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Essays in Commodity Price Shocks in Small Open Economies

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ABSTRACT

This thesis contributes to the understanding of the effect and transmission of commodity price shocks in commodity-exporting economies. The first chapter studies the role of domestic production linkages between the commodity sector and the rest of the economy. It starts by documenting a dampening effect over GDP generated by stronger connections of the commodity sector, either as customer or supplier of intermediate goods. To rationalize this fact, I build a real business cycle model with production linkages for a small open economy that produces commodity goods, with an empirical application for the commodity boom of the 2000s. The second chapter explores the interaction between nominal rigidities and production linkages for the transmission of the shock. The main finding is that, depending on the source of nominal rigidities, there could be either dampening of amplification, while production linkages unambiguously dampen the effect. The third chapter explores the role of household heterogeneity for the transmission of the shock. Building a two-agent New Keynesian model with non-homothetic preferences, I find that the response of consumption to an increase in the commodity price is dampened, relative to a model with homothetic preferences, because the increase in income induces a reallocation in the consumption basket towards more income-elastic and expensive goods.

RESUMEN

Esta tesis contribuye al entendimiento del efecto y canales de transmisión que tienen los *shocks* en precios de materias primas en países exportadores de estos productos. El primer capítulo estudia el rol de redes de producción doméstica entre el sector primario y el resto de la economía. Se documenta que un mayor nivel de interrelación del sector de materias primas con el resto de la economía, tanto como comprador o vendedor de bienes intermedios, amortigua el efecto del *shock* sobre el PIB. Para explicar este hecho, uso un modelo de ciclos reales con redes de producción, con una aplicación para el *boom* en el precio de materias primas en la década de 2000. El segundo capítulo explora la interacción entre rigideces nominales y redes de producción para la transmisión del *shock*. El resultado principal es que, dependiendo de la fuente de rigideces nominales, puede existir tanto amortiguación como amplificación del efecto, mientras que las redes productivas inambiguamente generan amortiguación. El tercer capítulo explora el rol de la heterogeneidad de hogares para la transmisión del *shock*. En base a un modelo Neo-Keynesiano de dos agentes con preferencias no homotéticas, encuentro que la respuesta del consumo ante un incremento en el precio de las materias primas exportadas es amortiguado en relación a un modelo con preferencias homotéticas, debido a que el incremento en ingreso induce una reasignación en la canasta de consumo hacia bienes ms elásticos al ingreso y más caros.

Preface

Fluctuations in the terms of trade have been understood as an important driver of the business cycle in small open economies. At the same time, the exporting structure of those countries is largely concentrated in few products, specially commodity goods. One key feature of these economies is that they are small relative to the rest of the world, so they take the price of these commodities as given in international markets. Therefore, the analysis of the effects of commodity price shocks and their propagation mechanisms is key to understand the business cycle in emerging countries. In this regard, the theoretical literature has remained silent about several empirical aspects that could affect such transmission mechanisms. This dissertation contributes by studying the consequences of incorporate such empirical elements into the analysis. In particular, my work studies the role of the domestic productive structure, through input-output linkages, its interaction with nominal rigidities, and the role of household heterogeneity for the transmission of commodity price shocks.

The first chapter start by documenting a novel empirical fact for a panel of commodity-exporting economies for the period 2000-2015: in countries in which the commodity sector is more connected to the rest of the economy through production linkages, the effect of commodity price shocks over GDP is dampened. This is, in an economy in which the commodity sector has a more important role either as customer or as supplier of intermediate goods to other sectors, the response of GDP to the shock is positive but smaller. To rationalize this fact, I build a static real business cycle model with production linkages (input-output) for a small open economy that produces commodity goods and explore the mechanism behind the dampening effect of those linkages. Under perfect competition, an increase in the commodity price raises wages, the marginal cost of production and the price of other sectors that demand commodity for production. Because the commodity sector is also a customer of the rest of the economy and a price taker of its good, the only way to keep its marginal cost sufficiently low is by decreasing its demand for inputs. In this way, a more linked economy experiences a milder increase on income relative to a less linked economy. On an application of the model to the commodity price boom of the 2000s under counterfactual scenarios for production linkages of the commodity sector, I find that the volatility of GDP in a representative commodity-exporting country would have been 6% lower if the commodity sector had been 10% more connected.

One of the main features in models with input-output linkages is the transmission of changes in prices from one sector to the other, affecting their marginal cost of production. Therefore, a natural question is how the transmission mechanism is affected when there exist frictions in the price setting of producers, induced by nominal rigidities. In the second chapter, I explore the interaction of nominal rigidities and production linkages in a New Keynesian model calibrated for Chile. One key element in my analysis is to differentiate between nominal rigidities in importable goods, which are necessary for domestic production, and in mainland sectors. A positive commodity price shock induces an increase in the demand for labor, so an increase in wages, and a real exchange appreciation. With production linkages such variations are transmitted across sectors influencing the marginal cost of customer sectors. When there are nominal rigidities, such transmission is incomplete because prices do not fluctuate one-to-one with marginal costs. Making the distinction about the sources of nominal rigidities matter for the aggregate effects of the shock: nominal rigidities in mainland sectors (such as manufactures and services) but not in imports, dampen the impact of the shock by 48% and 25% for consumption and output, respectively, while having nominal rigidities in the importable sector but not in mainland amplifies the impact by 72% and 46% in the same variables. As in the case of the model presented in the first chapter, production linkages dampen the responses to the shock. By analyzing an extension of the model with other sources of fluctuations, I find that shocks that directly affect marginal costs make production linkages and nominal rigidities more relevant to understand aggregate fluctuations.

Finally, the third chapter studies the role of household heterogeneity for the transmission of the shock. Using microdata from Chile and other commodity exporting economies, I document that low-income households spend relatively more on food, while high-income households spend relatively more on services. This evidence is consistent with literature documenting similar patterns in developed economies, or for some special emerging economies. Because the commodity price shock operates as an exogenous source of income in the economy, the previous empirical observation motivates me to analyze the role of household heterogeneity as a transmission mechanism. To keep the modeling approach as simple as possible and incorporate the consumption patterns that I observe in the data, I build a two-agent New Keynesian (TANK) model with non-homothetic preferences, in the spirit of Hanoch (1975) and Comin et al. (2019). The key element of these non-homothetic preferences is that expenditure shares not only depend on prices, but on the level of consumption of each household, hence on its level of income, with different income-elasticities for each good. With these preferences, the cost of an extra unit of consumption is not only given by the price of the consumption basket, which now is household-specific, but by the changes in the average income elasticity. A positive commodity price shock increases income for every household and generates a reallocation in consumption towards more income-elastic goods. In this regard, the shock induces a larger level of expenditures, which is more concentrated in income-elastic goods such as services. On the aggregate, I find that non-homothetic preferences dampen the effect of the commodity price shock: comparing with a counterfactual economy with homothetic preferences, the latter generates aggregate responses in consumption that are 50-80 percent larger.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

1

DOMESTIC LINKAGES AND THE TRANSMISSION OF COMMODITY PRICE SHOCKS

1.1 INTRODUCTION

Terms of trade and commodity price fluctuations are important driving forces in emerging economies. While Mendoza (1995) and Kose (2002) find that at least 30% of the variance of output is explained by terms of trade shocks, Fernandez et al. (2017) and Fernandez et al. (2018) show that commodity price shocks explain close to 50% of this variance. Two main characteristics help to explain these aggregate results. First, commodities represent half of the export basket of emerging countries on average. Therefore, fluctuations in commodity prices are very correlated to terms of trade fluctuations. At the same time, the commodity sector has been seen as relevant for its relative size in the economy. This is, the commodity sector in emerging economies is *central*. Second, these economies are small relative to the rest of the world, so they take the price of those products as given from abroad. This implies that fluctuations in commodity prices can be seen as exogenous shocks for these countries. In the light of the boom and bust cycle in commodity prices experienced in the 2000s decade, and the recent trade war between China and the U.S., the role of fluctuations in commodity prices in emerging economies has been revisited. However, beyond the consensus about their aggregate importance, the macroeconomic consequences of those fluctuations and their transmission channels are still a matter of discussion.

This paper contributes to the literature by studying the role of domestic input-output (IO) linkages in the transmission of commodity price shocks to GDP. On the empirical side, using a panel of commodity exporting emerging economies I find that commodity price fluctuations are positively correlated with GDP, in line with previous literature. At the same time, controlling by the centrality of the commodity sector proxied by its size, I find that when this sector is relatively more important for the overall economy, the effect of commodity prices is amplified. This is in line with the empirical literature that studies IO linkages and the role of centrality measures in the transmission of shocks (Acemoglu et al., 2016). Once I also interact commodity price fluctuations with measures of linkages between the commodity sector and the rest of the economy (i.e., how important is the commodity sector either as customer or as supplier), two results emerge. First, the importance of the centrality measure is severely reduced and becomes not statistically significant. Second, the interaction terms with measures of linkages is negative. That is, stronger linkages between the commodity sector and the mainland economy dampen the effect of commodity prices. This result holds even when controlling for unobserved factors (country and year fixed effects), and is robust to different specifications, treatments of the data, and definitions of variables.

To further understand the mechanisms behind this dampening effect, I build a theoretical model for a commodity exporting economy with IO linkages in sectoral production. It has three main ingredients. First, I assume that the country is a small open economy. This implies that the price of the commodity good that the economy produces and exports is exogenously given from abroad. Second, on the production side of the model, I assume a multisector economy with production linkages. Every industry in the economy is inhabited by a representative firm that produces a differentiated good and demands labor, an imported good and domestic materials. Every firm produces with a constant returns to scale technology and acts competitively (both in the market of inputs and in the market where it sales its output). Those domestic materials correspond to output from all the sectors in the economy and their relative importance is given by the IO structure of the economy. Finally, I assume that the linkages described by the IO matrix are technologically given and fixed. This implies that after any shock, the relative importance of each sector as supplier of others does not change.

In terms of the mechanism, the model operates as follows. An increase in the commodity price makes the sector to produce more, boosting the demand for factors and increasing the equilibrium wage. This represents a positive income effect, that is reflected as an increment in GDP. In an economy in which the commodity sector is linked, this is, is a customer and a supplier of other sectors, we observe additional forces at play. With the increase in the commodity price, the marginal cost of downstream sectors (i.e., those who demand commodity for production) increases. By perfect competition, their equilibrium price raises. This itself increases the marginal cost of production of the commodity sector, which is a customer of the rest of the economy. Because the commodity price is taken as given, the only way to keep its marginal cost sufficiently low (i.e., equal to its price), is by decreasing its demand for inputs, including labor. Therefore, an economy in which the commodity sector is more connected, a positive commodity price shock increases GDP but by less than in an economy with an isolated commodity sector. This is the dampening effect of IO linkages on commodity price fluctuations.

I provide some examples for a two-sector economy to illustrate the importance of IO linkages. In a baseline exercise with linkages between sectors, the elasticity of GDP to commodity prices is 1.31. In an economy without those linkages, this elasticity is 67% larger (2.18). By considering the case in which the commodity sector is only a customer of the rest of the economy, this elasticity is 21% larger than baseline (1.59). In the case in which the commodity sector is only a supplier, this elasticity is 53% larger (2.00). These back-of-the-envelope calculations show that this mechanism may have an important quantitative effect.

I further explore the implications of IO linkages of the commodity sector for GDP volatility. Focusing in the commodity boom of the 2000s, and assuming that the economy is subject to both commodity price and aggregate productivity shocks, I am able to recover the level of productivity shocks that exactly matches the observed GDP for each economy, conditional on the observed commodity prices. Then, given the fundamentals on each country, I use the model to simulate different counterfactuals for the degree of linkages with the mainland economy to analyze the impact of linkages over the volatility of GDP. The results highlight two key asymmetries in the role of IO linkages for the transmission from commodity prices to GDP. First, changes in the intensity of the commodity sector as customer generates larger differences in volatility than changes in the intensity as supplier, in line with the simple calculations described above: a 10% decrease in demand for domestic goods by the commodity sector, increase the volatility of GDP with respect to the baseline calibration by a factor of 2.1. The same percentage decrease in the demand of other sectors for commodity goods, imply a volatility only 1.5 times larger. Second, decreases in the intensity of linkages generate larger changes in volatility than increments: a 10% decrease in either of the measures considered reduce the volatility of GDP in less than 6%. These results highlight the quantitative importance of domestic linkages for the commodity sector with the rest of the economy for the transmission of commodity price shocks.

Relation to the literature This paper is related to two main strands of literature. First, it relates to studies that evaluate the impact of terms of trade and commodity price shocks in small open economies. Starting with the seminal contributions of Mendoza (1995) and Kose (2002), different papers have evaluated alternative channels to understand the transmission of these kinds of shocks.[1] Most related to my paper is the literature that has considered the possibility that the commodity sector demands factors from the non-commodity economy. Bergholt and Hoghaug Larsen (2016) studies a medium-scale New Keynesian model adapted for Norway, where the oil-supply sector produces using labor, capital and intermediate inputs. Those intermediates are a composite of manufactures and services. Something similar occurs in Fornero et al. (2016) for the case of Chile, in which the copper industry demands non-mining output to produce investment goods. Finally, Caputo and Irarrazaval (2017) study a real business cycle model that considers a composite of tradable and non-tradable goods as a productive factor of the copper industry in Chile. All these papers assume that the commodity output is fully exported abroad. My contribution to this literature is twofold. First, I consider the dual role of the commodity sector for the transmission of shocks (customer and supplier). Because previous papers do not consider the supplier role of the commodity sector, they do not take into ac-

[1]Some examples are the role of fiscal policy (Pieschacon, 2012; Céspedes and Velasco, 2014; Medina and Soto, 2016), monetary policy (Catão and Chang, 2015), prudential policies (Garcia-Cicco and Kawamura, 2015), exchange regimes (Broda, 2004; Edwards and Levy Yeyati, 2005; Céspedes and Velasco, 2012), trade imbalances (Kohn et al., 2020), and the financial implications of commodity price fluctuations (Shousha, 2016; Alberola and Benigno, 2017; Drechsel and Tenreyro, 2018).

count the direct effect that commodity prices have over marginal cost and prices of other sectors, beyond the general equilibrium effects of wages or the user cost of capital. In my paper, I consider this additional dimension of the commodity sector, showing that taking into account its role as supplier for the rest of the economy is both qualitative and quantitative important. Second, while the papers cited before only mention that the commodity sector buys inputs from the rest of the economy, they do not explore the importance of IO linkages for the transmission or amplification of shocks, neither theoretically nor empirically. Relative to these papers, I provide empirical evidence about the importance of domestic IO linkages for the transmission of commodity price shocks. To the best of my knowledge, this is the first paper that presents evidence about a dampening effect of linkages by focusing on the case of commodity price shocks in small open economies. Also, I explore a theoretical reasoning behind this phenomena, going beyond a quantitative exercise to understand this dampening effect.

Second, this paper applies the insights from the literature on the propagation of shocks in production networks for closed economies (Foerster et al., 2011; Acemoglu et al., 2012, 2016; Atalay, 2017) to a small open economy context. In particular, I construct a similar model to the one presented in the seminal contribution of Long and Plosser (1983), adapted to a commodity exporting economy. With this model, I study how IO linkages permeate commodity price shocks. While these papers have focused mostly in the transmission of technology shocks, both aggregate and sectoral, my work studies a foreign shock in the form of commodity price fluctuations. Even though is hard to interpret these shocks as supply or demand shocks, I show that they permeate both downstream and upstream, differently to the case of Acemoglu et al. (2016), in which demand shocks go only upstream (to input-supplying industries), while supply shocks go only downstream (to customer industries). Also, assuming perfect competition as other papers do, I show that IO linkages dampen commodity price shocks. Relative to these papers, I study how IO affect the amplification of shocks when rigidities are taken into account.[2] Another departure of this literature is the notion of the centrality of the sector as a sufficient statistic to understand the impact of shocks (Hulten, 1978). While other papers have broken Hulten's theorem,[3] in this paper this result does not hold because of the nature of the shock.

The rest of the paper is organized as follows. Section 1.2 describes the data used in the paper and presents motivating evidence about the importance of domestic linkages in the transmission of commodity price shocks, showing the dampening effect of linkages. Section 1.3 rationalizes this evidence by presenting the theoretical model. Section 1.4 explores in detail the mechanisms for a simplified economy and generalizes the dampening effect of linkages for commodity prices for an arbitrary economy. Section 1.5 provides a quantitative application of the model to analyze the impact of IO linkages of the commodity sector over the volatility of GDP. Finally, section 1.6 concludes.

1.2 EMPIRICAL EVIDENCE

This section documents the importance of commodity sector connections with the rest of the economy, in the form of input-output (IO) linkages, for the transmission of commodity price shocks. First, I describe the empirical setting to analyze the relation between business

[2]In this paper we can interpret the small open economy assumption for the commodity price as an extreme case of a rigidity in sectoral prices.

[3]For example, Baqaee and Farhi (2019) finds that impact of sectoral productivity shocks are better captured only by higher order approximations of the model. This is because up to a first order, all the nonlinearities inherited by the model are not relevant.

cycle fluctuations and domestic IO linkages. Then I present the data used in the analysis and the main empirical results.

1.2.1 *Setting*

Conventional wisdom says that commodity price shocks matter for business cycle fluctuations of a commodity exporting economy because this sector is large relative to the rest of the economy and an important source of income. In this sense, the commodity sector can be seen as *central* for the aggregate economy, an Hulten's theorem applies.[4] This fact is what has motivated most of the work studying emerging economies with a commodity sector.

However, this view ignores other channels by which commodity price shocks can affect the economy. In this paper I try to fill this gap by offering an alternative transmission mechanism: IO linkages. The existence of those linkages imply that after a positive commodity price shock there is an increase in demand for sectoral output of the mainland economy (i.e., all sectors but commodity), which has effects over aggregate activity and prices. The focus in this paper is to study the impact that commodity prices have over real GDP of small open economies, and how the domestic productive structure shape those responses. To investigate this, I estimate variants of the following reduced form equation

$$\begin{aligned} y_{it} = & \gamma_0 p_{it-1} + \gamma_1 (p_{it-1} \times \text{Size}_i) + \gamma_2 (p_{it-1} \times \text{Down}_i) \\ & + \gamma_3 (p_{it-1} \times \text{Up}_i) + \theta X_{it-1} + \delta_i + \delta_t + \nu_{i,t}, \end{aligned} \tag{1.1}$$

where y_{it} is a measure of real GDP of country i in year t, and p_{it} is a measure of the commodity price relevant for country i. Size measures the relative size of the commodity sector in the overall economy,

[4]This is, the relative size of a sector is a sufficient statistic to understand the effect of shocks over aggregate GDP. See Hulten (1978).

and proxies for its centrality. Vector X_{it} includes different controls, and δ_i and δ_t are country and year fixed effects respectively. To avoid any potential concern about endogeneity, I use all regressors lagged one period. To make the data stationary, I consider the log-deviation of each variable with respect to a quadratic trend. Later on, I show that the qualitative insights remain if we consider other alternatives.

Linkages. The key element under study is the relevance of IO linkages between the commodity sector and the rest of the economy. Those links are captured in reduced form by the terms Down_i and Up_i in (1.1)–downstream and upstream measures of connections. The downstream measure captures the importance of the commodity sector as *customer* of the rest, and is measured as the share of total commodity output coming from materials produced by other domestic sectors. On the other hand, the upstream measure captures the importance of the commodity sector as *supplier* for other industries, and is computed as the fraction of commodity output sold as materials to other industries. Let $\text{Sales}_{i,j\to co}$, $\text{Sales}_{i,co\to j}$ and $\text{Sales}_{i,co}$ denote sales from industry j to the commodity sector (co), sales from the commodity sector to industry j, and total sales of the commodity sector in country i, respectively. Then the downstream and upstream measures are constructed as

$$\text{Down}_i = \frac{\sum_{j\neq co}^{N} \text{Sales}_{i,j\to co}}{\text{Sales}_{i,co}} \quad \text{and} \quad \text{Up}_i = \frac{\sum_{j\neq co}^{N} \text{Sales}_{i,co\to j}}{\text{Sales}_{i,co}}.$$

There are three elements to emphasize in the computation of these measures. First, in the data the fraction of sales within a sector is not negligible across countries, which is captured my large diagonal elements in the IO tables. This is particularly important for the commodity sector: on average, sales within this sector represent 14% percent of its output (median value of 8%). Because I want to capture the pure relation between commodity and other sectors, I do not consider these within-commodity sector sales, so I impose the condition $j \neq co$ in the expressions above. Second, both downstream and upstream measures are intended to capture technological relations between sectors. Therefore, I consider a fixed measure for these two variables at the country-level. This is supported by the fact that the relations inherited in IO tables are stable over time, even though both the numerator and the denominator are changing.[5] Third, the focus of the paper is on the aggregate effect of commodity price shocks over GDP, and how production linkages between the commodity sector and the rest of the economy shape those responses. Therefore, even though the nature of the question is aggregate, the channel of interest has to be measured at the sectoral level. For this reason, I consider the downstream and upstream measures previously described just as proxies for connections between the commodity sector and the rest of the economy. As was mentioned before, while the downstream measure captures the importance of domestic materials for the commodity sector, the upstream measure captures the importance of sales to other sectors for the commodity sector. Therefore, these are proxies for the different roles that the commodity sector may have. In particular, they help to test how departures of the assumption of a commodity sector using just primary factors or an endowment commodity sector (in both cases Down would equal zero) and departures of the fully exported commodity sector (in which case Up would equal zero) shape aggregate responses to commodity price fluctuations.

Controls. The vector X_{it} includes several controls for the relation between commodity prices and GDP. In particular, it includes the log of GDP per capita, to capture the level of development of each country and as a proxy for the quality of institutions that could be affecting GDP. Motivated by the results in Broda (2004) and Edwards

[5] Qualitative results are robust to time-varying measures.

and Levy Yeyati (2005) that find that economies with more rigid exchange rate respond more to terms of trade shocks, it includes a measure of the exchange rate regime on each country to control for the differential effect of management in exchange rates, and how this isolates the economy from foreign shocks.[6] Finally, it considers the cyclical component of domestic CPI and bilateral exchange rate with respect to the US dollar to control for other covariates that might affect the business cycle of these small open economies. Additionally, I include country and time fixed effects to control for unobserved characteristics that could affect GDP. In particular, country fixed effects capture, among other things, macroeconomic policies, country-level aggregate volatility, country size and population, and the level of income. On the other hand, time fixed effects capture, among other things, common shocks across countries beyond the effect of commodity prices.

1.2.2 *Data*

Selection criteria. I construct a yearly panel of 34 emerging economies for the period 1990-2015. These countries are characterized by an average GDP per capita during 1990-2015 below 25,000 dollars, in line with the classification used in Schmitt-Grohe and Uribe (2018) for emerging economies. Also, for a country to be included, it must have information of real GDP per capita for the whole sample period. This is to avoid short-run panels that could bias the estimation. The countries selected are those mostly specialized in energy and mineral commodities, such as oil or copper. The list of countries and descriptive statistics is presented in Appendix 1.A.[7]

[6] I experiment with additional variables such as fiscal ciclicality (to control for institutional responses to the windfall behind commodity price shocks). While not reported, these controls do not change the main qualitative insights of the paper.

[7] The definition of a "Commodity Dependent Developing Country" in UNCTAD (2019) correspond to countries in which the share of commodity exports to

Macroeconomic data and controls. I obtain data of real GDP per capita, aggregate GDP, CPI and nominal exchange rates from the World Bank's World Development Indicators (WDI) database. The intensity of commodity exports comes from the UN's Comtrade Database. From this dataset, I obtain total exports and exports of different commodity products in order to mimic the basket of goods characterizing the commodity price index described below. To be consistent with the definitions for commodity prices on each country, I classify as commodity products those coming from the same set of goods defined by the commodity price index (see details below). Finally, for the exchange rate regime, I use the recently developed database by Ilzetzki et al. (2019) which summarizes de facto exchange rate arrangements. I combine their coarse classification codes to construct a dummy variable that takes a value of zero if a country-year pair has a peg exchange rate arrangement, and one otherwise (floating exchange rate).[8]

Commodity prices. Because the exporting structure is heterogeneous for this group of countries, it could be misleading to consider just one commodity price to run the analysis.[9] In my approach I use the International Monetary Fund (IMF) database constructed by Gruss and Kebhaj (2019). They build a single commodity price index

total merchandise exports is above 60 percent. My classification, although different, selects a similar set of countries.

[8]In concrete, pegs are defined by any of the following classification: no separate legal tender, pre announced pegs or crawling pegs, and de facto crawling pegs. Floating regimes are defined by any of the following: pre announced or de facto crawling bands, moving bands, managed floating or freely floating regimes.

[9]Recently, Fernandez et al. (2017) argue that only one foreign price, such as terms of trade, explains a small fraction of the variance in domestic output (less than 10 percent). Even though I consider a model with one price only, this summarizes in a better way the importance of individual commodity prices. This has been confirmed recently by Fernandez et al. (2018), which finds that the median share of the forecast error variance decomposition of GDP in Brazil, Chile, Colombia and Peru is 50%.

for 182 economies in the period 1962-2018 at a monthly and yearly frequencies. They use 45 commodity prices, broadly classified into agricultural raw materials, energy, food and beverages, and metals, which are aggregated using different possible weights. In my baseline exercise, I use the version of the index that aggregates individual prices using fixed weights over the 1962-2018 period, corresponding to the average of commodity export shares over total exports. As I will show latter on, my results are robust to aggregate with time-varying weights. For each commodity, real prices are constructed as the commodity price in US dollars divided by the IMF's unit value index for manufactured exports.

Input-output data. A key element in the analysis corresponds to IO tables to measure the downstream and upstream connections of the commodity sector with the rest of the domestic economy. Unfortunately, most of the databases that construct such information for different countries, do not take into account emerging economies but only developed economies and a "rest of the world" aggregate. This is the case, for example, of the OECD Input-Output tables (IOTs) and the World Input-Output Database (WIOD) (Timmer et al., 2015). One alternative would be to rely on country-level information about IO tables. However, domestic statistical agencies and central banks in general do not have this data in many of the countries considered in the paper. Also, when the data is available, the comparison across countries is difficult given different sectoral classifications. To overcome these issues, I use the Multi-region Input-Output table (MRIO) EORA 26 database (Lenzen et al., 2013). This provides a complete world IO table for 190 countries, using a harmonized 26-sector classification for the period 1990-2015. The main source of information for the construction of this database is the National Accounts Main Aggregates and Official Data by the United Nations. Therefore, EORA database is constructed in a consistent way that aggregate series both at the country and world levels. Note that data availability of this dataset restricts the time-series dimension of the whole data to the 1990-2015 period. Importantly, because of the heterogeneity in the goods composing the commodity price index and to keep consistency, I define the commodity sector as the sum of agriculture, fishing, and mining and quarrying. I consider that linkages are formed because of technological reasons, so for each country I use the average IO matrix in the 1990-2015 period to compute the downstream and upstream measures.

1.2.3 *Results*

Table 1.1 presents the main empirical results of the paper by running different variants of equation (1.1) using GDP per capita as the dependent variable. Column 1 shows the effect of commodity prices and the interaction with the size of the commodity sector without taking into account additional controls. On average, increases in commodity prices generate positive responses in GDP, with a coefficient of 0.07. This effect is both statistically and economically significant: a one standard deviation in commodity prices generate a 0.25 standard deviations in GDP. This is, commodity price shocks generates close to one quarter of the observed volatility in GDP per capita, having positive effects in the cycle of commodity exporting economies. On the other hand, the interaction with size is positive and significative, meaning that larger commodity sectors amplify the magnitude of the commodity shock over GDP.

In the second column I include additional interactions with measures of linkages. Two results emerge from this column. First, the magnitude and significance of the interaction between the commodity price and size is reduced about 50% and is no longer significant. Second, the interaction terms with linkages is negative (but only the upstream measure is significant). These results imply that in order to understand the impact of commodity price shocks, is not only relevant to take into account the relative size of the sector, but its connections with the rest of the economy.

In the remaining columns I run the same specification, but taking into account different sets of controls and fixed effects. Column 3 includes aggregate covariates such as the level of GDP (to capture the effect of development and institutions), past inflation and nominal exchange rate, as well as the exchange rate arrangement. The level effect of commodity prices remains strongly significant but its magnitude is reduced to 0.04. On the other hand, the interaction with Down and Up is still negative for both kinds of linkages and statistically significant for the two of them (size remains not significant). In columns 4 and 5, I also include country and year fixed effects to control for unobserved variables. Focusing in column 5, where both set of fixed effects are included, we note that the level effect of commodity price fluctuations is 0.09. On the other hand, the interaction terms are both negative and significative, with a magnitude close to -0.17 for each one. Results from this table establish that commodity price fluctuations are positively associated with GDP, and that more connected commodity sectors dampen these effects. All these results hold conditionally and unconditionally.

Robustness. In Appendix 1.B, I present robustness checks for the main empirical result. I show that the observed patterns are robust to other commodity price definitions and aggregate GDP as dependent variable.[10] The main qualitative message of the paper remains the same after these changes: commodity price shocks have a lower effect in countries with more connected commodity sectors.[11]

[10] Recall that baseline prices use fixed weights across the 1962-2018 period (share of commodity exports over total exports). In these robustness exercises, I consider the same kinds of weights but time-varying.

[11] In non reported results, I also show that these findings are qualitatively robust, but not significant, when I use either Hodrick-Prescot filtered data or Hamil-

Table 1.1: The effect of commodity price and linkages on GDP fluctuations

	(1)	(2)	(3)	(4)	(5)
Commodity price$_{it-1}$	0.066***	0.056***	0.040**	0.033*	0.090***
	(0.017)	(0.019)	(0.018)	(0.017)	(0.028)
Commodity price$_{it-1}$ $\times$ Size$_i$	0.708*	0.342	0.241	0.148	0.137
	(0.360)	(0.313)	(0.295)	(0.287)	(0.210)
Commodity price$_{it-1}$ $\times$ Down$_i$		-0.163	-0.199*	-0.172*	-0.168**
		(0.146)	(0.102)	(0.091)	(0.077)
Commodity price$_{it-1}$ $\times$ Up$_i$		-0.222**	-0.218**	-0.200**	-0.179**
		(0.093)	(0.091)	(0.086)	(0.074)
log(GDP$_{it-1}$)			0.001	0.043***	0.106***
			(0.001)	(0.011)	(0.018)
Inflation$_{it-1}$			-0.026	-0.022	-0.030
			(0.027)	(0.024)	(0.023)
NER$_{it-1}$			-0.028*	-0.024*	-0.012
			(0.016)	(0.013)	(0.012)
Floating$_{it-1}$			-0.014**	-0.023***	-0.023***
			(0.005)	(0.008)	(0.007)
Observations	806	806	684	684	683
R-squared	0.108	0.120	0.209	0.277	0.391
Number of countries	34	34	34	34	34
Number of years	25	25	25	25	24
Controls	No	No	Yes	Yes	Yes
Country FE	No	No	No	Yes	Yes
Year FE	No	No	No	No	Yes

NOTES: This table presents regressions on the cyclical component of log GDP per capita and the cyclical component of log commodity price index, conditional on commodity sector linkages and other control variables. Commodity sector defined as the sum of agriculture, fishing, and mining and quarrying. Size defined as the average relative size of the commodity sector. Inflation and NER denotes log-deviations around a quadratic trend for the consumer price index and nominal exchange rate (local currency per dollar), respectively. Floating is a dummy variable that takes a value of zero if a country-year pair has a peg exchange rate arrangement, and one otherwise (floating exchange rate). All columns, when corresponding, include the level of variable Size, Down and Up (not reported). Robust standard errors clustered at the country level in parentheses. ***, ** and * denote statistical significance at the 1, 5 and 10% levels, respectively.

To rationalize these results, in the next section I build a real business cycle model for a small open economy with production linkages and that produces commodity goods taking their price as given. With the help of the model, I provide the basic intuition behind the dampening effect of linkages to commodity price shocks.

ton (2018)'s filter.

1.3 BASELINE MODEL

This section studies a model for a small open economy that produces a commodity good which price is determined in international markets. The economy is populated by a representative household that supplies labor and consumes. On the production side, the economy is composed by N sectors that produce differentiated goods using the labor supplied by the household, an imported composite good and domestic materials from other sectors in the economy. These sectoral goods are demanded either for production purposes by other sectors, or by final good aggregators that combine them to get the consumption good and an exportable good. This is a version of the standard RBC model for small open economies proposed by Mendoza (1995) or Schmitt-Grohe and Uribe (2018), equipped with a multi-sector production side and domestic IO linkages at the sectoral level, in the spirit of Long and Plosser (1983).[12] To put the main theoretical contribution of the paper in perspective, I consider a static environment in which there is no debt nor capital.

1.3.1 *Households*

The representative household chooses consumption and labor to maximize its utility

$$U(C, L) = \frac{1}{1-\sigma}\left(C - \vartheta \frac{L^{1+\xi}}{1+\xi}\right)^{1-\sigma}, \tag{1.2}$$

subject to the budget constraint

$$wL = P^c C.$$

[12]Other papers with a similar theoretical framework for a closed economy are Acemoglu et al. (2016), Foerster et al. (2011) and Atalay (2017).

In terms of notation, C represents consumption, which has price P^c and L is the labor supply of the household which will be demanded by the productive sectors of the economy at a wage rate w. For simplicity, there is only one labor market and labor is perfectly mobile across sectors.

The maximization problem of the household gives the following labor supply and consumption schedules

$$L = \left(\frac{1}{\vartheta} \frac{w}{P^c} \right)^{1/\xi} \tag{1.3}$$

$$C = \left[\frac{1}{\vartheta} \left(\frac{w}{P^c} \right)^{\xi+1} \right]^{1/\xi} . \tag{1.4}$$

1.3.2 *Productive Sectors*

On the production side, the economy is inhabited by N sectors that produce differentiated goods. These sectors are indexed by j, in which $j = 1$ is normalized as the commodity sector. I assume that all sectors produce with a Cobb-Douglas technology of the form

$$Y_j = (Z_j) (L_j)^{\alpha_j} (V_j)^{\theta_j} \prod_{i=1}^{N} (M_{ij})^{\gamma_{ij}} . \tag{1.5}$$

Each sector j is characterized by a productivity term Z_j and demands labor (L_j) from the household, intermediate goods from other sectors (M_{ij}) and a composite imported good (V_j) which is the numeraire of the economy. The sub-index (ij) denotes the quantity of good produced by industry i demanded by industry j. The intensity in these productive linkages is captured by the parameter γ_{ij}, which are collected in the IO table of the economy, $\mathbf{\Gamma}$.[13] Note that this speci-

[13]For convenience, define $\widetilde{\mathbf{\Gamma}}$ as the IO table of the economy for the mainland economy (i.e., excluding the commodity sector). This component will be relevant for further analysis below.

fication takes into account the whole nature of IO linkages and not a single composite intermediate good.

I assume perfect competition and constant returns to scale in all sectors (i.e., $\alpha_j+\theta_j+\sum_{i=1}^{N}\gamma_{ij}=1, j=1,\ldots,N$), so they take prices as given both in the market for inputs and in the market where they sell their products. These sectors maximize profits choosing the optimal demands for labor, intermediates and imports, given their technologies. The key distinction between the commodity sector and the rest of industries in the economy, is that its price (P_1) is given internationally and not set in the equilibrium of the local economy, capturing in reduced form the fact that the country is small relative to the rest of the world.

Profit maximization of the representative firm of each sector generates the following demands for factors

$$L_j = \frac{\alpha_j P_j Y_j}{w} \tag{1.6}$$

$$V_j = \theta_j P_j Y_j \tag{1.7}$$

$$M_{ij} = \frac{\gamma_{ij} P_j Y_j}{P_i}. \tag{1.8}$$

From this demands and the technology on each sector, the equilibrium price must satisfy

$$P_j = (Z_j^{-1})(B_j)(w^{\alpha_j})\prod_{i=1}^{N}(P_i^{\gamma_{ij}}), \tag{1.9}$$

with $B_j \equiv (\alpha_j^{-\alpha_j})(\theta_j^{-\theta_j})\prod_{i=1}^{N}(\gamma_{ij}^{-\gamma_{ij}})$.

1.3.3 *Aggregators*

There are two final aggregators combining inputs from different sectors to generate a consumption good (c) and an exportable good (x).

The technology of these final sectors is assumed Cobb-Douglas with constant returns to scale

$$Y^c = \prod_{j=1}^{N} \left(\frac{A_j^c}{\mu_j} \right)^{\mu_j}, \quad \sum_{j=1}^{N} \mu_j = 1$$
$$Y^x = \prod_{j=1}^{N} \left(\frac{A_j^x}{\eta_j} \right)^{\eta_j}, \quad \sum_{j=1}^{N} \eta_j = 1,$$

where A_j^ℓ corresponds to the domestic absorption of sector j by aggregator $\ell \in \{c, x\}$. The demands for sectoral goods by each aggregator are

$$A_j^c = \frac{\mu_j P^c Y^c}{P_j} \quad \text{and} \quad A_j^x = \frac{\eta_j P^x Y^x}{P_j}, \quad \forall j = 1, \ldots, N.$$

These demands, combined with the technologies of each aggregator, generate the following prices for consumption and exportable goods

$$P^c = \prod_{j=1}^{N} P_j^{\mu_j} \quad \text{and} \quad P^x = \prod_{j=1}^{N} P_j^{\eta_j}.$$

1.3.4 *Market Clearing*

To close the model, I take into account the following market clearing conditions. First, labor supply must equal total labor demanded by all sectors

$$L = \sum_{j=1}^{N} L_j. \tag{1.10}$$

The second condition is that all the sectoral production must be fully demanded, implying the following set of equilibrium conditions

$$Y_j = A_j^c + A_j^x + \sum_{i=1}^{N} M_{ji}, \quad j = 1, \ldots, N. \tag{1.11}$$

At the same time, the use of final goods must equal their demands. First, the consumption good must be fully consumed domestically. Second, the exportable good is fully sold abroad. The latter requirement, combined with the demand of imports by each sector, defines an expression for the trade balance position of the country. These two conditions can be written as

$$\begin{aligned} Y^c &= C \\ TB_t &= P_t^x Y_t^x - \sum_{j=1}^{N} V_{jt}. \end{aligned}$$

Note that by assuming a static economy, the possibility of the household taken foreign debt is ruled out. Therefore, the trade balance condition must be equal to zero in every period.

Gross Domestic Product (GDP). The object of interest in the paper, GDP, is defined as the sum of sectoral value added. This is, the value of production net of imports and intermediate inputs from the N sectors in the economy

$$\text{GDP} = \sum_{j=1}^{N} P_j Y_j - \sum_{j=1}^{N} V_j - \sum_{j=1}^{N} \sum_{i=1}^{N} P_j M_{ji}. \tag{1.12}$$

In what follows, I exploit the equivalence between the previous definition and other approaches of getting GDP. In particular, given that the only productive factor generating value added is labor, the *income approach* of National Accounts establishes that GDP can be written as[14]

$$\text{GDP} = wL. \tag{1.13}$$

1.3.5 *Discussion*

Three comments about the model are in order. First, while standard in the RBC literature for small open economies, the use of GHH preferences (Greenwood et al., 1988) described in (1.2) imply a labor supply with no income effect. Therefore, this may exacerbate the response of labor supply to different kinds of shocks, relative to other kinds of preferences. As I show later on, similar qualitative insights can be obtained by using separable preferences.

Second, and related to the previous point, I exploit the equivalence in the definition of GDP for convenience and tractability. While using the income approach (equation 1.13) is less standard than using the value added approach (equation 1.12), the former allows me to get an expression for GDP knowing only the equilibrium conditions in the labor market. Combined with the GHH assumption of the previous point, I only need to solve for prices and wage to know how GDP varies with the commodity price shock. In this regard, these two assumptions give me closed form expressions and tractability to illustrate the mechanism.

Finally, I consider a simple static framework in comparison with most of the RBC literature for small open economies. While my approach is silent about interesting topics (such as the evolution of current account or investment dynamics after terms of trade shocks[15]),

[14]This expression can be obtained by combining the different equilibrium conditions of the model.

[15]See for example Fornero et al. (2016).

it allows me to show more clearly the relation between commodity prices and GDP, and how IO linkages shape this relation. As shown in the empirical section, these interactions are static in nature, so the best way to show the mechanism at play is in a static setting. Therefore, this paper intends to shed light on the mechanism behind IO linkages and commodity prices, more than an exhaustive quantitative assessment of the contribution of shocks.

1.4 THEORETICAL RESULTS

This section uses the model to study how domestic linkages shape the response of GDP after a commodity price shock. To build intuition, I start with a simplified version of the model, with only two sectors in the economy and only sales between sectors (i.e., diagonal elements in the IO matrix are zero, $\gamma_{ii} = 0$). Then, I extend these results to an arbitrary N-dimensional economy.

As mentioned in the discussion of the previous section, with GHH preferences, labor supply depends only on wage and the price of consumption but not on the quantity consumed. Therefore, I only need to solve for prices (wage, sectoral prices and the price of consumption) to get an expression for labor supply, and then for GDP. Importantly, note that in all the analysis, imports play the role of being a factor of production for each sector and being the numeraire of the economy. Therefore, all prices are in units of importable or foreign good, as in the empirical analysis.

1.4.1 *Two-sector Economy*

Consider the model presented in section 1.3 but assuming only two sectors: commodity and a composite sector for the mainland economy. For simplicity, I also name the composite sector as the mainland sector or the rest-of-the-economy sector. For consistency in notation, I index the commodity sector as $j = 1$ and the composite mainland sector as $j = 2$. This assumption reduce the heterogeneity in the production side of the economy to the minimum level necessary to understand the forces at play. At the same time, I assume that there are no sales within a sector (i.e., $\gamma_{11} = \gamma_{22} = 0$), so production linkages are characterized only by sales between sectors (given by parameters $\gamma_{12} > 0$ and $\gamma_{21} > 0$). This implies that the IO matrix of the economy is 2×2 with zero diagonal terms. For simplicity, in this example I ignore productivity, and consider as the only driver of the model fluctuations in the commodity price. Therefore, all the following results will show the elasticity of different variables with respect to commodity price, and ignore a constant term given by productivities.

Prices. Given the commodity price (P_1), I need to find the price of the composite mainland good (P_2) and the equilibrium wage (w). Using equation (1.9) for each sector in this specific case, I obtain the following expressions

$$\log P_1 = \alpha_1 \log w + \gamma_{21} \log P_2 \tag{1.14}$$

$$\log P_2 = \alpha_2 \log w + \gamma_{12} \log P_1. \tag{1.15}$$

These expressions are the supply equation of the commodity sector and of the composite mainland sector respectively. The only difference between the two correspond to the intensity of usage of labor (α) and materials from the other sector (γ). The fundamental element to take into account, is that the commodity price P_1 is determined in global markets, so taken as given by the small open economy. Therefore, the first equation pins-down wages in terms of commodity price.

What happens after a positive commodity price shock? First note that it boosts the demand for factors by the commodity sector. As consequence, this increases the equilibrium wage and the level of production of the rest of the economy. Second, it raises the cost of production in the rest of the economy because they also demand labor and commodity to produce ($\gamma_{12} > 0$). By perfect competition, this translates into a higher price in other sectors. The increase in P_2 has a feedback effect on the pricing equation of the commodity sector because these goods are necessary for the production of commodity ($\gamma_{21} > 0$). Because the economy takes the commodity price as given, the only possible margin of adjustment that satisfies this pricing equation and the small open economy assumption, corresponds to changes in the equilibrium wage. This can be seen by re-writing the supply equation of the commodity sector in (1.14) as

$$\alpha_1 \log w = \log P_1 - \gamma_{21} \log P_2,$$

in which P_1 is given and P_2 is increasing by the first-round effects of the commodity price shock. Because the commodity sector can not react to the increase in its cost of production by adjusting its price, the only way to compensate the effect is by reducing the amount of factors demanded (labor and materials) to the point in which the cost of production is now equal to the price of the good. This again will reduce the equilibrium wage and the price of the mainland sector, relative to the initial scenario.

In equilibrium, the relationship between wage and the price of the mainland economy and the commodity price can be written as

$$\log w = \left(\frac{1 - \gamma_{21}\gamma_{12}}{\alpha_1 + \alpha_2\gamma_{21}}\right) \log P_1$$
$$\log P_2 = \left(\frac{\alpha_2 + \alpha_1\gamma_{12}}{\alpha_1 + \alpha_2\gamma_{21}}\right) \log P_1,$$

by solving for w and P_2 using equations (1.14) and (1.15). Note that the presence of linkages (γ_{12} and γ_{21}) unambiguously dampen the effect of commodity prices over wages, while the effect over the price in the mainland sector depends on the relative strength of those linkages and the demand for labor. Interestingly, the price of the mainland sector also depends on the structural characteristics for production in the commodity sector (α_1 and γ_{21}).

Once we know the price of the commodity good and the price of the mainland good, we can obtain an expression for the price of consumption in terms of the commodity price. Note that we need such expression because the labor supply (1.3), hence GDP, depends on wages and the price of consumption. Given the aggregator for consumption, its price is given by

$$\begin{aligned}\log P^c &= \mu_1 \log P_1 + \mu_2 \log P_2 \\ &= \left(\frac{\alpha_1(\mu_1 + \mu_2\gamma_{12}) + \alpha_2(\mu_2 + \mu_1\gamma_{21})}{\alpha_1 + \alpha_2\gamma_{21}} \right) \log P_1.\end{aligned}$$

Note that the effect of changes in the commodity price over the price of consumption is positive, but the effect of linkages is ambiguous and depends on (i) the relative usage of labor by each sector and (ii) how much of each sector is demanded for final consumption (μ).

Labor supply and GDP. From previous analysis, an increment in the commodity price increases wages and (typically) sectoral prices in the economy. Hence, this shock increases the price of the consumption good. However, as was shown before, the presence of IO linkages dampen the effect over wages because the economy takes the commodity price as given, while the effect over sectoral prices and the price of consumption is ambiguous.

Using the expression for GDP given in equation (1.13) we have $\log \text{GDP} = \log w + \log L$, and from the GHH preferences we know that labor supply only depends on wages and the price of consumption, such that $\log L = (\log w - \log P^c - \log \vartheta)/\xi$. By replacing the expressions for wages and the price of consumption, we can obtain the elasticity of GDP with respect to the commodity price

$$\frac{\partial \log \mathrm{GDP}}{\partial \log P_1} = \left[\frac{(\xi+1)(1-\gamma_{21}\gamma_{12}) - \alpha_1(\mu_1+\mu_2\gamma_{12}) - \alpha_2(\mu_2+\mu_1\gamma_{21})}{\xi(\alpha_1+\alpha_2\gamma_{21})}\right]. \quad (1.16)$$

From this expression, linkages between the commodity sector and the rest of the economy dampen the effect of commodity price shocks. This is because the labor supply and therefore GDP, is increasing in wage and decreasing in the price of consumption, and as shown before, linkages diminish the impact of commodity price shocks in the former while amplify the impact in the latter.

It is important to highlight the fact that the positive commodity price shock does indeed stimulate the economy and have a positive impact on prices and wages. The main difference here is coming from IO linkages between the commodity sector and the rest of the economy, which dampen these positive effects, being the novel contribution of this paper.

Numerical examples. To gain intuition about the role of linkages in the transmission of commodity price fluctuations, I provide some simple numerical examples. The goal of this exercise is not to provide a careful quantitative evaluation of the framework, but to illustrate the relative magnitudes of changing the intensity in IO linkages on the extensive margin. From equation (1.16), the elasticity of GDP to the commodity price depends on (i) the (inverse of the) Frisch elasticity, ξ; (ii) sectoral labor share, α; (iii) sectoral shares in consumption good, μ; and (iv) linkages between sectors, γ. For illustration purposes, I focus in four cases: (i) linkages in both directions ($\gamma_{12} > 0$ and $\gamma_{21} > 0$); (ii) commodity being a customer but not a supplier ($\gamma_{12} = 0$ and $\gamma_{21} > 0$); (iii) commodity being a supplier but not a customer ($\gamma_{12} > 0$ and $\gamma_{21} = 0$); and (iv) commodity not connected with the rest of the economy ($\gamma_{12} = \gamma_{21} = 0$).

In all exercises, I choose a conservative value $\xi = 1$ to obtain a Frisch elasticity of 1, in the middle range of other studies. I assume that commodities represent only a 3% of the consumption basket, hence $\mu_1 = 0.03$ and $\mu_2 = 0.97$. I also assume that both sectors are equally imports-intensive, with a value of 10% ($\theta_1 = \theta_2 = 0.1$). Even though this parameter does not directly affect the expression in (1.16), it has to be set in order to satisfy constant returns on each sector. In the baseline case, I assume $\gamma_{12} = 0.19$, which is the average intensity of commodities in the production of the rest of sectors, and $\gamma_{21} = 0.31$ which is the average importance of other sectors in the production of commodity observed in the data. Combining this information with the imports-intensity parameter, I fix a labor share of $\alpha_1 = 0.59$ and $\alpha_2 = 0.71$ for the commodity sector and the rest of the economy. In what follows, when I turn-off one of the linkages parameters (γ), I assume that the economy is more imports-intense. This is, for every counterfactual economy, I keep fixed the value of α of each sector and increase the importance of imports, such that constant returns to scale are satisfied.

Table 1.2 presents the results of this exercise. Column (1) shows the elasticity with respect to the commodity price for a baseline economy with a commodity sector acting as supplier and customer. Columns (2) and (3) removes one of these roles, while column (4) shows the elasticity in the model with no production linkages. Even though the exercise is not designed to generate a realistic value for this elasticity, it helps to see the relative importance of production linkages. When the commodity sector only acts as a customer of the rest of the economy, the elasticity is 21% larger than the baseline case. When it only acts as supplier, this is almost 53% larger. This means that ignoring the role of the commodity sector as customer generates a greater deviation in the elasticity relative to a model taking into account these connections. When both are ignored (column 4), the elasticity is 67% larger. So even though both roles of the commodity sector contribute to understand the elasticity of GDP, we conclude that the role as customer seems to be more relevant for the example studied.

Table 1.2: GDP elasticity to commodity price in 2×2 example

	(1)	(2)	(3)	(4)
	Baseline ($\gamma_{12} > 0, \gamma_{21} > 0$)	Only customer ($\gamma_{12} = 0, \gamma_{21} > 0$)	Only supplier ($\gamma_{12} > 0, \gamma_{21} = 0$)	No links ($\gamma_{12} = 0, \gamma_{21} = 0$)
Elasticity	1.308	1.585	2.001	2.184
Relative to baseline	1.000	1.212	1.529	1.669

NOTES: This table presents the elasticity of GDP to commodity price for the 2×2 economy and different scenarios for linkages. Column (1) shows the baseline model with linkages in both directions (commodity as customer and supplier). Column (2) assumes that commodity sector is only a customer. Column (3) assumes that commodity sector is only a supplier. Column (4) assumes no links. Row 'Relative to baseline' presents the ratio of each column with respect to column (1).

1.4.2 *Multisectors Economy*

The previous result can be generalized to a N-sectors economy with arbitrary IO structure, as the following proposition shows.

Proposition 1. *The elasticity of GDP to commodity prices in a $N \times N$ economy with arbitrary IO linkages and GHH preferences is given by*

$$\frac{\partial \log \text{GDP}}{\partial \log P_1} = \frac{1}{\xi}\left\{(\xi+1)\Psi_1 - \mu_1 - \boldsymbol{\mu}'\widetilde{\mathbf{H}}(\boldsymbol{\alpha}\Psi_1 + \boldsymbol{\gamma}_{1j})\right\}, \tag{1.17}$$

where ξ is the inverse of the Frisch elasticity, $\widetilde{\mathbf{H}} = (\mathbf{I}_{N-1} - \widetilde{\mathbf{\Gamma}}')^{-1}$ is the transpose of the Leontief inverse in the mainland economy (i.e., all sectors but commodity) and $\Psi_1 \equiv \left(\frac{1-\gamma_{11}-\boldsymbol{\gamma}'_{j1}\widetilde{\mathbf{H}}\boldsymbol{\gamma}_{1j}}{\alpha_1+\boldsymbol{\gamma}'_{j1}\widetilde{\mathbf{H}}\boldsymbol{\alpha}}\right)$.

Proof. See Appendix 1.D. □

As in the two-sector economy, the elasticity of GDP depends on the Frisch elasticity, labor share in production, sectoral shares in consumption and the IO matrix. The main difference with respect to the simplest case comes from the presence of the Leontief inverse matrix, which captures all the sectoral interactions once we control for the intensity of the commodity sector either as customer or supplier (given by vectors $\boldsymbol{\gamma}_{j1}$ and $\boldsymbol{\gamma}_{1j}$). All in all, note that as in the 2×2 version, linkages unambiguously decrease the elasticity of GDP to commodity prices, dampening the effects of those shocks through parameters $\boldsymbol{\gamma}_{j1}$ and $\boldsymbol{\gamma}_{1j}$.

There are several points to mention from this result. First, in a model with no linkages (neither within mainland nor between mainland and the commodity sector), the effect of commodity prices over real GDP tends to be larger than in the case with linkages.[16] Second, in this model the effect of commodity price shocks could be even negative under certain conditions. In particular, this could happen when (i) the commodity sector is a key supplier of the rest of sectors (large elements in $\boldsymbol{\gamma}_{1j}$) or (ii) when it has a large share in the consumption basket (large μ_1), among other combinations. Therefore, there is the theoretical possibility that positive commodity prices could have negative effects over real GDP. Third, in general terms, the dampening effect of linkages is related to the effect over wages and sectoral prices. For positive values of Ψ_1, domestic linkages ameliorate the effect of commodity price shocks because of the general equilibrium effects over sectoral cost of production and wages. At the same time, this increase in costs makes the consumption good more expensive after a positive shock, which reinforces the previous effect. Finally, note that labor supply elasticity plays a key role in the effect. In this model, ξ is the inverse of the Frisch elasticity, therefore, the more responsive is the labor supply to changes in real wage, the higher the commodity price effect over real GDP.

[16] Note that without linkages, the elasticity is $\left[\frac{\xi+1-\boldsymbol{\mu}'\boldsymbol{\alpha}-\mu_1\alpha_1}{\alpha_1\xi}\right]$, which is typically larger than the expression in (1.17). As in the 2×2 case, this result assumes that the margin of adjustment is through imports, so an economy with no linkages will replace those materials with importable goods.

1.4.3 *Discussion*

Previous sub-sections show why domestic linkages may dampen the effect of commodity price shocks. Given the empirical evidence presented in section 1.2, the model is able to qualitatively replicate those results. However, as shown in Table 1.2, the elasticities that produces are too large compared to the data. In what follows, I discuss some potential modifications of the baseline model that might help to overcome these discrepancies.

First, the lack of income effect in labor supply may alter the response of the economy to commodity price fluctuations. By using GHH preferences, labor supply depends only on wages and the price of consumption but not on consumption itself. Therefore, any change in the latter variable will not affect the labor decision of the household. An alternative to this will be to consider separable preferences in which income effect plays a role. Assume, for example, an utility function of the form $U = \frac{C^{1-\sigma}-1}{1-\sigma} - \vartheta \frac{L^{1+\xi}}{1+\xi}$. In this case, labor supply will take the form $L = (\frac{1}{\vartheta}(\frac{W}{P^c})^{1-\sigma})^{\frac{1}{\sigma+\xi}}$. In Appendix 1.E, I show that in this version of the model, the elasticity of GDP to commodity price takes the form

$$\frac{\partial \log \text{GDP}}{\partial \log P_1} = \frac{1}{\sigma+\xi}\left\{(\xi+1)\Psi_1 - (1-\sigma)[\mu_1 + \boldsymbol{\mu}'\widetilde{\mathbf{H}}(\boldsymbol{\alpha}\Psi_1 + \boldsymbol{\gamma}_{1j})]\right\}.$$

It can be noticed that when $\sigma = 0$, (1.17) coincides with the previous expression. Note that σ plays a dual role in dampening the magnitude of the elasticity. On the one hand, it affects the importance of wages on labor supply, effect that is partially offset with wage itself (recall that GDP corresponds to wage times labor supply). On the other hand, it reduces the elasticity of GDP with respect to the price of consumption, so it reduces the impact of commodity prices. Note that this second channel holds for $\sigma \in (0, 1)$. For $\sigma > 1$, the second component in the expression for elasticity, which corresponds to the effect of consumption price on GDP, is now positive. In this case, the labor supply schedule exhibits a negative slope with respect to real wage. Thus, the role of the commodity sector as supplier may amplify the elasticity for certain configurations of parameters.

Second, the modelling approach followed here takes the commodity price as given to the small open economy, driving the equilibrium for the economy. This can be seen as an extreme case of rigidities in one sector (commodity), while the others can freely adjust their prices in order to be consistent to this level for the commodity price. An alternative framework would be an economy with linkages and nominal price/wage rigidities. In such environment, while prices would still react to the commodity price shock, due to its affect over sectoral marginal cost, this would generate a slower response in prices because they react sluggishly.[17] Also in such a framework, there is scope for (monetary) policy in order to contain inflation. To the extent that sectors also produce with capital, and the household wants (and can) smooth consumption through financial markets, the impact of the commodity price and the consequent inflation and policy response, will also affect the equilibrium elasticity of GDP. My conjecture is that in such a model, the elasticity of GDP would be closer to the empirical counterparts due to all this nominal rigidities.

1.5 QUANTITATIVE APPLICATION

During the decade of 2000, the world experienced a commodity price boom mostly fuelled by China's demand for these goods. Such boom generated a large increase in the volatility of commodity prices (measured as log deviations with respect to trend), going from 12.9% in the previous decade to 19.5%. Consistent with this, GDP in emerging economies also increase its volatility, partially due to the commod-

[17] As recently noticed by Pasten et al. (2019), heterogeneity in price adjustments at the sectoral level drive real effects of shocks.

ity boom. Motivated by these observations, this section evaluates to what extent the degree of production linkages between the commodity sector and the rest of the economy increased the volatility in GDP and what would has been the volatility in output under different levels of those linkages. To answer these questions, I use the static model presented in section 1.3 to study different counterfactual scenarios regarding the connection between the commodity sector and the rest of the economy.

To fully capture fluctuations in GDP, I proceed as follows. In order to account for the role of commodity linkages, I assume that, besides the commodity price shock, each country is subject to productivity shocks that are common across sectors ($Z_j = Z$) but country-specific. Then I use data for GDP per capita and commodity prices between 2000-2015 to recover the level of productivity that exactly matches GDP over the period. Using the level of productivity recovered by the model and the observed commodity price, I simulate counterfactual economies with different degrees of linkages between the commodity sector and the rest of the economy. This is done by changing the parameters contained in vectors $\boldsymbol{\gamma_{1j}}$ (the intensity of demand of other sectors for commodity goods) and $\boldsymbol{\gamma_{j1}}$ (the intensity of demand of the commodity sector for other domestic materials). Given the Cobb-Douglas nature of the productive side of the model, those changes connect naturally to the Down and Up measures presented in the empirical section. Then, for each counterfactual, I compute the standard deviation of the GDP predicted by the model to study the impact of linkages over volatility.

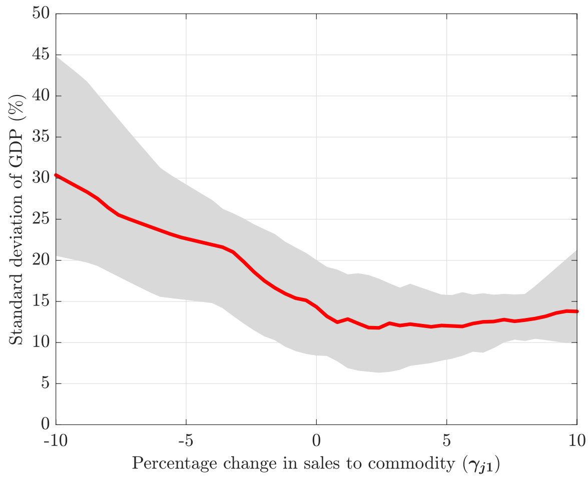
For simplicity and to avoid numerical problems with some of the parameters of the model, the exercise considers a three-sector economy: commodity, other tradables (e.g., manufactures) and non tradables (e.g., services). I consider the same 34 countries analyzed in section 1.2, with data on GDP per capita from WDI and commodity prices from the IMF. For the calibration of the model, I use country-by-country data from EORA database to recover the intensity of usage of materials (γ_{ij}), the usage of labor (α_j) and the relative importance of each sector for consumption and exportable goods (μ_j and η_j, respectively). Also, I assume a common value for the Frisch elasticity across countries of $\xi = 1$. With all this information, and as mentioned before, conditional on the baseline calibration that recovers the level of productivity that replicates GDP on each country, I re-compute GDP using the same level of productivity and commodity prices, but with different intensities in connections between the commodity sector and the rest of the economy. This recovers a counterfactual GDP, conditional on the new level of commodity linkages, and for each one of those counterfactuals, I compute the volatility of GDP.

Figure 1.1 presents the results of this exercise. On each panel, the solid red line corresponds to the median volatility across countries under the different counterfactuals, while the grey area denotes the interquartile range. The horizontal axis denotes the percentage change in either γ_{j1} or γ_{1j}, measuring changes in the intensity of the commodity sector as customer or supplier of the rest of the economy, respectively. For comparison purposes, note that the median volatility under the baseline calibration (when the percentage change equals zero) is 14.3%. Starting with panel (a), we observe that a decrease in the demand of commodity sector for domestic materials translates into more volatility of GDP. In particular, a counterfactual economy in which the demand for all domestic materials is 10 percent lower, would have have experienced a volatility of 30.4%, which is 2.1 times larger than in the baseline calibration. In the contrary, a 10 percent increase in the demand for materials would imply a volatility of 13.8% for GDP. Interestingly, the heterogeneity across countries is much larger under decreases in the demand for domestic materials than under increases, as reflected by the interquartile range. A different picture is presented in panel (b) where we compare different scenarios in the intensity of sales from commodity to the rest of the economy. In fact, decreasing such intensity would produce a volatility only 1.5 times larger than in the baseline, reaching a level of 21.9%, while a decrease in the domestic demand for commodity goods generates a volatility of 13.5%. Different to the case of panel (a), the interquartile range remains constant across the different experiments, implying a similar level of heterogeneity across countries under each scenario.

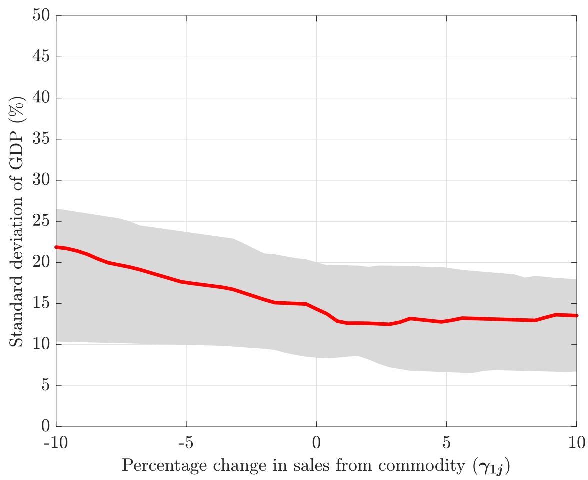
These results uncover two asymmetries about the importance production linkages between the commodity sector and the rest of the economy. First, changes in the role of commodity as customer generates larger differences in volatility than changes in its role as supplier, in line with the theoretical analysis. Second, conditional on changes in the intensity of commodity either as customer or as supplier, decreases in the degree of connections with the rest of the economy produce much larger differences with respect to the baseline economy than increases.

FIGURE 1.1: GDP volatility for counterfactual economies with different intensities in commodity linkages

(a) Changes in commodity as customer



(b) Changes in commodity as supplier



NOTES: This figure presents the standard deviation of GDP for the period 2000-2015 under counterfactual calibrations of the model. Panel (a) presents results under changes in the intensity of the commodity sector as customer (changes in γ_{j1}), while panel (b) presents results under changes in the intensity of the commodity sector as supplier (changes in γ_{1j}). On each figure, the horizontal axis shows the percentage change with respect to baseline calibration, while the vertical axis presents the standard deviation of GDP, in deviations from trend. Solid red line corresponds to the median volatility across countries, while grey corresponds to the interquartile range.

1.6 CONCLUSION

This paper analyzes the role of domestic IO linkages as a transmission mechanism of commodity price shocks in emerging economies. Focusing on the effect over GDP, I document that economies with more connected commodity sectors experience lower fluctuations from commodity price shocks. Therefore, linkages between the commodity sector and the rest of the economy dampen the effect of commodity price shocks. To rationalize this, I construct a RBC model for a small open economy that produces commodities, takes the commodity price as given, and has IO linkages in production. In the model, an increase in the commodity price not only represents a windfall, but also an increase in marginal cost of commodity sector's customers. Because they also act as suppliers of the commodity sector, this has second round effects in the production cost of the latter. However, because the economy takes the commodity price as given, the only way to keep marginal cost sufficiently low is by decreasing the demand for inputs. Therefore, the impact over GDP is lower than in a model without linkages. The predictions of the model are verified in a quantitative application, showing a monotone decreasing relation between the intensity of linkages (between the commodity sector and the rest of the economy) and the volatility of GDP. I conclude that taking into account the productive structure of a small open economy (i.e., IO linkages) is important to understand the impact of commodity price shocks on business cycles.

1

APPENDIX

1.A DATA APPENDIX

1.A.1 *Selected Countries*

Table 1.A.1 presents the countries selected in the empirical exercise. The first two columns show the relevance of commodity exports as a fraction of total exports and as a fraction of aggregate GDP. The last two columns show the level of linkages between the commodity sector and the rest of the economy, excluding transactions within the commodity sector itself. All these figures are averages between 1990 and 2015.

Table 1.A.1: Countries and descriptive statistics

		Commodity exports		Linkages commodity sector	
Country	ISO	Share Exports	Share GDP	Down	Up
Algeria	DZA	0.900	0.279	0.629	0.070
Angola	AGO	0.959	0.354	0.630	0.105
Bahrain	BHR	0.364	0.161	0.329	0.541
Bolivia	BOL	0.652	0.164	0.263	0.348

Continues on the next page

Table 1.A.1: Countries and descriptive statistics

		Commodity exports		Linkages commodity sector	
Country	ISO	Share Exports	Share GDP	Down	Up
Botswana	BWA	0.128	0.055	0.151	0.382
Cameroon	CMR	0.752	0.112	0.494	0.269
Chile	CHL	0.526	0.143	0.310	0.332
Colombia	COL	0.581	0.076	0.128	0.304
Congo, Rep.	COG	0.788	0.412	0.538	0.158
Ecuador	ECU	0.657	0.141	0.136	0.522
Gabon	GAB	0.838	0.415	0.613	0.101
Guinea	GIN	0.829	0.126	0.406	0.281
India	IND	0.129	0.014	0.131	0.341
Indonesia	IDN	0.460	0.118	0.100	0.445
Iran, Islamic Rep.	IRN	0.780	0.161	0.155	0.206
Iraq	IRQ	0.947	0.407	0.281	0.061
Malaysia	MYS	0.213	0.173	0.194	0.230
Mauritania	MRT	0.569	0.185	0.206	0.354
Mexico	MEX	0.174	0.036	0.173	0.510
Mongolia	MNG	0.578	0.202	0.294	0.190
Myanmar	MMR	0.397	0.080	0.174	0.070
Namibia	NAM	0.142	0.064	0.128	0.239
Nigeria	NGA	0.917	0.219	0.297	0.062
Oman	OMN	0.783	0.414	0.513	0.066
Papua New Guinea	PNG	0.668	0.273	0.442	0.086
Peru	PER	0.532	0.075	0.245	0.474
Poland	POL	0.145	0.033	0.298	0.413
Russian Federation	RUS	0.591	0.165	0.384	0.166
Saudi Arabia	SAU	0.836	0.353	0.480	0.144
South Africa	ZAF	0.396	0.084	0.361	0.485
Sudan	SDN	0.419	0.018	0.262	0.211
Trinidad and Tobago	TTO	0.642	0.309	0.329	0.046
Yemen, Rep.	YEM	0.827	0.237	0.290	0.104
Zambia	ZMB	0.756	0.222	0.278	0.533
Average		0.585	0.185	0.313	0.260

NOTES: This table shows the countries included in the empirical analysis and presents descriptive statistics. The first two columns show the relevance of commodities in terms of total exports and as a share of GDP. Last two columns show the downstream (commodity sector as customer) and upstream (commodity sector as supplier) level of linkages of the commodity sector. Commodity sector defined as the sum of agriculture, fishing, and mining and quarrying. All measures are averages over 1990-2015.

1.A.2 *Descriptive Statistics*

Table 1.A.2 presents additional descriptive statistics for the variables of interest in the cross-section of countries. To avoid the influence of outliers, I remove observations at the bottom 1 and top 99 percent. Panel A shows the relative importance of commodity exports. On

average, commodities represent more than 66% of total exports and around 18% of total GDP. Even though the variability in both measures is large, these products are quite important for all countries, reinforcing the notion that commodities are an important activity in emerging markets.

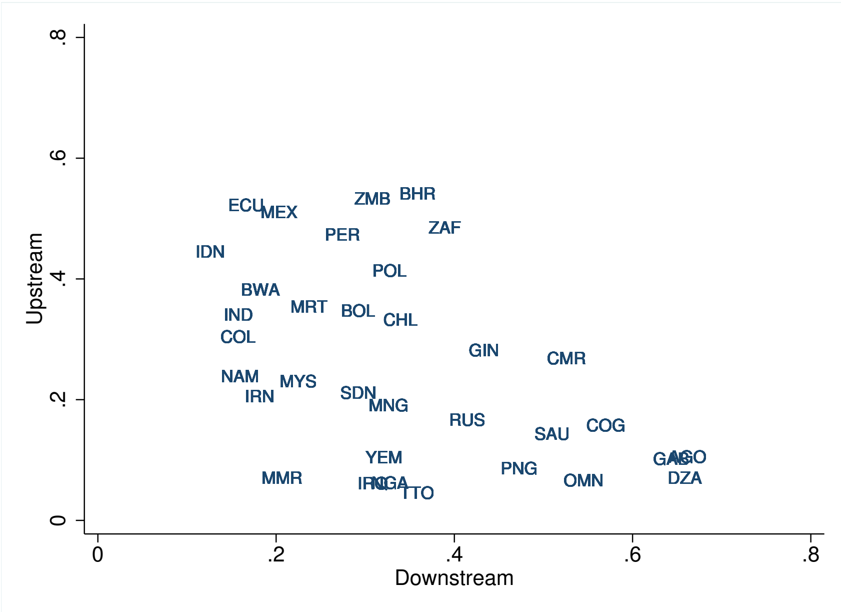
Panel B shows linkages between the commodity sector and the rest of the economy. On average, about one third of commodity output comes from domestic materials from the rest of the economy. A similar figure emerges for the use of commodity as materials for the rest of the economy (36% of commodity output is domestically sold in form of materials). Interestingly, these measures are heterogeneous across countries in both dimensions. This can be seen in Figure 1.A.1, which shows the average relationship between the downstream and the upstream measure at the country level. As we can see, there are no countries concentrated in either corner of the plot. This is, there are no countries in which the commodity sector is not connected in both directions.

Table 1.A.2: Descriptive statistics (cross-section)

	Mean	Median	SD	Min	Max
Panel A: Commodity share					
Over exports	0.662	0.705	0.244	0.150	0.975
Over GDP	0.178	0.141	0.120	0.008	0.425
Panel B: Commodity sector (links)					
Down	0.295	0.290	0.136	0.081	0.630
Up	0.357	0.340	0.203	0.046	0.743

NOTES: This table presents cross-sectional descriptive statistics of relevant variables in the analysis. Commodity sector defined as the sum of agriculture, fishing, and mining and quarrying. Panel A shows the relative importance in terms of total exports and GDP of selected commodity products. Panel B shows links of the commodity sector denoting its importance as customer (Down) and supplier (Up) relative to the rest of sectors in the economy (average 1990-2015). All variables trimmed at the 1% level.

FIGURE 1.A.1: Commodity sector linkages across countries



NOTES: This figure shows the importance of the commodity sector as customer (Downstream) and as supplier (Upstream). The x-axis shows the fraction of commodity output coming from domestic materials. The y-axis shows the fraction of commodity sales going to other domestic industries.

1.B ADDITIONAL EMPIRICAL RESULTS

Table 1.B.1 presents regressions using alternative commodity prices with time-varying weights, as well as time-varying commodity sector sizes. Table 1.B.2 presents the same kinds of results, but using aggregate GDP as dependent variable. As can be noted, all qualitative results remain under these alternative specifications.

Table 1.B.1: The effect of commodity price and linkages on GDP fluctuations: Robustness (1)

	Fixed price weights			Time-varying price weights		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: GDP per capita with average commodity size						
Commodity price$_{it-1}$	0.066***	0.056***	0.090***	0.061***	0.055***	0.085***
	(0.017)	(0.019)	(0.028)	(0.016)	(0.017)	(0.021)
Commodity price$_{it-1}$ $\times$ Size$_i$	0.708*	0.342	0.137	0.758**	0.511*	0.196
	(0.360)	(0.313)	(0.210)	(0.313)	(0.282)	(0.210)
Commodity price$_{it-1}$ $\times$ Down$_i$		-0.163	-0.168**		-0.112	-0.151**
		(0.146)	(0.077)		(0.135)	(0.073)
Commodity price$_{it-1}$ $\times$ Up$_i$		-0.222**	-0.179**		-0.143*	-0.131*
		(0.093)	(0.074)		(0.083)	(0.069)
Observations	806	806	683	807	807	684
R-squared	0.108	0.120	0.391	0.112	0.118	0.392
Number of countries	34	34	34	34	34	34
Number of years	25	25	24	25	25	24
Controls	No	No	Yes	No	No	Yes
Country FE	No	No	Yes	No	No	Yes
Year FE	No	No	Yes	No	No	Yes
Panel B: GDP per capita with time-varying commodity size						
Commodity price$_{it-1}$	0.074***	0.063***	0.092***	0.068***	0.062***	0.089***
	(0.016)	(0.018)	(0.029)	(0.015)	(0.016)	(0.023)
Commodity price$_{it-1}$ $\times$ Size$_{it-1}$	0.498*	0.169	-0.202	0.562**	0.318	-0.116
	(0.258)	(0.248)	(0.200)	(0.233)	(0.221)	(0.162)
Commodity price$_{it-1}$ $\times$ Down$_i$		-0.174	-0.215**		-0.132	-0.196**
		(0.152)	(0.097)		(0.142)	(0.090)
Commodity price$_{it-1}$ $\times$ Up$_i$		-0.220**	-0.236**		-0.147	-0.179*
		(0.103)	(0.101)		(0.094)	(0.094)
Observations	779	779	661	780	780	662
R-squared	0.107	0.118	0.389	0.110	0.116	0.388
Number of countries	34	34	34	34	34	34
Number of years	25	25	24	25	25	24
Controls	No	No	Yes	No	No	Yes
Country FE	No	No	Yes	No	No	Yes
Year FE	No	No	Yes	No	No	Yes

NOTES: This table presents regressions on the cyclical component of log GDP per capita and the cyclical component of log commodity price index, conditional on commodity sector linkages and other control variables. Panel A shows results using the time-series average relative size for the commodity sector. Panel B shows results using the lagged size relative size of the commodity sector. Commodity sector defined as the sum of agriculture, fishing, and mining and quarrying. Size defined as the average relative size of the commodity sector. Inflation and NER denotes log-deviations around a quadratic trend for the Consumer Price Index and nominal exchange rate (local currency per dollar), respectively. Floating is a dummy variable that takes a value of zero if a country-year pair has a peg exchange rate arrangement, and one otherwise (floating exchange rate). All columns, when corresponding, include the level of variable Size, Down and Up (not reported). Robust standard errors clustered at the country level in parentheses. ***, ** and * denote statistical significance at the 1, 5 and 10% levels, respectively.

Table 1.B.2: The effect of commodity price and linkages on GDP fluctuations: Robustness (2)

	Fixed price weights			Time-varying price weights		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: aggregate GDP with average commodity size						
Commodity price$_{it-1}$	0.066***	0.063***	0.073**	0.062***	0.062***	0.077***
	(0.017)	(0.016)	(0.029)	(0.017)	(0.015)	(0.021)
Commodity price$_{it-1}$ × Size$_i$	0.500	0.222	0.151	0.570*	0.357	0.150
	(0.319)	(0.301)	(0.213)	(0.295)	(0.286)	(0.204)
Commodity price$_{it-1}$ × Down$_i$		-0.152	-0.165**		-0.126	-0.173**
		(0.155)	(0.071)		(0.148)	(0.067)
Commodity price$_{it-1}$ × Up$_i$		-0.145	-0.167**		-0.090	-0.137**
		(0.087)	(0.069)		(0.082)	(0.065)
Observations	807	807	683	808	808	684
R-squared	0.090	0.096	0.399	0.096	0.101	0.406
Number of countries	34	34	34	34	34	34
Number of years	25	25	24	25	25	24
Controls	No	No	Yes	No	No	Yes
Country FE	No	No	Yes	No	No	Yes
Year FE	No	No	Yes	No	No	Yes
Panel B: aggregate GDP with time-varying commodity size						
Commodity price$_{it-1}$	0.065***	0.060***	0.074**	0.063***	0.061***	0.080***
	(0.018)	(0.017)	(0.030)	(0.017)	(0.016)	(0.022)
Commodity price$_{it-1}$ × Size$_{it-1}$	0.496*	0.228	-0.126	0.535**	0.310	-0.124
	(0.252)	(0.247)	(0.197)	(0.235)	(0.229)	(0.154)
Commodity price$_{it-1}$ × Down$_i$		-0.161	-0.202**		-0.143	-0.215**
		(0.158)	(0.095)		(0.152)	(0.086)
Commodity price$_{it-1}$ × Up$_i$		-0.162	-0.213**		-0.113	-0.183*
		(0.104)	(0.102)		(0.098)	(0.095)
Observations	780	780	661	781	781	662
R-squared	0.087	0.093	0.396	0.093	0.098	0.403
Number of countries	34	34	34	34	34	34
Number of years	25	25	24	25	25	24
Controls	No	No	Yes	No	No	Yes
Country FE	No	No	Yes	No	No	Yes
Year FE	No	No	Yes	No	No	Yes

NOTES: This table presents regressions on the cyclical component of log aggregate GDP and the cyclical component of log commodity price index, conditional on commodity sector linkages and other control variables. Panel A shows results using the time-series average relative size for the commodity sector. Panel B shows results using the lagged size relative size of the commodity sector. Commodity sector defined as the sum of agriculture, fishing, and mining and quarrying. Size defined as the average relative size of the commodity sector. Inflation and NER denotes log-deviations around a quadratic trend for the Consumer Price Index and nominal exchange rate (local currency per dollar), respectively. Floating is a dummy variable that takes a value of zero if a country-year pair has a peg exchange rate arrangement, and one otherwise (floating exchange rate). All columns, when corresponding, include the level of variable Size, Down and Up (not reported). Robust standard errors clustered at the country level in parentheses. ***, ** and * denote statistical significance at the 1, 5 and 10% levels, respectively.

1.C MODEL CHARACTERIZATION

In this section I provide the full solution of the model presented in Section 1.3. I proceed in steps as follows.

1. Set the fundamentals of the economy. This is, sectoral productivities Z_j, $\forall j = 1, \ldots, N$ and the commodity price, P_1.

2. Recall that the commodity sector is indexed by $j = 1$. Using the supply equation for each sector, find prices for all sectors but commodity and the wage rate. Taking logs in (1.9), and defining $\widetilde{Z}_j \equiv Z_j / B_j$ as sectoral productivity adjusted by the constant term B, obtain for $j = 1$

$$\alpha_1 \log w = \log \widetilde{Z}_1 + (1 - \gamma_{11}) \log P_1 - \boldsymbol{\gamma}'_{j1} \log \mathbf{P}, \tag{1.18}$$

where $\boldsymbol{\gamma}_{j1}$ is the $(N-1) \times 1$ vector containing the parameters γ_{j1}, indicating the importance of sector j as input for the commodity sector (summarizing the role of the commodity sector as customer), and $\mathbf{P}$ being the $(N-1) \times 1$ vector of sectoral prices for $j = 2, \ldots, N$. At the same time, for $j = 2, \ldots, N$ equation (1.9) generates the following system of equations

$$\begin{bmatrix} (1-\gamma_{22}) & -\gamma_{32} & \cdots & -\gamma_{N2} \\ -\gamma_{23} & (1-\gamma_{33}) & \cdots & -\gamma_{N3} \\ \vdots & \vdots & \ddots & \vdots \\ -\gamma_{2N} & -\gamma_{3N} & \cdots & (1-\gamma_{NN}) \end{bmatrix} \begin{bmatrix} \log P_2 \\ \log P_3 \\ \vdots \\ \log P_N \end{bmatrix} = - \begin{bmatrix} \log \widetilde{Z}_2 \\ \log \widetilde{Z}_3 \\ \vdots \\ \log \widetilde{Z}_N \end{bmatrix} + \begin{bmatrix} \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_N \end{bmatrix} \log w + \begin{bmatrix} \gamma_{12} \\ \gamma_{13} \\ \vdots \\ \gamma_{1N} \end{bmatrix} \log P_1$$

$$\log \mathbf{P} = \widetilde{\mathbf{H}}(-\log \widetilde{\mathbf{Z}}_j + \boldsymbol{\alpha} \log w + \boldsymbol{\gamma}_{1j} \log P_1), \tag{1.19}$$

where $\widetilde{\mathbf{H}} \equiv (\mathbf{I} - \widetilde{\mathbf{\Gamma}}')^{-1}$ is the (transpose) of the Leontief inverse matrix for the mainland economy (i.e., all sectors but commodity) of dimensions $(N-1) \times (N-1)$. Here $\boldsymbol{\gamma}_{1j}$ is the $(N-1) \times 1$ vector containing the parameters γ_{1j}, indicating the importance of sector j as customer for the commodity sector (summarizing the role of the commodity sector as supplier).

Using (1.18) and (1.19) we solve for wage and sectoral prices

$$
\begin{aligned}
\log w &= \left(\frac{\log \widetilde{Z}_1 + \boldsymbol{\gamma}'_{j1} \widetilde{\mathbf{H}} \log \widetilde{\mathbf{Z}}_j}{\alpha_1 + \boldsymbol{\gamma}'_{j1} \widetilde{\mathbf{H}} \boldsymbol{\alpha}} \right) + \left(\frac{1 - \gamma_{11} - \boldsymbol{\gamma}'_{j1} \widetilde{\mathbf{H}} \boldsymbol{\gamma}_{1j}}{\alpha_1 + \boldsymbol{\gamma}'_{j1} \widetilde{\mathbf{H}} \boldsymbol{\alpha}} \right) \log P_1 \\
&= \Psi_0 + \Psi_1 \log P_1 \qquad (1.20) \\
\log \mathbf{P} &= \widetilde{\mathbf{H}} [(\boldsymbol{\alpha} \Psi_0 - \log \widetilde{\mathbf{Z}}_j) + (\boldsymbol{\alpha} \Psi_1 + \boldsymbol{\gamma}_{1j}) \log P_1], \qquad (1.21)
\end{aligned}
$$

where the term Ψ_0 is a function of productivity shocks and parameters only.

3. Using (1.21) and the expression for the price of consumption and for the price of the exportable good, get

$$
\begin{aligned}
\log P^c &= \mu_1 \log P_1 + \boldsymbol{\mu}' \log \mathbf{P} \\
&= \boldsymbol{\mu}' \widetilde{\mathbf{H}} (\boldsymbol{\alpha} \Psi_0 - \log \widetilde{\mathbf{Z}}_j) + [\mu_1 + \boldsymbol{\mu}' \widetilde{\mathbf{H}} (\boldsymbol{\alpha} \Psi_1 + \boldsymbol{\gamma}_{1j})] \log P_1 \qquad (1.22) \\
\log P^x &= \eta_1 \log P_1 + \boldsymbol{\eta}' \log \mathbf{P} \\
&= \boldsymbol{\eta}' \widetilde{\mathbf{H}} (\boldsymbol{\alpha} \Psi_0 - \log \widetilde{\mathbf{Z}}_j) + [\eta_1 + \boldsymbol{\eta}' \widetilde{\mathbf{H}} (\boldsymbol{\alpha} \Psi_1 + \boldsymbol{\gamma}_{1j})] \log P_1. \qquad (1.23)
\end{aligned}
$$

4. Using the equilibrium level of wage and for the price of consumption, get labor supply and consumption from the household problem

$$
\log L = \frac{1}{\xi} (\log w - \log P^c - \log \vartheta)
$$

$$\log C = \frac{1}{\xi}[(\xi + 1)(\log w - \log P^c) - \log \vartheta].$$

5. From the market clearing condition of consumption goods, obtain the level of production of the consumption good, $Y^c = C$. Also, from the optimal demand of sectoral output by the consumption good aggregator, get

$$A_j^c = \frac{\mu_j P^c Y^C}{P_j}. \tag{1.24}$$

6. Solve for sectoral production and the quantity produced by the exportable good aggregator. For this, note that the market clearing condition at the sectoral level, given by equation (1.11), can be written in matrix form as

$$\mathbf{Y} = \widehat{\mathbf{H}}(\mathbf{A}^c + \widetilde{\boldsymbol{\eta}} Y^x), \tag{1.25}$$

where $\mathbf{Y} = [Y_1, \ldots, Y_N]'$ is the $N \times 1$ vector of sectoral output, $\mathbf{A}^c = [A_1^c, \ldots, A_N^c]$ is the $N \times 1$ vector of sales to the consumption good aggregator, $\widetilde{\boldsymbol{\eta}}$ is the $N \times 1$ vector of relative-price adjusted requirements from the exportable good aggregator (with characteristic element $\eta_j P^x / P_j$), and $\widehat{\mathbf{H}} \equiv (\mathbf{I} - \widehat{\boldsymbol{\Gamma}})^{-1}$ is the relative price-adjusted Leontief inverse of the whole economy, where

$$\widehat{\boldsymbol{\Gamma}} = \begin{bmatrix} \gamma_{11} & \gamma_{12}\frac{P_2}{P_1} & \cdots & \gamma_{1N}\frac{P_N}{P_1} \\ \gamma_{21}\frac{P_1}{P_2} & \gamma_{22} & \cdots & \gamma_{2N}\frac{P_N}{P_2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1}\frac{P_1}{P_N} & \gamma_{N2}\frac{P_2}{P_N} & \cdots & \gamma_{NN} \end{bmatrix}.$$

On the other hand, from the market clearing condition (1.10) we have

$$L = \widetilde{\boldsymbol{\alpha}}'\mathbf{Y}, \tag{1.26}$$

where $\widetilde{\boldsymbol{\alpha}}$ is the $N \times 1$ vector with characteristic element $\alpha_j P_j / w$. Equations (1.25) and (1.26) define a $(N+1) \times (N+1)$ system of equations. Replacing the former in the latter, get output of the exportable good aggregator

$$Y^x = \frac{L - \widetilde{\boldsymbol{\alpha}}'\widehat{\mathbf{H}}\mathbf{A}^c}{\widetilde{\boldsymbol{\alpha}}'\widehat{\mathbf{H}}\widetilde{\boldsymbol{\eta}}}. \tag{1.27}$$

Once computed the level of exports, recover all other variables recursively.

1.D PROOF OF PROPOSITION 1

Proof. From the definition of GDP we have GDP $= wL$, so we need to know the wage and labor in equilibrium. From the GHH assumption, we have that labor supply equals $L = (w/\vartheta P^c)^{1/\xi}$. Therefore, we need to characterize the equilibrium value of wage, and the price of consumption. We know that wage and sectoral prices are given by equations (1.20) and (1.21), repeated here for convenience

$$\begin{aligned}
\log w &= \left(\frac{\log \widetilde{Z}_1 + \boldsymbol{\gamma}'_{j1}\widetilde{\mathbf{H}} \log \widetilde{\mathbf{Z}}_j}{\alpha_1 + \boldsymbol{\gamma}'_{j1}\widetilde{\mathbf{H}}\boldsymbol{\alpha}} \right) + \left(\frac{1 - \gamma_{11} - \boldsymbol{\gamma}'_{j1}\widetilde{\mathbf{H}}\boldsymbol{\gamma}_{1j}}{\alpha_1 + \boldsymbol{\gamma}'_{j1}\widetilde{\mathbf{H}}\boldsymbol{\alpha}} \right) \log P_1 \\
&= \Psi_0 + \Psi_1 \log P_1 \\
\log \mathbf{P} &= \widetilde{\mathbf{H}}[(\boldsymbol{\alpha}\Psi_0 - \log \widetilde{\mathbf{Z}}_j) + (\boldsymbol{\alpha}\Psi_1 + \boldsymbol{\gamma}_{1j}) \log P_1].
\end{aligned}$$

This generate the following price of consumption, as function of productivity and commodity price

$$\begin{aligned}\log P^c &= \mu_1 \log P_1 + \boldsymbol{\mu}' \log \mathbf{P} \\ &= \boldsymbol{\mu}'\widetilde{\mathbf{H}}(\boldsymbol{\alpha}\Psi_0 - \log \widetilde{\mathbf{Z}}_j) + [\mu_1 + \boldsymbol{\mu}'\widetilde{\mathbf{H}}(\boldsymbol{\alpha}\Psi_1 + \boldsymbol{\gamma}_{1j})] \log P_1.\end{aligned}$$

Taking logs in the expression of GDP and using the labor supply equation we have

$$\log \text{GDP} = \left(\frac{\xi+1}{\xi}\right) \log w - \frac{1}{\xi} \log P^c - \frac{1}{\xi} \log \vartheta.$$

Replacing for wage and the price of consumption we get the following expression for GDP, which depends on sectoral productivity and the commodity price

$$\begin{aligned}\log \text{GDP} =& \left(\frac{\xi+1}{\xi}\right)(\Psi_0 + \Psi_1 \log P_1) - \frac{1}{\xi} \log \vartheta - \\ & \frac{1}{\xi}\left\{\boldsymbol{\mu}'\widetilde{\mathbf{H}}(\boldsymbol{\alpha}\Psi_0 - \log \widetilde{\mathbf{Z}}_j) + [\mu_1 + \boldsymbol{\mu}'\widetilde{\mathbf{H}}(\boldsymbol{\alpha}\Psi_1 + \boldsymbol{\gamma}_{1j})] \log P_1\right\} \\ =& \underbrace{\frac{1}{\xi}\left\{(\xi+1)\Psi_0 - \boldsymbol{\mu}'\widetilde{\mathbf{H}}(\boldsymbol{\alpha}\Psi_0 - \log \widetilde{\mathbf{Z}}_j) - \log \vartheta\right\}}_{\text{Constant terms + productivity}} \\ & + \underbrace{\frac{1}{\xi}\left\{(\xi+1)\Psi_1 - \mu_1 - \boldsymbol{\mu}'\widetilde{\mathbf{H}}(\boldsymbol{\alpha}\Psi_1 + \boldsymbol{\gamma}_{1j})\right\}}_{\text{Elasticity of GDP to commodity price}} \log P_1,\end{aligned}$$

with $\Psi_0 \equiv \left(\frac{\log \widetilde{Z}_1 + \boldsymbol{\gamma}'_{j1}\widetilde{\mathbf{H}} \log \widetilde{\mathbf{Z}}_j}{\alpha_1 + \boldsymbol{\gamma}'_{j1}\widetilde{\mathbf{H}}\boldsymbol{\alpha}}\right)$ and $\Psi_1 \equiv \left(\frac{1 - \gamma_{11} - \boldsymbol{\gamma}'_{j1}\widetilde{\mathbf{H}}\boldsymbol{\gamma}_{1j}}{\alpha_1 + \boldsymbol{\gamma}'_{j1}\widetilde{\mathbf{H}}\boldsymbol{\alpha}}\right)$. Taken the partial derivative of GDP with respect to the commodity price, we obtain the same expression as in the main text. □

1.E ELASTICITY OF GDP TO COMMODITY PRICES WITH SEPARABLE PREFERENCES

Proposition 2. *The elasticity of GDP to commodity prices in a $N \times N$ economy with arbitrary IO linkages and separable preferences is given by*

$$\frac{\partial \log \text{GDP}}{\partial \log P_1} = \frac{1}{\sigma + \xi}\left\{(\xi + 1)\Psi_1 - (1 - \sigma)[\mu_1 + \boldsymbol{\mu}'\widetilde{\mathbf{H}}(\boldsymbol{\alpha}\Psi_1 + \boldsymbol{\gamma}_{1j})]\right\}, \quad (1.28)$$

where the definition of each element is the same as in Proposition 1.

Proof. Given separable preferences of the form $U = \frac{C^{1-\sigma}-1}{1-\sigma} - \vartheta\frac{L^{1+\xi}}{1+\xi}$, labor supply adopts the form $L = (\frac{1}{\vartheta}(\frac{w}{P^c})^{1-\sigma})^{\frac{1}{\sigma+\xi}}$. This change in preferences does not alter either the definition of GDP as labor income, nor the equilibrium wage/prices in the economy. Thus, the same values for equilibrium values shown in Proposition 1 hold. Taking logs to the expression of GDP we get

$$\log \text{GDP} = \frac{1+\xi}{\sigma+\xi}\log w - \frac{1-\sigma}{\sigma+\xi}\log P^c - \frac{1}{\sigma+\xi}\log\vartheta.$$

Replacing the equilibrium values for wages and price of consumption and taking the partial derivative with respect to $\log P_1$

$$\frac{\partial \log \text{GDP}}{\partial \log P_1} = \frac{1}{\sigma + \xi}\left\{(\xi + 1)\Psi_1 - (1 - \sigma)[\mu_1 + \boldsymbol{\mu}'\widetilde{\mathbf{H}}(\boldsymbol{\alpha}\Psi_1 + \boldsymbol{\gamma}_{1j})]\right\},$$

which is the same expression as in the main text.

□

2

DOMESTIC LINKAGES, NOMINAL RIGIDITIES AND THE TRANSMISSION OF COMMODITY PRICE SHOCKS

2.1 INTRODUCTION

Terms of trade and commodity price shocks are important drivers of the business cycle in emerging economies (Mendoza, 1995; Kose, 2002; Fernandez et al., 2018). While typically assumed as an endowment, recent papers has shown that commodity sectors are directly connected to other sectors in the economy through input-output linkages (Cao and Dong, 2020; Romero, 2020a). Those connections imply direct effects of commodity price shocks beyond the standard general equilibrium effects. While these papers study frictionless economies, there is a growing literature analyzing how sectoral distortions are transmitted to other industries via input-output linkages.[1] One of

[1]Most of the literature focuses on closed economies. For example, Jones (2013), Liu (2019), Baqaee and Farhi (2020) and Bigio and La'O (2020) analyze how sectoral

those frictions correspond to nominal rigidities: changes in marginal costs are not directly reflected into sectoral prices, inducing real distortions at the sectoral level which have aggregate effects. Given that fluctuations in commodity prices induce changes in other macro prices that affect production costs (wages and the real exchange rate), this paper asks how the interaction between production linkages and nominal rigidities shape aggregate responses to commodity price shocks.

To answer this question, this paper builds a New Keynesian model for a commodity exporting economy with production linkages. In the model, sectors produce using labor, domestic materials and importable goods, and are subject to heterogeneous nominal rigidities. The model also considers nominal rigidities in the importable sector. This latter point is important because, even though it is standard to assume that the law of one price holds for importable goods (Dib, 2008), there is plenty of evidence showing that prices "at the dock" differ from the price paid by domestic firms and consumers (Campa and Goldberg, 2005). To account for this imperfect exchange rate pass-through, I extend the model to include the possibility of nominal rigidities for importable goods. This approach is well suited for the question at hand (the interaction between nominal rigidities and production linkages), given the theoretical foundations provided by the New Keynesian model.[2,3]

---

wedges are propagated through production networks and generate misallocation in general equilibrium. Pasten et al. (2019) focuses in how sectoral heterogeneities in nominal rigidities propagate monetary policy shocks. For a model on the role of production linkages in the transmission of sectoral distortions in the international context, see Baqaee and Farhi (2021).

[2]This approach has also been followed by Smets and Wouters (2002), Monacelli (2005) and Garcia-Schmidt and Garcia-Cicco (2020a), among others.

[3]This is not the only alternative in the literature to model imperfect exchange rate pass-through. Another popular alternative in the literature is to consider local distribution costs (Devereux and Engel, 2002; Corsetti and Dedola, 2005; Atkeson and Burstein, 2008). See Burstein and Gopinath (2014) for a survey of different modelling approaches.

I start the analysis by providing analytical partial equilibrium results for a simplified version of the model to illustrate the forces at play. A positive commodity price shock generates an increase in income (wages) and a real exchange rate appreciation, putting pressures over the marginal cost of domestic producers in opposite directions. These direct effects over the cost of production depends on the intensity of usage of labor and importables for production. These two competing forces are standard in models analyzing terms of trade shocks (Schmitt-Grohe and Uribe, 2018).

The novel aspect of the analysis comes with the introduction of production linkages, which propagate fluctuations in marginal costs (and prices) to customer sectors when prices are flexible. Note that, on top of the direct effect of the usage of labor and importables, production linkages generates an indirect pressure over marginal costs through the price of materials, from suppliers to customers, which is governed by the intensity of usage of domestic inputs and the direct connection between a pair of sectors. When there are nominal rigidities, the latter transmission is incomplete because (i) sectoral prices can only partially respond to the shock, and (ii) production linkages transmit those distortions to other sectors in the economy.

Distinguishing between nominal rigidities in mainland sectors (such as manufactures and services) and nominal rigidities in the importable sector matters for the aggregate response of the economy. When prices are flexible in mainland, the real exchange rate appreciation induced by the commodity price shock fully propagates to other sectors via production linkages. Because wages increase with the shock, such appreciation partially counteracts the increase in marginal costs and prices. If imports are subject to nominal rigidities, the real appreciation is only partially transmitted to the rest of the economy through the indirect channel and prices tend to increase. The opposite happens when domestic sectors are subject to nominal rigidities, but imports do not, because the effect of the real appreciation dominates (completely through the direct channel and partially through the indirect channel). The combination of these forces makes the response of sectoral prices and aggregate inflation different relative to a world with flexible prices and a horizontal economy (i.e., no production linkages). Such distinct response of inflation has direct implications over consumption and output because the domestic real interest rate responds differently, conditional on the monetary policy stance. This is, given the monetary policy rule at play, the response of inflation to the shock depends on relative nominal rigidities and the production network, thus affecting the real interest rate and the level of consumption and output. It is important to recall that the model does not consider domestic capital as a productive factor. Therefore, changes in the domestic interest rate are not reflected in the determination of sectoral prices. While this modelling approach excludes an interesting feedback effect of monetary policy, I keep the model as simple as possible in order to show the mechanisms at play in a transparent way.

Then I study an extended dynamic version of the model calibrated for the Chilean economy, a heavily commodity-dependent country. This version considers two mainland sectors (manufactures and services), as well as the commodity and the importable sectors, and focuses in aggregate responses of the economy to the commodity price shock. The goal is to study how different counterfactual scenarios for nominal rigidities and production linkages quantitatively shape the response of consumption and output. Starting with the case in which production networks are at play, my results indicate that the source of nominal rigidities generate sizeable differences in the response of aggregate variables. For example, when comparing the impact-response of a counterfactual with flexible prices in mainland sectors relative to the baseline economy with nominal rigidities in every sector, inflation has a response twice as large on impact, which translates into a higher level of the interest rate over time. Consequently, the response of consumption is 48% lower, while the response of output is 25% lower. In a second counterfactual in which import prices are flexible but mainland do not, inflation and the real interest rate have a much smaller response relative to baseline. Therefore, the responses of consumption and output are 72 and 46% larger than in the baseline case, respectively. In a final counterfactual in which all sectors have flexible prices, the response of consumption and output are 60 and 43% larger than in the baseline economy. Those numbers reflect the counteracting effect of the exchange rate appreciation and the increase in wages, translating into a higher level of consumption and output relative to baseline but lower than in the case of flexible prices for imports. All these conclusions qualitatively remain when analyzing cumulative responses over time.

When comparing with the case of an economy without production linkages, there are three differences to highlight. First, the impact of the shock is amplified relative to a world with production linkages because relative sectoral sizes change due to the shock (Bouakez et al., 2009; Romero, 2020a). In general, we observe responses in consumption and output at least twice as large relative to the case in which production networks are operative. Second, as in the case of the economy with production linkages, more flexible prices amplify the response of the economy to the shock because the real appreciation tends to be larger than the increase in wages, generating lower inflation and a lower real interest rate. In this regard, the same qualitative patterns of the economy with production linkages hold. Finally, the degree of distortions is lower in the economy without production linkages because nominal rigidities are not directly transmitted to other sectors.

While the focus of this paper is on commodity price shocks, I extend the analysis to other driving forces that the literature has identified as relevant in the determination of the business cycle of small open economies.[4] The goal of this exercise is to understand if the interaction between production linkages and nominal rigidities are also important when other variables drive the business cycle. I highlight the following result: the source of fluctuations matters in order to make production linkages and nominal rigidities relevant channels for the aggregate response of the economy. In particular, shocks that directly affect marginal costs (such as the commodity price shock, by affecting the real exchange rate, or an aggregate productivity shock), imply much larger differences between an economy with flexible prices and one subject to nominal rigidities. On the contrary, such differences are lower when the driving forces affect the overall economy but not directly marginal costs, as in the case of foreign demand and foreign interest rate shocks. Related to this last point, differences in responses between an economy with production linkages and one without them are milder for the foreign interest rate shock and the foreign demand shock. This implies that production linkages and nominal rigidities (and their interaction) are relevant transmission channels, conditional on the driving forces of the economy. This is a novel result in the literature of macro-network models because such papers take as given either the network structure or the degree of nominal rigidities, without analyzing the extensive margin of adjustment of those channels.[5]

Related literature. The main contribution of this paper is to analyze the interaction of domestic production linkages and nominal

[4]Specifically, I study the impact of (i) aggregate productivity shocks (Aguiar and Gopinath, 2007; Chang and Fernandez, 2013), (ii) foreign interest rate shocks (Neumeyer and Perri, 2005; Uribe and Yue, 2006), and (iii) foreign demand shocks (Dib, 2008; Garcia-Schmidt and Garcia-Cicco, 2020a).

[5]One exception corresponds to papers that study the dynamics of production networks which, by definition, analyze both the intensive and extensive margin of production linkages. See, for example, Huneeus (2018), Lim (2018) and Acemoglu and Azar (2020). However, in general those papers consider economies without nominal rigidities.

rigidities in shaping the aggregate response of the economy to commodity price shocks. In particular, it highlights the tension produced by the shock over sectoral marginal costs by increasing wages and appreciating the real exchange rate. On top of that, relative nominal rigidities between importable goods and domestic goods distort the transmission from marginal costs to prices, and such distortions are propagated through production networks to other sectors. This imply a differential response in aggregate inflation, the interest rate set by the central bank, and the final response of consumption and output.

In this regard, this paper contributes to two strands of literature. First, to the literature analyzing the role of terms of trade shocks for the business cycle in emerging economies. While most of the initial literature study RBC models (Mendoza, 1995; Kose, 2002),[6] there are recent contributions studying these shocks in the context of Keynesian economies with nominal rigidities. For example, Catão and Chang (2015) analyzes the design of optimal monetary policy for a commodity-importing economy. On the other hand, Drechsel et al. (2019) analyzes the design of optimal monetary policy in a model in which commodity output can be used as collateral to alleviate financial frictions. Other papers, such as Fornero et al. (2016) and Medina and Soto (2016) analyze the impact of commodity price shocks in a small open economy with nominal rigidities, to study the response of aggregate investment to the shock and the performance of alternative fiscal rules.

This paper builds on Chapter 1 of this thesis and Romero (2020a) to analyze the role of nominal rigidities, and makes two contributions relative to previous literature. First, it takes into account production linkages and how they shape aggregate responses of the

[6]Other papers analyzing the role of terms of trade and commodity price shocks in the context of RBC models are Caputo and Irarrazaval (2017), Drechsel and Tenreyro (2018), Schmitt-Grohe and Uribe (2018) and Kohn et al. (2020).

economy to the shock. My results indicate that the interaction between nominal rigidities and production linkages generate sizeable differences in the response of the economy, so are relevant channels to understand the aggregate consequences of those fluctuations. Second, previous literature either study the optimal monetary policy design given the shock or take the degree of nominal rigidities as given. In this paper, I take the monetary policy stance (i.e., the way the Taylor rule weights inflation and output growth to determine the nominal interest rate) as given and focus on how important are the interactions between nominal rigidities and production linkages for the aggregate response of the economy to the shock. In this regard, this paper helps us to understand the role of nominal rigidities, both in the extensive margin and on the sources of those rigidities, for business cycle fluctuations.

The closest paper to mine is the contemporaneous work of Cao and Dong (2020). They study the relative importance of production linkages (both domestic and with the rest of the world) in the transmission of commodity price shocks in Canada, and focus their analysis in the slowdown of the economy in the period 2014-2016. Relative to their work, this paper investigates the systematic consequences of commodity price shocks in an economy with production linkages, and emphasizes the interaction with nominal rigidities for the transmission of the shock.

This paper is also related to the literature analyzing multi-sector models in the New Keynesian framework. This literature analyzes the interaction between microeconomic heterogeneity, mostly in terms of differences in price rigidities, input-output linkages and productivity, to try to understand the network origin of aggregate fluctuations (Pasten et al., 2019), the microeconomic behavior of prices (Carvalho et al., 2020), the flattening of the Phillips curve over time (Höynck, 2020) or the optimal monetary policy that should be in place (Castro, 2019; Rubbo, 2020; La'O and Tahbaz-Salehi, 2020).[7] Different to these papers that focus the analysis in closed economy models, I analyze a multi-sector New Keynesian model for a small open economy, in which the driver is the price of commodity goods exported by the country. Also, my focus is to understand the interaction between nominal rigidities and production linkages for the transmission of the shock, while these previous literature are mostly concern in the design of monetary policy, given a level of sectoral nominal rigidities and production linkages. The key insight that my paper provides to this literature is the tension between fluctuations in wages and fluctuations in the real exchange rate, which by definition, are ignored in closed economy models. Two additional contributions of this paper relative to the previous literature are that (i) nominal rigidities and production linkages matter conditionally on the driving forces of the economy, and (ii) that the relative degree of nominal rigidities in importable goods versus domestic sectors are relevant to understand the dampening versus amplifying effects in the transmission of commodity price shocks.

The rest of the paper is organized as follows. Section 2.2 presents the model for the small open economy. Section 2.3 discusses the theoretical aspects of the interaction between nominal rigidities and production linkages. Section 2.4 presents the main quantitative results, while section 2.5 compares the transmission when there are other driving forces. Section 2.6 concludes.

[7] Wei and Xie (2020) studies optimal monetary policy in a New Keynesian model for a small open economy with global value chains. See also the seminal contribution of Gali and Monacelli (2005) that studies optimal monetary policy in a New Keynesian model for a small open economy.

2.2 MODEL

This section describes the theoretical model to study the impact of commodity price shocks in the small open economy and the interaction between production linkages and nominal rigidities. Figure 2.A.1 summarizes the structure and flows of the economy. It consists in a representative household, N mainland productive sectors, a commodity sector, an importable sector and an aggregator producing exportable goods.[8] The household consumes an aggregate good composed by the different mainland goods, supplies labor to all productive sectors and can take both domestic and foreign debt. Mainland sectors are characterized by a competitive aggregator combining a continuum of differentiated varieties whose producers are subject to nominal rigidities. The output of these sectors can be used either for domestic consumption, as materials for other industries (including mainland and the commodity sector) or to build and exportable good, as denoted by the red lines in the figure. The importable sector has similar characteristics (monopolistic competition and nominal rigidities), but produces by demanding goods from abroad and transforming them into different varieties. The commodity sector is characterized by a representative competitive firm who sold its entire output abroad. (Therefore, the country provides a composite exportable good built with mainland sectors and the commodity good to the rest of the world.) Because the economy is small and open, the commodity sector takes the price of the commodity good as given. In what follows, j and i denotes domestic mainland sectors, while the commodity sector is denoted by c.

[8] I use the term "mainland" to differentiate sectors that are indirectly related to the rest of the world (such as manufactures and services) to those that are directly connected (such as the commodity sector, imports and exports).

2.2.1 *Household*

The representative household maximizes its lifetime utility by choosing consumption and labor supplied to firms, subject to the budget constraint in (2.1). Formally, it solves the following problem

$$\max_{C_t, L_t, B_t, B_t^*} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \kappa \frac{L_t^{1+\varphi}}{1+\varphi} \right)$$

subject to

$$P_t C_t + B_t + S_t B_t^* = W_t N_t + R_{t-1} B_{t-1} + S_t R_{t-1}^* B_{t-1} + D_t, \quad (2.1)$$

where $\sigma \geq 1$ is the intertemporal elasticity of substitution, β is the discount factor, $\varphi \geq 0$ is the inverse of the Frisch elasticity, and κ is a constant that governs the desutility of labor. In terms of variables, C_t is the aggregate consumption bundle and L_t denotes total hours offered by the household at a wage rate W_t. In this open economy context, S_t denotes the nominal exchange rate (domestic currency per unit of foreign currency), while B_t and B_t^* are the amount of domestic and foreign bonds demanded at period t, with gross nominal return R_t and R_t^*, respectively. The term D_t collects all profits in the economy.

Intratemporal decisions. Each period the household must allocate its consumption expenditures across different sectors. The consumption aggregator C_t takes a Cobb-Douglas form

$$C_t = \delta \prod_{j=1}^{N} C_{jt}^{\vartheta_j}, \quad \text{with} \quad \sum_{j=1}^{N} \vartheta_j = 1,$$

where ϑ_j is the expenditure share of sectoral consumption j and δ is a constant term. Cost minimization derives the demands

$$C_{jt} = \frac{\vartheta_j P_t C_t}{P_{jt}} \quad \text{for every } j = 1, \ldots, N,$$

where $P_t = \prod_{j=1}^{N} P_{jt}^{\vartheta_j}$ is the ideal price index that depends on sectoral prices P_{jt}. I assume that the household only consumes aggregate sectoral goods and not the different varieties (see details below).[9]

Total hours are characterized by the following aggregator

$$L_t = \left(\sum_{j=1}^{N} L_{jt}^{\frac{1+\varrho}{\varrho}} + L_{ct}^{\frac{1+\varrho}{\varrho}} \right)^{\frac{\varrho}{1+\varrho}},$$

where $\varrho \geq 0$ denotes the intertemporal elasticity of substitution governing the degree of labor mobility across sectors. There are two important elements about this aggregator. First, it allows for imperfect labor mobility and keeps the representative household framework (Horvath, 2000). Different to the consumption aggregator bundle, this specification considers the possibility that the household supplies hours to both mainland and the commodity sectors. Second, having a less than perfect labor mobility will be relevant in the quantitative section below. Given a commodity price shock, this sector will increase its demand for factors, which induces an increase in its offered wage. Imperfect mobility guarantees that not all the labor supply reallocates towards commodity activities, so other sectors remain relevant after the shock.

The optimization problem of the household is to maximize total labor income, given sectoral wages and the labor aggregator. This derives the labor supply schedule

[9] This assumption is without loss of generality because each individual variety is demanded by a sectoral aggregator. For a multisector model in which the representative household directly consumes different varieties, see Pasten et al. (2019) and Carvalho et al. (2020).

$$L_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{\varrho} L_t, \quad \text{for every } j = 1, \ldots, N, \tag{2.2}$$

with $W_t \equiv \left(\sum_{j=1}^{N} W_{jt}^{1+\varrho} + W_{ct}^{1+\varrho}\right)^{\frac{1}{1+\varrho}}$ being the aggregate wage index that satisfies $W_t L_t = \sum_{j=1}^{N} W_{jt} L_{jt} + W_{ct} L_{ct}$.

Intertemporal decisions. On the intertemporal side, the household maximizes its lifetime utility subject to (2.1) by choosing sequences for consumption, total hours, and domestic and foreign bonds. The optimality conditions are summarized by

$$1 = \beta R_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1}{\Pi_{t+1}}\right] \tag{2.3}$$

$$1 = \beta R_t^* \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{\Pi_{t+1}^s}{\Pi_{t+1}}\right] \tag{2.4}$$

$$\frac{W_t}{P_t} = \kappa C_t^{\sigma} L_t^{\varphi}, \tag{2.5}$$

where $\Pi_t \equiv P_t/P_{t-1}$ and $\Pi_t^s \equiv S_t/S_{t-1}$ denote the gross inflation and the gross nominal depreciation rate, respectively. The first condition corresponds to the Euler equation for domestic bonds, while the second is the Euler equation for foreign bonds.[10] Finally, the last equation corresponds to the aggregate labor supply.

2.2.2 *Production*

I separate the problem between mainland and commodity production. Mainland sectors are characterized by two layers of produc-

[10] Combining (2.3) and (2.4) obtains a relation between the domestic and foreign interest rate, mediated by the expected nominal depreciation. Because the model will be solved around the deterministic steady-state, to induce stationarity I impose a debt-elastic foreign interest rate (Schmitt-Grohe and Uribe, 2003), so the uncovered interest parity condition (UIP) does not hold.

tion. At the top, there is a representative competitive firm aggregating the output of atomistic producers. The output of this firm is sold to households and other aggregators. At the bottom, there is a continuum of monopolistically competitive producers which are subject to nominal rigidities à la Calvo. The commodity sector is composed by a representative and competitive firm. Different to other sectors in the economy, it takes the international price of its good as given from foreign markets.

2.2.2.1 Mainland Sectors

Sectoral aggregator. Each sectoral good j is produced by a competitive firm that aggregates the output of a continuum of intermediate producers according to

$$Y_{jt} = \left(\int_0^1 Y_{jt}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $z \in [0,1]$ denotes the z-th variety of good j and ε is the elasticity of substitution across varieties, which is common across sectors. Profit maximization derives the following demand for variety z

$$Y_{jt}(z) = \left(\frac{P_{jt}(z)}{P_{jt}} \right)^{-\varepsilon} Y_{jt}, \tag{2.6}$$

where $P_{jt} \equiv \left(\int_0^1 P_{jt}(z)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$ is the price of the final sectoral good j. The output of this sector is used either for consumption of each household, exports, or as an input for other sectors of the economy.

Sectoral producers. Each producer $z \in [0,1]$ in sector j operates in a monopolistically competitive environment. Technology is common to all firms within a sector but differs across sectors. The production function is given by

$$Y_{jt}(z) = \delta_j L_{jt}(z)^{\alpha_j} M_{jt}(z)^{\mu_j} V^m_{jt}(z)^{1-\alpha_j-\mu_j},$$

where L_{jt}, M_{jt} and V^m_{jt} denote labor, domestic materials and an importable good, respectively, and where δ_j is a sector-specific constant. The bundle of materials is sector-specific and given by

$$M_{jt} = \delta^m_j \prod_{i=1}^{N} M_{ijt}^{\gamma_{ij}}, \quad \text{with} \quad \sum_{i=1}^{N} \gamma_{ij} = 1,$$

where M_{ijt} denotes sales of materials from sector i to sector j in period t. The elements $\gamma_{ij} \geq 0$ can be collected in the matrix $\mathbf{\Gamma}$, which establishes the production linkages between sectors in the economy, conditional on the level of usage of materials, μ_j. In particular, entry γ_{ij} denotes the fraction of intermediates from sector i to sector j, per units of total materials demanded by sector j.[11] In principle, sectoral producers could demand the commodity good in the form of materials for production. However, the main role of the commodity sector is as customer of other sectors rather than supplier. To keep the analysis focused, I do not consider sales from the commodity sector to the rest of the economy.[12]

Because technology and factor prices are common across producers within a sector, the marginal cost is the same across monopolists. Given this, by cost minimization, the demands for labor, materials, and imports at the *sectoral* level are

[11]Note that the combination of the intensity of usage of materials μ_j with the matrix $\mathbf{\Gamma}$, characterize the input-output matrix of the economy, denoting the amount of intermediates from sector i necessary to produce \$1 of output in sector j.

[12]Also, because the model is applied to the Chilean experience, most of domestic sales are within the same sector, while sales to mainland sectors are negligible. See Cao and Dong (2020) and Romero (2020a) for models in which the commodity sector also acts as supplier of mainland industries.

$$L_{jt} = \alpha_j \frac{MC_{jt}Y_{jt}}{W_{jt}} \Delta_{jt}$$
$$M_{jt} = \mu_j \frac{MC_{jt}Y_{jt}}{P^m_{jt}} \Delta_{jt}$$
$$V^m_{jt} = (1 - \alpha_j - \mu_j) \frac{MC_{jt}Y_{jt}}{P^m_{vt}} \Delta_{jt},$$

where $A_{jt} \equiv \int_0^1 A_{jt}(z)dz$ for $A = \{L, M, V\}$, $P^m_{jt} = \prod_{i=1}^N P_{it}^{\gamma_{ij}}$ is the sector-specific price of intermediate goods, P^m_{vt} is the (common) price of the importable good, and MC_{jt} denotes the marginal cost of production. Also by cost minimization, the demand for materials from sector i is given by $M_{ijt} = \gamma_{ij} P^m_{jt} M_{jt} / P_{it}$. In the previous expressions, Δ_{jt} is the sectoral price dispersion coming from nominal rigidities and firms' pricing rule, which I describe next.

Nominal rigidities. Firms face price stickiness à la Calvo, implying that with probability $1 - \theta_j$ they can reset their price in any given period, regardless of their last update. Therefore, a fraction θ_j of producers keep their price unchanged from last period, while the remaining fraction $1 - \theta_j$ can set their desired price (a constant markup over marginal costs). Note that this probability is common to firms within a sector but not necessarily the same across sectors. This assumption captures the importance of heterogeneity in nominal rigidities to understand aggregate fluctuations, as shown by Pasten et al. (2019).

The desired price of any monopolist z in sector j is chosen by maximizing profits subject to the demand of the sectoral aggregator, taking into account that this price can remain over time with probability θ_j

$$\max_{\widetilde{P}_{jt}(z)} \sum_{\tau=0}^{\infty} \theta_j^\tau \mathbb{E}_t \Big\{ Q_{t,t+\tau} \Big[\widetilde{P}_{jt}(z) Y_{jt+\tau|t}(z) - MC_{jt+\tau} Y_{jt+\tau|t}(z) \Big] \Big\},$$

subject to (2.6), where $\widetilde{P}_{jt}$ denotes the desired price and $Q_{t,t+\tau}$ is the stochastic discount factor of the representative household.[13] In the previous expression, $Y_{jt+\tau|t}(z)$ denotes the demand of the sectoral aggregator in period $t+\tau$, conditional on the last price reset of firm z in period t.

The optimality condition is given by

$$\frac{\widetilde{P}_{jt}}{P_t} = \frac{\varepsilon}{\varepsilon - 1}\frac{S_{jt}}{F_{jt}} \tag{2.7}$$

with

$$\begin{aligned} S_{jt} &= C_t^{-\sigma} Y_{jt} \frac{MC_{jt}}{P_t} + \theta_j \beta \mathbb{E}_t [\Pi_{jt+1}^{\varepsilon} S_{jt+1}] \\ F_{jt} &= C_t^{-\sigma} Y_{jt} + \theta_j \beta \mathbb{E}_t [\Pi_{t+1}^{-1} \Pi_{jt+1}^{\varepsilon} F_{jt+1}], \end{aligned}$$

where Π_{jt} denotes sector j's gross inflation. Note that when prices are flexible ($\theta_j = 0$), the model collapses to the familiar case in which the desired price is a constant markup over marginal cost. Because the marginal cost is common across firms within a sector, the optimal desired price is the same across firms.

Given the properties of the Calvo pricing setup, sectoral prices and the sectoral price dispersion evolve as

$$\begin{aligned} 1 &= (1-\theta_j)\left(\frac{\widetilde{P}_{jt}}{P_{jt}}\right)^{1-\varepsilon} + \theta_j \Pi_{jt}^{\varepsilon-1} \\ \Delta_{jt} &= (1-\theta_j)\left(\frac{\widetilde{P}_{jt}}{P_{jt}}\right)^{-\varepsilon} + \theta_j \Pi_{jt}^{\varepsilon} \Delta_{jt-1}. \end{aligned}$$

At this point, it worths discussing how production linkages interact with nominal rigidities (see more details in section 2.3). From

[13] In particular, $Q_{t,t+\tau} \equiv \beta^{\tau} \left(\frac{C_{t+\tau}}{C_t}\right)^{-\sigma} \frac{1}{\Pi_{t+\tau}}$.

equation (2.7), the optimal price of firms imperfectly depend on their marginal cost, a standard result in the New Keynesian model: due to nominal rigidities, the desired price is not a markup over marginal costs. On the other hand, production linkages change the structure of marginal cost by including not only the price of primary factors, such as labor, but also the price of other sectors. This is one of the transmission channels of production linkages (Acemoglu et al., 2016). Note however that, because of nominal rigidities, the transmission from changes in marginal costs in sector i to prices in sector j is incomplete, and depends on the relative degree of nominal rigidities across sectors (Basu, 1995; Pasten et al., 2019).

There is one important difference relative to these papers though. While previous research studies closed economies, my setup is characterized by (i) a small open economy in which sectors produce with importable goods, and (ii) the main driving force (a commodity price shock) directly impacts the price of those imports. Together, these two forces imply that the commodity price shock impacts not only wages but the price of importable goods, moving marginal costs in opposite directions. This effect is transmitted to other sectors through production linkages but the final effect not only depends on the strength of production linkages and relative sectoral rigidities, but on the exchange rate pass-through of the economy. As we will see below, if there exist nominal rigidities in the importable sector, we will observe a differential effect of the shock that interact with sectoral nominal rigidities and production linkages, effect that has not been analyzed in the literature.

2.2.2.2 Commodity Sector

The commodity sector is characterized by a representative firm that produces with the following Cobb-Douglas technology

$$Y_{ct} = \delta_c (L_{ct})^{\alpha_c} (M_{ct})^{\mu_c} (V^m_{ct})^{1-\alpha_c-\mu_c},$$

where δ_c is a constant term. As in mainland sectors, L_c denotes labor, M_c the amount of domestic materials demanded by the sector, and V^m_c denotes the demand for the importable good. I assume that the law of one price holds for the commodity price, implying that the price in domestic currency is $P_{ct} = S_t P^*_{ct}$.

The commodity sector is competitive, so profit maximization derives the following demands for factors

$$L_{ct} = \alpha_c \frac{P_{ct} Y_{ct}}{W_{ct}}, \quad M_{ct} = \mu_c \frac{P_{ct} Y_{ct}}{P^m_{ct}}, \quad \text{and} \quad V^m_{ct} = (1 - \alpha_c - \mu_c) \frac{P_{ct} Y_{ct}}{P^m_{vt}}.$$

As in the case of mainland sectors, materials take a Cobb-Douglas form

$$M_{ct} = \delta^m_c \prod_{j=1}^{N} M_{jct}^{\gamma_{jc}}, \quad \text{with} \quad \sum_{j=1}^{N} \gamma_{jc} = 1.$$

Cost minimization implies a demand for specific sectors of the form $M_{jct} = \gamma_{jc} P^m_{ct} M_{ct} / P_{jt}$, with the price of materials given by $P^m_{ct} = \prod_{j=1}^{N} P_{jt}^{\gamma_{jc}}$.

As mentioned before, the commodity sector takes the international price as given. I assume this follows an autoregressive form, and its the only source of fluctuations in the baseline analysis of the economy

$$\log(P^*_{ct}) = (1 - \rho_c) \log(P^*_c) + \rho_c \log(P^*_{ct-1}) + \nu^c_t.$$

2.2.3 *Exportable Good*

There is a competitive firm combining inputs from different mainland sectors to produce an exportable good, which is fully sold abroad. The technology of this sector is given by $Y_t^x = \delta_x \prod_{j=1}^{N} (Y_{jt}^x)^{\psi_j}$, where δ_x is a constant term and $\sum_{j=1}^{N} \psi_j = 1$ holds. The price of exports and the demand for inputs from each sector are given by

$$P_t^x = \prod_{j=1}^{N} P_{jt}^{\psi_j} \qquad \text{and} \qquad Y_{jt}^x = \psi_j \frac{P_t^x}{P_{jt}} Y_t^x.$$

The foreign demand for the exportable good takes the form $Y_t^x = \omega^x \left(\frac{P_t^x}{S_t P_t^*} \right)^{-\eta^*} Y_t^*$, where P_t^* and Y_t^* denote the foreign CPI and the foreign level of output, respectively.

2.2.4 *Importable Good*

In the model, sectoral producers are the only agents who import goods from abroad.[14] Therefore, external prices and the real exchange rate play a major role in determining the cost of production of firms. In the business cycle literature for small open economies is standard to assume that the law of one price holds for foreign goods, which is particularly used in models analyzing the role of terms of trade shocks.[15] However, there is plenty of evidence indicating that the law of price holds only "at the dock" for imported goods in the short run, so there is only a partial exchange rate pass-through to the rest of the economy at business cycle frequencies. For

[14]This assumption is without loss of generality. A model in which the representative household also imports would be isomorphic to a version of this model in which (i) one sector only produces with imports, and (ii) this sector sells to the household, with an appropriate weight ϑ capturing the degree of home bias.

[15]See, among others, Dib (2008), Caputo and Irarrazaval (2017), Drechsel et al. (2019) and Cao and Dong (2020).

example, Campa and Goldberg (2005) documents that the exchange rate pass-through to imported prices is only 46% across a sample of OECD countries in the period 1975-2003. Those results have been recently confirmed by Brun-Aguerre et al. (2012) and Choudhri and Hakura (2015), showing that the exchange rate pass-through to imported goods lies between 0 and 1 in a sample of developed and emerging countries. Similar figures have been shown by Gopinath and Rigobon (2008) and Gopinath et al. (2010) for the U.S., with a pass-through to imported prices of 22%, and by Campa and Gonzalez Minguez (2006) for the Euro Area.[16]

To model this limited exchange rate pass-through, I follow the literature and incorporate monopolistic competition and nominal rigidities for the importable good sector (Smets and Wouters, 2002; Monacelli, 2005; Garcia-Schmidt and Garcia-Cicco, 2020a).[17] As we will see later in the theoretical and quantitative analysis, the interaction of nominal rigidities in mainland and imported sectors, as well as production linkages, will generate rich interactions in the determination of aggregate variables.

There exist an importable-good sector with similar features as mainland sectors: there is a continuum of monopolistically competitive firms demanding imports from abroad and transforming these good into differentiated varieties. The output of these firms is combined by a monopolistically competitive firm in a CES fashion and sold to mainland and commodity sectors for production[18]

[16]Recent additional evidence for developed and emerging economies on exchange rate pass-through to other prices can be found, among many others, in Choudhri and Hakura (2006) and Kohlscheen (2010). See Burstein and Gopinath (2014) for a survey.

[17]An alternative approach to model the limited exchange rate pass-through is to consider local distribution costs. See for example Devereux and Engel (2002), Corsetti and Dedola (2005) and Atkeson and Burstein (2008).

[18]Note that V_t^m is the total quantity produced by the aggregator while $V_t(z)$ denotes the quantity of a specific variety. At the same time, $P_{vt}(z)$ is the price of variety z, while P_{vt}^m is the price at which the aggregator sales to other domestic firms.

$$V_t^m = \left(\int_0^1 V_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \text{with demand}$$
$$V_t(z) = \left(\frac{P_{vt}(z)}{P_{vt}^m} \right)^{-\varepsilon} V_t^m.$$

Total output of this sector must be equal to the amount demanded by other sectors for production, $V_t^m = V_{ct}^m + \sum_j V_{jt}$. For simplicity, I assume that the elasticity of substitution among varieties are the same as in mainland sectors.

The monopolistically competitive firms $z \in [0, 1]$ produce with the linear technology $V_t(z) = V_t^*(z)$. These firms import a homogenous good from abroad and transform it into differentiated varieties. As in mainland sectors, these firms are subject to nominal rigidities à la Calvo, with a probability of price adjustment given by $1 - \theta_v$. The optimization problem, similar to the one presented for mainland sectors, is to maximize profits subject to demand, by choosing the optimal price $\widetilde{P}_{vt}(z)$. Letting P_{vt}^* to be the foreign price of imports and assuming that the law of one price holds, the domestic price for inputs in the importable sector is equal to the nominal exchange rate times the foreign price, $P_{vt} = S_t P_{vt}^*$. This latter expression corresponds to the price "at the dock of imports." Assuming that the price of the importable good is the same as the foreign CPI, the marginal cost of production in the importable sector in units of the domestic consumption good corresponds to the real exchange rate. Because of nominal rigidities, fluctuations in the real exchange rate are imperfectly passed to mainland sectors.

2.2.5 *Aggregation, Monetary Policy and Market Clearing*

Total exports are given by the value of commodity goods plus the value of the exportable good, $X_t = P_{ct} Y_{ct} + P_t^x Y_t^x$. On the other hand, total imports are given by $V_t = \Delta_{vt} V_t^m$, where Δ_{vt} is the price dispersion term in the importable sector given by nominal rigidities. Combining these expressions, the trade balance is given by $TB_t = X_t - V_t$.

Gross domestic product (GDP) is defined as the sum of aggregate consumption and the trade balance, $GDP_t = C_t + TB_t$.

Finally, the domestic nominal rate is set by the central bank using the following Taylor rule

$$\frac{R_t}{R} = (\Pi_t)^{\phi_\pi} \left(\frac{GDP_t}{GDP_{t-1}} \right)^{\phi_y}, \tag{2.8}$$

where R is the long-run value of the gross nominal rate.

For equilibrium, we have to impose market clearing conditions for every sectoral good

$$Y_{jt} = C_{jt} + Y_{jt}^x + \sum_{i=1}^{N} M_{jit} + M_{jct}. \tag{2.9}$$

By combining the different market clearing conditions of the model, the domestic bond market clearing ($B_t = 0$) and the aggregate resource constraint (given by the budget constraints of the household), the financial position of the economy is given by

$$S_t B_t^* = TB_t + S_t R_{t-1}^* B_{t-1}^*. \tag{2.10}$$

To close the model, I consider the following specification for the foreign interest rate

$$R_t^* = R_t^w \times \exp\left[\phi_b \left(\bar{b} - \frac{S_t B_t^*}{GDP_t} \right) \right], \tag{2.11}$$

where $\phi_b > 0$ and R_t^w is the world interest rate and the term $\exp[\cdot]$ is a risk premium that the country pays over the risk-free rate. This premium is affected by the deviations of total debt of the country, relative to GDP, with respect to a long-run determined value, $\bar{b}$. This guarantees the stationarity of the model (Schmitt-Grohe and Uribe, 2003).

2.3 INSPECTING THE MECHANISM

This section explores the effect of a commodity price shock in the small open economy and how the interaction between nominal rigidities and production linkages shape aggregate responses. To simplify the analysis I assume (i) an economy in financial autarky, implying trade balance at every period, and (ii) the commodity good is an endowment normalized to one. Later on, in the quantitative analysis, I relax those assumptions in line with the full model presented in the previous section. In what follows, all lowercase prices denote prices relative to the aggregate consumption good.

Commodity prices and the real exchange rate. Given the assumptions of financial autarky and a commodity endowment, the trade balance condition ($TB_t = 0$) pins down the real exchange rate (Corsetti et al., 2010)

$$rer_t = \left(\frac{\sum_{j=1}^{N} V_{jt} - p_{ct}}{\omega^x Y_t^* (p_t^x)^{1-\eta^*}} \right)^{\frac{1}{\eta^*-1}}. \tag{2.12}$$

In this simplified economy, to the extent that the foreign demand elasticity for domestic goods is larger than one ($\eta^* > 1$), the commodity price shock will induce a real appreciation. Even though this is just a partial equilibrium effect, because the price of exports and the quantities imported also depend on the real exchange rate, the previous expression helps to illustrate one of the direct implications of a commodity price shock over aggregate prices. Next, I analyze the implications of this real appreciation over marginal costs when there are production linkages and nominal rigidities.

Marginal costs, linkages and nominal rigidities. The marginal cost in any sector j reads as

$$mc_{jt} = (w_{jt})^{\alpha_j}(p^m_{jt})^{\mu_j}(p^m_{vt})^{\phi_j},$$

where $\phi_j \equiv 1 - \alpha_j - \mu_j$ and $p^m_{jt} = \prod_{i=1}^{N} p_{it}^{\gamma_{ij}}$ is the price of materials for sector j. Consider first the case in which all price are flexible (both in mainland sectors and in the importable sector). In such scenario, the price of sector j and the price of the imported good are $p_{jt} = \left(\frac{\varepsilon}{\varepsilon-1}\right)mc_{jt}$ and $p^m_{vt} = \left(\frac{\varepsilon}{\varepsilon-1}\right)rer_t$, respectively. Taking a first order approximation around the steady-state for marginal costs and replacing the expression for the price of materials, we get the following N-dimensional system of equations for sectoral prices

$$\widehat{p}_{jt} = \alpha_j\widehat{w}_{jt} + \mu_j\sum_{i=1}^{N}\gamma_{ij}\widehat{p}_{it} + \phi_j\widehat{rer}_t, \tag{2.13}$$

where hat-variables corresponds to approximated values around the steady-state. From (2.13) we observe three forces by which the initial commodity price shock affect the determination of sectoral prices. First, the commodity price shock induces an increase in income in the economy, which is translated into higher demand and more production. Therefore, we expect that wages increase at the sectoral level, increasing equilibrium prices. Second, the real exchange rate appreciation (see equation 2.12) directly decreases marginal costs, effect that is governed by the intensity of imports in the sector, ϕ_j. Note that these two forces correspond to the direct channels of the commodity price shock over sectoral marginal costs. Finally, a third force at play is coming from the transmission of variations in the marginal cost of suppliers. This latter effect is governed by the intensity of usage in materials (μ_j), as well as the direct link with specific sectors (γ_{ij}). All these forces can be summarized by solving (2.13) for prices

$$\widehat{\mathbf{p}}_t = \mathbf{H}(\boldsymbol{\alpha} \odot \widehat{\mathbf{w}}_t + \boldsymbol{\phi}\widehat{rer}_t), \tag{2.14}$$

where $\odot$ denotes the Hadamard (entrywise) product and $\boldsymbol{\alpha}$ and $\boldsymbol{\phi}$ are vectors with characteristic elements α_j and ϕ_j, respectively. Equation (2.14) characterizes the vector of sectoral prices, $\widehat{\mathbf{p}}_t = [\widehat{p}_{1t}, \ldots, \widehat{p}_{Nt}]'$, in terms of parameters, sectoral wages, $\widehat{\mathbf{w}}_t = [\widehat{w}_{1t}, \ldots, \widehat{w}_{Nt}]'$, and the real exchange rate. The matrix $\mathbf{H} \equiv [I_N - (\boldsymbol{\mu} \otimes \iota'_N)\mathbf{\Gamma}']^{-1}$, with ι_N being a N-dimensional unitary vector, corresponds to the (transpose of the) Leontief inverse of the mainland economy, with characteristic element $h_{ij} \geq 0$. This matrix captures the multiplier effect of variations in wages and the real exchange rate to domestic prices, and corresponds to the indirect channel of the commodity price shock over costs and prices. In this case, variations in the sectoral wage and in the real exchange rate not only affect the price of sector j by its direct effect, but it also has an impact by changing the marginal cost of production and the price of its suppliers. Note that when there are no IO linkages, $\mu_j = 0, \forall j$, the Leontief inverse collapses to the identity matrix and only the direct labor and imports intensity matter for the transmission of fluctuations of wages and the real exchange rate to prices.

Note however that such transmission is incomplete in a world with nominal rigidities because sectoral prices are no longer equal to its desired price (a markup over marginal costs). In general, the price in any sector j will be given by (see appendix 2.D for the derivation)

$$\widehat{p}_{jt} = \theta_j(\widehat{p}_{jt-1} - \pi_t) + (1-\theta_j)(1-\theta_j\beta)\widehat{mc}_{jt} + (1-\theta_j)\theta_j\beta\mathbb{E}_t[\widehat{\widehat{p}}_{jt+1}]. \tag{2.15}$$

Equation (2.15) shows that current price is a weighted average of three terms. First, it depends on the difference between past sectoral price and aggregate inflation ($\widehat{p}_{jt-1} - \pi_t$), with a weight given by the degree of nominal rigidities in the sector, θ_j. This term reflects the degree of persistence in (relative) sectoral prices, adjusted by aggregate inflation. In a world where prices are completely rigid ($\theta_j = 1$), this implies that the nominal sectoral price remains constant over time. Second, it depends on the evolution of marginal costs, with a lower relative importance than in the flexible price scenario because $(1 - \theta_j)(1 - \theta_j\beta) < 1$. Finally, it depends on the expected optimal price in the following period (denoted by $\widehat{\widetilde{p}}_{jt+1}$), which at the same time depends on the evolution of marginal costs and aggregate inflation.[19] From this expression is clear that sectoral prices are distorted by nominal rigidities. To the extent that there exist IO linkages, those price distortions are transmitted to customer sectors, even if those customer sectors have flexible prices.

Up to this point, we have only mentioned the role played by nominal rigidities in mainland sectors. However, is also important to distinguish the case in which the imported good is also subject to this friction. As mentioned before, wages and the exchange rate move in opposite directions after the commodity price shock. When imports are subject to nominal rigidities but mainland prices are flexible, the dampening effect of the real appreciation is lower than with flexible prices and the upward pressure of wages dominates over sectoral prices. On the contrary, when the price of imports is flexible and mainland prices are rigid, the real appreciation has a larger impact by directly affecting sectors and by being transmitted through IO linkages. Therefore, the origin of nominal rigidities matters for the small

[19] While the impact of past sectoral prices is increasing in the degree of nominal rigidities, the opposite happens for current marginal costs: the lower the level of rigidities, the higher its importance in determining prices. On the other hand, the weight of the expected optimal price follows an inverted u-shape with a maximum at $\theta_j = 1/2$.

open economy. This can be seen as an open economy extension of the results in Pasten et al. (2019), by highlighting the importance not only of mainland sectoral rigidities, but relative rigidities with respect to imports.

Sectoral sizes. Production linkages also affect the transmission of the shock by changing sectoral sizes. This can be seen by looking at the sectoral market clearing condition given in equation (2.9). When there are no IO linkages, $M_{ijt} = 0$ for any pair (i, j) and any period t, the only features determining the relative size of sector j are (i) the relative importance of consumption and trade balance in the determination of output, and (ii) the relative importance of sector j in the composition of consumption and net exports (given by the parameters ϑ_j and ψ_j in the definition of each aggregator). When IO linkages are active, then the intensity of usage of materials of customer sectors (given by μ_i) as well as the direct strength in those links (given by γ_{ji}) matter to determine sectoral sizes and how a shock is transmitted across the economy.

Inflation, monetary policy and aggregate variables. All the previous features will have a direct impact over the price of domestic consumption and exports, which are functions of sectoral prices. Recall that the price of consumption, which is the numeraire of the economy, is defined by $P_t = \prod_{j=1}^{N} P_{jt}^{\vartheta_j}$, implying that aggregate inflation is given by $\pi_t = \sum_{j=1}^{N} \vartheta_j \pi_{jt}$, where π_{jt} denotes the inflation rate in sector j. At the same time, the price of exports is given by $p_t^x = \prod_{j=1}^{N} p_{jt}^{\psi_j}$, so sectoral price distortions will have implications in the exchange rate determination in equilibrium (see equation 2.12).

Conditional on the monetary policy stance, this is, on the relative importance of inflation and output in the determination of the nominal interest rate, sectoral price distortions will have implications for the determination of the real interest rate (see equation 2.8). This is because sectoral prices and the aggregate inflation will respond differently depending on nominal rigidities and IO linkages, affecting expected inflation and how the nominal rate fluctuates. Given that consumption is fully determined by the path of the real rate, as can be seen from the Euler equation of the household in (2.3), all these margins will have different aggregate implications. Understanding how quantitatively important are each one of these features is the subject of the next section.

2.4 QUANTITATIVE ANALYSIS

This section explores the quantitative properties of the model. I start by describing the calibration strategy for the Chilean economy to then analyze the impact of commodity price shocks under different configurations for production linkages and nominal rigidities.

2.4.1 *Calibration*

The frequency of the model is set to be a quarter and the number of mainland sectors is set to $N = 2$ to replicate an economy characterized by commodities, manufactures ($j = m$) and services ($j = s$). I partition the parameter space into two groups. First, there is a subset of predetermined parameters which are standard in the literature or taken from the Chilean data and governs the steady-state of the model. The second group of parameters is chosen in order to improve the fit of the model in comparison with an empirical impulse-response function to a commodity price shock.

Table 2.B.1 summarizes the calibration of the model. The parameters of risk aversion and the (inverse of the) Frisch elasticity are set to one, which are standard values in the New Keynesian literature. The discount factor is set to get a domestic interest rate of 5.8 percent at a yearly frequency, in line with Garcia-Schmidt and Garcia-Cicco

(2020a). The labor supply elasticity is set to 1, implying limited labor mobility across sectors.

With respect to sectors, the elasticity of substitutions across varieties is set to 10, implying a steady-state level of sectoral profits close to 11 percent, among the values in the New Keynesian literature. The parameters associated to the production functions of mainland and commodity sectors are taken from the Input-Output (IO) table for Chile in the year 2017. For this, I aggregate the 111 sectors in the IO table to get an aggregate of commodity (corresponding to mining activities), manufactures and services. Services are more labor intensive than the rest of sectors ($\alpha_s = 0.48$), while commodity and manufactures have a similar labor intensity ($\alpha_c = 0.22$ and $\alpha_m = 0.24$). On the other hand, domestic materials are intensively used by the three sectors ($\mu_c = 0.63$, $\mu_m = 0.59$ and $\mu_s = 0.45$), leaving an usage of imports that varies between 8 percent for services and 17 percent for manufactures. This is consistent with the notion that services is a non tradable sector, while commodities and manufactures are tradable. The intensity in the usage of specific materials (input-output) also comes from the IO (2017), and fluctuates between 0.35 and 0.65. Finally, manufactures represents 83 percent of the composition of the exportable good.

To calibrate the level of price rigidities, I follow Pasten et al. (2019) and compute the average frequency of price adjustment for goods in the different categories. For this, I rely on the microdata underlying the construction of the official Chilean CPI for the period 2010-2018, and compute the ratio between the number of price changes over the total number of periods available for each product. Then, I aggregate those individual products to get the behavior of manufactures and services (for more details, see chapter 3). Finally, the level of price rigidity is computed as one minus the obtained level. With this procedure I obtain values of $\theta_m = 0.46$ and $\theta_s = 0.59$, implying that the price of manufactures is relatively more flexible than the price of services. Due to lack of information, I set $\theta_v = 0.75$ in the baseline analysis, a standard value.

The values for the monetary policy rule are standard in the literature, with a central bank reacting aggressively to movements in inflation ($\phi_\pi = 1.5$) and more moderately to output fluctuations ($\phi_y = 0.125$). The world interest rate is taken from Garcia-Schmidt and Garcia-Cicco (2020a) and set to 4.5 percent in annual terms. The long-run value of foreign bonds is set to match a ratio of trade balance-to-GDP of 8 percent in steady-state.

The final set of parameters correspond to the persistence of the commodity price, the parameters governing the foreign demand for the exportable good, and the sensitivity of the interest rate premium to fluctuations in debt. To calibrate them, first I run a structural VAR (SVAR) model at quarterly frequency between 1996 and 2018 including (i) the real foreign price of copper (nominal price in dollars divided by the U.S. CPI); (ii) consumption; (iii) output; (iv) trade balance; (v) CPI; and (vi) the real exchange rate. All variables are in logs, except the trade balance which is included as a fraction of total output. The model includes a linear and a quadratic trend as exogenous controls. The identifying assumption in the SVAR model is that Chile is a small open economy with respect to the commodity price, so this price does not react to domestic conditions. For parsimonym the model is estimated with only one lag. Then, I set the values of the theoretical parameters in order to match the theoretical and empirical responses of consumption and the trade balance to the commodity price shock.[20] This procedure gives the values $\omega^x = 0.90$ and $\eta^* = 2.93$ for the foreign demand, $\phi_b = 0.009$ for the in-

[20]Formally, this procedure corresponds to a Simulated Method of Moments (SMM), in which the vector of parameters θ solve $\arg\min_{\theta\in\Theta}(\widehat{\gamma}_T - f(\theta))' W_T (\widehat{\gamma}_T - f(\theta))$. In this expression, $\widehat{\gamma}_T$ is the structural impulse-response function obtained from a sample of dimension T, $f(\cdot)$ defines the theoretical response, and W_T is a weighting matrix given by the inverse of the interquartile range of the empirical impulse-response.

terest rate premium, and $\rho_c = 0.712$ for the autoregressive parameter of the commodity price. Figure 2.B.1 in the appendix compares the impulse-response function of the model and in the data for selected aggregate variables, showing that it replicates the main features of a commodity price shock.

2.4.2 *The Role of Nominal Rigidities*

This section studies the importance of nominal rigidities in the transmission of commodity price shocks when there are IO linkages. Note that there exist different potential margins of adjustment: flexible prices in mainland sectors and flexible prices for importable goods. Figure 2.1 compares the response of the main aggregate variables to a 10 percent increase in the commodity price under different scenarios.

Flexible prices in mainland. Let start the analysis by comparing the baseline case (nominal rigidities in all sectors) with the case in which mainland sectors (manufactures and services) have flexible prices, but the importable good is subject to nominal rigidities (these cases are illustrated in the figure with the solid black line and the dashed red line respectively). In the counterfactual case of flexible prices in the mainland economy, fluctuations in sectoral prices move one to one with fluctuations in marginal costs, but the real exchange rate is imperfectly transmitted to costs. This implies that the real appreciation generated by the commodity price shock has a lower effect on prices that in the baseline scenario, so the increase in wages dominates and inflation is amplified in about 58%, as noticed in panel (c). This directly implies a higher increase in the real interest rate over time, as shown in panel (d). Even though the difference in the response of the real rate might seem small in comparison to the baseline scenario, recall that consumption depends on the full path of the real rate. This generate an impact response of consumption 48 percent lower than in the baseline case, as shown by panel (b), with some persistence over time. The response in the trade balance is about four percent below relative to baseline, but those differences quickly converge over time, so domestic nominal rigidities are not very relevant for the response of this variable to the shock. The response of output weights the responses of consumption and the trade balance, with an impact effect 25% lower than in baseline case, as shown in panel (a). Finally, note that the real exchange rate exhibits a larger appreciation than in the baseline case, because domestic goods become relatively more expensive.

Flexible prices in imports. A second case of interest is when the price of imports is flexible, so the real exchange rate is directly transmitted to marginal costs in other sectors, but domestic prices exhibit some degree of rigidity. The responses of this counterfactual economy are presented in blue dashed-dotted lines in Figure 2.1. In this scenario, given the strong appreciation, marginal costs will decrease and the potential increasing pressure of wages transmits to other sectors only imperfectly. Therefore, prices and inflation are about 87% lower than in the baseline case, implying a response of monetary policy 30 percent lower than in the baseline case, as illustrated in panels (c) and (d). Opposite to the case of flexible prices in mainland sectors, the response of consumption is now 72% larger and the difference is more persistent over time. This also has implications for the trade balance, which is now 20% higher. Combining these two expansionary effects, the response of aggregate output is 46% larger than in the baseline case, as shown in panel (a).

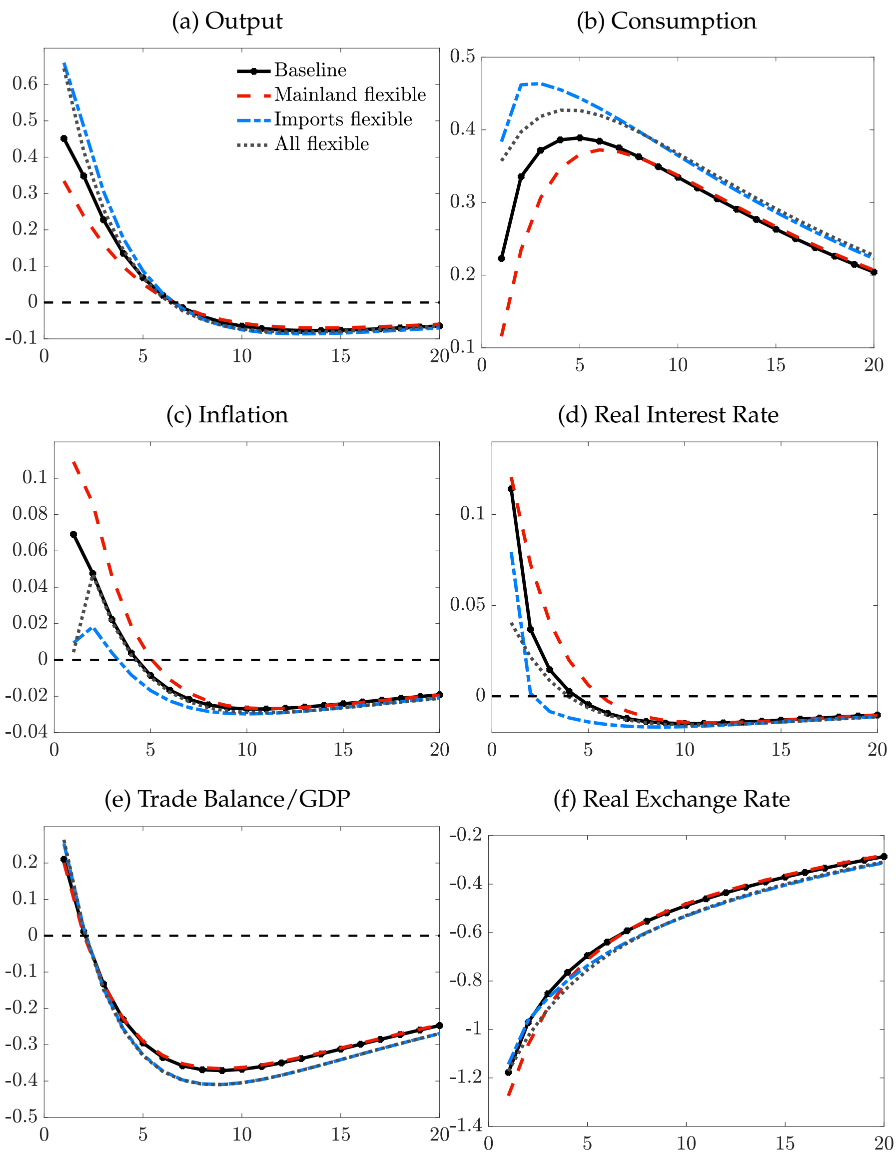
Flexible prices in all sectors. The previous analysis showed that the source of nominal rigidities differentially affects the response of aggregate variables to the shock: while flexible prices in manufactures and services dampen the impact of the commodity price shock, flexible prices for the importable sector generate amplification. Therefore, it is of interest to analyze the combined case in which all prices are fully flexible, as it would be in a RBC model (Cao and Dong, 2020; Romero, 2020a). This case is illustrated in Figure 2.1 with grey dotted lines. Starting with inflation in panel (c), note that it initially responds by less than in the baseline case, to then follow a similar path. Therefore, at the beginning, the real appreciation dominates and dampens the effect of the commodity price shock. Consistent with this, the real interest rate has a lower reaction (65% relative to the baseline reaction), so consumption has a 60% larger response than in the baseline case, but lower than in the scenario with flexible importable prices. Given that the trade balance has a similar reaction than in the baseline case, this implies that output has a response 43% larger relative to baseline, but a bit lower over time, in comparison to the case when imports are flexible.

2.4.3 *The Role of Production Linkages*

This section extends the previous analysis by exploring the interactions between production linkages and nominal rigidities. For this, we compare the response of aggregate variables to a commodity price shock under different degrees of nominal rigidities (either in mainland sectors or for imported goods), conditional on the existence or not of production linkages. This exercise allows to understand how different is the transmission of the shock through production networks when there are different degrees of nominal rigidities.

Table 2.1 compares the cumulative response of selected aggregate variables to the commodity price shock in a horizon of 20 quarters. Panel A presents the responses in the baseline model with full links and different degrees of nominal rigidities, in which "Baseline" denotes the model with nominal rigidities in all sectors. To ease

FIGURE 2.1: Nominal rigidities vs flexible prices



NOTES: This figure compares the response of aggregate variables to a 10 percent commodity price shock under different configurations of the model. Baseline model is the one with nominal rigidities in all sectors. Mainland flexible is the model with flexible prices in manufactures and services. Imports flexible is the model with flexible prices on importable goods. All flexible is the model with flexible prices in all sectors. All variables in percentage deviations with respect to steady-state, except for trade balance/GDP which is measured in deviations.

comparison, numbers in brackets corresponds to the ratio of the response of each counterfactual relative to the baseline specification. As discussed in the previous section, when mainland sectors (manufactures and services) are flexible and imports are not, we observe a lower response of the economy to the shock (5 percent lower for consumption). The opposite happens when imports are flexible and mainland sectors are subject to nominal rigidities (15 percent larger for consumption). With fully flexible prices, all responses are larger than in the baseline economy but lower than in the case with flexible prices of imports, illustrating the competing effects of the increase in wages and the real appreciation.

How does these responses interact with production linkages? As discussed in section 2.3, IO linkages transmit the effect of fluctuations in wages and on the price of imports to other sectors (customers). The same is true for sectoral distortions given by nominal rigidities. Without production linkages, only the direct effect of wages and the price of imports matter in the determination of marginal costs and sectoral prices (how intensive is the usage of labor and imports by the sector).

Panel B of Table 2.1 presents the same exercises as Panel A, but considering an economy in which there are no production linkages (neither within mainland sectors nor between the commodity sector and the rest of the economy). There are three observations to make for the comparison with the case with links. First, responses in all variables are larger (in absolute value) than in the world with full links, conditional on the degree of nominal rigidities. This is particularly important for consumption, the trade balance and the real exchange rate, in which the response in the model without linkages is two to three times larger. This observation implies that production linkages dampen the effect of the commodity price shock.[21] Sec-

[21] A similar observation has been made by Bouakez et al. (2009) in the context of monetary policy shocks for a closed economy.

ond, in the model without links we observe the same qualitative patterns for nominal rigidities as in the model with full linkages. This is, when mainland sectors are flexible (rigid) but imports are rigid (flexible), there is a lower (higher) response than in the baseline model. Finally, event though the degree of distortions implied by nominal rigidities are higher in the model with production linkages, the differences between flexible prices and nominal rigidities are larger in the horizontal economy, as can be seen in the number with square brackets. To see this, observe that responses of the model with flexible prices relative to the model with full rigidities are larger in the scenario without linkages, with the exception of the real interest rate. For example, consumption is 26% larger in the case with no linkages, while in the case with linkages is only 13% times larger. Those differences are lower for other variables but still quantitatively important.

Table 2.1: Comparison between price rigidities and IO linkages

	Output	Consumption	Inflation	Real Interest Rate	Trade Balance	Real Exchange Rate
Panel A: Full links						
Baseline	0.355	6.101	-0.217	-0.029	-5.286	-10.915
Mainland flexible	0.084	5.783	-0.078	0.082	-5.243	-11.133
	[0.237]	[0.948]	[0.359]	[-2.828]	[0.992]	[1.020]
Imports flexible	0.735	7.017	-0.385	-0.174	-5.779	-11.519
	[2.070]	[1.150]	[1.774]	[6.000]	[1.093]	[1.055]
All flexible	0.582	6.869	-0.315	-0.143	-5.785	-11.682
	[1.639]	[1.126]	[1.452]	[4.931]	[1.094]	[1.070]
Panel B: No links						
Baseline	0.435	14.786	-0.906	-0.464	-13.203	-27.844
Mainland flexible	0.282	14.639	-0.822	-0.432	-13.209	-28.044
	[0.648]	[0.990]	[0.907]	[0.931]	[1.000]	[1.007]
Imports flexible	1.776	18.635	-1.534	-0.978	-15.511	-31.328
	[4.083]	[1.260]	[1.693]	[2.108]	[1.175]	[1.125]
All flexible	1.791	18.625	-1.522	-1.030	-15.487	-31.260
	[4.117]	[1.260]	[1.680]	[2.220]	[1.173]	[1.123]

NOTES: This table compares the cumulative response over 20 quarters across different versions of the model. Panel A corresponds to the baseline model with IO linkages. Panel B corresponds to a multisector model without IO linkages. Number in square brackets corresponds to the ratio with respect to the baseline model with nominal rigidities. Trade balance is normalized by GDP.

2.5 EXTENSION TO OTHER DRIVING FORCES

While the main focus of this paper is the impact of commodity price shocks in an emerging economy, the transmission channels of production linkages and nominal rigidities might also be important in the context of other shocks. How do they interact when there are other driving forces at play?

This section analyzes the interaction of production linkages and nominal rigidities when the economy is subject to (i) aggregate technology shocks; (ii) shocks to the foreign interest rate; and (iii) foreign demand shocks. These are standard shocks considered in previous literature as relevant for the business cycle of emerging economies.[22] I assume that all these shocks follow an AR(1) process in logs. The goal of this exercise is not to provide a fully calibrated analysis but to illustrate the importance and implications of the two channels at play (IO linkages and nominal rigidities) in the transmission of shocks. In appendix 2.C, Table 2.C.1 compares the cumulative responses of selected aggregate variables to these different shocks in the model with IO linkages, while Table 2.C.2 presents the responses when the model does not consider production linkages. To save space, it only compares between the baseline case with nominal rigidities in all sectors with the case of fully flexible prices. To ease comparison with the previous analysis, panel A on each table repeats the results for commodity price shock. In what follows, I summarize the main results for each shock.

Aggregate TFP shocks. Panel B of Table 2.C.1 presents the results for an aggregate productivity shock. Conceptually, this shock affects all sectors in the economy by reducing their marginal cost of production and prices. As we can see from the table, inflation strongly

[22]See for example Neumeyer and Perri (2005), Uribe and Yue (2006), Fernandez and Gulan (2015), Kohn et al. (2020) and Garcia-Schmidt and Garcia-Cicco (2020a).

decreases with the shock, which is exacerbated in the case with flexible prices. This implies a strong decrease in the real interest rate, stimulating aggregate consumption and total output. The decrease in prices associated to the shock, generates a lower exchange rate appreciation and a lower deficit in the trade balance, relative to the case with commodity price shocks. In general, we observe that flexible prices are more expansionary for TFP shocks than in the case of commodity price shocks. What happens in the economy without linkages? Interestingly, the conclusions of the previous analysis are partially reverted in this case: even though more flexible prices amplify the response of the economy to the shock, the differences in the case without production linkages are now lower (see panel B in Table 2.C.2). This is because the productivity shock has a direct impact on marginal costs, so in the model with linkages such effect is amplified through the use of domestic materials.

Foreign interest rate shocks. A positive foreign interest rate shock implies an increase in the domestic rate, as can be observed by approximating and combining equations (2.3) and (2.4). This larger interest rate postpones consumption and increases savings today, which translates into a larger surplus in the trade balance position. At the same time, the larger expansion in the trade balance depreciates the real exchange rate, which combines with a positive response in inflation. Panel C of Table 2.C.1 presents the results for this shock, with responses in line with the previous discussion. When prices are flexible, we observe a higher increase in the real interest rate and inflation, implying a higher decrease in consumption and a dampened response in total output. Therefore, different to previous shocks, flexible prices dampen the response of total output to a shock because the trade balance moves in the opposite direction of consumption. How does these effect compare in the model without linkages? There are two observations to make from panel C of Table 2.C.2. First, the magnitude in the response of each variable is similar to the model with IO linkages. Therefore, production linkages are less relevant for the transmission of the shock. Second, and in line with the previous point, the differences between the model with or without nominal rigidities are similar to the case with IO linkages. These are intuitive results because previous driving forces (commodity price and productivity shocks) have a direct impact over marginal costs (in the case of the commodity price shock, by affecting the real exchange rate and the price of imported goods). In the case of the foreign interest rate shock, there is no effect over marginal costs but all the implications are through aggregate demand in general equilibrium.

Foreign demand shock. Finally, I analyze the impact of an increase in the foreign demand for domestic goods. This can be seen, either as an increase in the taste shifter for domestic goods, or as an increase in total foreign output. As shown in panel D of Table 2.C.1, this shock increases the value of total exports in the economy, so we observe a lower trade balance deficit in comparison to a commodity price shock. This also reflects in a less pronounced exchange rate appreciation. Interestingly, the shock does not affect much the rest of the economy. Even though it translates into more income and more consumption, the magnitudes are way lower than in the case of the commodity price shock. At the same time, we observe that, even though flexible prices amplify the response of the economy to the shock, the differences with the case of nominal rigidities are much lower than in the case of other shocks. Similar to the case of foreign interest rate shocks, an increase in foreign demand generates similar responses regardless of the presence of production linkages or not (see panel D of Table 2.C.2). Again, this is because these shocks do not have a direct impact over marginal costs, so all the transmission applies only trough general equilibrium forces.

2.6 CONCLUSION

While commodity price shocks are an important foreign source of fluctuations for small open economies, their aggregate effects crucially depend on structural features of the economy.

This paper studies how domestic production linkages and nominal rigidities affect the transmission of commodity price shocks to the overall economy. Building a New Keynesian model for a commodity exporting small open economy with production linkages, I find that nominal rigidities tend to dampen the effect of the shocks by distorting the transmission of sectoral prices to the marginal cost of other sectors. I also show that the source of nominal rigidities matter for this result. When mainland sectors, such as manufactures or services, are subject to nominal rigidities but the price of imports is flexible, the previous result is reverted because the real exchange rate appreciation generated by the shock is amplified by production networks. Comparing with a version of the model without production linkages, these qualitative features remain but the magnitude of the responses is larger relative to the baseline model, by changing relative sectoral sizes, and the degree of distortions generated by nominal rigidities decrease, because there are no linkages that directly transmit them to other sectors.

Extending the analysis to other relevant shocks for a small open economy, I find that those directly affecting the marginal cost of production (such as the commodity price shock, by changing the real exchange rate, or an aggregate productivity shock), generate larger differences (i) between a model with and without production linkages, and (ii) between a model with and without nominal rigidities. These results highlight that the relative importance of production networks and nominal rigidities as transmission channels is conditional on the source of fluctuations in the small open economy.

2

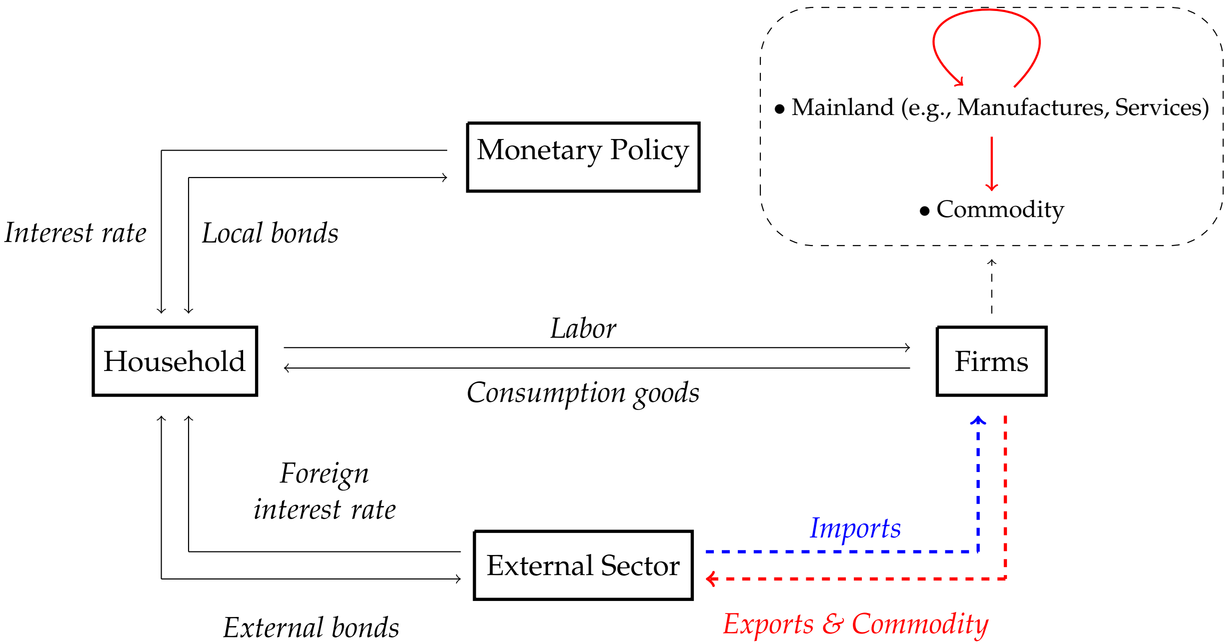
APPENDIX

2.A FLOWS IN THE SMALL OPEN ECONOMY

Figure 2.A.1 presents the diagram flow for the commodity exporting small open economy described in section 2.2.

As mentioned in the main text, the representative household provides labor to firms in different sectors, consumes and trade both domestic and foreign bonds. Firms production is used either to generate consumption goods, exportable goods or as materials for other firms in the economy. The production side of the economy is connected to the rest of the world by buying importable goods and supplying and exportable good and commodities.

FIGURE 2.A.1: Diagram flow in the commodity exporting SOE



2.B CALIBRATION AND COMPARISON WITH EMPIRICAL RESPONSES

Table 2.B.1 presents the calibration of the baseline model.

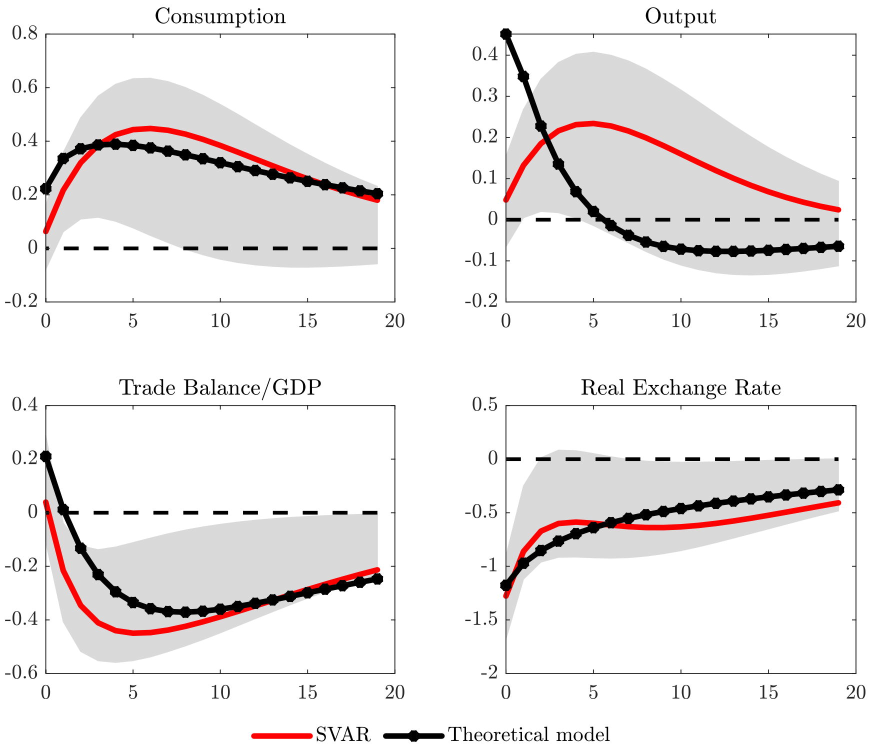
Table 2.B.1: Calibrated parameters

Variable	Parameter	Value	Source/Target
Panel A: Household			
σ	Risk aversion	1	Standard value
φ	Frisch elasticity	1	Standard value
β	Discount factor	0.986	Garcia-Schmidt and Garcia-Cicco (2020a)
ϱ	Elasticity of substitution (labor)	1	Standard value
ϑ_j	Sectoral consumption shares	$\{\vartheta_m = 0.4; \vartheta_s = 0.6\}$	IO (2017)
Panel B: Sectors			
ε	Elasticity of substitution (varieties)	10	Standard value
α_j	Labor share	$\{\alpha_c = 0.22; \alpha_m = 0.24; \alpha_s = 0.48\}$	IO (2017)
μ_j	Materials share	$\{\mu_c = 0.63; \mu_m = 0.59; \mu_s = 0.45\}$	IO (2017)
γ_{ij}	Production linkages		IO (2017)
θ_j	Price rigidities (Calvo)	$\{\theta_m = 0.46; \theta_s = 0.59\}$	Chilean microdata
ψ_j	Share in exports	$\{\psi_m = 0.83; \psi_s = 0.17\}$	IO (2017)
Panel C: Monetary policy and aggregates			
ϕ_π	Weight on inflation	1.5	Standard value
ϕ_y	Weight on GDP	0.125	Standard value
R^w	World interest rate	1.045	Garcia-Schmidt and Garcia-Cicco (2020a)
$\bar{b}$	Interest rate premium	-5.636	TB/GDP = 0.08
ϕ_b	Interest rate premium	0.009	SVAR calibration
ω^x	Foreign demand for exports	0.903	SVAR calibration
η^*	Foreign demand for exports	2.932	SVAR calibration
ρ_c	Persistence commodity price	0.712	SVAR estimation
θ_v	Price rigidities (imports)	0.75	SVAR estimation

NOTES: This table presents the set of parameters of the model. See the main text of Section 2.4.1 for details.

Figure 2.B.1 compares the empirical and theoretical responses to a positive 10 percent commodity price shock. As can be noticed, the model replicates the response to the commodity price shock for most variables. The exception is the case of output, which exhibits a larger impact response with a relatively constant decreasing rate. However, most of the response is in between of the empirical ranges.

FIGURE 2.B.1: Impulse-response function to a 10% commodity price shock



NOTES: This figure compares the impulse-response function of the data versus the model. Grey areas correspond to 95% confidence intervals computed with bootstrap with 100,000 replications. The SVAR model includes a constant and linear and quadratic time trends. Horizontal axes correspond to quarters. Vertical axes corresponds to percentage deviations with respect to trend (for empirical model) and percentage deviations with respect to steady-state (for model-implied responses), except for trade balance/GDP which is measured in deviations.

2.C OTHER DRIVING FORCES

This section presents results comparing the IRF under different driving forces. Table 2.C.1 presents results for the model with full linkages (within mainland sectors and between mainland and the commodity sector), while table 2.C.2 presents results for the model without linkages. On each table, panel A presents results for the commodity price shock, panel B for an aggregate TFP shock, panel C for a foreign interest rate shock, and panel D for a foreign demand shock. Each panel compares the baseline economy with nominal rigidities in mainland and importable sectors (row "Baseline"), with a counterfactual economy with flexible prices (row "All flexible".)

Table 2.C.1: Other shocks in the model with linkages

	Output	Consumption	Inflation	Real Interest Rate	Trade Balance	Real Exchange Rate
Panel A: Commodity price shocks						
Baseline	0.355	6.101	-0.217	-0.029	-5.286	-10.915
All flexible	0.582	6.869	-0.315	-0.143	-5.785	-11.682
	[1.639]	[1.126]	[1.452]	[4.931]	[1.094]	[1.070]
Panel B: Aggregate TFP shocks						
Baseline	2.011	5.188	-1.074	-1.065	-2.923	-3.533
All flexible	4.114	8.149	-1.913	-1.970	-3.712	-3.178
	[2.046]	[1.571]	[1.781]	[1.850]	[1.270]	[0.900]
Panel C: Foreign interest rate shocks						
Baseline	7.475	-13.348	1.483	0.943	19.158	22.354
All flexible	5.183	-17.623	2.301	1.628	20.981	23.835
	[0.693]	[1.320]	[1.552]	[1.726]	[1.095]	[1.066]
Panel D: Foreign demand shocks						
Baseline	0.020	0.713	-0.016	0.015	-0.638	-1.233
All flexible	-0.001	0.732	-0.004	0.022	-0.674	-1.327
	[-0.050]	[1.027]	[0.250]	[1.467]	[1.056]	[1.076]

NOTES: This table compares the cumulative response over 20 quarters across different versions of the model and different driving forces when IO linkages are active. Baseline corresponds to the model version with all nominal rigidities. All flexible corresponds to the model with all flexible prices. Panel A presents responses for commodity price shocks. Panel B presents responses for aggregate productivity shocks. Panel C presents responses for interest rate premium shocks. Panel D presents responses for foreign demand shocks. Number in square brackets corresponds to the ratio with respect to the baseline model with nominal rigidities. Trade balance is normalized by GDP.

Table 2.C.2: Other shocks in the model without linkages

	Output	Consumption	Inflation	Real Interest Rate	Trade Balance	Real Exchange Rate
Panel A: Commodity price shocks						
Baseline	0.435	14.786	-0.906	-0.464	-13.203	-27.844
All flexible	1.791	18.625	-1.522	-1.030	-15.487	-31.260
	[4.117]	[1.260]	[1.680]	[2.220]	[1.173]	[1.123]
Panel B: Aggregate TFP shocks						
Baseline	0.986	2.980	-0.670	-0.701	-1.834	-2.428
All flexible	1.905	4.195	-1.005	-1.042	-2.107	-2.272
	[1.932]	[1.408]	[1.500]	[1.486]	[1.149]	[0.936]
Panel C: Foreign interest rate shocks						
Baseline	9.063	-11.242	1.279	0.812	18.681	23.148
All flexible	7.225	-15.978	2.081	1.451	21.347	26.817
	[0.797]	[1.421]	[1.627]	[1.787]	[1.143]	[1.159]
Panel D: Foreign demand shocks						
Baseline	-0.077	0.452	0.008	0.032	-0.487	-1.047
All flexible	-0.080	0.532	0.002	0.020	-0.563	-1.196
	[1.039]	[1.177]	[0.250]	[0.625]	[1.156]	[1.142]

NOTES: This table compares the cumulative response over 20 quarters across different versions of the model and different driving forces when IO linkages are not active. Baseline corresponds to the model version with all nominal rigidities. All flexible corresponds to the model with all flexible prices. Panel A presents responses for commodity price shocks. Panel B presents responses for aggregate productivity shocks. Panel C presents responses for interest rate premium shocks. Panel D presents responses for foreign demand shocks. Number in square brackets corresponds to the ratio with respect to the baseline model with nominal rigidities. Trade balance is normalized by GDP.

2.D DERIVATIONS

2.D.1 *Price Evolution*

First note that from the properties of Calvo pricing, the nominal price in sector j is given by $P_{jt}^{1-\varepsilon} = \theta_j P_{jt-1}^{1-\varepsilon} + (1-\theta_j)\widetilde{P}_{jt}^{1-\varepsilon}$, where $\widetilde{P}_{jt}$ is the desired price in period t. When written in terms of the aggregate consumption price, P_t, this expression reads as $p_{jt}^{1-\varepsilon} = \theta_j p_{jt-1}^{1-\varepsilon}\Pi_t^{\varepsilon-1} + (1-\theta_j)\widetilde{p}_{jt}^{1-\varepsilon}$, where lower case variables denote prices relative to aggregate consumption. Taking a first order approximation around the steady-state, we get

$$\widehat{p}_{jt} = \theta_j(\widehat{p}_{jt-1} - \pi_t) + (1-\theta_j)\widehat{\widetilde{p}}_{jt}, \tag{2.16}$$

with hat variables denoting percentage deviations with respect to steady-state, and π_t denotes the inflation rate associated to consumption price.

Now we want to express the sectoral price in terms of the marginal cost. For this, we can replace for the optimal price of equation (2.7) into (2.16). Taking a first order approximation to (2.7), the optimal price can be expressed as

$$\widehat{\widetilde{p}}_{jt} = (1-\theta_j\beta)\sum_{\tau=0}^{\infty}(\theta_j\beta)^{\tau}\mathbb{E}_t[\widehat{mc}_{jt+\tau} + \pi_{t+\tau-1}],$$

which can be written recursively as

$$\widehat{\widetilde{p}}_{jt} = (1-\theta_j\beta)\widehat{mc}_{jt} + \theta_j\beta\mathbb{E}_t[\widehat{\widetilde{p}}_{jt+1}]. \tag{2.17}$$

Replacing (2.17) into (2.16)

$$\widehat{p}_{jt} = \theta_j(\widehat{p}_{jt-1} - \pi_t) + (1-\theta_j)(1-\theta_j\beta)\widehat{mc}_{jt} + (1-\theta_j)\theta_j\beta\mathbb{E}_t[\widehat{\widetilde{p}}_{jt+1}],$$

which is the expression in the main text.

3

CONSUMPTION HETEROGENEITY AND COMMODITY PRICE SHOCKS

3.1 INTRODUCTION

Typically, an increase in commodity prices for a commodity exporting economy generates a windfall for the country, which is translated into higher wages, output and consumption, and an increase in overall well-being. While the literature has focused mostly in the aggregate effects of such shocks, either by studying representative household models or by empirically analyzing its aggregate consequences, little is known about the microeconomic impact of those fluctuations and how they are transmitted to the aggregate economy. Are the gains induced by increases in commodity prices equally distributed across the population? Can these shocks generate differences in consumption across the income distribution and a differential impact in well-being? Are the differences induced by household heterogeneity relevant to understand aggregate fluctuations?

In this paper, I contribute to the literature by analyzing the role of household heterogeneity in the transmission of commodity price shocks. I analyze household heterogeneity in terms of differential consumption expenditures. Using Chilean microeconomic data, I document the following empirical fact: while the share of expenditures in manufactured goods is flat across the income distribution, low-income households spent proportionally more of their income in food relative to high-income households and less in services. This behavior is monotone, implying a decreasing (increasing) pattern of consumption in food (services) across the income distribution. Even though there exist differences in levels, I also document that these patterns remain stable over time and in the definition of income, and can be observed in other emerging economies as well.[1]

Motivated by these observations, I build a model for a commodity exporting small open economy with household heterogeneity. For tractability, I consider a model with two households that differ in their access to financial markets and ownerships of firms, being these the only sources of income heterogeneity. The first kind of household neither has access to financial markets nor owns firms in the economy. It behaves in a hand-to-mouth fashion and only consumes their labor income on every period. This household is denoted as *restricted* or *constrained*. The remaining fraction of the population, besides owning profits coming from firms in the economy, can take debt both domestically and with the rest of the world, and are denoted as *unconstrained*. To model consumption heterogeneity, I follow Comin et al. (2019) and assume that households have non-homothetic preferences given by an implicitly additive non-homothetic CES function.[2] While all households have the same preferences over domes-

[1]Other papers documenting heterogeneity in consumption expenditures across the income distribution, especially in the U.S., are Costa (2001), Hamilton (2001), Almas (2012), Almas and Kjelsrud (2017), Kaplan and Schulhofer-Wohl (2017), Clayton et al. (2018), Dobrew (2018), Cravino and Levchenko (2017) and Cravino and Sotelo (2019), among others. For a recent summary specialized in the consequences for inflation heterogeneity, see Jaravel (2021).

[2]Those preferences have been introduced by Hanoch (1975) and used recently

tic goods, their expenditure shares are explicitly dependent on their level of income. Consistent with the empirical observations, food is less income elastic than manufactures, which at the same time are less elastic than services. In this way, high-income households devote a relatively larger fraction of their total consumption in services, while low-income households spend a larger fraction in food.

The rest of the model is an extension of the New Keynesian (NK) framework for a multi-sector small open economy with non-homothetic preferences, and builds on the model presented in Chapter 2. The domestic production side (given by food, manufactures and services) is characterized by a continuum of monopolistically competitive firms producing with labor, domestic materials and imports, and who are subject to heterogeneous price rigidities as in Calvo (1983). The output of those firms is combined by a competitive firm that sells to domestic and foreign consumers, as well as to other sectors in the form of materials. As in the baseline NK model, output is demand determined, implying a key role for the different sources of demand. There is also a competitive firm producing the commodity good with a similar technology as mainland producers. This firm takes the international commodity price as given and fully exports its output. Variations in the international price of the commodity good is the only source of fluctuations in the economy. Finally, there is a central bank that sets the domestic nominal interest rate following a Taylor rule that depends on inflation and output growth. This mechanism is relevant because allows an endogenous adjustment in the interest rate, which provides a better adjustment in the responses of the model.

I use the model to study both the aggregate and microeconomic impact of a commodity price shock in the context of income heterogeneity and non-homothetic preferences. First, the model generates

in the context of structural transformation and trade (see, for example, Comin et al., 2019; Cravino and Sotelo, 2019; Matsuyama, 2019; Redding and Weinstein, 2020).

sensitive responses in aggregate variables. Consumption and output increase in response to a positive commodity price shock, which is accompanied by a trade balance deficit and a real exchange rate appreciation. These results are in line with the empirical responses for the Chilean economy and previous empirical literature (Fornero et al., 2016; Caputo and Irarrazaval, 2017).

Then I investigate the effect of a commodity price shock and the role of non-homothetic preferences and income heterogeneity for the Chilean economy. In this context, a positive commodity price shock increases sectoral output and the demand for factors, such as labor and materials. This stimulates other sectors through production linkages and increases household income through wages and profits. At this point, non-homothetic preferences and market incompleteness (i.e., the fraction of constrained agents) operate in different directions. On the one hand, with non-homothetic preferences, the marginal cost of an additional unit of consumption is not only given by the price of the consumption basket but on how this extra unit changes its composition. This induces a change in the average income elasticity of households, mechanism that is not present in a standard model with homothetic preferences. With the increase in income induced by the commodity price shock, households shift consumption towards services, which are more income-elastic but also more expensive. This dampen the response of consumption both at the household and aggregate levels, relative to a model with constant income elasticity. On the other hand, market incompleteness gives a role to a fraction of agents that have a higher marginal propensity to consume, so this mechanism amplifies the response of aggregate consumption to the shock. As shown in the quantitative section of the paper, by modelling similar labor supplies across households, the second mechanism plays a minor role and we observe that aggregate consumption is less reactive to the shock. On the microeconomic side, even though consumption of both households increase after the shock, their response is quite different. While the unconstrained household can smooth consumption over time given its access to financial markets, the restricted household must consume any additional income. In this regard, the impact response of the latter is stronger and decays faster. Hours worked are reduced for both households after the increase in the commodity price, due to the income effect in the labor supply.

Related literature. First, this paper is related to the literature studying the effects of terms of trade shocks in emerging economies. Since the seminal contributions of Mendoza (1995) and Kose (2002), a recent wave of papers analyzes the particular role of commodity prices on aggregate fluctuations and potential transmission mechanisms for the shock.[3] For example, while some papers have focused on the links between financial frictions and commodity prices (Shousha, 2016), others have analyzed the aggregate implications of booms and busts in emerging economies (Drechsel and Tenreyro, 2018), the differences in aggregate productivity generated by those swings (Kohn et al., 2020), and the role of production networks for the transmission of the shock (Romero, 2020a). Another strand of this literature has focused on the role of policy. Catão and Chang (2015) and Drechsel et al. (2019) study the role of monetary policy for commodity importers and exporters, respectively, while Pieschacon (2012) and Caputo and Irarrazaval (2017) analyze the role of fiscal policy. Different to these papers, which build on a representative consumer setup, I consider a framework with heterogeneous households and non-homothetic preferences to investigate the importance of income and consumption heterogeneity for the transmission of the shock, and its distributive consequences. While the aggregate effects of the shock are qualitatively similar, this paper offers a novel view about the role of household heterogeneity as a transmission mechanism, and pro-

[3] See Fernandez et al. (2017) for empirical evidence.

vides an evaluation of the microeconomic impact of commodity price fluctuations.

This paper is also related to the literature analyzing multi-sector models in the New Keynesian framework. This literature analyzes the interaction between microeconomic heterogeneity, mostly in terms of differences in price rigidities, input-output linkages and monetary policy, to try to understand the network origin of aggregate fluctuations (Pasten et al., 2020), the microeconomic behavior of prices (Carvalho et al., 2020), the flattening of the Phillips curve over time (Höynck, 2020) or the optimal monetary policy that should be in place (Castro, 2019; Rubbo, 2020; La'O and Tahbaz-Salehi, 2020).[4,5] Different to these papers which model a closed economy, I analyze a multi-sector New Keynesian model for a small open economy, in which the driver is the price of commodity goods exported by the country. This paper builds on Romero (2020b) and the model presented in Chapter 2. Also, these papers study models with a representative household and focus on the production side of the economy and its interaction with monetary policy, while my paper analyzes an environment with heterogeneous agents with non-homothetic preferences. While not key for the qualitative results of the paper, I allow for input-output linkages and heterogeneous price rigidities, as in previous literature. Therefore, my analysis complements this literature by studying the domestic aggregate consequences of foreign shocks in a context of nominal rigidities, production networks and household heterogeneity, to analyze the differential gains of households after a commodity price shock and the aggregate consequences of such heterogeneity.

Finally, this paper contributes to the literature that analyzes the

[4] Wei and Xie (2020) studies optimal monetary policy in a New Keynesian model for a small open economy with global value chains.

[5] See also the seminal contribution of Gali and Monacelli (2005) that studies optimal monetary policy in a New Keynesian for a small open economy.

role of household heterogeneity and the transmission of shocks. On the one hand, Clayton et al. (2018) and Cravino et al. (2020) study the effect of monetary policy shocks across the income distribution, taking into account consumption heterogeneity. While these papers analyze closed economies in which consumption heterogeneity is exogenously given by assuming differences in consumption baskets, my paper analyzes the consequences of a foreign shock and endogenize consumption heterogeneity by modelling non-homothetic preferences which are common across households. Also related to this paper are Cravino and Levchenko (2017) and Cugat (2019). The first of them studies the impact of the Tequila crisis and the devaluation the Mexican peso in 1994. The authors analyze the distributional consequences of the devaluation, by showing that the cost of living of households at the bottom decile of the distribution increased more than for the top decile. In this paper, I do not focus on a specific event, but on the systematic consequences of commodity price shocks and their distribution consequences measured as relative responses of consumption across the income distribution. On the other hand, Cugat (2019) also studies the consequences of the sudden-top associated to the Tequila crisis, considering household heterogeneity by means of limited asset market participation. I also consider this mechanism by means of the presence of the restricted and the unconstrained household but, different to that paper, I analyze the role of consumption heterogeneity to understand the aggregate and microeconomic consequences of a shock.

The rest of the paper is organized as follows. Section 3.2 describes the heterogeneous consumption patterns of Chilean households that motivates the study of this dimension for the transmission of commodity price shocks, with an extension to other emerging economies. Section 3.3 presents the model and analyzes the role of non-homothetic preferences, while section 3.4 shows the quantitative effects of the commodity price shock at the micro and aggregate levels. Section 3.5 presents sensitivity analysis of the baseline results. Section 3.6 concludes.

3.2 MOTIVATING FACTS: CONSUMPTION HETEROGENEITY ACROSS THE INCOME DISTRIBUTION

This section documents the heterogeneous expenditure patterns across the income distribution of households that motivate the theoretical analysis. First, I describe the data and facts for Chile, because the model will be specialized to this country. Then I extend the evidence for other emerging economies to show that similar patterns emerge.

3.2.1 *Consumption Expenditure Survey*

The Consumption Expenditure Survey (*Encuesta de Presupuestos Familiares* in Spanish, or EPF) is a cross-sectional survey that measures consumption patterns across households in the Chilean economy, together with households' characteristics (demographics and income). Its most recent version is EPF VIII for the year 2017, which is used as the baseline for this paper. Previous versions of the survey have been taken in the years 1996, 2007 and 2013. The main goal of EPF is to serve as the base for the construction of the Consumer Price Index (CPI).

To the extent that different households have heterogeneous consumption expenditures, this might have implications for the microeconomic and aggregate responses to a shock.[6] I use EPF data to characterize consumption expenditure patterns across the income distribution. For this, I start by defining the measure of income, the house-

[6] In particular, the interest in this paper is a commodity price shock, which directly represents an increase in income for the overall economy.

hold classification and which goods are being considered in the analysis.

Income measure. EPF VIII provides information of different sources of income, such as labor income, rents from assets and real estate, and imputed rentals for home owners. As baseline, I use total disposable income per capita of each household (sum of all income sources net of transfers), and classify each household according to its percentile in this distribution.[7] Appendix 3.A.2 shows that the general patterns presented below are similar by using the distribution of labor income instead. Also, using a more narrow classification, such as deciles, do not change the general picture presented in this section.

Consumption categories. Consumption in EPF VIII is structured into five different levels of aggregation. From more to less disaggregated, these groups are defined by 1,186 products, 285 subclasses, 126 classes, 59 groups and 12 divisions. In the analysis, I aggregate the 12 divisions to generate three categories, which correspond to food and beverages, manufactured goods, and services. Table 3.A.1 gives more details about their components.

3.2.2 *Expenditure Patterns*

Figure 3.1 presents the main empirical observation that motivates the analysis of consumption heterogeneity. Each panel shows the expenditure share for the three aggregate goods (food and beverages, manufactured goods and services) relative to total expenditures of each percentile. The figure reveals that there are clear patterns of consumption across the income distribution. First, low-income households spent relatively more than rich households in food and bev-

[7] Considering imputed rentals of home owners in the income classification is a standard practice in the literature (Cravino et al., 2020). Income classification and overall results are robust to exclude imputed rentals.

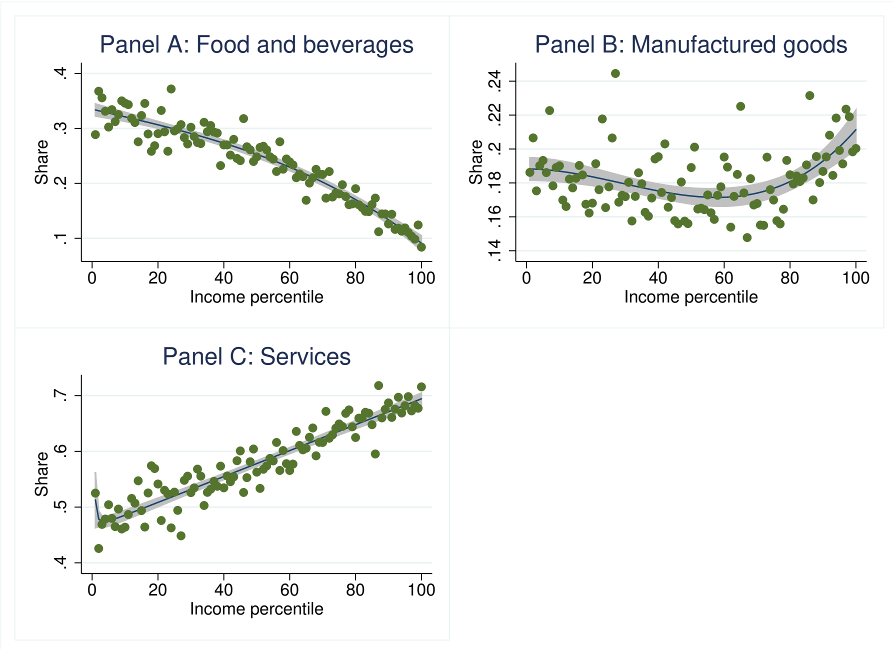
erages. While a household in the bottom decile spent 35 percent of their income in food, a household in the top decile spent only 13 percent. In between these two points there is a monotone decreasing pattern. Second, even though it seems that manufactured goods has a U-shaped form, the pattern of expenditures is relatively constant across the distribution. For example, the lowest decile spent 19 percent on these goods, which is the same as the top decile and the median household. Interestingly, note that the expenditure pattern on these goods is more volatile than in the case of food and services.[8] Finally, even though services is an important category for all households in the distribution, there is a clear increasing pattern in expenditures: while the lowest decile spent around 46 percent on these categories, the top decile spent 69 percent.[9]

Interestingly, while this evidence is based on cross-sectional data, it is consistent with aggregate patterns documented in the literature of structural transformation (see, for example, Garcia-Santana et al., 2020). In general, this literature has found (i) decreasing expenditures in agricultural goods, which are proxied by food and beverages in my approach; and (ii) increasing expenditures in services across the level of development (which in my case corresponds to the income percentile). The main difference comes with manufactured goods. While aggregate evidence shows a hump-shaped pattern of expenditures, in my case the distribution is largely flat across income.

[8]My classification of manufactured goods include also expenditures in utilities (water, electricity, gas and other fuels), so they can be generally understood as industry goods.

[9]Table 3.A.1 provides additional details of consumption expenditures across these aggregate categories for selected percentiles. As can be seen, the decreasing (increasing) pattern for food and beverages (services) remain at the more disaggregated level, so these three categories capture the behavior of the more microeconomic expenditures categories.

FIGURE 3.1: Aggregate consumption patterns across income distribution



NOTES: This figure presents the share in consumption expenditures for households in each percentile of the income distribution, considering three aggregate expenditure groups. On every panel, each dot corresponds to a percentile. Solid line corresponds to the local polynomial fit. Grey area denotes 95% confidence interval.

How stable are these patterns across time? Ideally, we would like to observe a panel of households over time to study in more depth their consumption expenditure patterns over the cycle. However, the Chilean data only provides cross-sectional evidence at different points in time. Fortunately, the data allows me to compare the aggregate three goods classification used in the baseline analysis, even though the components of each kinds of aggregate goods is different mostly due to changes in technological progress that make some goods and services obsolete over time. Figure 3.A.2 compares the distribution of expenditures across different waves of EPF survey. I consider versions V (taken in 1996) to VII (taken in 2013), which covers the periods before and after the commodity price booms of the mid-2000s. The figure reveals that the decreasing (increasing) expenditure pattern for food (services) across the income distribution is quite stable over time. Note that the data exhibits a high correlation for both measures (above 80 percent), and that the level of these expenditure shares is also quite similar. Most of the differences, however, come for the comparison between EPF V of 1996 and the baseline wave, EPF VIII of 2017. As mentioned before, an obvious explanation of this is the time difference between these two surveys, and the large changes in technological developments, as well as the changes in income over these 20 years. What is certainly different is the expenditure share in manufactured goods, which is positively but weakly correlated across surveys. This is the category that most likely has change its components over time, as well as the prices of the individual goods on each sub-category.

3.2.3 *Cross-country Evidence*

How representative are the consumption expenditure patterns of the Chilean economy? In principle, we may be concern about the possibility that the Chilean economy is special regarding the decreasing (increasing) pattern of consumption in food (services), so the theoretical analysis would be only locally valid. To alleviate this concern, I also study expenditure patterns across the income distribution in other emerging economies, using data from the World Bank's Global Consumption Database. This data combines microeconomic data from different consumption surveys across 90 emerging economies for the year 2010, and contains information about 107 products/services.[10,11] Unfortunately, the data is grouped into four categories of income per capita and not at a more disaggregated level. The lowest income group corresponds to the bottom half if the distribution (50th percentile and below); the low group to the 51th-75th percentiles; the middle group to the 76th-90th percentiles; and the high group to the 91th percentile and above. I aggregate expenditure shares in a classification consistent with the three goods used for Chile (food, manufactures and services).

Figure 3.2 presents consumption patterns across the income distribution in the pooled data. As we can see, the same patterns presented for Chile in Figure 3.1 emerge in the group of emerging economies: there is a decreasing (increasing) pattern in food (services) expenditures across the income distribution, while expenditures in manufactured goods are relatively flat. The main difference with the Chilean case corresponds to the level of this expenditure shares. While in Chile the average level of expenditures on food for the bottom half of the distribution is below 30 percent, in the group of emerging economies of Figure 3.2 this is close to 60 percent (the opposite for the case of services). This is because the data contains information about very poor countries in which food expenditures corresponds to most of total expenditures.[12] Therefore, we can conclude that the heterogeneous consumption expenditure patterns presented for Chile are also a common feature for other economies.

Motivated by this evidence, the next section builds a model with

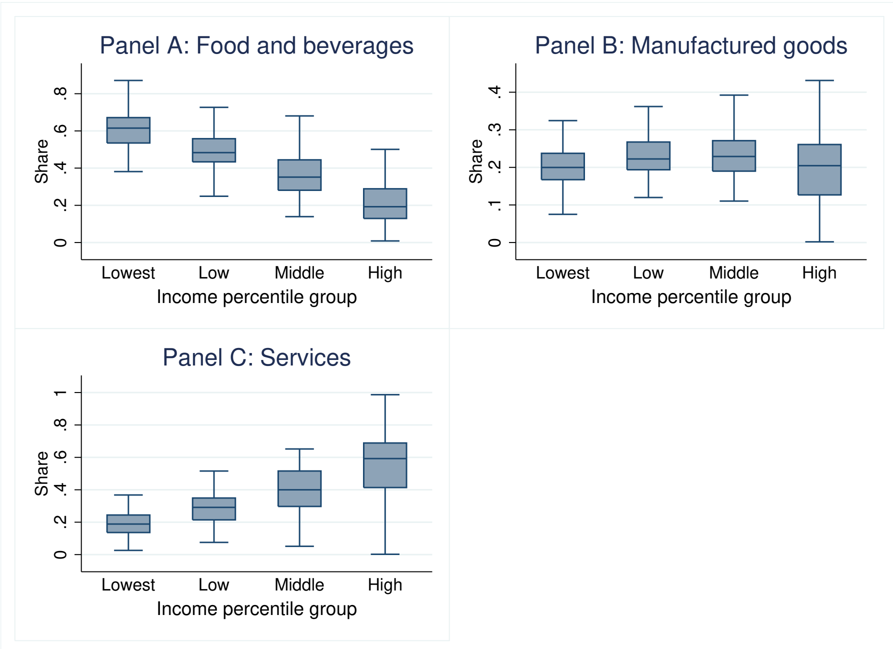
[10]All the data is standardized across commodity classifications, and compared in PPP terms. Even though there still could be differences in the level of consumption given by income and differences in goods/services available in different countries, this is less of a concern in my context because I am comparing expenditure shares and not levels.

[11]For details of the data see `http://datatopics.worldbank.org/consumption/`.

[12]Table 3.A.2 presents descriptive statistics separating between groups of countries (dependent and non-dependent on commodity exports as defined by UNCTAD (2019)). All the qualitative results remain on each subsample.

household heterogeneity for a small open economy that exports commodity goods to analyze the aggregate and microeconomic impact of a commodity price shock.

FIGURE 3.2: Consumption patterns across income distribution in other emerging economies



NOTES: This figure presents expenditure shares across the income distribution in 90 emerging economies, considering three aggregate expenditure groups (food and beverages, manufactured goods and services). Each figure shows the expenditure share across the income distribution according to the World Bank's Global Consumption Database. The lowest consumption segment corresponds to the bottom half of the global distribution, or the 50th percentile and below; the low consumption segment to the 51th-75th percentiles; the middle consumption segment to the 76th-90th percentiles; and the higher consumption segment to the 91st percentile and above. Each box presents the median and interquartile range.

3.3 MODEL

This section presents a model for a small open economy that produces commodity goods, building on Romero (2020b) and the model presented in Chapter 2. It has three main distinctions with respect to previous literature. First, and most important, it considers non-homothetic preferences to capture the heterogeneity in consumption patterns across different households. Second, to generate income heterogeneity I assume that the population is composed by two kinds of households who differ in their access to financial markets and ownership of firms. Finally, there are N domestic sectors in the economy indexed by j (besides the commodity sector which is indexed by c) subject to nominal rigidities in setting their prices, which gives a more relevant role to demand forces. For future reference, under-scripts denote sectors in the economy, while upper-scripts denote type of households.

3.3.1 *Households*

There are two households in the economy indexed by h. The first household ($h = r$) is financially constrained, meaning that it only has access to its labor income: it does not receive profits from firms in the economy and can not borrow/lend in financial markets. Therefore, this household behaves in a hand-to-mouth fashion and is denoted as *restricted* or *constrained*. This household represents a fraction λ of the population, which is fixed over time. The remaining fraction $1-\lambda$ corresponds to the *unconstrained* household ($h = u$). In addition to its labor income, this household owns the firms in the economy, so receives their profits, and can take both domestic and foreign debt, which allows them to smooth consumption over time.

I start by describing the intratemporal problem of a generic household, which consists in deciding their consumption and labor supply allocations. Then, I describe the intertemporal problem, which is household-specific. Detailed derivations are presented in Appendix 3.B.1.

3.3.1.1 Intratemporal Problems

Consumption allocation. Households derive utility from consumption of the N different mainland goods in the economy (households do not consume the commodity good). Even though the model is written in general terms, we can think about sectors being food, manufactures and services, in line with the empirical section. The consumption aggregator of every household is denoted by C_t^h and takes the form of an implicitly additive non-homothetic CES function. It defines total consumption in period t by[13]

$$1 = \sum_{j=1}^{N} (\omega_j (C_t^h)^{\varepsilon_j})^{\frac{1}{\sigma}} (C_{jt}^h)^{\frac{\sigma-1}{\sigma}},$$

where C_{jt}^h denote the consumption of good j by household h in period t, ω_j is a taste parameter for good j, σ is the constant elasticity of substitution between sectoral goods, and ε_j is the constant elasticity of consumption of sectoral good j with respect to the consumption index C_t^h that allows preferences to be non-homothetic. This latter parameter is also understood as the income elasticity with respect to individual consumption goods. These preferences were introduced by Hanoch (1975) and recently used in the macroeconomic literature by Comin et al. (2019), Cravino and Sotelo (2019), Matsuyama (2019) and Redding and Weinstein (2020), among others, mostly in the context of trade and structural change. Note that in the particular case of $\varepsilon_j = 1 - \sigma$ for every j, we recover the standard homothetic CES specification.

Given a level of total expenditures E_t^h, the intratemporal optimization problem derives the following conditions

[13] Comin et al. (2019) shows that with these preferences the intertemporal and intratemporal allocation problems can be separated, as in the case of standard CES utility function.

$$C_{jt}^h = \omega_j \left(\frac{P_{jt}}{P_t^h}\right)^{-\sigma} (C_t^h)^{\varepsilon_j+\sigma} \tag{3.1}$$

$$s_{jt}^h \equiv \frac{P_{jt}C_{jt}^h}{E_t^h} = \omega_j \left(\frac{P_{jt}}{P_t^h}\right)^{1-\sigma} (C_t^h)^{\varepsilon_j-(1-\sigma)}. \tag{3.2}$$

Equation (3.1) corresponds to the household demand for good j. Different to a standard CES specification, the dependance of sectoral demand to aggregate consumption is heterogeneous across goods and given by the term ε_j. The higher this term, the more consumption- (hence, income-) elastic is the good. Equation (3.2) characterizes the expenditure shares, where s_{jt}^h is the fraction of expenditures of household h in good j in period t, and $E_t^h = P_t^h C_t^h$ denotes total expenditures of household h in period t.

Note that these preferences define a household-specific CPI of the form

$$P_t^h = \left[\sum_{j=1}^{N} (\omega_j P_{jt}^{1-\sigma})^{\vartheta_j} (s_{jt}^h {E_t^h}^{1-\sigma})^{1-\vartheta_j}\right]^{\frac{1}{1-\sigma}}, \tag{3.3}$$

with $\vartheta_j \equiv (1-\sigma)/\varepsilon_j$. From these expressions is clear that both the expenditure share (3.2) and the CPI of each household (3.3) depend on the level of consumption/expenditure on every period. Note again that in the case of homothetic preferences ($\varepsilon_j = 1-\sigma$), the expenditure shares do not depend on the level of consumption, and the CPI is common across households because only depends on observed prices and not on the level of expenditures ($\vartheta_j = 1$). At the same time, with these non-homothetic preferences the demand for each good is non-linearly related to aggregate consumption, and such relation depends on the income elasticity of each good, ε_j. For future reference, let $\bar{\varepsilon}_t^h \equiv \sum_{j=1}^N s_{jt}^h \varepsilon_j$ be the average (expenditure-weighted) income elasticity.

As noticed by Comin et al. (2019), the predictions of the model for observables remain invariant to any scaling of all income elasticities and taste shifters (ε_j and ω_j) by a constant factor. Therefore, we can normalize all these parameters relative to a base good. Let $j = b$ to denote such base good, which will be normalized to one (i.e., $\varepsilon_b = \omega_b = 1$). Using (3.2), this implies that we can write the real consumption index as $C_t^h = s_{bt}^h (P_{bt}/E_t^h)^{\sigma-1}$. Substituting this expression back in (3.2) for any $j \neq b$, the expenditure share in good j relative to the base good for household h in period t can be written as

$$\begin{aligned} \log\left(\frac{s_{jt}^h}{s_{bt}^h}\right) =& (\varepsilon_j - 1)\log(s_{bt}) + (1-\sigma)\log\left(\frac{P_{jt}}{P_{bt}}\right) \\ &+ (\varepsilon_j - 1)(1-\sigma)\log\left(\frac{E_t^h}{P_{bt}}\right), \quad \forall j \neq b, \end{aligned} \tag{3.4}$$

which defines a $N-1$ system of demand equations. The key element to notice from (3.4) is that provides an expression for relative consumption shares of all other goods different than b in terms of observables, so it will be the base for the empirical estimation in the quantitative section of the paper.

Labor supply. Each household can work in any of the domestic sectors, as well as in the commodity sector. I assume there is imperfect mobility of labor between sectors. Following Horvath (2000), the aggregator labor bundle is given by

$$N_t^h = \left(\sum_{j=1}^{N} N_{jt}^{\frac{1+\varrho^h}{\varrho^h}} + N_{ct}^{\frac{1+\varrho^h}{\varrho^h}}\right)^{\frac{\varrho^h}{1+\varrho^h}}. \tag{3.5}$$

The parameter ϱ^h is the intratemporal elasticity of substitution and governs the degree of labor mobility across sectors. When

$\varrho^h \to 0$ ($\to \infty$), labor is completely immobile (mobile). For $\varrho^h < \infty$, the economy displays a limited degree of labor mobility and sectoral wages are not equalized. In principle, the elasticity ϱ^h might differ across households, introducing heterogeneity in labor income.

The optimization problem of the household consists in maximize total labor income, given wages and the labor aggregator. This derives the labor supply schedule

$$N_{jt}^h = \left(\frac{W_{jt}}{W_t^h}\right)^{\varrho^h} N_t^h, \tag{3.6}$$

with $W_t^h \equiv \left(\sum_{j=1}^N W_{jt}^{1+\varrho^h} + W_{ct}^{1+\varrho^h}\right)^{\frac{1}{1+\varrho^h}}$ being a household-specific wage index. Note that this index satisfies $W_t^h N_t^h = \sum_{j=1}^N W_{jt} N_{jt}^h + W_{ct} N_{ct}^h$.

3.3.1.2 Intertemporal Problems

Per-period utility. The per-period utility function, common across households, is separable in consumption and labor, and given by

$$U(C_t^h, N_t^h) = \frac{1}{1-\varsigma}(C_t^h)^{1-\varsigma} - \frac{\kappa^h}{1+\varphi}(N_t^h)^{1+\varphi}, \tag{3.7}$$

where $\varsigma > 0$ is the household's coefficient of relative risk aversion (or the inverse of the intertemporal elasticity of substitution), $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply and κ^h scales the disutility of labor. I assume that all these parameters are common across households, except for the latter, which allows me to impose the same number of total hours worked in steady-state.

Unconstrained households. The mass $1 - \lambda$ of unconstrained households u, maximizes their lifetime utility by choosing aggregate consumption and labor supply. As an additional source of income, they receive all the profits in the economy. Also, they have access to financial markets in the form of a domestic bond B_t that pays the (gross) nominal interest rate R_t, and a foreign bond B_t^* paying the (gross) nominal interest rate R_t^*. The former bond pays in domestic currency while the latter pays in units of foreign currency. The latter payments are transformed in domestic currency by the nominal exchange rate, S_t. Formally, the optimization problem of these households is

$$\max_{C_t^u, N_t^u, B_t, B_t^*} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(C_t^u, N_t^u)$$

subject to the budget constraint

$$E_t^u + \frac{B_t^h}{1-\lambda} + S_t \frac{B_t^*}{1-\lambda} = W_t^h N_t^h + R_{t-1} \frac{B_{t-1}}{1-\lambda} + S_t R_{t-1}^* \frac{B_{t-1}^*}{1-\lambda} + \frac{D_t}{1-\lambda},$$

where $U(C_t^u, N_t^u)$ is defined in (3.7) with the labor aggregator defined in (3.5) for unconstrained households, E_t^u denotes total expenditures (which is a function of total consumption of the household), and D_t is the total amount of profits coming from all firms in the economy. For simplicity, I assume that profits are equally distributed across all unconstrained households in the economy.

The solution of this problem is given by

$$1 = \beta R_t \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^u}{C_t^u} \right)^{-\varsigma} \frac{\overline{\varepsilon}_t^u}{\overline{\varepsilon}_{t+1}^u} \frac{P_t^u}{P_{t+1}^u} \right\} \tag{3.8}$$

$$1 = \beta R_t^* \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^u}{C_t^u} \right)^{-\varsigma} \frac{\overline{\varepsilon}_t^u}{\overline{\varepsilon}_{t+1}^u} \frac{P_t^u}{P_{t+1}^u} \frac{S_{t+1}}{S_t} \right\} \tag{3.9}$$

$$\frac{W_t^u}{P_t^u} = \kappa^u (C_t^u)^{\varsigma} (N_t^u)^{\varphi} \frac{\overline{\varepsilon}_t^u}{1-\sigma}. \tag{3.10}$$

Equation (3.8) denotes the Euler equation for unconstrained households. This characterizes the (nonlinear) relationship between real and nominal consumption over time at different levels of income, which the household incorporate in the intertemporal allocation problem. There is a wedge between the marginal cost of real consumption and the aggregate price index. The size of this wedge depends on the average income elasticity across sectors (time-varying and households-specific) and on the composition of expenditures. Something similar is observed in equation (3.9), corresponding to the optimality condition with respect to foreign bonds. The only difference with the previous expression is that now the intertemporal trade-off must take into consideration also the expected nominal depreciation of domestic currency. On the other hand, equation (3.10) characterizes the aggregate amount of labor, which depends on the average wage received. Different to other specifications with homothetic preferences, the labor supply depends on a real wage that is household specific and on the wedge induced by the average income elasticity. Once we have the desired amount of consumption and total amount of hours supplied, we can recover the sector-specific demands and labor supplies from (3.1) and (3.6), respectively.

Constrained households. The mass λ of constrained households has no access to financial markets and do not receive profits from firms. In this scenario, their optimization problem is static and they choose consumption and labor supply period by period. Formally, they solve

$$\max_{C_t^r, N_t^r} U(C_t^r, N_t^r) \quad \text{subject to} \quad W_t^r N_t^r = E_t^r,$$

where E_t^r denotes total expenditures of the restricted household. The solution of this problem is given by the following expression for total hours and consumption expenditures

$$\frac{W_t^r}{P_t^r} = \kappa^r (C_t^r)^{\varsigma} (N_t^r)^{\varphi} \frac{\overline{\varepsilon}_t^r}{1-\sigma} \tag{3.11}$$

$$W_t^r N_t^r = E_t^r. \tag{3.12}$$

As in the case of the unconstrained household, the characterization of the problem is completed with the sector-specific demands for consumption goods and labor supplies given by (3.1) and (3.6).

3.3.2 *Production*

I separate the problem of production between mainland sectors and the commodity sector. Mainland sectors are characterized by two layers of production. At the top, there is a representative competitive firm aggregating the output of atomistic producers. The output of this firm is sold to households and other aggregators. At the bottom, there is a continuum of monopolistically competitive producers which are subject to nominal rigidities à la Calvo. Further derivations are presented in Appendix 3.B.2. The commodity sector is composed by a representative and competitive firm. Distinct to other sectors in the economy, it takes the price of its good as given from foreign markets.

3.3.2.1 Mainland Sectors

Sectoral aggregator. Each sectoral good j is produced by a competitive firm that aggregates the output of a continuum of intermediate producers according to

$$Y_{jt} = \left(\int_0^1 Y_{jt}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $z \in [0,1]$ denotes the z-th variety of good j. As standard, profit maximization derives the following demand faced by a producer z

$$Y_{jt}(z) = \left(\frac{P_{jt}(z)}{P_{jt}}\right)^{-\varepsilon} Y_{jt}, \tag{3.13}$$

where $P_{jt} = \left(\int_0^1 P_{jt}(z)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$ is the price of the final sectoral good j. The output of this sector is used either for consumption of each household, exports, or as an input for other sectors of the economy (see details below).

Sectoral producers. Each producer $z \in [0,1]$ in sector j, operates in a monopolistically competitive environment. Technology is common to all firms within a sector but differs across sectors. The production function is given by

$$Y_{jt}(z) = \delta_j L_{jt}(z)^{\alpha_j} M_{jt}(z)^{\mu_j} V_{jt}(z)^{1-\alpha_j-\mu_j},$$

where L_{jt}, M_{jt} and V_{jt} denote labor, domestic materials and imports, respectively, and δ_j is a sector-specific constant.

The bundle of materials is sector-specific and given by

$$M_{jt} = \delta_j^m \prod_{i=1}^{N} M_{ijt}^{\gamma_{ij}}, \quad \text{with} \quad \sum_{i=1}^{N} \gamma_{ij} = 1,$$

where M_{ijt} denotes sales of materials from sector i to sector j in period t. The elements $\gamma_{ij} \geq 0$ can be collected into the input-output (IO) matrix of the economy ($\mathbf{\Gamma}$), which establishes the production linkages between sectors in the economy. In particular, entry γ_{ij} denotes the amount of intermediates from sector i necessary to produce \$1 of output in sector j.[14]

[14]Because the model is applied to the Chilean experience, I do not consider sales from the commodity sector to mainland sectors because these amounts are negligible. This might not be true for other commodity exporting economies, as shown by Romero (2020a).

Let P^*_{vt} be the foreign price of imports. Assuming that the law of one price holds, the domestic price of this good is equal to the nominal exchange rate times the foreign price, $P_{vt} = S_t P^*_{vt}$.[15] Because the technology and factor prices are common across producers within a sector, the marginal cost is the same across monopolists. Given this, by cost minimization, the demands for labor, materials, and imports at the *sectoral* level are

$$
\begin{aligned}
L_{jt} &= \alpha_j \frac{MC_{jt} Y_{jt}}{W_{jt}} \Delta_{jt} \\
M_{jt} &= \mu_j \frac{MC_{jt} Y_{jt}}{P^m_{jt}} \Delta_{jt} \\
V_{jt} &= (1 - \alpha_j - \mu_j) \frac{MC_{jt} Y_{jt}}{P_{vt}} \Delta_{jt},
\end{aligned}
$$

where $P^m_{jt} = \prod_{i=1}^N P_{it}^{\gamma_{ij}}$ is the sector-specific price of intermediate goods, and MC_{jt} denotes the marginal cost of production. By cost minimization, the demand for materials is given by $M_{ijt} = \gamma_{ij} P^m_{jt} M_{jt} / P_{it}$. In the previous expressions, Δ_{jt} is the sectoral price dispersion which comes from nominal rigidities (see details below).

Nominal rigidities. Firms face price stickiness à la Calvo, implying that with probability $1 - \theta_j$ they can reset their price in any given period, regardless of their last update. With this, a fraction θ_j of producers keep their price unchanged from last period, while the remaining fraction can set their desired price. I allow for sectoral heterogeneity in this probability.

The desired price is chosen by maximizing profits subject to the demand of the sectoral aggregator, taking into account that this price

[15] Different to the model presented in Chapter 2, for simplicity I abstract from nominal rigidities in importable goods.

can remain for an undetermined period time, which is captured by the probability θ_j[16]

$$\max_{\widetilde{P}_{jt}(z)} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_j \beta)^{\tau} \left[\widetilde{P}_{jt}(z) Y_{jt+\tau|t}(z) - MC_{jt+\tau} Y_{jt+\tau|t}(z) \right]$$

subject to (3.13), where $\widetilde{P}_{jt}$ denotes the desired price. In the previous expression $Y_{jt+\tau|t}(z)$ denotes the demand of the sectoral aggregator in period $t+\tau$, conditional on the last price reset of firm z in period t.

The optimality condition is given by

$$\widetilde{P}_{jt} = \frac{\varepsilon}{\varepsilon - 1} \frac{S_{jt}}{F_{jt}}$$

with

$$\begin{aligned} S_{jt} &= Y_{jt} MC_{jt} + \theta_j \beta \mathbb{E}_t [\Pi_{t+1} \Pi_{jt+1}^{\varepsilon} S_{jt+1}] \\ F_{jt} &= Y_{jt} + \theta_j \beta \mathbb{E}_t [\Pi_{jt+1}^{\varepsilon} F_{jt+1}], \end{aligned}$$

where Π_t denotes aggregate gross inflation and $\Pi_{jt} = \frac{P_{jt}}{P_{jt-1}} \Pi_t$ denotes sectoral gross inflation. In an abuse of notation, $\widetilde{P}_{jt}$ corresponds to the real sectoral desired prices (i.e., nominal price relative to price of aggregate consumption), while MC_{jt} will be understood from now on as the real marginal cost. Note that when prices are

[16]The pricing problem of firms typically considers the stochastic discount factor of the household. However, because households are heterogeneous in terms of their access to financial markets and consumption patterns, their stochastic discount factor is not the same. In this case, in which only unconstrained households own the firms, it would be natural to consider their discount factor. However, in an extended version of the model with more households which own the firms and exhibit non-homotheticities, it is not obvious which one is the relevant for the price setting. For simplicity, and because the model will be solved around its steady-state, I consider only the common deterministic discount factor β as the relevant element for pricing.

flexible ($\theta_j = 0$), the model collapses to the familiar case in which the desired price is a markup over marginal cost.

Given the properties of the Calvo pricing, sectoral prices and sectoral price dispersion evolve as

$$1 = (1-\theta_j)\left(\frac{\widetilde{P}_{jt}}{P_{jt}}\right)^{1-\varepsilon} + \theta_j \Pi_{jt}^{\varepsilon-1}$$
$$\Delta_{jt} = (1-\theta_j)\left(\frac{\widetilde{P}_{jt}}{P_{jt}}\right)^{-\varepsilon} + \theta_j \Pi_{jt}^{\varepsilon}\Delta_{jt-1}.$$

3.3.2.2 Commodity Sector

The commodity sector is characterized by a representative firm that produces with the following Cobb-Douglas technology

$$Y_{ct} = \delta_c (L_{ct})^{\alpha_c} (M_{ct})^{\mu_c} (V_{ct})^{1-\alpha_c-\mu_c},$$

where δ_c is a constant term. In this specification, L_c denotes labor, M_c the amount of domestic materials demanded by the sector, and V_c denotes imports. I assume that the law of one price also holds for the commodity prices, implying that the domestic price is $P_{ct} = S_t P^*_{ct}$.

The commodity sector is competitive, so profit maximization derives the following demands for factors

$$L_{ct} = \alpha_c \frac{P_{ct}Y_{ct}}{W_{ct}}, \quad M_{ct} = \mu_c \frac{P_{ct}Y_{ct}}{P^m_{ct}}, \quad \text{and} \quad V_{ct} = (1-\alpha_c-\mu_c)\frac{P_{ct}Y_{ct}}{P_{vt}}.$$

As in the case of mainland sectors, materials take a Cobb-Douglas form

$$M_{ct} = \delta^m_c \prod_{j=1}^{N} M_{jct}^{\gamma_{jc}}, \quad \text{with} \quad \sum_{j=1}^{N} \gamma_{jc} = 1.$$

Cost minimization implies a demand for specific sectors of the form $M_{jct} = \gamma_{jc} P_{ct}^m M_{ct}/P_{jt}$, with the price of materials given by $P_{ct}^m = \prod_{j=1}^N P_{jt}^{\gamma_{jc}}$.

As mentioned before, the commodity sector takes the international price as given. I assume this follows an autoregressive form, and its the only source of fluctuations in the economy

$$\log(P_{ct}^*) = (1-\rho_c)\log(P_c^*) + \rho_c \log(P_{ct-1}^*) + \nu_t^c.$$

3.3.3 *Exportable Good*

There is a competitive firm combining inputs from different mainland sectors to produce an exportable good, which is fully sold abroad. The technology of this sector is given by $Y_t^x = \delta_x \prod_{j=1}^N (Y_{jt}^x)^{\psi_j}$, where δ_x is a constant term and $\sum_{j=1}^N \psi_j = 1$ holds. The price of exports and the demand for inputs from each sector are given by

$$P_t^x = \prod_{j=1}^N P_{jt}^{\psi_j} \qquad \text{and} \qquad Y_{jt}^x = \psi_j \frac{P_t^x}{P_{jt}} Y_t^x.$$

The foreign demand for the exportable good takes the form $Y_t^x = \omega^x \left(\frac{P_t^x}{S_t P_t^*}\right)^{-\eta^*} Y_t^*$, where P_t^* and Y_t^* denote the foreign CPI and the foreign level of output, respectively.

3.3.4 *Aggregation, Monetary Policy and Market Clearing*

Aggregation and monetary policy. Total consumption expenditures are defined as the sum consumption of the restricted and unconstrained households

$$P_t C_t = \lambda P_t^r C_t^r + (1-\lambda) P_t^u C_t^u.$$

The numeraire of the economy is aggregate consumption ($P_t = 1$).[17] Working the previous expression, total consumption can also be written as $C_t = \sum_{j=1}^{N} P_{jt} C_{jt}$, with $C_{jt} = \lambda C_{jt}^{c} + (1-\lambda) C_{jt}^{u}$ being total sectoral consumption.

Total exports are given by the value of commodity goods plus the value of the exportable good, $X_t = P_{ct} Y_{ct} + P_t^x Y_t^x$. On the other hand, total imports are given by the sum of sectoral imports, $V_t = P_{vt}\big(\sum_{j=1}^{N} V_{jt} + V_{ct}\big)$. Combining these expressions, the trade balance is given by $TB_t = X_t - V_t$.

Gross domestic product (GDP) is defined as the sum of aggregate consumption and the trade balance, $GDP_t = C_t + TB_t$.

Finally, the domestic nominal rate is set by the central bank by using a Taylor rule

$$\frac{R_t}{R} = (\Pi_t)^{\phi_\pi} \left(\frac{GDP_t}{GDP_{t-1}} \right)^{\phi_y},$$

where R is the long-run value of the gross nominal rate.

Market clearing. For equilibrium, we have to impose market clearing conditions in labor markets and for every sectoral good

$$L_{jt} = \lambda N_{jt}^{r} + (1-\lambda) N_{jt}^{u}$$
$$L_{ct} = \lambda N_{ct}^{r} + (1-\lambda) N_{ct}^{u}$$
$$Y_{jt} = C_{jt} + Y_{jt}^{x} + \sum_{i=1}^{N} M_{jit} + M_{jct}.$$

By combining the different market clearing conditions of the model, the domestic bond market clearing ($B_t = 0$), as well as the

[17] To make this expression operative, I apply the normalization $P_t = \sum_{j=1}^{N} \omega_j P_{jt}^{1-\sigma} = 1$. This is the same expression that would hold in a world with CES homothetic preferences.

aggregate resource constraint (given by the combined budget constraints of different households), the financial position of the economy is given by

$$S_t B_t^* = TB_t + S_t R_{t-1}^* B_{t-1}^*. \tag{3.14}$$

To close the model, I consider the following specification for the foreign interest rate

$$R_t^* = R_t^w \times \exp\left[\phi_b \left(\bar{b} - \frac{S_t B_t^*}{GDP_t}\right)\right], \tag{3.15}$$

where $\phi_b > 0$ and R_t^w is the world interest rate and the term $\exp[\cdot]$ is a risk premium that the country pays over the risk-free rate. This premium is affected by the deviations of total debt of the country, relative to GDP, with respect to a long-run determined value, $\bar{b}$. This guarantees the stationarity of the model (Schmitt-Grohe and Uribe, 2003).

3.3.5 *The Role of Non-homotheticities: Analytical Results*

What is the role of non-homothetic preferences? To analyze the impact of this channel, first I study the behavior of individual consumption and hours supplied. I start with the general consumption decision problem to then evaluate the implications for each specific household.

Consumption decision. When making consumption decisions, households equalize the marginal utility of an extra unit with its marginal cost. In general terms, this trade-off for a generic household h is characterized by

$$U_C(C_t^h, N_t^h) = \lambda_t^h \frac{\partial E(C_t^h)}{\partial C_t^h},$$

where $U_C(\cdot)$ denotes the marginal utility of consumption, $\frac{\partial E(C_t^h)}{\partial C_t^h}$ is the marginal effect of consumption over expenditures and λ_t^h is the household specific Lagrange multiplier capturing the shadow value of wealth. The second term can be written as $\frac{\partial E(C_t^h)}{\partial C_t^h} = P_t^h \frac{\bar{\varepsilon}_t^h}{1-\sigma}$, where, as mentioned before, $\bar{\varepsilon}_t^h = \sum_{j=1}^N s_{jt}^h \varepsilon_j$ denotes the average income elasticity and s_{jt}^h is the expenditure share in good j for household h (see Appendix 3.B.1 for derivations). Note that this average elasticity is household-specific and time-varying because it depends on the consumption expenditures shares.

In models with homothetic preferences, the marginal effect of an extra unit of consumption over expenditures is only given by the price of the consumption basket because the income elasticity of each good is common and equal to $1-\sigma$. This reflects the fact that the composition of the consumption basket does not change with the level of consumption itself. Therefore, in such case, $\frac{\partial E(C_t^h)}{\partial C_t^h} = P_t = 1$, which is the aggregate (and common) price of consumption. With non-homothetic preferences, however, the income elasticity is good-specific and given by ε_j. In this case, the average income elasticity is not trivial. Conceptually, this term reflects the fact that how the extra unit of consumption is allocated among different goods matters to capture the marginal effect of consumption over expenditures. In particular, as total consumption grows, this additional unit is allocated towards more income elastic goods (i.e., services) and the cost of living grows in a nonlinear way. In what follows, I analyze how this elasticity differentially affects each household, depending on their access to financial markets.

Unconstrained household. Using the specific form of separable preferences given in Equation (3.7), consumption of the unconstrained household is determined by its Euler equation (3.8). Taking a log-linear approximation around the steady-state, this equation can be written as

$$c_t^u = \underbrace{\mathbb{E}_t\{c_{t+1}^u\} - \varsigma^{-1}(r_t - \mathbb{E}_t\{\pi_{t+1}\})}_{\text{Standard Euler}} + \underbrace{\varsigma^{-1}\mathbb{E}_t\{\Delta\bar{\varepsilon}_{t+1}^u\}}_{\text{Income elasticity}} - \underbrace{\varsigma^{-1}\mathbb{E}_t\{\pi_{t+1} - \pi_{t+1}^u\}}_{\text{Expected inflation}}, \tag{3.16}$$

where lowercase letters denote log-deviations with respect to steady-state. The previous expression contains three arguments on the right-hand side.[18] The first term corresponds to the standard expression for the Euler equation. Consumption today is positively associated to consumption tomorrow and negatively to the real interest rate ($r_t - \mathbb{E}_t\{\pi_{t+1}\}$). This latter term is the standard intertemporal substitution effect for consumption, in which a larger real rate provides incentives to increase savings and postpone consumption.

On top of that, the model with non-homotheticities change the response of consumption of the unconstrained in two ways. On the one hand, the second term on the right-hand side captures the effect of changes in the income elasticity over consumption decisions. As mentioned before, an increase in such elasticity today reduces current consumption by changing the composition of expenditures: an extra unit of consumption not only changes the cost of living by its direct price effect, but also by the compositional effect of the consumption basket, which is moving towards more income elastic and expensive goods. In this sense, when the average income elasticity is expected to decrease over time (i.e., the average income elasticity increases today), is optimal for the household to postpone consumption. On the other hand, the last term captures a differential in the expected inflation for tomorrow. When the unconstrained household

[18]In an abuse of notation, in the previous expression $\bar{\varepsilon}_t$ denotes the log-deviation of the average income elasticity with respect to steady-state and not its level.

experiences higher inflation than the aggregate, it is optimal to advance consumption for today.

How does labor supply reacts? Approximating (3.10) around the steady-state, gives the expression

$$n_t^u = \frac{1}{\varphi}(w_t^u - p_t^u) - \frac{\varsigma}{\varphi}c_t^u - \frac{1}{\varphi}\bar{\varepsilon}_t^u.$$

As usual, labor supply is increasing in the real wage, which in the case of non-homotheticities considers the household-specific CPI $(w_t^u - p_t^u)$. On top of that, there is the standard income effect of labor supply, given by c_t^u, which reduces hours. In the non-homothetic model, the compositional effect given by changes in the income elasticity operates as an additional income effect by reducing hours even further.

Restricted household. The case of the restricted household shares the dampening features of the problem of the unconstrained given by the non-homothetic preferences. We can get expressions for consumption and hours in closed form by solving (3.11)-(3.12), which can be approximated as

$$\begin{aligned} c_t^r &= \frac{1+\varphi}{\varphi+\varsigma}(w_t^r - p_t^r) - \frac{1}{\varphi+\varsigma}\bar{\varepsilon}_t^r \\ n_t^r &= \frac{1-\varsigma}{\varphi+\varsigma}(w_t^r - p_t^r) - \frac{1}{\varphi+\varsigma}\bar{\varepsilon}_t^r. \end{aligned}$$

Consumption of the restricted household increases with its real wage, but such impact is dampened by the compositional effect captured by the average income elasticity. The same happens for hours supplied. Interestingly, hours will respond to fluctuations even in the log-utility case ($\varsigma = 1$) where labor supply does not depend on wages, which is a standard assumption in models with constrained agents

(Bilbiie, 2008). Therefore, non-homothetic preferences not only modify the intertemporal elasticity of substitution, as in the case of the unconstrained household, but has intratemporal consequences as well.

Non-homothetic preferences, heterogeneity and aggregate consumption. Recall from that aggregate consumption corresponds to a weighted average between expenditures of each household. Therefore, it is clear that non-homothetic preferences will dampen the response of this variable to a shock. On the other hand, heterogeneity in the form of market incompleteness operates in the opposite direction. This is because a higher degree of market incompleteness implies a more important role for constrained agents in determining aggregate consumption, and these agents have a higher marginal propensity to consume. For this latter channel to dominate over the dampening effect of non-homothetic preferences, it must be the case that income of the two households fluctuates differently over the cycle. As we will see in the following section, by assuming a similar degree of labor mobility across households, hence a similar response of labor income over the cycle, we minimize the role of heterogeneity and the dampening effect of preferences dominates.

3.4 QUANTITATIVE ANALYSIS

This section explores the quantitative properties of the model. First, I describe the calibration strategy for the Chilean economy, which also considers the estimation of the demand system. Then, I analyze the impact of commodity price shocks in the context of non-homothetic preferences and heterogeneity.

3.4.1 *Calibration*

The frequency of the model is set to be a quarter and the number of mainland sectors is set to $N = 3$, in order to replicate the empirical characterization of consumption. This implies that the model is composed by the sectors of food and beverages ($j = f$), manufactures ($j = m$), and services ($j = s$), besides the commodity sector. To parameterize the economy, I partition the parameter space into three groups in a similar fashion as in Chapter 2. First, there is a subset of predetermined parameters which are standard in the literature or taken from the Chilean data and governs the steady-state of the model. A second set of parameters, which are related to the demand system of households, is estimated in order to match the features presented in Section 3.2. Finally, there is a third group of parameters chosen in order to improve the fit of the model.

Predetermined parameters. Table 3.B.1 summarizes the set of predetermined parameters of the model. The parameters of risk aversion and the (inverse of the) Frisch elasticity are set to one, which are standard values in the New Keynesian literature. The discount factor is set to get a domestic interest rate of 5.8 percent at a yearly frequency, in line with Garcia-Schmidt and Garcia-Cicco (2020b). The demand taste shifters are set to match a steady-state relative consumption shares of 0.20, 0.19 and 0.61 for food, manufactures and services, respectively (see Table 3.A.1). In the baseline analysis, labor supply elasticity is set equal to 1 for both households ($\varrho^r = \varrho^u = 1$), implying limited labor mobility across sectors and the same wage index, which minimizes the role of income heterogeneity. In the baseline analysis, I use a value $\lambda = 0.3$ for the fraction of constrained agents in the economy. This number is in line with the estimates in Kaplan et al. (2014) for the U.S. and it can be taken as a lower bound for an emerging economy.

With respect to sectors, the elasticity of substitutions across varieties is set to 10, implying a steady-state level of sectoral profits close to 11 percent, a standard value in the New Keynesian literature. The parameters associated to the production functions of mainland and commodity sectors are taken from the Input-Output (IO) table for Chile of year 2017. In line with the empirical application of Section 3.2, I aggregate the 111 sectors in the IO table to get the same sectors (food, manufactures and services).[19] Services are more labor intensive than the rest of sectors ($\alpha_s = 0.48$), while food is the sector that uses less labor ($\alpha_f = 0.18$). On the other hand, the usage of imports for production is more intensive in the food sector and significantly less relevant for services. This is consistent with the notion that services is a non tradable sector, while food and manufactures are tradable. The parameters that characterize the production function of the commodity sector are also taken from IO data.

To calibrate the level of price rigidities, I follow Pasten et al. (2020) and compute the average frequency of price adjustment for goods in the different categories. For this, I rely on the microdata underlying the construction of the official Chilean CPI for the period 2010-2018. I compute the ratio between the number of price changes over the total number of periods available for each product. Then, I aggregate at the three-sector classification level to get the behavior of food, manufactures and services. Finally, the level of price rigidity is computed as one minus the obtained level. With this procedure I obtain values of $\theta_f = 0.27$, $\theta_m = 0.46$ and $\theta_s = 0.59$, implying that the price of food and beverages are relatively more flexible than the price of manufactures and services.

The values for the monetary policy rule are standard in the literature, with a central bank reacting aggressively to movements in inflation. The world interest is taken from Garcia-Schmidt and Garcia-Cicco (2020b) and set to 4.5 percent in annual terms. The long-run value of foreign bonds is set to match a ratio of trade balance-to-GDP of 8 percent in steady-state. Finally, the persistence of the autoregressive process for the commodity price is set to 0.7123.

[19]The commodity sector corresponds to all mining activities in the economy. Agricultures and fishing are not considered.

Demand system estimation. To estimate the parameters associated to the demand system (3.2), recall that the predictions of the model remain invariant to any scaling of the elasticities ε_j and the taste parameters ω_j as shown in Equation (3.4). Therefore, I set manufactures as the baseline good, and normalized those values to one ($\varepsilon_m = \omega_m = 1$). Therefore, the demand system can be written as

$$\begin{aligned}\log\left(\frac{s_j^h}{s_m^h}\right) =& (\varepsilon_j - 1)\log(s_m) + (1-\sigma)\log\left(\frac{P_j}{P_m}\right) \\ &+ (\varepsilon_j - 1)(1-\sigma)\log\left(\frac{E^h}{P_m}\right) + \nu^h, \quad j \in \{f, s\},\end{aligned} \tag{3.17}$$

where s_{jt}^h denotes the expenditure share of household h in good j, and E_t^h denotes total expenditures of household h. I estimate this empirical specification with the cross-sectional data presented in Section 3.2 in order to replicate the expenditure patterns observed across the income distribution. As Comin et al. (2019), I assign a percentile-specific price for the three goods, which captures in an imperfect way the fact that different households might not face the same prices. The previous expression denotes a system of $N-1$ equations, and I impose that across them the parameters are the same. The estimation is carried out using Feasible Generalized Nonlinear Least Squares (FGNLS) as Herrendorf et al. (2013) and Cravino and Sotelo (2019).[20] As in Comin et al. (2019) and Cravino and Sotelo (2019), the identification assumption is that shocks to income and relative prices are not correlated to changes in demand shifters, ω_j, so preferences do not change over time, other than by the income effect.

Table 3.1 reports the results. The elasticity of substitution σ is significantly below one and close to zero, implying a high degree of complementarities in consumption. On the other hand, the income

[20]For the estimation I constraint the elasticities to be positive to ensure that the consumption aggregator is concave.

elasticity of food is close to zero, while the income elasticity of services is above one. These results indicate that services are more income elastic than manufactures and food.[21]

Table 3.1: Demand system estimates

	Coefficient	Std. Error
σ	0.271^{***}	(0.023)
ε_f	0.000	$(\cdot)$
ε_s	1.113^{***}	(0.036)
Observations	100	

NOTES: This table presents the estimates of the demand system given by equation (3.17). σ denotes the elasticity of substitution between goods, while ε_j denotes the income elasticity of good $j \in \{f, s\}$. Robust standard errors reported in parenthesis. *, ** and *** denote statistical significance at the 1, 5 and 10% levels, respectively.

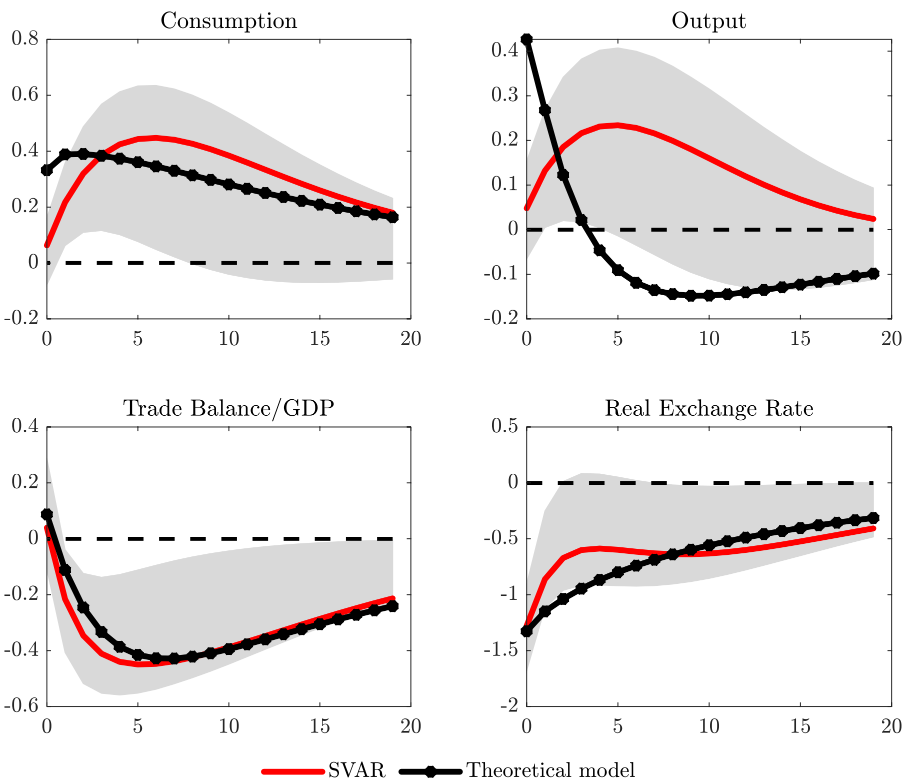
Other parameters. The final set of parameters corresponds to the foreign demand shifter and elasticities of demand (ω^x and η^*), and the risk-premium of the interest rate (ϕ_b). In order to choose values for those parameters, I run an empirical structural VAR of length one, that includes the real value of copper (dollars deflated by US CPI) as the relevant commodity price for Chile, gross domestic product, consumption, the trade balance to GDP ratio, inflation and the real exchange rate. The model is estimated with quarterly data between 1996 and 2018 and is set such that the commodity price does not react to domestic conditions. To estimate those parameters, I minimize the distance between the empirical and the model-implied responses for consumption and the trade-balance to GDP, using as a weighting matrix the inverse of the interquartile range used to compute the confidence interval of the empirical response. This procedure delivers the following values: $\omega^x = 0.90$, $\eta^* = 2.93$ and $\phi_b = 0.011$.

[21] For comparison purposes, note that Comin et al. (2018) finds $\sigma = 0.26$, $\varepsilon_f = 0.2$ and $\varepsilon_s = 1.65$ for the U.S., using panel data from the Consumption Expenditure Survey (CEX) for the period 1999-2010.

3.4.2 *Model vs Data*

Figure 3.3 shows the impulse-response functions to a 10 percent increase in the commodity price shock and compares with the empirical responses derived from the SVAR model. The theoretical model captures most of the responses of aggregate variables to the commodity price shock. On the one hand, consumption increases by 0.3 percent, with a long lasting effect. The trade balance shows a positive reaction on impact of 0.09, to then run a sustained deficit over the next quarters. This is a standard response for small open economies, in which the increase in terms of trade (here given by the increase in the commodity price) imply a higher expenditure in imports over time. Finally, there is a persistent appreciation of the real exchange rate, with an impact effect of 1.3 percent. Note that the model-implied responses of all these variables replicate the shape and level of the empirical responses over time and are between the confidence intervals. The one exception is given by output, in which case the model generates a stronger impact response than in the data, with a less persistent effect given by the income effect of labor supply. Overall, the figure shows that the model considering non-homothetic preferences generate sensitive responses of aggregate variables to a commodity price shock.

FIGURE 3.3: Impulse-response function to a 10% commodity price shock



NOTES: This figure compares the impulse-response function of the data versus the model. Grey areas correspond to 95% confidence intervals computed with bootstrap with 100,000 replications. The SVAR model includes a constant and linear and quadratic time trends. Horizontal axes correspond to quarters. Vertical axes corresponds to percentage deviations with respect to trend (for empirical model) and percentage deviations with respect to steady-state (for model-implied responses), except for trade balance/GDP which is measured in deviations.

3.4.3 *The Role of Non-homotheticities*

The relative contribution of heterogeneity and non-homotheticities. How important are non-homothetic preferences? As shown in the analytical results (see section 3.3.5), consumption and hours worked of both households should be lower than in the case with homothetic preferences. Moreover, aggregate consumption also responds by less to shocks, due to the dampened response in individual consumptions. At the same time, the model takes into account income heterogeneity between the households, given by profits and the access to financial markets.

To further investigate the relative importance of these mechanisms, I focus on the response of aggregate consumption to a commodity price shock, and compare the evolution on its response for different calibrations of the model.[22] Figure 3.4 shows the response to a commodity price shock under four specifications of the model. The solid black line corresponds to the baseline case, characterized by two agents (restricted and unconstrained) and non-homothetic preferences. The effect of income heterogeneity (given by market incompleteness) can be inferred by comparing with a representative agent model with non-homothetic preferences (grey solid line denoted as RANK+NH). As can be seen, differences between the two models are mild: on impact (cumulative terms), the baseline model has a response 3.5 (2.2) percent larger. This result is generated by the conservative value for the share of constrained agents in the baseline model ($\lambda = 0.3$) and the assumption of equal labor supply elasticity of both households.[23] The larger response in the baseline model relative to RANK+NH is because the former model has a fraction of the population with a higher marginal propensity to consume. With the commodity price shock, output is growing fast on impact and the nominal (and real) interest rate is increasing, which dampens the response of unconstrained households. Because in RANK models all the population is subject to fluctuations in the interest rate, this explains the lower response relative to baseline.

To infer the contribution of non-homothetic preferences, now

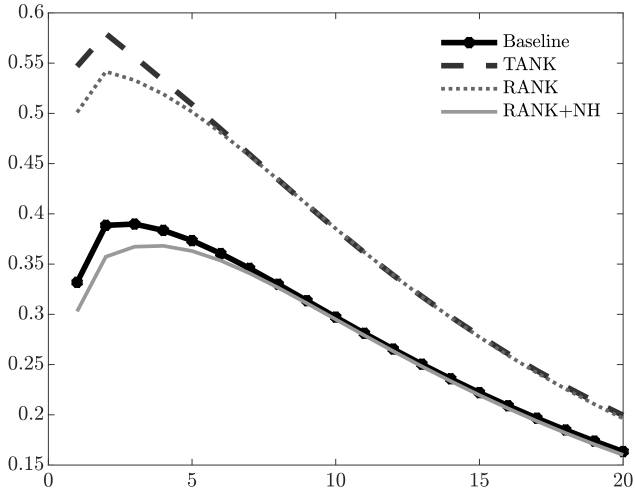
[22] Additional analysis for other aggregate and household-specific variables is presented in Appendix 3.C.1.

[23] For example, in a model in which the degree of labor mobility of the unconstrained household is equal to zero, the difference between these models in cumulative terms would be of 50 percent.

compare the response of the baseline model with the specification with two agents and homothetic preferences (dashed line, corresponding to the TANK model). Now the differences between the two models are way larger. On impact, the response of the TANK model is 79 percent larger, while in cumulative terms is 34 percent larger. These differences are in line with the theoretical analysis presented in section 3.3.5, in which non-homothetic preferences dampens the response of each individual household, as well as the aggregate level of consumption.

Finally, we can compare the response with respect to a version of the model with a representative agent and homothetic preferences (dotted line, denoted as RANK). This case is one in which both channels (income heterogeneity and non-homothetic preferences) are turned-off, so constitutes a good benchmark in line with the literature. Relative to this specification, the baseline case generate a response 70 and 31 percent lower on impact and in cumulative terms, respectively. This combines both, the dampening effect of non-homothetic preferences and the expansive effect of income heterogeneity, where the former effects dominates.

FIGURE 3.4: Aggregate consumption under different specifications



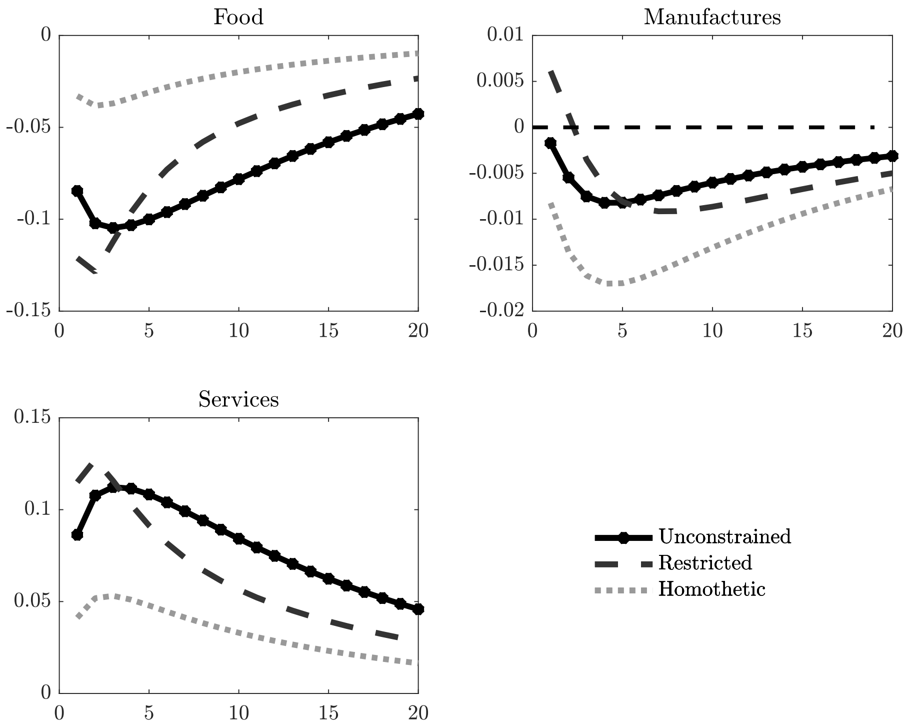
NOTES: This figure compares the impulse-response function of aggregate consumption under different specifications of the model. Baseline (black-solid line) corresponds to the baseline specification with two agents and non-homothetic preferences. TANK (dashed line) corresponds to a model with two agents and homothetic preferences. RANK (dotted line) corresponds to a model with a representative agent and homothetic preferences. RANK+H (grey-solid line) corresponds to a model with a representative agent and non-homothetic preferences. All responses are in percentage deviations with respect to steady-state.

Are the differences induced by non-homothetic preferences too large? In a different context, Ravn et al. (2006) documents similar differences between a homothetic and a non-homothetic model. Their paper studies a closed economy subject to technology shocks (which is the most comparable to the commodity shock studied here) with monopolistic competition, in which non-homothetic preferences are modelled as "deep habits": the representative household has Stone-Geary preferences at the goods level (and not at the level of aggregate consumption), implying that each good has different subsistence levels and income elasticity. In their setup, they find that a positive technology shock in a world with homothetic preferences generates a response four times larger in consumption relative to the non-homothetic case. This differences remain large over time. Therefore, even though the context and shock analyzed is different, there is precedent in the literature with findings of similar magnitudes.

Changes in consumption expenditures. One of the key elements of the non-homothetic preferences is that expenditure shares not only depend on prices but on the level of income. How different are consumption expenditures in this case?

The theoretical analysis indicates that in the non-homothetic case, expenditure shares vary with the level of income, which is governed by the good-specific income elasticity. The empirical estimates for these elasticities indicate that services have a higher elasticity than manufactures, which is also more elastic than food. Therefore, we expect to observe a decrease in expenditures of food and an increase in expenditures of services. This is confirmed in Figure 3.5. Each panel presents the response of the expenditure share of the three goods composing the consumption basket (food, manufactures and services), and compares the differential response of each household (unconstrained and restricted), as well as the response in the homothetic world. There is a large decrease in food expenditures, as well as an increase in the expenditure of services. In the case of the restricted household, food decreases in 0.12, while the increase in services is 0.11. For the unconstrained, the decrease in food expenditures is 0.08 and the increase in services has a similar magnitude. Compared to the homothetic case, the response in those expenditure shares are more than doubled (food decreases by 0.03 and services increases by 0.04). Comparing between the two households, on impact the restricted household has a larger response because its increase in income must be fully consumed. Interestingly, the restricted household initially increases its consumption on manufactures, to then reduce it over time, while the unconstrained household always decrease such expenditure. These business-cycle responses are in line with the structural transformation literature, documenting a decrease (increase) in the consumption of food (services) when there is an increase in income. From this exercise we conclude that non-homothetic preferences largely modify consumption patterns, and that those changes are heterogeneous across the income distribution.

FIGURE 3.5: Expenditure shares



NOTES: This figure compares the impulse-response function of expenditure shares of different households. Each panel presents the evolution of the expenditure share in food, manufactures and services consumption for each household (unconstrained and restricted), and compares with the response in a homothetic model in which those responses are no longer household specific. All responses are in deviations with respect to steady-state.

3.5 SENSITIVITY ANALYSIS

This section studies the sensitivity of the main results to key parameters in the model. As in the previous section, I focus on the response of aggregate consumption to a commodity price shock, considering different parameter configurations. In particular, I analyze the role played by the degree of labor mobility of households, the degree of market incompleteness (captured by the fraction of restricted agents in the economy), the elasticity of substitution among goods in the consumption aggregator, and the income elasticity of services.[24]

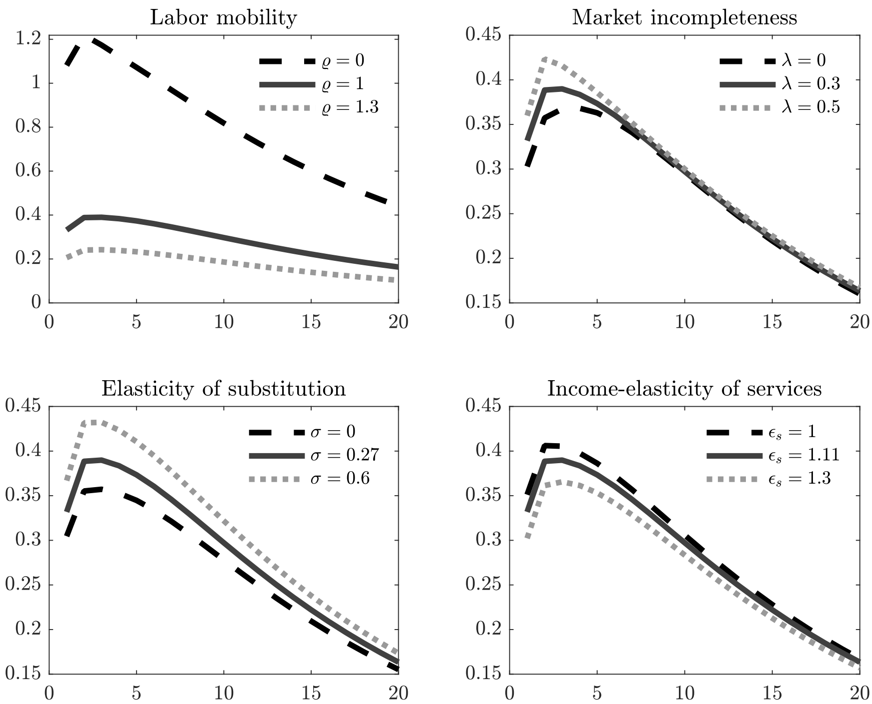
Degree of labor mobility. Panel A in Figure 3.6 compares the responses under different values of the degree of labor mobility, ϱ. As in the baseline scenario, I assume that this parameter is common across household in every counterfactual. As we can see, the higher the degree of labor mobility (larger value of ϱ), the lower the response of consumption. This is because, after the increase in the commodity price, all sectors would like to hire more due to the increase in demand. This is specially relevant for services, which is an important supplier to the rest of the economy (for both the commodity sector and mainland sectors), is more income elastic for households, and is more labor intensive. When labor mobility is reduced ($\varrho \to 0$) there is no possible sectoral reallocation of workers because sectoral labor supply is completely inelastic and households provide the same amount of labor in every sector. This impose a large increase in sectoral wages and household income, which translates in more consumption. In particular, the case $\varrho = 0$ implies an impact response of consumption three times larger than baseline.

When labor is more mobile ($\varrho = 1.3$ in our counterfactual), labor supply becomes more sensitive to wages. In the limit, sectoral labor supplies become completely elastic, and each household supplies labor only to the sector that offers the largest wage rate. Given the commodity price shock, the commodity sector is the one that offers the highest wage and households would like to offer all their hours only to this sector, affecting all other industries. The result is that

[24] Additional analysis for other aggregate and household-specific variables is presented in Appendix 3.C.2.

there is strong reallocation of labor towards the commodity sector, which is less labor intensive. As a consequence, this sector can produce more without having to increase wages as much, other sectors in the economy shrink their activity, and the overall level of household wages increases by less than in the case with no labor mobility. This effect implies a lower level of income and consumption, with an impact response 38 percent lower than baseline.

FIGURE 3.6: Aggregate consumption–Sensitivity



NOTES: This figure compares the impulse-response function of aggregate consumption under different calibrations for key parameters. On each panel, the solid line corresponds to the baseline calibration. All responses are in percentage deviations with respect to steady-state.

Degree of market incompleteness. The degree by which markets are incomplete is captured by the fraction of agents in the economy that have no access to financial markets to smooth consumption. In the model, this corresponds to the fraction of restricted agents, λ: the larger this value, the higher the degree of market incompleteness. How this margin affects aggregate consumption? Conceptually, changes in the share of constrained agents will modify the relative importance of each household in order to compute sectoral labor and sectoral consumption.[25] This differential composition has two associated effects. On the one hand, a higher degree of market incompleteness implies that restricted households are relatively more important in the overall economy, and they receive a larger fraction of labor income. But on the other hand, the larger is the amount of constrained households, the lower is the income per capita they receive (and the higher for the unconstrained households that now represent a lower fraction of the population). Both forces operate in opposite directions, so in the end which one dominates is a quantitative question.

The second panel of Figure 3.6 compares the responses by changing the fraction of constrained agents in the economy. As can be seen, an increase in the fraction of constrained agents increases the response of aggregate consumption, supporting the notion that a larger fraction of income is distributed towards a group with higher marginal propensity to consume. Under the baseline calibration for the labor supply–in which both kinds of households provide the same hours with the same elasticity–an increase from $\lambda = 0.3$ to $\lambda = 0.5$ generates 8 percent amplification in consumption. Even though these differences are not large, implying a minor role for market incompleteness under this configuration of the model as discussed before, larger differences would be obtained by imposing differences in labor supply at the household level.

[25] Again, these changes should be more relevant for the services sector, because it is the more labor intensive.

Elasticity of substitution. The third panel in Figure 3.6 presents results of changing the elasticity of substitution between the three goods consumed by households. An increase in such elasticity implies that food, manufactures and services are less complementary; in the limit ($\sigma \to 1$), the consumption basket converges to the Cobb-Douglas case. In this case, the expenditure shares become less sensitive to changes in prices and income. Because in the model the price of services increases, while the price of food and manufactures decrease, such lower reallocation generates an increase in the cost of living of each household, and an increase in their degrees of expenditures. This is reflected in the figure with a larger response in aggregate consumption expenditures: on impact (cumulative terms), an elasticity of substitution of 0.6 generates a response 11 (9) percent higher than in the baseline scenario.

Income elasticity of services. Finally, I explore the consequences of changes in the income elasticity of goods. I focus in the case of services because they are the more important in terms of weight in the consumption basket and also the more income-elastic. An increase in such elasticity makes services more sensitive to increments in income and generates a larger difference in terms of valuation with respect to manufactures. Therefore, in a world with a higher income elasticity, the expenditure share of services (manufactures) increases (decreases) by more, implying an increase in the average income elasticity of households. As discussed in Section 3.3.5, this variable is key to understand the role of consumption over time, because it represents an additional cost for consumption. Therefore, we expect to observe a decrease in the response of both household-specific and aggregate consumption expenditures. The last panel in Figure 3.6 shows the response of aggregate consumption after the commodity price shock under different values of the income elasticity of services. In line with the previous discussion, the response is decreasing in terms of that parameter. In particular, an elasticity of $\varepsilon_s = 1.3$ generates a response 9 percent lower on impact (in cumulative terms is five percent below) than in the baseline case.

3.6 CONCLUSION

This paper analyzes the role of household heterogeneity in the transmission of commodity price shocks, analyzing a novel channel of adjustment: consumption heterogeneity. I document using data from Chile and other emerging economies, that low-income households spend relatively more on food, while high income households spend more on services.

Motivated by these observations, I build a model for a commodity exporting economy with non-homothetic preferences, in which expenditure shares are endogenous and depend on the level of income. In the model, households have differential access to financial markets, so not all of them can smooth consumption, which induces heterogeneity in income and expenditures. After a positive commodity price shock, all households increase their income and can consume more, but the gains are biased towards agents that have access to financial markets and receive profits from firms. In terms of the mechanisms, non-homothetic preferences dampen the microeconomic and aggregate impact of the shock because they induce a reallocation effect on the consumption basket towards more income elastic goods (services), while heterogeneity amplifies the responses by giving a more relevant role to a fraction of agents with a higher marginal propensity to consume.

These findings contribute not only to the our understanding on the effects and transmission of terms of trade and commodity price shocks to small open economies, but also on the role that heterogeneity (either in terms of income or in terms of consumption) play in shaping the response to those shocks.

3

APPENDIX

3.A ADDITIONAL EMPIRICAL RESULTS

3.A.1 *Consumption Patterns Across a More Disaggregated Level of Consumption*

Table 3.A.1 presents additional evidence for selected percentiles in the income distribution and a more disaggregated level of consumption categories.

Table 3.A.1: Consumption expenditure of households

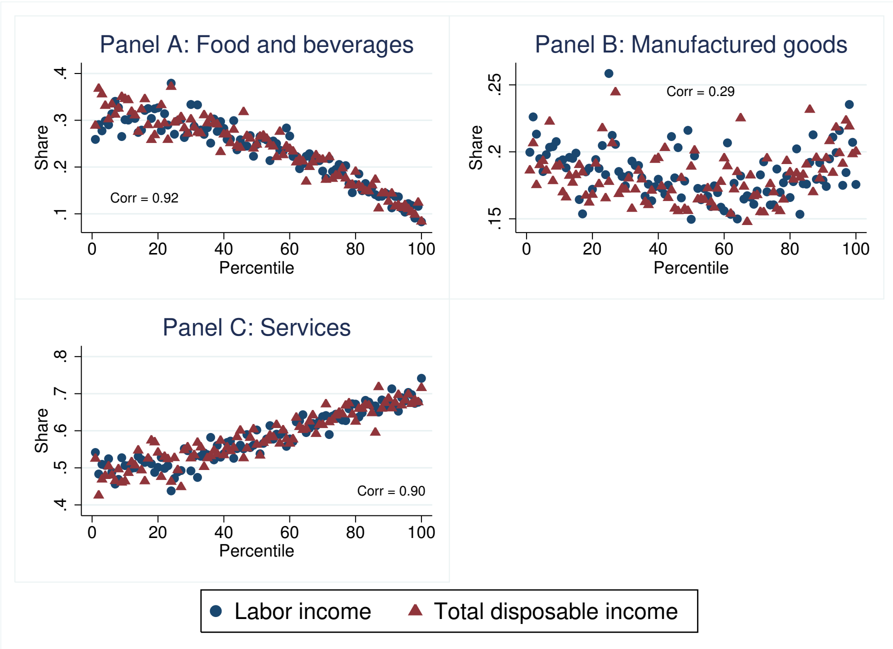
Code	Consumption division	Income percentile					Average
		P10	P25	P50	P75	P90	
Panel A: Food and beverages							
01	Food and non-alcoholic beverages	0.325	0.276	0.232	0.160	0.110	0.186
02	Alcoholic beverages, tobacco	0.021	0.020	0.016	0.021	0.016	0.018
	Total	0.346	0.295	0.248	0.181	0.126	0.204
Panel B: Manufactures							
03	Clothing and footwear	0.047	0.043	0.037	0.032	0.033	0.035
04	Housing, water, electricity, gas and other fuels	0.100	0.098	0.110	0.078	0.068	0.087
05	Furnishings, household equipment	0.043	0.037	0.042	0.060	0.086	0.062
	Total	0.190	0.178	0.189	0.170	0.187	0.185
Panel D: Services							
04.1	Rentals	0.043	0.034	0.035	0.053	0.062	0.053
06	Health	0.036	0.076	0.094	0.072	0.069	0.072
07	Transport	0.093	0.137	0.122	0.165	0.159	0.156
08	Communication	0.050	0.058	0.069	0.063	0.044	0.052
09	Recreation and culture	0.058	0.053	0.064	0.115	0.133	0.086
10	Education	0.088	0.053	0.063	0.032	0.051	0.046
11	Restaurants and hotels	0.045	0.046	0.043	0.069	0.078	0.067
12	Miscellaneous goods and services	0.052	0.070	0.073	0.080	0.090	0.078
	Total	0.464	0.527	0.563	0.649	0.687	0.611

NOTES: This table presents the share in consumption expenditures for households in selected percentiles of the income distribution, considering the 12 division of expenditure groups. Codes corresponds to the 12 divisions in the Classification of Individual Consumption by Purpose (COICOP). Panel A presents the decomposition for Food and Beverages categories. Panel B presents the decomposition for Manufactures, housing and utilities. Panel C presents the decomposition for Services. Each column denotes percentiles 10, 25, 50, 75 and 90, and average consumption, respectively.

3.A.2 *Consumption Patterns Across Labor Income Distribution*

Figure 3.A.1 compares consumption expenditures between labor income and total income distributions. As can be seen, both present a similar picture, in which low-income households (measured either by labor income or total income) spent a larger fraction of their income in food and beverages, while richer households spent more on services. While both distributions are closely correlated for those goods (above 90 percent), larger differences are presented for manufactured goods, in which the correlation is just 23 percent.

FIGURE 3.A.1: Consumption patterns across: aggregate vs labor income distribution



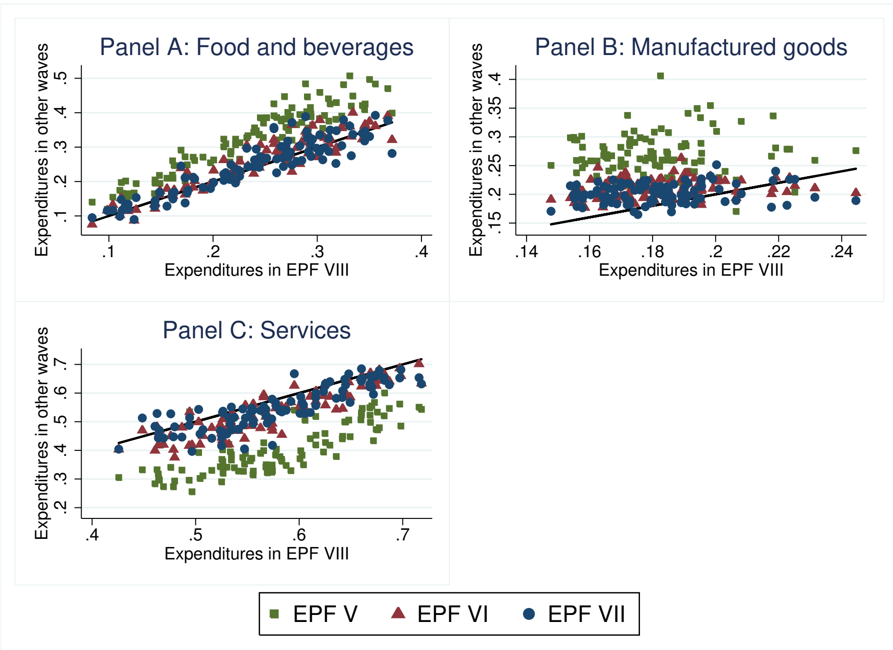
NOTES: This figure compares the share in consumption expenditures for households in each percentile of the income distribution vs percentiles in labor income distribution, considering three aggregate expenditure groups. On every panel, each dot/triangle corresponds to a percentile.

3.A.3 *Consumption Patterns Across Different Waves of the Consumption Expenditure Survey*

Figure 3.A.2 compares the distribution of consumption expenditures across different waves of the Consumption Expenditure Survey, EPF. The baseline sample corresponds to EPF VIII of the year 2017 (x-axis on each panel), while the alternative samples corresponds to EPF V of 1996 (green squares on each panel), EPF VI of 2006 (red triangles on each panel), and EPF VII of 2014 (blue dots on each panel). On each panel, every point shows the expenditure share in the respective kind of good in EPF VIII against other waves of the survey. As can

be seen, with the exception of manufactured goods, the expenditure patterns documented in the main text are relatively stable over time. In the case of food and beverages, correlations with respect to the baseline year are above 0.9, while for services they are above 0.8.[26] Note that the largest differences in levels are observed with respect to EPF V, because of the distance in time with EPF VIII (20 years). In particular, food and beverages account for a greater fraction of expenditures in EPF V, while the opposite happens in services.

FIGURE 3.A.2: Consumption patterns across different waves of the Consumption Expenditure Survey



NOTES: This figure compares the share in consumption expenditures for households in each percentile of the income distribution between different waves of the Consumption Expenditure Survey (EPF). On every panel, each point corresponds to a percentile. Black solid line denotes 90 degree line.

[26]More precisely, the correlation of food are 0.93, 0.94 and 0.91 for EPF V, EPF VI, and EPF VII, respectively. For services, those correlations are 0.83, 0.91 and 0.86.

3.A.4 *Consumption Patterns in Other Emerging Economies*

Table 3.A.2 presents descriptive statistics on consumption expenditures across the income distribution in 90 emerging economies. Panel A shows statistics for the full set of countries. Panel B shows statistics for commodity dependent economies (55 countries) defined by UNCTAD (2019), while Panel C shows statistics for non-dependent economies (35 countries). As can be noticed, the decreasing (increasing) patterns in consumption expenditures in food (services) across the income distribution is stable across these different sets of countries.

Table 3.A.2: Consumption expenditure across the income distribution–Emerging economies

	Food and beverages				Manufactures				Services			
	Lowest	Low	Middle	Higher	Lowest	Low	Middle	Higher	Lowest	Low	Middle	Higher
Panel A: All countries												
Mean	0.600	0.491	0.372	0.242	0.206	0.226	0.232	0.211	0.194	0.283	0.396	0.547
Median	0.615	0.483	0.352	0.193	0.200	0.222	0.229	0.205	0.188	0.291	0.400	0.593
Std. Dev.	0.106	0.111	0.140	0.179	0.062	0.058	0.072	0.121	0.083	0.096	0.138	0.200
Panel B: Commodity dependent countries												
Mean	0.605	0.488	0.370	0.232	0.208	0.231	0.234	0.198	0.186	0.281	0.395	0.570
Median	0.616	0.479	0.348	0.178	0.210	0.229	0.233	0.191	0.183	0.292	0.406	0.625
Std. Dev.	0.097	0.110	0.151	0.193	0.063	0.060	0.074	0.113	0.071	0.092	0.139	0.201
Panel C: Non-commodity dependent countries												
Mean	0.593	0.494	0.374	0.255	0.202	0.219	0.229	0.230	0.205	0.286	0.397	0.515
Median	0.612	0.485	0.357	0.223	0.196	0.206	0.209	0.210	0.206	0.291	0.391	0.547
Std. Dev.	0.121	0.112	0.123	0.160	0.062	0.055	0.069	0.129	0.100	0.104	0.139	0.198

NOTES: This table presents descriptive statistics on the consumption expenditure shares across the income distribution in 90 emerging economies. Each column present expenditure shares in food and beverages, manufactured goods and services. Each consumption group is separated across the income distribution according to the World Bank's Global Consumption Database: the lowest consumption segment corresponds to the bottom half of the global distribution, or the 50th percentile and below; the low consumption segment to the 51th-75th percentiles; the middle consumption segment to the 76th-90th percentiles; and the higher consumption segment to the 91st percentile and above.

3.B MODEL DERIVATIONS

3.B.1 *Households' Problem*

This section summarizes the optimization problems of households and their optimality conditions.

3.B.1.1 Intratemporal Consumption Allocation

The expenditure minimization problem of household h is

$$\min_{C_{jt}^h} \sum_{j=1}^{N} P_{jt}C_{jt}^h + \lambda_t^h \left[1 - \sum_{j=1}^{N} (\omega_j (C_t^h)^{\varepsilon_j})^{\frac{1}{\sigma}} (C_{jt}^h)^{\frac{\sigma-1}{\sigma}} \right],$$

where the Lagrange multiplier λ_t^h is household specific.

The first order condition for any good j reads as

$$P_{jt}C_{jt}^h = \lambda_t^h \left(\frac{\sigma-1}{\sigma} \right) (\omega_j (C_t^h)^{\varepsilon_j})^{\frac{1}{\sigma}} (C_{jt}^h)^{\frac{\sigma-1}{\sigma}}.$$

Using this condition and the definition of the aggregator, total expenditure is given by

$$E_t^h = \sum_j P_{jt}C_{jt}^h = \lambda_t^h \left(\frac{\sigma-1}{\sigma} \right),$$

so the expenditure share of good j for household h is

$$s_{jt}^h \equiv \frac{P_{jt}C_{jt}^h}{E_t^h} = (\omega_j (C_t^h)^{\varepsilon_j})^{\frac{1}{\sigma}} (C_{jt}^h)^{\frac{\sigma-1}{\sigma}},$$

which implies that each summand in the consumption aggregator corresponds to the equilibrium expenditure share.

Using the latter expression, we can get the demand for each sectoral good and its expenditure share

$$C_{jt}^h = \omega_j \left(\frac{P_{jt}}{P_t^h} \right)^{-\sigma} (C_t^h)^{\varepsilon_j+\sigma}$$
$$s_{jt}^h = \omega_j \left(\frac{P_{jt}}{P_t^h} \right)^{1-\sigma} (C_t^h)^{\varepsilon_j-(1-\sigma)},$$

where I use the fact that P_t^h is the relevant price index for household h in period t, such that $E_t^h = P_t^h C_t^h$. By replacing this demand in the aggregator, the price index for the household is defined by

$$P_t^h = \left[\sum_{j=1}^{N} \omega_j P_{jt}^{1-\sigma} (C_t^h)^{\varepsilon_j+\sigma-1} \right]^{\frac{1}{1-\sigma}}.$$

Defining $\vartheta_j \equiv \frac{1-\sigma}{\varepsilon_j}$, a more intuitive expression for the price index, that only depends on observables, can be written as follows

$$P_t^h = \left[\sum_{j=1}^{N} (\omega_j P_{jt}^{1-\sigma})^{\vartheta_j} (s_{jt}^h {E_t^h}^{1-\sigma})^{1-\vartheta_j} \right]^{\frac{1}{1-\sigma}}$$

On the other hand, the expenditure function reads as

$$E_t^h = \left[\sum_{j=1}^{N} \omega_j P_{jt}^{1-\sigma} (C_t^h)^{\varepsilon_j} \right]^{\frac{1}{1-\sigma}}$$

Importantly, note that the elasticity of expenditure with respect to aggregate consumption is

$$\begin{aligned} \eta_C^E \equiv \frac{\partial E_t^h}{\partial C_t^h} \frac{C_t^h}{E_t^h} &= \frac{1}{1-\sigma} \sum_{j=1}^{N} \varepsilon_j \omega_j \left(\frac{P_{jt}}{E_t^h} \right)^{1-\sigma} (C_t^h)^{\varepsilon_j} \\ &= \frac{1}{1-\sigma} \sum_{j=1}^{N} s_{jt}^h \varepsilon_j = \frac{\bar{\varepsilon}_t^h}{1-\sigma}, \end{aligned} \tag{3.18}$$

where the third equality comes from the definition of the expenditure share (using expenditures instead of the aggregate price), and where $\bar{\varepsilon}_t^h \equiv \sum_{j=1}^{N} s_{jt}^h \varepsilon_j$ is the expenditure-weighted average of income elasticity, which is time-varying and household-dependent.

For comparison, note that in the homothetic case, $\varepsilon_j = 1 - \sigma$ for every j, and all the previous conditions collapse to the familiar CES demand system, in which the expenditure share of each good and the CPI do not depend on the level of consumption. Also note that the elasticity of expenditure equals one in every period and for every household.

3.B.1.2 Labor Supply Allocation

The optimization problem of each household consists in maximizing total labor income subject to the labor supply aggregator

$$\max_{N_{jt}^h, N_{ct}^h} \sum_{j=1}^{N} W_{jt} N_{jt}^h + W_{ct} N_{ct}^h + \lambda_t^h \left[N_t^h - \left(\sum_{j=1}^{N} N_{jt}^{\frac{1+\varrho^h}{\varrho^h}} + N_{ct}^{\frac{1+\varrho^h}{\varrho^h}} \right)^{\frac{\varrho^h}{1+\varrho^h}} \right].$$

Note that this formulation admits the possibility of differential wages across sectors in the economy.

The first order condition for any sector is

$$W_{jt} = \left(\frac{W_{jt}}{\lambda_t^h} \right)^{\varrho^h} N_t^h.$$

Replacing these supplies in the aggregator we get

$$\lambda_t^h \equiv W_t^h = \left(\sum_{j=1}^{N} W_{jt}^{1+\varrho^h} + W_{ct}^{1+\varrho^h} \right)^{\frac{1}{1+\varrho^h}},$$

which defines a household specific wage index. With this, sectoral labor supply for any sector reads as

$$N_{jt}^h = \left(\frac{W_{jt}}{W_t^h} \right)^{\varrho^h} N_t^h.$$

3.B.1.3 Intertemporal Problem for the Unconstrained Household

The Lagrangian of this problem can be written as

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ \frac{1}{1-\varsigma}(C_t^u)^{1-\varsigma} - \frac{\kappa^u}{1+\varphi}(N_t^u)^{1+\varphi}$$

$$+ \lambda_t^u \Bigg[W_t^u N_t^u + R_{t-1}\frac{B_{t-1}}{1-\lambda} + S_t R_{t-1}^* \frac{B_{t-1}^*}{1-\lambda} + \frac{D_t}{1-\lambda}$$

$$- E_t^u - \frac{B_t}{1-\lambda} - S_t \frac{B_t^*}{1-\lambda} \Bigg] \Bigg\},$$

where λ_t^u is the Lagrange multiplier. The first order conditions of this problem are

$$C_t^u: \qquad (C_t^u)^{-\varsigma} = \lambda_t^u \frac{\partial E(C_t^u)}{\partial C_t^u}$$

$$B_t: \qquad \lambda_t^u = \beta R_t \mathbb{E}_t[\lambda_{t+1}^u]$$

$$B_t^*: \qquad \lambda_t^u = \beta R_t^* \mathbb{E}_t \left[\lambda_{t+1}^u \frac{S_{t+1}}{S_t} \right]$$

$$N_t^u: \qquad \kappa^u (N_t^u)^{\varphi} = \lambda_t^u W_t^u.$$

From equation (3.18) and the fact that $E_t^h = P_t^h C_t^h$, we get $\frac{\partial E_t^h}{\partial C_t^h} = P_t^h \frac{\bar{\varepsilon}_t^h}{1-\sigma}$, which implies that Lagrange multiplier can be written as $\lambda_t^u = (C_t^u)^{-\varsigma} \left(\frac{1-\sigma}{\bar{\varepsilon}_t^u} \right) \frac{1}{P_t^u}$. Replacing in the rest of optimality conditions we get

$$1 = \beta R_t \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^u}{C_t^u} \right)^{-\varsigma} \frac{\bar{\varepsilon}_t^u}{\bar{\varepsilon}_{t+1}^u} \frac{P_t^u}{P_{t+1}^u} \right\}$$

$$1 = \beta R_t^* \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^u}{C_t^u} \right)^{-\varsigma} \frac{\bar{\varepsilon}_t^u}{\bar{\varepsilon}_{t+1}^u} \frac{P_t^u}{P_{t+1}^u} \frac{S_{t+1}}{S_t} \right\}$$

$$\frac{W_t^u}{P_t^u} = \kappa^u (C_t^u)^{\varsigma} (N_t^u)^{\varphi} \frac{\overline{\varepsilon}_t^u}{1-\sigma},$$

as in the main text.

3.B.1.4 Optimization Problem for the Restricted Household

As mentioned in the main text, the restricted household must solve for consumption and labor supply in a period-by-period basis. The Lagrangian is

$$\mathcal{L} = \frac{1}{1-\varsigma}(C_t^r)^{1-\varsigma} - \frac{\kappa^r}{1+\varphi}(N_t^r)^{1+\varphi} + \lambda_t^r \left[W_t^r N_t^r - E_t^r \right].$$

The first order conditions are

$$\begin{aligned} C_t^r &: & (C_t^r)^{-\varsigma} &= \lambda_t^r \frac{\partial E(C_t^r)}{\partial C_t^r} \\ N_t^r &: & \kappa^r (N_t^r)^{\varphi} &= \lambda_t^r W_t^r. \end{aligned}$$

As in the unconstrained problem, the Lagrange multiplier can be expressed as $\lambda_t^r = (C_t^r)^{-\varsigma} \left(\frac{1-\sigma}{\overline{\varepsilon}_t^r} \right) \frac{1}{P_t^r}$, so the labor supply equation is

$$\frac{W_t^r}{P_t^r} = \kappa^r (C_t^r)^{\varsigma} (N_t^r)^{\varphi} \frac{\overline{\varepsilon}_t^r}{1-\sigma}.$$

Together with the budget constraint, this equation characterize the optimality conditions of the constrained household.

3.B.2 *Producers' Problem*

3.B.2.1 Calvo Pricing

Optimal price. The pricing problem is given by

$$\max_{\widetilde{P}_{jt}(z)} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_j \beta)^{\tau} \left[\widetilde{P}_{jt}(z) Y_{jt+\tau|t}(z) - MC^n_{jt+\tau} Y_{jt+\tau|t}(z) \right]$$

subject to

$$Y_{jt+\tau|t}(z) = \left(\frac{P_{jt}(z)}{P_{jt+\tau}} \right)^{-\varepsilon} Y_{jt+\tau},$$

where MC^n denotes nominal marginal cost.

The first order condition reads as

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_j \beta)^{\tau} Y_{jt+\tau|t}(z) \left[\widetilde{P}_{jt}(z) - \frac{\varepsilon}{\varepsilon - 1} MC^n_{jt+\tau} \right] = 0.$$

Replacing the constraint in the objective function and re-ordering, this expression can be written as

$$\begin{aligned} \widetilde{P}_{jt}(z) &= \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{\sum_{\tau=0}^{\infty} (\theta_j \beta)^{\tau} MC^n_{jt+\tau} P^{\varepsilon}_{jt+\tau} Y_{jt+\tau}}{\sum_{\tau=0}^{\infty} (\theta_j \beta)^{\tau} P^{\varepsilon}_{jt+\tau} Y_{jt+\tau}} \\ \frac{\widetilde{P}_{jt}(z)}{P_t} &= \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{\sum_{\tau=0}^{\infty} (\theta_j \beta)^{\tau} MC_{jt+\tau} \Pi_{t,t+\tau} \Pi^{\varepsilon}_{j,t,t+\tau} Y_{jt+\tau}}{\sum_{\tau=0}^{\infty} (\theta_j \beta)^{\tau} \Pi^{\varepsilon}_{j,t,t+\tau} Y_{jt+\tau}}. \end{aligned}$$

In the latter expression, $\Pi_{t,t+\tau}$ is the cumulative gross inflation between period t and $t + \tau$. This is, $\Pi_{t,t+\tau} = \frac{P_{t+\tau}}{P_t}$.

We can work the numerator and denominator of the latter expression, which have the following recursive forms

$$\begin{aligned} S_{jt} &= Y_{jt} MC_{jt} + \theta_j \beta \mathbb{E}_t [\Pi_{t+1} \Pi^{\varepsilon}_{jt+1} S_{jt+1}] \\ F_{jt} &= Y_{jt} + \theta_j \beta \mathbb{E}_t [\Pi^{\varepsilon}_{jt+1} F_{jt+1}]. \end{aligned}$$

Therefore, the optimal sectoral price is given by

$$\widetilde{P}_{jt} = \frac{\varepsilon}{\varepsilon - 1} \frac{S_{jt}}{F_{jt}}.$$

The evolution of sectoral price. Recall the definition of sectoral price, $P_{jt} = \left(\int_0^1 P_{jt}(z)^{1-\varepsilon} dz \right)^{\frac{1}{1-\varepsilon}}$. By the Calvo property, we can partition the space of firms between those who can update their price and those that can not. Therefore, the sectoral price is reads as

$$P_{jt} = \left[(1-\theta_j)\widetilde{P}_{jt}^{1-\varepsilon} + \theta_j P_{jt-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$
$$1 = (1-\theta_j)\left(\frac{\widetilde{P}_{jt}}{P_{jt}} \right)^{1-\varepsilon} + \theta_j \Pi_{jt}^{\varepsilon-1}.$$

Price dispersion. The aggregation of production of each atomistic monopolist derives in a price dispersion term given by nominal rigidities. As it is well known, this term is given by $\Delta_{jt} = \int_0^1 \left(\frac{P_{jt}(z)}{P_{jt}} \right)^{-\varepsilon} dz$. Again, by using the properties of Calvo pricing, we can partition the space between firms that can update their price and those that can not

$$\begin{aligned}
\Delta_{jt} &= \int_0^{\theta_j} \left(\frac{P_{jt-1}(z)}{P_{jt}} \right)^{-\varepsilon} dz + \int_{\theta_j}^1 \left(\frac{\widetilde{P}_{jt}}{P_{jt}} \right)^{-\varepsilon} dz \\
&= \Pi_{jt}^{\varepsilon} \theta_j \int_0^1 \left(\frac{P_{jt-1}(z)}{P_{jt-1}} \right)^{-\varepsilon} dz + (1-\theta_j) \left(\frac{\widetilde{P}_{jt}}{P_{jt}} \right)^{-\varepsilon} \\
&= (1-\theta_j) \left(\frac{\widetilde{P}_{jt}}{P_{jt}} \right)^{-\varepsilon} + \theta_j \Pi_{jt}^{\varepsilon} \Delta_{jt-1}.
\end{aligned}$$

Note that price dispersion naturally appears when aggregating individual firms' decisions at the sectoral level. For example, the optimal demand for labor of a firm z in sector j is

$$L_{jt}(z) = \alpha_j \frac{MC_{jt} Y_{jt}(z)}{W_{jt}},$$

which depends on the level of output of this firm. Aggregating this expression across all firms in the sector, we get

$$L_{jt} \equiv \int_0^1 L_{jt}(z)dz = \int_0^1 \alpha_j \frac{MC_{jt}Y_{jt}(z)}{W_{jt}} dz = \alpha_j \frac{MC_{jt}}{W_{jt}} \int_0^1 Y_{jt}(z)dz.$$

By replacing the demand of the sectoral aggregator given by (3.13), we get

$$L_{jt} = \alpha_j \frac{MC_{jt}Y_{jt}}{W_{jt}} \int_0^1 \left(\frac{P_{jt}(z)}{P_{jt}}\right)^{-\varepsilon} dz = \alpha_j \frac{MC_{jt}Y_{jt}}{W_{jt}} \Delta_{jt}.$$

Analogous expressions are obtained for the demand of other productive inputs.

3.B.3 *Baseline Calibration*

Table 3.B.1 presents the calibration of predetermined parameters of the model.

Table 3.B.1: Predetermined parameters

Variable	Parameter	Value
Panel A: Households		
ς	Risk aversion	1
φ	Frisch elasticity	1
β	Discount factor	$R = 1.058$
ω_j	Taste shifter	Avg. consumption expenditures
λ	Share constrained agents	0.3
ϱ^h	Elasticity of substitution (labor)	1
χ_j^h	Labor supply shifter	1
Panel B: Sectors		
ε	Elasticity of substitution (varieties)	10
α_j	Labor share	IO (2017)
μ_j	Materials share	IO (2017)
γ_{ij}	Production linkages	IO (2017)
θ_j	Price rigidities (Calvo)	Frequency of price adjustment
α_c	Labor share (commodity)	IO (2017)
μ_c	Materials share (commodity)	IO (2017)
ψ_j	Share on exports	IO (2017)
Panel C: Monetary policy and aggregates		
ϕ_π	Weight of inflation	1.5
ϕ_y	Weight of GDP	0.125
R^w	World interest rate	1.045
$\bar{b}$	Interest rate premium	TB/GDP=0.08
ρ_c	Persistence commodity price	0.7123

NOTES: This table presents the set of predetermined parameters of the model. See the main text of Section 2.4.1 for details.

3.C ADDITIONAL QUANTITATIVE RESULTS

3.C.1 *The Role of Non-homotheticities and Heterogeneity*

Table 3.C.1 compares the response of aggregate and household-specific variables to a commodity price shock, distinguishing between the impact and the cumulative effect in a horizon of 20 quarters. The table presents the ratio of the response of different specifications with respect to the baseline model with two agents and non-

homothetic preferences. Columns TANK refer to a version with two agents and homothetic preferences. Columns RANK (RANK+NH) refers to a version with a representative agent and homothetic (non-homothetic) preferences.

Given that in our baseline calibration non-homotheticicities are more relevant than heterogeneity, Figures 3.C.1 and 3.C.2 compare aggregate and household specific responses to a commodity price shock, between the baseline model and the one with homothetic preferences. In the case of aggregate variables, the impact response of total consumption is 80 percent larger in the homothetic model, with a persistent effect over time. In the case of output, the difference is 50 percent stronger in the homothetic model, and differences with respect the non-homothetic version remain relatively constant over time. Interestingly, there are no large differences, neither for the trade balance nor the real exchange rate, between the two versions of the model. This means that movements in the trade balance are cancelled in a similar proportion to movements in aggregate output, which imply a similar response in the real exchange rate in both versions of the model.

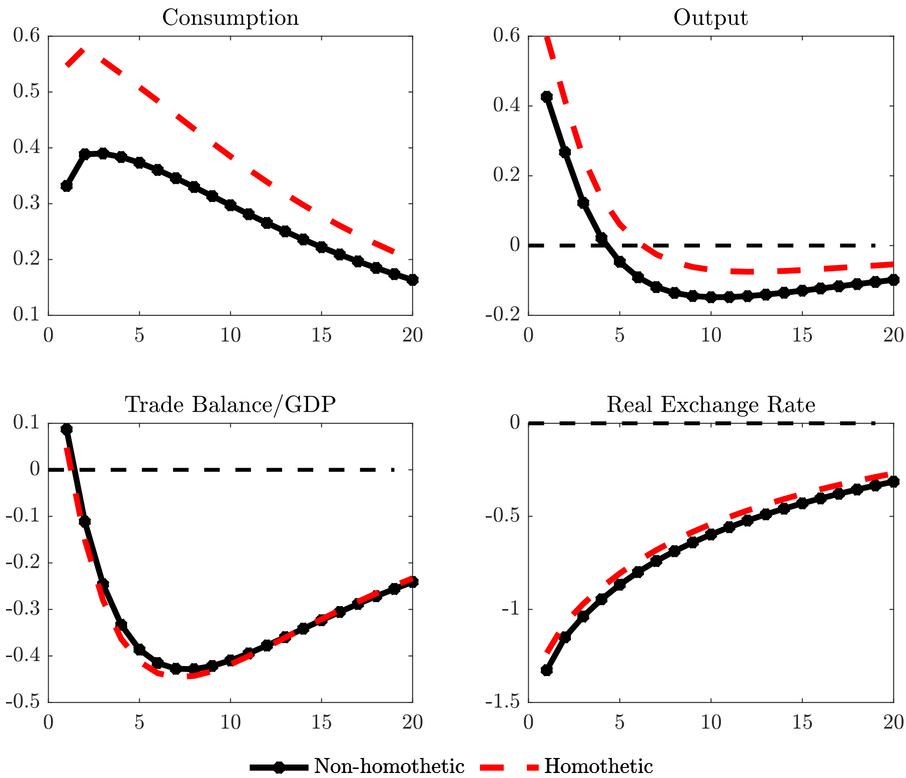
Figure 3.C.2 compares the responses of consumption and hours of each household between the non-homothetic and the homothetic version of the model. In line with the analytical results of section 3.3.5, non-homotheticicities dampens the response of consumption for both agents. On impact, the homothetic model almost doubles the response for both households: while the consumption of the unconstrained moves from 0.27 to 0.47 percent, the consumption of the restricted household changes from 0.36 to 0.63 percent. In the case of hours of the unconstrained, while on impact the homothetic model has a response four times larger (0.16 vs 0.04 percent), the behavior afterwards stabilizes with a slightly stronger decrease in the non-homothetic model. For the restricted household, hours in the homothetic version do not respond given the log-utility calibration.

Table 3.C.1: Response to a commodity price shock under different model specifications

	Impact effect			Cumulative effect		
	TANK	RANK	RANK + NH	TANK	RANK	RANK + NH
Panel A: Aggregate variables						
Consumption	1.648	1.510	0.913	1.340	1.315	0.973
Output	1.404	1.398	1.020	-0.548	-0.395	1.165
Trade Balance	0.550	1.004	1.396	1.035	1.038	1.004
Real Exchange Rate	0.930	0.906	0.977	0.912	0.920	1.009
Panel B: Household-specific variables						
Consumption (u)	1.667	1.671	1.019	1.397	1.207	0.871
Hours (u)	2.247	1.701	0.622	0.834	0.619	0.785
Consumption (r)	1.624	1.519	0.952	1.366	1.333	0.979
Hours (r)	0.000	0.000	0.971	0.000	0.000	1.000
Expenditure gap	1.699			1.079		
Consumption gap	1.465			1.379		

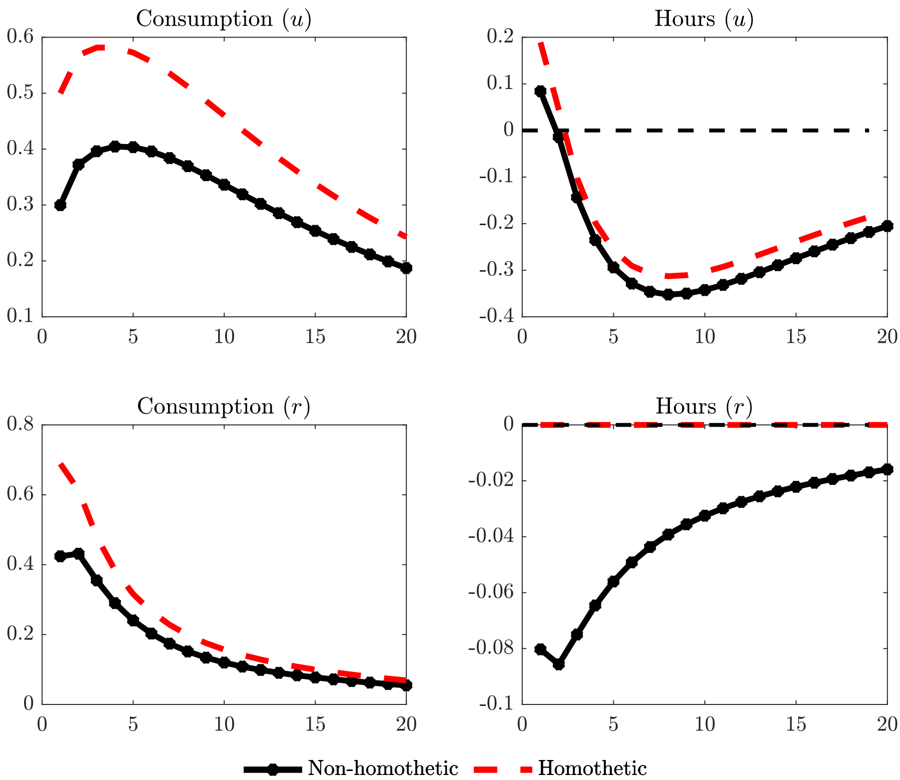
NOTES: This table compares the impact and cumulative (in a horizon of 20 quarters) response of aggregate and household-specific variables to a commodity price shock. Columns TANK correspond to a specification with two agents and homothetic preferences. Columns RANK (RANK+NH) correspond to a specification with a representative agent and (non-)homothetic preferences. All numbers corresponds to the response of a variable-model relative to the baseline model with two agents and non-homothetic preferences.

FIGURE 3.C.1: The role of non-homotheticities: Aggregate responses



NOTES: This figure compares the impulse-response function of aggregate variables after a 10% increase in the commodity price between the model with and without non-homotheticities. Horizontal axes correspond to quarters. Vertical axes corresponds to percentage deviations with respect to steady-state, except for trade balance (measured in deviations).

FIGURE 3.C.2: The role of non-homotheticities: Household-specific responses



NOTES: This figure compares the impulse-response function of selected household variables after a 10% increase in the commodity price between the model with and without non-homotheticities. Horizontal axes correspond to quarters. Vertical axes corresponds to percentage deviations with respect to steady-state.

3.C.2 *Sensitivity Analysis*

This section presents additional sensitivity results. In particular, it shows cumulative responses of selected aggregate and household-specific variables to a commodity price shock, comparing responses under alternative calibrations and between the homothetic and non-homothetic version of the model. Table 3.C.2 presents results for aggregate variables, while Table 3.C.3 presents results for selected household-specific variables.

The results can be summarized as follows

- An increase in the degree of labor mobility makes the economy less responsive. This happens both at the aggregate and microeconomic levels. This result are particularly important for the trade balance and the real exchange rate.

- An increase in the degree of market incompleteness does not generate large differences in aggregate terms. This result is in line with the discussion in main text: by assumption there are no large differences in income across households because they have the same degree of labor mobility and work in the same places. At the microeconomic level, an increase in market incompleteness is more beneficial for unconstrained households which now receive a larger amount of income in per capita terms.

- An increase in the elasticity of substitution in the consumption basket does not generate large differences in the real exchange rate or the trade balance. At the microeconomic level, this increment implies a lower response in consumption of each household but a higher expenditure level. These changes are particularly large for the unconstrained household, implying a reduction in the inequality between households.

- Finally, an increment in the income elasticity of services generates aggregate responses only in consumption and output. The trade balance and the real exchange rate do not react as much, because the services sector is relatively not tradable (does not import much materials and is mostly domestically consumed). At the household level, this increment implies a decrease in the level of consumption expenditures by moving towards more expensive goods.

Table 3.C.2: Sensitivity–Aggregate variables

	Consumption		Output		Trade Balance		Real Exchange Rate	
	H	NH	H	NH	H	NH	H	NH
Panel A: Degree of labor mobility								
$\varrho = 0$	23.329	16.200	0.293	-5.732	-21.193	-20.178	-38.858	-42.347
$\varrho = 1$	7.635	5.697	0.602	-1.097	-6.471	-6.251	-11.886	-13.035
$\varrho = 1.3$	4.733	3.564	0.388	-0.639	-3.997	-3.867	-7.416	-8.121
Panel B: Degree of market incompleteness								
$\lambda = 0$	7.489	5.541	0.434	-1.278	-6.491	-6.274	-11.992	-13.146
$\lambda = 0.3$	7.635	5.697	0.602	-1.097	-6.471	-6.251	-11.886	-13.035
$\lambda = 0.5$	7.803	5.874	0.795	-0.894	-6.448	-6.226	-11.764	-12.913
Panel C: Elasticity of substitution								
$\sigma = 0$	7.670	5.306	0.590	-1.447	-6.514	-6.213	-12.002	-13.295
$\sigma = 0.27$	7.635	5.697	0.602	-1.097	-6.471	-6.251	-11.886	-13.035
$\sigma = 0.6$	7.595	6.197	0.616	-0.652	-6.421	-6.301	-11.752	-12.709
Panel D: Income-elasticity of services								
$\varepsilon_s = 1$	7.635	5.885	0.602	-0.909	-6.471	-6.250	-11.886	-12.845
$\varepsilon_s = 1.11$	7.635	5.697	0.602	-1.097	-6.471	-6.251	-11.886	-13.035
$\varepsilon_s = 1.3$	7.635	5.400	0.602	-1.387	-6.471	-6.244	-11.886	-13.306

NOTES: This table compares the cumulative response in a horizon of 20 quarters for selected aggregate variables after a 10% increase in the commodity price. Trade balance is normalized by GDP. All numbers correspond to percentage deviations with respect to steady-state, except the trade balance which is in deviations. Each set of columns compares the response in the homothetic model (H) vs the non-homothetic model (NH). Panel A compares responses for different degrees of labor mobility (baseline corresponds to $\varrho = 1$). Panel B compares responses for different degrees of market incompleteness, which corresponds to the fraction of restricted households in the economy (baseline corresponds to $\lambda = 0.3$). Panel C compares responses for different degrees of complementarity between consumption goods (baseline corresponds to $\sigma = 0.27$). Panel D compares responses for different degrees of income-elasticity of services (baseline corresponds to $\varepsilon_s = 1.11$).

Table 3.C.3: Sensitivity–Household-specific variables

	Unconstrained Consumption		Restricted Consumption		Consumption Gap		Unconstrained Hours		Restricted Hours	
	H	NH	H	NH	H	NH	H	NH	H	NH
Panel A: Degree of labor mobility										
$\varrho = 0$	27.015	18.723	12.346	8.503	11.486	8.296	-14.669	-17.276	0.000	-2.512
$\varrho = 1$	8.671	6.206	4.503	3.296	3.218	2.334	-4.169	-4.999	0.000	-0.781
$\varrho = 1.3$	5.367	3.856	2.812	2.072	1.970	1.430	-2.555	-3.066	0.000	-0.480
Panel B: Degree of market incompleteness										
$\lambda = 0$	7.489	5.404	4.394	3.228	2.565	1.851	-3.096	-3.924	0.000	-0.782
$\lambda = 0.3$	8.671	6.206	4.503	3.296	3.218	2.334	-4.169	-4.999	0.000	-0.781
$\lambda = 0.5$	10.052	7.132	4.625	3.368	3.838	2.811	-5.426	-6.257	0.000	-0.781
Panel C: Elasticity of substitution										
$\sigma = 0$	8.710	8.054	4.526	4.282	3.230	2.785	-4.184	-5.237	0.000	-1.116
$\sigma = 0.27$	8.671	6.206	4.503	3.296	3.218	2.334	-4.169	-4.999	0.000	-0.781
$\sigma = 0.6$	8.627	3.638	4.477	1.905	3.204	1.536	-4.150	-4.671	0.000	-0.404
Panel D: Income-elasticity of services										
$\varepsilon_s = 1$	8.671	6.767	4.503	3.610	3.218	2.484	-4.169	-4.956	0.000	-0.759
$\varepsilon_s = 1.11$	8.671	6.206	4.503	3.296	3.218	2.334	-4.169	-4.999	0.000	-0.781
$\varepsilon_s = 1.3$	8.671	5.409	4.503	2.857	3.218	2.100	-4.169	-5.073	0.000	-0.824

NOTES: This table compares the cumulative response in a horizon of 20 quarters for selected household-specific variables after a 10% increase in the commodity price. All numbers correspond to percentage deviations with respect to steady-state, except the consumption gap which is in deviations. Each set of columns compares the response in the homothetic model (H) vs the non-homothetic model (NH). Panel A compares responses for different degrees of labor mobility (baseline corresponds to $\varrho = 1$). Panel B compares responses for different degrees of market incompleteness, which corresponds to the fraction of restricted households in the economy (baseline corresponds to $\lambda = 0.3$). Panel C compares responses for different degrees of complementarity between consumption goods (baseline corresponds to $\sigma = 0.27$). Panel D compares responses for different degrees of income-elasticity of services (baseline corresponds to $\varepsilon_s = 1.11$).

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