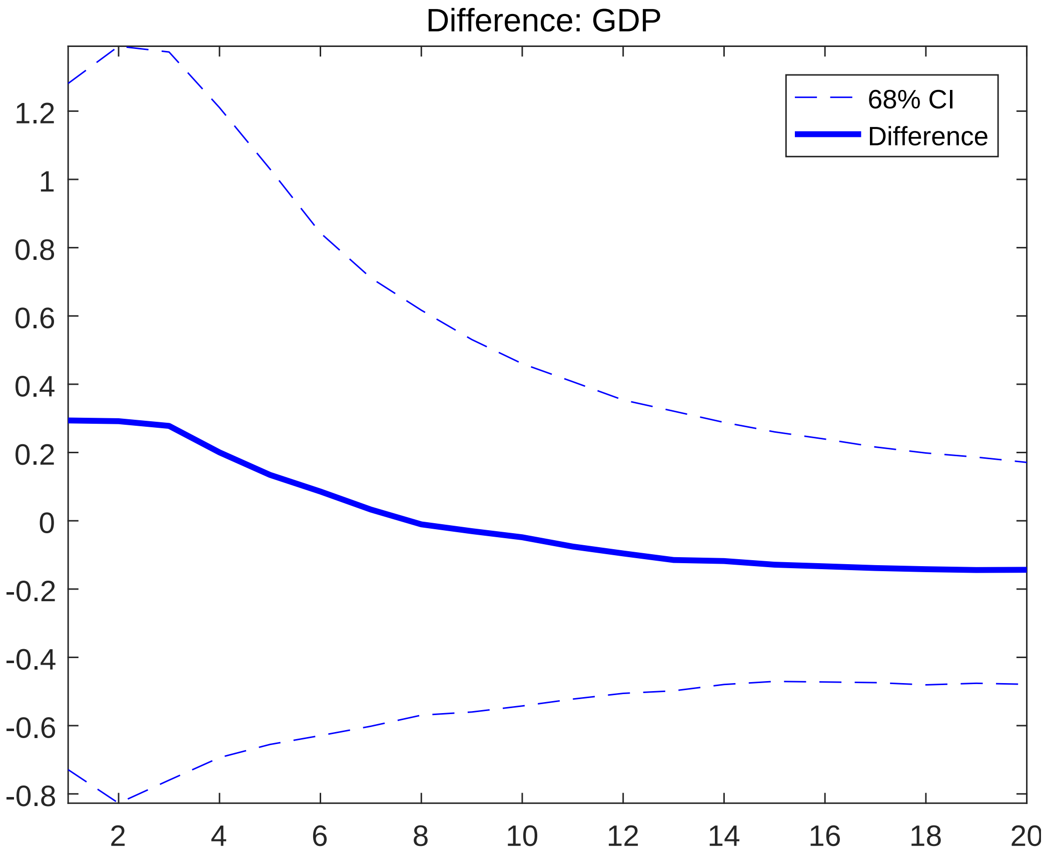
Notes: The figure presents the difference in the impulse response of the real GDP to the government spending shock between the pre-Volcker period and the Volcker period. The pre-Volcker period is defined as 1974Q1 to 1979Q2. The Volcker period is defined as 1979Q4 to 1987Q2. The blue solid line is the median estimate of the difference and blue dashed line is the 68% confidence band.

Figure 1.13: Impulse Response to the Government Spending Shock: Pre-ZLB vs. ZLB Period in Japan



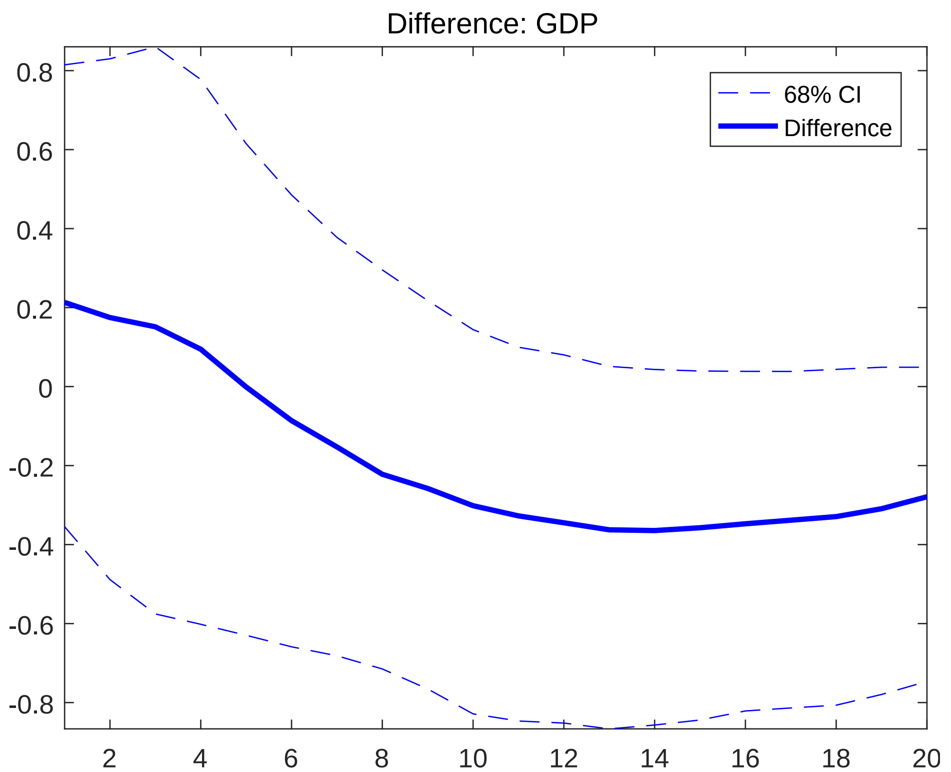
Notes: The figure presents the difference in the impulse response of the real GDP to the government spending shock between the pre-ZLB period and ZLB period in Japan. The pre-ZLB period is defined as 1980Q1 to 1995Q3. The ZLB period is defined as 1995Q4 to 2014Q1. The blue solid line is the median estimate of the difference and blue dashed line is the 68% confidence band.

Figure 1.14: Impulse Response to the Government Spending Shock: Pre-ZLB vs. ZLB Periods



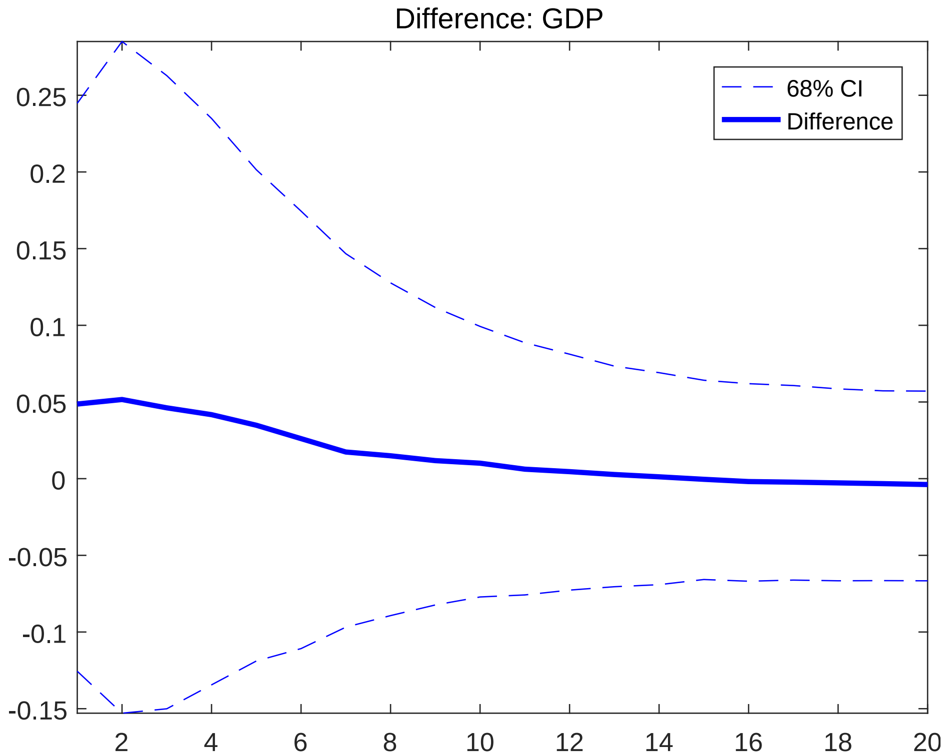
Notes: The figure presents the difference in the impulse response of the real GDP to the government spending shock between the pre-ZLB period and the ZLB period. The pre-ZLB period is defined as 2002Q1 to 2008Q4 and the ZLB period is defined as 2009Q1 to 2015Q4. The blue solid line is the median estimate of the difference and blue dashed line is the 68% confidence band.

Figure 1.15: Impulse Response to the Government Spending Shock



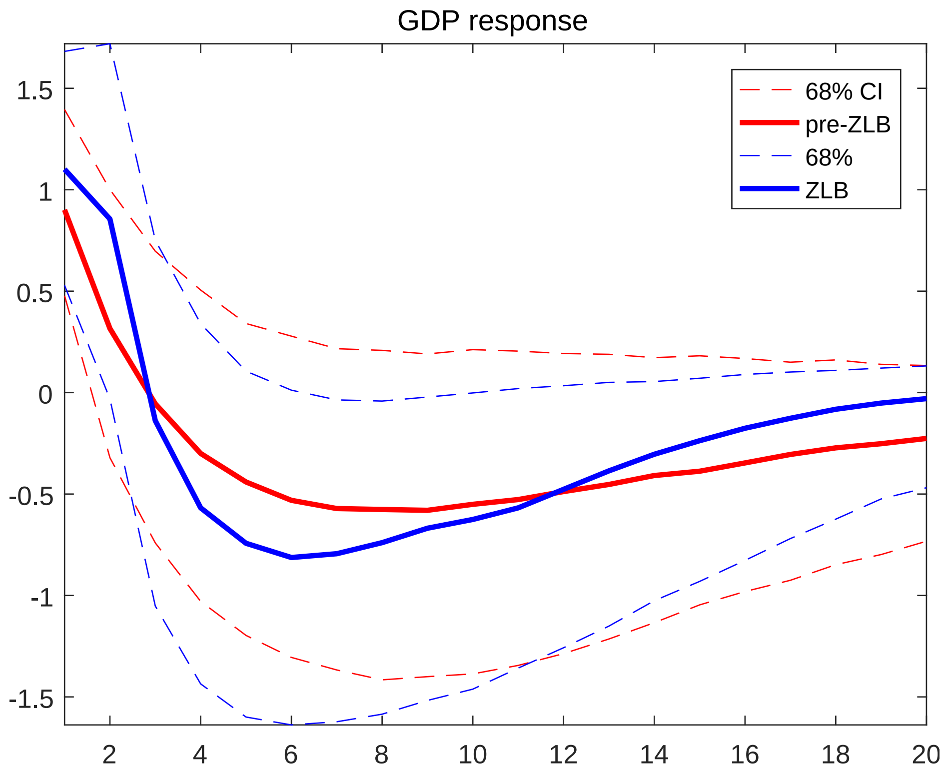
Notes: The figure presents the difference in the impulse response of the real GDP to the government spending shock between the pre-ZLB period and the ZLB period. The pre-ZLB period is defined as 2002Q1 to 2008Q4 and the ZLB period is defined as 2009Q1 to 2015Q4. The blue solid line is the median estimate of the difference and the blue dashed line is the 68% confidence.

Figure 1.16: Impulse Response to the Government Spending Shock



Notes: The figure presents the difference in the output response to the government spending shock. The pre-ZLB period is defined as 2002Q1 to 2008Q4 and the ZLB period is defined as 2009Q1 to 2015Q4. The blue solid line is the median estimate of the difference and blue dashed line is the 68% confidence band.

Figure 1.17: Impulse Response to the Government Spending Shocks



Notes: The figure presents the impulse response of the real GDP to the government spending shocks during the pre-ZLB (red line) and the ZLB period (blue line) respectively. The pre-ZLB period is defined as 1985Q1 to 2008Q4 and the ZLB period is defined as 2009Q1 to 2015Q4. The dashed line is the 68% confidence band.

1.7.3 DSGE Model

Estimation of DSGE model: MCMC algorithm

Estimation I simply present the steps of MCMC algorithm to draw the posterior distribution of the parameters. I initialize the algorithm at the posterior mode.

1. Draw $\theta^{'}$ from the distribution: $N(\theta, c\hat{\Sigma})$.

2. Draw α from uniform distribution, if $\alpha < min\left\{P(\theta^{'})/P(\theta), 1\right\}$, accept $\theta^{'}$. Otherwise, $\theta^{'} = \theta$.

3. Iterate the above procedure.

$\hat{\Sigma}$ is the inverse of the Hessian computed at the posterior mode. c is the parameter to control the acceptance rate. $P(\theta)$ is the posterior evaluated at θ.

Convergence Checks The convergence check is based on the Gelman-Rubin Potential Scale Reduction Factor. Table 1.2 reports the Gelman-Rubin Potential Scale Reduction Factor (PSRF) for five chains of 200000 draws each. I discard the first 50000 draws and keep the remaining 150000 draws. Values below 1.2 are regarded as indicative of convergence.

Parameters	PSRF	Parameters	PSRF	Parameters	PSRF
$\rho_r(\zeta = 1)$	1.0007	$\sigma_z(\zeta = 1)$	1.0009	ζ_p	1.0003
$\phi_\pi(\zeta = 1)$	1.0007	$\sigma_r(\zeta = 1)$	1.0009	ν	1.0005
$\phi_y(\zeta = 1)$	1.0008	$\sigma_{\zeta_d}(\zeta = 1)$	1.004	400γ	1.0002
$\rho_r(\zeta = 2)$	1.0009	$\sigma_g(\zeta = 1)$	1.001	400π	1.0012
$\phi_\pi(\zeta = 2)$	1.0001	$\sigma_z(\zeta = 2)$	1.0002	$400GS_{10}$	1.0009
$\phi_y(\zeta = 2)$	1.0004	$\sigma_r(\zeta = 2)$	1.0001		
ρ_z	1.0014	$\sigma_{\zeta_d}(\zeta = 2)$	1.0022		
ρ_{ζ_d}	1.0012	$\sigma_g(\zeta = 2)$	1.0002		
ρ_g	1.002	σ_{obs}	1.0003		

Table 1.2: The table reports the Gelman-Rubin Potential Scale Reduction Factor (PSRF) for five chains of 200000 draws each. I discard the first 50000 draws and keep the remaining 150000 draws. Values below 1.2 are regarded as indicative of convergence.

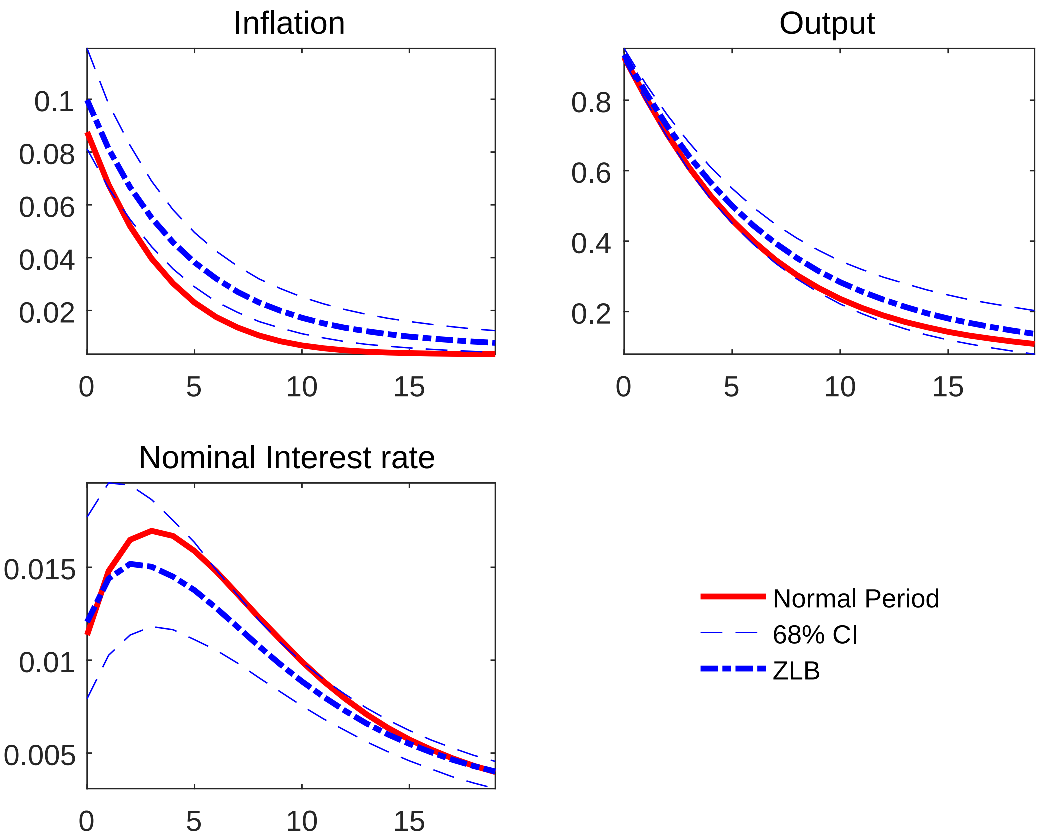
1.7.4 Robustness Checks

Table 1.3 and 1.4 presents the estimated results of the DSGE model using Wu-Xia shadow rate and Commercial Bank Interest Rate on Credit Card Plans summarizing both conventional and unconventional monetary policy respectively. Figure 1.18 and 1.20 present the impulse responses of output, inflation and the nominal interest rate to a government spending shock. Figure 1.19 and 1.21 displays the difference in the output response to the government spending shock. As explained in the paper, there is no significant change in the Wu-Xia shadow rate rule and Commercial Bank Interest Rate on Credit Card Plans rule between normal times and ZLB times. As a result, there is no significant difference in the size of government spending multiplier between the pre-ZLB period and the ZLB period.

	Posterior				Prior		
Parameter	Mode	Mean	$\%5$	$\%95$	Distr.	Mean	St. Dev.
$\rho_r(\zeta = 1)$	0.8551	0.8548	0.8146	0.8896	B	0.5	0.2
$\phi_\pi(\zeta = 1)$	2.3102	2.3237	1.9468	2.7101	N	1.5	0.3
$\phi_y(\zeta = 1)$	0.1506	0.1598	0.0317	0.3046	N	0.25	0.1
$\rho_r(\zeta = 2)$	0.8326	0.7998	0.6668	0.8991	N	0.5	0.2
$\phi_\pi(\zeta = 2)$	1.2657	1.3922	0.7811	2.0254	N	0.5	0.5
$\phi_y(\zeta = 2)$	0.1646	0.1713	0.0347	0.3240	N	0.15	0.1
ρ_z	0.1027	0.1689	0.0379	0.3623	B	0.5	0.2
ρ_{ζ_d}	0.8448	0.8436	0.7477	0.9247	B	0.5	0.2
ρ_g	0.9622	0.9572	0.9238	0.9842	B	0.5	0.2
$\sigma_z(\zeta = 1)$	0.0118	0.0116	0.0093	0.0140	IG1	0.1	2
$\sigma_r(\zeta = 1)$	0.0019	0.0020	0.0017	0.0023	IG1	0.1	2
$\sigma_{\zeta_d}(\zeta = 1)$	0.0301	0.0321	0.0255	0.0399	IG1	0.1	2
$\sigma_g(\zeta = 1)$	0.0168	0.0173	0.0137	0.0211	IG1	0.1	2
$\sigma_z(\zeta = 2)$	0.0113	0.0118	0.0084	0.0160	IG1	0.1	2
$\sigma_r(\zeta = 2)$	0.0040	0.0045	0.0032	0.0062	IG1	0.1	2
$\sigma_{\zeta_d}(\zeta = 2)$	0.0233	0.0268	0.0170	0.0393	IG1	0.1	2
$\sigma_g(\zeta = 2)$	0.0209	0.0222	0.0164	0.0295	IG1	0.1	2
$\sigma_o bs4$	0.0021	0.0022	0.0017	0.0027	IG1	0.1	2
ζ_p	0.7465	0.7427	0.6868	0.7883	B	0.5	0.05
ν	1.5542	1.5585	1.1825	1.9902	G	2	0.3
400γ	1.8284	1.8766	1.3464	2.4548	G	2	0.5
400π	1.8108	1.8499	1.4318	2.2878	G	2	0.5
$400GS_{10}$	2.8745	2.9159	2.2584	3.6085	G	3	0.5

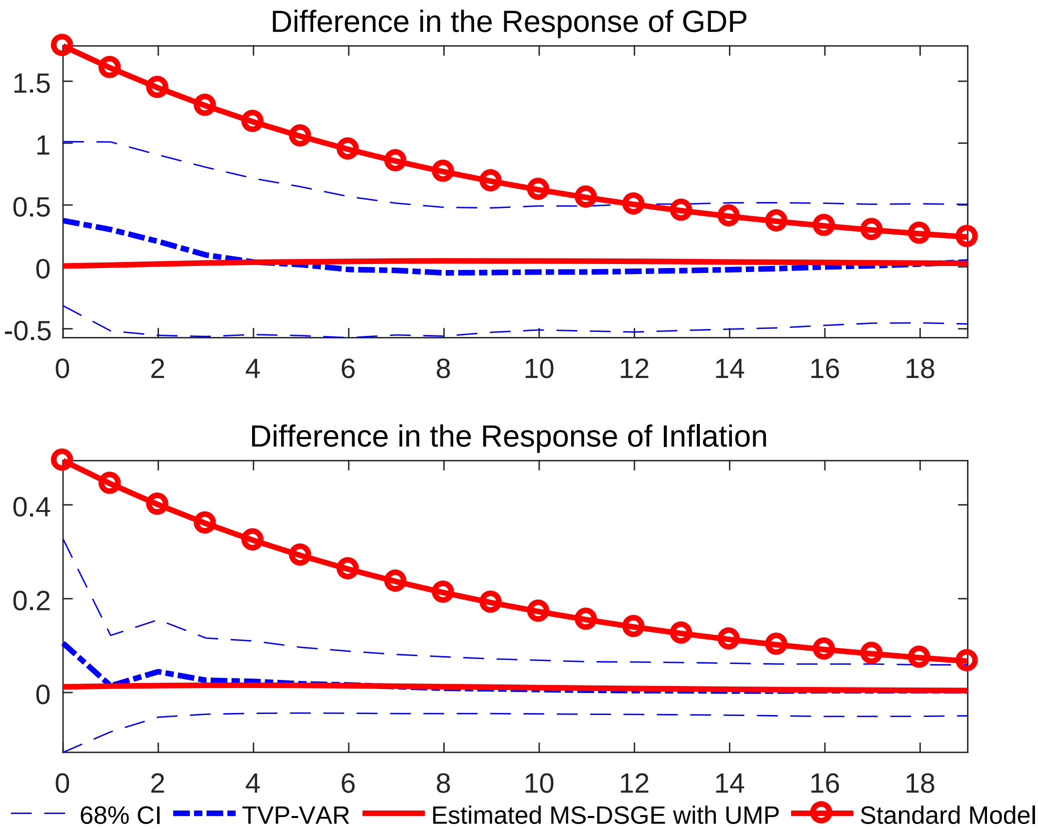
Table 1.3: Modes, Mean, 90% error bands, and prior distributions of the parameters of the Markov-switching DSGE model. $\zeta = 1$ is the normal periods. $\zeta = 2$ is the ZLB periods

Figure 1.18: Impulse Response to the Government Spending Shock in the Estimated Model



Note: The figure presents the impulse response of the output, inflation and nominal interest rate to a government spending shock in the estimated model. The response of inflation and nominal interest rate is expressed as a percentage. The response of output is denoted in dollars, corresponding to one dollar-change government spending. The blue dash-dot line is the median impulse response and the blue dashed line is the 68% confidence band for the ZLB periods 2009Q1-2015Q4. The red solid line is the median impulse response for the normal periods 1985Q1-2008Q4.

Figure 1.19: Compare VAR results with that of the Estimated MS-DSGE Model

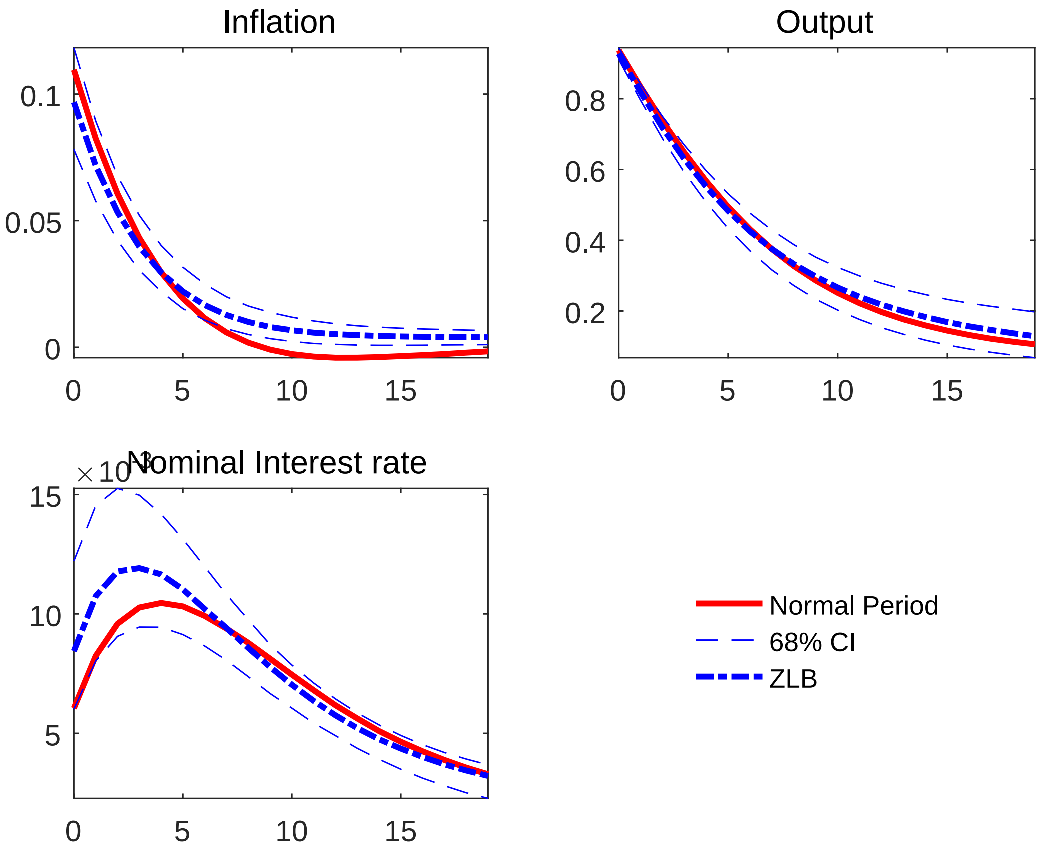


Note: The figure presents the difference in output and inflation response to a government spending shock in the TVP-VAR model, the calibrated model and the estimated MS-DSGE model with unconventional monetary policy. The blue dash-dot line is the median estimate of the response and blue dashed line is the 68% confidence band. The red solid line is the median difference of the impulse response in the estimated MS-DSGE model with unconventional monetary policy. The red circle line is the difference in the impulse response from the calibrated theoretical model at the ZLB.

	Posterior				Prior		
Parameter	Mode	Mean	$\%5$	$\%95$	Distr.	Mean	St. Dev.
$\rho_r(\zeta=1)$	0.9432	0.9389	0.9183	0.9557	B	0.5	0.2
$\phi_\pi(\zeta=1)$	2.2328	2.2997	1.9043	2.7257	N	1.5	0.3
$\phi_y(\zeta=1)$	0.2064	0.2101	0.0615	0.3660	N	0.25	0.1
$\rho_r(\zeta=2)$	0.8870	0.8680	0.7981	0.9198	N	0.5	0.2
$\phi_\pi(\zeta=2)$	1.5171	1.6850	1.0310	2.4302	N	0.5	0.5
$\phi_y(\zeta=2)$	0.1347	0.1544	0.0247	0.3054	N	0.15	0.1
ρ_z	0.1291	0.1954	0.0506	0.4082	B	0.5	0.2
ρ_{ζ_d}	0.9816	0.9800	0.9677	0.9891	B	0.5	0.2
ρ_g	0.9650	0.9583	0.9229	0.9863	B	0.5	0.2
$\sigma_z(\zeta=1)$	0.0132	0.0133	0.0104	0.0167	IG1	0.1	2
$\sigma_r(\zeta=1)$	0.0024	0.0026	0.0020	0.0032	IG1	0.1	2
$\sigma_{\zeta_d}(\zeta=1)$	0.0429	0.0482	0.0321	0.0729	IG1	0.1	2
$\sigma_g(\zeta=1)$	0.0190	0.0194	0.0148	0.0244	IG1	0.1	2
$\sigma_z(\zeta=2)$	0.0102	0.0108	0.0075	0.0148	IG1	0.1	2
$\sigma_r(\zeta=2)$	0.0046	0.0052	0.0037	0.0071	IG1	0.1	2
$\sigma_{\zeta_d}(\zeta=2)$	0.0513	0.0597	0.0376	0.0916	IG1	0.1	2
$\sigma_g(\zeta=2)$	0.0221	0.0232	0.0171	0.0305	IG1	0.1	2
$\sigma_o bs4$	0.0031	0.0033	0.0025	0.0044	IG1	0.1	2
ζ_p	0.7206	0.7161	0.6532	0.7713	B	0.5	0.05
ν	1.4954	1.5107	1.1460	1.9136	G	2	0.3
400γ	1.8175	1.9497	1.3218	2.6543	G	2	0.5
400π	1.2329	1.2570	0.7829	1.8425	G	2	0.5
$400GS_{10}$	3.7456	3.9574	2.9312	5.1680	G	3	0.5

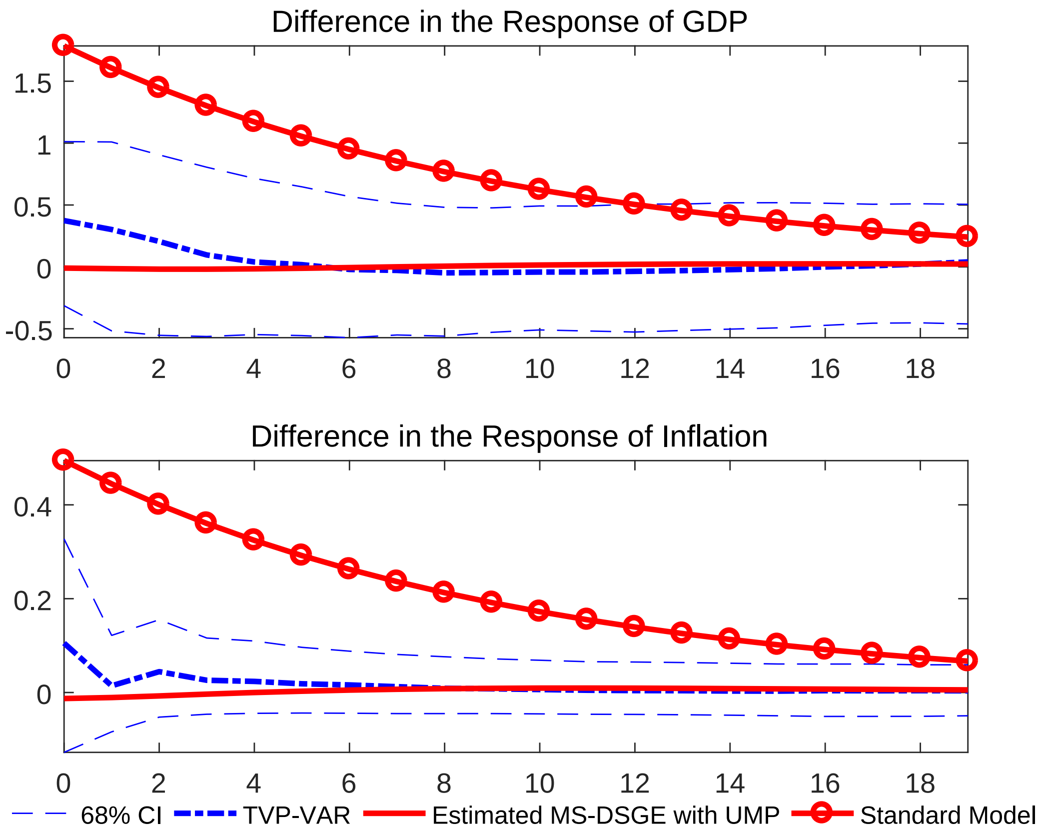
Table 1.4: Modes, Mean, 90% error bands, and prior distributions of the parameters of the Markov-switching DSGE model. $\zeta = 1$ is the normal periods. $\zeta = 2$ is the ZLB periods

Figure 1.20: Impulse Response to the Government Spending Shock in the Estimated Model



Note: The figure presents the impulse response of the output, inflation and nominal interest rate to a government spending shock in the estimated model. The response of inflation and nominal interest rate is expressed as a percentage. The response of output is denoted in dollars, corresponding to one dollar-change government spending. The blue dash-dot line is the median impulse response and the blue dashed line is the 68% confidence band for the ZLB periods 2009Q1-2015Q4. The red solid line is the median impulse response for the normal periods 1995Q2-2008Q4.

Figure 1.21: Compare VAR results with that of the Estimated MS-DSGE Model



Note: The figure presents the difference in output and inflation response to a government spending shock in the TVP-VAR model, the calibrated model and the estimated MS-DSGE model with unconventional monetary policy. The blue dash-dot line is the median estimate of the response and blue dashed line is the 68% confidence band. The red solid line is the median difference of the impulse response in the estimated MS-DSGE model with unconventional monetary policy. The red circle line is the difference in the impulse response from the calibrated theoretical model at the ZLB.

1.7.5 Model details

Firms. I next turn to the firms' side. The final good is produced by a representative firm combining intermediate goods $Y_{i,t}, i \in [0,1]$ with CES technology:

$$Y_t = \left(\int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

Assuming the perfect competition of final goods markets, the profit maximization leads to demand function for intermediate goods i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \tag{1.24}$$

for all $i \in [0,1]$. The aggregate price index of one unit of final goods is obtained by integrating over the intermediate goods price $p_t(i)$:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \tag{1.25}$$

The intermediate good i is produced by the monopolistic firm with production technology:

$$Y_t(i) = Z_t L_t(i)^{1-\alpha} \tag{1.26}$$

where Z_t is the technology level of intermediate goods producing firm. The intermediate goods market is monopolistic competition. Following Calvo (1983a), in each period, the individual monopolist are allowed to reoptimize the price wtih probability ζ_p. The ζ_p fraction of price-setting firms set the price to maxizmize the expected present discounted value of future profits:

$$E_t \sum_{s=0}^{\infty} (\beta \zeta_p)^s \frac{\lambda_{t+s}}{\lambda_t} \frac{P_t(i)\pi^s}{P_{t+s}} \left(Y_{t+s}(i) - W_{t+s} N_{t+s}(i) \right) \tag{1.27}$$

Subject to the demand function 3.15 and production function 3.16. λ_t is the household marignal utility of nominal income. The remaining fraction ,$1 - \zeta$ firms, would set the price with full indexation $p_t(i) = \pi p_{t-1}(i)$ where π is steady state level of inflation rate that central bank targets.

Chapter 2

AN EMPIRICAL ASSESSMENT OF THE FED'S TARGETING RULE AND LOSS FUNCTION

2.1 Introduction

Central banks play a crucial role in modern economies. They counter-react to economic fluctuations to ensure stability. For instance, the Federal Reserve (the Fed thereafter) of the United States claims a dual mandate, ”two goals of price stability and maximum sustainable employment”, as its official policy objective. While the announced objective is clear qualitatively, it is ambiguous quantitatively. Facing a tradeoff between price stability and employment (or output gap), what is the Fed's ”desired allocation”? To be more concrete, what is the reduction in output gap (in growth) that the Fed is willing to take for inflation that is one percentage point higher than the target? The answer to this question defines the central bank's targeting rule, see, e.g., Svensson (1999) and Svensson and Woodford (2004).

The goal of this paper is to estimate central banks' targeting rules empirically. Understanding a central bank's targeting rule (desired allocation) is extremely important for consumers', professionals' and firms' optimization problems. Unfortunately, the Fed has been opaque regarding this. And there is a lack of empirical studies on this matter in the academic literature. This paper fills this gap.

The estimation of a central bank's targeting rule is a non-trivial exercise as it is subject to a simultaneity problem. Theoretically, as we will show later through the lens of a standard New Keynesian model (NK model thereafter),[1] a central bank's desired allocation between inflation and output gap (in growth rate) is negatively related to the New Keynesian Phillips Curve (NKPC thereafter). To find the central bank's targeting rule, one might project inflation on the output gap. This is problematic because the equilibrium outcomes of the economy (i.e. inflation and the output gap) are jointly determined by a negatively sloped targeting rule and a positively slopped New Keynesian Philips Curve. Thus, OLS estimates are subject to simultaneity bias. See McLeay and Tenreyro (2018) for a detailed discussion of this simultaneity issue.

To overcome this simultaneity problem, we construct an environment in which the central bank's targeting curve is fixed over time. In this environment, the realized equilibrium outcomes are driven by shocks that shift the NKPC. In other words, output gap growth and inflation are distributed along with the central bank's targeting rule. This makes the estimation of the latter possible. We construct this desired environment by first identifying cost-push shocks and second construct "cleaned" output gap and inflation measures by purging out all the other shocks. The identification assumption is that cost-push shocks, for an example price markup shocks, only shifts the NKPC and do not move the targeting rule. The identification of cost-push shocks is achieved by relying on sign restrictions implied by the NK model.

We apply this methodology to the data in the U.S. The following results standout. First, in the presence of cost-push shocks, the Fed is willing to drop the output gap growth by roughly two and a half percentage points for each percentage point increase in (quarterly) inflation. Qualitatively, this suggests that indeed the Fed has a dual mandate. Quantitatively, this magnitude is higher than the implication of the welfare loss function derived in a standard NK model. Second, the data is in favor of optimal policy under commitment against the discretionary case.

Third, the Fed's targeting rule remains stable around the pre and post-Volcker' periods. It is well known that the change in the Fed's conduct of monetary policy, i.e. Taylor rule, since Volcker's appointment is a plausible source of the

[1]The same result is derived in Galí (2015).

Great Moderation since the 1980s and the Fed's insufficient reaction to inflation is blamed for the great inflation in the 1970s, see, e.g., Taylor (1999), Clarida et al. (2000b), Lubik and Schorfheide (2004), Boivin and Giannoni (2006) and Coibion and Gorodnichenko (2011). On the other hand, Orphanides (2001, 2002, 2004) challenges this view of the break in the Taylor rule. He argues that there is no change in the conduct of the Fed's policy if real-time data is used. Note that one should not view our results as evidence against the break in Taylor rule. As we will discuss later, for each targeting rule there are infinite Taylor rules that are consistent with the equilibrium outcomes. It is entirely plausible that the implementation rule (Taylor rule) has changed while the targeting rule remains unchanged.

Lastly, we provide an estimate for the deep parameter that characterizes a central bank's preference in a standard NK model. That is the relative weight that the central bank attaches to the volatilities of the output gap and inflation in its welfare loss function.[2] Our empirical results suggest that the Fed attaches a weight of 0.18 on the output gap volatility relative to the (annualized) inflation volatility.[3] This again confirms our previous finding that indeed the Fed has a dual mandate. And this estimate is much higher than the reference value 0.048 calibrated in Woodford (2011a), and smaller than the optimal value 1.042 calculated by Debortoli et al. (2017) based on a medium scale Dynamic Stochastic General Equilibrium (DSGE thereafter) model. Theoretically, the central bank's loss function is derived as the second order approximation of households' utility function. In other words, central banks' objective is assumed to be aligned with the consumer's welfare. This is not necessarily the case in the data. The discussion about the optimality of a central bank's welfare loss function is beyond the scope of this paper. In fact, the literature has not agreed on the correct calibration. Our estimate provides guidance for the calibrations of central banks' welfare loss functions.

Theoretical discussions of the targeting rule are vast, see, e.g., Rudebusch and Svensson (1999), Svensson (1999) and Svensson and Woodford (2004) for the discussions of the targeting rule against instrument rules. To the best of our knowledge, this is the first paper that provides an empirical estimate for a central

[2] More specifically, we characterize the λ in a central bank's period welfare loss function $L_t = \pi_t^2 + \lambda x_t^2$, where π_t and x_t are inflation and output gap respectively.

[3] The weight (0.18) has been adjusted for the annualized inflation in the welfare loss function to be comparable with the literature.

bank's targeting rule.

This paper contributes to the literature that estimates the welfare loss function of a central bank, see e.g., Debortoli et al. (2017), Dennis (2006), Favero and Rovelli (2003), Ilbas (2012), Lakdawala (2016) and Ozlale (2003).[4] Our paper extends this literature in the following dimensions. First, instead of estimating the deep parameters in the welfare loss function, we provide an estimate for the central bank's targeting rule. Moreover, the estimation of the deep parameter is also possible using our methodology. Second, due to the nature of the central bank's preference defined in the literature, the estimation is mostly based on the DSGE models. In this paper, the estimation is conducted using a Structural Vector Autoregressive (SVAR thereafter) model that relies on less structural assumptions. Exceptionally, Cecchetti et al. (2002) and Cecchetti and Ehrmann (1999) estimate a central bank's preference in a VAR model. However, they do not address the simultaneity issue. The reliability of their estimates relies crucially on the assumption that policymakers *always* act optimally. In other words, it is *assumed* that the targeting curve is fixed over time and the simultaneity issue is absent by assumption. Surico (2007) estimates the Fed's preference using GMM, however as it is documented by Mavroeidis et al. (2014) and we will show later, that those instruments are weak.

This paper is closely related to the literature that estimates central banks' instrument rules, see, e.g., seminal works by Taylor (1993) for the interest rate rule and by McCallum (1988) for the money supply rule. The estimated instrument rules such as the Taylor rule is merely an approximation of a central bank's reaction function, whereas the targeting rule describes a central bank's preference. As it is shown by Svensson (1997), a targeting rule can be re-written in the form of a Taylor rule. However, the estimated Taylor coefficients are not sufficient to derive the underlining preference of a central bank as it is discussed by Svensson (1997) and Favero and Rovelli (2003). Our estimated targeting rule complements existing instrument rules as it provides a direct measure of a central bank's "desired allocation" and it is more informative about the central bank's preference.

[4]A related term is the sacrifice ratio, which describes the output cost of disinflation policy, see, e.g., Gordon et al. (1982), Ball (1994), King and Watson (1994), Cecchetti and Rich (2001), Benati (2015) and most recently Barnichon and Mesters (2018a,b). The sacrifice ratio is related to the NKPC, whereas the targeting rule is a description of a central bank's preference.

The methodology used in this paper is similar to that of Cecchetti and Rich (2001), Barnichon and Mesters (2018a,b) and Galí and Gambetti (2018). Those papers purge out shocks that shift the NKPC to estimate the empirical NKPC or the sacrifice ratio. In contrast, our paper identifies and relies on NKPC shifters to identify central banks' targeting rule.

The remaining of the paper is organized as follows. Section (2.2) explains the definition of a targeting rule, the simultaneity problem associated with its estimation and its relationship to the Taylor rule. Section (2.3) describes our estimation methodology. Section (2.4) provides empirical results. Section (2.5) estimates the central bank's preference. And Section (2.6) concludes.

2.2 The Targeting Rule

Section (2.2.1) briefly discusses the central bank's targeting rule and the simultaneity problem associated with the estimation through the lens of a New Keynesian model discussed in Galí (2015), Chapter 5. For the simultaneity problem, one can find similar and more formal discussions of the same issue in McLeay and Tenreyro (2018).

2.2.1 The Targeting Rule and the Simultaneity Problem

In a standard NK model, the period welfare loss function can be derived as the second order approximation of the representative household's utility function[5]:

$$L_t = \pi_t^2 + \lambda x_t^2,$$

where π_t denotes inflation, x_t is output gap. λ is the deep parameter that characterizes the central bank's preference. It describes the relative weight that the central bank attaches to the volatilities of the output gap and inflation in its loss function. Theoretically, the central bank's targeting rule — the growth rate of the output gap that a central bank is willing to take for inflation that is 1 percentage

[5]In the literature, it is often assumed exogenous as well. That is λ is calibrated separately rather than a function of parameters that describe the household's preference.

point higher than its target, is closely related to λ. To see this we solve for the central bank's optimization problem.

The optimization problem of the central bank is:

$$\min \; E_t \sum_{t=0}^{\infty} L_t = E_t \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2)$$

subject to the sequence of constraint, namely NKPC:

$$\pi_t = \beta E \pi_{t+1} + \kappa x_t + \epsilon_t^{\pi}, \forall t \geq 0 \tag{2.1}$$

The solution to this problem under commitment is:

$$\pi_t = -\frac{\lambda}{\kappa} \triangle x_t, \tag{2.2}$$

The equation (2.2) is labeled as the central bank's targeting rule. It describes the central bank's "desired allocation" in the presence of cost-push shocks. The parameter of interest is $-\frac{\lambda}{\kappa}$, which is interpreted as the following: the central bank is willing to lower the output gap growth by $\frac{\kappa}{\lambda}$ percentage points for each percentage point increase in inflation. The goal of the remaining paper is to provide an empirical estimate of $\frac{\lambda}{\kappa}$ as well as its decomposition λ and κ.

As discussed by McLeay and Tenreyro (2018), while (2.2) is the "desired allocation" of the central bank, it might deviate from this allocation because of 1) measurement errors that are exogenous 2) exogenous unknown reasons. In other words, the central bank's targeting rule is subject to monetary shocks. For illustration, let's consider the first case: in real time the central bank only observes a $\hat{x}_t$ that is a noisy measure of the unobservable x_t, i.e., $\hat{x}_t = x_t + e_t$. Therefore the targeting rule is:

$$\pi_t = -\frac{\lambda}{\kappa} \triangle x_t + \epsilon_t^m, \tag{2.3}$$

where $\epsilon_t^m \equiv -\frac{\lambda}{\kappa} \triangle e_t$ denotes the monetary shock. This relaxes the assumption that the central bank is aways capable of achieving the desired allocation. Notice that

ϵ_t^m is serially correlated. Even if e_t were i.i.d, ϵ_t^m would follow a $MA(1)$ process. In general, if the real time measurement error e_t follows a $AR(p)$ process, the monetary shock ϵ_t^m follows a $ARMA(p,1)$ process.

The equilibrium outcomes of the economy are jointly determined by the NKPC (3.20) and the central bank's targeting rule (2.3). As a solution, π_t and $\triangle x_t$ depend on both ϵ_t^π and ϵ_t^m. The econometrician who estimates (2.3) directly will obtain a biased estimate as the regressors and residuals are correlated.

Intuitively, this problem arises if the central bank's targeting curve shifts over time. Therefore, the solution to this problem is to shut down the central bank's targeting rule shifters and allowing only for NKPC shifters in the data. We achieve this goal by identifying cost-push shocks.[6]

In theory, alternatively one could estimate (2.3) using external instruments. Candidates for instruments include the lagged inflation and lagged output gap. This approach is subject to two drawbacks. First, it is well known, see e.g., Mavroeidis et al. (2014) and we will show later, that those instruments are weak. Second, the exclusion assumption is likely to be violated in the use of the first few lags as instruments. As it is discussed above, ϵ_t^m follows a moving average process with at least one lag and therefore $E(\epsilon_t^m x_{t-1}) \neq 0$ in other words x_{t-1}, π_{t-1} are not valid instruments. Similarly x_{t-2} or π_{t-2} cannot be used as instruments if the real-time measurement error is serially correlated. Overall, it is not clear which lagged variables satisfy the exclusion restriction. What is more, excluding the first few lags as instruments worsen the first problem (relevance). Zhang and Clovis (2010) address this issue formally for the estimation of the NKPC.

2.3 Estimation Methodology

Our estimation methodology can be summarized in the following steps. First, we identify cost-push shocks (shocks to the NKPC) by means of sign restrictions. Second, given the identified shocks we purge out all other shocks from the

[6]It is clear that monetary policy shocks are the central bank's targeting rule shifters and price markup shocks are NKPC shifter. It is less clear for example how demand and technology shocks shift those curves. In fact, in the basic model studied above, those shocks are perfectly stabilized by the central bank and therefore none of those two curves shift. To use as less structural assumptions as possible, we focus on cost-push shocks.

raw data. The resulting inflation and output are generated from shocks that only shift the Philips Curve. Therefore the generated data is free of the simultaneity problem. Lastly, we project the "cleaned" output gap growth on the "cleaned" inflation.

In the first step, we construct cost-push shocks by relying on sign restrictions.In our baseline, we consider a VAR with the following variables $Y_t = \begin{bmatrix} \pi_t & x_t & i_t \end{bmatrix}'$, which denote inflation, output gap and nominal interest rate respectively. The system is assumed to follow a VAR with p lags:

$$Y_t = A_1 Y_{t-1} + ... + A_p Y_{t-p} + U_t \tag{2.4}$$

where $U_t = \begin{bmatrix} u_t^\pi & u_t^x & u_t^i \end{bmatrix}'$ denotes the reduced form residues, which are linear combinations of structural shocks:

$$\begin{bmatrix} u_t^\pi \\ u_t^x \\ u_t^i \end{bmatrix} = B_0^{-1} \begin{bmatrix} \epsilon_t^\pi \\ \epsilon_t^x \\ \epsilon_t^i \end{bmatrix} \tag{2.5}$$

where $U_t = \begin{bmatrix} \epsilon_t^\pi & \epsilon_t^x & \epsilon_t^i \end{bmatrix}'$ denote the structural shocks. One could compute the structural shocks once B_0^{-1} is estimated. Under the normalization assumption that structural shocks have unit variances then $(B_0^{-1})' B_0^{-1} = \Sigma_u$, where Σ_u is the covariance matrix of the reduced form residues. It is trivial to get an unbiased estimate $\hat{\Sigma}_u$, however, the identification issue arises because there are more parameters to be estimated in B_0^{-1} than the number of knowns contained in $\hat{\Sigma}_u$.

Structural assumptions on B_0^{-1} are required to overcome this identification issue. The goal is to identify NKPC shifters, i.e. price markup shocks. To this end, we impose the following sufficient restrictions:

$$B_0^{-1} = \begin{bmatrix} + & * & * \\ - & * & * \\ * & * & * \end{bmatrix}, \tag{2.6}$$

The first shock is named as the cost-push shock, which is identified as a shock that pushes the output gap and inflation in the opposite direction. One example of such a shock is the markup shock. For robustness check, we identify markup shocks in a larger VAR and results are similar to those we obtain here in the baseline. Crucial to the estimation of the targeting rule is that those identified shocks do not contain targeting rule shifters, which we interpret as monetary policy shocks and generate positive comovement of output gap and inflation.

As we identify the shocks using sign restrictions, we obtain a set of the structural matrix B_0^{-1} and thus shock series consistent with the identification restrictions("set identified"). In the second step, for each time series of cost-push shocks j contained in the set identified above, we purge out the remaining shocks by constructing a new dataset j that is only driven by markup shocks. That is, all the remaining shocks are set to be zero in the whole sample.

In the third step, we estimate the following regression using the purged dataset:

$$\pi_t(j) = \alpha(j) + \beta(j)\triangle x_t(j) + e_t(j) \tag{2.7}$$

This is done for each dataset j constructed from each time series of cost-push shocks j that belong to the set identified in the first step. The confidence interval for $\hat{\beta}$ is constructed by taking into account that markup shocks are "set identified" and that parameter estimate is uncertain in the third step using the bootstrap technique.

2.4 Estimation Results

We proceed with a description of the data used in this paper. The NIPA variables are drawn from the FRED database from the period 1960Q1-2017Q4. We use variables as follows: (log) real GDP (y_t), (log) GDP deflator (p_t), (log) hours of all persons in the nonfarm business sector (n_t), 10-year constant maturity yield rate (i_t), (log) wage earnings of production and nonsupervisory workers (w_t). The labor productivity is defined as the difference between (log) real GDP and (log) hours, i.e. $\triangle(y_t - n_t)$. The price inflation and wage inflation is defined as the log first differences of the GDP deflator and the wage earnings of production and nonsupervisory workers. We use 10-year constant maturity yield rate to avoid the problem associated with the zero lower bound in the recent periods. The price markup is defined as the difference between labor productivity and real wage, i.e. $\mu_t = (y_t - n_t) - (w_t - p_t)$.

We conduct the empirical exercise with two models. First, we take a small scale VAR model with three variables as our baseline model, output gap, inflation, and the 10-year constant maturity yield rate. The (log) real efficient GDP is proxied by the Congressional Budget Office (CBO). The output gap is the difference of (log) real GDP and the (log) real efficient GDP. The underlying assumption here is that CBO potential real GDP is a noisy measurement of the true efficient GDP where the measurement error is orthogonal to cost-push shocks. All data are in quarterly frequency and four lags are chosen in the VAR. Second, to avoid the above assumption that the measurement error of efficient GDP is orthogonal to the markup shocks, we identify price markup shocks that are orthogonal to the efficient output in a larger VAR model and estimate a modified specification without using the efficient GDP in the estimation procedure.

2.4.1 The estimated central banks' targeting rule

Figure (2.1) presents the impulse response functions of the output gap and inflation to an exogenous cost-push shock and a monetary shock respectively. Inflation increases and the output gap decreases when there is a positive markup shock. In response to a positive monetary policy shock, inflation and the output gap decreases simultaneously. These results reflect our identification scheme with sign restrictions.

Table (2.1) reports the estimation results based on time series that are generated by cost-push shocks. The first three columns report the mean estimate and 90% confidence interval for β using the simple OLS approach. The middle columns report the two stages least square estimate using the first four lags of inflation and output gap as instruments. The last three columns show the conditional estimate using our methodology. Confidence interval for the conditional estimate is calculated by taking into account that shocks are "set identified" by sign restrictions and the bootstrap is applied in the second step. The OLS estimate of the targeting rule (β) is smaller than our conditional estimate in absolute value. This is consistent with the fact that the OLS estimate is biased downward (in absolute value). Since it is subject to the simultaneity problem and the NKPC is positively sloped. In the IV approach, the first stage F-statistic is 4.28 smaller than the rule-of-thumb value 10 proposed by Stock and Yogo (2005), which suggests that instruments are weak. Our estimate, the conditional estimate, of the targeting rule is -0.39 suggesting that the Fed is willing to take a 2.5 percentage points decrease in the growth rate of output gap for each percentage point increase in inflation.

Table 2.1: $\pi_t = \alpha + \beta \triangle x_t + e_t$

	OLS			IV				Conditional estimate		
		90% Interval			90% Interval		First Stage		90% Interval	
	Mean	Low	High	Median	Low	High	F-statistics	Median	Low	High
$\hat{\beta}$	-0.12**	-0.20	-0.04	-0.36*	-0.71	-0.00	4.28	-0.39***	-1.37	-0.15

Notes: confidence interval for the conditional estimate is constructed based on 200 000 draws times 1000 bootstrap for each draw.
*** 1% , ** 5% , * 10%

2.4.2 Discretion v.s Commitment

So far we have estimated the targeting rule under commitment. However, it is not entirely clear whether the Fed has been conducting the optimal policy under discretion or commitment. Under discretionary optimal policy the targeting rule is:

$$\pi_t = -\frac{\lambda}{\kappa} x_t. \tag{2.8}$$

We test the null hypothesis that the central bank conducts optimal policy under discretion. To this end, we estimate the following model:

$$\pi_t = \alpha + \beta_1 x_t + \beta_2 x_{t-1} + e_t \tag{2.9}$$

Under the null hypothesis, β_2 is equal to zero.

Table (2.2) reports the results. For an easy comparison, we report the baseline parameter estimate $\hat{\beta}$ together with $\hat{\beta}_1$ and $\hat{\beta}_2$. The null hypothesis is significantly rejected. Surprisingly, $\hat{\beta}_1$ is neither economically nor statistically different from $\hat{\beta}_2$ in absolute value, which is close to the $\hat{\beta}$ estimated in the baseline. We interpret this as an evidence suggesting that the Fed has been conducting optimal policy under commitment. This result is also consistent with the empirical evidence in the literature that exists interest smoothing component in the Taylor rule and that the probability of commitment is estimated to be about 0.80 in the DSGE model as found by Debortoli and Lakdawala (2016).

Table 2.2: Discretionary v.s Commitment
$\pi_t = \alpha + \beta \triangle x_t + e_t$ and $\pi_t = \alpha + \beta_1 x_t + \beta_2 x_{t-1} + e_t$
$H_0 : \beta_2 = 0$

	Baseline			**Discretion v.s Commitment**		
		90% Interval			90% Interval	
	Median	Low	High	Median	Low	High
$\hat{\beta}$	-0.39***	-1.37	-0.15			
$\hat{\beta}_1$				-0.43***	-1.57	-0.14
$\hat{\beta}_2$				0.37***	0.15	1.38

Notes: confidence interval is constructed based on 200 000 draws times 1000 bootstrap for each draw.
*** 1% , ** 5% , * 10%

2.4.3 Has the Fed Changed its Targeting rule?

It is well known that the change in the Fed's conduct of monetary policy since Volcker's appointment as the Fed's chairman is a plausible source of the Great Moderation since the 1980s and the Fed's insufficient reaction to the inflation is blamed for the great inflation in the 1970s, see, e.g., Taylor (1999), Clarida et al. (2000b), Lubik and Schorfheide (2004), Boivin and Giannoni (2006) and Coibion and Gorodnichenko (2011). On the other hand, Orphanides (2001, 2002, 2004) challenges this view of the break in the Taylor rule. He argues that there is no change in the conduct of the Fed's policy if the real time data is used. The natural question arises: has Fed changed its targeting rule since Volcker's appointment?

To address this question, we repeat our exercise using the following subsamples: pre-Volcker 1960Q1-1979 and post-Volcker 1979Q1-2017Q4 as well as the sample that excludes the recent recession 1979Q1-2008.

Table (2.3) reports the results. As one can see, there is no significant change in the targeting rule for the subsample analyzed. However, one should not view this as evidence against the break in Taylor rule. Actually for each targeting rule there exist infinitely many supporting Taylor rules. It is entirely plausible that the implementation rule (Taylor rule) has changed however the targeting rule remains unchanged. The detailed discussion about the relationship between the targeting rule and the Taylor rule can be found the Appendix 2.7.3.

Table 2.3: Subsample Analysis
Regression: $\pi_t = \alpha + \beta_1 x_t + \beta_2 x_{t-1} + e_t$

	60-1979			**79-2018**			**79-2008**		
		90% Interval			90% Interval			90% Interval	
	Median	Low	High	Median	Low	High	Median	Low	High
$\hat{\beta}_1$	-0.49**	-1.12	-0.18	-0.56***	-1.34	-0.20	-0.52***	-1.09	-0.22
$\hat{\beta}_2$	0.41**	0.16	0.95	0.39***	0.10	1.11	0.34***	0.06	0.93

Notes: confidence interval is constructed based on 200 000 draws times 1000 bootstrap for each draw.
*** 1% , ** 5% , * 10%

2.4.4 Evidence from a Larger VAR

The larger VAR studied in this section serves two purposes. First, we identify a particular source of cost-push shock— price markup shock by including a measure of markup in the VAR. Second, we relax the assumption that is made in the baseline: the measurement error of the real efficient GDP is orthogonal to cost-push shocks.

In this section, we avoid the above assumption using a modified specification and identification scheme. To illustrate that, consider a targeting rule that we will estimate:

$$\pi_t = \beta\Delta(y_t - y_{t,e}) + \epsilon_{m,t} \tag{2.10}$$

where β is the true parameter we will estimate, $\pi_t, y_t, y_{t,e}, \epsilon_{m,t}$ are the inflation, output, efficient output and monetary policy shocks. Rewrite the above equation as follows:

$$\pi_t = \beta \Delta y_t + (-\beta \Delta y_{t,e} + \epsilon_{m,t}) \tag{2.11}$$

In the following, first we will identify a price markup shock that would be orthogonal to the efficient output $y_{t,e}$. Second, we construct the "cleaned" data series for inflation and output driven by the price markup shocks. Last, we simply project the inflation on the output growth rate to obtain the unbiased estimate as previous. Notice that the efficient output is the first best allocation that is orthogonal to distortions in the economy such as the price markup shock, therefore we can use output growth instead of the output gap growth. In other words, we rely on the fact that $E(-\beta \Delta y_{t,e} + \epsilon_{m,t} | \text{markup shocks}) = 0$ to get an unbiased estimate of the targeting rule β.

We construct price markup shocks using a combination of long run and sign restrictions. Consider a VAR with the following variables $Y_t = \begin{bmatrix} \triangle(y_t - n_t) & \pi_t & x_t & i_t & \mu_t \end{bmatrix}'$, which denote the first difference in labor productivity, inflation, output, nominal interest rate and price markup respectively. The system is assumed to follow a VAR with p lagged variables:

$$Y_t = A_1 Y_{t-1} + ... + A_p Y_{t-p} + U_t \tag{2.12}$$

where $U_t = \begin{bmatrix} u_t^a & u_t^\pi & u_t^x & u_t^i & u_t^\mu \end{bmatrix}'$ denotes the reduced form residuals, which are linear combinations of structural shocks:

$$\begin{bmatrix} u_t^a \\ u_t^\pi \\ u_t^x \\ u_t^i \\ u_t^\mu \end{bmatrix} = B_0^{-1} \begin{bmatrix} \epsilon_t^a \\ \epsilon_t^\pi \\ \epsilon_t^x \\ \epsilon_t^i \\ \epsilon_t^\mu \end{bmatrix} \tag{2.13}$$

where $U_t = \begin{bmatrix} \epsilon_t^a & \epsilon_t^\pi & \epsilon_t^x & \epsilon_t^i & \epsilon_t^\mu \end{bmatrix}'$ denote the structural shocks. We impose the following restrictions to identify the price markup shocks:

$$B_0^{-1} = \begin{bmatrix} * & * & * & * & * \\ * & * & * & * & + \\ * & * & * & * & - \\ * & * & * & * & + \\ * & * & * & * & + \end{bmatrix},\ A_L \equiv \left(I - \sum_{i=1}^{p} A_i \right)^{-1} B_0^{-1} = \begin{bmatrix} * & * & * & * & 0 \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix} \tag{2.14}$$

The last shock is named as the price markup shock, which is identified using the following assumptions. First, a markup shock has no impact on labor productivity in the long run. This corresponds to the zero restriction imposed on the last entry of the first raw of the long-run effect matrix A_L. Second, markup shocks have positive impacts on inflation, nominal interest rate, and price markup and they affect output negatively that is imposed on the impact matrix B_0^{-1}. Those restrictions are consistent with predictions of New Keynesian models, which is a standard practice in the SVAR literature: see for example Galí and Gambetti (2018) for a recent application. While the long-run restriction and the sign restriction on markup itself are cheap assumptions, the other three sign restrictions deserve a short discussion. A price markup shock affects inflation positively because price resetting firms set higher prices, which results in positive inflation. The central bank stabilizes inflation by raising the nominal interest rate. Consequently, it leads to a negative output. Those sign restrictions are specific to markup shocks, therefore, the constructed shocks are not confounding with other structural shocks.[7]

Table (2.4) reports the conditional estimates based on the markup shocks.

[7] See Galí and Gambetti (2018) for sign restrictions that are required for the remaining structural shocks.

Table 2.4: Price Markup Shocks
$\pi_t = \alpha + \beta \triangle x_t + e_t$ and $\pi_t = \alpha + \beta_1 x_t + \beta_2 x_{t-1} + e_t$

	Baseline			**Discretion v.s Commitment**		
		90% Interval			90% Interval	
	Median	Low	High	Median	Low	High
$\hat{\beta}$	-0.35**	-0.88	-0.14			
$\hat{\beta}_1$				-0.38**	-1.12	-0.14
$\hat{\beta}_2$				0.33***	0.14	0.86

Notes: Unemployment gap that is taken from the CBO. Confidence interval is constructed based on 200 000 draws times 1000 bootstrap for each draw.
*** 1% , ** 5% , * 10%

2.4.5 Robustness Checks

We briefly report several robustness checks in this section. The details are in the Appendix 2.7.1.

First, we use real time data in the estimation and find that the result is largely unchanged.

Second, we use an alternative measure of the output gap. The (log) real efficient GDP is estimated as a sixth degree polynomial for the logarithm of real GDP. Table (2.6) shows the results using this measure. Results remain the same.

Third, the results presented above are robust to the use of the unemployment gap instead of the output gap, see Table (2.7). The fed is willing to accept roughly a one percentage point increase in the unemployment rate for each percentage point increase in inflation.

Last, Table (2.8) presents the estimation results when the Fed Funds rate (FFR) is used instead of the long-term interest rate. Due to the fact that the FFR was at its zero lower bound from 2008 to 2016, for this robustness check we use data up to 2008. Results are unchanged.

2.5 Central Bank's Preference

We have shown in section (2.2) that a central bank's targeting rule (β) depends on the slope of the NKPC (κ) and the deep parameter (λ) that characterizes the loss function ($L_t = \pi_t^2 + \lambda x_t^2$) of the central bank. In particular, $\beta = -\frac{\lambda}{\kappa}$. While the interpretation of β is intuitive, the knowledge of λ might be more desirable in other circumstances. For instance, a macroeconomist might want to calibrate it to study the optimal monetary policy or the transmission of shocks. In this section, we provide an estimate of λ. To this end, it is sufficient to estimate the slope of the NKPC.

The estimation of the NKPC is subject to the same simultaneity problem mentioned above. To overcome this issue, in a similar spirit of Barnichon and Mesters (2018a,b) and Galí and Gambetti (2018), we identify the central bank's Targeting rule shifter and purge out all the other shocks. In this environment, the NKPC is fixed and the realizations of the equilibrium outcomes are solely driven by shifting the central bank's targeting rule. Therefore, applying the methodology described above the NKPC is identified.

For the estimation of the NKPC, a proxy for $E_t(\pi_{t+1})$ is required. We use the professionals' forecast of GDP deflator taken from the Survey of Professional Forecasters. Then we consider a VAR consisting of four variables: $Y_t = \begin{bmatrix} \pi_t & x_t & i_t & E_t(\pi_{t+1}) \end{bmatrix}'$. We identify the targeting rule shifters by relying on the following sign restrictions;

$$B_0^{-1} = \begin{bmatrix} + & * & * & * \\ + & * & * & * \\ * & * & * & * \\ + & * & * & * \end{bmatrix}, \tag{2.15}$$

It can be easily verified that shocks to the targeting rule push the output gap, inflation and expected inflation towards the same direction. Crucial for the unbiasedness of parameter estimate is that shocks to the NKPC (cost-push shocks) are not captured in the identified series of shocks. This is insured by our identification strategy since in response to a cost-push shock, output gap and inflation move towards the opposite direction.

In the next, we purge out all the other shocks and estimate the following regression:

$$\pi_t = \alpha_1 + \kappa x_t + \gamma E_t \pi_{t+1} + u_t. \tag{2.16}$$

The second raw in Table (2.9) reports the estimated slope of the NKPC. This estimate is comparable to the findings of the existing literature, see, e.g. Barnichon and Mesters (2018b)[8]. The slope of the NKPC together with the targeting rule pin down the deep parameter (λ) that characterizes the Fed's loss function. This is computed and shown in the last raw in Table (2.9). On average, the relative weight that Fed attached to the volatility of output gap is roughly 0.2— a value that is one magnitude larger than the calibration suggested in Woodford (2011a)[9].

2.6 Concluding Remarks

The present paper estimates the central bank's targeting rule. The direct estimation is challenging due to the simultaneity problem. We overcome this issue by purging out shocks that shift a central bank's targeting rule. The following results stand out. First, the Fed is willing to decrease the growth rate of the output gap by two and a half percentage points for each percentage point increase in inflation. Second, the data rejects optimal discretionary policy in favor of optimal policy under commitment. Third, the Fed's targeting rule remains stable around the pre and post-Volcker' periods. The last but not least, we provide an empirical estimate of the deep parameter that characterizes a central bank's loss function. On average, the relative weight that the Fed attaches to the volatility of the output gap is 0.2.

[8]Note that most of the other studies using annual inflation, therefore our estimate of κ needs to be multiplied by a factor of 4 in order to be comparable.

[9]Note that following Woodford (2011a) $\hat{\lambda}$ is multiplied by 16 for a central bank corresponding to the loss function in which the inflation is annualized.

2.7 Appendices

2.7.1 Tables

Table 2.5: Robustness Check: Real Time Data
$\pi_t = \alpha + \beta \triangle x_t + e_t$ and $\pi_t = \alpha + \beta_1 x_t + \beta_2 x_{t-1} + e_t$

	Baseline			**Discretion v.s Commitment**		
		90% Interval			90% Interval	
	Median	Low	High	Median	Low	High
$\hat{\beta}$	-0.43***	-1.15	-0.17			
$\hat{\beta}_1$				-0.42***	-1.40	-0.14
$\hat{\beta}_2$				0.42***	0.18	1.30

Notes: Output gap is constructed using the potential GDP provided by the CBO. Confidence interval is constructed based on 200 000 draws times 1000 bootstrap for each draw.
*** 1% , ** 5% , * 10%

Table 2.6: Robustness Check: Alternative Measure of Output Gap

$\pi_t = \alpha + \beta \triangle x_t + e_t$ and $\pi_t = \alpha + \beta_1 x_t + \beta_2 x_{t-1} + e_t$

	Baseline			**Discretion v.s Commitment**		
		90% Interval			90% Interval	
	Median	Low	High	Median	Low	High
$\hat{\beta}$	-0.41**	-1.18	-0.18			
$\hat{\beta}_1$				-0.46**	-1.42	-0.17
$\hat{\beta}_2$				0.36**	0.16	1.01

Notes: Confidence interval is constructed based on 200 000 draws times 1000 bootstrap for each draw.

*** 1% , ** 5% , * 10%

Table 2.7: Robustness Check: Unemployment Gap
$\pi_t = \alpha + \beta \triangle x_t + e_t$ and $\pi_t = \alpha + \beta_1 x_t + \beta_2 x_{t-1} + e_t$

	Baseline			**Discretion v.s Commitment**		
		90% Interval			90% Interval	
	Median	Low	High	Median	Low	High
$\hat{\beta}$	1.13***	3.51	0.43			
$\hat{\beta}_1$				1.13***	3.51	0.43
$\hat{\beta}_2$				1.04***	0.41	3.28

Notes: Unemployment gap that is taken from the CBO. Confidence interval is constructed based on 200 000 draws times 1000 bootstrap for each draw. *** 1% , ** 5% , * 10%

Table 2.8: Robustness Check: Fed Funds Rate
$\pi_t = \alpha + \beta \triangle x_t + e_t$ and $\pi_t = \alpha + \beta_1 x_t + \beta_2 x_{t-1} + e_t$

	Baseline			**Discretion v.s Commitment**		
		90% Interval			90% Interval	
	Median	Low	High	Median	Low	High
$\hat{\beta}$	-0.35***	-0.98	-0.14			
$\hat{\beta}_1$				-0.38***	-1.11	-0.14
$\hat{\beta}_2$				0.32***	0.13	0.97

Notes: we use the fed funds rate instead of the 10-year constant maturity yield rate. We use the sample up to 2008 to avoid the zero lower bound. Confidence interval is constructed based on 200 000 draws times 1000 bootstrap for each draw.
*** 1% , ** 5% , * 10%

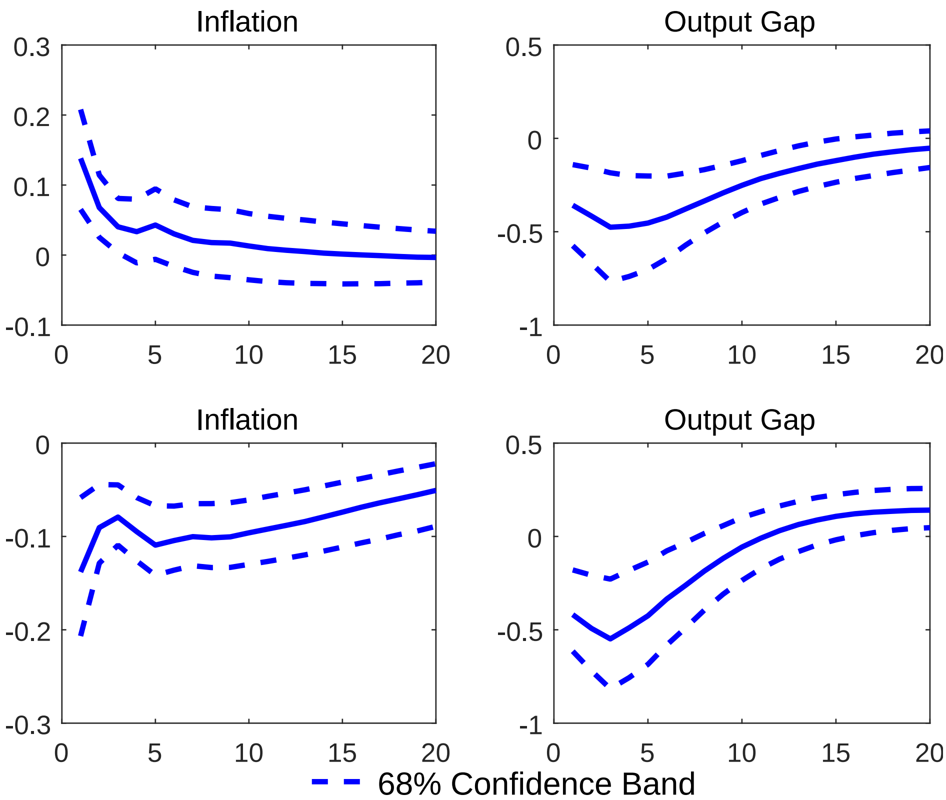
Table 2.9: The Fed's Preference: $\pi_t = \alpha + \beta \triangle x_t + e_t$ and $\pi_t = \alpha_1 + \kappa x_t + \gamma E \pi_{t+1} + u_t$

		68% Interval	
	Median	Low	High
$\hat{\beta}$	-0.44**	-1.01	-0.14
$\hat{\kappa}$	0.03	0.00	0.06
$\hat{\lambda}$	0.18	0.01	0.51

Notes: confidence interval is constructed based on 200 000 draws times 1000 bootstrap for each draw. Note that we multiply λ by 16 so that the inflation rate in the welfare loss function is annualized as it is done by Woodford (2011a).
*** 1% , ** 5% , * 10%

2.7.2 Figures

Figure 2.1: Impulse response



Notes: The figure presents the impulse response of the inflation and output gap to the identified shock. The first row corresponds to the response to the markup shock and the second row is the response to the monetary policy shock. The blue solid line is the median estimate and the blue dashed line is the 68% confidence band.

2.7.3 The Relationship between the Targeting Rule and the Taylor Rule

We now discuss the relation between the targeting rule and the Taylor rule. For illustration, we consider the targeting rule under a discretionary policy. That is, the central minimize its period loss function:

$$\min\ L_t = \pi_t^2 + \lambda x_t^2$$

subject to :

$$\pi_t = \kappa x_t + v_t$$

where $v_t \equiv \beta E\pi_{t+1} + \epsilon_t^\pi$. The solution to this problem is:

$$\pi_t = \lambda\Psi\epsilon_t^\pi,\ x_t = -\kappa\Psi\epsilon_t^\pi,$$

where $\Psi = \frac{1}{\kappa^2+\lambda(1-\beta\rho_{\epsilon,\pi})}$. To solve for the equilibrium nominal interest rate, we make use of the Dynamic IS equation:

$$x_t = E(x_{t+1}) - \frac{1}{\sigma}(r_t - r_t^e),$$

where $r_t^e = \rho + \sigma E\triangle y_{t+1}^e$ denotes the efficient real interest rate that follows an exogenous process. The nominal interest rate under this targeting rule is:

$$i_t = r_t^e + \Psi_i\epsilon_t^\pi, \tag{2.1}$$

where $\Psi_i = \Psi[\kappa\sigma(1-\rho_{\epsilon,\pi}) + \lambda\rho_{\epsilon,\pi}]$. Equation (2.1) is merely the solution of the model, it does not represent the interest rate rule (Taylor rule) that is employed by the central bank to achieve the target. In fact, if the central bank were setting interest rates following (2.1), there would be multiple equilibria. In theory, there are infinitely many Taylor rules of the following form that satisfy the equilibrium allocations derived above:

$$i_t = r_t^e + \Psi_i\epsilon_t^\pi + \phi_\pi(\pi_t - \lambda\Psi\epsilon_t^\pi) + \phi_x(x_t + \kappa\Psi\epsilon_t^\pi), \tag{2.2}$$

for any combinations of ϕ_π and ϕ_x that satisfy Blanchard and Kahn (1980) conditions. Therefore, the Taylor rule is not informative about the targeting rule. Even if the central bank has an unique mandate of inflation stabilization, i.e., $\lambda = 0$, the Taylor rule coefficient on output gap ϕ_x could be different from zero.

Chapter 3

INFORMATION FRICTIONS AND THE PARADOX OF PRICE FLEXIBILITY

3.1 Introduction

Electronic shelf labels (ESL) permits retailers to set price digitally without any costs that would otherwise occur using paper price tags. Over the past decade, we have witnessed an expansion in the usage of digital price tags thanks to the growing affordability of ESL.[1] The introduction of digital price tags may facilitate price adjustment and reduce the degree of nominal rigidity in the economy. Is such a technological progress welfare improving?

We address this research question in a NK model with both nominal rigidity and dispersed information. Both frictions are shown in the literature to be relevant empirically.[2] We highlight a new channel — *dispersed beliefs channel* that is relevant to understand the welfare consequence of a reduction in nominal rigidity.

[1]See the report by Global Market Insights: https://www.globenewswire.com/news-release/2018/09/18/1572161/0/en/Electronic-Shelf-Label-ESL-Market-to-hit-1bn-by-2024-Global-Market-Insights-Inc.html

[2]See for an example Nakamura and Steinsson (2008) for empirical evidence on nominal rigidity from the micro level prices data. Empirical evidences supporting the presence of information frictions and dispersed beliefs are abundant, see for examples Coibion and Gorodnichenko (2012, 2015), Andrade et al. (2016) and Coibion et al. (2018) for a recent survey.

In a standard NK model, dispersions in prices and quantities arise due to price stickiness. Nominal rigidity such as staggered prices $\grave{a}\ la$ Calvo (1983b) creates dispersion in prices across those firms because of staggered price setting. We denote this as the *Calvo channel*. Such dispersions are inefficient because goods matter for households' utility symmetrically and production technology are identical. Dispersed information that gives birth to imperfect common knowledge or dispersed beliefs creates another channel through which price dispersion arises. Firms have different assessments about the state of the economy due to information frictions. Therefore, in contrast to a standard model with perfect information, price dispersion emerges among those firms who can reset prices. We denote this as the *dispersed beliefs channel*.

To fully understand the welfare consequence of a change in price flexibility, we derive the welfare loss function around the perfect information steady state and decompose it into three components. The first component, through the *Calvo channel*, is proportional to the price dispersion across all firms as if the newly price resetting firms are restricted to set the same average price. This component is a hump shape function of price rigidity. The second component, arising from the dispersed belief channel, is proportional to the price dispersion across the newly set prices. The associated welfare losses increase monotonically in the degree of price flexibility. In one extreme case, if the price is fully rigid, there is no dispersion in prices even if firms have different assessments about the state of the economy. In another extreme case when the prices are fully flexible, firms disagree with each other and would set the price based on their private information. Then the welfare losses associated with the price dispersion originating from the imperfect common knowledge would be maximized. The third component is proportional to the output gap volatility, caused by both nominal and information frictions, which is increasing in the price rigidity. The aggregate effect of an increased price flexibility on welfare is thus ambiguous.

In a static model, we derive analytically the conditions under which the dispersed beliefs channel dominates. Consequently, under those conditions an improved pricing technology that facilitates price adjustment may be associated with bigger welfare losses — the paradox of price flexibility. Two parameters are crucial to this finding. Not surprisingly, the first is the signal-to-noise ratio that characterizes the degree of information frictions, and thus the degree of disagreement among firms. The latter leads to a proportionally higher price dispersion among price resetting firms. The second is the parameter that characterizes the relative importance of price dispersion and output gap volatility in the welfare loss function. This is the case because the price dispersion through the *dispersed belief channel*, is decreasing in the price rigidity while the output gap volatility is increasing in the rigidity due to the *Calvo channel*. The relative weight of price dispersion in the welfare loss function is proportional to the degree of competition in the goods market as the latter amplifies the welfare losses for a given price dispersion. The degree of signal-to-noise ratio and market competition that are required to generate the paradox of price flexibility are satisfied according to empirical estimates of Kalman gain conducted in Coibion and Gorodnichenko (2012, 2015) and empirical estimates of markup, which reflects the degree of competition, studied by Loecker and Eeckhout (2017).

We extend our static model into a Dynamic Stochastic General Equilibrium model with nominal rigidity and dispersed beliefs as in Nimark (2008). Our finding survives in a dynamic model. Qualitatively, the same results hold independent of whether the underlining shocks that drive the business cycle fluctuations are i) technology shocks, ii) preference shocks.

Our results point to market inefficiency. In our model, from an individual firm's perspective, it would be better off if the firm was allowed to adjust its price more frequently as they would be able to respond to shocks on a timely manner. Therefore, firms would pay for new pricing technologies. However, if all individual firms adopt the new technology to facilitate the price adjustment, the aggregate welfare losses may increase as we discussed above. Thus, the constrained social planner allowing for the presence of dispersed information would not introduce the new technology.

Literature This paper is related to two branches of the literature. The theoretical framework is related to the literature that incorporates dispersed beliefs into business cycle models (e.g. Lucas (1972), Woodford (2001), Nimark (2008), Lorenzoni (2009), Angeletos and Jennifer (2009), Hellwig and Venkateswaran (2009), Angeletos and La'O (2011, 2013), Huo and Takayama (2015a,b), Melosi

(2016), Angeletos et al. (2016), Angeletos and Lian (2018), Huo and Pedroni (2019)).[3] We build on a NK model with dispersed beliefs and derive different components of welfare loss function explicitly.

Our paper contributes to the literature that studies the implications of a reduction in nominal rigidity. The idea that an increase in price flexibility may increase output volatility dates back to Keynes (1936), was formalized by Long and Summers (1986) and recently it is revisited by Bhattarai et al. (2018). Galí (2013) concludes that a reduction in wage rigidity improves welfare only if the central bank reacts to inflation sufficiently aggressive in a closed-economy New Keynesian model featuring both price rigidity and wage rigidity. Following similar reasoning, in an open economy with the fixed exchange rate (Galí and Monacelli (2016)) or a closed economy when the monetary policy is constrained by the Zero Lower Bound (Amano and Gnocchi (2017) and Billi et al. (2018)), a labor market reform that results in a more flexible wage is not necessarily welfare improving.

The main contribution of our paper is to combine these two branches of literature, and discuss the role of information frictions or dispersed beliefs in the welfare analysis of nominal rigidity. With this, we introduce a new channel that was previously ignored in the literature: the dispersed belief channel. We show that a more flexible price is welfare detrimental even if i) it does not increase output volatility ii) monetary policy strongly reacts to the inflation.

The remainder of the paper is organized as follows. Section 3.2 presents a static model and derive the analytical solution to shed light on the role of dispersed information to understand the welfare implications of nominal rigidity. Section 3.3 builds a dynamic model to quantitative evaluate the welfare gains/losses of the reduction in nominal rigidity. Section 3.4 concludes.

3.2 Model

3.2.1 Static Model

In this section, we present a static model to explain the key mechanisms.

[3] See Mankiw and Reis (2002), Sims (2003), Mackowiak and Wiederholt (2009) for other applications of information frictions for business cycle analysis.

Household There is a representative household with the following period utility function:

$$U = log(C) - L,$$

where C is the amount of goods that the household consume and L is the number of hours that the consumer works.

The household maximizes her utility by choosing the optimal amount of final goods C and total labor L subject to the following budget constraint:

$$PC = WL + T,$$

where, P is the nominal price of goods, W is the nominal wage and T is the firm's profit net of lump-sum taxes paid to the government. The household's optimization problem leads to the following labor supply equation:

$$PC = W \tag{3.1}$$

Firms There is a continuum of monopolistic firms producing differentiated intermediate goods using a homogenous production technology. An individual firm i's production function is the following:

$$Y(i) = exp(a)L(i),$$

where a is the log of level productivity that is draw from a normal distribution: $N(0, \sigma_a^2)$. A firm i hires labor $L(i)$ from the representative household. The intermediate goods are aggregated into the final consumption good according to the Dixit-Stiglitz aggregator:

$$Y = \left(\int_0^1 Y(i)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon > 1$ measures the degree of substitution among varieties and $\frac{\epsilon}{\epsilon-1}$ is the desired markup charged by firms. In the limiting case, $\epsilon \to \infty$, firms engage into a competitive market. In general, the degree of market competition is increasing in ϵ.

Consumers's optimal expenditure allocation yields the following demand curve that a firm i faces:

$$Y(i) = \left(\frac{P(i)}{P}\right)^{-\epsilon} Y, \tag{3.2}$$

where $P \equiv \left(\int_0^1 P(i)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$ denotes the aggregate price.

Firms cannot perfectly observe the technology level a and each firm receives an idiosyncratic signal $s(i)$:

$$s(i) = a + e(i), \tag{3.3}$$

where $e(i)$ is an idiosyncratic noise drawn from a normal distribution $N(0, \sigma_e^2)$. The structure of the model and processes of shocks are common knowledge among all firms.

We assume that only a fraction $1-\theta$ of firms are allowed to set the price. Each price resetting firm i solves the following maximization problem conditional on its signal:

$$\max_{P(i)} E\left\{P(i)Y(i) - \frac{W}{Pexp(a)}|s(i)\right\} \tag{3.4}$$

subject (3.2). Combine firms' first order condition with goods and labor market clearing conditions, and linearize it to obtain:

$$p^\star(i) = E(p + y - a|s(i)), \tag{3.5}$$

where $p^\star(i)$, p and y is the log-deviation from the steady state. p is the aggregate price across all firms defined as $p = \int_i p(i)di$. The remaining fraction θ of firms keep their prices at initial value zero. Therefore, the aggregate price is:

$$p = (1-\theta)\int p^\star(i)di \tag{3.6}$$

Monetary Policy The central bank determines an exogenous amount of money in the economy:

$$m = 0. \tag{3.7}$$

In addition, we assume an ad-hoc money demand equation:

$$m - p = y. \tag{3.8}$$

The Timing of the Model The model consists of three stages. In stage one, the nature draws a fundamental shock a, and each firm i receives a private signal s_i about a. Moreover, the structure of the model, including the monetary policy rule and the distribution of shocks are common knowledge across firms. In stage two, each firm i forms a belief and decides a price setting plan. In stage 3, the representative household observes the state of the economy and makes the consumption and labor decision. At the same stage, the goods, labor and money markets clear.

Model Solution The solution of the model is characterized by (3.5), (3.6), (3.7), (3.8) and the following Bayes' rule that characterizes firms' beliefs updating:

$$E(a|I_i) = Ks(i), \tag{3.9}$$

with $K \equiv \frac{\sigma_a^2}{\sigma_a^2+\sigma_e^2}$. Solving those equations, we obtain the following aggregate allocations:

$$p = -(1-\theta)Ka, \tag{3.10}$$

$$y = (1-\theta)Ka, \tag{3.11}$$

$$\tilde{y} = (1-\theta)Ka - a, \tag{3.12}$$

where $\tilde{y}$ denotes the output gap.

Welfare Losses The object of interest is the representative household's welfare. To this end, we derive the welfare loss function as the second order approximation of the household's utility function:

$$L = \epsilon var_i p(i) + \tilde{y}^2 \tag{3.13}$$

In the appendix 3.5.2, by combining the welfare loss function with the solution of the model, we show that the ex-ante expected welfare losses can be written as:

$$E(L) = \underbrace{\epsilon \left\{ \underbrace{(1-\theta)K^2\sigma_e^2}_{\text{Dispersed belief}} + \underbrace{(1-\theta)\theta K^2\sigma_a^2}_{\text{Calvo}} \right\}}_{\text{Price dispersions}} + \underbrace{\left((1-\theta)K-1\right)^2\sigma_a^2}_{\text{Output gap volatility}}. \tag{3.14}$$

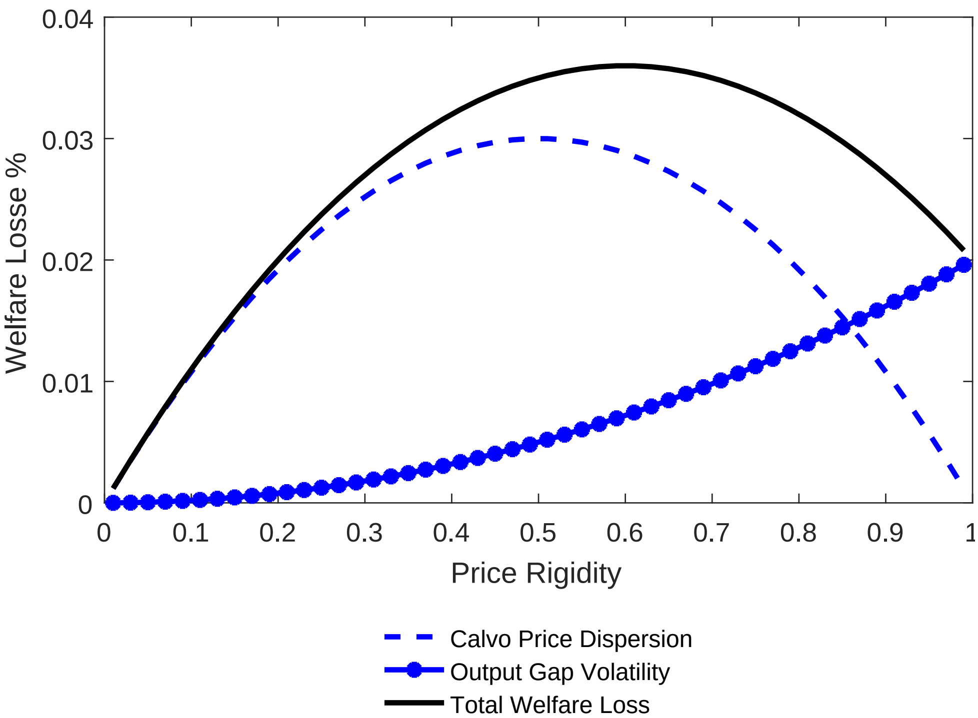
The welfare losses consist of two components: the part that is proportional to the price dispersions and the one that is proportional to the output gap volatility. Such dispersions and volatilities reflect the degree of inefficiency of the economy because goods matter for households' utility symmetrically and production technologies are identical.

The novelty, in a dispersed belief model as compared to the perfect information case, is that there are two channels that drive price dispersions. The first is the standard *Calvo channel* that leads to price dispersions across the group of firms who can reset prices with those who cannot reset prices. The second is the *Dispersed belief channel* that generates price dispersions *within* the group of price resetting firms. Firms form different beliefs about the state of the economy, therefore, in contrast to a standard model with perfect information, price dispersion emerges among those firms who can reset prices.

We are now ready to discuss the welfare implication of an increase in price flexibility. We start with the special case in which there is no information imperfection.

Proposition 1 *In the special case in which firms have perfect information, i.e.* $\sigma_\epsilon = 0$*, the welfare loss as a function of* θ *is hump-shaped, the economy reaches the maximum welfare loss when* $\theta^{per} = \frac{1}{2}\frac{\epsilon}{\epsilon-1}$ *and the welfare loss is minimized if price were flexible.*

Figure 3.1: Welfare Losses Decomposition: Perfect Information



Note: This figure plots each component of welfare losses and total welfare losses as a function of price rigidity for the case with perfect information. The elasticity of substitution across goods ϵ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock σ_a^2 is calibrated to be 0.02.

With perfect information, the welfare loss function collapses to

$$E(L) = \epsilon(1-\theta)\theta\sigma_a^2 + \theta^2\sigma_a^2,$$

the Calvo price dispersion component plus the output gap volatility component. It is trivial that the output gap volatility component is strictly increasing in the degree of price rigidity θ. This is the case because the bigger is the nominal rigidity the larger response of the output gap as it is shown in the solution of the model. Price dispersion originating from the Calvo pricing friction is a hump-shaped function of nominal rigidity due to the fact that the Calvo price dispersion peaks if there are as many firms who can reset price as those who cannot, i.e. if $\theta = 0.5$. Therefore, overall, the total welfare losses peak at a value of θ that is greater than 0.5. The value of the price rigidity that achieves the maximum welfare losses also depends on the ϵ, the elasticity of substitution across goods as it measures the relative importance of price dispersions against output gap volatility in the welfare loss function.

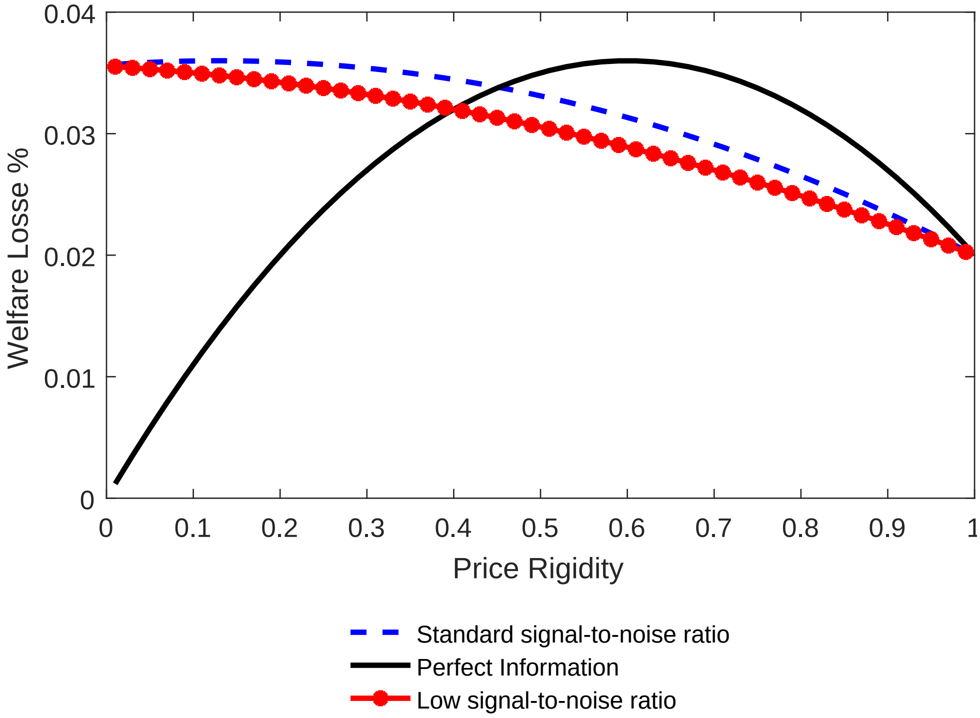
Figure (3.1) provides a visual representation of those results by plotting the components of welfare loss as a function of price rigidity. The elasticity of substitution across goods ϵ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock σ_a^2 is calibrated to be 0.02. Those results suggest that, in a model with perfect information, if the introduction of digital price tags reduces price rigidity θ towards the left hand side of 0.6, it will be welfare improving.

The presence of information frictions and dispersed beliefs changes the result dramatically:

Proposition 2 *In the presence of information frictions and dispersed beliefs, the economy reaches the maximum welfare losses at* $\theta^{imper} = \frac{1}{2}\frac{\epsilon}{\epsilon-1} - \frac{\epsilon-2}{2(\epsilon-1)}\frac{\sigma_e^2}{\sigma_a^2}$. *Moreover, given a reasonably degree of competition ($\epsilon > 2$), θ^{imper} is increasing in the signal-to-noise ratio $\frac{\sigma_a}{\sigma_e}$. And when the signal-to-noise ratio is sufficiently low, in particular if $\frac{\sigma_a^2}{\sigma_e^2} < \frac{\epsilon-2}{\epsilon}$, the welfare losses are monotonically decreasing in* $\theta \in [0, 1]$.

Figure (3.3) demonstrates proposition 2 visually. It plots the welfare losses as a function of price rigidity under different scenarios. The solid black line presents the perfect information case. The dashed blue line plots the prediction of the model with dispersed beliefs and the signal-to-noise ratio calibrated to match the empirical Kalman gain of 0.46 estimated by Coibion and Gorodnichenko (2015). The red line with circle shows the result if the signal-to-noise ratio is calibrated to $\frac{\epsilon-2}{\epsilon}$, which corresponds to a Kalman gain of $\frac{1}{3}$. Surprisingly, under the reasonable calibration of information frictions (the dashed blue line in Figure 3.3), in contrast to the perfect information case (solid black line), reducing price rigidity is no longer welfare improving! In fact, the welfare loss is minimized if price were fully rigid.

Figure 3.2: Welfare Losses: Dispersed Beliefs

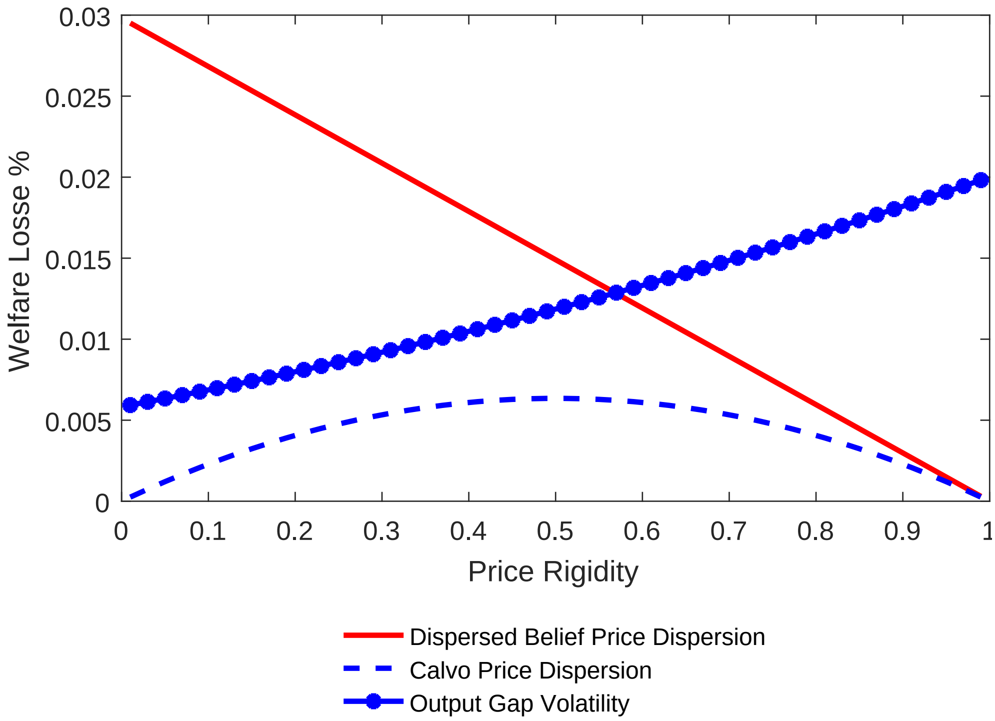


Note: This figure plots the welfare losses as a function of price rigidity under different scenarios. The solid black line presents the perfect information case. The dashed blue line plots the prediction of the model with dispersed beliefs with and signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The red line with circle shows the result if the signal-to-noise ratio is calibrated to $\frac{\epsilon-2}{\epsilon}$, which corresponds to a Kalman gain of $\frac{1}{3}$. The elasticity of substitution across goods ϵ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock σ_a^2 is calibrated to be 0.02.

The above result is entirely driven by the dispersed beliefs channel. With information imperfection and dispersed beliefs, dispersion in prices arises within the group of firms who can reset prices. As one can see in equation (3.14), the dispersed belief component of welfare losses are strictly increasing in the price flexibility $(1-\theta)$. This is verified in Figure (3.3), which plots each component of welfare losses as a function of price rigidity for the case with dispersed beliefs: the solid red line is strictly decreasing in price rigidity. It is worth to note that if the output gap volatility component (blue line with circles) were sufficiently large, the total welfare losses would be increasing in nominal rigidity. As discussed above, in this model the relative importance of price dispersion v.s output gap volatility depends on ϵ. This explains why our result requires a ϵ that is greater than two. However, in our view this is not a binding condition. There is a consensus in the literature that ϵ is calibrated to be between 3 and 11 that correspond to a desired markup in between 1.5 and 1.1. Those numbers are in line with average markup estimated using micro data: see e.g. Berry et al. (1995) and Nevo (2001). More recently, Loecker and Eeckhout (2017)' estimate suggests that the average markup in the U.S. has risen to 1.5, yet it is still far below our condition (a markup of 2).

To conclude this section, in a static model we derive analytically that a reduction in nominal rigidity is not welfare improving when firms possess heterogenous beliefs. In the next section, we extend our baseline static model to a dynamic model and quantitatively evaluate whether our results hold true.

Figure 3.3: Welfare Losses Decomposition: Dispersed Beliefs



Note: This figure plots the components of welfare losses as a function of price rigidity for the case with dispersed beliefs. The signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The elasticity of substitution across goods ϵ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock σ_a^2 is calibrated to be 0.02.

3.3 Dynamic Model

In the current section, we present a dynamic model to quantitatively evaluate the welfare gains/losses by reducing the price rigidity. The model is based on Nimark (2008), featuring the firms' imperfect knowledge about the state of the economy.

Household The representative household maximizes the lifetime utility function:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t exp(d_t) \left\{ \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\nu}}{1+\nu} \right\}$$

subject to the budget constraint:

$$P_t C_t + B_{t+1} = B_t R_t + W_t N_t + T_t.$$

Again C_t denotes the aggregate consumption, N_t is the hours, W_t is the nominal wage, T_t is the firm's profit net of lump-sum taxes paid to the government. B_{t+1} is the quantity of the one-period bond households buy at period t, R_t is the interest rate of the one period bond. d_t is the inter-temporal preference shock that follows an autoregressive process:

$$d_t = \rho_d d_{t-1} + \epsilon_d$$

Where ϵ_d represents an i.i.d. shock with mean zero and constant variance σ_d^2 that is drawn from a normal distribution, i.e. $\epsilon_d \sim N(0, \sigma_d)$.

The household has perfect information. Therefore, the consumer's optimality conditions: the Euler equation and the labor supply equation are standard.

Firm The final goods producer who operates in a perfect competitive market combines intermediate goods $Y_{i,t}$ for $\forall i \in [0, 1]$ according to a CES technology:

$$Y_t = \left(\int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

The profit maximization of the final goods producer leads to the demand function for each intermediate goods i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t \tag{3.15}$$

for all $i \in [0, 1]$.

There is a continuum of monopolistic competitive firms producing the intermediate goods with the following production technology:

$$Y_t(i) = e^{a_t} L_t(i) \tag{3.16}$$

where e^{a_t} is the technology level and a_t evolves according to an autoregressive process $a_t = \rho_a a_{t-1} + \epsilon_a$, where ϵ_a is a Gaussian innovations:$\epsilon_a \sim N(o, \sigma_a^2)$.

Following Calvo (1983b), in each period firms are only allowed to re-optimize their prices with a probability $1 - \theta$. Among the faction of firms who can re-optimize they set their prices to maxizmize the expected present discounted value of their future profits:

$$E_t \left\{ \sum_{s=0}^{\infty} (\beta\theta)^s \frac{\lambda_{t+s}}{\lambda_t} \frac{P_t(i)}{P_{t+s}} \left(Y_{t+s}(i) - W_{t+s} N_{t+s}(i)\right) | I_{i,t} \right\}$$

subject to its demand function (3.15) and production function (3.16). λ_t is the household marignal utility of nominal income. The remaining θ fraction of firms, would set the price with $p_t(i) = p_{t-1}(i)$. $I_{i,t}$ is the information set of firm i which we will specify in the following.

Firm's information set Firms are subject to information frictions. In particular, each individual firm cannot perfectly observe the state of the economy and receive noisy signals about aggregate shocks.

$$s_{d,i,t} = d_t + \nu_{d,i,t} \tag{3.17}$$

$$s_{a,i,t} = a_t + \nu_{a,i,t} \tag{3.18}$$

where $s_{d,i,t}$ and $s_{a,i,t}$ are imprecise signals about preference shocks, monetary policy shocks and technology shocks respectively that each individual firm receive. $\nu_{d,i,t}$ and $\nu_{a,i,t}$ are the corrsponding noise drawn from the normal distrition $N(0, \sigma_{e,d}^2)$ and $N(0, \sigma_{e,a}^2)$ respectively. Firms set the price in advance before production takes place based on their information set. Then household supply the labor and consume final product goods. The firms' information set is defined by:

$$I_{i,t} = \{s_{d,j}, s_{a,j}, p_{i,j-1}, y_{i,j-1} : j \leq t\}$$

Monetary Policy The central bank sets the short term nominal interest rate and follows a Taylor rule:

$$R_t = (\frac{\pi_t}{\pi_{ss}})^{\phi_\pi} (\frac{y_t}{y_{ss}})^{\phi_y}$$

where π_{ss} is the steady state inflation, y_t is the output and y_{ss} is the steady state level of output.

The Timing of the Model The timing of model is the same with that we describe in the static model.

Equilibrium We log-linearize the model around the perfect information steady state. The equilibrium conditions are:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma}(\hat{\imath}_t - E_t \hat{\pi}_{t+1}) - (\rho_d - 1) d_t \tag{3.19}$$

$$\hat{\pi}_t = (1-\theta)(1-\beta\theta) \sum_{k=0}^{\infty} (1-\theta)^k \hat{mc}_{t|t}{}^{(k)} + \beta\theta \sum_{k=0}^{\infty} (1-\theta)^k \hat{\pi}_{t+1|t}^{k+1}, \forall k \geq 0 \tag{3.20}$$

$$\hat{mc}_{t|t}{}^{(k)} = (\sigma + \nu) \hat{y}_t^{(k+1)} - (1+\nu) a_t^{(k+1)}, \forall k \geq 0 \tag{3.21}$$

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \tag{3.22}$$

The equation 3.19 is the standard (log-linearized) Euler equation derives from the household optimization problem. Note that the E is the expectation operator assuming that the household has perfect information. The equation 3.20 is the (log-linearized) New Keynesian phillips curve derived from the firm's price setting equation relying on the assumption that firms cannot perfectly observe the fundamental shocks and set the price based on their information sets as defined in the previous paragraph. $\hat{\pi}^k_{t+1|t}$ is the k-th order expectations about the next period's inflation rate and defined as $\hat{\pi}^k_{t+1|t} \equiv \underbrace{\int E_{i,t} ... \int E_{i,t}}_{k} \hat{\pi}_{t+1} di...di$. $E_{i,t}$ denotes the expectation operator conditional on the firm i's information set at time t. $\hat{mc}^{(k)}_{t|t}$ is the k-th order expectations about the marginal cost and defined as $\hat{mc}^k_{t|t} \equiv \underbrace{\int E_{i,t} ... \int E_{i,t}}_{k} \hat{mc}_t di...di$. Equation 3.21 defines the marginal cost from the combination of the labor supply equation, the production equation and the definition of marginal cost. Equation 3.22 is the (log-linearized) monetary policy rule.

3.3.1 Solving the model

Following the literature, e.g. Woodford (2001), Nimark (2008) and Melosi (2016), we solve the model by the method of undetermined coefficients. We conjecture that the vector of average expectation of state variables $X^{(0:\infty)}_{t|t}$ follows a VAR(1):

$$\mathbf{X}^{(0:\infty)}_{t|t} = M\mathbf{X}^{(0:\infty)}_{t-1|t-1} + N\epsilon_{\mathbf{t}} \tag{3.23}$$

where $X^{(0:\infty)}_{t|t} \equiv \left[d^{(s)}_t, a^{(s)}_t : s = 0, 1, ...\infty\right]'$. ϵ_t is the vector of exogenous shocks, i.e. $\epsilon_t \equiv [\epsilon_{dt}, \epsilon_{at}]$.

Each individual firm observes a vector of imprecise signal $S_{i,t}$ about the state variable $\mathbf{X}^{(0:\infty)}_{t|t}$:

$$S_{i,t} = D\mathbf{X}^{(0:\infty)}_{t|t} + Qe_{i,t} \tag{3.24}$$

Where $S_{i,t} = [s_{d,i,t}, s_{a,i,t}]$ and $e_{i,t} = [\nu_{d,i,t}, \nu_{a,i,t}]$. Given equation 3.23 and 3.24, each agent forms an estimate about the state variable $X^{(0:\infty)}_{t|t}$. Averaging the individual estimate of the economy, we have a updated law of motion about the state variables.

The vector of endogenous variables $Y_t \equiv [\hat{y}_t, \hat{\pi}_t, \hat{\imath}_t]$ evolves as follows:

$$\mathbf{Y_t} = H\mathbf{X}_{t|t}^{(0:\infty)}$$

We solve for matrix M, N, H such that the equilibrium conditons hold from 3.19 to 3.22 hold. To keep the solution strategy tractable, consistent with the literature, we truncate the infinite order of average expectations up to order $k > 0$. More details are in the Appendix 3.5.3.

3.3.2 Welfare Losses

Under the assumption of the efficient steady state, the unconditional period welfare losses with information friction, up to a second order approximation are:

$$E(L) = E\left\{\frac{\epsilon}{\Theta}\frac{1}{1-\beta\theta}(1-\theta)\int_i (p^\star_{i,t} - p^\star_t)^2 di + \frac{\epsilon}{\Theta}\frac{1}{1-\beta\theta}\frac{\theta}{1-\theta}\pi_t^2 + (\sigma + \frac{\nu+\alpha}{1-\alpha})\tilde{y_t}^2\right\} \tag{3.25}$$

The first term, arising from the dipsersed belief in the model, is proportional to the price dispersion within price setting firms. $p^\star_{i,t}$ is the newly set price of firm i in period t and $p^\star_t$ is the average level of the newly set price. The second term captures the welfare losses associated with the inflation volatility that reflects the price dispersion across all firms as if the newly reset prices are restricted to set the average level $p^\star_t$. The last term is proportional to the output gap volatility. In a model with perfect information, the first term will disappear and the welfare loss function will be equivalent to the standard welfare lossses in the texbook (e.g. Galí (2015)). The detailed derivations and calculations are in the Appendix 3.5.2.

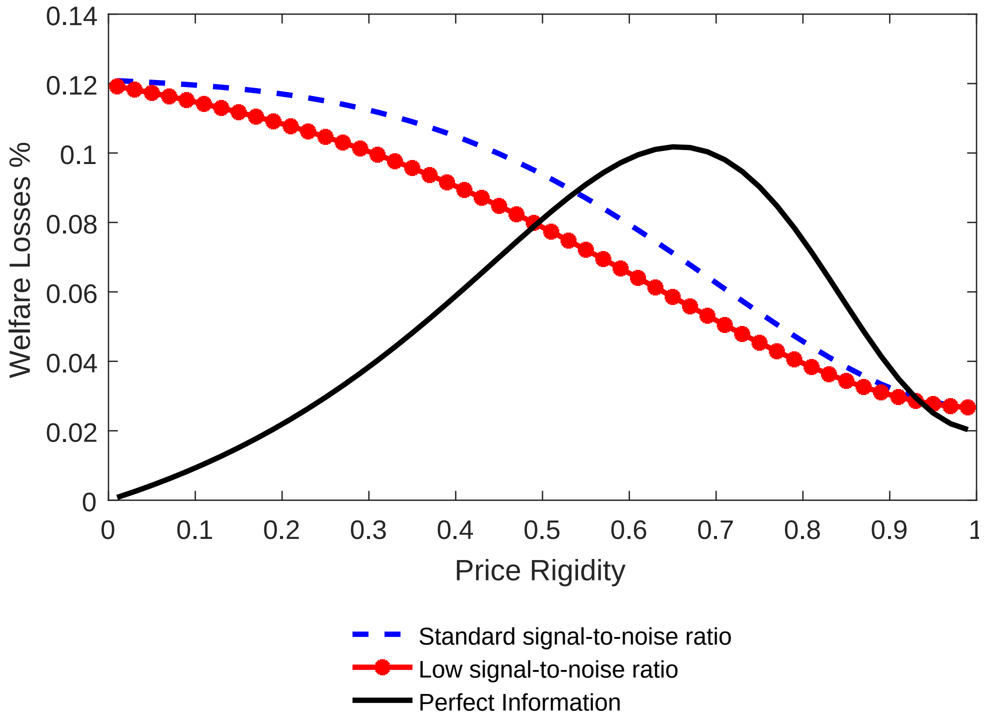
3.3.3 Calibration

The model is calibrated as follows. Each period in the model corresponds to a quarter. The discount factor is 0.99. The inverse Frisch elasticity ν is 1. The constant relative risk aversion σ is 3. The elasticity of substitution across differentiated goods ϵ is 6. The Taylor coefficient on inflation is 1.5 and on the output is 0.125. All the volatilities of shocks are set to be 0.02. Consistent with our static model, in our baseline case, the variances of noise are calibrated such that the Kalman gain is 0.46 to be consistent with the estimate of Coibion and Gorodnichenko (2015). We also consider a low signal-to-noise ratio case under which the Kalman gain is 0.3. These parameters are common in the broad business cycle literature.

3.3.4 Results

Figure 3.4 plots the welfare losses against the price rigidity conditional on technology shocks under different scenarios. The welfare losses in the perfect information model are hump-shaped as we described in the static model. The welfare loss peaks when the price rigidity (probability of remaining the previous price) is around 0.65 which is close to the standard calibration of the price rigidity. According the perfect information model, there is a welfare gain from an increase in the flexibility starting from the rigidity of 0.65. In contrast, based on the impefect information model, there are welfare losses from an increase in price flexibility in both the standard signal-to-noise ratio case (red circled line) and the low signal-to-noise ratio case (blue dashed line). This contrasting result gives rise to quite different policy implications on whether to introduce the new technology to facilitate the price adjustment. If the above perfect information model is the true model, the social planner should encourage firms to undertake the new techonology to faciliate the adujustment cost. As a result, social welfare improves. Constrastly, if the information friction model is right, the social planner would suggest preventing the firms from adopoting the technology.

Figure 3.4: Welfare losses conditional on technology shocks: dispersed belief

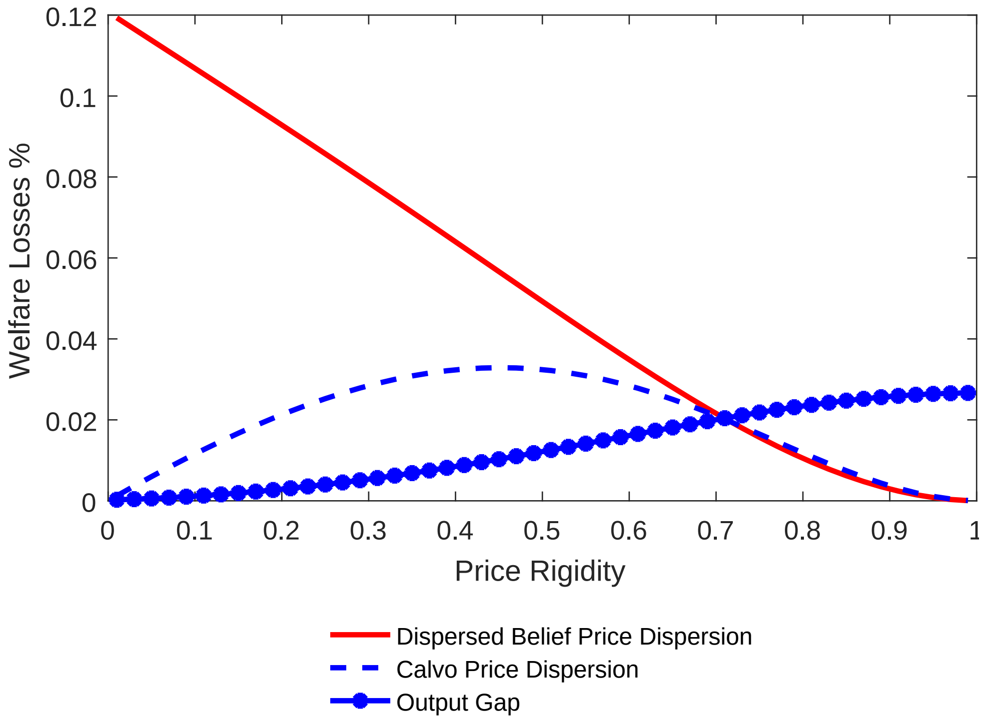


Note: This figure plots the welfare losses as a function of price rigidity under different scenarios. The solid black line presents the perfect information case. The dashed blue line plots the prediction of the model with dispersed beliefs with and signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The red line with circle shows the result if the signal-to-noise ratio is calibrated to match the Kalman gain of $\frac{1}{3}$. The elasticity of substitution across goods ϵ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock σ_a^2 is calibrated to be 0.02.

Figure 3.5 reports each component of the welfare loss function for different values of nominal rigidity. Similar to the static model, the welfare losses associated with the price dispersion within the price resetting firms through the *dispersed belief channel* increases significantly from 0.02% to 0.12% of steady state consumption when the price rigidity is reduced from 0.65 to 0.01. The welfare losses associated with the average inflation volatility and output gap volatility are hump-shaped and increases with the price rigidity respectively, in line with the predictions of the standard textbook model (e.g. Galí (2015)). In our quantitative model, the dispersed belief channel dominates, which generates a welfare loss from an increase in price flexibility.

The welfare losses conditional on the preference shocks share similar patterns and the reasoning. The detailed results are in the Appendix 3.5.1.

Figure 3.5: Welfare losses decomposition conditional on technology shocks



Note: This figure plots the components of welfare loss as a function of price rigidity for the case with dispersed beliefs. The signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The elasticity of substitution across goods ϵ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock σ_a^2 is calibrated to be 0.02.

Discussions There are two issues worth to discuss in our quantitative model. The first is that our results hold even when the monetary policy response is sufficiently strong. In the model of Galí (2013) and Galí and Monacelli (2016), the result that a reduction in wage rigidity can worsen the welfare occurs when the inflation coefficient in the Taylor rule in the closed-economy model is close to unit or when the exchange rate is fixed in a small open economy model. Both models rely on the mechanism that insufficient response of monetary policy will greatly amplify the employment gap volatility when the wage rigidity decreases. However, our results are based on the increase in the welfare losses associated with the price dispersion within the price resetting firms through dispersed belief channel and the output gap volatility is even decreasing when the price rigidity decreases.

Second, our results point to market inefficiency. In our model, from an individual firm's perspective, it would be better off if the firm was allowed to adjust its price more frequently as they would be able to respond to shocks on a timely manner. Therefore, firms would pay for new pricing technologies. However, if all individual firms adopt the new technology to facilitate the price adjustment, the aggregate welfare losses increase as we discussed above. Thus, the constrained social planner, at the presence of the dispersed information, would not introduce the new technology.

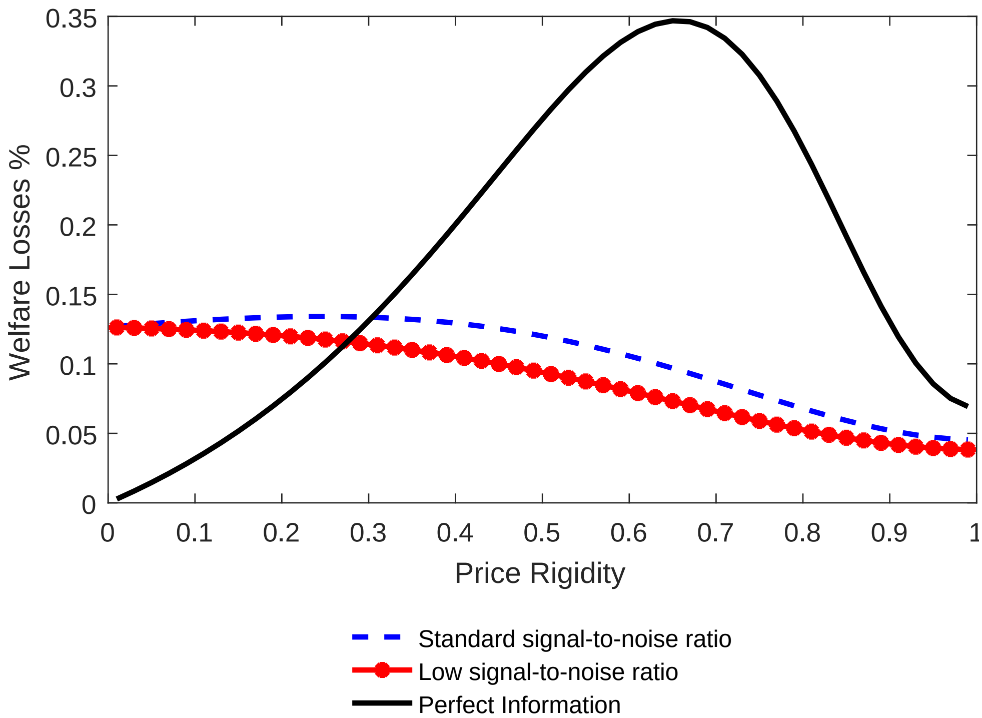
3.4 Concluding Remarks

Is the introduction of digital price tags that may facilitate price adjustment welfare improving? In a New Keynesian model featuring both price rigidity and dispersed information, we show that the answer is no. The dominant underlying mechanism is that a reduction in the price rigidity will amplify the welfare losses associated with the price dispersion within price resetting firms when they have heterogeneous beliefs about the economy. These results add caution to the introduction of the new technology to decrease the price rigidity (e.g. digital price tags).

3.5 Appendices

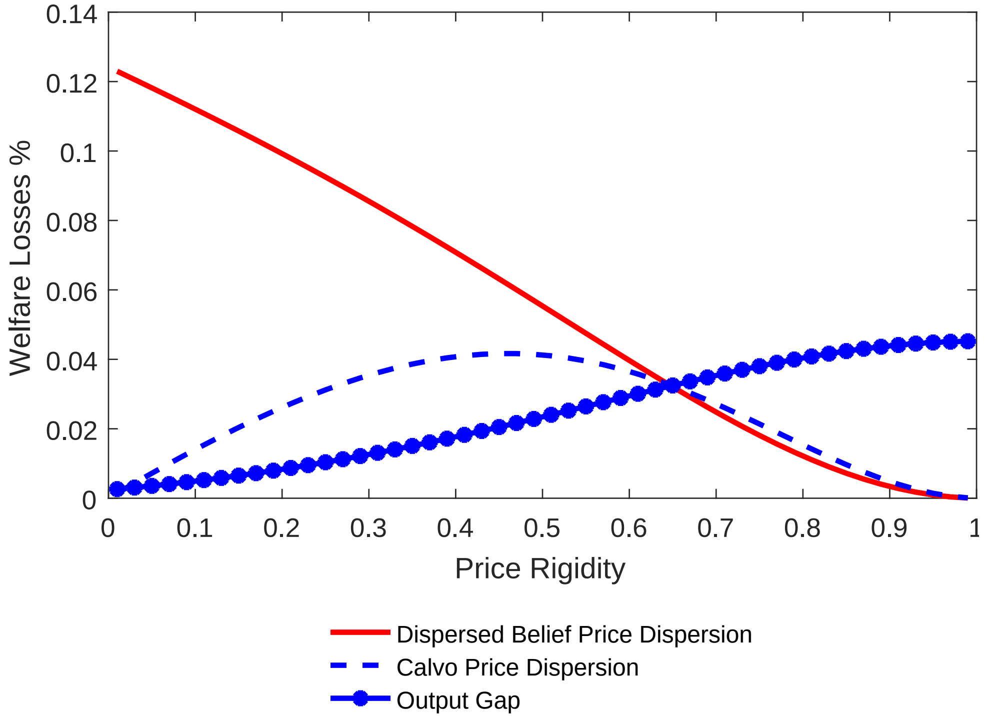
3.5.1 Figures

Figure 3.6: Welfare losses conditional on preference shocks: dispersed belief



Note: This figure plots the welfare loss as a function of price rigidity under different scenarios. The solid black line presents the perfect information case. The dashed blue line plots the prediction of the model with dispersed beliefs with and signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The red line with circle shows the result if the signal-to-noise ratio is calibrated to match the Kalman gain of $\frac{1}{3}$. The elasticity of substitution across goods ϵ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock σ_a^2 is calibrated to be 0.02.

Figure 3.7: Welfare losses decomposition conditional on preference shocks



Note: This figure plots the components of welfare loss as a function of price rigidity for the case with dispersed beliefs. The signal-to-noise ratio calibrated to match a Kalman gain of 0.46. The elasticity of substitution across goods ϵ is calibrated to be 6, which corresponds to a desired markup of 1.2. The volatility of shock σ_a^2 is calibrated to be 0.02.

3.5.2 Welfare Losses

Static Model

The second-order approximation to the consumer's welfare losses can be written and express as a fraction of steady state consumption as

$$L = \epsilon var_i p(i) + \tilde{y}^2 \tag{3.26}$$

We first derive the price dipsersion component $var_i p(i)$.

$$\begin{aligned}
var_i p(i) &= \int_i (p(i) - p)^2 di && (3.27)\\
&= (1-\theta)\int_i (p^\star(i) - p)^2 di + \theta p^2 && (3.28)\\
&= (1-\theta)\left(\int_i (p^\star(i) - p^\star)^2 di + (p^\star - p)^2\right) + \theta p^2 && (3.29)\\
&= (1-\theta)K^2 \int_i e^2(i) di + (1-\theta)\theta p^{\star 2} && (3.30)\\
&= (1-\theta)K^2\sigma_e^2 + (1-\theta)\theta K^2 a^2 && (3.31)
\end{aligned}$$

From the first to the second line, we have used the fact that a fraction of $1-\theta$ of firms set their prices to $p_t^\star(i)$, and a fraction of θ firms keep the original price 0. From the third line to the fourth line, we have used $p^\star(i) = Ks(i)$, $p^\star = Ka$ and $p = (1-\theta)p^\star$.

$$\tilde{y}^2 = \left((1-\theta)K - 1\right)^2 a^2 \tag{3.32}$$

$$E(L) = \epsilon\left\{(1-\theta)K^2\sigma_e^2 + (1-\theta)\theta K^2\sigma_a^2\right\} + \left((1-\theta)K-1\right)^2\sigma_a^2. \tag{3.33}$$

Dynamic Model

The second-order approximation to the consumer's welfare losses can be written and express as a fraction of steady state consumption as

$$\mathbf{W} = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{\epsilon}{\Theta}var_i p_t(i) + (\sigma + \frac{\phi+\alpha}{1-\alpha})\tilde{y_t}^2\right)$$

In the following, fisrt, We will derive the term $E_0\sum_{t=0}^{\infty}\beta^t\frac{\epsilon}{\Theta}var_i p_t(i)$. Second, I will decompose it into two components: the price dispersion within the price resetting firms and that across all firms as if the newly price resetting firms are restricted to set the same average price.

Proposition 3

$$E_0\sum_{t=0}^{\infty}\beta^t\frac{\epsilon}{\Theta}var_i p_t(i) = E_0\sum_{t=0}^{\infty}\beta^t\frac{\epsilon}{\Theta}\frac{1}{1-\beta\theta}[(1-\theta)U_t - \pi_t^2] \tag{3.34}$$

where $U_t = (1-\theta)\int_i (p^{\star}_{i,t} - p_{t-1})^2 di$.

Proof: Let $\Delta_t = var_i p_t(i)$, then

$$\Delta_t = var_i(p_t(i) - p_{t-1}) \tag{3.35}$$

$$= \int_i (p_{i,t} - p_{t-1})^2 di - \left[\int_i (p_{i,t} - p_{t-1})di\right]^2 \tag{3.36}$$

$$= \theta\int_i (p_{i,t-1} - p_{t-1})^2 di + (1-\theta)\int_i (p^{\star}_{i,t} - p_{t-1})^2 di - \pi_t^2 \tag{3.37}$$

$$= \theta\Delta_{t-1} + (1-\theta)\int_i (p^{\star}_{i,t} - p_{t-1})^2 di - \pi_t^2 \tag{3.38}$$

Finally, from Equation 3.38, $E_0\sum_{t=0}^{\infty}\beta^t\frac{\epsilon}{\Theta}var_i p_t(i) = E_0\sum_{t=0}^{\infty}\beta^t\frac{\epsilon}{\Theta}\frac{1}{1-\beta\theta}[(1-\theta)\int_i (p^{\star}_{i,t} - p_{t-1})^2 di - \pi_t^2]$.

Next, we will derive $E(U) = E\int_i (p^{\star}_{i,t} - p_{t-1})^2$.

Lemma 3.5.1

$$p^{\star}_{i,t} - p_{t-1} = \frac{1}{1-\theta}H^{\pi}\mathbf{X}^i_{t|t} \tag{3.39}$$

where $\mathbf{X}^i_{t|t} = [S_{i,t}, E(\mathbf{X}^{(0:\infty)}_{t|t}|I_{i,t})]'$, $S_{i,t} = D\mathbf{X}^{(0:\infty)}_{t|t} + Qe_{i,t}$ *and* H^π *is the row vetor in the H matrix corrsponding to the response of aggregate inflation.*

Proof: Let $p^\star_t = \int_i p^\star_{i,t} di$ and $p^\star_t - p_{t-1} = \frac{\pi_t}{1-\theta} = \frac{1}{1-\theta} H^\pi \mathbf{X}^{(0:\infty)}_{t|t}$. Integrating equation 3.39 in both left hand side and right-hand side gives rise to the above equation.

Note $S_{i,t}$ is the set of signals firm i receive and $E(\mathbf{X}^{(0:\infty)}_{t|t}|I_{i,t})$ is the conditional estimate of the higherachy of average expectations of the state based on each individual firm's information set. Combing measurement equation 3.5.1 and the law of motion for the vector of average expectation, and denote $\mathbf{X}^{(0:\infty)}_{t|t}$ as X_t. Using Kalman filter, we obtain,

$$E(X_t|I_j) \equiv X_{t|t}(j) = MX_{t-1|t-1}(j) + K(S_{t,j} - DMX_{t-1|t-1}(j)) \quad (3.40)$$

$$= (M - KDM)X_{t-1|t-1}(j) + KS_{t,j} \quad (3.41)$$

Then,

$$E\int_i (p^\star_{i,t} - p_{t-1})^2 di = E\int_j (\frac{1}{1-\theta} H^\pi \mathbf{X}^j_{t|t})^2 dj \quad (3.42)$$

$$= \frac{1}{(1-\theta)^2} E\int_j H^\pi \mathbf{X}^j_{t|t} \mathbf{X}'^j_{t|t} H^{\pi'} dj \quad (3.43)$$

$$= \frac{1}{(1-\theta)^2} H^\pi EE_0(\mathbf{X}^j_{t|t} \mathbf{X}'^j_{t|t}) H^{\pi'} \quad (3.44)$$

$$= \frac{1}{(1-\theta)^2} H^\pi \begin{bmatrix} EE_0(S_{j,t}S'_{j,t}) & EE_0(S_{j,t}X_{t|t}(j)') \\ EE_0(S_{j,t}X_{t|t}(j)')' & EE_0(X_{t|t}(j)X_{t|t}(j)') \end{bmatrix} H^{\pi'} \quad (3.45)$$

Where the notation $E_0(X_i)$ is defined as $\int_i X_i di$. In the following derivation, for simplicity, we abuse the notation a bit and E means EE_0. Next we will derive each submatrix in the block matrix $E(\mathbf{X}^{j}_{t|t}\mathbf{X}'^{j}_{t|t})$.

$$
\begin{aligned}
E(S_{j,t}S^{'}_{j,t}) &= DVar(\mathbf{X}^{0:\infty}_{t|t})D' + QQ && (3.46)\\
E(X_{t|t}(j)S^{'}_{j,t}) &= E\left\{\left[(M-KDM)X_{t-1|t-1}(j) + KS_{j,t}\right]S^{'}_{j,t}\right\} && (3.47)\\
&= E\left\{(M-KDM)X_{t-1|t-1}(j)S^{'}_{j,t} + KS_{j,t}S^{'}_{j,t}\right\} && (3.48)\\
&= (M-KDM)E\left[X_{t-1|t-1}(j)S^{'}_{j,t}\right] + KE(S_{j,t}S^{'}_{j,t}) && (3.49)\\
&= (M-KDM)E\left[X_{t-1|t-1}(j)(DMX_{t-1} + DN\epsilon_t + Qe_{j,t})'\right] + KE(S_{j,t}S^{'}_{j,t}) && (3.50)\\
&= (M-KDM)E(X_{t-1|t-1}(j)X^{'}_{t-1})M'D' + KE(S_{j,t}S^{'}_{j,t}) && (3.51)\\
E(X_{t|t}(j)X^{'}_{t}) &= E\left[(M-KDM)X_{t-1|t-1}(j) + K(DX_t + Qe_{j,t})\right]X^{'}_{t} && (3.52)\\
&= E\left[(M-KDM)X_{t-1|t-1}(j)(MX_{t-1})^{'} + KDX_tX^{'}_{t}\right] && (3.53)\\
&= (M-KDM)E(X_{t-1|t-1}(j)X^{'}_{t-1})M' + KDE(X_tX^{'}_{t}) && (3.54)
\end{aligned}
$$

$$
\begin{aligned}
E(X_{t|t}(j)X_{t|t}(j)^{'}) &= E\left[(M-KDM)X_{t-1|t-1}(j) + KS_{j,t}\right]\left[(M-KDM)X_{t-1|t-1}(j) + KS_{j,t}\right]^{'} && (3.55)\\
&= (M-KDM)EX_{t-1|t-1}(j)X_{t-1|t-1}(j)^{'}(M-KDM)' && (3.56)\\
&+ (M-KDM)EX_{t-1|t-1}(j)S^{'}_{j,t}K' && (3.57)\\
&+ KES_{j,t}X_{t-1|t-1}(j)^{'}(M-KDM)' && (3.58)\\
&+ KE(S_{j,t}S^{'}_{j,t})K' && (3.59)
\end{aligned}
$$

where

$$
\begin{aligned}
EX_{t-1|t-1}(j)S^{'}_{j,t} &= E(X_{t-1|t-1}(j)X^{'}_{t-1})M'D' && (3.60)\\
Var(\mathbf{X}^{0:\infty}_{t|t}) &= E(X_tX^{'}_{t}) && (3.61)\\
Var(\mathbf{X}^{0:\infty}_{t|t}) &= MVar(\mathbf{X}^{0:\infty}_{t|t})M' + NVar(\epsilon_t)N' && (3.62)
\end{aligned}
$$

We can further decompose the price dispersion into two components: the price dispersion within the price resetting firms and that across all firms as if the newly price resetting firms are restricted to set the same average price.

$$\Delta_t = \theta\Delta_{t-1} + (1-\theta)\int_i (p^\star_{i,t} - p^\star_t + p^\star_t - p_{t-1})^2 di - \pi_t^2 \tag{3.63}$$

$$= \theta\Delta_{t-1} + (1-\theta)\int_i (p^\star_{i,t} - p^\star_t)^2 di + (1-\theta)\int_i (p^\star_t - p_{t-1})^2 di - \pi_t^2 \tag{3.64}$$

$$= \theta\Delta_{t-1} + (1-\theta)\int_i (p^\star_{i,t} - p^\star_t)^2 di + \frac{\theta}{1-\theta}\pi_t^2 \tag{3.65}$$

The term $\int_i (p^\star_{i,t} - p^\star_t)^2 di$ is the price dispersion among price setting firms. Then,

$$\sum_{t=0}^{\infty}\beta^t\frac{\epsilon}{\Theta}var_i p_t(i) = \sum_{t=0}^{\infty}\beta^t\frac{\epsilon}{\Theta}\frac{1}{1-\beta\theta}[(1-\theta)U_t - \pi_t^2] \tag{3.66}$$

$$= \sum_{t=0}^{\infty}\beta^t\frac{\epsilon}{\Theta}\frac{1}{1-\beta\theta}[(1-\theta)U_t - \frac{1}{1-\theta}\pi_t^2 + \frac{\theta}{1-\theta}\pi_t^2] \tag{3.67}$$

$$= \underbrace{\sum_{t=0}^{\infty}\beta^t\frac{\epsilon}{\Theta}\frac{1}{1-\beta\theta}[(1-\theta)U_t - \frac{1}{1-\theta}\pi_t^2]}_{\text{Dispersed Belief}} + \underbrace{\sum_{t=0}^{\infty}\beta^t\frac{\epsilon}{\Theta}\frac{1}{1-\beta\theta}\frac{\theta}{1-\theta}\pi_t^2}_{\text{Calvo}} \tag{3.68}$$

The first term in the above equation measures the welfare losses induced by the dispersed belief and the second one corresponds to the loss by Calvo pricing friction.

3.5.3 Solution Method

The observation equation is

$$Z_t(i) = L\mathbf{X}_t + Q\nu_t(i), \tag{3.69}$$

where $Z_t(i) = [s_{dt}(i)\ s_{\eta t}(i)\ s_{at}(i)]'$,$\nu_t(i) = [\nu_{dt}(i)\ \nu_{\eta t}(i)\ \nu_{at}(i)]'$and Q is

$$Q \equiv \begin{bmatrix} \sigma_{\nu_d} & 0 & 0 \\ 0 & \sigma_{\nu_a} & 0 \\ 0 & 0 & \sigma_{\nu_\eta} \end{bmatrix}.$$

As is decribed earlier, the law of motion for state is

$$\mathbf{X}_t = M\mathbf{X}_{t-1} + N\epsilon_t \tag{3.70}$$

Define variance matrix for the noise

$$\Sigma_{\nu\nu} \equiv QQ^{'}.$$

Define variance matrix for the shock

$$\Sigma_{\epsilon\epsilon} \equiv N \begin{bmatrix} \sigma^2_{\epsilon_d} & 0 & 0 \\ 0 & \sigma^2_{\epsilon_a} & 0 \\ 0 & 0 & \sigma^2_{\epsilon_\eta} \end{bmatrix} N'.$$

Equations (3.69) and (3.70) consist of linear space which calls for the Kalman filter.

The steady state Kalman filter gives

$$\begin{aligned} \mathbf{X}^{(1)}_{t|t}(i) &= (M - KLM)\mathbf{X}^{(1)}_{t-1|t-1}(i) + KZ_t(i), && (3.71) \\ &= (M - KLM)\mathbf{X}^{(1)}_{t-1|t-1}(i) + K[LM\mathbf{X}_{t-1} + LN\epsilon_t + Q\mathbf{e}_t(i)], && (3.72) \end{aligned}$$

where

$$K_l = PL'(LPL' + \Sigma_{\nu\nu})^{-1},$$

with

$$P = M(P - PL'(LPL' + \Sigma_{\nu\nu})^{-1}LP)M^{'} + \Sigma_{\epsilon\epsilon}.$$

Take average over individual expectation, we have

$$\mathbf{X}_{t|t}^{(1)} = (M - KLM)\mathbf{X}_{t-1|t-1}^{(1)} + KZ_t \quad (3.73)$$
$$= (M - KLM)\mathbf{X}_{t-1|t-1}^{(1)} + K[LM\mathbf{X}_{t-1} + LN\epsilon_t] \quad (3.74)$$

Recall that the shocks evolve as

$$\begin{bmatrix} d_t \\ \eta_t \\ a_t \end{bmatrix} = \rho \begin{bmatrix} d_{t-1} \\ \eta_{t-1} \\ a_{t-1} \end{bmatrix} + R\epsilon_t, \quad (3.75)$$

where $\rho \equiv \begin{bmatrix} \rho_d & 0 & 0 \\ 0 & \rho_\eta & 0 \\ 0 & 0 & \rho_a \end{bmatrix}$ and $R \equiv \begin{bmatrix} \sigma_{\epsilon_d} & 0 & 0 \\ 0 & \sigma_{\epsilon_a} & 0 \\ 0 & 0 & \sigma_{\epsilon_\eta} \end{bmatrix}$.

Equation (3.73) and equation (3.75) can fully characterize the matrices M and N as follows

$$M = \begin{bmatrix} \rho & \mathbf{0}_{3\times 3J} \\ \mathbf{0}_{3J\times 3} & (M - KLM)\mid_{(1:3J,1:3J)} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times 3(J+1)} \\ KLM\mid_{(1:3J,1:3(J+1))} \end{bmatrix}, \quad (3.76)$$

and

$$N = \begin{bmatrix} R \\ \mathbf{0}_{3J\times 3} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times 3} \\ KLN\mid_{(1:3J,1:3)} \end{bmatrix}. \quad (3.77)$$

When firms don't hvae endogenous signals, the problem of solving the law of motion for state and policy function can be seperated. Equation (3.76) and (3.77) can be used to iterate to obtain a fixed point for M and N.

Given the law of motion for state, we can compute for policy functions. Let $\hat{y}_t = \mathbf{a}\mathbf{X}_{t|t}^{(0:J)}$ and $\hat{\pi}_t = \mathbf{b}\mathbf{X}_{t|t}^{(0:J)}$ as output and inflation are a linear function of hierachy of state variables. Let $\mathbf{s}_t \equiv [\hat{y}_t\ \hat{\pi}_t]'$ and $\mathbf{v}_0 \equiv \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}$.

Define matrix $T^{(s)}$ as follows:

$$T^{(s)} = \begin{bmatrix} \mathbf{0}_{3(J-s+1)\times 3s} & I_{3(J-s+1)} \\ \mathbf{0}_{3s\times 3s} & \mathbf{0}_{3s\times 3(J-s+1)} \end{bmatrix}.$$

Then we have the following important results for higher order expectation of endogenous variables

$$
\begin{aligned}
\mathbf{s}_{t|t}^{(s)} &= \mathbf{v}_0 T^{(s)} X_{t|t}^{(0:J)}, \\
\mathbf{s}_{t+h|t}^{(s)} &= \mathbf{v}_0 M^h T^{(s)} X_{t|t}^{(0:J)}.
\end{aligned}
$$

for any $0 \leq s \leq J$.

Let

$$\tau \equiv [1\ 0\ 0\ \mathbf{0}_{1\times 3J}],$$

$$G \equiv [\mathbf{0}_{1\times 3s}\ 0\ 1\ 0\ \mathbf{0}_{1\times 3(J-s)}],$$

$$\chi \equiv [0\ 0\ 1\ \mathbf{0}_{1\times 3J}].$$

Recall that IS curve and NKPC after the substitution of policy function are

$$\mathbf{a}X_{t|t}^{(0:J)} = \mathbf{a}MT^{(1)}X_{t|t}^{(0:J)} - \frac{1}{\sigma}(\phi_\pi \mathbf{b}X_{t|t}^{(0:J)} + \phi_y \mathbf{a}X_{t|t}^{(0:J)} + \chi X_{t|t}^{(0:J)} - \mathbf{b}MT^{(1)}X_{t|t}^{(0:J)}) + (\rho_d - 1)\gamma X_{t|t}^{(0:J)}, \tag{3.78}$$

$$\mathbf{b}X_{t|t}^{(0:J)} = (1-\theta)(1-\beta\theta)\sum_{j=1}^{J}(1-\theta)^{j-1}((\sigma+\nu)\mathbf{a}T^{(j)}X_{t|t}^{(0:J)} - (1+\nu)GX_{t|t}^{(0:J)}) + \beta\theta\sum_{j=1}^{J}(1-\theta)^{j-1}\mathbf{b}MT^{(j)}X_{t|t}^{(0:J} \tag{3.79}$$

which lead to

$$\mathbf{a} = \mathbf{a}MT^{(1)} - \frac{1}{\sigma}(\phi_\pi \mathbf{b} + \phi_y \mathbf{a} + \chi - \mathbf{b}MT^{(1)}) + (\rho_d - 1)\tau, \tag{3.80}$$

$$\mathbf{b} = (1-\theta)(1-\beta\theta)\sum_{j=1}^{J}(1-\theta)^{j-1}((\sigma+\nu)\mathbf{a}T^{(j)}-(1+\nu)G)+\beta\theta\sum_{j=1}^{J}(1-\theta)^{j-1}\mathbf{b}MT^{(j)}. \tag{3.81}$$

Iterating over $\mathbf{a}$ and $\mathbf{b}$ according to equation (3.80) and (3.81) until they converge gives rise to the solution for policy function.

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