Universitat Autònoma de Barcelona

Essays on Fiscal Policy

José de Anchieta Semedo Neves

Essays on Fiscal Policy

Author: José de Anchieta Semedo Neves

Supervisors: Prof. Albert Marcet
Prof. Francesc Obiols-Homs

Departament d'Economia i d'Història Econòmica
Universitat Autònoma de Barcelona
International Doctorate in Economic Analysis - IDEA
Spain

June 29th, 2021.

In memoriam of my mother Anita Semedo Neves and
my grandmother Olindina da Conceição Neves.

Dedicated to my family, especially my parents Anita and José, my wife Marizete,
my sons Anita y Leonardo, and my pet Marvin.

Acknowledgements

First of all, I would like to thank God for all the opportunities during my life journey.

Thanks to my parents Anita and José, for life principles and opening my eyes to education's importance as the main way to change our life history. My parents are heroes in supporting their three sons' education, an opportunity that they did not have. They have an incomplete primary education level but succeed as doctors in educating their sons.

I am very grateful to the Brazilian National Treasury for approving the doctoral license to improve my professional capability. I would also like to thank the IDEA program for the partial financial wave in the tuition fees to 2014/2015 and 2015/2016.

Thanks to professors Albert Marcet and Francesc Obiols-Homs for the advising and availability. During my stay in Barcelona, you opened your office doors to help and give me important feedback. After coming to Brazil, I know that it became more challenging to appreciate your enhancing guidance more closely. However, there is something that I fixed in my mind: the relevance of starting doing simple things before advance to a more complex problem. I thank professor Ada Ferrer-i-Carbonel for helping me with the methodological approach to the third chapter. And my special thanks to all the faculty members of IDEA, especially the professors of the first two years of master's classes.

I could not forget to mention the kind helps of Àngels López García and Mercè Vicente during the whole Master and Ph.D. track. Even before moving to Barcelona, I can not count the number of e-mails I sent them asking for help. And they always gave me prompt clarifying answers. Moltes gràcies!

The decision to apply to a challenging Ph.D. program after eight years apart from the academic environment was not easy. Thanks to professors Arlindo Villaschi Filho, Marcelo Savino Portugal, and Miriam de Magdala Pinto for all the guidance during my academic life. You always believed a lot in me and gave me pieces of advice for my personal life.

I am very proud to have friends that are like brothers in my life. Especially thanks to Sérgio Manhans, Rafael Estanislau, Daniel de Araujo e Borges, and Helano Borges Dias. I can not finish this without mention Leonardo Moreto Elias, a family friend that we met in Barcelona after a kind e-mail contacting me. We have had beautiful moments in Barcelona.

Let me register my family's enormous affection for the Catalan and Spanish people that were part of our stay in Cerdanyola del Vallès. When we decide to cite names, we incur the risk of forgetting someone important during this period. And many people were. Special thanks to Ana Carretero Ocaña, Emilio, and Oriol for their warmness. They were lovely from the beginning to the end of our stay and made our lives easier in Cerdanyola. We did great friends: Oscar, Cintia and Sofia; Cristian, Lidia, Sara and Julia; Vanessa, Cristian, Ivete and Nuria; Eva, Ascensión and Lucia; Ana, Ruben, Eleonor, Julian, Ema and Lara; Mossèn Joan Carles Montserrat; Maria Jose, Miguel Angel and Alex; Silvia, Àngel, Abril and Laia; teachers and monitors of Llar d'Infants Patufet; teachers and monitors of Escola Bressol Municipal Turonet; teacher Eulàlia and families of Tortugas (P3 - 2016/2017) and Pinguins (2017/2018) school groups in Escola Turó de Guiera; teachers Gemma Baños and Monica, and families with which we enjoy the Anita and Leonardo cos i veu courses from 2014 to 2018.

Some friends are like a family extension. We could not say something different to Cintia, Oscar, and Sofia; Maria Jose, Miguel Angel, and Alex. We had memorable meetings with Vanessa, Sergi, Ariadna, and Ona, including our visit to the Penedès bodegas. An enormous hug to our abuela Elena, an angel in our life, and her family (Elena, Jose, Sergio, and Miguel) for their big heart. Jaume, from Gosgast, for his tenderness with our pet Marvin and for the time we spent talking together about interesting and joking things. I devote a special thanks to Doctor Magda Ros Monllao for her lovely prenatal medical care. We thank the angel Doctor Corominas for the four years being our pediatrician in Spain, for his love and warmness with our babies, me, and my wife. Thank you all for being part of our life. These are people that we had a closer relation. Many other people were special during this stay in Cerdanyola, whether walking in the streets or stopping to talk about any topic that brings good people together. We love the city of Cerdanyola.

Finally, I do not have words to thank my family for their patience and love during these very challenging years. I love Mari, Anita, Leonardo, and Marvin!!! My life does not have any sense without you!

Contents

List of Figures

List of Tables

Introduction

In this thesis, I work on three independent essays concerning fiscal policy. Even though the motivation relates to the Brazilian challenges, the results serve as references to other developing economies. In the first chapter, I discuss policy responses in a deep recession context, assessing distinct fiscal rules. In the second chapter, I study optimal fiscal policy when a country experiences an abrupt increase in fiscal revenues due to a nonrenewable natural resource windfall. Chapter 3 explores the impact of public expenditures on citizens' well-being in distinct government action areas. Together, these chapters bring a contribution to an ample spectrum of fiscal policy discussions. In assessing distinct fiscal rules and their implications for policy decisions, I explore the relevance to open fiscal space to a government implement a countercyclical policy, even finding evidence that this policy is not desirable as a general rule. In addition, by studying optimal policies when dealing with a natural resource windfall, I show the importance of responsible government savings behavior to challenge uncertainty in intergenerational wealth management. Finally, studying the impact of public expenditures on people's well-being highlights that maybe governments are failing to use in an efficient way citizens' tax payments.

The main objective of the first chapter is to evaluate the dynamic of public expenditures composition under distinct fiscal rules in an economy that experiences a preference shock. The motivation grounds in the Brazilian implementation of a cap on real expenditure fiscal rule in 2016. I allow the government to spend on two categories of public expenditures: consumption goods and investment. Then, I consider two kinds of fiscal rules: a cap on real expenditure and a discretionary rule that responds to the output gap. Mixes of optimal fiscal policy with either optimal or sub-optimal monetary policy allow me to cover a couple of Ramsey's problems. Finally, I present the welfare results to the distinct fiscal environments and policy mix a government can choose. The results highlight the importance of reforms that permits a government to enhance its capability to implement a countercyclical fiscal policy. However, I show that this countercyclicality is not a general rule as a policy instrument.

The second chapter's main objective is to propose an alternative approach to analyze optimal fiscal policy in an environment where a country faces a positive primary revenue shock due to discovering and exploring a nonrenewable natural resource. The motivation comes from Brazil's discovery of oil in the pre-salt layer, announced in 2007. This paper solves an infinite-horizon stochastic Ramsey problem without assuming that the model turns out to a deterministic environment at some finite period in time. To this end, I apply the Parameterized Expectation Approach to approximate expectations before and after the windfall term. The baseline model predicts the relevance of precautionary saving during the windfall to cope with uncertainty in the natural resource revenue. The optimal policy suggests adopting an austere fiscal policy during almost a half period of the windfall. The reward is an attainable level of private and public consumption significantly higher than in the model without uncertainty during the transition to and in the long-run equilibrium.

The main objective of the third chapter is to assess the impact of public expenditures on people's happiness in the Brazilian States. I propose a two-step approach to work with cross-sectional surveys that, at successive releases, draw new and independent samples. In the first step, I regress individuals' self-reported life-satisfaction on objective variables. In the second step, I regress the States' averaged residuals of life satisfaction obtained from the first step on distinct categories of public expenditures and control variables. This paper contributes to the incipient literature that evaluates the relationship between public spending and life satisfaction using panel data. I find evidence that there is a negative (or at most null) impact of public expenditures on happiness.

Chapter 1

Optimal Composition of Public Expenditures in a Recession under Distinct Fiscal Rules

1.1 Introduction

The international financial crisis that emerged in the second semester of 2008 had severe negative externalities on the economic activity in the following years. Many countries worldwide started to launch packs of stimuli, including both fiscal and monetary policies measures. Brazil shared this behavior with other developing and developed countries, adopting anti-crisis measures from the second semester of 2009.

In the following years, economic activity response was relatively positive, registering, after a decline of 0.13% in 2009, an average economic growth of 4.11% in 2010-2013. The base nominal interest rate (Selic) reached a floor of 7.25% in April 2013, from 13.75% in the middle of September 2008. Annual CPI from 2009 to 2013 averaged 5,69%, staying inside the inflation target range. Regarding fiscal policy, the federal government launched a mix of measures with and without direct impact in the primary balance indicator (the one government needs to accomplish yearly due to the Fiscal Responsibility Law).

Treasury loans to the National Bank of Economic and Social Development (BNDES) between 2008 and 2014 amounted to BRL 424 billion, without immediate impact on the primary balance. However, the widening of tax incentives to labor-intensive sectors, together with expanding spending measures (social programs, public investments) and the difficulty to revert some of these policies, culminated in the deterioration of the principal fiscal indicators from 2014 (Figure 1.1).

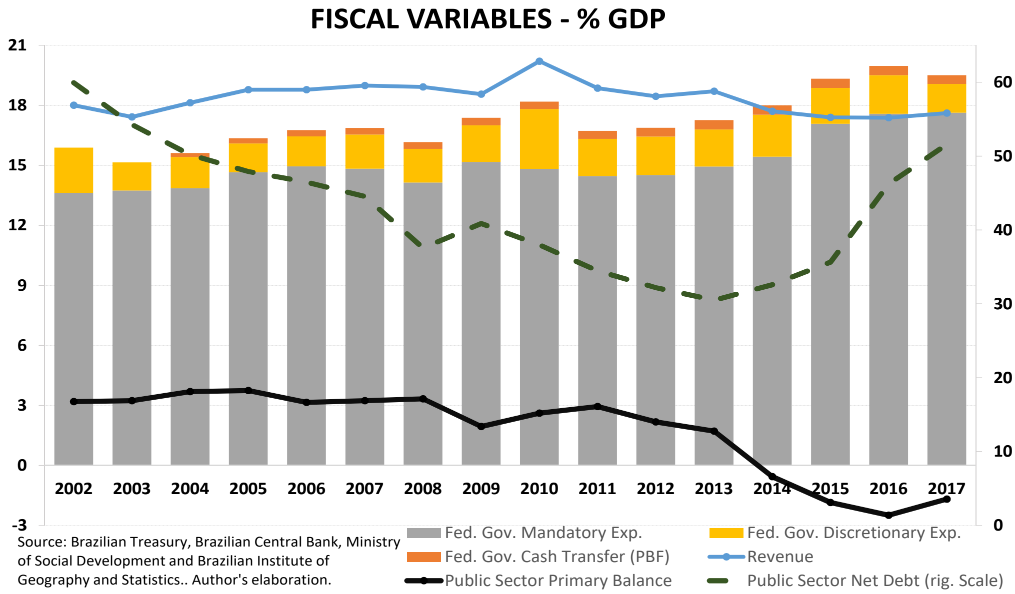

Figure 1.1: Fiscal Main Indicators

Historically, revenues increases helped to maintain the increasing trajectory of public expenditures. From 2014 on, things seemed to be different due to a lack of resilience in economic activity. The main fiscal variables, primary balance and public debt, have registered an inflection on their trajectories.

Under this environment, in June 2016, the federal government sent to the National Congress a Constitution Amendment Law (PEC 241/2016), approved in December of 2016, that established the "Brazilian New Fiscal Regime". The Motivation Letter 83/2016 brought the following two sentences:

"This instrument aims to revert, in the medium and long terms, the deterioration of fiscal performance over the last years".

"Reestablish confidence in public expenditures and debt sustainability".

Summarily, the law aims to discipline the evolution of federal government spending. It states that total nominal public primary expenditures could not exceed the previous period adjusted by inflation[1]:

$$P_t G_t \leq (1 + \pi_{t-1})(P_{t-1} G_{t-1})$$

[1]The concept of primary expenditures excludes interest net payments. The most important exclusion is interest on public debt.

Given that this rule does not allow a mechanism to adjust for economic growth, it represents an extra effort to implement a countercyclical fiscal policy. Supposing the inflation measure (CPI) is closer to the GDP price deflator and a relatively stable inflation dynamics, the public expenditure to GDP ratio would reduce in economic booms, with the revenue naturally increasing. It would raise the primary balance in good times. The reverse would happen in periods of economic downturns.

In this chapter, I assume that the previous rule is binding. Historically, it is not a flawed assumption, as one can observe in Figure 1.2. Moreover, there is an economic reason why this is a reasonable assumption under the Brazilian fiscal environment. At the federal government, about 90% of primary public expenditures are mandatory[2], which means the government can not adopt a contingent policy regarding the majority of that kind of spending.

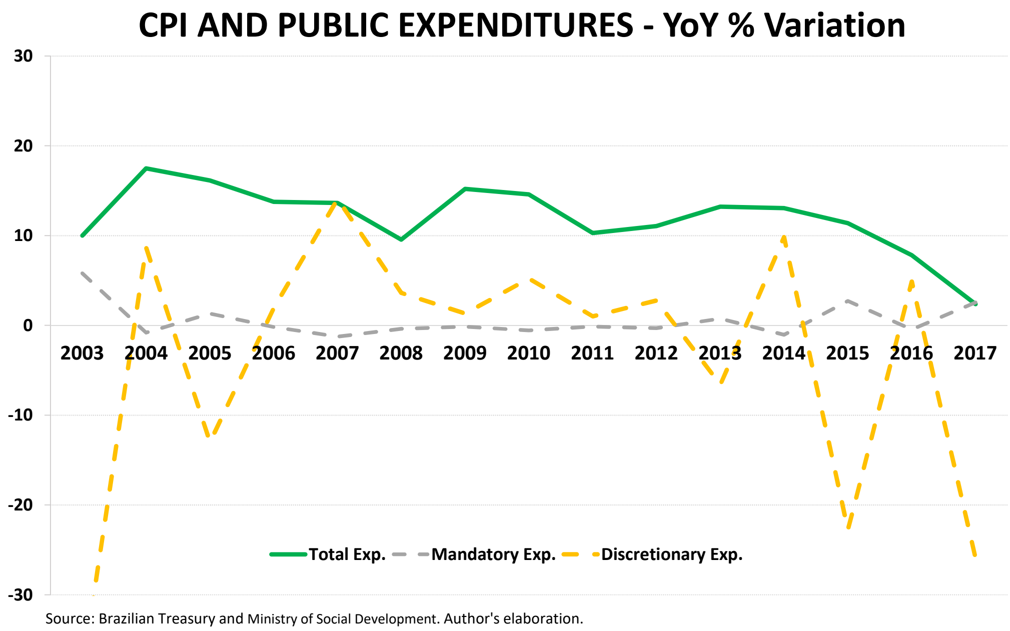

Figure 1.2: CPI and Public Expenditure Variations

The contribution of this chapter is threefold. First, I evaluate public expenditures composition dynamics concerning a preference shock under distinct fiscal rules in a private equilibrium economy. Beyond the cap on real expenditures growth rule, I study an output gap responding fiscal rule, the latter encompassing a non-reactive rule when the sensibility parameters to output gap are null.

Second, I solve a couple of Ramsey Problems to distinct mixes of monetary and fiscal

[2]This number reduces to close to 75% at the general government, which also accounts for State and Municipalities.

policies setups and assess public expenditures composition dynamics following that demand shock. Here, I show that when the monetary policy is set sub-optimally (Taylor rule), the Ramsey solution under the output gap responding fiscal rule delivers the same result as that of the private equilibrium economy.

The third contribution consists of a welfare analysis of different mixes of monetary and fiscal policies. The benchmark model to which it is possible to derive first-best results is used to compare the welfare losses across the distinct policies mixes.

In one simulation, I perform a sensitivity analysis on the output gap rule when the monetary policy follows a Taylor rule to evaluate the importance of reforms that enhance the government's capability to implement a countercyclical fiscal policy.

There is a range of public consumption and investment elasticities to the output gap that generates lower welfare losses than the private equilibrium economy under the cap on expenditure growth constraint. However, it does not generate a welfare loss lower than the Ramsey solution to the mix of sub-optimal monetary policy and optimal fiscal policy under the cap. Still, the output gap responding rule becomes attractive from a policymaker's perspective because it also characterizes an optimal Ramsey policy [3].

In another simulation, I find that it is not possible, over an acceptable range of elasticities, to generate a welfare loss to the mix of optimal monetary and fiscal policies under the output gap rule lower than that attainable following the same mix of policies under the cap rule. This finding suggests a limit to the pursuance of a countercyclical spending policy beyond that attainable under the cap rule, supposing the government implements optimal fiscal and monetary policies.

I organize this chapter into six sections beyond this introduction. Section 1.2 presents a (nonexhaustive) literature review. In section 1.3, I describe a standard New-Keynesian model with public capital. I define distinct Ramsey Problems in section 1.4, varying according to the mix of monetary and fiscal policies environments. Section 1.5 brings a quantitative exercise aiming to compare the dynamic of economic variables in response to a demand shock in each of the environments shown in section 1.4. I develop a welfare analysis in section 1.6. The 7th section concludes this work.

[3]The private equilibrium under the output gap fiscal rule and the monetary policy following a Taylor rule coincides with the Ramsey solution

1.2 Literature Review

The literature that relates fiscal policy to economic growth is not recent. Until the end of the 1980s, the neoclassical growth theory was the workhorse of such analysis (Solow, 1956; Swan, 1956; Solow, 1957). The role of fiscal policy in influencing long-run growth is null in that class of models (Arrow and Kurz, 1969). More recently, some articles evaluate the optimal composition of public expenditures in those models (Carboni and Medda, 2011; Bajo-Rubio, 2000; Fisher and Turnovsky, 1998). The predominant impact occurs in the short run, during the transition path to the steady-state, with exogenous factors (technological progress, population growth) determining the growth rate.

The development of endogenous growth models in the 1980s (Romer, 1986; Lucas, 1988) benefited the literature's surge about fiscal policy and long-run growth. The most influential paper was that of Barro (1990), which analyzes the effects on the growth and saving rates of tax-financed government services provided to the productive sector and consumers. The author extends the kind of constant returns to scale endogenous growth models to account for a broad concept of capital (human and nonhuman capital), giving space to the influence of productive government services. The model assumes diminishing returns on private capital but constant returns in the whole inputs for production, the principal channel to affect steady-state growth.

The main conclusion is that while services to the private sector increase growth and saving rates, at least initially (when government size is not large and inefficient), the utility-type expenditures reduce them. Assuming a Cobb-Douglas technology and a flat income tax, the share of government production services over output that optimizes growth is the one that equals the share that would prevail whether a competitive market of inputs provides those services. Even satisfying a condition for productive-efficiency, the decentralized equilibrium results in lower growth and saving rates than the planner solution.

Barro (1990) brings beyond the results on growth and saving and discusses the government's size and public expenditures composition. When providing other types of expenditures, the choice of production services' share becomes below the productive-efficiency share. Then the growth and saving rates are lower than if the government was providing just inputs to production. The provision of other expenditures than those that raise production does not directly impact private sector productivity.

However, it reduces individuals' rewards on their investment return since the government needs to increase the income tax rate.

Concerning empirical results, Barro (1990) mentions some applications to government expenditures and growth. Kormendi and Meguire (1985) find a non-significant relation between GDP growth and public consumption expenditures. Grier and Tullock (1989) support a significant negative relation between those two variables, mainly attributed to OECD countries (see also Landau, 1983). Barth and Bradley (1987) find a similar relation for public consumption but a non-significant effect of government investment on GDP growth, although with a positive point estimate. This finding on investment suggests that countries act to optimize production-type expenditures.

Devajaran, Swaroop and Zou (1996) settle an endogenous growth model to derive conditions under which changes in the composition of public expenditures imply higher growth rates. Such conditions depend on both physical productivity and initial shares. They consider two types of public expenditures, productive and unproductive expenditures, entering into the production function. Based on a group of 43 developing economies over 20 years, they find evidence that current expenditures, often considered unproductive, positively affect growth rates. Otherwise, (possibly excessive) expenditures considered productive (including the capital, transport, communication, education, health) have a negative and significant effect. The authors argue that developing countries have misallocated expenditures in favor of an excessive unproductive level of capital expenditures. These results have implications for recurrent policy recommendations: widespread public investment to boost growth.

Glomm and Ravikumar (1997) work with an overlapping generation model (Diamond, 1965), assuming two kinds of public spending provisions: infrastructure (roads, airports, R&D) and expenditures that enhance investment technologies (education). Their model delivers an economy converging to a sustainable growth path, with long-run growth rates across countries varying accordingly to their technology and preference parameters. They present a review of the empirical literature. The conclusion is that public capital seems to have long-run effects on output, even without a significant short-run impact. Regarding education, results are not conclusive, with no relevant impact on growth. The authors argue that growth rate differences across countries can be significant, explained by variations in schools' quality.

Also grounded on a one-sector endogenous growth model, Chen (2006) studies the optimal composition of two types of public expenditures, consumption and investment, and its relationship with economic growth. He derives a unique interior solution to the share of investment in public expenditures, determined by the policy and structural parameters. This finding would have implications for the case of East Asian growth miracles, noting that these countries registered both higher public investment share and economic growth than countries of other regions as a result of optimizing choices in favor of public investment.

Ghosh and Gregoriu (2008) widen the work of Devarajan et al. (1996) to choose the share of productive and unproductive public expenditures under an optimality perspective. They also experiment with the optimal setup of the income tax to solve the model. Their empirical results, grounded over 28 years of data on 15 developing countries, are very similar to those of Devarajan et al. (1996), with a rising in current expenditures associated with higher growth rates and increasing capital experiments reducing growth. The authors argue not necessarily in favor of overspending on capital to the detriment of current expenditure but that capital expenditures did not increase productivity as expected.

The literature on endogenous growth models sheds light on the potential implications of different types of public expenditures on economic growth. Aschauer (1989) argues that infrastructure expenditures (highways, airports, streets) are relevant when they have implications for private production, using US data from 1949-1985. Munnell and Cook (1990) and Garcia-Milà and McGuire (1992) find evidence of public capital's importance to explain differences in economic performance across the US States. Otherway around, Evans and Karras (1994) and Holtz-Eakin (1994) do not find evidence, to a panel of 48 US States, of a positive impact of public capital expenditures on the economic performance.

Easterly and Rebelo (1993) get evidence on the association between economic growth and investment in transport and communication in developing countries. Barro (1991) relates the increase in nonproductive government consumption and lower *per-capita* growth but does not find a significant association between public investment and growth for a mix of 98 countries in the period 1960-1985. Gupta, Clements, Baldacci and Mulas-Granados (2005) find evidence to a sample of 39 low-income countries that fiscal consolidation through the sacrifice of current and preserving capital expenditures is associated with higher economic growth.

Under a Real Business Cycle (RBC) general equilibrium approach, Baxter and King (1993) model fiscal policy with government financing basic public expenditures, that increase utility without altering private consumption and production decisions, and government investments to accumulate public capital, that affects the production function as an externality and also increases households' utility. The study investigates the multiplier effects of temporary and permanent changes in basic expenditures and permanent changes in public investment. The authors also explore public expenditures' government financing decisions.

They point to a plausible possibility of a long-run output multiplier higher than one for basic expenditures under lump-sum taxation. In the short run, the multiplier size can still be above one but depends directly on the size of the labor supply's intertemporal elasticity. Regarding the temporary effects of changes in these basic expenditures, they sustain that they are lower than the effects of permanent changes, in contrast with Barro (1981) and Hall (1980). When the authors assume distortionary taxation, the results are entirely distinct, with plausible negative values of multipliers. In the presence of that kind of tax, the agents have less incentive to work and invest when the government raises tax to finance more basic expenditures. Back to lump-sum taxation, the authors evaluate the impact of permanent changes in public investment. There is a direct effect on public capital, keeping fixed private capital and labor inputs, and a supply-side effect due to induced changes in capital and labor choices. Both effects increase in the public capital's productivity parameter, with capital and labor responses key determinants of output response to public investment changes.

Ambler and Paquet (1996) analyze the optimal composition of public expenditure under the stochastic RBC approach. A benevolent government optimizes public investment and nonmilitary expenditures. The former serves to increase public capital in the production function while the latter serves as utility-type expenditures. The third kind of public expenditure is exogenous and does not affect private marginal utility and productivity. Public financing comes with a fixed income tax rate and a lump-sum tax. The authors evaluate responses of aggregate macroeconomic variables to technological shocks and exogenous military spending. In their findings, the model predicts well that government current and investment expenditures types should behave like their private sector counterparts. Current private and government consumption are less volatile than output, while private and public investments are much more volatile than output. The model predictions of correlations among government expenditures types and output overestimate those observed in the data. Possible explanations for these findings are measurement errors and the delay between the shocks and the endogenous government reaction.

Aimed at endogenizing the available fiscal policy instruments, Lansing (1998) widens the RBC framework to account for endogenous public expenditures (utility-type and production-type public capital), tax, and debt. Indeed, beyond technological shocks, he also assesses the role of preference shocks for public consumption goods. On the revenue side, the model follows Chari, Christiano and Kehoe (1994, 1995), while the expenditure side is similar to that of Ambler and Paquet (1996). A Ramsey government optimizes a representative household utility function under the assumption of a commitment technology. The model that accounts for both kinds of shocks can reproduce key features of US fiscal policy. The author argues that even the optimal fiscal variables present substantial variability over the business cycle, it does not imply that they function as automatic stabilizers. Optimal tax rates are countercyclical and public expenditures procyclical. Accordingly, these results reinforce doubts about the desirability of government stabilizing policies.

The study of public spending and growth also encounters relevant developments in the class of New-Keynesian dynamic stochastic general equilibrium (DSGE) models (Smets and Wouters, 2003; Schmitt-Grohé and Uribe (2005, 2007); Smets and Wouters, 2007; Galí, Lopez-Salido and Vallés, 2007; Cogan, Cwik, Taylor and Wieland, 2010). The optimal choice of public spending is the object of, for example, Galí and Monacelli (2008).

Existing literature using such a class of models focuses on assessing the multiplier effect and the welfare perspective regarding public expenditures' desirability but restricting attention to government spending on consumption goods (not necessarily increasing households' utility). The findings support that under a liquidity trap (and deep recession), the multiplier is high (Christiano, Eichenbaum and Rebelo, 2011; Woodford, 2011) and that the optimal fiscal response, at least temporally, is to increase public spending (Woodford, 2011; Schmidt, 2013; Nakata, 2017; Bhattarai and Egorov, 2016). Some papers that consider public investment, but without going through optimal policy concerns, are Leeper, Walker and Yang (2010), Leduc and Wilson (2013), and Bouakez, Guillard and Roulleau-Pasdeloup (2017a).

Sims and Wolf (2018) explore the output and welfare effects of government consumption and investment expenditures in a medium-scale New-Keynesian DSGE model. A quantitative exercise shows that households would prefer a government more engaged in public investment once the government's size is optimal. However, they note that this result is quite sensible to the weight of government consumption and public investment productivity. They conclude that countercyclical government spending should not be a general prescription rule. Especially in the presence of distortionary taxation. Notwithstanding, the fiscal stimulus can be relevant under a passive monetary policy, as in zero lower bounds (ZLB) periods. They also evaluate the inclusion of "rule of thumb" households. The higher the fraction of such agents, the higher are the output and the welfare multipliers.

A formal investigation of the optimal composition between utility-type expenditures and capital-type expenditures is the object of Bouakez, Guillard and Roulleau-Pasdeloup (2019) in a context of a deep recession with zero lower bound interest rate constraint. They use a New-Keynesian model approach to investigate the impact of a preference shock that leaves the economy in a ZLB episode. The authors show a role for expanding public spending under this circumstance, with a prominent place for investments to raise public capital. Restricting the monetary policy instrument to a sub-optimal Taylor rule, public expenditures related to the stabilization motive for consumption and investment amount to -1.0% and 6.0% of the steady-state output in cumulative terms.

1.3 The Model

I am following Bouakez, Guillard and Roulleau-Pasdeloup (2017b)[4] in setting up the main model characteristics. The New-Keynesian economy, without private capital, is composed of four types of agents: infinitely living households, competitive final-good firms, monopolistic competitive intermediate-goods firms, and government.

1.3.1 Households

There is a continuum of infinitely living identical households, indexed by $i \in (0,1)$, that derive utility from private (C_t) and public (G_t^c) consumption and dislike labor (N_t). The lifetime utility function of a representative household follows from:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Big[U\big(C_t, N_t\big) + V\big(G_t^c\big) \Big] \tag{1.1}$$

[4] This is a previous version of Bouakez et al. (2019).

where β is the discount factor. The function $U(.)$ is increasing and concave in C_t and decreasing and concave in N_t, while $V(.)$ is increasing and concave in G_t^c. Households are endowed with 1 unit of time, which they allocate between labor and leisure $(1 - N_t)$. The logarithm of the preference shock ξ_t follows an AR(1) process, such that:

$$\ln(\xi_t) = \rho \ln(\xi_{t-1}) + \epsilon_t^\xi, \text{ with } \epsilon_t^\xi \overset{i.i.d.}{\sim} \mathcal{N}(0, 1) \tag{1.2}$$

Households trade among themselves a risk-free one period bond (B_t) in zero net supply. A representative household enters in period t with B_{t-1} units of nominal bonds carried from the previous period, receive from intermediate-goods firms a nominal wage W_t from renting its labor in a competitive input market and dividends D_t corresponding to his ownership share in each of those firms[5], and pays a lump-sum tax T_t to government. This net income finance private consumption and the acquisition of bonds. The flow budget constraint is given by:

$$P_t C_t + \frac{1}{1 + R_t} B_t = W_t N_t + D_t + B_{t-1} - T_t \tag{1.3}$$

where R_t is the nominal interest rate prevailing in the economy and set by the monetary authority. Households maximize (1.1) subject to (1.3) and the following *non-Ponzi* condition:

$$lim_{j \to \infty} \left(\frac{1}{(1 + R_t)} \right)^{j+1} B_{t+j} \leq 0 \tag{1.4}$$

The FOCs conditions with respect to C_t, N_t and B_t implies the following two equilibrium conditions:

$$[C_t] \text{ and } [N_t] : \quad \frac{W_t}{P_t} = -\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} \tag{1.5a}$$

$$[B_t] : \quad \frac{1}{(1 + R_t)} = \beta E_t \left\{ \frac{\Gamma_{t,t+1}}{(1 + \pi_{t+1})} \right\} \tag{1.5b}$$

where $\beta^k \Gamma_{t,t+k} \equiv \beta^k \left(\frac{\xi_{t+k}}{\xi_t} \right) \left(\frac{U_C(C_{t+k}, N_{t+k})}{U_C(C_t, N_t)} \right)$ is the discount factor and $\pi_t = P_t / P_{t-1} - 1$ the inflation rate.

[5] Indexing the intermediate-goods firms by $z \in (0, 1)$ implies that $D_t = \int_0^1 D_t(z) dz$.

1.3.2 Final-good Firms

The final-good firms operate under a competitive market, taking as given the price of the final good (P_t) and the prices of inputs provided by the intermediate-goods firms ($P_t(z)$). The technology is a CES over the continuum $z \in [0,1]$ inputs ($X_t(z)$) provided by intermediate-goods firms.

$$Y_t = \left[\int_0^1 X_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{1.6}$$

The maximization problem is the following:

$$\max_{X_t(z)} \qquad P_t Y_t - \int_0^1 P_t(z) X_t(z) dz \tag{1.7}$$

$$\text{s.t.: equation (1.6) is satisfied.}$$

The FOC to this problem delivers the demand schedule for the firm z intermediate good:

$$X_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta} Y_t \tag{1.8}$$

The competitive market structure implies that the zero-profit condition is satisfied. Then, this results in the following dynamic of the final good price:

$$P_t = \left[\int_0^1 P_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \tag{1.9}$$

1.3.3 Intermediate-goods Firms

These firms operate in a monopolistic competitive market, offering heterogeneous intermediate goods to the final-good firms. Under this market structure, they offer any amount of inputs demanded by the final-good firms. A typical intermediate-good firm demands labor ($N_t(z)$) and faces positive externalities due to the provision of public capital (K_t^g), that is provided without any additional charge. The input choice comes with the solution of a minimization problem subject to the technology constraint.

$$\min_{N_t(z)} \quad W_t N_t(z)$$
$$\text{s.t.:} \quad Y_t(z) = a_t F(N_t(z), K_t^g) \tag{1.10}$$

where a_t is a productivity factor assumed to be equal across firms z. This factor evolves exogenously according to the following $AR(1)$ process:

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \epsilon_t^a, \text{ with } \epsilon_t^a \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_a^2) \tag{1.11}$$

Defining $\lambda_t(z)$ as the Lagrange multiplier associated with the production technology of firm z, the FOC is:

$$\lambda_t(z) = \frac{W_t}{a_t F_N(N_t(z), K_t^g)} \tag{1.12}$$

Then, $\lambda_t(z)$ represents the nominal marginal cost of firm z. Since $z \in (0,1)$ faces the same labor factor competitive market and share the same technology, under constant return to scale (CRS) technology on private input, the nominal marginal cost is the same across firms ($\lambda_t(z) = \lambda_t, \forall z$). From now on, I will refer to the nominal marginal cost with the notations ($MC_t(z)$) and (MC_t), instead of ($\lambda_t(z)$) and (λ_t).

Under this model setup, all the firms distribute profits to their owners by[6]:

$$\begin{aligned} D_t(z) =& (1+\tau)P_t(z)Y_t(z) - W_t N_t(z) - P_t \Xi_t(z) \\ =& (1+\tau)P_t(z)Y_t(z) - MC_t Y_t(z) - P_t \Xi_t(z) \end{aligned} \tag{1.13}$$

where $\tau = 1/(\theta - 1)$ is a subsidy to intermediate-goods producers aimed to correct monopolistic competition distortion in steady-state and $\Xi_t(z)$ is a price adjustment-cost paid by firm z, in terms of the real final good (see the next subsection).

Price Setting

The dynamic of price setting is assumed to be dictated by the presence of a price-adjustment cost (Rotemberg, 1982). In resetting its price $P_t(z)$ each period, an intermediate-good firm z incurs in a quadratic adjustment cost, measured in terms

[6]The second equality comes under the assumption of CRS technology on labor.

of the final good:

$$\Xi_t(z) = \frac{\psi}{2}\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2 Y_t \tag{1.14}$$

where $\psi > 0$ measures the degree of importance of this source of inefficiency.

Under this setup to the price dynamics of the monopolistic competitive intermediate-goods firms, the choice of $P_t(z)$ involves the maximization of the real market value in the whole time path, constrained on the intermediate good z demand schedule (1.6) and the market-clearing condition ($X_t(z) = Y_t(z), \forall z$).

$$\begin{aligned} &\max_{\{P_{t+k}(z)\}_{k=0}^{\infty}} \quad E_t\left\{\sum_{k=0}^{\infty}\beta^k \Gamma_{t,t+k}\frac{D_{t+k}(z)}{P_{t+k}}\right\} \\ &\text{s.t.:} \quad Y_{t+k}(z) = X_{t+k}(z) = \left(\frac{P_t(z)}{P_{t+k}}\right)^{-\theta} Y_{t+k} \end{aligned} \tag{1.15}$$

The FOC condition associated with this problem is the following:

$$\begin{aligned} &-\theta\left(\frac{P_t(z)}{P_t}\right)^{-\theta}\left(\frac{Y_t}{P_t}\right) + \theta\left(\frac{MC_t}{P_t}\right)\left(\frac{P_t(z)}{P_t}\right)^{-\theta-1}\left(\frac{Y_t}{P_t}\right) + \\ &-\psi\left\{\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)\left(\frac{Y_t}{P_{t-1}(z)}\right) + \right. \\ &\left. -\beta E_t\left[\Gamma_{t,t+1}\left(\frac{P_{t+1}(z)}{P_t(z)} - 1\right)\left(\frac{P_{t+1}(z)}{P_t(z)}\right)\left(\frac{Y_{t+1}}{P_t(z)}\right)\right]\right\} = 0 \end{aligned} \tag{1.16}$$

In equilibrium, intermediate-goods firms set the same prices ($P_t(z) = P_t \forall z$) and production ($Y_t(z) = Y_t, \forall z$) levels. Defining the inflation rate as $\pi_t = P_t/P_{t-1} - 1$ and the real marginal cost as MC_t/P_t, I can rewrite (1.16) as:

$$\begin{aligned} &\theta(mc_t - 1) - \psi\left\{(1 + \pi_t)\pi_t + \right. \\ &\qquad \left. -\beta E_t\left[\Gamma_{t,t+1}(1 + \pi_{t+1})\pi_{t+1}\left(\frac{Y_{t+1}}{Y_t}\right)\right]\right\} = 0 \end{aligned} \tag{1.16'}$$

1.3.4 Government

The government in this model concerns fiscal and monetary authorities. The former operates under a cap on real expenditure growth fiscal rule or a discretionary policy according to which public spending follows an output gap responding rule. Public expenditures serve to different proposals, which are financed by a lump-sum tax (T_t). In the absence of public debt, the budget constraint is balanced each period:

$$G_t + \tau Y_t = G_t^c + G_t^i + \tau Y_t = \frac{T_t}{P_t} \tag{1.17}$$

Under the cap on real expenditure growth fiscal rule, total expenditures evolve according to the following rule:

$$G_t = \left(\frac{1 + \pi_{t-1}}{1 + \pi_t} \right) G_{t-1} \tag{1.18}$$

Alternatively, following the output gap responding rule, public expenditures deviate from their equilibrium values according to the output gap dynamics:

$$\left(\frac{G_t^j}{G^j} \right) = \left(\frac{\Delta_t Y_t}{Y} \right)^{\rho_{Gj}}, \text{ for } j = c \text{ and } i \tag{1.19}$$

where $\rho_j \lesseqgtr 0$ measures the sensibility of public consumption-goods and investment to the output gap. A countercyclical fiscal rule concerning a kind of public expenditure follows when $\rho_j < 0$. In turn, $\rho_j > 0$ characterizes a procyclical fiscal rule. The case of $\rho_j = 0$ is what I will call a non-reactive fiscal policy when discussing the simulations' results.

Following whatever these rules constraints, the government plays with the allocation of G_t between public consumption (G_t^c) and public investments (G_t^i), beyond that necessary to eliminate monopolistic competition distortion in steady-state.

The law of motion of public capital follows assuming a convex adjustment cost function S[7]:

$$\begin{gathered} K_{t+T}^g = (1-\delta) K_{t+T-1}^g + \left(1 - S\left(\frac{G_t^i}{G_{t-1}^i} \right) \right) G_t^i \\ \text{where:} \\ S\left(\frac{G_t^i}{G_{t-1}^i} \right) = \frac{\overline{\omega}}{2} \left(1 - \frac{G_t^i}{G_{t-1}^i} \right)^2 \end{gathered} \tag{1.20}$$

The presence of $T >= 0$ turns it possible to consider the time to build constraint regarding the productivity of public capital (Kydland and Prescott, 1982; Bouakez et al., 2017a).

[7]Specifically, $S(1) = S'(1) = 0$, which ensures no adjustment cost in case of zero variation on public investment.

The role of monetary authority is to set the nominal interest rate prevailing in the economy. I will investigate two possibilities. In the first one, the Central Bank determines this policy optimally. In the other, the authority follows a simple Taylor rule:

$$1 + R_t = \frac{(1 + \pi_t)^{\theta_\pi}}{\beta} \epsilon_t^R \tag{1.21}$$

where $\theta_\pi > 1$ (Taylor principle) and ϵ_t^R is an exogenous $AR(1)$ process monetary shock:

$$\ln(\epsilon_t^R) = \rho_R \ln(\epsilon_{t-1}^R) + \varepsilon_t^R, \text{ with } \varepsilon_t^R \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_R^2) \tag{1.22}$$

1.3.5 Market Clearing and Resource Constraint

Under the representative agent setup, the risk-free bond is in zero net supply ($B_t = 0$). Also, the demand for labor by the intermediate-goods sector matches the labor supply:

$$N_t = \int_0^1 N_t(z)dz = N_t^d \tag{1.23}$$

The production of a differentiated good of a monopolistic competitive firm $Y_t(z)$ matches the demand from the final-good firms ($X_t(z)$):

$$X_t(z) = Y_t(z) \tag{1.24}$$

To derive the resource constraint, I first aggregate the dividend equation, in real terms, across firms:

$$\begin{aligned} \frac{D_t}{P_t} &= \frac{\int_0^1 D_t(z)dz}{P_t} \\ &= (1 + \tau)Y_t \left(\frac{P_t(z)}{P_t} \right)^{(1-\theta)} - \frac{W_t}{P_t} N_t - \int_0^1 \Xi_t(z)dz \end{aligned} \tag{1.25}$$

In equilibrium, $P_t(z) = P_t \; \forall z$:

$$\frac{D_t}{P_t} = (1 + \tau)Y_t - \frac{W_t}{P_t} N_t - \frac{\psi}{2} \pi_t^2 Y_t \tag{1.26}$$

The derivation of the economy feasibility constraint follows after the summation of households and government budget constraints.

$$C_t + G_t + \tau Y_t = \frac{W_t}{P_t} N_t + \frac{D_t}{P_t} \tag{1.27}$$

Substituting (1.26) in (1.27) provides the identity between the aggregate demand and the output of this economy, the latter adjusted by a price distortion term $\Delta_t = (1 - \frac{\psi}{2}\pi_t^2)$:

$$\Delta_t Y_t = \Delta_t a_t F(N_t, K_t^g) = C_t + G_t \tag{1.28}$$

1.3.6 Competitive Equilibrium and Ramsey Problem

Definition 1: A Competitive Equilibrium to this economy, considering the stochastic patterns of exogenous shocks $\{\epsilon_t^{\xi}, \epsilon_t^{a}, \epsilon_t^{R}\}_{t=0}^{\infty}$, the initial government indebtedness B_{-1} and the initial stock of public capital K_{-T}^g, is characterized by a feasible allocation $\{C_t, N_t, K_t^g\}_{t=0}^{\infty}$, a price system $\{\pi_t, W_t/P_t, mc_t\}_{t=0}^{\infty}$ and a government policy $\{\tau, T_t/P_t, G_t^c, G_{t-T}^i, R_t\}_{t=0}^{\infty}$, such that: *i)* given the government policy and the price system, the allocation solves the households' and firms' problems; and *ii)* given the allocation and the price system, the government policy satisfies government budget constraint.

Definition 2: Provided that the government has a commitment technology, the Ramsey Problem solution to this economy are fiscal and monetary policies that maximize the government objective function $E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[U\big(C_t, N_t\big) + V\big(G_t^c\big) \right]$ constrained to the private economy's competitive equilibrium conditions.

1.4 Monetary and Fiscal Policies Environments

1.4.1 Benchmark Result

The benchmark model is the one at which the monetary policy is set optimally and there is no constraint on the evolution of public expenditures, like those imposed by equations (1.18) and (1.19). Given that the only distortion is the one coming from price rigidity, once the monopolistic competition is eliminated by the subsidy τ in the intermediate-goods sector, the monetary authority can replicate the flexible-price result setting the nominal interest rate equal to its natural efficient level. In the literature, this is called "divine coincidence" (Blanchard and Galí, 2007), when the policy can fully stabilize inflation and output gap every period (Bouakez et al., 2017b).

Solving for the first-best allocation gives the solution to this version of the model:

$$
\begin{aligned}
L(C_t, N_t, G^c_t, G^i_t, K^g_{t+T}, \lambda_{1,t}, \lambda_{2,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \Big[U\big(C_t, N_t\big) + V\big(G^c_t\big) \Big] + \\
- \lambda_{1,t} \Big[C_t + G^c_t + G^i_t - a_t F(N_t, K^g_t) \Big] + \\
- \lambda_{2,t} \Bigg[K^g_{t+T} - (1-\delta) K^g_{t+T-1} - \Bigg(1 - S\bigg(\frac{G^i_t}{G^i_{t-1}}\bigg)\Bigg) G^i_t \Bigg] \Bigg\}
\end{aligned}
\tag{1.29}
$$

The FOCs follows:

$$
[C_t]: \quad \xi_t U_C(C_t, N_t) - \lambda_{1,t} = 0 \tag{1.30a}
$$

$$
[N_t]: \quad a_t F_N(N_t, K^g_t) + \frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = 0 \tag{1.30b}
$$

$$
[G^c_t]: \quad V_{G^c}(G^c_t) = U_C(C_t, N_t) \tag{1.30c}
$$

$$
\begin{aligned}
[G^i_t]: \quad & -\lambda_{1,t} + \lambda_{2,t} \Bigg[1 - \frac{\overline{\omega}}{2} \bigg(1 - \frac{G^i_t}{G^i_{t-1}}\bigg)^2 + \overline{\omega} \bigg(1 - \frac{G^i_t}{G^i_{t-1}}\bigg) \bigg(\frac{G^i_t}{G^i_{t-1}}\bigg) \Bigg] + \\
& - \beta E_t \Bigg\{ \lambda_{2,t+1} \overline{\omega} \bigg(1 - \frac{G^i_{t+1}}{G^i_t}\bigg) \bigg(\frac{G^i_{t+1}}{G^i_t}\bigg)^2 \Bigg\} = 0
\end{aligned}
\tag{1.30d}
$$

$$
\begin{aligned}
[K_{t+T}]: \quad & -\lambda_{2,t} + \beta(1-\delta) E_t \big\{ \lambda_{2,t+1} \big\} + \\
& + \beta^T E_t \Big\{ \lambda_{1,t+T} a_{t+T} F_K(N_{t+T}, K^g_{t+T}) \Big\} = 0
\end{aligned}
\tag{1.30e}
$$

$$
[\lambda_{1,t}]: \quad a_t F(N_t, K^g_t) = C_t + G^c_t + G^i_t \tag{1.30f}
$$

$$
[\lambda_{2,t}]: \quad K^g_{t+T} = (1-\delta) K^g_{t+T-1} + \Bigg(1 - \frac{\overline{\omega}}{2} \bigg(1 - \frac{G^i_t}{G^i_{t-1}}\bigg)^2 \Bigg) G^i_t \tag{1.30g}
$$

1.4.2 Optimal Monetary and Fiscal Policies under the Cap on Real Expenditure Growth Constraint

The problem of the Ramsey planner is to find optimal allocations constrained to the competitive equilibrium conditions, including the cap on real expenditure growth.

To this goal, the Lagrangian problem is the following:

$$
\begin{aligned}
L(C_t, N_t, G_t^c, G_t^i, K_{t+T}^g, \pi_t, R_t, \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \Bigg[U\big(C_t, N_t\big) + \\
+ V\big(G_t^c\big)\Bigg] - \lambda_{1,t} \Bigg[\beta \Gamma_{t,t+1} \left(\frac{1+R_t}{1+\pi_{t+1}} \right) - 1 \Bigg] + \\
- \lambda_{2,t} \Bigg[\psi \Bigg[(1+\pi_t)\pi_t - \beta \Gamma_{t,t+1} (1+\pi_{t+1})\pi_{t+1} \\
\left(\frac{a_{t+1} F(N_{t+1}, K_{t+1}^g)}{a_t F(N_t, K_t^g)} \right) \Bigg] + \theta \left(1 + \frac{U_N(C_t, N_t)}{U_C(C_t, N_t) a_t F_N(N_t, K_t^g)} \right) \Bigg] + \\
- \lambda_{3,t} \Bigg[C_t + G_t^c + G_t^i - \Delta_t a_t F(N_t, K_t^g) \Bigg] + \\
- \lambda_{4,t} \Bigg[K_{t+T}^g - (1-\delta) K_{t+T-1}^g - \left(1 - \frac{\varpi}{2} \left(1 - \frac{G_t^i}{G_{t-1}^i} \right)^2 \right) G_t^i \Bigg] + \\
- \lambda_{5,t} \Bigg[\left(\frac{1+\pi_{t-1}}{1+\pi_t} \right) (G_{t-1}^c + G_{t-1}^i) + \tau \Bigg(\left(\frac{1+\pi_{t-1}}{1+\pi_t} \right) a_{t-1} F(N_{t-1}, K_{t-1}^g) + \\
- a_t F(N_t, K_t^g) \Bigg) - G_t^c - G_t^i \Bigg] \Bigg\}
\end{aligned}
\tag{1.31}
$$

1.4.3 Sub-optimal Monetary Policy and Optimal Fiscal Policy under the Cap on Real Expenditure Growth Constraint

The problem is the same as that established at (1.31), except that now I am constraining the set of actions the monetary authority can implement. Instead of setting the interest rate optimally, the Central Bank now set R_t following the Taylor rule (1.21). Thus, the Ramsey Planner solves the following problem:

$$
\begin{aligned}
L(C_t, N_t, G_t^c, G_t^i, K_{t+T}^g, \pi_t, R_t, \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \Bigg[U\big(C_t, N_t\big) + \\
+ V\big(G_t^c\big)\Bigg] - \\
(...) \\
- \lambda_{6,t} \Bigg[\frac{(1+\pi_t)^{\theta_\pi}}{\beta} \epsilon_t^R - 1 - R_t \Bigg] \Bigg\}
\end{aligned}
\tag{1.32}
$$

1.4.4 Optimal Monetary and Fiscal Policies under the Output Gap Responding Fiscal Rule

The Ramsey problem is similar to that established in (1.31), except that the fiscal rule now follows the output gap responding rule. The relevant Lagrangian is:

$$L(C_t, N_t, G_t^c, G_t^i, K_{t+T}^g, \pi_t, R_t, \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \Bigg[U\big(C_t, N_t\big) +$$

$$+ V\big(G_t^c\big)\Bigg] - \lambda_{1,t}\Bigg[\beta\Gamma_{t,t+1}\left(\frac{1+R_t}{1+\pi_{t+1}}\right) - 1\Bigg] +$$

$$\begin{aligned}
& - \lambda_{2,t}\Bigg[\psi\Bigg[(1+\pi_t)\pi_t - \beta\Gamma_{t,t+1}(1+\pi_{t+1})\pi_{t+1} \\
& \left(\frac{a_{t+1}F(N_{t+1}, K_{t+1}^g)}{a_t F(N_t, K_t^g)}\right)\Bigg] + \theta\left(1 + \frac{U_N(C_t, N_t)}{U_C(C_t, N_t) a_t F_N(N_t, K_t^g)}\right)\Bigg] + \\
& - \lambda_{3,t}\Bigg[C_t + G_t^c + G_t^i - \Delta_t a_t F(N_t, K_t^g)\Bigg] + \\
& - \lambda_{4,t}\Bigg[K_{t+T}^g - (1-\delta)K_{t+T-1}^g - \left(1 - \frac{\overline{\omega}}{2}\left(1 - \frac{G_t^i}{G_{t-1}^i}\right)^2\right) G_t^i\Bigg] + \\
& - \lambda_{5,t}\Bigg[G^c\left(\frac{\Delta_t Y_t}{Y}\right)^{\rho_{G^c}} - G_t^c\Bigg] + \\
& - \lambda_{6,t}\Bigg[G^i\left(\frac{\Delta_t Y_t}{Y}\right)^{\rho_{G^i}} - G_t^i\Bigg]\Bigg\}
\end{aligned} \tag{1.33}$$

1.4.5 Sub-optimal Monetary Policy and Optimal Fiscal Policy under the Output Gap Responding Fiscal Rule

The Ramsey Planner problem differs from (1.33) only by including one additional constraint, the Taylor rule, that discipline the way the monetary policy is conducted:

$$L(C_t, N_t, \pi_t, R_t, \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \Bigg[U\big(C_t, N_t\big) +$$

$$+ V\big(G_t^c\big)\Bigg] -$$

$$(...) \tag{1.34}$$

$$- \lambda_{7,t}\left[\frac{(1+\pi_t)^{\theta_\pi}}{\beta}\epsilon_t^R - 1 - R_t\right]\Bigg\}$$

1.5 Quantitative Exercise

This section aims to develop a quantitative exercise to assess public expenditures composition dynamics in response to a demand shock under distinct fiscal rules. I analyze both the private equilibria economies and Ramsey problems that mix monetary and fiscal policies environments.

1.5.1 Calibration and Estimation Approach

As in Bouakez et al. (2017b), I assume that the representative household's utility function (and of the benevolent government) is additively separable in time and follows a Cobb-Douglas in consumption and labor. Otherwise, the momentary utility function is separable on public consumption goods.

$$U(C_t, N_t, G_t^c) = \frac{\left(C_t^\gamma (1-N_t)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{c\ 1-\sigma_g}}{1-\sigma_g} \tag{1.35}$$

where σ, σ_g and χ are positive[8] and $\gamma \in (0,1]$. The parameter σ measures the (constant) relative risk aversion for the mix of private consumption and leisure, while γ captures their relative importance. Regarding σ_g, it is the counterpart of σ, but for the public consumption good. The role of χ is to capture the relevance of public consumption goods in household's utility.

Concerning the intermediate-goods production function, I assume a CRS in private input (labor) that benefits from the public capital provision's externality:

$$Y_t(z) = a_t N_t(z) K_t^{g\ b} \tag{1.36}$$

On the topside of Table 1.1, I replicate the parametrization presented in Table 1 of Bouakez et al. (2017b), to which I refer for further calibration details. These values align with international DSGE literature and are inside admissible parameters' regions to study the Brazilian economy (Vereda and Cavalcanti, 2010; Carvalho and Valli, 2011; Cavalcanti and Vereda, 2011; Mussolini and Teles, 2012).

[8] The case $\sigma = 1$ degenerates to the time and arguments separable utility function.

The downside includes parameters not presented in Bouakez et al. (2017b) model setup. They are related to the output gap fiscal rule parameters that measure the sensibility of public consumption and investment to the output gap. I calibrate these parameters such that the elasticities match their historical levels of 0.25 and 3.00, respectively. These values are consistent with a public budget characterized by a high percentage of mandatory expenditures, leaving investment expenditure highly susceptible to adjustments in the business cycle. In the welfare results (see next section), I perform a sensibility analysis to explore alternatives couple of elasticities, showing the importance of reforms that enhance the government's capability to implement a countercyclical fiscal policy. I am not showing parameters related to productivity and monetary shocks, as I am not concerned with shocks in these variables[9].

Table 1.1: Parameters calibration

Discount factor	$\beta = 0.99$
Relative risk aversion: C and $(1 - N)$	$\sigma = 2$
Relative risk aversion G_t^c	$\sigma_g = 2$
Relative weight of C and $(1 - N)$	$\gamma = 0.29$
Relative importance of G_t^c	$\chi = 0.054$
Elasticity of output w.r.t. K_t^g	$b = 0.08$
Elasticity of substitution between intermediate goods	$\theta = 6$
Time-to-build delay	$T = 16$
Price-adjustment-cost parameter	$\psi = 200$
Depreciation rate of public capital	$\delta = 0.02$
Investment adjustment-cost parameter	$\overline{\omega} = 2.5$
Steady-state ratio $g \equiv (G^c + G^i)/Y$	$g = 0.2$
Autocorrelation of the preference shock	$\rho = 0.9$
Sensibility of R_t to inflation	$\theta_\pi = 1.5$
Output gap fiscal rule parameter to public consumption	$\rho_{G^c} = 0.25$
Output gap fiscal rule parameter to public investment	$\rho_{G^i} = 3.00$

The estimation approach assumes the economy is in the steady-state when a preference shock hits the system of nonlinear equations. Furthermore, as standard in this literature (Smets and Wouters, 2003; Christiano et al., 2011), I assume agents have perfect foresight over the simulation horizon, which means the system does not challenge any shock in the future, converging to a steady-state in the long run.

[9]It is the same to assume that the productivity and monetary shocks have null variation and always keep in their unitary equilibria values.

1.5.2 Dynamic response to a demand shock

1.5.2.1 First-Best and Private Equilibria Environments

A preference shock hits the economy initially in the steady-state equilibrium. Figure 1.3 plots the response of three possible private equilibria environments. The blue line represents an economy where the fiscal policy does not react to the shock through public expenditures changes. Thus the share of public investment is kept constant during the role period at which the economy fluctuates before returning to the steady-state equilibrium. The black and red lines represent the private equilibria when fiscal policy functions obey, respectively, a cap on real expenditure growth rule and a discretionary policy according to which public spending follows an output gap responding rule. The green line shows the result attainable when a central planner implements the first-best allocation and serves as a reference for comparisons. The size of the shock is such that under a non-reactive fiscal policy, the GDP decreases exactly one percent below its steady-state level.

Public expenditures stay at their equilibria level and no change in composition proceeds when fiscal policy is non-reactive. Deflation and a reduction in the nominal interest rate follow. Real marginal cost (and real wage) shrinks as labor demand reduces, impacting the hours devoted to the labor market. Private consumption reduces by 1.0% on impact (by the construction of the preference shock). The GDP gap is negative for an extended period, converging from below to steady-state as the monetary policy responds to the inflation dynamics. The public sector's share in the economy still increases under this non-reactive fiscal policy, about 1.02% on impact, which is also the peak of this share.

When fiscal policy follows a cap on real expenditure rule, as inflation stays below its steady-state level, public consumption and investment respond countercyclically, increasing on impact 0.07% and 0.02% of equilibrium output. As a result, the public sector share in the economy increases by 1.41%. Due to this reaction of public expenditures, the GDP decrease is less pronounced than in a non-reactive fiscal policy environment, and the demand for workers will be less affected. Even working more, private consumption on impact is below the case of a non-reactive fiscal policy, characterizing the crowding-out effect of public spending. That case of a lower private consumption under the cap fiscal policy reverts from $T > 16$, when the public investment starts affecting public capital accumulation and increase production capacity. At this point, 80% of the public expenditures' initial expansion was reverted.

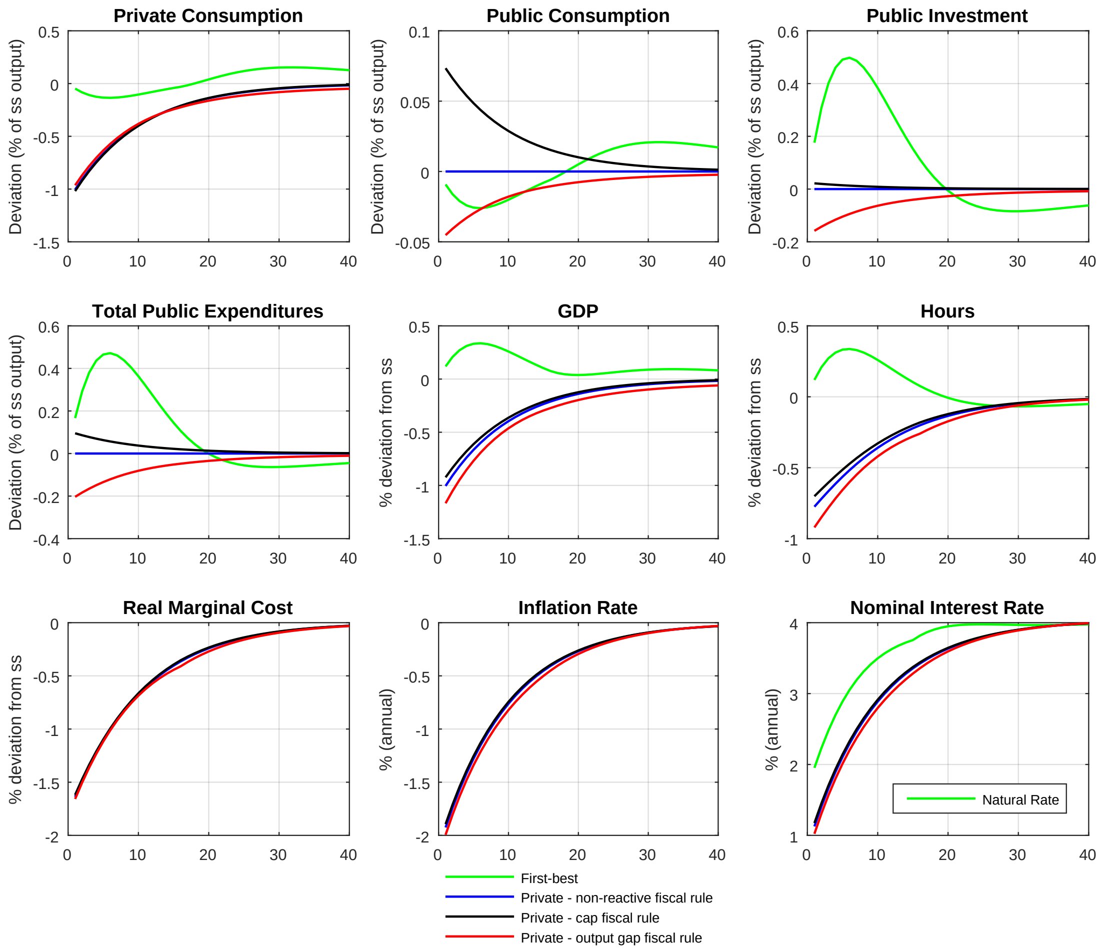

Figure 1.3: Response to a preference shock: first-best and private equilibria

The case of a discretionary procyclical fiscal policy that responds to the output gap also fails in avoiding the transmission of a preference shock to the economic activity. The GDP decrease under this environment is about 17.0% higher than that of a non-reactive fiscal policy. The amount of public expenditures decreases by 0.2% of equilibrium output on impact, with public investment responding by almost 80% of that contraction, implying a decrease in its share on public expenditures. Thus, the contractionary fiscal policy persists for an extended period, as is the case of economic activity. Regarding the share of the public sector in the economy, given that the relative decrease in public expenditures is lower than that of private consumption, it still observes a tiny increase of about 0.15% of steady-state output. In contrast to the cap rule, the output gap responding policy allows a crowding-in effect lasting about the first four years of the recession. Even working less than in a non-reactive fiscal policy, private consumption accumulates a lower relative loss in the first thirteen quarters, close to 0.30% percent of equilibrium output.

As a critical concern of this study is about optimal monetary and fiscal policies under distinct fiscal rules environments, the first-best allocation appears in Figure 1.3 to serve as a reference. The preference shock induces an intertemporal reallocation of households' consumption, increasing current savings to obtain more private goods in the future. It is possible due to the accumulation of physical public capital, in this model delayed by $T = 16$ periods, that widen producing capacity over time. The efficient policy points to a desirable huge increase in public investment in the face of that shock. The on impact increase is 0.18%, peaking 0.50% one and a half years after the shock. Public consumption follows a similar path to that of the private one, which aligns with the Samuelson optimality condition (see equation 1.30c).

The impact on the economic activity of an efficient response to the shock succeeds to be positive, increasing in annual terms 0.50% on impact and reaching 1.35% after one and a half years, converging to the steady-state from above. Households work more at the beginning, even consuming less compared to the equilibrium allocation. The payback comes after period $t > 16$, when the optimal policy implies a consumption above the steady-state a long period before the shock's complete dissipation.

From the results, I can explore some implications concerning the first-best and the private equilibria environments. First, a change in the composition of public expenditure, in favor of an increase in public investment share, follows when an efficient response to the shock is attainable (see Figure 1.4). This change lasts up to five years after the shock.

In equilibrium, public investment responds by 22,86% of government expenditures. When an efficient response is attainable, this share peaks at 24.76% one and a half years after experiencing the shock. Before the shock's complete dissipation, the share of public investment experiences a level below its equilibrium. In the opposite direction, the output gap responding rule predicts a decrease of 2.47% on the share of investment in total public spending. A level below the steady-state persists for an extended period until the whole shock's dissipation. I should note that the result of the output gap responding rule is very dependent on the sensitivity parameters ρ_j, here calibrated to match with Brazilian historical data. Finally, by assumption, under the cap on real expenditure growth and the non-reactive rules, no changes in the composition dynamics should be observed in the decentralized economy[10].

[10] Under the cap on real expenditure growth fiscal rule in the decentralized economy, I assume that the share of public investment on total public spending is constant, with this total amount varying according to the cap rule.

Second, in terms of the fiscal expansion's size following the shock, the first-best policy accounts for a cumulative impact of 3.41% of equilibrium output, with public investment responding for 88.64% of that expansion. The cap on expenditure growth policy shows a modest expansion, 0.96% of equilibrium output, with the public investment responding with its exactly steady-state share on this expansion. While spending response is nil by definition under the non-reactive fiscal policy, the case of the procyclical output gap responding rule reveals a contraction of 2.76% in terms of steady-state output, from which public investment is responsible by 77.9% of spending cuts.

Third, similarly to the definition in Bouakez et al. (2017b), I will define *stimulus spending* as the difference between the public spending level under private (or Ramsey optimal solution, like will be the case ahead) equilibrium and that prevailing under the first-best economy. So then, a stimulus refers to deviations from the level that would prevail under a full-flexible price economy.

Figure 1.4 shows that public consumption stimulus spending is positive about five years under the cap on real expenditure and the non-reactive rules. After this period, it becomes negative up to the complete dissipation of the shock. The stimulus spending under the cap rule peaks at 0.08% of equilibrium output on impact and reaches 0.36% in cumulative terms. The non-reactive fiscal policy response is almost null on impact and accumulates a negative stimulus (-0.39%). Under the output gap responding rule, public consumption stimulus is negative over the years, except a tiny stimulus period during the 7th and 11th quarters, accumulating 1.0% of equilibrium output.

Concerning public investment, the stimulus spending component of the fiscal expansion is not that relevant. During the first five years, the stimulus is negative, with modest positive numbers before the shock's dissipation. In cumulative terms, none of the fiscal policy rules support public investment for stimulus purposes. The output gap responding rule accumulates a level of spending 5.17% (in terms of equilibrium output) below the one that would prevail under the fully-flexible price equilibrium. As for the non-reactive and cap rules, these numbers are, respectively, 3.02% and 2.80%.

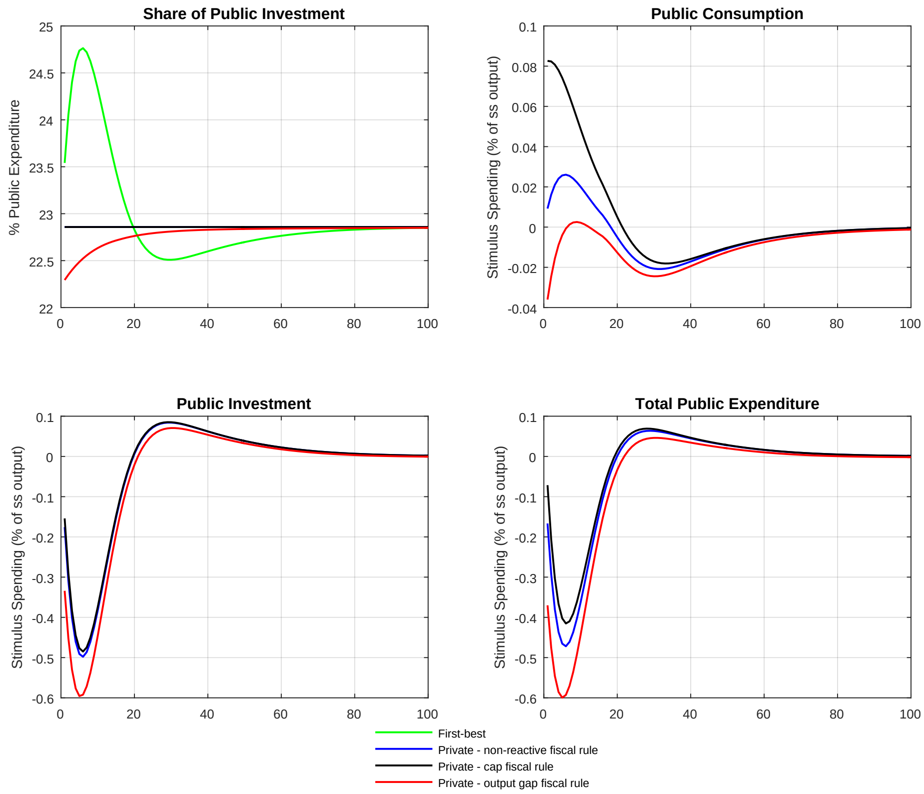

Figure 1.4: Public Expenditure Composition and Stimulus Spending

Summing up the results to the total public expenditures, all the fiscal rules studied in this work show pronounced negative stimulus numbers. Beyond that, this portrait grounds most intensively on the poor role of investment-type spending, which goes in the opposite direction to the efficient response. The most noticeable picture prevails under the output gap responding rule, to which negative stimulus amounts to 6.17% of equilibrium output. The non-reactive policy and the cap rule sum negative stimuli of 2.44% and 3.41%, respectively.

1.5.2.2 Ramsey Problems Mixing Monetary and Fiscal Policies

1.5.2.2.1 Optimal Monetary and Fiscal Policies under the Cap on Real Expenditures

In the absence of a cap on real expenditures, and considering that the zero lower bound is not a concern in this study, the Ramsey planner would succeed in replicating the efficient allocation. It is the case because the government would eliminate the monopolistic competition distortion with the subsidy τ. When implementing an optimal monetary policy, the government would have sufficient instruments to avoid inflation deviations from the steady-state. Nevertheless, the presence of a cap on real expenditure brings inefficiency to the economy.

The monetary authority nominal interest rate response stays above the natural interest rate during the first and a half years, contributing to a tiny deflation that lasts about two years. The initial impact of the shock in the intertemporal allocation of consumption is lower than that of the efficient policy during the first four and a half years, potentially benefiting from the interest rate policy. Even a tinny deflation induces the government to implement an active fiscal policy, with the prevalence of public investment. As the Samuelson condition is not valid under this environment, the government can still reduce public consumption more than the efficient response. However, there is not much appeal to an increase in public investment, as is the case under the first-best allocation.

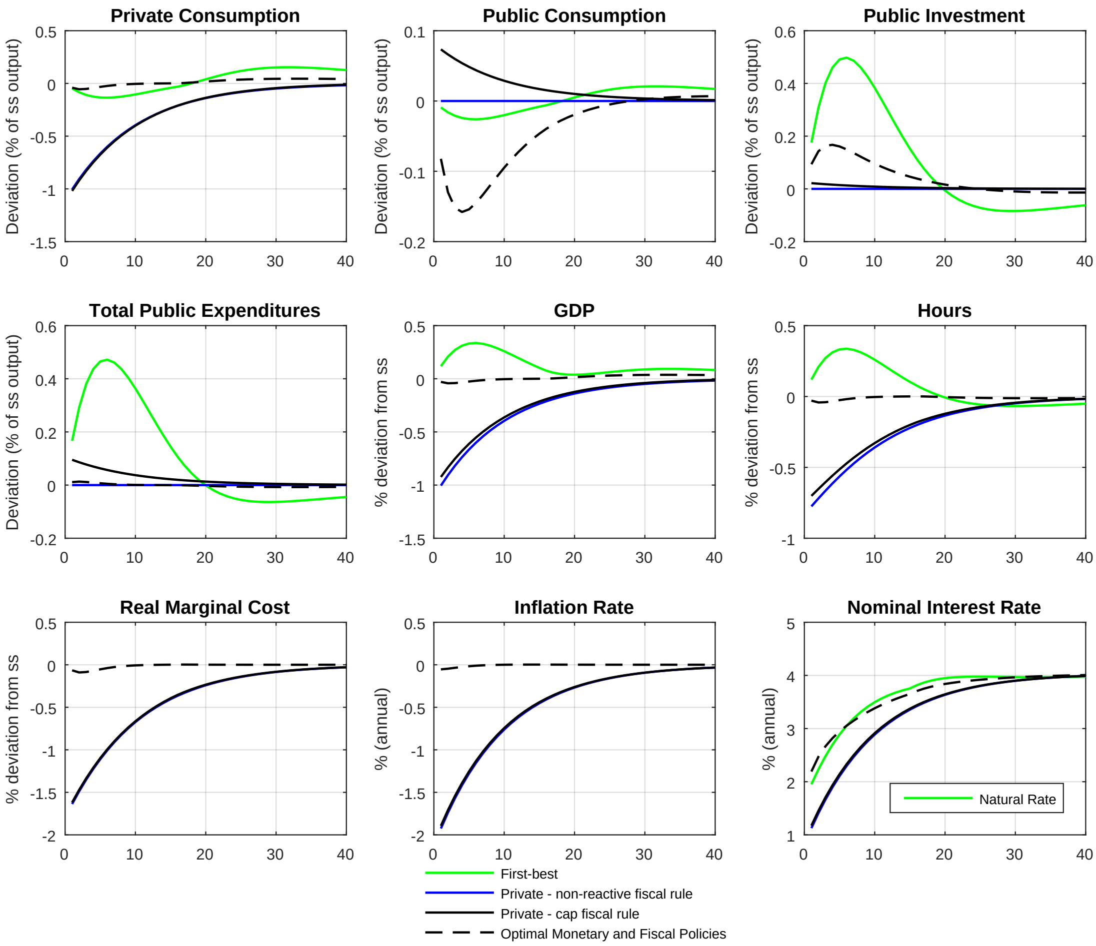

Figure 1.5: Response to a preference shock: Optimal monetary and fiscal policies under the cap

Total public spending shows a tiny deviation as a share of equilibrium output, about 0.01%, while the first-best response points to an increase of 0.16% in public spending. Then, the Ramsey planner policy entails an almost complete reallocation of public spending, from consumption to investment.

Even constrained to implement an efficient response to the preference shock, public spending reallocation allows the Ramsey policy to succeed in avoiding large deviations from steady-state to the whole set of economic variables except the nominal interest rate.

1.5.2.2.2 Sub-optimal Monetary Policy and Optimal Fiscal Policy under the Cap on Real Expenditures

As a consequence of the preference shock, it follows a huge decrease in consumption, marginally higher than that observed under the private equilibrium environment under the cap (1.13% against 1.02%). A sub-optimal monetary policy implies a nominal interest rate below the one implied by an efficient response, one consequence of a deflationary economy.

Public consumption on impact is positive, 0.04% of equilibrium output, an opposite sign to the efficient response and the optimal monetary and fiscal policies under the cap. The second quarter to the second and a half years show a decrease more pronounced than that of the first-best response. Regarding public investment, during the first and a half years, it stays above the efficient response, but the ceiling point happens at the end of the first year, two quarters before the peak of the efficient response. It stays marginally lower than the efficient response from the seventh quarter on, and before a complete convergence to steady-state, it eventually becomes higher.

Under this policy mix, the Ramsey plan response implies a crowding-out effect that lasts for almost five years, with the share of public expenditures increasing 2.37% on impact, representing the peak of that share. GDP follows the private equilibrium environments closely during the first nine quarters. As a result of the increase in the production capacity due to public investment, the economy registers a GDP higher than the equilibrium for at least five years in the medium term, with consumers working less than the equilibrium while consuming more.

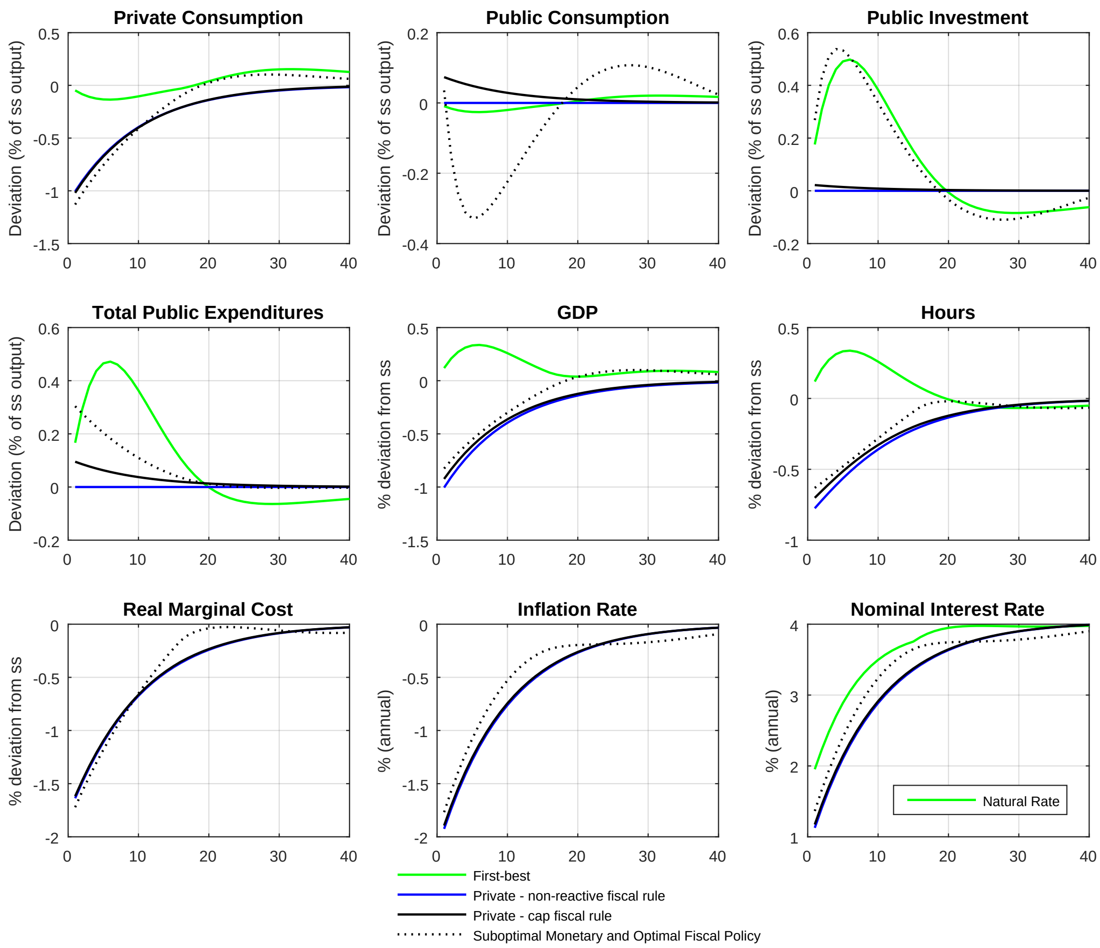

Figure 1.6: Response to a preference shock: sub-optimal monetary policy and optimal fiscal policy under the cap

1.5.2.2.3 Ramsey Composition and Stimulus Spending under the Cap on Real Expenditures

In the two mixes of monetary and fiscal policies studied above, the composition of public spending changed in favor of public investment. That change persists about five years, experimenting with the share of public investment below the equilibrium before the shock's complete dissipation. It is lower in the mix of optimal monetary and fiscal policies under the cap. In this mix, this share increases to 23.31% on impact and peaks 23.68% at the end of the first year. The mix of sub-optimal monetary policy and optimal fiscal policy shows that the weight on public investment should be more pronounced, reaching 23.84% on impact and peaking 25.26% in the fifth quarter. The efficient response is between those mixes, pointing to an on impact increase of 23.54% and a peak of 24.76% one and a half years after the shock. The equilibrium level of this share, as mentioned before, is 22.86%.

While the mix of optimal monetary and fiscal policies recommends a contractionary fiscal policy amounting to 0.26% of equilibrium output, the mix of sub-optimal monetary policy and optimal fiscal policy prescribes an expansionary one that reaches 2.15%. In the former, public investment increases 1.24%, while consumption reduces 1.50%, then characterizing the relevance of public resources' reallocation under this mix. Under the Taylor rule, public consumption shrinks 1.30%, and public investment increases 3.45% of equilibrium output.

Figure 1.7 shows that when monetary policy is set optimally, public consumption stimulus does not cross to the positive side, accumulating a negative stimulus of 1.89% of steady-state output. In the case monetary policy follows a Taylor rule, cumulative stimulus spending is negative (1.69%), but there is space for stimulus spending before the shock's complete dissipation. That the Ramsey optimal solutions ask for a strong contractionary policy during the first five years dominates the total effect.

In the two Ramsey mixes of monetary and fiscal policies, there is a place for periods of positive stimulus spending regarding public investment. However, when monetary policy is set optimally, the contractionary investment decisions over the first five years preponderates, implying, in cumulative terms, a negative stimulus of 1.78% of equilibrium output. Following a Taylor rule, investment spending stimulus accumulates a positive stimulus (0.43%), showing that the higher output gap compared to when the monetary policy is optimal asks for a prominent role of public investment in addressing the preference shock.

Summing up, stimulus spending results for total public expenditure show that both Ramsey mixes of policies present negative numbers in cumulative terms, even with periods when a stronger fiscal policy response is recommended compared to the efficient policy. The mix of optimal monetary and fiscal policies accumulates -3.67% of equilibrium output, and the results show that it can eventually increase investment above the prescribed by the efficient response. In turn, for the Ramsey policy mix of a Taylor rule and optimal fiscal policy, the negative stimulus spending is 1.26%. However, here both public consumption and investment can eventually surpass the full-flexible price allocations, especially investment.

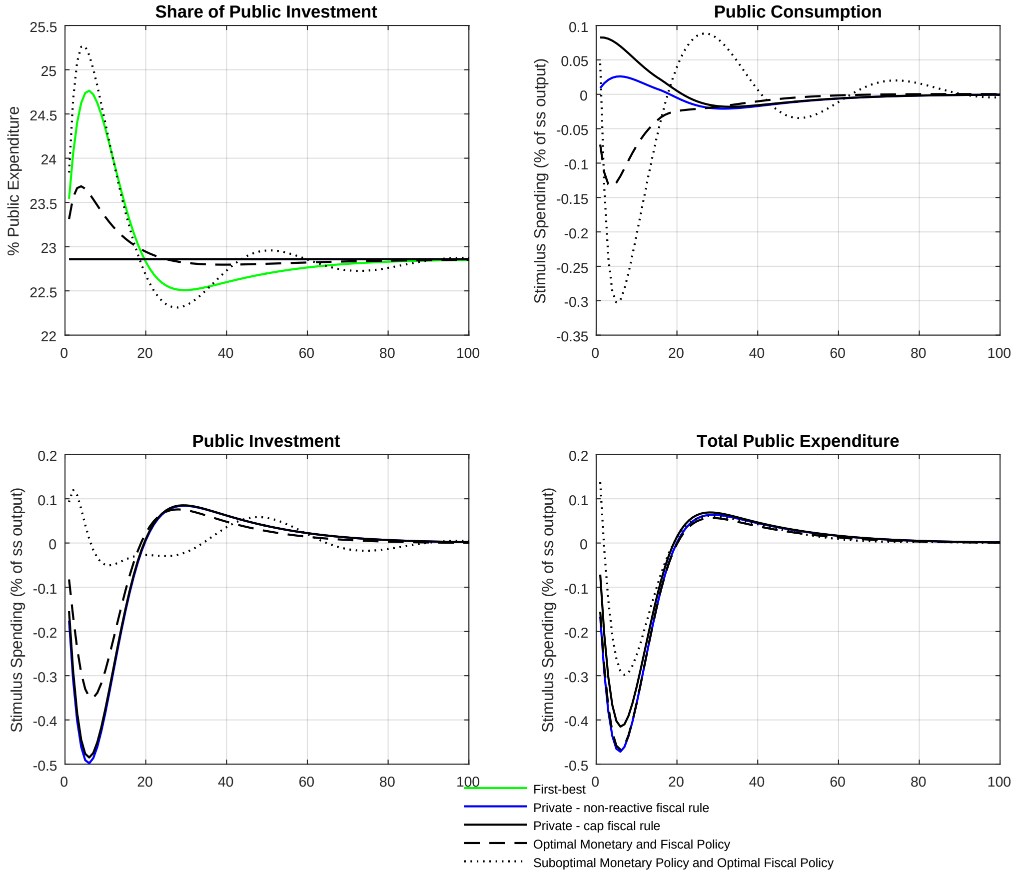

Figure 1.7: Public Expenditure Composition and Stimulus Spending: Ramsey policy under the cap

1.5.2.2.4 Optimal Monetary and Fiscal Policies under the Output Gap Responding Fiscal Rule

The output gap responding fiscal policy also represents a source of inefficiency in the economy, as the cap on real expenditure growth rule. In both cases, the Ramsey planner could not fully implement the first-best solution, even having the possibility to set the monetary policy optimally. Figure 1.8 shows that, in general, similar to the case of the mix of optimal monetary and fiscal policies under the cap rule, the deviations relative to the equilibrium values of the economic variables are lower than those of the efficient policy, excepting the nominal interest rate.

Contrary to the results so far, the initial impact on consumption is positive. It persists up to the 7th quarter following the shock, accumulating over-consumption of 0.40% of equilibrium output. Public expenditures scaled up in the same direction as private consumption, but its response is tiny and accumulates about 0.09% of steady-state output during the first six quarters following the shock. About 80% of public spending response is due to public investment, giving the higher sensibility of its parameter to the output gap.

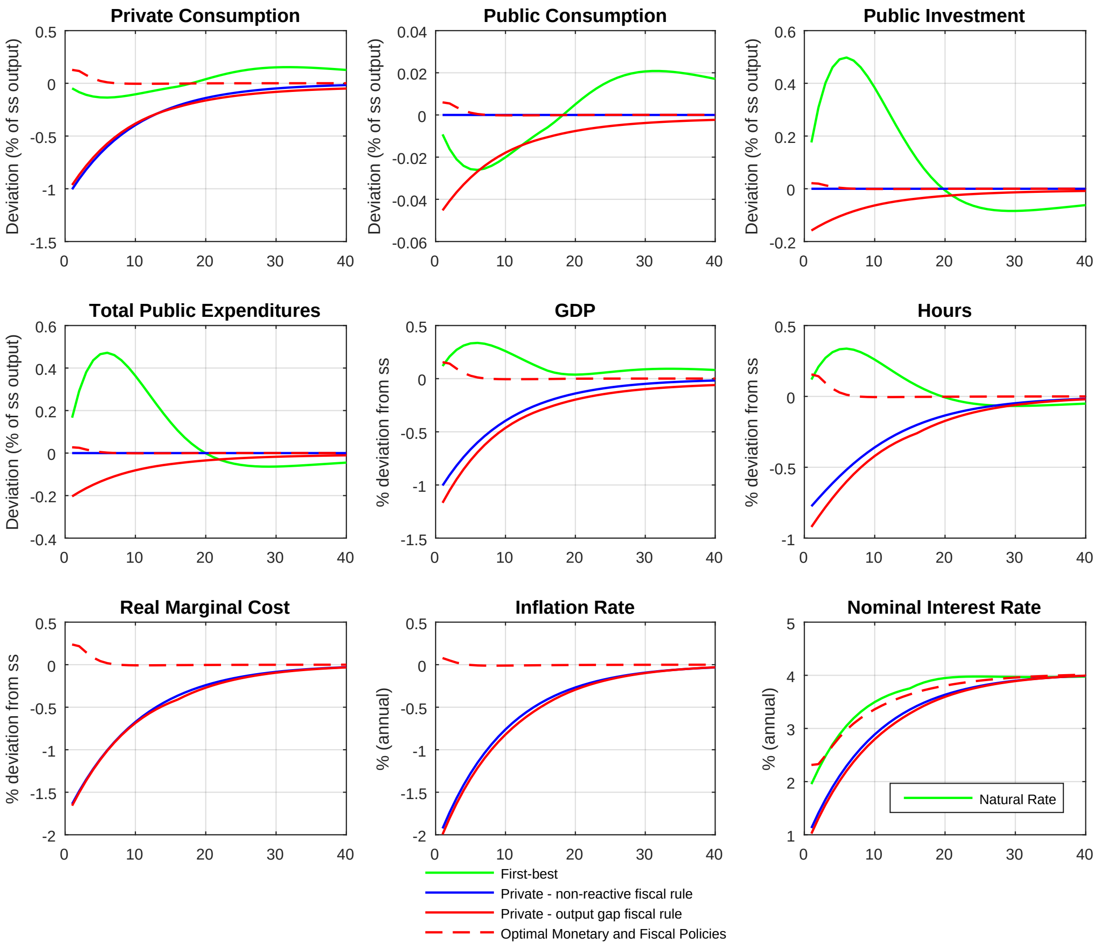

Figure 1.8: Response to a preference shock: Optimal monetary and fiscal policies under the output gap rule

The economic activity on impact stays 0.16% above the equilibrium level, exceeding about 30.54% the increase attained by the first-best response. After that peak, it follows a convergence path to equilibrium, with deviations being close to zero along the second year.

During the first three quarters, the monetary policy response following the shock is lower than the natural rate due to an inflationary transition that lasts about one year. This inflation dynamics reflects the positive response of private consumption. The interest rate registers a transitional period below the natural rate, and inflation shows a tiny negative gap relative to its equilibrium before the shock's complete dissipation. As a consequence of the positive portrait in the economic activity, hours of work respond positively (0.16% on impact) and persists seven quarters following the shock, after which it registers a tiny negative deviation from equilibrium. Real marginal cost increases on impact 0.24%, in similar dynamics as that of the hours of work.

1.5.2.2.5 Sub-optimal Monetary Policy and Optimal Fiscal Policy under the Output Gap Responding Fiscal Rule

Now the Ramsey Planner set the monetary policy according to a Taylor rule. The fiscal policy is set optimally following the output gap responding rule. From the FOCs derived in Appendix E.2, I can prove the following proposition.

Proposition: Consider an economy well described by the New-Keynesian model settled up in section 1.3. Suppose the fiscal authority follows a discretionary policy according to which public spending responds to the output gap (equation 1.19), and the monetary authority sets interest rate following a Taylor rule that responds to inflation deviation from its equilibrium (equation 1.21). Then the Ramsey problem responses to a preference shock coincide with those of the decentralized economy.

Proof: The private equilibrium economy has a closed solution because the number of equilibrium equations ($1.5b, 1.16', 1.19$ for G_t^c and $G_t^i, 1.20, 1.21, 1.28$) equals the number of unknowns variables ($C_t, N_t, G_t^c, G_t^i, K_{t+T}^g, \pi_t, R_t$). Once Blanchard-Kahn (Blanchard and Kahn, 1980) condition for uniqueness is satisfied, the solution to the Ramsey problem is the same as that of the private equilibrium.

1.5.2.2.6 Ramsey Composition and Stimulus Spending under the Output Gap Responding Fiscal Rule

The baseline parametrization of the output gap responding rule fixes a higher degree of sensibility to public investment (elasticity 3) than public consumption (0.25). The implication is that the Ramsey solution to the mix of sub-optimal monetary policy and optimal fiscal policy under the output gap rule, which coincides with the decentralized economy equilibrium, is grounded on adjustments in the investment level. I refer to section 1.5.2.1, which discusses the decentralized economy dynamic after challenging a preference shock. Summarily, the share of public investment reduces on impact by 2.47% and stays below the equilibrium up to the shock's dissipation. As a result, the contraction on the total public spending reaches 2.76% of the equilibrium output, with 77.9% due to investment cuts.

When the monetary policy is set optimally, the optimal solution predicts a tiny increase in the public investment share that lasts about eight quarters following the shock. Fiscal expansion sums to 0.10% of steady-state output, with investment accounting for almost 80% of that public resources.

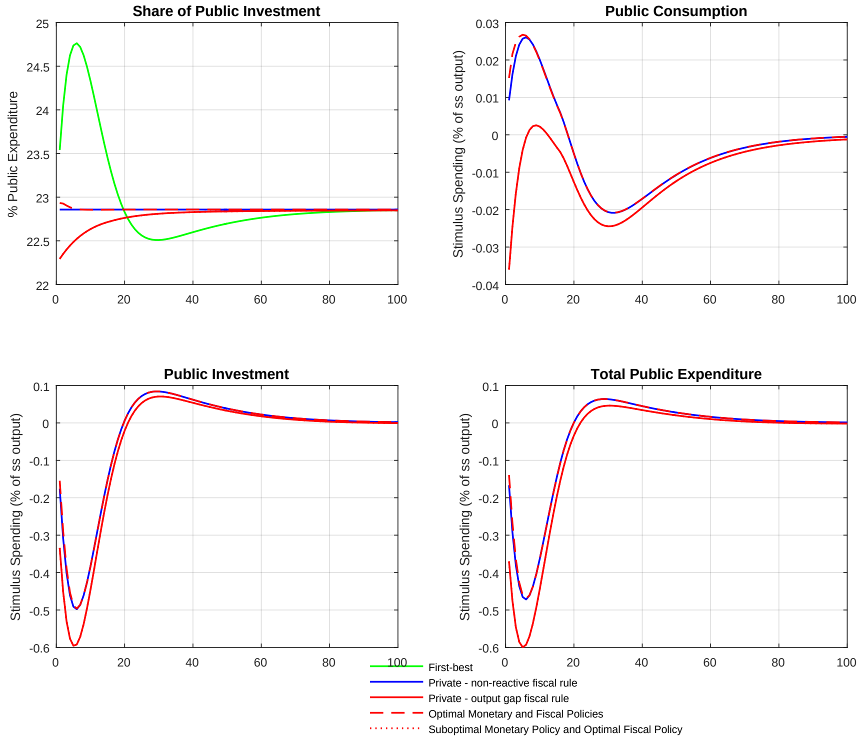

Figure 1.9: Public Expenditure Composition and Stimulus Spending: Ramsey policy under the output gap rule

Concerning stimulus spending, one can see that both consumption and investment have moments of positive stimuli. In being optimal monetary policy attainable, just after the shock follows about four years of tiny positive public consumption stimulus spending. Then, a negative stimulus appears up to the convergence to the equilibrium. In turn, in general, public investment shows a negative stimulus that lasts until the third quarter of the fifth year. A period of positive stimulus follows before the shock's dissipation. Even with public investment increasing its share in spending composition, it is clearly below the one prescribed under the full-flexible price economy. When monetary policy is set sub-optimally, public consumption situates below the efficient full-flexible price equilibrium almost all the time, except a tiny positive stimulus between the seventh quarter and the end of the third year. The investment stimulus spending behavior is similar to the case monetary policy is set optimally, with a tiny higher negative stimulus until the third quarter of the fifth year and a lower positive stimulus after that period.

In cumulative terms, when the monetary authority follows a Taylor rule, the Ramsey optimal policy predicts a government negative spending stimulus of 6.17% of equilibrium output, most of which (about 84%) due to a response of public investment in the opposite direction to the first-best policy. Whether the monetary authority sets interest rate optimally, spending stimulus is still negative, by 3.31% of equilibrium output. The picture is similar to the case the authority follows a Taylor rule, with investment responding by 89% of that number.

1.6 Welfare Analysis

This section explores the welfare gains associated with the efficient solution's attainability against the private equilibrium and the Ramsey optimal policies explored in the previous section. I follow the Bouakez et al. (2017b) approach, widely used in the literature, and compute the compensating variation in private consumption under an alternative economic environment targeting households as satisfied as to the consumption attainable when the first-best solution is implementable. Table 1.2 summarizes the results.

One can note that, among the economic environments studied in this chapter, the highest welfare gain of the first-best policy is obtained against the private equilibrium environment (and the Ramsey solution to the mix of sub-optimal monetary policy and optimal fiscal policy) under the output gap responding rule. This gain is around 13.48% higher than that of the private equilibrium under a non-reactive fiscal policy rule. Compared to the private equilibrium under the cap on real expenditure growth rule, this gain is 17.71% higher. These numbers have implications for choosing a fiscal rule when the policymaker also weighs the impact on households' welfare to decide among alternative policies.

Still, the Ramsey solution to the mix of optimal monetary and fiscal policies under the cap delivers the lowest welfare loss compared to the efficient solution in terms of welfare. This loss is 34.39% lower than the one of the Ramsey solution to the mix of optimal monetary and fiscal policies under the output gap responding rule. These results imply that the output gap responding rule represents a more relevant inefficiency source when the planner can optimally set monetary policy.

The picture is similar when monetary policy is set sub-optimally. Whether the fiscal authority follows the cap on real expenditure growth rule, the Ramsey solution imposes a welfare loss of 34.41% lower than the Ramsey solution whether the fiscal policy obeys an output gap rule. Compared to the decentralized economy solution when fiscal authority follows the cap rule, the Ramsey solution represents a welfare loss of 22.80% lower.

Table 1.2: Welfare Results

Model	First-Best Gain
Decentralized equilibrium under the non-reactive fiscal rule	0.0155
Decentralized equilibrium under the cap on real expenditure growth	0.0150
Decentralized equilibrium under the output gap responding rule	0.0176
Ramsey mixing optimal monetary and fiscal policies under the cap rule	0.0004
Ramsey mixing sub-optimal monetary policy and optimal fiscal policy under the cap rule	0.0116
Ramsey mixing optimal monetary and fiscal policies under the output gap responding rule	0.0006
Ramsey mixing sub-optimal monetary policy and optimal fiscal policy under the output gap responding rule	0.0176

The result that the private equilibrium at which the monetary authority sets nominal interest rate according to a Taylor rule and the fiscal policy follows an output gap responding rule coincides with the Ramsey solution seems very attractive from a policymaker perspective. It is so because it conciliates a simple and understandable fiscal rule with an optimal policy rule.

Then, it is worth evaluating the possibility of reducing its associated relative welfare loss. I perform a sensibility analysis on the output gap responding rule parameters ρ_j to investigate the behavior of this welfare loss compared to the efficient (first-best) allocation. To this aim, I implement a grid search on the welfare gain varying the domain of the ρ_j parameters such that: $(\rho_{G^i}, \rho_{G^c}) = \mathbb{R}^2 \backslash \{(\rho_{G^i}, \rho_{G^c}) : \rho_{G^i} \in [-5, 5];$ and $\rho_{G^c} \in [-3, 3]\}$. These ranges are sufficiently wide to cover admissible elasticities to public consumption and investment.

Figure 1.10 shows the simulation results over the decentralized economy at which the monetary authority follows a Taylor rule and the fiscal authority pursues an output gap responding rule. Again, this decentralized economy presents an equilibrium that coincides with the optimal Ramsey policy mixing sub-optimal monetary policy and optimal fiscal policy under the output gap rule. I highlight three points in this figure.

The "baseline calibration" one corresponds to the pair of parameters $[\rho_{G^i}, \rho_{G^c}]$ used to perform the results presented so far. They coincide with historical evidence on Brazilian data and are consistent with a procyclical public spending policy.

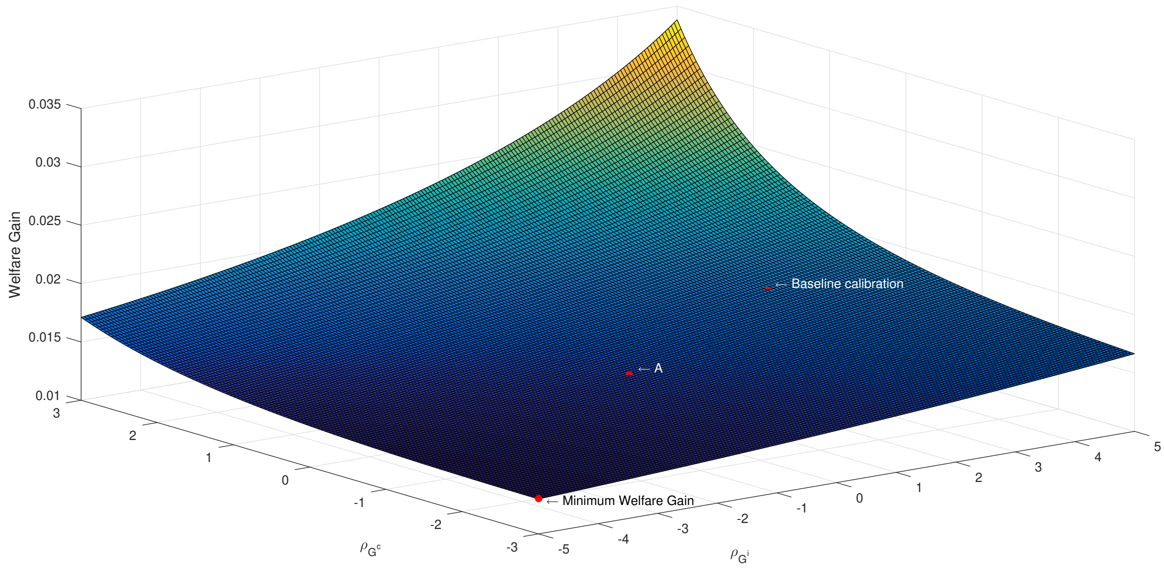

Figure 1.10: Welfare Gain: sensibility ρ_{G^c} and ρ_{G^i} - Decentralized Economy under the output gap rule

The point "minimum welfare gain" is the one that minimizes the welfare loss against the first-best allocation (equal to 0.0130). The relation is clear: the welfare loss is inversely proportional to a countercyclical fiscal policy, at least to the range of parameters ρ_j considered in this simulation. It does not mean this relationship is true to whatever set of parameters ρ_j associated with that countercyclical policy. Here I limit the range of the simulation to cover even more than what would be reasonable to expect whether a government has sufficient capability to challenge output gap deviations from equilibrium. Note that even though the range is quite wide to capture parameters that would cover a high degree of countercyclical fiscal policy, the results do not generate a welfare loss lower than that attainable under the Ramsey mix of sub-optimal monetary policy and optimal fiscal policy under the cap rule (equal to 0.0116 according to table 1.2).

The point "A" represents the maximum welfare loss (under the output gap rule) lower than the decentralized economy's welfare loss under the cap on expenditure growth rule. It means, *ceteris paribus*. that the decentralized economy (or Ramsey mix) with sub-optimal monetary policy and output gap fiscal rule instrumentalized with countercyclical pair of parameters $[\rho_{G^i}, \rho_{G^c}] = [-0.4, -0.6]$ would allow a level of welfare loss lower than that attainable by the decentralized economy operating the cap rule.

Next, I perform the same analysis for the Ramsey mix of optimal monetary and fiscal policies under the output gap responding rule. The aim is to check whether it is possible to get a pair of parameters $[\rho_{G^i}, \rho_{G^c}]$ to which the optimal solution is superior to the one attainable with the same mix of Ramsey policies but under the cap on real expenditure growth rule. Figure 1.11 summarizes the results.

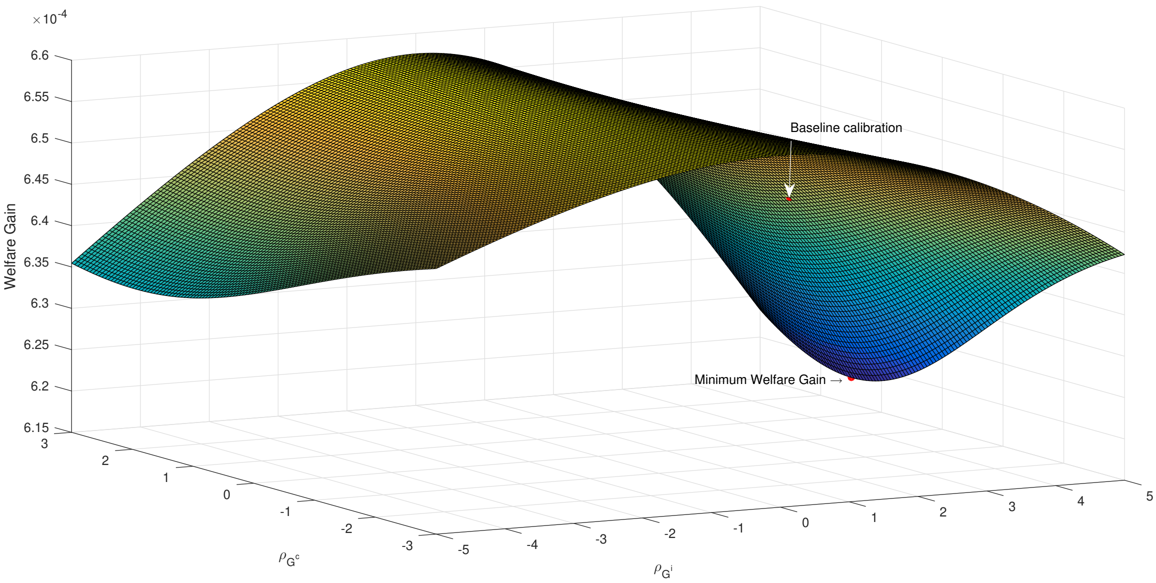

Figure 1.11: Welfare Gain: sensibility ρ_{G^c} and ρ_{G^i} - Optimal Monetary and Fiscal Policies under the output gap rule

I highlight two points in this figure. The first one is the baseline parameters ρ_j. The other is the "minimum welfare gain," which gives the minimum loss compared to the first-best welfare allocation. And here, this point corresponds to $[\rho_{G^i}, \rho_{G^c}] = [5, 1.5]$. Some comments on this result. First, it is not the case that the sensibility parameters ρ_j necessarily need to converge to higher negative numbers to deliver a lower welfare loss. Second, when monetary and fiscal policies under the output gap rule are optima, the Ramsey solution's welfare results show a non-linearity concerning that pair of sensibility parameters. Third, this Ramsey mix does not generate a welfare loss inferior to that attainable under the Ramsey mix of optimal monetary and fiscal policies under the cap, considering the domain I simulate the combination of sensibility parameters.

These simulations allow me to argue that it is not possible to ensure that, in general, to pursue a countercyclical spending policy necessarily results in welfare gains. While this seems to be the case to the mix of sub-optimal monetary and optimal fiscal policy under the output gap responding rule, it can still generate worse welfare results when optimal (Ramsey) monetary and fiscal policies are attainable. These results are in line with the Sims and Wolf (2018) findings that do not prescribe the adoption of a countercyclical spending policy as a general rule.

1.7 Final Remarks

In this work, I extended the small-scale New-Keynesian model of Bouakez et al. (2017b). However, unlike them, I abstracted from the liquidity trap under a deep recession. The focus of this work was on the assessment of public spending composition dynamics under distinct fiscal rules. My motivation was the adoption of a cap on real expenditure growth fiscal rule in Brazil in 2016. Beyond the cap rule, I explore a non-reactive rule and an output gap responding rule.

First, I explored the decentralized equilibria environments mixing that different fiscal rules with a monetary authority following a Taylor rule to sustain decisions on the establishment of nominal interest rates. Second, I worked on Ramsey solutions mixing optimal or sub-optimal monetary policy and optimal fiscal policy under the cap and the output gap responding rules. Finally, I assessed welfare results and performed simulations to a range of sensibility parameters of public consumption and investment in the output gap responding rule.

Concerning the cyclical behavior, while the cap rule features a countercyclical spending rule, the output gap responding policy depends on public expenditures' sensibility parameters. My baseline calibration for Brazil for the latter rule suggests that a procyclical spending policy matches the historical data. This characteristic determines the response of fiscal policy after a preference shock hitting the economy. Under the cap, the on impact economic activity's response is almost 21% lower than under the output gap responding policy. In terms of fiscal expansion following the shock, the picture for the three private equilibria environments studied shows that only the cap rule prescripts an expansionary policy, but less than 30% of the one suggested by the efficient policy. Notwithstanding, regarding stimulus spending, defined as the expansion beyond that under the efficient full-flexible price economy, the results show that even under the cap rule, it is both negative and grounded on a poor role of public investment as an instrument to challenge the shock.

The Ramsey solutions to the mix of optimal or sub-optimal monetary policy and optimal fiscal policy under the cap point that, when monetary policy follows a Taylor rule, the response of public investment is more pronounced than when the monetary authority sets the nominal interest rate optimally. In the latter, public investment accumulates a negative stimulus spending of 1.78% of equilibrium output, while in the former, it is positive (0.43%). These results suggest that the higher output gap the government challenges, when constrained to implement optimal monetary policy, opens space to public investment to address the preference shock more effectively. Total public spending stimulus is negative whatever the monetary policy is set optimally or not. That means the increase in public investment is attainable by reallocating public spending.

Regarding the Ramsey problems under the output gap responding rule, I showed that the Ramsey solution when the monetary authority follows a Taylor rule coincides with the decentralized economy counterpart. The results reveal that the role of public investment is tiny when monetary policy is set optimally and contractionary when a Taylor rule dictates the establishment of the nominal interest rate. Comparing to the full-flexible price economy response, it is the case that stimulus spending is negative whatever the setup of monetary policy and that about 80% comes from the investment-type public expenditure.

The welfare analysis suggests that the first-best allocation obtains a higher welfare gain against the private equilibrium under the output gap responding rule, which coincides with the Ramsey solution to the mix of sub-optimal monetary policy and optimal fiscal policy. Among the cases studied here, the lower welfare loss is attainable when the monetary and fiscal policies are set optimally under the cap on real expenditure rule.

By performing some simulations, I found that it is possible to variate the sensibility parameters of the output gap responding rule under the decentralized economy (identical to the mix of sub-optimal monetary policy and optimal fiscal policy) and get a welfare loss lower than that under the private equilibrium under the cap on real expenditure rule. To this end, the sensibility parameters need to respond countercyclically to the output gap. However, simulations do not support extending that finding to the Ramsey solution mixing sub-optimal monetary policy and optimal fiscal policy under the cap.

In another simulation, I performed the same exercise to verify whether I could support a similar conclusion when comparing the Ramsey solution to the mix of optimal monetary and fiscal policies under the output gap responding rule against the same mix of policies under the cap rule. I could not find a pair of sensibility parameters to the output gap rule, on a reasonable range, that supports a welfare loss lower than that obtained to the Ramsey problem with the cap rule. Beyond that, I found that I can reduce the welfare loss under the output gap (against its baseline calibration), even increasing the sensibility parameters' procyclicality feature. It suggests that a countercyclical spending rule can not be a general prescription as a policy instrument.

For future research, I highlight the relevance of a more complex setup of the fiscal instance financing side (debt, distortionary taxation), an aspect commented in Bouakez et al. (2017b). Specifically, to assess distinct fiscal rules, I mention the importance of studying the transition dynamics when a government promotes a fiscal consolidation by the spending side, contrasting with a consolidation invoking an increase in taxation. I also leave for future research policy responses to other kinds of shocks, like productivity and monetary ones.

Appendix: Steady-state computations

A: Benchmark Model

A.1: Steady-state

$$\text{From } (1.30c): \quad \frac{\gamma\big(C^{\gamma}(1-N)^{1-\gamma}\big)^{1-\sigma}}{C} = \chi {G^{c}}^{-\sigma}$$
$$\text{From } (1.30b): \quad a\frac{Y}{N} = \frac{1-\gamma}{\gamma}\frac{C}{1-N}$$
$$\text{From } (1.30d) \text{ and } (1.30e): \quad b\frac{Y}{K^g} = \frac{1-\beta(1-\delta)}{\beta^T}$$
$$\text{From } (1.30f): \quad Y = N^a {K^g}^b$$
$$\text{Still from } (1.30f): \quad Y = C + G^c + G^i$$
$$\text{From } (1.30g): \quad G^i = \delta K^g$$

This benchmark problem is a system of 6 equations on 6 unknowns variables $\{C, G^c, G^i, K^g, N, Y\}$. The set of parameters are $\{a, b, \gamma, \sigma, \chi, \beta, \delta, T\}$. Following Bouakez et al. (2017b) I calibrate the parameters χ and γ to match the steady-state fraction of work N and the steady-state GDP share of public spending on G^c and G^i:

$$g \equiv \frac{G^c + G^i}{Y}$$

B: Optimal Monetary and Fiscal Policies under the Cap on Real Expenditure Growth Constraint

The problem of the Ramsey planner is to find optimal allocations along the competitive equilibrium. To this goal, the Lagrangian problem is the following:

$$L(C_t, N_t, G^c_t, G^i_t, K^g_{t+T}, \pi_t, R_t, \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \Bigg[U\big(C_t, N_t\big) +$$

$$+ V\big(G^c_t\big)\Bigg] - \lambda_{1,t}\Bigg[\beta\Gamma_{t,t+1}\bigg(\frac{1+R_t}{1+\pi_{t+1}}\bigg) - 1\Bigg] +$$

$$- \lambda_{2,t}\Bigg[\psi\Bigg[(1+\pi_t)\pi_t - \beta\Gamma_{t,t+1}(1+\pi_{t+1})\pi_{t+1}$$

$$\bigg(\frac{a_{t+1}F(N_{t+1}, K^g_{t+1})}{a_t F(N_t, K^g_t)}\bigg)\Bigg] + \theta\bigg(1 + \frac{U_N(C_t, N_t)}{U_C(C_t, N_t) a_t F_N(N_t, K^g_t)}\bigg)\Bigg] +$$

$$- \lambda_{3,t}\Bigg[C_t + G^c_t + G^i_t - \Delta_t a_t F(N_t, K^g_t)\Bigg] +$$

$$(1.31)$$

$$- \lambda_{4,t}\Bigg[K^g_{t+T} - (1-\delta)K^g_{t+T-1} - \bigg(1 - \frac{\overline{\omega}}{2}\bigg(1 - \frac{G^i_t}{G^i_{t-1}}\bigg)^2\bigg)G^i_t\Bigg]\Bigg\} +$$

$$- \lambda_{5,t}\Bigg[\bigg(\frac{1+\pi_{t-1}}{1+\pi_t}\bigg)\big(G^c_{t-1} + G^i_{t-1}\big) + \tau\bigg(\bigg(\frac{1+\pi_{t-1}}{1+\pi_t}\bigg)a_{t-1}F(N_{t-1}, K^g_{t-1}) +$$

$$- a_t F(N_t, K^g_t)\bigg) - G^c_t - G^i_t\Bigg]$$

B.1: Equilibrium Conditions

Ramsey FOCs:

$$[C_t]: \quad \xi_t U_{C,t} + \bigg(\frac{U_{CC,t}}{U_{C,t}}\bigg)(\lambda_{1,t} - \beta^{-1}\lambda_{1,t-1}) - \psi\bigg(\frac{U_{CC,t}}{U_{C,t}}\bigg)\Bigg[\lambda_{2,t}\beta E_t\Bigg\{\Gamma_{t,t+1}$$

$$(1+\pi_{t+1})\pi_{t+1}\bigg(\frac{a_{t+1}F_{t+1}}{a_t F_t}\bigg)\Bigg\} - \lambda_{2,t-1}\Gamma_{t-1,t}(1+\pi_t)\pi_t\Big(\frac{a_t F_t}{a_{t-1}F_{t-1}}\Big)\Bigg] +$$

$$+ \theta\lambda_{2,t}\Big(\frac{U_{NC,t}}{U_{N,t}} - \frac{U_{CC,t}}{U_{C,t}}\Big)mc_t - \lambda_{3,t} = 0 \qquad (1.31a)$$

$$[N_t]: \quad \xi_t U_{N,t} + \bigg(\frac{U_{CN,t}}{U_{C,t}}\bigg)(\lambda_{1,t} - \beta^{-1}\lambda_{1,t-1}) - \psi\Bigg[\lambda_{2,t}E_t\Bigg\{\beta\Gamma_{t,t+1}(1+\pi_{t+1})\pi_{t+1}$$

$$\bigg(\frac{a_{t+1}F_{t+1}}{a_t F_t}\bigg)\Bigg\} - \lambda_{2,t-1}\Gamma_{t-1,t}(1+\pi_t)\pi_t\bigg(\frac{a_t F_t}{a_{t-1}F_{t-1}}\bigg)\Bigg]\bigg(\frac{U_{CN,t}}{U_{C,t}} + \frac{F_{N,t}}{F_t}\bigg) +$$

$$+ \theta\lambda_{2,t} mc_t\Bigg[\frac{U_{NN,t}}{U_{N,t}} - \frac{U_{CN,t}}{U_{C,t}} - \frac{F_{NN,t}}{F_{N,t}}\Bigg] + \lambda_{3,t}\Delta_t a_t F_{N,t} +$$

$$+ \tau a_t F_{N,t}\Bigg[\lambda_{5,t} - \beta E_t\Bigg\{\lambda_{5,t+1}\bigg(\frac{1+\pi_t}{1+\pi_{t+1}}\bigg)\Bigg\}\Bigg] = 0 \qquad (1.31b)$$

$$[G^c_t]: \quad \xi_t V_{G^c,t} - \lambda_{3,t} + \Bigg[\lambda_{5,t} - \beta E_t\Bigg\{\lambda_{5,t+1}\bigg(\frac{1+\pi_t}{1+\pi_{t+1}}\bigg)\Bigg\}\Bigg] = 0 \qquad (1.31c)$$

$$[G_t^i]: \quad -\lambda_{3,t} + \lambda_{5,t} - \beta E_t\Big\{\lambda_{5,t+1}\Big(\frac{1+\pi_t}{1+\pi_{t+1}}\Big)\Big\} + \lambda_{4,t}\Big[1 - \frac{\overline{\omega}}{2}\Big(1 - \frac{G_t^i}{G_{t-1}^i}\Big)^2 +$$

$$+ \overline{\omega}\Big(1 - \frac{G_t^i}{G_{t-1}^i}\Big)\Big(\frac{G_t^i}{G_{t-1}^i}\Big)\Big] +$$

$$- \beta E_t\Big\{\lambda_{4,t+1}\overline{\omega}\Big(1 - \frac{G_{t+1}^i}{G_t^i}\Big)\Big(\frac{G_{t+1}^i}{G_t^i}\Big)^2\Big\} = 0 \tag{1.31d}$$

$$[K_{T+t}^g]: \quad -\lambda_{4,t} + \beta(1-\delta)E_t\lambda_{4,t+1} - \beta^T\theta E_t\Big\{\lambda_{2,t+T}mc_{t+T}\Big(\frac{F_{NK,t+T}}{F_{N,t+T}}\Big)\Big\} +$$

$$- \beta^T\psi\Big[\beta E_t\Big\{\lambda_{2,t+T}\Gamma_{t+T,t+T+1}(1+\pi_{t+T+1})\pi_{t+T+1}\Big(\frac{a_{t+T+1}F_{t+T+1}}{a_{t+T}F_{t+T}}\Big)$$

$$\Big(\frac{F_{K,t+T}}{F_{t+T}}\Big)\Big\} - E_t\Big\{\lambda_{2,t+T-1}\Gamma_{t+T-1,t+T}(1+\pi_{t+T})\pi_{t+T}\Big(\frac{a_{t+T}F_{t+T}}{a_{t+T-1}F_{t+T-1}}\Big)$$

$$\Big(\frac{F_{K,t+T}}{F_{t+T}}\Big)\Big\}\Big] + \beta^T E_t\{\lambda_{3,t+T}\Delta_{t+T}a_{t+T}F_{k,t+T}\} +$$

$$+ \tau\beta^T E_t\Big\{a_{t+T}F_{k,t+T}\Big[\lambda_{5,t+T} - \beta\lambda_{5,t+T+1}\Big(\frac{1+\pi_{t+T}}{1+\pi_{t+T+1}}\Big)\Big]\Big\} = 0$$

$$\tag{1.31e}$$

$$[\pi_t]: \quad \frac{\lambda_{1,t-1}}{\beta(1+\pi_t)} - \psi(2\pi_t+1)\Big[\lambda_{2,t} - \lambda_{2,t-1}\Gamma_{t-1,t}\Big(\frac{a_tF_t}{a_{t-1}F_{t-1}}\Big)\Big] +$$

$$- \psi\lambda_{3,t}\pi_t a_t F_t + \Big(\frac{1}{1+\pi_t}\Big)\Big[\lambda_{5,t}(G_t^c + G_t^i + \tau a_t F_t) +$$

$$- \beta E_t\Big\{\lambda_{5,t+1}(G_{t+1}^c + G_{t+1}^i + \tau a_{t+1}F_{t+1})\Big\}\Big] = 0 \tag{1.31f}$$

$$[R_t]: \quad -\lambda_{1,t}E_t\Big\{\beta\Gamma_{t,t+1}\Big(\frac{1}{1+\pi_{t+1}}\Big)\Big\} = 0 \tag{1.31g}$$

$$[\lambda_{1,t}]: \quad E_t\Big\{\beta\Gamma_{t,t+1}\Big(\frac{1+R_t}{1+\pi_{t+1}}\Big)\Big\} = 1 \tag{1.31h}$$

$$[\lambda_{2,t}]: \quad \psi(1+\pi_t)\pi_t - \psi\beta E_t\Big\{\Gamma_{t,t+1}(1+\pi_{t+1})\pi_{t+1}\Big(\frac{a_{t+1}F_{t+1}}{a_tF_t}\Big)\Big\} +$$

$$+ \theta(1 - mc_t) = 0 \tag{1.31i}$$

$$[\lambda_{3,t}]: \quad C_t + G_t^c + G_t^i - \Delta_t a_t F_t = 0 \tag{1.31j}$$

$$[\lambda_{4,t}]: \quad K_{t+T}^g = (1-\delta)K_{t+T-1}^g + \Big[1 - \frac{\overline{\omega}}{2}\Big(1 - \frac{G_t^i}{G_{t-1}^i}\Big)\Big]G_t^i \tag{1.31k}$$

$$[\lambda_{5,t}]: \quad G_t^c + G_t^i = \Big(\frac{1+\pi_{t-1}}{1+\pi_t}\Big)(G_{t-1}^c + G_{t-1}^i) +$$

$$+ \tau\Big(\Big(\frac{1+\pi_{t-1}}{1+\pi_t}\Big)a_{t-1}F(N_{t-1}, K_{t-1}^g) - a_tF(N_t, K_t^g)\Big) \tag{1.31l}$$

B.2: Steady-state

In a zero-inflation steady-state, we can write:

$$
\begin{aligned}
&\text{From}[C]: && U_C + \lambda_1\left(1-\frac{1}{\beta}\right)\frac{U_{CC}}{U_C} + \theta\lambda_2\left[\frac{U_{NC}}{U_N} - \frac{U_{CC}}{U_C}\right] - \lambda_3 = 0 \\
&\text{From}[N]: && U_N + \lambda_1\left(1-\frac{1}{\beta}\right)\frac{U_{CN}}{U_C} + \theta\lambda_2\left[\frac{U_{NN}}{U_N} - \frac{U_{CN}}{U_C} - \frac{F_{NN}}{F_N}\right]mc+ \\
& && + \lambda_3 F_N + \lambda_5 \tau F_N(1-\beta) = 0 \\
&\text{From}[G^c]: && V_{G^c} = \lambda_3 - \lambda_5(1-\beta) \\
&\text{From}[G^i]: && \lambda_4 = \lambda_3 - \lambda_5(1-\beta) \\
&\text{From}[K^g]: && \lambda_4[1-\beta(1-\delta)] = -\beta^T\theta\lambda_2 mc\frac{F_{NK^g}}{F_N} + \\
& && + \beta^T\lambda_3 F_{K^g} + \tau\beta^T\lambda_5(1-\beta)F_{K^g} \\
&\text{From}[\pi]: && \lambda_5(1-\beta)(G^c + G^i + \tau F) = 0 \\
&\text{From}[R]: && \lambda_1\beta = 0 \\
&\text{From}[\lambda_1]: && 1 + R = \frac{1}{\beta} \\
&\text{From}[\lambda_2]: && mc = 1 \\
&\text{From}[\lambda_3]: && Y = C + G^c + G^i \\
&\text{From}[\lambda_4]: && G^i = \delta K^g \\
&\text{From}[\lambda_5]: && G^c + G^i = G^c + G^i
\end{aligned}
$$

From $[\pi]$, $\lambda_5 = 0$, and from $[R]$, $\lambda_1 = 0$. This implies, from $[C]$ and $[N]$, that $\lambda_2 = 0$. And from $[C]$, $[G^c]$ and $[G^i]$, follows: $U_C = V_{G^c} = \lambda_3 = \lambda_4$. Finally, from K^g, you get the steady-state marginal product of public capital:

$$F_{K^g} = \frac{1-\beta(1-\delta)}{\beta^T}$$

The equilibrium allocations is then identical to the first-best ones, which does not surprise given that under the zero-inflation set up any distortion vanishes in equilibrium.

C: Sub-optimal Monetary Policy and Optimal Fiscal Policy under the Cap on Real Expenditure Growth Constraint

The problem is the same as that established at (1.31), except that now I am constraining the set of actions monetary authority is able to implement. Instead of setting optimally the interest rate, now Central Bank set R_t following the Taylor rule (1.21). The Ramsey Planner solves the following problem:

$$L(C_t, N_t, G_t^c, G_t^i, K_{t+T}^g, \pi_t, R_t, \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \Bigg[U\big(C_t, N_t\big) +$$
$$+ V\big(G_t^c\big)\Bigg] - \lambda_{1,t}\Bigg[\beta\Gamma_{t,t+1}\Bigg(\frac{1+R_t}{1+\pi_{t+1}}\Bigg) - 1\Bigg] +$$
$$- \lambda_{2,t}\Bigg[\psi\Bigg[(1+\pi_t)\pi_t - \beta\Gamma_{t,t+1}(1+\pi_{t+1})\pi_{t+1}$$
$$\Bigg(\frac{a_{t+1}F(N_{t+1}, K_{t+1}^g)}{a_t F(N_t, K_t^g)}\Bigg)\Bigg] + \theta\Bigg(1 + \frac{U_N(C_t, N_t)}{U_C(C_t, N_t) a_t F_N(N_t, K_t^g)}\Bigg)\Bigg] +$$
$$- \lambda_{3,t}\Bigg[C_t + G_t^c + G_t^i - \Delta_t a_t F(N_t, K_t^g)\Bigg] + \tag{1.32}$$
$$- \lambda_{4,t}\Bigg[K_{t+T}^g - (1-\delta)K_{t+T-1}^g - \Bigg(1 - \frac{\overline{\omega}}{2}\Bigg(1 - \frac{G_t^i}{G_{t-1}^i}\Bigg)^2\Bigg)G_t^i\Bigg]\Bigg\} +$$
$$- \lambda_{5,t}\Bigg[\Bigg(\frac{1+\pi_{t-1}}{1+\pi_t}\Bigg)(G_{t-1}^c + G_{t-1}^i) + \tau\Bigg(\Bigg(\frac{1+\pi_{t-1}}{1+\pi_t}\Bigg)a_{t-1}F(N_{t-1}, K_{t-1}^g) +$$
$$- a_t F(N_t, K_t^g)\Bigg) - G_t^c - G_t^i\Bigg] +$$
$$- \lambda_{6,t}\Bigg[\frac{(1+\pi_t)^{\theta_\pi}}{\beta}\epsilon_t^R - 1 - R_t\Bigg]\Bigg\}$$

C.1: Equilibrium Conditions

Ramsey FOCs:

$$[C_t]: \quad \xi_t U_{C,t} + \Bigg(\frac{U_{CC,t}}{U_{C,t}}\Bigg)(\lambda_{1,t} - \beta^{-1}\lambda_{1,t-1}) - \psi\Bigg(\frac{U_{CC,t}}{U_{C,t}}\Bigg)\Bigg[\lambda_{2,t}\beta E_t\Bigg\{\Gamma_{t,t+1}$$
$$(1+\pi_{t+1})\pi_{t+1}\Bigg(\frac{a_{t+1}F_{t+1}}{a_t F_t}\Bigg)\Bigg\} - \lambda_{2,t-1}\Gamma_{t-1,t}(1+\pi_t)\pi_t\Big(\frac{a_t F_t}{a_{t-1}F_{t-1}}\Big)\Bigg] +$$
$$+ \theta\lambda_{2,t}\Big(\frac{U_{NC,t}}{U_{N,t}} - \frac{U_{CC,t}}{U_{C,t}}\Big)mc_t - \lambda_{3,t} = 0 \tag{1.32a}$$

$$[N_t]: \quad \xi_t U_{N,t} + \Bigg(\frac{U_{CN,t}}{U_{C,t}}\Bigg)(\lambda_{1,t} - \beta^{-1}\lambda_{1,t-1}) - \psi\Bigg[\lambda_{2,t}E_t\Bigg\{\beta\Gamma_{t,t+1}(1+\pi_{t+1})\pi_{t+1}$$
$$\Bigg(\frac{a_{t+1}F_{t+1}}{a_t F_t}\Bigg)\Bigg\} - \lambda_{2,t-1}\Gamma_{t-1,t}(1+\pi_t)\pi_t\Bigg(\frac{a_t F_t}{a_{t-1}F_{t-1}}\Bigg)\Bigg]\Bigg(\frac{U_{CN,t}}{U_{C,t}} + \frac{F_{N,t}}{F_t}\Bigg) +$$
$$+ \theta\lambda_{2,t}mc_t\Bigg[\frac{U_{NN,t}}{U_{N,t}} - \frac{U_{CN,t}}{U_{C,t}} - \frac{F_{NN,t}}{F_{N,t}}\Bigg] + \lambda_{3,t}\Delta_t a_t F_{N,t} +$$
$$+ \tau a_t F_{N,t}\Bigg[\lambda_{5,t} - \beta E_t\Bigg\{\lambda_{5,t+1}\Bigg(\frac{1+\pi_t}{1+\pi_{t+1}}\Bigg)\Bigg\}\Bigg] = 0 \tag{1.32b}$$

$$[G_t^c]: \quad \xi_t V_{G^c,t} - \lambda_{3,t} + \left[\lambda_{5,t} - \beta E_t\left\{\lambda_{5,t+1}\left(\frac{1+\pi_t}{1+\pi_{t+1}}\right)\right\}\right] = 0 \tag{1.32c}$$

$$\begin{aligned}[G_t^i]: \quad & -\lambda_{3,t} + \lambda_{5,t} - \beta E_t\left\{\lambda_{5,t+1}\left(\frac{1+\pi_t}{1+\pi_{t+1}}\right)\right\} + \lambda_{4,t}\left[1 - \frac{\overline{\omega}}{2}\left(1 - \frac{G_t^i}{G_{t-1}^i}\right)^2 + \right. \\ & \left. + \overline{\omega}\left(1 - \frac{G_t^i}{G_{t-1}^i}\right)\left(\frac{G_t^i}{G_{t-1}^i}\right)\right] + \\ & - \beta E_t\left\{\lambda_{4,t+1}\overline{\omega}\left(1 - \frac{G_{t+1}^i}{G_t^i}\right)\left(\frac{G_{t+1}^i}{G_t^i}\right)^2\right\} = 0 \end{aligned} \tag{1.32d}$$

$$\begin{aligned}[K_{T+t}^g]: \quad & -\lambda_{4,t} + \beta(1-\delta)E_t\lambda_{4,t+1} - \beta^T\theta E_t\left\{\lambda_{2,t+T}mc_{t+T}\left(\frac{F_{NK^g,t+T}}{F_{N,t+T}}\right)\right\} + \\ & - \beta^T\psi\left[\beta E_t\left\{\lambda_{2,t+T}\Gamma_{t+T,t+T+1}(1+\pi_{t+T+1})\pi_{t+T+1}\left(\frac{a_{t+T+1}F_{t+T+1}}{a_{t+T}F_{t+T}}\right)\right.\right. \\ & \left.\left(\frac{F_{K^g,t+T}}{F_{t+T}}\right)\right\} - E_t\left\{\lambda_{2,t+T-1}\Gamma_{t+T-1,t+T}(1+\pi_{t+T})\pi_{t+T}\left(\frac{a_{t+T}F_{t+T}}{a_{t+T-1}F_{t+T-1}}\right)\right. \\ & \left.\left.\left(\frac{F_{K^g,t+T}}{F_{t+T}}\right)\right\}\right] + \beta^T E_t\{\lambda_{3,t+T}\Delta_{t+T}a_{t+T}F_{K^g,t+T}\} + \\ & + \tau\beta^T E_t\left\{a_{t+T}F_{K^g,t+T}\left[\lambda_{5,t+T} - \beta\lambda_{5,t+T+1}\left(\frac{1+\pi_{t+T}}{1+\pi_{t+T+1}}\right)\right]\right\} = 0 \end{aligned} \tag{1.32e}$$

$$\begin{aligned}[\pi_t]: \quad & \frac{\lambda_{1,t-1}}{\beta(1+\pi_t)} - \psi(2\pi_t+1)\left[\lambda_{2,t} - \lambda_{2,t-1}\Gamma_{t-1,t}\left(\frac{a_tF_t}{a_{t-1}F_{t-1}}\right)\right] + \\ & - \psi\lambda_{3,t}\pi_t a_t F_t + \left(\frac{1}{1+\pi_t}\right)\left[\lambda_{5,t}(G_t^c + G_t^i + \tau a_t F_t) + \right. \\ & \left. - \beta E_t\left\{\lambda_{5,t+1}(G_{t+1}^c + G_{t+1}^i + \tau a_{t+1}F_{t+1})\right\}\right] + \\ & - \lambda_{6,t}\frac{\theta_\pi(1+\pi_t)^{\theta_\pi - 1}}{\beta}\epsilon_t^R = 0 \end{aligned} \tag{1.32f}$$

$$[R_t]: \quad -\lambda_{1,t}E_t\left\{\beta\Gamma_{t,t+1}\left(\frac{1}{1+\pi_{t+1}}\right)\right\} + \lambda_{6,t} = 0 \tag{1.32g}$$

$$[\lambda_{1,t}]: \quad E_t\left\{\beta\Gamma_{t,t+1}\left(\frac{1+R_t}{1+\pi_{t+1}}\right)\right\} = 1 \tag{1.32h}$$

$$\begin{aligned}[\lambda_{2,t}]: \quad & \psi(1+\pi_t)\pi_t - \psi\beta E_t\left\{\Gamma_{t,t+1}(1+\pi_{t+1})\pi_{t+1}\left(\frac{a_{t+1}F_{t+1}}{a_tF_t}\right)\right\} + \\ & + \theta(1 - mc_t) = 0 \end{aligned} \tag{1.32i}$$

$$[\lambda_{3,t}]: \quad C_t + G_t^c + G_t^i - \Delta_t a_t F_t = 0 \tag{1.32j}$$

$$[\lambda_{4,t}]: \quad K_{t+T}^g = (1-\delta)K_{t+T-1}^g + \left[1 - \frac{\overline{\omega}}{2}\left(1 - \frac{G_t^i}{G_{t-1}^i}\right)\right]G_t^i \tag{1.32k}$$

$$\begin{aligned}[\lambda_{5,t}]: \quad & G_t^c + G_t^i = \left(\frac{1+\pi_{t-1}}{1+\pi_t}\right)(G_{t-1}^c + G_{t-1}^i) + \\ & + \tau\left(\left(\frac{1+\pi_{t-1}}{1+\pi_t}\right)a_{t-1}F(N_{t-1}, K_{t-1}^g) - a_tF(N_t, K_t^g)\right) \end{aligned} \tag{1.32l}$$

$$[\lambda_{6,t}]: \quad 1 + R_t = \frac{(1+\pi_t)^{\theta_\pi}}{\beta}\epsilon_t^R \tag{1.32m}$$

C.2: Steady-state

Under the zero-inflation steady-state, again the equilibrium allocations are identical to the ones under the first-best benchmark.

$$\text{From}[C]: \quad U_C + \lambda_1\left(1 - \frac{1}{\beta}\right)\frac{U_{CC}}{U_C} + \theta\lambda_2\left[\frac{U_{NC}}{U_N} - \frac{U_{CC}}{U_C}\right] - \lambda_3 = 0$$

$$\text{From}[N]: \quad U_N + \lambda_1\left(1 - \frac{1}{\beta}\right)\frac{U_{CN}}{U_C} + \theta\lambda_2\left[\frac{U_{NN}}{U_N} - \frac{U_{CN}}{U_C} - \frac{F_{NN}}{F_N}\right]mc +$$

$$+ \lambda_3 F_N + \lambda_5 \tau F_N(1-\beta) = 0$$

$$\text{From}[G^c]: \quad V_{G^c} = \lambda_3 - \lambda_5(1-\beta)$$

$$\text{From}[G^i]: \quad \lambda_4 = \lambda_3 - \lambda_5(1-\beta)$$

$$\text{From}[K^g]: \quad \lambda_4[1-\beta(1-\delta)] = -\beta^T\theta\lambda_2 mc\frac{F_{NK^g}}{F_N} +$$

$$+ \beta^T\lambda_3 F_{K^g} + \tau\beta^T\lambda_5(1-\beta)F_{K^g}$$

$$\text{From}[\pi]: \quad \lambda_5(1-\beta)(G^c + G^i + \tau F) = \lambda_1\left(\theta_\pi - \frac{1}{\beta}\right)$$

$$\text{From}[R]: \quad \lambda_6 = \beta\lambda_1$$

$$\text{From}[\lambda_1]: \quad 1 + R = \frac{1}{\beta}$$

$$\text{From}[\lambda_2]: \quad mc = 1$$

$$\text{From}[\lambda_3]: \quad Y = C + G^c + G^i$$

$$\text{From}[\lambda_4]: \quad G^i = \delta K^g$$

$$\text{From}[\lambda_5]: \quad G^c + G^i = G^c + G^i$$

$$\text{From}[\lambda_6]: \quad 1 + R = \frac{1}{\beta}$$

From $[C]$ and $[N]$, one can get $\lambda_1 = \lambda_2 = \lambda_5 = 0$. This implies, from $[R]$, that $\lambda_6 = 0$. And from $[C]$, $[G^c]$ and $[G^i]$, follows: $U_C = V_{G^c} = \lambda_3 = \lambda_4$. Finally, from K^g, you get the steady-state marginal product of public capital:

$$F_{K^g} = \frac{1-\beta(1-\delta)}{\beta^T}$$

D: Optimal Monetary and Fiscal Policies under the Output Gap Responding Fiscal Rule

The relevant Lagrangian is:

$$
\begin{aligned}
L(C_t, N_t, G_t^c, G_t^i, K_{t+T}^g, \pi_t, R_t, \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \Bigg[U\big(C_t, N_t\big) + \\
+ V\big(G_t^c\big) \Bigg] - \lambda_{1,t} \Bigg[\beta \Gamma_{t,t+1} \Big(\frac{1+R_t}{1+\pi_{t+1}} \Big) - 1 \Bigg] + \\
- \lambda_{2,t} \Bigg[\psi \Bigg[(1+\pi_t)\pi_t - \beta \Gamma_{t,t+1} (1+\pi_{t+1}) \pi_{t+1} \\
\Big(\frac{a_{t+1} F(N_{t+1}, K_{t+1}^g)}{a_t F(N_t, K_t^g)} \Big) \Bigg] + \theta \Big(1 + \frac{U_N(C_t, N_t)}{U_C(C_t, N_t) a_t F_N(N_t, K_t^g)} \Big) \Bigg] + \\
- \lambda_{3,t} \Bigg[C_t + G_t^c + G_t^i - \Delta_t a_t F(N_t, K_t^g) \Bigg] + \\
- \lambda_{4,t} \Bigg[K_{t+T}^g - (1-\delta) K_{t+T-1}^g - \Big(1 - \frac{\overline{\omega}}{2} \Big(1 - \frac{G_t^i}{G_{t-1}^i} \Big)^2 \Big) G_t^i \Bigg] + \\
- \lambda_{5,t} \Bigg[G^c \Big(\frac{\Delta_t a_t F(N_t, K_t^g)}{Y} \Big)^{\rho_{G^c}} - G_t^c \Bigg] + \\
- \lambda_{6,t} \Bigg[G^i \Big(\frac{\Delta_t a_t F(N_t, K_t^g)}{Y} \Big)^{\rho_{G^i}} - G_t^i \Bigg] \Bigg\}
\end{aligned}
\tag{1.33}
$$

D.1: Equilibrium Conditions

Ramsey FOCs:

$$
\begin{aligned}
[C_t]: \quad & \xi_t U_{C,t} + \Big(\frac{U_{CC,t}}{U_{C,t}} \Big) (\lambda_{1,t} - \beta^{-1} \lambda_{1,t-1}) - \psi \Big(\frac{U_{CC,t}}{U_{C,t}} \Big) \Bigg[\lambda_{2,t} \beta E_t \Bigg\{ \Gamma_{t,t+1} \\
& (1+\pi_{t+1}) \pi_{t+1} \Big(\frac{a_{t+1} F_{t+1}}{a_t F_t} \Big) \Bigg\} - \lambda_{2,t-1} \Gamma_{t-1,t} (1+\pi_t) \pi_t \big(\frac{a_t F_t}{a_{t-1} F_{t-1}} \big) \Bigg] + \\
& + \theta \lambda_{2,t} \big(\frac{U_{NC,t}}{U_{N,t}} - \frac{U_{CC,t}}{U_{C,t}} \big) mc_t - \lambda_{3,t} = 0
\end{aligned}
\tag{1.33a}
$$

$$
\begin{aligned}
[N_t]: \quad & \xi_t U_{N,t} + \left(\frac{U_{CN,t}}{U_{C,t}}\right)(\lambda_{1,t} - \beta^{-1}\lambda_{1,t-1}) - \psi\Bigg[\lambda_{2,t} E_t \Bigg\{\beta\Gamma_{t,t+1}(1+\pi_{t+1})\pi_{t+1} \\
& \left(\frac{a_{t+1}F_{t+1}}{a_t F_t}\right)\Bigg\} - \lambda_{2,t-1}\Gamma_{t-1,t}(1+\pi_t)\pi_t\left(\frac{a_t F_t}{a_{t-1}F_{t-1}}\right)\Bigg]\left(\frac{U_{CN,t}}{U_{C,t}} + \frac{F_{N,t}}{F_t}\right) + \\
& + \theta\lambda_{2,t} mc_t \left[\frac{U_{NN,t}}{U_{N,t}} - \frac{U_{CN,t}}{U_{C,t}} - \frac{F_{NN,t}}{F_{N,t}}\right] + \lambda_{3,t}\Delta_t a_t F_{N,t} + \\
& - \lambda_{5,t}\left(\frac{G^c}{Y^{\rho_{G^c}}}\right)\rho_{G^c}(\Delta_t a_t F_t)^{\rho_{G^c}-1}\Delta_t a_t F_{N,t} + \\
& - \lambda_{6,t}\left(\frac{G^i}{Y^{\rho_{G^i}}}\right)\rho_{G^i}(\Delta_t a_t F_t)^{\rho_{G^i}-1}\Delta_t a_t F_{N,t} = 0 \qquad (1.33b)
\end{aligned}
$$

$$
[G^c_t]: \quad \xi_t V_{G^c,t} - \lambda_{3,t} + \lambda_{5,t} = 0 \qquad (1.33c)
$$

$$
\begin{aligned}
[G^i_t]: \quad & -\lambda_{3,t} + \lambda_{4,t}\left[1 - \frac{\overline{\omega}}{2}\left(1 - \frac{G^i_t}{G^i_{t-1}}\right)^2 + \right. \\
& \left. + \overline{\omega}\left(1 - \frac{G^i_t}{G^i_{t-1}}\right)\left(\frac{G^i_t}{G^i_{t-1}}\right)\right] + \\
& - \beta E_t\left\{\lambda_{4,t+1}\overline{\omega}\left(1 - \frac{G^i_{t+1}}{G^i_t}\right)\left(\frac{G^i_{t+1}}{G^i_t}\right)^2\right\} + \lambda_{6,t} = 0 \qquad (1.33d)
\end{aligned}
$$

$$
\begin{aligned}
[K^g_{t+T}]: \quad & -\lambda_{4,t} + \beta(1-\delta)E_t\lambda_{4,t+1} - \beta^T\theta E_t\left\{\lambda_{2,t+T} mc_{t+T}\left(\frac{F_{NK^g,t+T}}{F_{N,t+T}}\right)\right\} + \\
& - \beta^T\psi\Bigg[\beta E_t\Bigg\{\lambda_{2,t+T}\Gamma_{t+T,t+T+1}(1+\pi_{t+T+1})\pi_{t+T+1}\left(\frac{a_{t+T+1}F_{t+T+1}}{a_{t+T}F_{t+T}}\right) \\
& \left(\frac{F_{K^g,t+T}}{F_{t+T}}\right)\Bigg\} - E_t\Bigg\{\lambda_{2,t+T-1}\Gamma_{t+T-1,t+T}(1+\pi_{t+T})\pi_{t+T}\left(\frac{a_{t+T}F_{t+T}}{a_{t+T-1}F_{t+T-1}}\right) \\
& \left(\frac{F_{K^g,t+T}}{F_{t+T}}\right)\Bigg\}\Bigg] + \beta^T E_t\{\lambda_{3,t+T}\Delta_{t+T}a_{t+T}F_{K^g,t+T}\} + \\
& - \beta^T\lambda_{5,t+T}\left(\frac{G^c}{Y^{\rho_{G^c}}}\right)\rho_{G^c}(\Delta_{t+T}a_{t+T}F_{t+T})^{\rho_{G^c}-1}\Delta_{t+T}a_{t+T}F_{K^g,t+T} + \qquad (1.33e) \\
& - \beta^T\lambda_{6,t+T}\left(\frac{G^i}{Y^{\rho_{G^i}}}\right)\rho_{G^i}(\Delta_{t+T}a_{t+T}F_{t+T})^{\rho_{G^i}-1}\Delta_{t+T}a_{t+T}F_{K^g,t+T} = 0
\end{aligned}
$$

$$
\begin{aligned}
[\pi_t]: \quad & \frac{\lambda_{1,t-1}}{\beta(1+\pi_t)} - \psi(2\pi_t + 1)\left[\lambda_{2,t} - \lambda_{2,t-1}\Gamma_{t-1,t}\left(\frac{a_t F_t}{a_{t-1}F_{t-1}}\right)\right] + \\
& - \psi\lambda_{3,t}\pi_t a_t F_t + \lambda_{5,t}\left(\frac{G^c}{Y^{\rho_{G^c}}}\right)\rho_{G^c}(\Delta_t a_t F_t)^{\rho_{G^c}-1}\psi\pi_t a_t F_t + \\
& + \lambda_{6,t}\left(\frac{G^i}{Y^{\rho_{G^i}}}\right)\rho_{G^i}(\Delta_t a_t F_t)^{\rho_{G^i}-1}\psi\pi_t a_t F_t = 0 \qquad (1.33f)
\end{aligned}
$$

$$
[R_t]: \quad -\lambda_{1,t}E_t\left\{\beta\Gamma_{t,t+1}\left(\frac{1}{1+\pi_{t+1}}\right)\right\} = 0 \qquad (1.33g)
$$

$$[\lambda_{1,t}]: \quad E_t\left\{\beta\Gamma_{t,t+1}\left(\frac{1+R_t}{1+\pi_{t+1}}\right)\right\} = 1 \tag{1.33h}$$

$$[\lambda_{2,t}]: \quad \psi(1+\pi_t)\pi_t - \psi\beta E_t\left\{\Gamma_{t,t+1}(1+\pi_{t+1})\pi_{t+1}\left(\frac{a_{t+1}F_{t+1}}{a_t F_t}\right)\right\}+ + \theta(1-mc_t) = 0 \tag{1.33i}$$

$$[\lambda_{3,t}]: \quad C_t + G_t^c + G_t^i - \Delta_t a_t F_t = 0 \tag{1.33j}$$

$$[\lambda_{4,t}]: \quad K_{t+T}^g = (1-\delta)K_{t+T-1}^g + \left[1 - \frac{\overline{\omega}}{2}\left(1 - \frac{G_t^i}{G_{t-1}^i}\right)\right]G_t^i \tag{1.33k}$$

$$[\lambda_{5,t}]: \quad \left(\frac{G_t^c}{G^c}\right) = \left(\frac{\Delta_t a_t F_t}{Y}\right)^{\rho_{G^c}} \tag{1.33l}$$

$$[\lambda_{6,t}]: \quad \left(\frac{G_t^i}{G^i}\right) = \left(\frac{\Delta_t a_t F_t}{Y}\right)^{\rho_{G^i}} \tag{1.33m}$$

D.2: Steady-state

Under the zero-inflation steady-state, again the equilibrium allocations are identical to the ones under the first-best benchmark.

$$\text{From}[C]: \quad U_C + \lambda_1\left(1-\frac{1}{\beta}\right)\frac{U_{CC}}{U_C} + \theta\lambda_2\left[\frac{U_{NC}}{U_N} - \frac{U_{CC}}{U_C}\right] - \lambda_3 = 0$$

$$\text{From}[N]: \quad U_N + \lambda_1\left(1-\frac{1}{\beta}\right)\frac{U_{CN}}{U_C} + \theta\lambda_2\left[\frac{U_{NN}}{U_N} - \frac{U_{CN}}{U_C} - \frac{F_{NN}}{F_N}\right]mc+ + \lambda_3 F_N - \lambda_5\left(\frac{G^c}{Y}\right)\rho_{G^c}F_N - \lambda_6\left(\frac{G^i}{Y}\right)\rho_{G^i}F_N = 0$$

$$\text{From}[G^c]: \quad V_{G^c} = \lambda_3 - \lambda_5$$

$$\text{From}[G^i]: \quad \lambda_4 = \lambda_3 - \lambda_6$$

$$\text{From}[K^g]: \quad \lambda_4[1-\beta(1-\delta)] = -\beta^T\theta\lambda_2 mc\frac{F_{NK^g}}{F_N}+ + \beta^T\lambda_3 F_{K^g} - \beta^T\lambda_5\left(\frac{G^c}{Y}\right)\rho_{G^c}F_{K^g} - \beta^T\lambda_6\left(\frac{G^i}{Y}\right)\rho_{G^i}F_{K^g}$$

$$\text{From}[\pi]: \quad \lambda_1 = 0$$

$$\text{From}[R]: \quad \beta\lambda_1 = 0$$

$$
\begin{aligned}
\text{From}[\lambda_1]: &\quad 1 + R = \frac{1}{\beta} \\
\text{From}[\lambda_2]: &\quad mc = 1 \\
\text{From}[\lambda_3]: &\quad Y = C + G^c + G^i \\
\text{From}[\lambda_4]: &\quad G^i = \delta K^g \\
\text{From}[\lambda_5]: &\quad G^c = G^c \\
\text{From}[\lambda_6]: &\quad G^i = G^i
\end{aligned}
$$

From $[C]$ and $[N]$, one can get $\lambda_1 = \lambda_2 = \lambda_5 = \lambda_6 = 0$. And from $[C]$, $[G^c]$ and $[G^i]$, follows: $U_C = V_{G^c} = \lambda_3 = \lambda_4$. Finally, from K^g, you get the steady-state marginal product of public capital:

$$
F_{K^g} = \frac{1 - \beta(1-\delta)}{\beta^T}
$$

E: Sub-optimal Monetary Policy and Optimal Fiscal Policy under the Output Gap Responding Fiscal Rule

The relevant Lagrangian is:

$$
\begin{aligned}
L(C_t, N_t, G_t^c, G_t^i, K_{t+T}^g, \pi_t, R_t, \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}, \lambda_{7,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \Bigg[U\big(C_t, N_t\big) + \\
+ V\big(G_t^c\big) \Bigg] - \lambda_{1,t} \Bigg[\beta \Gamma_{t,t+1} \bigg(\frac{1+R_t}{1+\pi_{t+1}} \bigg) - 1 \Bigg] + \\
- \lambda_{2,t} \Bigg[\psi \Bigg[(1+\pi_t)\pi_t - \beta \Gamma_{t,t+1} (1+\pi_{t+1})\pi_{t+1} \\
\bigg(\frac{a_{t+1} F(N_{t+1}, K_{t+1}^g)}{a_t F(N_t, K_t^g)} \bigg) \Bigg] + \theta \bigg(1 + \frac{U_N(C_t, N_t)}{U_C(C_t, N_t) a_t F_N(N_t, K_t^g)} \bigg) \Bigg] + \\
- \lambda_{3,t} \Bigg[C_t + G_t^c + G_t^i - \Delta_t a_t F(N_t, K_t^g) \Bigg] +
\end{aligned}
\tag{1.34}
$$

(...)

$$- \lambda_{4,t}\left[K^g_{t+T} - (1-\delta)K^g_{t+T-1} - \left(1 - \frac{\overline{\omega}}{2}\left(1 - \frac{G^i_t}{G^i_{t-1}}\right)^2\right)G^i_t\right] +$$

$$- \lambda_{5,t}\left[G^c\left(\frac{\Delta_t a_t F(N_t, K^g_t)}{Y}\right)^{\rho_{G^c}} - G^c_t\right] +$$

$$- \lambda_{6,t}\left[G^i\left(\frac{\Delta_t a_t F(N_t, K^g_t)}{Y}\right)^{\rho_{G^i}} - G^i_t\right] +$$

$$- \lambda_{7,t}\left[\frac{(1+\pi_t)^{\theta_\pi}}{\beta}\epsilon^R_t - 1 - R_t\right]\Bigg\}$$

E.1: Equilibrium Conditions

Ramsey FOCs:

$$[C_t]: \quad \xi_t U_{C,t} + \left(\frac{U_{CC,t}}{U_{C,t}}\right)(\lambda_{1,t} - \beta^{-1}\lambda_{1,t-1}) - \psi\left(\frac{U_{CC,t}}{U_{C,t}}\right)\Bigg[\lambda_{2,t}\beta E_t\Bigg\{\Gamma_{t,t+1}$$
$$(1+\pi_{t+1})\pi_{t+1}\left(\frac{a_{t+1}F_{t+1}}{a_t F_t}\right)\Bigg\} - \lambda_{2,t-1}\Gamma_{t-1,t}(1+\pi_t)\pi_t\left(\frac{a_t F_t}{a_{t-1}F_{t-1}}\right)\Bigg] +$$
$$+ \theta\lambda_{2,t}\left(\frac{U_{NC,t}}{U_{N,t}} - \frac{U_{CC,t}}{U_{C,t}}\right)mc_t - \lambda_{3,t} = 0 \tag{1.34a}$$

$$[N_t]: \quad \xi_t U_{N,t} + \left(\frac{U_{CN,t}}{U_{C,t}}\right)(\lambda_{1,t} - \beta^{-1}\lambda_{1,t-1}) - \psi\Bigg[\lambda_{2,t}E_t\Bigg\{\beta\Gamma_{t,t+1}(1+\pi_{t+1})\pi_{t+1}$$
$$\left(\frac{a_{t+1}F_{t+1}}{a_t F_t}\right)\Bigg\} - \lambda_{2,t-1}\Gamma_{t-1,t}(1+\pi_t)\pi_t\left(\frac{a_t F_t}{a_{t-1}F_{t-1}}\right)\Bigg]\left(\frac{U_{CN,t}}{U_{C,t}} + \frac{F_{N,t}}{F_t}\right) +$$
$$+ \theta\lambda_{2,t}mc_t\left[\frac{U_{NN,t}}{U_{N,t}} - \frac{U_{CN,t}}{U_{C,t}} - \frac{F_{NN,t}}{F_{N,t}}\right] + \lambda_{3,t}\Delta_t a_t F_{N,t} +$$
$$- \lambda_{5,t}\left(\frac{G^c}{Y^{\rho_{G^c}}}\right)\rho_{G^c}(\Delta_t a_t F_t)^{\rho_{G^c}-1}\Delta_t a_t F_{N,t} +$$
$$- \lambda_{6,t}\left(\frac{G^i}{Y^{\rho_{G^i}}}\right)\rho_{G^i}(\Delta_t a_t F_t)^{\rho_{G^i}-1}\Delta_t a_t F_{N,t} = 0 \tag{1.34b}$$

$$[G^c_t]: \quad \xi_t V_{G^c,t} - \lambda_{3,t} + \lambda_{5,t} = 0 \tag{1.34c}$$

$$[G^i_t]: \quad -\lambda_{3,t} + \lambda_{4,t}\left[1 - \frac{\overline{\omega}}{2}\left(1 - \frac{G^i_t}{G^i_{t-1}}\right)^2 +\right.$$
$$\left. + \overline{\omega}\left(1 - \frac{G^i_t}{G^i_{t-1}}\right)\left(\frac{G^i_t}{G^i_{t-1}}\right)\right] +$$
$$- \beta E_t\left\{\lambda_{4,t+1}\overline{\omega}\left(1 - \frac{G^i_{t+1}}{G^i_t}\right)\left(\frac{G^i_{t+1}}{G^i_t}\right)^2\right\} + \lambda_{6,t} = 0 \tag{1.34d}$$

$$
\begin{aligned}
[K^g_{t+T}]: \quad & -\lambda_{4,t} + \beta(1-\delta)E_t\lambda_{4,t+1} - \beta^T\theta E_t\left\{\lambda_{2,t+T}mc_{t+T}\left(\frac{F_{NK^g,t+T}}{F_{N,t+T}}\right)\right\}+ \\
& - \beta^T\psi\left[\beta E_t\left\{\lambda_{2,t+T}\Gamma_{t+T,t+T+1}(1+\pi_{t+T+1})\pi_{t+T+1}\left(\frac{a_{t+T+1}F_{t+T+1}}{a_{t+T}F_{t+T}}\right)\right.\right. \\
& \left.\left(\frac{F_{K^g,t+T}}{F_{t+T}}\right)\right\} - E_t\left\{\lambda_{2,t+T-1}\Gamma_{t+T-1,t+T}(1+\pi_{t+T})\pi_{t+T}\left(\frac{a_{t+T}F_{t+T}}{a_{t+T-1}F_{t+T-1}}\right)\right. \\
& \left.\left.\left(\frac{F_{K^g,t+T}}{F_{t+T}}\right)\right\}\right] + \beta^T E_t\{\lambda_{3,t+T}\Delta_{t+T}a_{t+T}F_{K^g,t+T}\}+ \\
& - \beta^T\lambda_{5,t+T}\left(\frac{G^c}{Y^{\rho_{G^c}}}\right)\rho_{G^c}(\Delta_{t+T}a_{t+T}F_{t+T})^{\rho_{G^c}-1}\Delta_{t+T}a_{t+T}F_{K^g,t+T}+ \\
& - \beta^T\lambda_{6,t+T}\left(\frac{G^i}{Y^{\rho_{G^i}}}\right)\rho_{G^i}(\Delta_{t+T}a_{t+T}F_{t+T})^{\rho_{G^i}-1}\Delta_{t+T}a_{t+T}F_{K^g,t+T} = 0
\end{aligned} \tag{1.34e}
$$

$$
\begin{aligned}
[\pi_t]: \quad & \frac{\lambda_{1,t-1}}{\beta(1+\pi_t)} - \psi(2\pi_t+1)\left[\lambda_{2,t} - \lambda_{2,t-1}\Gamma_{t-1,t}\left(\frac{a_tF_t}{a_{t-1}F_{t-1}}\right)\right]+ \\
& - \psi\lambda_{3,t}\pi_t a_t F_t + \lambda_{5,t}\left(\frac{G^c}{Y^{\rho_{G^c}}}\right)\rho_{G^c}(\Delta_t a_t F_t)^{\rho_{G^c}-1}\psi\pi_t a_t F_t+ \\
& + \lambda_{6,t}\left(\frac{G^i}{Y^{\rho_{G^i}}}\right)\rho_{G^i}(\Delta_t a_t F_t)^{\rho_{G^i}-1}\psi\pi_t a_t F_t+ \\
& - \lambda_{7,t}\frac{\theta_\pi(1+\pi_t)^{\theta_\pi-1}}{\beta}\epsilon^R_t = 0
\end{aligned} \tag{1.34f}
$$

$$
[R_t]: \quad -\lambda_{1,t}E_t\left\{\beta\Gamma_{t,t+1}\left(\frac{1}{1+\pi_{t+1}}\right)\right\} + \lambda_{7,t} = 0 \tag{1.34g}
$$

$$
[\lambda_{1,t}]: \quad E_t\left\{\beta\Gamma_{t,t+1}\left(\frac{1+R_t}{1+\pi_{t+1}}\right)\right\} = 1 \tag{1.34h}
$$

$$
\begin{aligned}
[\lambda_{2,t}]: \quad & \psi(1+\pi_t)\pi_t - \psi\beta E_t\left\{\Gamma_{t,t+1}(1+\pi_{t+1})\pi_{t+1}\left(\frac{a_{t+1}F_{t+1}}{a_tF_t}\right)\right\}+ \\
& + \theta(1-mc_t) = 0
\end{aligned} \tag{1.34i}
$$

$$
[\lambda_{3,t}]: \quad C_t + G^c_t + G^i_t - \Delta_t a_t F_t = 0 \tag{1.34j}
$$

$$
[\lambda_{4,t}]: \quad K^g_{t+T} = (1-\delta)K^g_{t+T-1} + \left[1 - \frac{\overline{\omega}}{2}\left(1 - \frac{G^i_t}{G^i_{t-1}}\right)\right]G^i_t \tag{1.34k}
$$

$$
[\lambda_{5,t}]: \quad \left(\frac{G^c_t}{G^c}\right) = \left(\frac{\Delta_t a_t F_t}{Y}\right)^{\rho_{G^c}} \tag{1.34l}
$$

$$
[\lambda_{6,t}]: \quad \left(\frac{G^i_t}{G^i}\right) = \left(\frac{\Delta_t a_t F_t}{Y}\right)^{\rho_{G^i}} \tag{1.34m}
$$

$$
[\lambda_{7,t}]: \quad 1 + R_t = \frac{(1+\pi_t)^{\theta_\pi}}{\beta}\epsilon^R_t \tag{1.34n}
$$

E.2: Steady-state

Under the zero-inflation steady-state, again the equilibrium allocations are identical to the ones under the first-best benchmark.

$$
\begin{aligned}
\text{From}[C]: \quad & U_C + \lambda_1\left(1-\frac{1}{\beta}\right)\frac{U_{CC}}{U_C} + \theta\lambda_2\left[\frac{U_{NC}}{U_N} - \frac{U_{CC}}{U_C}\right] - \lambda_3 = 0 \\
\text{From}[N]: \quad & U_N + \lambda_1\left(1-\frac{1}{\beta}\right)\frac{U_{CN}}{U_C} + \theta\lambda_2\left[\frac{U_{NN}}{U_N} - \frac{U_{CN}}{U_C} - \frac{F_{NN}}{F_N}\right]mc + \\
& + \lambda_3 F_N - \lambda_5\left(\frac{G^c}{Y}\right)\rho_{G^c}F_N - \lambda_6\left(\frac{G^i}{Y}\right)\rho_{G^i}F_N = 0 \\
\text{From}[G^c]: \quad & V_{G^c} = \lambda_3 - \lambda_5 \\
\text{From}[G^i]: \quad & \lambda_4 = \lambda_3 - \lambda_6 \\
\text{From}[K^g]: \quad & \lambda_4[1-\beta(1-\delta)] = -\beta^T\theta\lambda_2 mc\frac{F_{NK^g}}{F_N} + \\
& + \beta^T\lambda_3 F_{K^g} - \beta^T\lambda_5\left(\frac{G^c}{Y}\right)\rho_{G^c}F_{K^g} - \beta^T\lambda_6\left(\frac{G^i}{Y}\right)\rho_{G^i}F_{K^g}
\end{aligned}
$$

$$
\begin{aligned}
\text{From}[\pi]: \quad & \lambda_1 = \lambda_7\theta_\pi \\
\text{From}[R]: \quad & \beta\lambda_1 = \lambda_7 \\
\text{From}[\lambda_1]: \quad & 1 + R = \frac{1}{\beta} \\
\text{From}[\lambda_2]: \quad & mc = 1 \\
\text{From}[\lambda_3]: \quad & Y = C + G^c + G^i \\
\text{From}[\lambda_4]: \quad & G^i = \delta K^g \\
\text{From}[\lambda_5]: \quad & G^c = G^c \\
\text{From}[\lambda_6]: \quad & G^i = G^i \\
\text{From}[\lambda_7]: \quad & 1 + R = \frac{1}{\beta}
\end{aligned}
$$

From $[C]$ and $[N]$, one can get $\lambda_1 = \lambda_2 = \lambda_5 = \lambda_6 = 0$. This implies, from $[R]$, that $\lambda_7 = 0$. And from $[C]$, $[G^c]$ and $[G^i]$, follows: $U_C = V_{G^c} = \lambda_3 = \lambda_4$. Finally, from K^g, you get the steady-state marginal product of public capital:

$$F_{K^g} = \frac{1-\beta(1-\delta)}{\beta^T}$$

Chapter 2

Optimal Fiscal Policy under a Temporary Windfall of Nonrenewable Resources

2.1 Introduction

This chapter investigates optimal fiscal policy design in an environment where a government experiences a positive structural change in its revenue due to a natural resource source. The objective is to derive useful lessons about how government should distribute the windfall over time.

The motivation to investigate optimal fiscal policy in this challenging environment comes from the pre-salt's oil discovery in Brazil, dating from 2007. Official figures show that the daily pre-salt oil production increased from an average of 41 thousand barrels per day (bpd) in 2010 to 1 million bpd in 2016 and 1.9 million in March 2020. Current forecasts point to the pre-salt layer producing 4.2 million bpd in 2030, responding for 80% of total production[1]. In the same direction, expectations point to a jump in the primary revenues from the oil sector (royalties, special participation in production and profit oil revenue), that after registering R$ 21.4 billion in 2010 and R$ 45.0 billion in 2020, is expected to increase to R$ 75.0 billion in 2024 and

[1]According to the National Agency for Petroleum, Natural Gas and Biofuels (ANP), proved oil reserves registered 14 billion barrels (Gbbl) in 2010 and 12 Gbbl in 2020. The Brazilian Energy Research Office (EPE) forecasts that reserves will reach 34 Gbbl in 2030. Forecasts date from 2020.

R$ 160 billion in 2030.[2].

Given the potential of revenue collection involved in this new milestone of the oil sector in Brazil, a struggle for "appropriation" of resources to different government action areas was triggered. In September 2013 (Law nº. 12.858/2013), it was settled the allocation of the entire resources due to royalties and to special participation in production for education (75%) and health (25%), for contracts signed from December 3, 2012. In this context, Brazil needs to design a fiscal restructuring strategy aimed at the intertemporal management of a nonrenewable source of wealth.

This work's main contribution is to solve an infinite-horizon stochastic Ramsey problem of a benevolent government that faces a temporary windfall of natural resource revenue. The model setup is a small open economy variant founded on Lucas and Stokey (1983), but with incomplete markets (Aiyagari, Marcet, Sargent and Seppälä (2002).

I do not assume uncertainty disappearance after a finite period in time to solve the model numerically. Doing so justifies because an oil producer country probably will continue being a producer even after the windfall ends. For example, in the case of Brazil, the windfall is due to oil discovery in the pre-salt layer. Once this windfall ceases, the exploration and production based on deep waters and onshore basins will probably remain, and the country will continue to collect revenue from the natural resource sector. Moreover, even with an eventual resource exhaustibility, other sources of uncertainty could exist, like technological and monetary shocks.

I apply the Parameterized Expectation Approach (PEA - den Haan and Marcet, 1990) to solve a nonlinear stochastic dynamic model with a structural break. It allows me to evaluate the optimal policy response to cope with the nonrenewable resource volatility and the transition dynamics towards the stationary solution once the windfall ceases.

The advantage of the PEA approach is that it allows resolving structural breaks in one (or more) state variables without the necessity to assume that uncertainty disappears in a finite moment in time, a shortcut commonly used in the literature. Also, with the PEA, the 'curse of dimensionality' problem is not a concern when it is necessary to increase the model in complexity by adding more state variables

[2]Data for production from the Brazilian Energy Research Office (EPE) and data for primary revenues from the National Agency for Petroleum, Natural Gas and Biofuels (ANP), Pré-Sal Petróleo and FGV Energia. Forecasts date from 2020.

(Marcet and Lorenzoni (1998); and Faraglia, Marcet, Oikonomou and Scott (2014)). However, an important concern to guarantee the convergence of the PEA algorithm is the setup of good initial conditions for the parameters in the polynomial that approximates Euler's equation conditional expectation. This concern amplifies if the deterministic version of the model does not have a steady-state. Remarkably, applying PEA to solve the model with structural breaks, one should care about good initial conditions regarding pre and post-break periods. I challenge this problem using a linearized solution to the model pre and post the structural break (Christiano and Fisher, 2000).

The environment with a windfall of nonrenewable resource income, considering the stochastic nature behind the flows of revenue (mainly driven by commodity price volatility), allows me to approximate agents expectation when they know that at a specific moment in time, there would be a structural shift in one of the state variables (here resource revenue). One can generalize this approach to the case of more than one break and unknown break periods. For now, I am evaluating the accuracy of this approach with a model setup already analyzed in the literature. Future work plans to advance in the model setup's complexity to reflect more realistic characteristics of the Brazilian economy.

I organize this chapter into five sections beyond this introduction. Section 2 presents a (nonexhaustive) literature review. The third section describes a benchmark Permanent Income Hypothesis (PIH) deterministic model and derives the Ramsey Problem first-order conditions (FOCs). Section 4 introduces uncertainty in the model through a resource revenue exogenous stochastic process. In section 5, I perform a quantitative exercise, including a sensitivity analysis. Finally, I conclude this work in the sixth section.

2.2 Literature Review

The literature concerning the design of fiscal rules to oil-producing countries is vast and, generally, followed by applications to specific countries. See, for example, Caceres, Cevik, Fenochietto and Gracia (2015), Harding and Ploeg (2013), Iacono (2017), Carcillo, Leigh and Villafuerte (2007), Leigh and Olters (2006), Olters (2007), Segura (2006) and Gobetti (2009).

Lessons brought from the literature, generally, derive suitable recommendations to policymakers that have a foundation either on theoretical models or existing practicing rules (sometimes *ad hoc* rules). Examples are in Medas and Zakharova (2009), Barnett and Ossowski (2002), Baunsgaard, Villafuerte, Poplawski-Ribeiro and Richmond (2012), Segal (2012), Chalk (1998), and Sturm, Gurtner and Alegre (2009).

Regarding a theoretical understanding of the optimal fiscal policy in resource-rich countries, the papers of Engels and Valdés (2000) and Maliszewski (2009) are relevant readings. They have a comprehensive review of the kind of models used in the literature and propose new approaches.

The benchmark framework, called Permanent Income Hypothesis (PIH) model, can be summarized as the choice of the optimal *per capta* consumption level, considering that a Social Welfare Function (SWF) is defined as follows:

$$U(u_0, u_1, ..., u_\infty) = \sum_{t=0}^{\infty}[\beta(1+n)]^t \frac{u(g_t, c_t)^{1-\rho}}{1-\rho}$$

where β is the discount factor, n is the population growth rate, ρ is the intertemporal elasticity of substitution and c_t and g_t are the *per capta* consumption level of private and public goods.

Assuming Constant Elasticity of Substitution (CES) between c_t and g_t allows to obtain $u(g_t, c_t)$ proportional to c_t. The government maximizes the SWF contrained on its intertemporal budget constraint:

$$\begin{aligned} \underset{\{c_t\}_{t=0}^{\infty}}{\text{maximize}} \quad & U(u_0, u_1, ..., u_\infty) = \sum_{t=0}^{\infty}[\beta(1+n)]^t \frac{c_t^{1-\rho}}{1-\rho} \\ \text{s.t.:} \quad & \sum_{t\geq 0} R^{-t} G_t = F_0 + \sum_{t\geq 0} R^{-t}[T_t + Y_t^{oil}] \equiv W_0^G \end{aligned}$$

where G_t is the total government expenditure, R is the real interest rate, F_0 is the government's initial net foreign asset, and T_t and Y_t^{oil} are, respectively, the tax over the non-oil sector and the oil revenue. W_0^G denotes the government's total wealth at time 0. If the government is not restricted in its power to tax the non-oil economy, one can substitute T_t by $Y_t^{non-oil}$ and, then, W_0^G can be interpreted as the wealth of the whole economy (W_0). Assuming $\beta R = 1$ as standard to solve for consumption smoothing [3], the solution to this problem gives the following optimal

[3] If $\beta R < 1$, which characterizes an impatient society, individuals will optimally consume more

per capta consumption:

$$\bar{c} = \left(1 - \frac{1+n}{R}\right) W_0$$

The right-hand side is defined in the literature as the permanent income, due to Friedman (1957), and is the indefinitely maintainable highest level of *per capta* consumption.

An alternative approach, called Permanent Oil Income Hypothesis (POIH) model, is obtained restricting the wealth that the government can distribute across generations only to the natural resource income (W_0^{oil} instead of W_0). The POIH preserves the idea of basing the consumption (of public goods) on the permanent income, restricted to that of the natural resource source.

Maliszewski (2009) proposed modifying the POIH model marginally to consider taxation on the non-oil sector. This proposal originated the Modified POIH (MPOIH) model, which is not as restrictive as the POIH but not as flexible as the PIH regarding the government's instrument to promote intergenerational wealth transfers.

Ploeg and Venables (2011) shed some light on how to challenge the problem of interest rate differential in developing countries. They suggest that in this environment, it is optimal: to reduce the level of savings and allow for a skewed pattern path of consumption in favor of the current generation; that the investment in assets should involve a mix between investing domestically in public infrastructure and the reduction of foreign public debt; and the reduction of distortionary taxes.

Engels and Valdés (2000) bring to the analysis uncertainty in a restricted form[4] (to consider precautionary saving) and adjustment cost concerning changes in the government expenditures level due to, for example, new discoveries or the exhaustion of reserves.

Up to my knowledge, just after almost one decade later, uncertainty became more frequent in analyzing optimal policy in resource-rich countries. The benevolent government's problem is similar to the deterministic case and, with few exceptions (van

in the present. See Engels and Valdés (2000).

[4]Under a CES utility function, the authors collapse all the future uncertainty about oil price in a one-period uncertainty. To this end, they assume that the oil price risk is diversified away, with the government recurring to privatization in $t+1$. In this way, they arrive at a two-period optimization problem.

der Ploeg, 2010), the uncertainty is concerning commodity prices. Under convenient utility assumptions, one can find analytical solutions when solving finite-horizon models (van der Ploeg, 2010; Iacono, 2017). The solution of finite-horizon models numerically can be found in van der Ploeg and Venables (2012) and Cherif and Hasanov (2013). Regarding solving infinite-horizon models in continuous or discrete time, the assumption is that uncertainty disappears after a finite period in time due to the exhaustion of the natural resource windfall. This shortcut facilitates the model's numerical solution (Bems and Carvalho Filho, 2011; van den Bremer and van der Ploeg, 2013; van der Ploeg, 2014; IMF, 2015).

A common reason to include uncertainty in the analysis is to evaluate the impact of precautionary saving on government policies to deal with short and medium-term adjustments necessary to cope with volatility (mainly) in commodity prices. Some papers go beyond and consider countries that face capital scarcity and absorption constraints to justify raising an investment fund.

An alternative approach to assessing fiscal policy in countries granted by a natural resource windfall is the Dynamic Stochastic General Equilibrium (DSGE) model. Pieschacón (2009) implements a DSGE model, calibrating for Mexico, to analyze the effects of distinct sustainable fiscal rules in a near-to-exhaustion oil sector. Berg, Portillo, Yang and Zanna (2013) set up a three-sector DSGE model to analyze the effects of public investments financed with natural resources revenue. Furthermore, Agenor (2016) implements a DSGE model to evaluate optimal fiscal policy response to transitory shocks in the commodities price, dealing with optimization over a social loss function considering both volatility of private consumption and one indicator of macroeconomic stability.

The models briefly described above are derived under optimality. Notwithstanding, *ad hoc* rules are also present in the literature about the design and evaluation of fiscal policy for oil-producing countries.

The "bird-in-hand" rule (Bjerkholt, 2002), used in Norway, states that the *per capta* government expenditure should base on the returns of the net financial assets at each period. The goal is to prevent an abrupt decrease in the amount of government expenditure. This rule asks for a very patient society, which can restrict its implementation in developing countries that are more demanding for short-term public expenditures and/or have negative initial assets.

Another *ad hoc* rule is the "spending current oil revenue", which states that government should spend all the oil revenue obtained each period. According to Maliszewski (2009), this rule occurs when it is impossible to control for political pressures or in the case of a very optimistic scenario about the non-oil sector economic growth. A way to minimize the shortages of this rule is to avoid mandatory government expenditures. This prudence in establishing expenditures is also important to manage a slowdown in oil prices.

The main lesson from the literature review is that there is a variety of work under deterministic environments. However, there is much to be done to deal with uncertainty, not only to the price of commodities but also to the possibility of other sources of uncertainty, like technological shocks in the non-resource sector and/or a possible exogenous stochastic process on current public expenditures (that is apart from public investments to promote the non-resource sector). Specifically to Brazil's case, it would be important to consider analyzing the interest rate differential between the debt and asset positions. Moreover, the domestic households are the main holders of government debt outstanding, which has implications regarding an optimal policy that concerns debt repayment. The channel is through the impact of the exchange rate appreciation on the non-resource tradable sector, so-called "Dutch Disease".

2.3 Deterministic Economy: a benchmark model

The objective of this section is to derive the solution for a deterministic economy. I use this benchmark to confront the results I obtain ahead under uncertainty. The households in this model derive utility from private and government consumption. The government provides the latter at no cost to households. The private consumption comes from the net non-natural resource sector income that, to simplify the analysis, I assume it is constant $\{y_t = \bar{y}\}_{t=0}^{\infty}$. The government charge this income with a time-varying tax rate ($\{\tau_t\}_{t=0}^{\infty}$). The households do not have access to the financial market and their optimal choice is to consume the whole net income ($\{c_t = (1-\tau_t)\bar{y}\}_{t=0}^{\infty}$).

Household's Problem:

$$
\begin{aligned}
&\underset{\{c_t\}_{t=0}^{\infty}}{\text{maximize}} \quad \sum_{t=0}^{\infty} \beta^t [u(c_t) + z(g_t)] \\
&\text{s.t.:} \qquad\quad c_t = (1 - \tau_t)\bar{y}
\end{aligned}
\tag{2.1}
$$

The functions $u(.)$ and $z(.)$ are strictly increasing in their argument and strictly concave.

The government finances its expenditure and repay the principal and interest over the bonds issued in the previous period charging the non-oil sector, collecting the flow of the natural resource sector revenue (N_t), and issuing new bonds in the international market at an exogenous and assumed constant interest rate (r^*). I analyze optimal fiscal policy when the government faces a relevant increase in the natural resource sector's revenue flows during a finite period (let me call it T). From time T on, I assume that these flows decrease to a significantly lower magnitude. The government budget constraint is:

$$
\begin{aligned}
&g_t + (1 + r^*)b_{t-1}^g = \tau_t \bar{y} + b_t^g + N_t, \text{with:} \\
&N_t = \begin{cases} N, \text{if } t \leq T. \\ n, \text{if } t > T. \end{cases}
\end{aligned}
\tag{2.2}
$$

where b_t^g is the government's bonds issued at time t.

The feasibility constraint of this single good small open economy is:

$$
c_t + g_t + tb_t = \bar{y} + N_t \tag{2.3}
$$

where tb_t represents the trade balance at time t.

Finally, the international financial flows dictate the debt dynamics as follows:

$$
tb_t = (1 + r^*)b_{t-1}^g - b_t^g \tag{2.4}
$$

2.3.1 The Ramsey Problem

As my goal is to state the Ramsey problem using the primal approach, I first derive the present value of the government budget constraint. After a forward substitution of equation (2.2) and the use of the *non-Ponzi* condition

$$lim_{j\to\infty}\left(\frac{1}{(1+r^*)}\right)^{j+1} b^g_{t+j} \leq 0 \ , \tag{2.5}$$

I substitute the income tax as a function of consumption allocation and the income process derived from the households' optimality. Then, I get the following budget constraint's present value, at time t:

$$b^g_{t-1} = \sum_{j=0}^{\infty}\left(\frac{1}{(1+r^*)}\right)^{j+1}\left[y_{t+j} + N_{t+j} - c_{t+j} - g_{t+j}\right] \equiv \sum_{j=0}^{\infty}\left(\frac{1}{(1+r^*)}\right)^{j+1} s_{t+j} \tag{2.6}$$

where I use s_{t+j} to account for the government's primary fiscal balance at time $t+j$[5].

Assuming that the subjective discount factor equals the market discount factor ($\beta(1+r^*) = 1$), the Ramsey Problem allocations arise from the solution to the following Lagrangian:

$$L(c_t, g_t, b_t, \Delta) = \sum_{t=0}^{\infty}\beta^t\left\{\left[u(c_t) + z(g_t)\right] + \beta\Delta\left[y_t + N_t - c_t - g_t\right]\right\} - \beta\Delta b^g_{-1} \tag{2.7}$$

Ramsey FOCs:

$$[c_t]: \quad u'(c_t) = \Delta \tag{2.8a}$$

$$[g_t]: \quad z'(g_t) = \Delta \tag{2.8b}$$

$$[Implementability\ Constraint]: b_{-1} = \sum_{t=0}^{\infty}\beta^{t+1}s_t \tag{2.8c}$$

The optimal allocations that arise from equations (2.8) point to a constant private and government consumption. Given the streams of incomes from the natural and non-natural resource sectors, the evolution of the trade balance (and, then, of the government debt position) will be such that it accommodates the optimal allocations. I look for a Δ such that the system of equations formed by (2.8a)-(2.8b) delivers me the allocations $\{c_t = \bar{c}, g_t = \bar{g}, b_t\}_{t=0}^{\infty}$ and equation (2.8c) is satisfied. The optimal tax policy is also constant $\{\tau_t = \bar{\tau} = 1 - \bar{c}/\bar{y}\}_{t=0}^{\infty}$ from the optimal constant consumption allocation and the assumption of constant non-oil sector income.

[5]The primary fiscal balance does not include the interest over the bonds issued in the previous year.

2.4 Adding uncertainty to the benchmark model

To assess the role of uncertainty in the optimal fiscal policy problem, I depart from the previous model assuming an exogenous stochastic process to the natural resource revenue $N_t(s_t)$. I refer to the history of the state variable throughout as s^t. To highlight the $N_t(s_t)$ stochastic process in the case of the oil commodity, I can say that it assembles a mix of uncertainty about prices, production and big producers offer policy, for example[6]. The natural resource windfall will last for T periods, after which I assume that this source of revenue shifts to a lower level. I model $N_t(s_t)$ as a mean-reverting $AR(1)$ process:

$$N_t(s_t) = \begin{cases} \bar{\mu} + \rho N_{t-1}(s^{t-1}) + \epsilon_t(s_t), \text{ if } t \leq T. \\ \text{And for } t > T: \\ \underline{n}, \text{ if } \bar{\bar{\mu}} + \rho N_{t-1}(s^{t-1}) + \epsilon_t(s_t) < \underline{n} \\ \bar{\bar{\mu}} + \rho N_{t-1}(s^{t-1}) + \epsilon_t(s_t), \text{ if } \underline{n} \leq \bar{\bar{\mu}} + \rho N_{t-1}(s^{t-1}) + \epsilon_t(s_t) \leq \overline{n} \\ \overline{n}, \text{ if } \bar{\bar{\mu}} + \rho N_{t-1}(s^{t-1}) + \epsilon_t(s_t) > \overline{n} \end{cases} \tag{2.9}$$

$$\text{with } \bar{\mu} = N(1-\rho) > \bar{\bar{\mu}} = n(1-\rho).$$

The noise is such that $\epsilon_t(s_t) \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2)$. The boundaries on $N_t(s^t)$ to the post-windfall period match with observed data, as will be explicit in the calibration section ahead[7].

The government issues a one-period maturity risk-free bond, being a price taker of the international interest rate. I will assume that the government faces symmetric (in absolute terms) boundaries in the value of the bonds it issues ($\underline{M}$ and $\overline{M}$)[8]. To

[6] In the Appendix, I apply a simple Vector Autoregressive exercise to show the short-run dynamics between the oil revenue and the international oil price in the Brazilian case.

[7] Such boundaries are not applicable for $t \leq T$, since these lower and upper bounds are strictly lower than whatever observation during the windfall under the magnitude of change I assume to the mean of $N_t(s_t)$, from n to N. See the calibration section.

[8] In the deterministic setup, I did not need to care about these boundaries, because $\beta(1+r^*) = 1$ guarantees a compact asset space. However, under uncertainty, without these boundaries, the asset space is not compact under $\beta(1+r^*) = 1$. These boundaries are also important to implement the numerical solution using the Parameterized Expectation Approach.

avoid redundancy, I set the Ramsey Problem directly:

$$
\begin{aligned}
L(c_t(s^t), g_t(s^t), b_t^g(s^t), \delta_t(s^t), \mu_{l_t}(s^t), \mu_{u_t}(s^t)) = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \Bigg[u(c_t(s^t)) + \\
+ z(g_t(s^t)) \Bigg] - \delta_t(s^t) \Bigg[g_t(s^t) + c_t(s^t) + (1+r^*) b_{t-1}^g(s^{t-1}) - \bar{y} - N_t(s_t) - \\
- b_t^g(s^t) \Bigg] - \mu_{l_t}(s^t) \Bigg[\underline{M} - b_t^g(s^t) \Bigg] - \\
- \mu_{u_t}(s^t) \Bigg[b_t^g(s^t) - \overline{M} \Bigg] \Bigg\}
\end{aligned}
\tag{2.10}
$$

Ramsey FOCs:

$$[c_t]: \quad u'(c_t(s^t)) = \delta_t(s^t) \tag{2.11a}$$

$$[g_t]: \quad z'(g_t(s^t)) = \delta_t(s^t) \tag{2.11b}$$

$$[b_t^g]: \quad \delta_t(s^t) = \beta(1+r^*) E_t[\delta_{t+1}(s^{t+1})] - \mu_{l_t}(s^t) + \mu_{u_t}(s^t) \tag{2.11c}$$

$$[Feasibility\ Constraint]: \quad c_t(s^t) + g_t(s^t) + tb_t(s^t) = \bar{y} + N_t(s_t) \tag{2.11d}$$

To solve this Ramsey Problem I proceed with the Parameterized Expectation Approach to approximate the expectational term $E_t[\delta_{t+1}(s^{t+1})]$ as a function of the state variables of the model $\big(X_t(s_t) = \{t\mathbb{1}\{t \leq T\}, b_{t-1}(s^{t-1}), N_t(s_t)\}_{t=0}^{\infty}\big)$, where $\mathbb{1}\{t \leq T\}$ is an indicator function[9]:

$$
\begin{gathered}
E_t[\delta_{t+1}(s^{t+1})] = \Phi^i(X_t(s_t), \omega_{pea}^i), \\
\text{with } i = 1(t \leq T) \text{ and } i = 2(t > T).
\end{gathered}
\tag{2.12}
$$

I parameterize $\Phi^i(X_t(s_t), \omega_{pea}^i)$ as a linear polynomial on $X_t(s_t)$. To start the algorithm, I simulate $N_t(s_t)$, assume an initial condition to the debt position and initialize $\omega_{pea}^{i,0}$ solving a linearized version of the model. Then, I follow three steps to compute the optimal paths for the allocations and fiscal policy:

(1) Assume $\underline{M} \leq b_{t+1}^g(s^t) \leq \overline{M}$. Under the parameterization (2.12), I recover $c_t(s^t)$, $g_t(s^t)$ and $\delta_t(s^t)$ from (2.11a)-(2.11c) and $b_t^g(s^t)$ from (2.11d). The multipliers $\mu_{l_t}(s^t)$ or $\mu_{u_t}(s^t)$ are equal to zero;

(2) In case the optimal $b_t^g(s^t)$ is inside the boundaries, I go ahead to the next period. Otherwise, I set the optimal value for $b_t^g(s^t)$ as equal to the bound it hints and solve

[9] The steps I follow to solve the model are grounded on Faraglia et al. (2014), specially section 3. Adaptations were needed to solve a model with structural breaks.

the following system of equations (from the FOCs) to find the optimal $c_t(s^t)$ and $g_t(s^t)$:

$$u'(c_t(s^t)) - z'(g_t(s^t)) = 0 \tag{2.13a}$$

$$c_t(s^t) + g_t(s^t) + (1+r^*)b_t^g(s^{t-1}) - b_t^g(s^t) - \bar{y} - N_t(s_t) = 0 \tag{2.13b}$$

(3) I recover the optimal path for $\delta_t(s^t)$ from (2.11a)[10] and $\mu_{l_t}(s^t)$ or $\mu_{u_t}(s^t)$ from (2.11c).

2.4.1 Ramsey Problem

Let me define a feasible allocation at t as $x_t = (c_t(s^t))$. And $X = \{x_t\}_{t=0}^{\infty}$ as the sequence of allocations for the whole time path. In the same way, I define a fiscal policy to a particular t as $\phi_t = \big(\tau_t(s^t), g_t(s^t), b_t^g(s^t), N_t(s_t)\big)$ and the respective time path sequence as $\Phi = \{\phi_t\}_{t=0}^{\infty}$.

Definition 1: A Competitive Equilibrium to this small open economy, considering the pattern of realizations of $N_t(s_t)$ and the initial government indebtedness b_{-1}^g, are sequences of fiscal policy Φ and feasible allocations X, such that: i) given a fiscal policy, the feasible allocation solves the representative household's problem; ii) given a feasible allocation, the fiscal policy satisfies government's budget constraint; iii) the domestic and the rest-of-the-world feasibility constraints' hold.

Definition 2: The Ramsey Problem to this economy, considering the pattern of realizations of $N_t(s^t)$, is to choose allocations from the set of Competitive Equilibria, such that the government maximizes the objective function $E_0 \sum_{t=0}^{\infty} \beta^t \{[u(c_t(s^t)) + z(g_t(s^t))]$, subject to the government implementability constraint (that encompasses the households one)[11].

[10] I can equally recover $\delta_t(s^t)$ from (2.11b).

[11] From the household budget constraint, $\tau_t(s^t) = 1 - \frac{c_t(s^t)}{y_t(s^t)}$. But under the Ramsey primal approach, plugging this expression for $\tau_t(s^t)$ in the household budget constraint delivers that $c_t(s^t) = c_t(s^t)$. Then, to include the household implementability constraint in the Ramsey problem is useless to this problem specifically.

2.5 Quantitative Exercise

This section aims to develop a quantitative exercise to analyze optimal fiscal policy response to an exhaustible windfall of natural resources. I will abbreviate the notation whenever possible since no prejudice to the understanding of the derivations appears.

2.5.1 Calibration and Estimation Approach

I assume that the representative household's utility function (and of the benevolent government) is additively separable in time and isoelastic in private and public consumption.

$$u(c, g) = \frac{c^{\sigma_c+1} - 1}{\sigma_c + 1} + \psi_g \frac{g^{\sigma_g+1} - 1}{\sigma_g + 1} \tag{2.14}$$

where σ_c and σ_g are negative and ψ_g is positive. The absolute value of σ_c and σ_g measures the constant relative risk aversion and ψ_g captures the relative importance between government and private consumption. The coefficient of relative prudence is constant and equal to $1 - \sigma_c$, that is greater than zero. It means the optimal policy dealing with uncertainty regarding future revenues is prompted to save more today to avoid the effect of successive adverse chocks. By the Jensen's Inequality, $E_t[u'(c_{t+1})] > u'(E_t(c_{t+1}))$. From this and equations (2.11a) and (2.11c):

$$u'(c_t) > \beta(1 + r^*)u'(E_t(c_{t+1})) - \mu_{l_t}(s^t) + \mu_{u_t}(s^t) \tag{2.15}$$

Under the assumption that $\beta(1 + r^*) = 1$, equation (2.15) implies an expected upward shift trajectory for consumption ($c_t < E_t(c_{t+1})$), provided optimal asset is inside the boundaries $[\underline{M}\ , \overline{M}]$.

The steady-state in the scenario without oil windfall matches the Brazilian economy data, from 2000 to 2015, adjusted to the absence of capital. I normalize the gross domestic product (GDP) in steady-state to 100, with $\bar{y} = 99$ and $N_t = n = 1$. I assume $T = 70$ and that the initial debt position is 65% of the GDP.

Regarding the windfall, I suppose a shift in the oil revenue to GDP ratio from 1% to approximately 9%. Under this assumption, the participation of oil revenue in the federal government revenue (excluding social security) would increase from 4.2% to

30.6%. A country is classified as resource-rich in the literature when more than 20% of fiscal revenues or exports is based on natural resources. Additionally, after T, oil revenue varies between 0.54% and 1.60% of GDP in steady-state. I assume $T = 70$ and, as in the model without a windfall, the initial debt position is set at 65% of GDP.

Table 2.1: Parameters calibration

β	r^*	ψ_g	σ_c	σ_g	$\bar{\mu}$	$\bar{\bar{\mu}}$	ρ	σ_ϵ^N	$\underline{n}$	$\overline{n}$
0.95	0.0526	0.5565	-0.5	-0.5	3.5	0.35	0.65	0.072	0.54	1.60

The estimation approach attempts to solve a model that deals with a structural break at time T. I suppose the period at which the break occurs is known (see Figure 2.1). The general case accounts for the possibility of more than one break and unknown break periods.

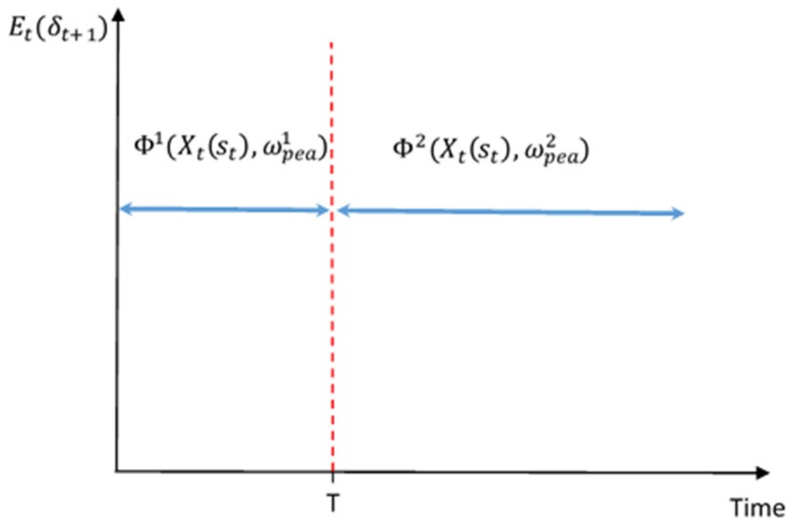

Figure 2.1: PEA estimation and structural break

A common approach in the literature to deal with this concern is to assume that after T any uncertainty disappears. Bems and Carvalho Filho (2011) solve the problem with the dynamic program approach. First, they solve forward the deterministic period (from $T + 1$ on). Then, facing a value function with only B_{T+1} as a state variable, they do a grid on the value of B_{T+1}. Then, they solve backward from T to 0 to obtain the optimal solution trajectories. IMF (2015) adapted the Carroll (2006) endogenous gridpoint method to solve the problem without detailing its approach.

The problem I propose to solve is more general as I do not stand on the deterministic environment after period T. The approach I follow is an application of PEA to solve rational expectation models with structural breaks. In this specific application, once agents know that in period T there will occur a shift in the level of one state variable, they need to incorporate this information when forming expectations in the period before that change. The solution to link the pre and post-structural break periods includes one additional state variable (time) when solving forward up to T. Then, I take the asset state variable's value at time T (B_T) to solve forward from the period $T+1$. With the risk of notation abuse, let me call this a $forward_{\{0,T\}}$-$forward_{\{T+1,\infty\}}$ approach.

In practice, I parameterize $\Phi^i(X_t(s_t), \omega_{pea})$ in a three-step approach. First, I solve the problem before time T, to approximate $\Phi^1(X_t(s_t), \omega^1_{pea})$, implementing 550 simulations of size 70 (this is the period oil windfall lasts). I extract two results from this first step: (i) the mean across samples of ω^1_{pea}; (ii) and the mean of debt position at time T, b^g_T. In the second step, I implement $10,000$ simulations to approximate $\Phi^2(X_t(s_t), \omega^2_{pea})$, taking b^g_T as given (the mean from the first step). The result I extract from this step is ω^2_{pea} . Finally, the third step solves the whole path, consisting of 1,000 simulations of size 10,000, using the estimates of ω^i_{pea} I found in the previous steps. I parameterize $\Phi^i(X_t(s_t), \omega^i_{pea})$ as a linear polynomial on $X_t(s_t)$:

$$\Phi^1(X_t(s_t), \omega^1_{pea}) = \omega^1_0 + \omega^1_1 time + \omega^1_2 \left[\frac{(N_t - N)}{N}\right] + \omega^1_3 \left[\frac{b^g_{t-1}}{\overline{M}}\right] \tag{2.16a}$$

And

$$\Phi^2(X_t(s_t), \omega^2_{pea}) = \omega^2_0 + \omega^2_1 \left[\frac{(N_t - n)}{n}\right] + \omega^2_2 \left[\frac{b^g_{t-1}}{\overline{M}}\right] \tag{2.16b}$$

2.5.2 Discussing the results

In Figure 2.2, I show the simulated series to the deterministic and stochastic natural resource processes, together with the optimal allocation of private consumption and the optimal fiscal policy response[12]. The primary fiscal balance and trade balance variables are the same in this simple model. The same applies to the relation between the overall fiscal balance and the current account. I show the results up to period 800 to highlight the most important features.

[12]To the stochastic series, I use the means from the third step described in the previous paragraph. By the law of large numbers, the difference between the blue and black lines in the oil revenue panel is almost imperceptible.

First, let me comment on some aspects concerning the deterministic model. In the absence of a natural resource windfall, the optimal policy is to keep the initial debt position (dotted blue line) and smooth the allocations and fiscal policy paths. The implications for optimal policy, when challenging a natural resource revenue windfall, are the following: implement an austere fiscal policy with positive primary balance during the windfall; sustain a higher level of public expenditure, as government gains fiscal space; repay public debt up to obtain a relevant asset position; and reduce the tax charge on the non-resource sector. Consequently, the latter induces an increase in private consumption, given the perfect correlation between tax and consumption in the household budget constraint. Under the optimal policy, the government ensures an asset position sufficient to sustain, after the windfall, a smoothing policy in tax and private and public consumption.

Regarding the presence of uncertainty in the resource revenue, the upward paths in the stochastic series of consumption and public expenditure potentially highlight that the model captures the precautionary saving motive in the right way. However, in the scenario with a windfall, this upward behavior reverts once that windfall ceases. Here, precautionary saving is fundamental to allow for the accumulation of assets, turning possible an optimal policy that targets a high level of public and private consumption at the price of a more austere policy in the first years.

The dotted black line refers to the model with uncertainty but no windfall. I solve this model without the necessity to deal with a structural break. The model predicts well an upward path for consumption and the accumulation of assets to cope with volatility. After debt repayment, the government sustains an asset position sufficient to guarantee policy smoothness in the long run, with a consumption level above the deterministic model without a windfall. It is possible due to precautionary saving.

In the model under uncertainty and windfall, the solid black line, the simulation points to an upward shift in the tax variable and a downward shift in the consumption and public expenditures simultaneously to the oil windfall exhaustion. When facing a lower level of resource revenue, the government immediately increases taxation on the non-resource sector and reduces public spending. The primary balance panel shows that the optimal policy is to proceed with an austere policy until accumulating a robust asset position, reverting to imbalanced outcomes from the thirty-third year. Due to the accumulated assets, the austere policy allows for a positive overall balance until the windfall term.

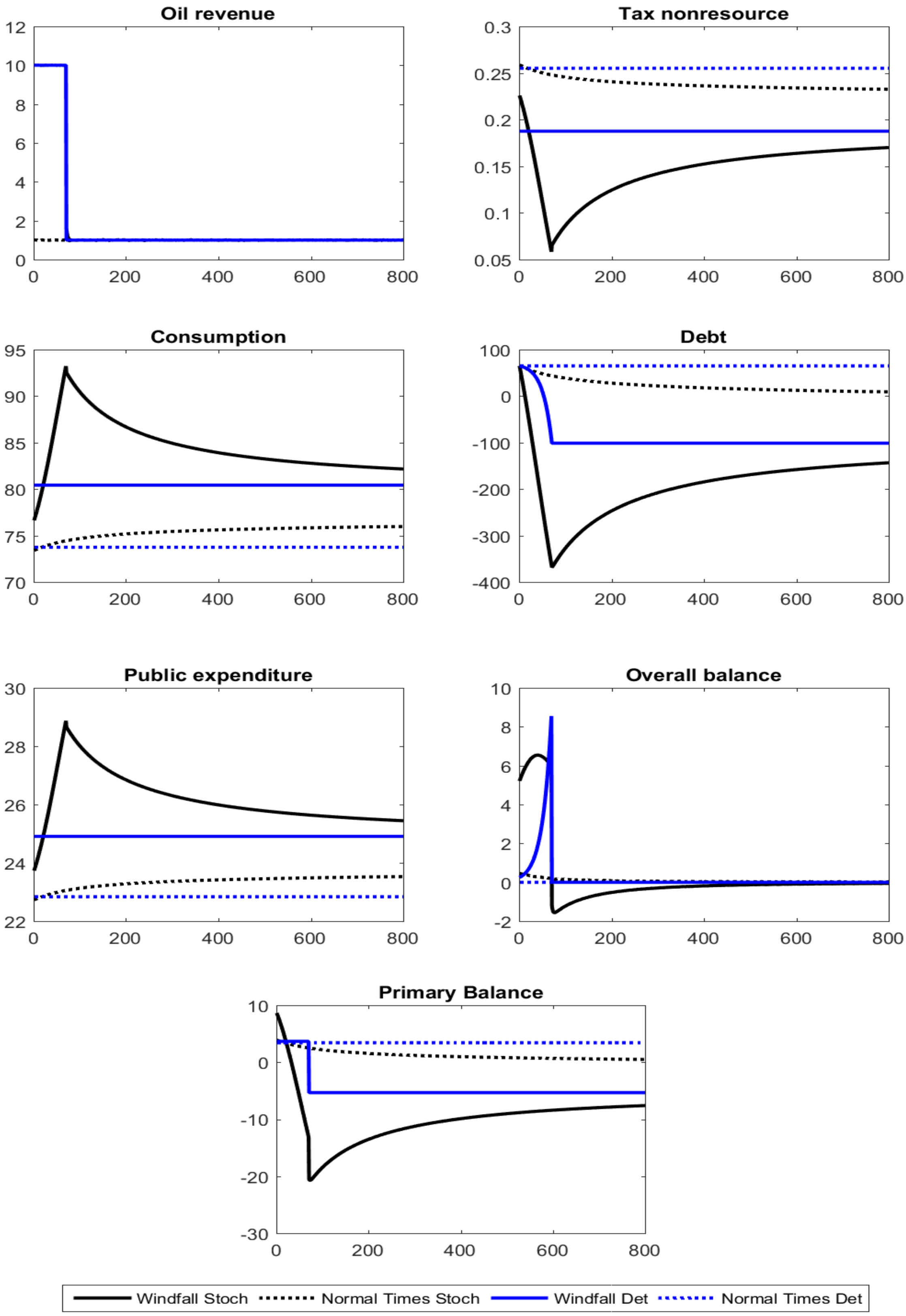

Figure 2.2: Simulated trajectories

Solid and dotted blue lines correspond to the deterministic model with and without the windfall. The same to black lines, but to the stochastic model.

The optimal policy provides a transition with several years (before and after the windfall exhaustion) of a significantly higher public and private consumption level compared to the model without uncertainty, possible through the precautionary saving. Additionally, a higher level of public and private consumption in the model with uncertainty also prevails in the long-run equilibrium.

Figure 2.3 decomposes the destination of the additional natural resource revenue source. The change in the primary balance starts responding for about 50% percent of that additional oil revenue but follows a downward trend. From period 22, the primary balance in the model with windfall becomes smaller than the one prevailing in the absence of windfall. This result reflects the accumulation of financial assets due to the windfall, allowing the possibility to conjugate a lower level of primary fiscal balance with a reduction of taxation in the non-resource sector and increased public expenditure.

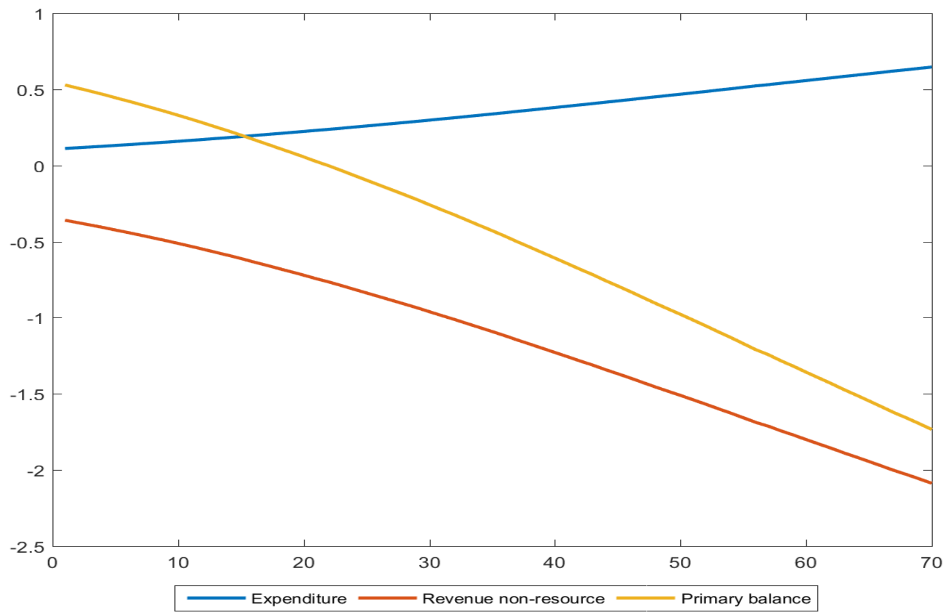

Figure 2.3: Decomposition of changes on resource revenue during the windfall

The standard measure of precautionary saving in the literature refers to the difference between the asset accumulation paths under the stochastic and deterministic models. The asset accumulation in the deterministic model has an association with an intergenerational distribution fund. Moreover, the additional asset accumulation verified in the stochastic model refers to a liquid/stabilization fund to deal with volatility in the resource revenue. Note that the model by itself does not distinguish between the two types of funds. For example, all the assets can be held abroad in a sovereign wealth fund.

Figure 2.4 shows precautionary saving for two models: one at which a country benefits from a natural resource windfall; and the other in the absence of such a windfall. The optimal policy guides to a more robust wealth accumulation when challenging a windfall. This wealth peaks in period 64, at a level almost 14 times higher than in the model without a windfall. This higher asset accumulation persists for an extended period (up to 636).

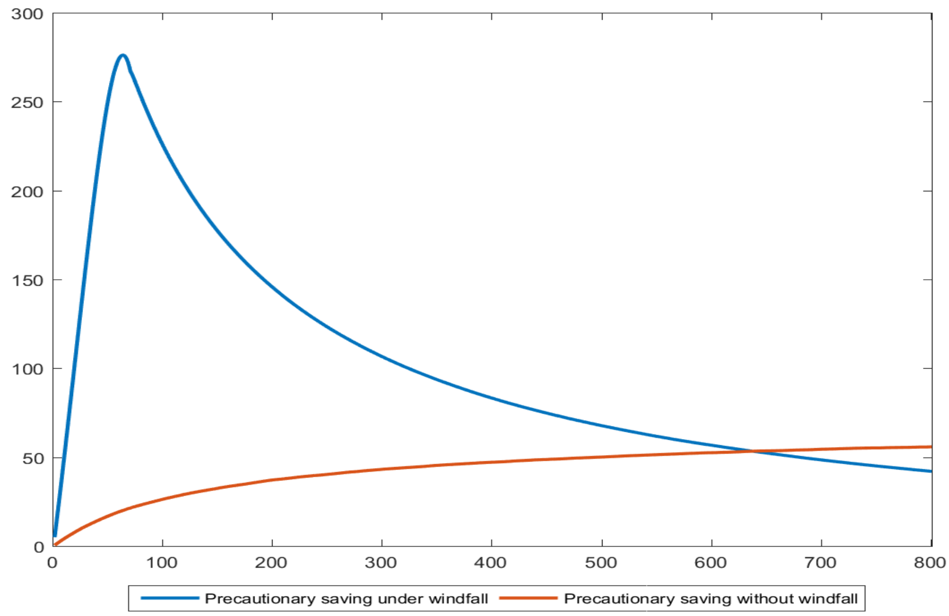

Figure 2.4: Precautionary saving in the models with and without a windfall

2.5.3 Sensitivity Analysis

In the previous subsection, I showed that the optimal policy response supports the accumulation of financial assets to cope with volatility in the natural resource revenue process. The precautionary saving is still more relevant whether a country challenges a windfall. Constraining access to financial markets has a direct impact on a country's possibility to smooth consumption. Figure 2.5 shows the sensitivity to alternative debt limits to the model with a windfall.

Reducing these limits implies a lower level of attainable asset accumulation. Furthermore, this imposes a negative externality on the consumption pattern of the generations following the windfall period. When the windfall ceases, the immediate drop in consumption is about 0.75% in the baseline model. The reduction reaches 4.90% and 7.68% when facing debt limits of $+/-350$ and $+/-200$, respectively.

In the long term, the equilibrium consumption in the baseline model is 0.45% and 0.85% higher than those under those alternative debt limits. During the whole transition, the consumption difference between the baseline and the one with a debt limit of $+/-350$ accumulates almost 12 times the output. This number increases to about 18 when the debt limit reduces to $+/-200$.

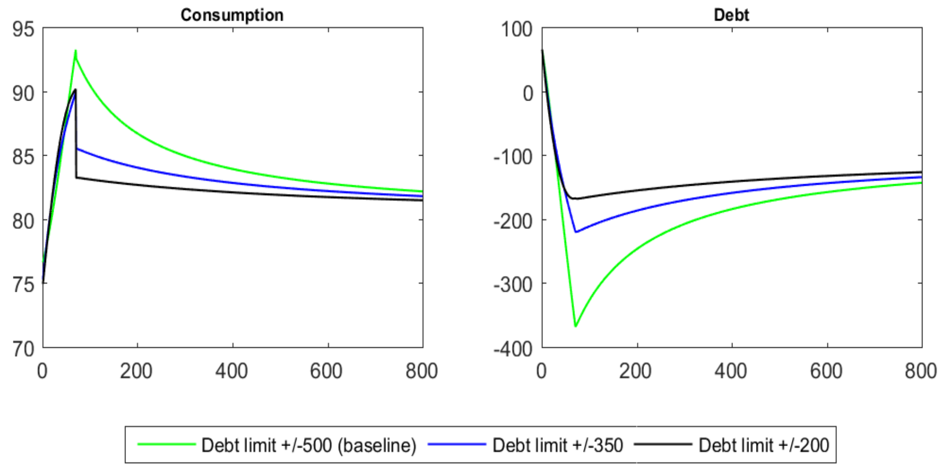

Figure 2.5: Sensitivity to debt limits parameter

Figure 6 presents the sensitivity to the persistence parameter of the natural resource process. The impact is tiny, but pointing that a higher persistence parameter constrains the level of assets the optimal policy can achieve at the windfall term. Concerning consumption, a higher persistence implies a higher departure from the baseline model in the whole path. As a result of a lower level of asset accumulation, the higher persistence parameter implies a lower consumption in the periods surrounding the time at which the windfall ceases. These findings do not contradict the rationale that a higher level of precautionary saving due to the longer duration of the shocks follows a higher persistence parameter of the resource revenue process. Note that in the long-run equilibrium, the solid black line ($\rho = 0.95$) exhibits a tiny higher level of asset accumulation and consumption than the baseline model (and also the model parameterized with $\rho = 0.80$).

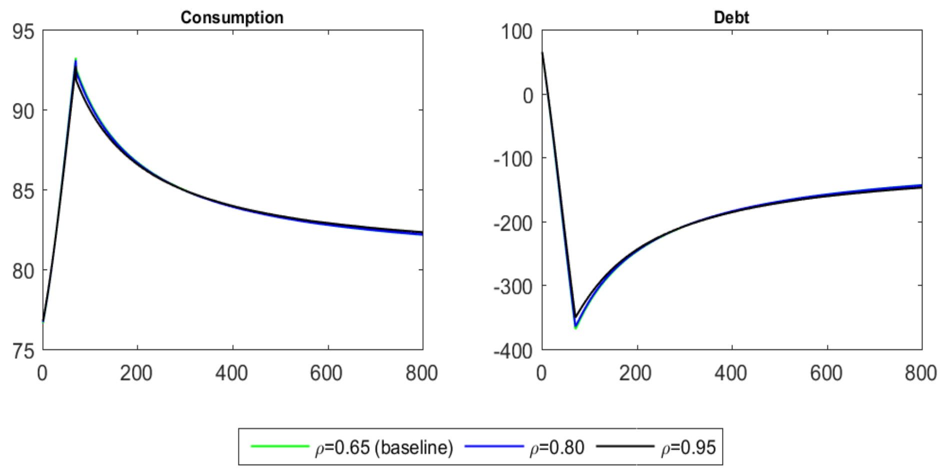

Figure 2.6: Sensitivity to the natural resource persistence parameter

The asset accumulation dynamics relates to the preference parameters on private and public consumption, respectively, σ_c and σ_g. Considering the type of CRRA utility used to calibrate the model, the coefficient of relative prudence increases with the preference parameters' absolute values. So then, I should expect a higher precautionary saving when increasing the absolute value of those parameters.

In Figure 7, one observes that an increase in the absolute value to 2 implies an acceleration in the asset accumulation along the first years of the windfall. It sustains a higher level of consumption during that period. This increase in the precautionary saving does not translate to a higher asset accumulation than the baseline model. The convergence to the long-run equilibrium occurs relatively quickly, with a lower level of financial assets, translating to the level of consumption in equilibrium.

The sensitivity to smaller preference parameters (in absolute values) delivers a lower financial assets level in the years preceding the windfall term. However, the peak of asset accumulation surpasses the parametrization under the preference parameters absolute values equal to 2. Thus, the transition to the long-run equilibrium allows for an extended period of higher consumption than the model with parameters equal to 2. Indeed, in steady-state, the levels of assets and consumption are higher than the baseline model (tiny in the graph due to the slight difference of the preference parameters compared to the baseline).

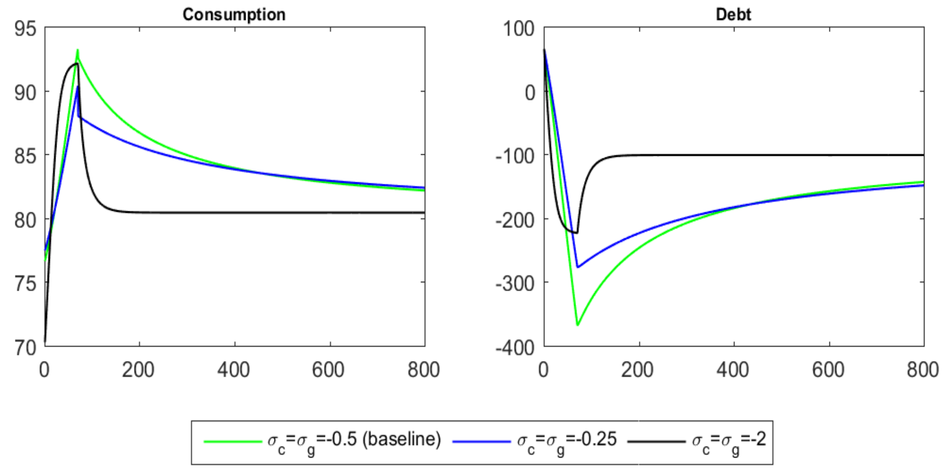

Figure 2.7: Sensitivity to the preference parameters σ_c and σ_g

The last sensitivity performed is on the subjective discount factor. As in the case for the preference parameters σ_c and σ_g, the interpretation is not trivial. Changes in the discount factor have two forces in opposite directions: a lower(higher) β implies an increase(a decrease) in consumption as well as a decrease (an increase) in savings. The interest rate in the risk-free bonds increases (decreases). And the precautionary saving increases (decreases) due to a higher (lower) income dependence on the uncertain resource revenue. The net effect when reducing the baseline discount factor to 0.94 is a less intensive propensity to save compensated by a higher precautionary saving that allows a long-term level of consumption higher than in the baseline model. In the case of an increase in β, the net effect points to a tiny increase in the propensity to accumulate assets and a tiny decrease in consumption in the first windfall years. The lower precautionary saving causes a relevant drop-down in the level of consumption in the post-windfall period.

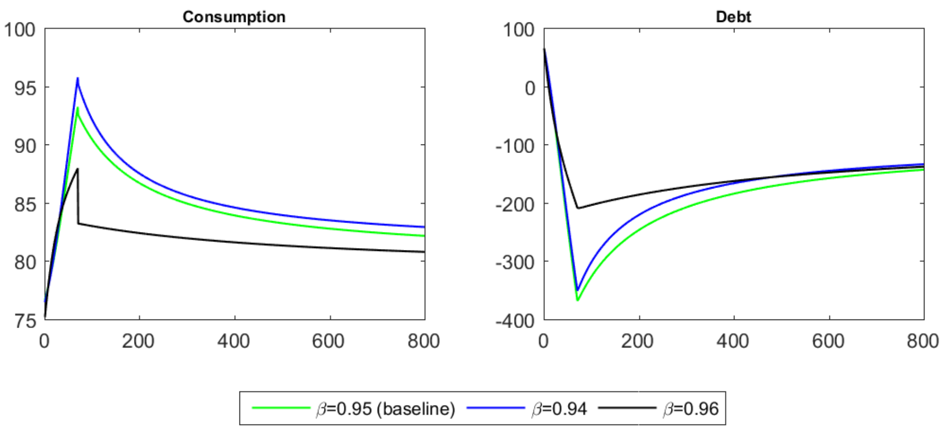

Figure 2.8: Sensitivity to the subjective discount factor

2.6 Final Remarks

I proposed applying the Parameterized Expectation Approach (PEA) to solve an infinite-horizon stochastic problem in this work. The goal was to analyze optimal fiscal policy in an environment where a country faces a temporary revenue windfall from a natural resource. The main concern was about optimal spending and savings behaviors to fairly distribute the windfall across generations. The most acknowledged model in the literature is adherent to the permanent income hypothesis theory, to which optimal policy implies smoothing consumption and the accumulation of assets abroad in an intergenerational fund.

The majority of literature does not model uncertainty when assessing the implications of a revenue windfall. Up to my knowledge, Engel and Valdés (2000) were the first to include uncertainty in the analysis. More recently, other contributions appeared regarding assessing the government's precautionary saving due to uncertainty (almost always on the natural resource price) and the necessity to raise a liquidity/stabilization fund. Another branch of the recent literature analyzes the role of domestic capital assets (public and private), mainly to promote the non-resource sector. The implication is that resource-rich countries that face capital scarcity and absorption constraints should raise an investment fund.

The model I proposed in this work is standard in the literature. The strategy to start with this basic setup serves to evaluate the adherence of the approach I proposed in reproducing the optimal policy response commonly found in the literature. A direct comparison is not perfect, given that here I considered that uncertainty lasts to the whole time path, while in the literature, the standard approach assumes that uncertainty disappears in a finite moment in time.

I leave for future research to extend the model to capture aspects particular to the Brazilian economy. First, considering that the relevant debt position is held on domestic bonds and the asset position on international reserves at the central bank. Then, contextualizing the challenging fiscal environment with around 90% of public expenditures on the federal budget being mandatory and the other 10% including sectoral subsidies, investment, and contingency reserves.

Regarding the first point, there are two fundamental aspects to highlight. One refers to the interest rate differential between debt and asset positions, with interest on debt meaningfully higher. The other is about optimal policy reduction of the net debt position. On one side, the amortization of liabilities reduces the charge of interest payments in the public finance but potentially implies an appreciation of the exchange rate and, consequently, adverse effects on the non-resource tradable sector ("Dutch Disease"). On the other side, accumulating reserves at the central bank or even investing abroad avoids concerns about the exchange rate and the tradable sector but does not reduce the weight that the interest on debt plays on the government budget.

The second point, about the weight of mandatory spendings, implies that I can abstract from endogenizing public consumption but instead assume a stochastic exogenous process to that spending (generating or not generating utility to the agent) and include the nonmandatory expenditures in the production function of the non-resource sector (infrastructure, health and education investments). Under this assumption, the policy adjustment to the natural resource's post-exhaustibility is more implementable (reduce mandatory expenditures is hard). Another departure from the usual literature assumptions would be considering the household's labor choice impact on the economic dynamics.

Appendix: Short-run Dynamics: primary oil revenue and international oil price

This appendix introduces the short-run dynamics of the Brazilian primary oil revenue regarding oil production and the international oil price. Figure A.1 plots together quarterly data on primary oil revenue, WTI international oil price, and oil production[13].

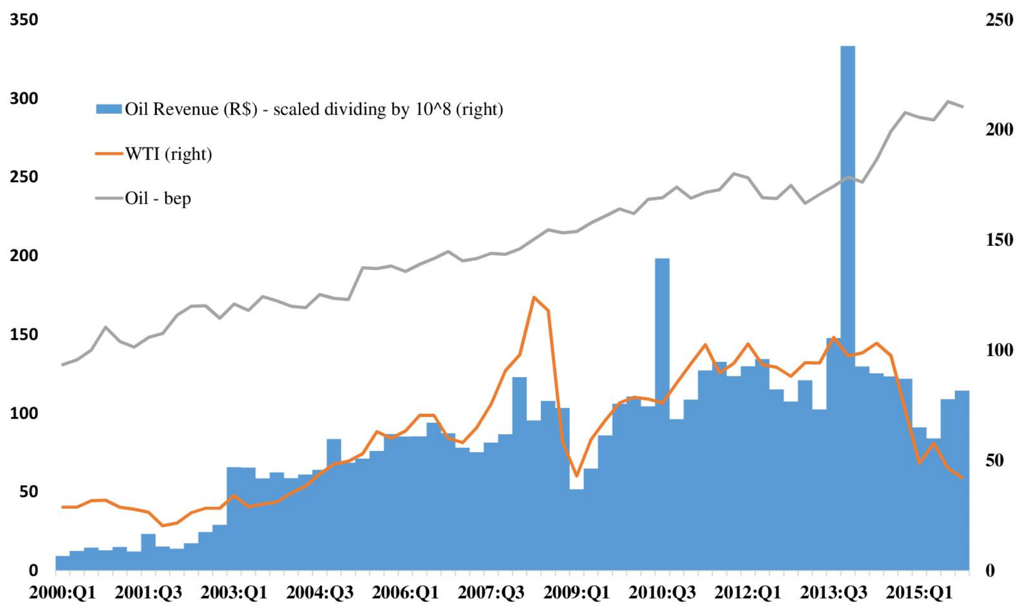

Figure A.1: international oil price and Brazilian oil revenue and oil production

It is suggestive that the WTI and the primary oil revenue behavior are very related. However, this relation is not clear between WTI and oil production. The coefficient of variation of the oil revenue is 60%, while those of WTI and production are, respectively, 45% and 21%. The international prices' behavior is crucial to producers in elaborating the medium and long-run business plan since the expected future price is a fundamental variable to determine competitive projects' economic viability. In the short run, the production is less responsive to shocks in the oil price. A direct implication is that oil price uncertainty predominates over the production one in the short-run dynamic.

[13]Due to data availability, the primary revenue here includes only *royalties*, special participation on production, economic contribution (Cide-Combustíveis), signing bonuses and retention payments to areas of production and exploration.

I explore this dynamic using a VAR(1) on the variables oil price and primary oil revenue[14]. The identification comes with the following steps:

- Revenue does not affect contemporaneously the oil price;

- Impulse response functions (IRFs) under a Cholesky decomposition approach.

The IRFs are in Figure A.2. The revenue response to a shock in the international oil price reflects that the oil tax base is highly referenced on the international oil price. The accumulated response of the variation on oil primary revenue is around 13% to a shock of one standard deviation in the variation of WTI price (to this sample 15%). Notwithstanding, the absorption of this shock is almost immediate and disappears after the third quarter. I observe the same absorption pattern in the oil revenue dynamics after its shock.

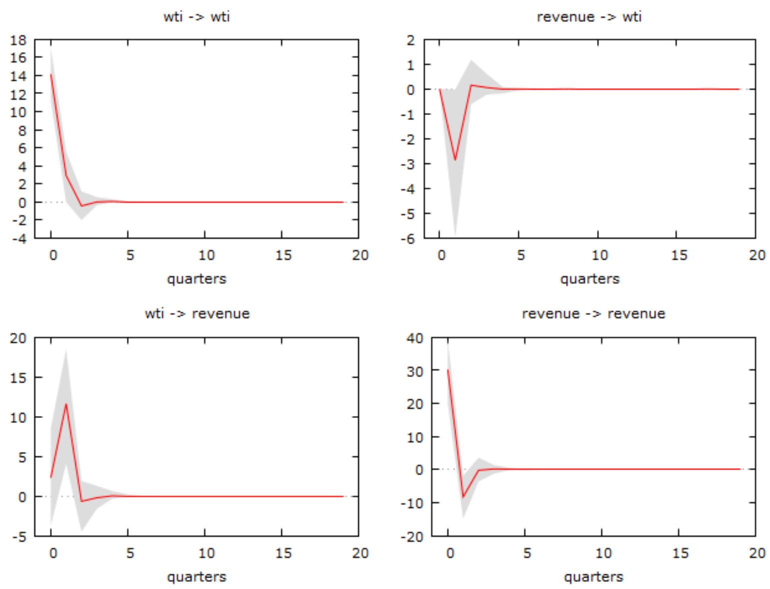

Figure A.2: IRFs: primary oil revenue and oil price

This exercise highlights why the resource production path is not usually the object of uncertainty analysis in the literature. Instead, the focus is predominantly on the volatility of international prices.

[14] Actually, I started with a VAR(1) including, additionally, oil production, oil trade balance, and GDP. However, I opted to show this parsimonious result founded on the block exogeneity test concerning these variables. In both cases, I chose the VAR order optimally based on the BIC criterion.

Chapter 3

The Impact of Public Expenditures on Happiness: evidence from the Brazilian States

3.1 Introduction

This chapter concerns the analysis of public expenditures' impact on people's happiness in the Brazilian States. The rationale is that happiness depends on a set of events, like job, earn, health, leisure, education, security, urban mobility. Through the allocative role of the public sector, governments can take actions that impact the way people experience the realization of these kinds of events. For example, a government can allocate resources to the construction of hospitals, which will allow people to reduce the time in the line for surgery. In turn, it will positively impact people's feelings of happiness. Concerning security, the role of government actions is more prominent, given that this is a pure public good (people cannot fight against crime on their own).

From a theoretical perspective, one can think about a society in which agents derive utility from both private and public goods, an assumption that is quite reasonable. One approach to measuring how important public goods are on individuals' utility is to evaluate their degree of happiness. There is literature support to the assumption that subjective well-being (SWB) is a *proxy* for utility. Rabin (1998) connects happiness and experienced utility, with the last representing the pleasure derived from consumption (Kahneman and Thaler, 1991). Benjamin, Heffetz, Kimball and Rees-Jones (2012) show evidence, from a laboratory study, that individuals' choices are oriented to optimize their SWB. Liberini, Redoano and Proto (2017) use SWB measure as a *proxy* for the utility to analyze individuals voter's behavior.

Except for data availability, there is no restriction working on Federal, States or Municipalities levels. However, I should consider that, depending on the government sphere, there are important aspects regarding the fiscal federalism framework to keep in consideration. For example, in the case of Brazil, the Federal government is the main responsible for financing the Social Security System (social insurance, social assistance, and health). Therefore, even though States and Municipalities can structure their social insurance policy raising statutory regimes for civil servants and implementing social assistance programs (like cash transfers to poor people), it is not a high amount compared to federal spending.

Regarding health and education spending, the three spheres should follow constitutional rules that establish a minimum amount of financing according to primary revenues. In the case of security spending, constitutional attributions to each level determine the importance of this category in the public administrations' budget at each government level. The Municipalities' role is very delimited, which makes security spending relatively low. Among the three levels of government, spending on security weighs more on the States' public finance. Spending on infrastructure is challenging at any government level. The reason is that spending on mandatory expenditures (social insurance, health, education, payroll, for example) and public debt heavily squeeze the governments' budget.

The literature about the impact of public expenditures on happiness (or any other measure of SWB) has increased from 2000. And panel data studies, particularly from the 2010s decade. The evidence is ambiguous in both the cross-sectional or the panel data approach.

Bjornskov, Dreher and Fischer (2007) find evidence of a negative impact of overall government consumption spending on individual life satisfaction in a cross-sectional study of 74 countries (see also Ovaska and Takashima, 2006; and Yamamura, 2009). However, when considering capital formation and welfare spending, they find a null impact on life satisfaction. The authors work with data on the central government and mention that it is impossible to extrapolate the conclusions to subnational governments.

Kacapyr (2008) reports a non-significant impact of government expenditures as a share of GDP on life satisfaction in a cross-section of 63 countries in the 1990s. Ram (2009) finds no significant effect of government consumption on happiness. He stresses that the sign is indeed positive even that not significant, being concerned about policy recommendations that one can derive from results that point to a negative association, like in Bjornskov et al. (2007). Ott (2010 and 2011), grounded on a comparison of 127 and 130 countries in 2006, finds that positive correlations between government expenditures and happiness depends on the government's technical quality (a measure of governance).

The cross-section literature on specific categories of public expenditures, as for the aggregate concept, also offers ambiguous results. Ono and Lee (2013) study 29 countries with 2002 data and do not find evidence that social-related public expenditures increase happiness, even that social democratic welfare states use that expenditure to redistribute happiness across citizens groups (see also Veenhoven,2000; and Ouweneel, 2002). O'Connnor (2017) supports a positive association between social protection expenditures and life satisfaction on a cross-section data for 104 countries averaged over 2005-2012 (see also Kotakorpi and Laamanen, 2010; and Flavin, Pacek and Radcliff, 2011).

Livani and Graham (2018) assess how social protection programs impact subjective well-being in Iraq with 2012 data. Their findings support that the relationship differs according to the type of social protection programs. For example, pensions and programs directed to all citizens seem to have a null impact for citizens in general but a positive one for the poorest people. In turn, programs related to individuals' disadvantages aspects show negative signs. Rodríguez-Pose and Maslauskaite (2012) find evidence on the negative impact of total government expenditures on happiness in 10 Central and Eastern European countries. For specific spending categories, they find a positive effect of health care, unemployment benefits and pensions, once controlling for corruption in 1999. However, their findings for the last two types of expenditures do not hold in 2008.

Concerning panel data studies, Perovic and Golem (2010) find a positive impact of government expenditures on happiness for 13 transition economies from Central and Eastern Europe (see also Kasmaoui and Bourhaba, 2017). Regarding specific expenditures categories, Di Tella, MacCulloch and Oswald (2003) find a positive impact of unemployment benefits on well-being for European countries over 1975-1992 (see also Pacek and Radcliff, 2008; and Flavin, Pacek and Radcliff, 2014). Knoll and Pitlik (2016) study 25 European countries from 2002 to 2012 and find a null or negative impact of government expenditures categories (total, health, education, and social protection) on well-being, with this impact being worse to high-income groups.

Hessami (2010) studies the relationship between government spending and well-being for 12 European countries during 1990-2000 and finds mixed evidence. He finds an inverse U-shape relation when analyzing total expenditures (positive linear term impact and negative quadratic term impact). As for specific categories, his findings suggest that while education and social protection positively impact the well-being, this does not hold regarding health expenditures (see also Kiya, 2012).

Nordheim and Martinussem (2020) study the effects of social spending on life satisfaction in OECD countries between 1980-2012, finding a null correlation when analyzing total expenditures. In contrast, when analyzing distinct categories, the evidence is mixed. They find a positive impact for health care and poverty reduction-related spending and a negative one for unemployment benefits and active labor market programs (see also Flavin, 2019).

In this chapter, I propose a two-steps approach to work with cross-sectional surveys that, on successive releases, draw new and independent samples. First, I regress individuals' self-reported happiness on objective variables, using microdata from Latinobarómetro. Then, in the second step, I regress the States' averaged residuals of life-satisfaction from the first step on distinct categories of public expenditures and control variables.

This approach differs from the previous studies using panel data for not treating different individuals from each round of cross-sectional surveys as equal ones to proceed with the panel data method. The two-steps approach allows me to implement a panel on States nor individuals. Once States are the same, prevails the panel analysis in the strict sense[1].

The chapter contributes to the incipient literature that assesses the relationship between distinct categories of public expenditures and happiness using panel data.

[1] In the Eurobarometer FAQ, one of the questions is about the longitudinal/panel characteristic of the survey (which applies to the Latinobarómetro and other similar surveys). The answer is the following: "The Eurobarometer is a series of cross-sectional surveys. For each round, a completely new and independent sample is drawn. As a consequence, panel analyses in the strict sense are possible only for aggregated data." Available on (question 7): `https://www.gesis.org/en/eurobarometer-data-service/faq`

The main findings suggest a negative effect (at most null) of States' spending, both at the aggregate level and disaggregating on security, education, health, infrastructure, and other expenditures. Furthermore, these findings are robust to the exclusion of some controls grounded on multicollinearity analysis and the method used to estimate the model (fixed effects, random effects, pooled, and between estimators).

Following this introduction and literature review, I organize this work into five sections. Section 3.2 describes the data, and the 3rd section brings the methodology. I present the main empirical results in section 3.4. In Section 3.5, I proceed with some robustness checks. The 6th section concludes this paper. The tables and figures are all in the Appendix.

3.2 Data Description

Nowadays, there is no availability of a panel survey on happiness in Brazil. Latinobarómetro applies annually a cross-sectional survey in Latin American countries. Specifically, data on happiness (or life satisfaction) started in 1997. However, at least in the case of Brazil, until 2002, the sampling design did not include a countrywide representative sample. Other institutional alternatives, free of charge, were considered at the beginning. The Interamerican Development Bank (IDB) has data on happiness only from 2005, and the World Value Survey (WVS) allows me to work with data for the 2006 and 2015 waves. Considering these restrictions, I work with Latinobarómetro data that comprehends 2002-2013, except 2012 (survey not conducted). See Table A.1 for a list of variables that I use from Latinobarómetro.

Regarding public expenditures at the States-level, I use annual data from the Brazilian National Treasury. Due to legal statements, all government spheres need to publish lots of information regarding revenues and expenditures. I use the functional classification on the expenditure side, which presents data according to the spending characteristics, like security and education. Since 2002, these expenditures are classified into 28 distinct functions. After analyzing their relative individual importance and aggregating some related categories, I elect to work with four main government functions: security, education, health, and infrastructure.

The first three are individual functions, while the infrastructure includes spending on urbanization, housing, sanitation, transport, communication, and energy. Also, I include in the analysis, for completeness, two additional categories: other and total expenditures. It is important to mention that I am not considering public debt spending (both amortization and interest). All expenditure data are in *per capta* 2013 value. First, I inflated nominal expenditures to 2013 real value by the General Price Index (IGP-DI), produced by the Brazilian Institute of Economics - Getulio Vargas Foundation (IBRE-FGV). Then, I divided by each States' populations from the Annual Population Estimates (2002 to 2013, except 2010) and Population Census (2010), both produced by the Brazilian National Institute of Geography and Statistics (IBGE).

Another source of data for this work comes from the National Household Sample Survey (PNAD) and the Regional Accounts, the last specifically to States' GDP, both available by the IBGE. The data I use from IBGE are relevant in the analysis between expenditures and happiness to control factors that can become a confounder on identifying the expenditures' impact. Among candidates to controls, some would be interesting but are not available to all Brazilian States (unemployment rate and inflation) and a considerable time horizon (inequality). While unemployment rate and inflation are available only to the main metropolitan areas from some representative States of the Brazilian regions, the inequality indicator is available only every ten years in the Population Census. Considering this constraint, I construct some variables using annual microdata from PNAD, 2002 to 2013 (except 2010), and Population Census (2010). See Table A.1 for a description of these variables.

In Table A.2, I present the sample size regarding IBGE and Latinobarómetro data. Acre is the only State to which there is no observation in the Latinobarómetro annual survey. Amapá and Roraima were included only in one annual survey, and Mato Grosso do Sul in six surveys. Regarding the PNAD, the sample includes households (and individuals) from all the Brazilian States. The same is true in the case of the Population Census. Note that when I refer to the sample in the Census case, it is specifically about the sample results of the decennial Census. These results come from the application of a more detailed questionnaire on a sample of households and individuals. Beyond this sample questionnaire, there is a basic questionnaire that covers all the population.

3.3 Methodological Description

The empirical analysis proceeds into two steps. In the first step, I run at each year (11 in total) a cross-sectional regression (OLS estimator) of life satisfaction on some individual objective variables (see the descriptions in Table A.1). At this stage, I use only Latinobarómetro data. Then, I store the residuals from these regressions, which I call ***res_lifesat***. Finally, I collapse the average regressions' residuals (***res_lifesat***) by States. This first step delivers a total of 261 observations on 26 Brazilian States.

In the second step, I work with data at States-level, using the panel data approach. I have observations for 26 States and 11 years. The main interest is to evaluate the impact of public expenditure on happiness. To this goal, I estimate the following specification:

$$res\_lifesat_{it} = \alpha_i + \boldsymbol{\beta x_{it}} + \boldsymbol{\gamma z_{it}} + \epsilon_{it}$$

where:
i = 1,...,26 are the individual States and t=1,...,11 are the time index;
α_i are random States-specific effects;
$\boldsymbol{\beta}$ is a vector $1xk$ of parameters on the public expenditure $kX1$ vector $\boldsymbol{x}_{it}$;
$\boldsymbol{\gamma}$ is a vector $1xm$ of parameters on the set of $mX1$ control variables $\boldsymbol{z}_{it}$;
ϵ_{it} is a white-noise term that represents idiosyncratic shocks on State i at time t.

Even considering that the panel fixed-effects empirical model is better suitable to this chapter's research interest, I do not discard other kinds of specifications at the outset. Then, I also perform, for completeness and robustness checks, OLS and FGLS pooled regressions, random-effects regressions, and between-estimator regressions. This procedure aims to verify whether the main findings based on a specific model specification are sensitive to other alternative approaches.

I am not concerned about reverse causality between (residual) life satisfaction and government expenditures due to two main reasons. First, government actions need to respect budget constraints. The reality of the Brazilian States' public finance environment is of great difficulty in finding fiscal space to accommodate additional demands. Second, currently, there is no regular survey conducted on the Brazilian States that focuses on evaluating the impact of distinct categories of public expenditures on individual happiness. One can rationalize the role of this kind of survey as a source of information to the incumbent government to reallocate spending across distinct functions to get benefits, like government approval and the maintenance of a political coalition in power (including reelection).

Regarding the omitted variable bias, I should care about it. The problem is that I do not have much flexibility to include other relevant variables in the analysis. Notwithstanding, working with States within the same country reduces the impact of omitting some important variables like inflation and unemployment. These restrictions bias the understanding that the fixed-effects panel approach is more appropriate to address the research proposal. In the end, I show that the results are robust to any method I use to estimate the parameters.

3.4 Empirical Results

Before analyzing the regressions' results, let me make general considerations about the pattern of States' expenditures both across years (Figure A.1) and regions (Figure A.2) in Brazil. Across years, the average spending on security, education, health, and infrastructure responds to 57% of States' expenditures. From which education and health are, respectively, 21% and 15%. The general pattern presents a non-monotonically but slightly increase in the distinct categories of *per capta* public spending across years. Also, education and health show a relatively lower variance than the other groups, given that these expenditures have their behavior dictated by legal rules that establish a minimum percentage of these expenditures regarding States' revenues.

Regarding regional patterns, the Midwest (MW) presents the most relevant volatility, except for security. It is notable also that the Northeast (NE) region is the one that, in general, has the lowest medians in the distinct categories. Indeed, this region shows relatively low dispersion, together with the South (SO). Notwithstanding, while the NE is the region that concentrates the massive poor population in Brazil, the SO is the less unequal region, according to the PNAD 2014. The North (NO) and Southeast (SE) are intermediate cases, with the NO being more closely related to NE and SE to SO.

The first-step regressions results are in Table A.3. The goal here is to extract the variation in life satisfaction regarding objective variables. The constraint is that the availability of this kind of variable in the Latinobarómetro survey is relatively small compared with the number of subjective measures surveyed. Therefore, I include the variables surveyed with continuity of at least a couple of years in the first-step regressions.

The degree of adjustment, measured by the R^2, is pretty low (around 4.0% across the years). In general, all the explanatory variables play some significant effects on life satisfaction, yet not present in all the years (except the individuals' age). Figure A.3 shows the (residual) life satisfaction relationship with distinct categories of public expenditures, averaged over 2002-2013. A first look suggests the absence of a clear directional relation whatever spending category one considers.

In Table A.4, I present the results of the second step, grounded on a fixed-effect panel specification. First, I estimate the most general model in which I include all expenditures and control variables (second column). Then, I move on to show the results to specifications that include each of the expenditure categories. Indeed, model (7) considers total public expenditures (net of debt). All the results consider standard error corrections to challenge the potential Moulton problem (Moulton, 1986) and correlations concerns in the context of few clusters (Liang and Zeger, 1986; Donald and Lang, 2007), which improves statistical inferences reliability in this kind of applications. The first relevant finding is that only the control over the percentage of households with bathrooms is statistically significant. Concerning the main question I propose to challenge in this work, the first model suggests no impact of public expenditures, in general, on average life satisfaction at States-level. Models from (2) to (7) suggest a negative sign at the distinct public spending categories. Model (7) goes beyond any distinction of expenditure's functional classification.

3.5 Robustness checks

To perform some robustness checks, let me consider potential multicollinearity problems regarding both expenditures and controls. Table A.5 presents the bivariate correlations. The first part of the table shows high levels of correlation between distinct expenditures functional classifications. In the second part of the table, there are the bivariate correlations among the controls. The main concerns with this part are the high levels of correlations among the infrastructure controls (***water***, ***wc***, ***garbage*** and ***electric***).

Deepening the analysis, I show in Table A.6 the Variance Inflation Factor (VIF) multicollinearity test, to which I implement an OLS regression. There is no wide consensus about the level of VIF that can guide conclusions on multicollinearity. The rule-of-thumb is that a VIF superior to ten and a mean VIF meaningfully greater than one can justify concerns about the problem (Chatterjee and Hadi, 2012). The left side of the table confirms the suspicion about a high level of collinearity. It is true for expenditures when including all the categories together, and some infrastructure variables, mainly on ***garbage*** and ***wc***. Note that in the case I work with models that include the distinct expenditures alone, the concerns about multicollinearity become a constrain only on the controls. On the right side of the table I exclude ***garbage*** and ***wc*** from the analysis. Again, the evidence is that working with individual expenditures categories is a better identification option.

After considering this multicollinearity brief analysis, I replicate the model specification of Table A.4, except that I exclude the infrastructure variables ***garbage*** and ***wc*** from the controls. I keep the first model specification just for comparability. Note that the sign and magnitude of the estimates on expenditures coefficients are very robust to those of Table A.4. Indeed, no improvement in the power of the remaining controls in terms of statistical significance. The results are in Table A.7.

Ahead with some robustness check, I replicate Table A.4, again excluding variables ***garbage*** and ***wc***, but allowing for other methods than not just the fixed-effect approach. I show the results to each expenditure category and total expenditure on Tables A.8 to A.14. The main message, regarding the sign and significance, is the same from Table A.4, which points in general to a negative impact of public spending when I constrain the analysis including distinct expenditures categories individually, which seems more appropriate from the multicollinearity analysis. The last robustness check is concerning the inclusion of time dummies variables. The conclusions still stay the same. To avoid another bunch of tables, I do not attach the results. They are available upon request.

3.6 Final Remarks

This chapter assessed the impact of distinct categories of public expenditures on the average happiness of Brazilian States' citizens. To accomplish this task, I performed a two-step approach due to the nonavailability of a panel survey about life satisfaction.

In the first step, I worked with microdata at the individuals-level, available from Latinobarómetro annual cross-sectional surveys. First, I regressed happiness (life satisfaction) on objective variables, like age, years of school, marital status, and head of a family. The findings suggest a pretty low explanation power of these objective variables on individuals' life satisfaction, around 4%. Then, I proceeded to collapse the average regressions' residuals by the Brazilian States.

In the second step, I regressed the States' averaged residuals of life-satisfaction on distinct categories of public expenditures and control variables. I used some controls, like the percentage of the elderly population, urban rate, and GDP *per capta*. However, these controls were not that effective, at least in terms of statistical significance. The main finding was that, in general, the impact of distinct categories of public expenditures exhibit a negative effect (at most null) on life satisfaction in the Brazilian States.

Theoretically, one rationale for these findings is that the Brazilian States, in general, reached an optimum welfare state level, from which citizens' well-being would reduce with the increase of welfare policies. However, as peers developing countries, Brazilian States challenge bottlenecks in different areas, like infrastructure, poverty, health, security, and education. Therefore, an alternative explanation can pass through inefficiencies in allocating public expenditures (Oliveira, Rocha, Duarte, Pereira. and Gadelha, 2014; and Rocha, Oliveira, Duarte, Gadelha, and Pereira, 2017).

Concerning policy implications, these findings suggest caution on policies that aim to transfer financial resources to subnational governments. It is suggestive that these policies should be accompanied by mechanisms that promote the efficient use of public resources. If this is not the case, governments can fail to improve their citizens' quality of life.

In future developments, it would be interesting to investigate the theoretical and empirical implications of the application of panel data methods in annual cross-sectional surveys instead of the two-step approach developed in this chapter. The use of panel data methods is widespread in the literature that evaluates public expenditures' impact on life satisfaction. Another branch for future developments concerns the design of incentive mechanisms to enhance fiscal federalism implications of economic policies that aim at financial support from federal to subnational governments (or from States to Municipalities).

Appendix: Tables and Figures

Table A.1: Variables description.

Variables	Description
	Latinobarómetro (1st step)
lifesat	Satisfaction with life (scale 1 to 4).[1]
age and age2	Respondent's age and age squared.
yearseduc	Years of education.
socioeconom_hig	Indicator 1 if the socioeconomic status (perception of the interviwer) is 1 and 2, in a scale from 1 to 5 in descending order.
male	Indicator 1 if male.
married	Indicator 1 if married.
head	Indicator 1 if head of the household.
student	Indicator 1 if student.
selfemployed	Indicator 1 if selfemployed.
retired	Indicator 1 if retired.
res_lifesat	Residual from the OLS regression of **lifesat** on: **age, age2, yearseduc, socioeconom_hig, male, married, head, student, selfemployed and retired.**
	IBGE: PNAD and Population Census (2nd step)
lowstud	Percentage of people with less than 4 years of study.
old65	Percentage of people with more than 65 years old.
formalemployment	Percentage of people with formal employment[2]
children	Average number of alive children.
water	Percentage of households with at least one room that has piped water supply.
wc	Percentage of households that has a bathroom.
garbage	Percentage of households with garbage collected directly or indirectly.
electric	Percentage of households with energy source used for lighting from electric network, solar or generator power.
urban	Urbanization rate.
gdppc	GDP {per capta}.
	Brazilian National Treasury
securityexp	Security {per capta} expenditure, at 2013 value.
educationexp	Education {per capta} expenditure, at 2013 value.
infrastructureexp	Infrastructure {per capta} expenditure, at 2013 value.
othersexp	Other {per capta}expenditures, at 2013 value.
totalexp	Total {per capta} expenditure, net of public debt, at 2013 value.

[1] In the survey of 2002, the question about life satisfaction was not in the questionnaire. I use the question about happiness, defined on the same scale as life satisfaction. The surveys of 2002 and 2008 presented a question specifically about happiness.

[2] IBGE's definition: legally employed (all employees except public servants and those in the armed forces).

Table A.2: Surveys sample size.

Year	IBGE[1]		Latinobarómetro
	Individuals	Households	Individuals
2002	385.431	129.705	1.000
2003	384.834	133.255	1.200
2004	399.354	139.157	1.204
2005	408.148	142.471	1.204
2006	410.241	145.547	1.204
2007	399.964	147.851	1.204
2008	391.868	150.591	1.204
2009	399.387	153.837	1.204
2010	6.192.332	20.635.472	1.204
2011	358.919	146.207	1.204
2013	362.554	148.696	1.204

[1] For 2002-2013 (except 2010): PNAD. For 2010: Population Census.

Table A.3: First-step regressions estimates

Explanatory Variables	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2013
age	-1.68e-02**	-1.58e-02**	-2.66e-02***	-2.58e-02***	-4.48e-03	-1.72e-02**	-1.23e-02**	-2.59e-02***	-1.61e-02***	-4.49E-03	-2.18e-02***
St. Error	(8.25e-03)	-7.30E-03	(6.39e-03)	(7.07e-03)	(6.08e-03)	(7.10e-03)	(5.27e-03)	(6.22e-03)	(5.47e-03)	-6.19E-03	-6.68E-03
age2	1.76e-04*	1.54e-04**	2.50e-04***	2.75e-04***	4.17e-05	2.01e-04**	1.64e-04***	2.96e-04***	1.99e-04***	6.09E-05	2.08e-04***
St. Error	(9.34e-05)	-7.79E-05	(6.95e-05)	(8.07e-05)	(6.40e-05)	(7.96e-05)	(5.70e-05)	(6.77e-05)	(6.29e-05)	-6.72E-05	-7.42E-05
yearseduc	-6.25e-03	-2.54e-02***	-2.18e-02***	-7.90e-03	-9.12e-03	1.27e-03	-5.99e-03	9.16e-03	1.25e-03	-1.03E-03	-2.79E-03
St. Error	(7.89e-03)	-7.37E-03	(6.68e-03)	(6.66e-03)	(6.28e-03)	(5.95e-03)	(4.98e-03)	(6.18e-03)	(5.18e-03)	-4.97E-03	-5.79E-03
socioeconom_hig	2.03e-01***	1.18e-01***	1.85e-01***	9.71e-02**	3.32e-02	1.40e-01***	1.29e-01***	7.27e-02*	5.65e-02*	1.97e-01***	7.84e-02**
St. Error	(4.93e-02)	-4.11E-02	(3.93e-02)	(4.31e-02)	(4.04e-02)	(4.16e-02)	(3.38e-02)	(3.95e-02)	(3.34e-02)	-3.67E-02	-3.99E-02
male	1.17e-01**	2.12E-03	2.89e-02	2.32e-02	6.24e-02	9.21e-02**	9.11e-02**	1.80e-02	4.48e-02	4.23E-02	3.13E-02
St. Error	(5.07e-02)	-4.46E-02	(4.30e-02)	(4.24e-02)	(4.41e-02)	(4.24e-02)	(3.61e-02)	(4.24e-02)	(3.61e-02)	-3.80E-02	-4.31E-02
married	7.33e-02	9.21e-02**	1.27e-01***	9.78e-02**	4.44e-03	1.02e-01**	1.18e-02	1.50e-01***	1.11e-01***	-1.24E-02	3.15E-02
St. Error	(4.97e-02)	-4.51E-02	(4.38e-02)	(4.43e-02)	(4.05e-02)	(4.30e-02)	(3.50e-02)	(4.19e-02)	(3.45e-02)	-3.67E-02	-4.17E-02
head	4.84e-02	-4.52E-02	-3.82e-02	-6.54e-02	-6.95e-02	-7.82e-02	2.25e-02	-1.94e-02	-8.63e-02**	2.65E-02	-6.82E-02
St. Error	(5.56e-02)	-4.92E-02	(4.72e-02)	(4.96e-02)	(4.95e-02)	(4.92e-02)	(3.97e-02)	(4.73e-02)	(4.01e-02)	-4.13E-02	-4.81E-02
student	4.97e-02	1.87e-01*	4.78e-02	5.73e-02	3.30e-02	2.49e-01***	-3.95e-02	1.18e-01	1.80e-01**	5.70E-02	-9.29E-02
St. Error	(9.61e-02)	-9.85E-02	(9.63e-02)	(9.35e-02)	(1.00e-01)	(8.97e-02)	(7.39e-02)	(9.75e-02)	(8.45e-02)	-1.11E-01	-1.14E-01
selfemployed	-1.26e-01**	6.29E-02	-2.51e-03	5.64e-03	-5.21e-02	1.86e-03	2.24e-02	3.14e-03	-3.25e-02	2.38E-02	-1.29e-01***
St. Error	(4.93e-02)	-4.54E-02	(4.24e-02)	(4.37e-02)	(4.12e-02)	(4.40e-02)	(3.61e-02)	(4.21e-02)	(3.70e-02)	-3.76E-02	-4.61E-02
retired	-1.02e-01	1.56e-01**	1.33e-02	-1.75e-01*	8.34e-02	2.31e-02	-1.38e-01**	2.11e-02	-1.80e-01**	-3.76E-02	-3.71E-02
St. Error	(1.00e-01)	-7.87E-02	(7.75e-02)	(9.18e-02)	(7.68e-02)	(8.29e-02)	(6.27e-02)	(8.31e-02)	(7.36e-02)	-6.76E-02	-7.69E-02
_cons	3.07e+00***	3.08e+00***	3.21e+00***	3.22e+00***	2.94e+00***	2.92e+00***	2.98e+00***	3.10e+00***	3.10e+00***	2.82e+00***	3.28e+00***
St. Error	(1.92e-01)	-1.87E-01	(1.55e-01)	(1.62e-01)	(1.59e-01)	(1.64e-01)	(1.32e-01)	(1.60e-01)	(1.32e-01)	-1.48E-01	-1.63E-01
R2	0.05	0.04	0.04	0.02	0.02	0.04	0.04	0.04	0.03	0.04	0.03
N	923	1,084	1,071	1,092	1,064	1,072	1,035	1,050	1,031	1,036	1,035

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table A.4: Second-step regressions estimates

Explanatory Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
securityexp	5.42e-05	-2.48e-05**					
St. Error	(1.07e-04)	(9.67e-06)					
educationexp	-1.71e-05		-1.24e-05***				
St. Error	(7.60e-05)		(2.31e-06)				
healthexp	-6.90e-05			-2.23e-05***			
St. Error	(2.33e-04)			(5.13e-06)			
infrastructureexp	-1.27e-05				-2.81e-05***		
St. Error	(1.13e-04)				(5.81e-06)		
othersexp	1.71e-05					-6.95e-06**	
St. Error	(3.86e-05)					(2.42e-06)	
totalexp							-3.01e-06***
St. Error							(7.27e-07)
lowstud	-8.48e-02	-5.72e-02	-5.68e-02	-5.82e-02	-6.59e-02	-5.67e-02	-5.69e-02
St. Error	(3.72e-01)	(3.61e-01)	(3.63e-01)	(3.61e-01)	(3.60e-01)	(3.62e-01)	-3.62E-01
old65	1.28e+00	1.32e+00	1.32e+00	1.34e+00	1.40e+00	1.29e+00	1.33E+00
St. Error	(2.18e+00)	(2.21e+00)	(2.21e+00)	(2.22e+00)	(2.24e+00)	(2.22e+00)	-2.22E+00
formalemployment	1.36e-01	1.64e-01	1.70e-01	1.67e-01	1.56e-01	1.67e-01	1.68E-01
St. Error	(5.17e-01)	(5.05e-01)	(5.06e-01)	(5.05e-01)	(5.03e-01)	(5.08e-01)	-5.07E-01
children	-1.12e-01	-1.29e-01	-1.23e-01	-1.25e-01	-1.21e-01	-1.24e-01	-1.24E-01
St. Error	(1.93e-01)	(1.86e-01)	(1.84e-01)	(1.85e-01)	(1.85e-01)	(1.85e-01)	-1.85E-01
water	6.96e-01	6.75e-01	6.76e-01	6.77e-01	6.78e-01	6.78e-01	6.76E-01
St. Error	(6.52e-01)	(6.40e-01)	(6.37e-01)	(6.40e-01)	(6.37e-01)	(6.39e-01)	-6.39E-01
wc	-7.59e-01*	-8.77e-01*	-8.64e-01*	-8.55e-01*	-8.27e-01	-8.75e-01*	-8.65e-01*
St. Error	(4.01e-01)	(4.69e-01)	(4.65e-01)	(4.68e-01)	(4.70e-01)	(4.68e-01)	-4.68E-01
garbage	-3.88e-01	-4.34e-01	-4.24e-01	-4.33e-01	-3.99e-01	-4.44e-01	-4.31E-01
St. Error	(6.51e-01)	(6.17e-01)	(6.19e-01)	(6.23e-01)	(6.15e-01)	(6.25e-01)	-6.23E-01
electric	-6.57e-01	-4.50e-01	-4.94e-01	-4.68e-01	-5.72e-01	-4.46e-01	-4.73E-01
St. Error	(8.93e-01)	(8.69e-01)	(8.61e-01)	(8.64e-01)	(8.58e-01)	(8.57e-01)	-8.63E-01
urban	-5.19e-01	-4.36e-01	-4.70e-01	-4.54e-01	-4.88e-01	-4.37e-01	-4.55E-01
St. Error	(5.74e-01)	(5.63e-01)	(5.65e-01)	(5.64e-01)	(5.67e-01)	(5.60e-01)	-5.65E-01
gdppc	-1.42e-06	-2.12e-06	-1.93e-06	-1.91e-06	-1.94e-06	-1.88e-06	-1.94E-06
St. Error	(3.73e-06)	(3.62e-06)	(3.46e-06)	(3.51e-06)	(3.52e-06)	(3.56e-06)	-3.53E-06
_cons	1.72e+00	1.65e+00	1.68e+00	1.65e+00	1.71e+00	1.64e+00	1.66E+00
St. Error	(1.08e+00)	(1.11e+00)	(1.11e+00)	(1.11e+00)	(1.10e+00)	(1.11e+00)	-1.11E+00
R2	0.04	0.03	0.04	0.04	0.04	0.03	0.04
N	261	261	261	261	261	261	261

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table A.5: Correlations of the explanatory variables.

Variables	**securityexp**	**educationexp**	**healthexp**	**infrastructureexp**	**othersexp**	**totalexp**
securityexp	1					
educationexp	0.9418	1				
healthexp	0.9351	0.9755	1			
infrastruc∼p	0.8635	0.9384	0.9413	1		
othersexp	0.9267	0.957	0.9692	0.8895	1	
totalexp	0.9528	0.9882	0.9899	0.9403	0.9857	1

Variables	**lowstud**	**old65**	**formalemployment**	**children**	**water**	**wc**	**garbage**	**electric**	**urban**	**gdppc**
lowstud	1									
old65	0.2133	1								
formalemployment	-0.5594	0.2759	1							
children	0.0716	-0.4032	-0.7041	1						
water	-0.2432	0.2949	0.7904	-0.8372	1					
wc	-0.1747	0.1316	0.6496	-0.7327	0.8622	1				
garbage	-0.3705	0.2164	0.7732	-0.7795	0.8547	0.8488	1			
electric	-0.1213	0.3345	0.6261	-0.6554	0.7738	0.8447	0.7668	1		
urban	-0.4249	0.2249	0.7478	-0.7231	0.759	0.673	0.8815	0.576	1	
gdppc	-0.1547	0.1978	0.5852	-0.7295	0.5968	0.4899	0.5813	0.432	0.5771	1

Table A.6: Variance Inflation Factor from OLS regression

Explanatory Variables	**A: All explanatory variables**							**B: except *wc* and *garbage***						
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**
securityexp	**13.45**	1.05						**13.08**	1.04					
educationexp	**39.19**		1.05					**38.7**		1.04				
healthexp	**53.65**			1.11				**53.65**			1.11			
infrastructureexp	**16.95**				1.15			**16.77**				1.14		
othersexp	**21.39**					1.09		**21.20**					1.09	
totalexp							1.07							1.07
lowstud	3.12	2.88	2.88	2.89	2.91	2.89	2.89	3.02	2.77	2.77	2.79	2.81	2.78	2.78
old65	1.90	1.86	1.86	1.86	1.86	1.86	1.86	1.50	1.46	1.46	1.46	1.46	1.46	1.46
formalemployment	6.80	5.69	5.71	5.73	5.83	5.72	5.72	6.77	5.66	5.67	5.70	5.81	5.69	5.69
children	7.29	6.41	6.41	6.43	6.42	6.41	6.41	6.95	6.10	6.09	6.11	6.09	6.09	6.09
water	8.27	7.97	7.97	7.99	7.98	8.01	7.99	6.76	6.46	6.45	6.48	6.47	6.5	6.48
wc	8.74	8.56	8.52	8.53	8.53	8.53	8.53							
garbage	**11.95**	**11.21**	**11.24**	**11.22**	**11.29**	**11.21**	**11.22**							
electric	5.21	4.75	4.75	4.76	4.77	4.76	4.75	2.95	2.63	2.63	2.64	2.63	2.64	2.63
urban	5.94	5.87	5.88	5.89	5.85	5.9	5.89	3.62	3.33	3.29	3.32	3.26	3.36	3.32
gdppc	4.66	2.35	2.39	2.49	2.56	2.43	2.43	4.61	2.31	2.35	2.45	2.51	2.39	2.39
Mean VIF	**13.9**	**5.33**	**5.33**	**5.35**	**5.38**	**5.35**	**5.34**	**13.81**	**3.53**	**3.53**	**3.56**	**3.58**	**3.55**	**3.55**

Table A.7: Robust 1: second-step regressions estimates (excluding ***garbage*** and ***wc***)

Explanatory Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
securityexp	6.68E-05	-2.48e-05**					
St. Error	-1.07E-04	-1.04E-05					
educationexp	-3.04E-06		-1.27e-05***				
St. Error	-7.30E-05		-2.38E-06				
healthexp	-8.81E-05			-2.29e-05***			
St. Error	-2.43E-04			-5.17E-06			
infrastructureexp	-4.12E-05				-3.03e-05***		
St. Error	-1.16E-04				-5.94E-06		
othersexp	1.94E-05					-6.86e-06**	
St. Error	-4.09E-05					-2.72E-06	
totalexp							-3.06e-06***
St. Error							-7.63E-07
lowstud	-8.02E-02	-3.48E-02	-3.41E-02	-3.57E-02	-4.33E-02	-3.47E-02	-3.43E-02
St. Error	-3.74E-01	-3.67E-01	-3.69E-01	-3.67E-01	-3.66E-01	-3.68E-01	-3.69E-01
old65	1.49E+00	1.47E+00	1.48E+00	1.50E+00	1.57E+00	1.45E+00	1.49E+00
St. Error	-2.16E+00	-2.16E+00	-2.16E+00	-2.17E+00	-2.19E+00	-2.18E+00	-2.17E+00
formalemployment	8.09E-02	1.22E-01	1.32E-01	1.26E-01	1.23E-01	1.23E-01	1.28E-01
St. Error	-5.00E-01	-4.93E-01	-4.94E-01	-4.92E-01	-4.91E-01	-4.94E-01	-4.94E-01
children	-2.22E-02	-2.21E-02	-1.76E-02	-1.98E-02	-2.18E-02	-1.59E-02	-1.82E-02
St. Error	-1.65E-01	-1.62E-01	-1.61E-01	-1.61E-01	-1.61E-01	-1.62E-01	-1.61E-01
water	5.79E-01	5.20E-01	5.22E-01	5.25E-01	5.30E-01	5.24E-01	5.23E-01
St. Error	-6.42E-01	-6.19E-01	-6.17E-01	-6.19E-01	-6.16E-01	-6.18E-01	-6.18E-01
electric	-1.45E+00	-1.31E+00	-1.34E+00	-1.31E+00	-1.38E+00	-1.31E+00	-1.32E+00
St. Error	-9.87E-01	-1.01E+00	-1.00E+00	-1.01E+00	-9.90E-01	-1.00E+00	-1.00E+00
urban	-6.55E-01	-5.70E-01	-6.02E-01	-5.90E-01	-6.18E-01	-5.74E-01	-5.89E-01
St. Error	-5.15E-01	-5.11E-01	-5.13E-01	-5.12E-01	-5.16E-01	-5.08E-01	-5.13E-01
gdppc	-7.60E-07	-1.44E-06	-1.27E-06	-1.25E-06	-1.32E-06	-1.19E-06	-1.27E-06
St. Error	-3.66E-06	-3.50E-06	-3.34E-06	-3.39E-06	-3.40E-06	-3.44E-06	-3.41E-06
_cons	1.39E+00	1.22E+00	1.25E+00	1.22E+00	1.31E+00	1.20E+00	1.23E+00
St. Error	-1.09E+00	-1.12E+00	-1.11E+00	-1.11E+00	-1.11E+00	-1.11E+00	-1.11E+00
R2	0.03	0.02	0.03	0.03	0.03	0.02	0.03
N	261	261	261	261	261	261	261

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table A.8: Robust 2: second-step regressions estimates - multiple approaches (excluding ***garbage*** and ***wc*** - All Expenditures)

Variable	All_POLS	All_PGLS	All_FE	All_BE	All_RE
securityexp	3.10E-05	3.02E-05	6.68E-05	-3.11E-04	3.10E-05
educationexp	7.90E-06	8.76E-06	-3.04E-06	-6.35E-05	7.90E-06
healthexp	-2.40E-04	-2.45e-04*	-8.81E-05	-3.75E-04	-2.40E-04
infrastructureexp	5.76E-05	5.87E-05	-4.12E-05	3.56E-05	5.76E-05
othersexp	4.72E-05	4.80E-05	1.94E-05	2.18E-04	4.72E-05
lowstud	-2.46E-01	-2.45E-01	-8.02E-02	2.24E+00	-2.46E-01
old65	1.45e+00*	1.44e+00**	1.49E+00	-3.07E+00	1.45e+00**
formalemployment	-3.97e-01***	-3.99e-01***	8.09E-02	-1.58E-01	-3.97e-01***
children	1.05E-01	1.07E-01	-2.22E-02	1.18E-01	1.05E-01
water	4.60e-01*	4.62e-01**	5.79E-01	-5.39E-02	4.60e-01**
electric	-1.30E-01	-1.16E-01	-1.45E+00	-8.33E-01	-1.30E-01
urban	5.45E-02	5.65E-02	-6.55E-01	1.51E+00	5.45E-02
gdppc	1.94E-06	1.97E-06	-7.60E-07	1.08E-06	1.94E-06
_cons	-4.79E-01	-4.99E-01	1.39E+00	-1.03E+00	-4.79E-01
N	261	261	261	261	261
r2	0.06		0.03	0.69	
r2_o			0.01	0.01	0.06
r2_b			0.00	0.69	0.30
r2_w			0.03	0.00	0.01

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table A.9: Robust 2: second-step regressions estimates - multiple approaches (excluding ***garbage*** and ***wc*** - Security Expenditures)

Variable	Sec_POLS	Sec_PGLS	Sec_FE	Sec_BE	Sec_RE
securityexp	-2.51e-05**	-2.44e-05**	-2.48e-05**	-6.55E-05	-2.51e-05**
lowstud	-2.31E-01	-2.32E-01	-3.48E-02	1.96e+00*	-2.31E-01
old65	1.65e+00*	1.66e+00*	1.47E+00	-2.08E+00	1.65e+00*
formalemployment	-3.80e-01***	-3.56e-01***	1.22E-01	-3.77E-01	-3.80e-01***
children	6.25E-02	5.71E-02	-2.21E-02	4.36E-02	6.25E-02
water	4.67e-01**	4.60e-01**	5.20E-01	1.29E-01	4.67e-01**
electric	-3.27E-01	-4.00E-01	-1.31E+00	-4.29E-01	-3.27E-01
urban	4.38E-02	3.02E-02	-5.70E-01	9.35e-01*	4.38E-02
gdppc	6.79E-07	5.60E-07	-1.44E-06	3.05E-06	6.79E-07
_cons	-1.68E-01	-7.53E-02	1.22E+00	-7.75E-01	-1.68E-01
N	261	261	261	261	261
r2	0.05		0.02	0.47	
r2_o			0.01	0.00	0.05
r2_b			0.00	0.47	0.21
r2_w			0.02	0.00	0.01

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table A.10: Robust 2: second-step regressions estimates - multiple approaches (excluding ***garbage*** and ***wc*** - Education Expenditures)

Variable	Edu_POLS	Edu_PGLS	Edu_FE	Edu_BE	Edu_RE
educationexp	-1.11e-05***	-1.10e-05***	-1.27e-05***	-3.57E-05	-1.11e-05***
lowstud	-2.34E-01	-2.35E-01	-3.41E-02	1.90e+00**	-2.34E-01
old65	1.65e+00*	1.66e+00*	1.48E+00	-1.98E+00	1.65e+00*
formalemployment	-3.86e-01***	-3.62e-01***	1.32E-01	-4.39E-01	-3.86e-01***
children	6.66E-02	6.07E-02	-1.76E-02	5.57E-02	6.66E-02
water	4.64e-01**	4.58e-01**	5.22E-01	1.54E-01	4.64e-01**
electric	-3.26E-01	-4.03E-01	-1.34E+00	-3.12E-01	-3.26E-01
urban	5.28E-02	3.71E-02	-6.02E-01	9.13e-01**	5.28E-02
gdppc	8.78E-07	7.49E-07	-1.27E-06	3.92E-06	8.78E-07
_cons	-1.87E-01	-8.63E-02	1.25E+00	-9.00E-01	-1.87E-01
N	261	261	261	261	261
r2	0.05		0.03	0.48	
r2_o			0.01	0.00	0.05
r2_b			0.00	0.48	0.21
r2_w			0.03	0.00	0.01

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table A.11: Robust 2: second-step regressions estimates - multiple approaches (excluding ***garbage*** and ***wc*** - Health Expenditures)

Variable	Hea_POLS	Hea_PGLS	Hea_FE	Hea_BE	Hea_RE
healthexp	-2.42e-05***	-2.32e-05***	-2.29e-05***	-7.56E-05	-2.42e-05***
lowstud	-2.44E-01	-2.44E-01	-3.57E-02	1.93e+00***	-2.44E-01
old65	1.63e+00*	1.63e+00*	1.50E+00	-2.08E+00	1.63e+00*
formalemployment	-3.95e-01***	-3.72e-01***	1.26E-01	-4.20E-01	-3.95e-01***
children	7.10E-02	6.53E-02	-1.98E-02	7.84E-02	7.10E-02
water	4.76e-01**	4.69e-01**	5.25E-01	1.66E-01	4.76e-01**
electric	-3.07E-01	-3.77E-01	-1.31E+00	-3.69E-01	-3.07E-01
urban	4.00E-02	2.72E-02	-5.90E-01	9.10e-01**	4.00E-02
gdppc	1.14E-06	9.93E-07	-1.25E-06	4.87E-06	1.14E-06
_cons	-2.10E-01	-1.18E-01	1.22E+00	-9.37E-01	-2.10E-01
N	261	261	261	261	261
r2	0.05		0.03	0.50	
r2_o			0.01	0.00	0.05
r2_b			0.00	0.50	0.21
r2_w			0.03	0.00	0.01

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table A.12: Robust 2: second-step regressions estimates - multiple approaches (excluding ***garbage*** and ***wc*** - Infrastructure Expenditures)

Variable	Inf_POLS	Inf_PGLS	Inf_FE	Inf_BE	Inf_RE
infrastructureexp	-2.31e-05***	-2.32e-05***	-3.03e-05***	-3.30E-05	-2.31e-05***
lowstud	-2.47E-01	-2.49E-01	-4.33E-02	1.84e+00***	-2.47E-01
old65	1.64e+00*	1.64e+00*	1.57E+00	-1.65E+00	1.64e+00*
formalemployment	-4.05e-01***	-3.80e-01***	1.23E-01	-4.16E-01	-4.05e-01***
children	6.78E-02	6.09E-02	-2.18E-02	6.63E-02	6.78E-02
water	4.70e-01**	4.64e-01**	5.30E-01	1.29E-01	4.70e-01**
electric	-3.57E-01	-4.35E-01	-1.38E+00	-4.01E-01	-3.57E-01
urban	6.54E-02	4.75E-02	-6.18E-01	9.10e-01**	6.54E-02
gdppc	1.14E-06	9.89E-07	-1.32E-06	4.36E-06	1.14E-06
_cons	-1.65E-01	-5.86E-02	1.31E+00	-8.57E-01	-1.65E-01
N	261	261	261	261	261
r2	0.05		0.03	0.46	
r2_o			0.01	0.00	0.05
r2_b			0.00	0.46	0.19
r2_w			0.03	0.00	0.01

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table A.13: Robust 2: second-step regressions estimates - multiple approaches (excluding ***garbage*** and ***wc*** - Other Expenditures)

Variable	Oth_POLS	Oth_PGLS	Oth_FE	Oth_BE	Oth_RE
othersexp	-5.68E-06	-5.72e-06*	-6.86e-06**	-9.82E-06	-5.68E-06
lowstud	-2.33E-01	-2.34E-01	-3.47E-02	1.82e+00*	-2.33E-01
old65	1.65e+00*	1.65e+00*	1.45E+00	-1.68E+00	1.65e+00*
formalemployment	-3.85e-01***	-3.60e-01***	1.23E-01	-4.07E-01	-3.85e-01***
children	6.70E-02	6.12E-02	-1.59E-02	5.60E-02	6.70E-02
water	4.69e-01**	4.63e-01**	5.24E-01	1.40E-01	4.69e-01**
electric	-3.28E-01	-4.06E-01	-1.31E+00	-3.20E-01	-3.28E-01
urban	4.63E-02	3.09E-02	-5.74E-01	8.57e-01*	4.63E-02
gdppc	8.96E-07	7.66E-07	-1.19E-06	3.77E-06	8.96E-07
_cons	-1.85E-01	-8.47E-02	1.20E+00	-8.57E-01	-1.85E-01
N	261	261	261	261	261
r2	0.04		0.02	0.46	
r2_o			0.00	0.00	0.04
r2_b			0.00	0.46	0.20
r2_w			0.02	0.00	0.01

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table A.14: Robust 2: second-step regressions estimates - multiple approaches (excluding ***garbage*** and ***wc*** - Total Expenditures)

Variable	Tot_POLS	Tot_PGLS	Tot_FE	Tot_BE	Tot_RE
totalexp	-2.72e-06***	-2.68e-06***	-3.06e-06***	-6.66E-06	-2.72e-06***
lowstud	-2.37E-01	-2.38E-01	-3.43E-02	1.88E+00	-2.37E-01
old65	1.65e+00*	1.65e+00*	1.49E+00	-1.87E+00	1.65e+00*
formalemployment	-3.89e-01***	-3.64e-01***	1.28E-01	-4.17E-01	-3.89e-01***
children	6.71E-02	6.12E-02	-1.82E-02	5.83E-02	6.71E-02
water	4.70e-01**	4.63e-01**	5.23E-01	1.48E-01	4.70e-01**
electric	-3.27E-01	-4.04E-01	-1.32E+00	-3.53E-01	-3.27E-01
urban	4.73E-02	3.21E-02	-5.89E-01	8.91E-01	4.73E-02
gdppc	9.39E-07	8.06E-07	-1.27E-06	4.03E-06	9.39E-07
_cons	-1.84E-01	-8.52E-02	1.23E+00	-8.62E-01	-1.84E-01
N	261	261	261	261	261
r2	0.05		0.03	0.47	
r2_o			0.01	0.00	0.05
r2_b			0.00	0.47	0.20
r2_w			0.03	0.00	0.01

Significance levels : $*$: 10% $**$: 5% $***$: 1%

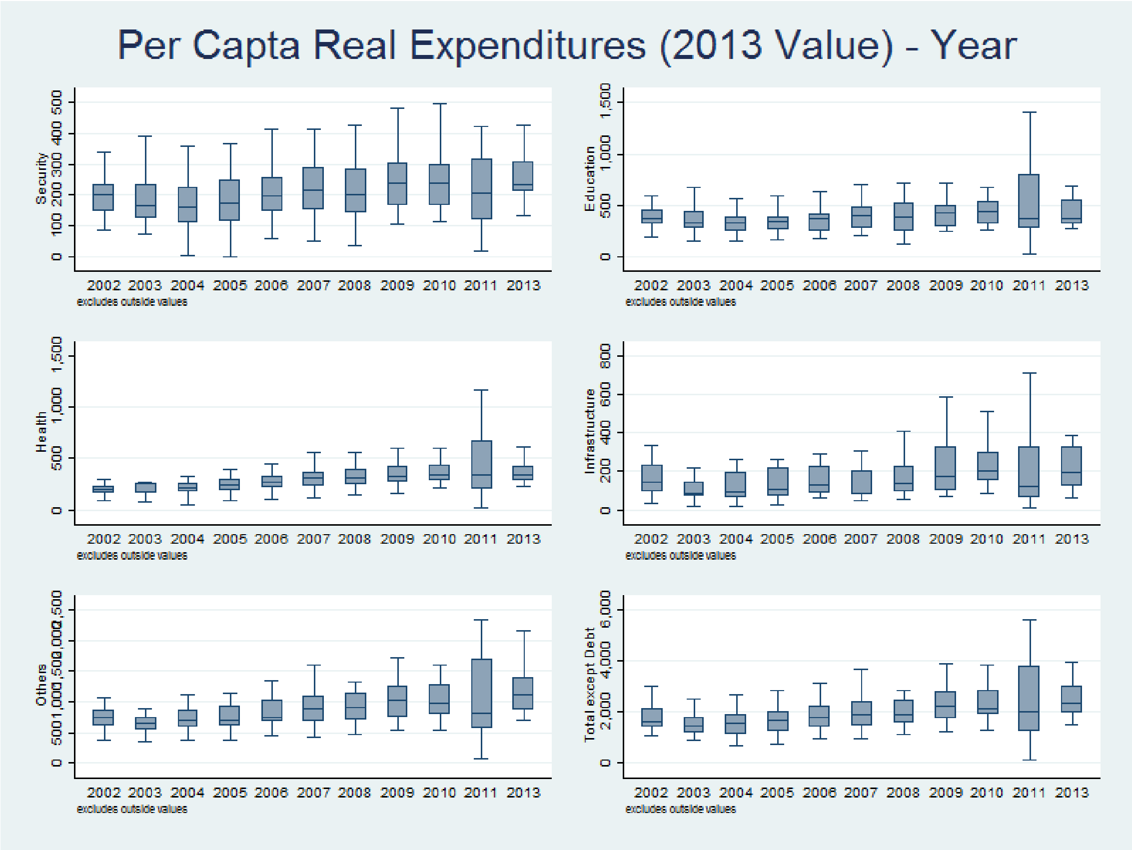

Figure A.1: Boxplot cross years.

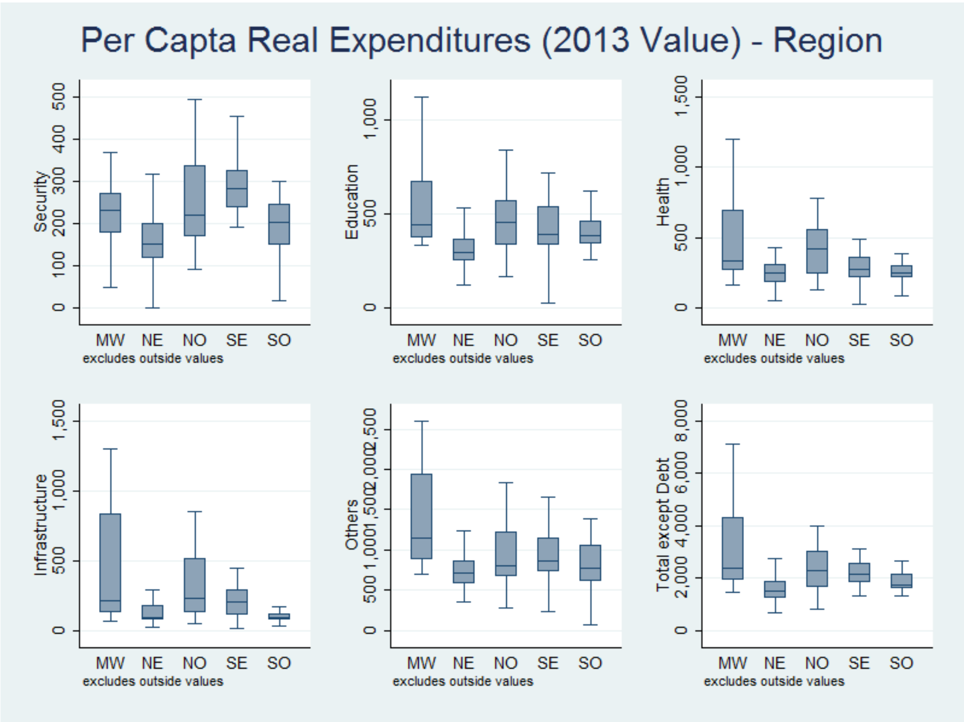

Figure A.2: Boxplot cross regions.

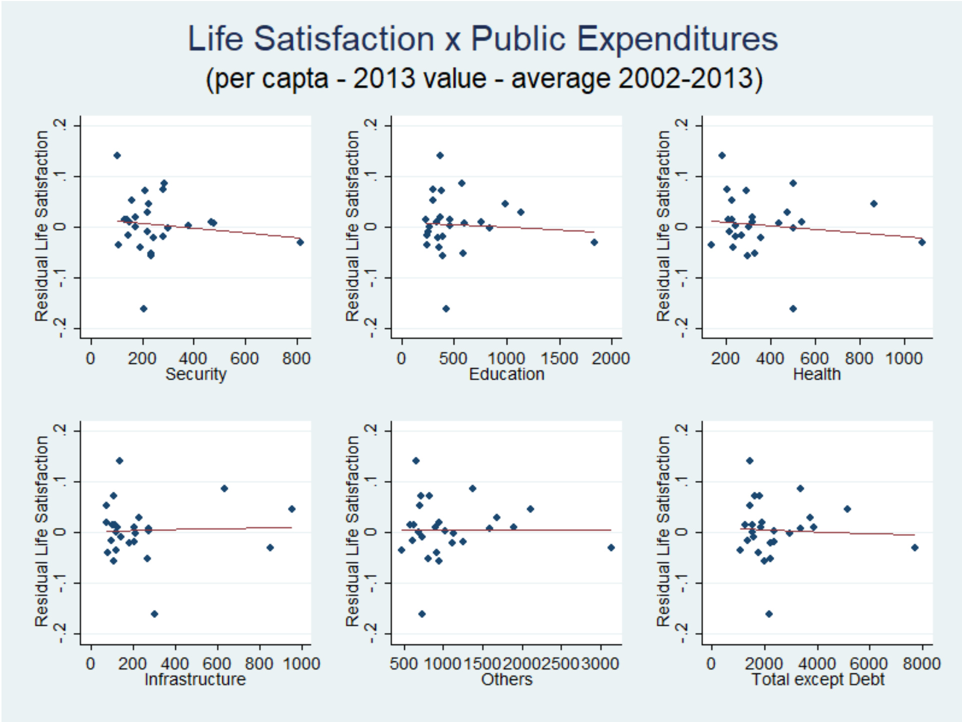

Figure A.3: Average residual life satisfaction and per capta government expenditures.

References

Agénor, P.-R. (2016). “Optimal fiscal management of commodity price shocks”, Journal of Development Economics, vol. 122, 183–196.

Aiyagari, S. R.; Marcet, A.; Sargent, T. J.; and Seppälä, J. (2002). “Optimal Taxation without State-Contingent Debt”, Journal of Political Economy, vol. 110(6), 1220–1254.

Ambler, S.; and Paquet; A. (1996). Fiscal spending shocks, endogenous government spending, and real business cycles. Journal of Economic Dynamics and Control, 20, 237-56.

Arrow, K. J.; and Kurz, M. (1969). Optimal public investment policy and controllability with fixed private savings ratio. Journal of Economic Theory, 1, 141–177.

Aschauer, D. (1989). Is government spending productive? Journal of Monetary Economics, v. 23, 1, 77-200.

Bajo-Rubio, O. (2000). A further generalization of the Solow growth model: the role of the public sector. Economics Letters, v. 68, 1, 79–84.

Barnett, S.; and Ossowski, R. (2002). “Operational aspects of fiscal policy in oil-producing countries”, International Monetary Fund Working Paper, WP/02/177.

Barro, R. J. (1981). Output Effects of Government Purchases. Journal of Political Economy, v. 89, 1086-1121.

Barro, R. J. (1990). Government spending in a simple model of endogenous growth. Journal of Political Economy, v. 98, 5, 103-126.

Barro, R. J. (1991). Economic growth in a cross section of countries. Quarterly Journal of Economics, v. 106, 407-444.

Barth, J. R.; and Bradley, M. D. (1987). The Impact of Government Spending on

Economic Activity. Manuscript. Washington: George Washington University.

Baunsgaard, T.; Villafuerte, M.; Poplawski-Ribeiro, M.; and Richmond, C. (2012). "Fiscal Frameworks for Resource Rich Developing Countries", International Monetary Fund Staff Discussion Note, SDN/12/04.

Baxter, M.; and King, R. (1993). Fiscal Policy in General Equilibrium. American Economic Review, v. 83, 315-334.

Bems, R.; de Carvalho Filho, I. (2011). "The current account and precautionary savings for exporters of exhaustible resources", Journal of International Economics, vol. 84(1), 48–64.

Benjamin, D. J.; Heffetz, O.; Kimball, M. S.; Rees-Jones, A. (2012). What do you think would make you happier? What do you think you would choose?. American Economic Review, 102(5), p. 2083-2110.

Bhattarai, S.; and Egorov, K. (2016). Optimal monetary and fiscal policy at the zero lower bound in a small open economy. Globalization and Monetary Policy Institute Working Paper 260, Federal Reserve Bank of Dallas.

Bjornskov, C.; Dreher, A.; Fischer, J. A. V. (2007). The bigger the better? Evidence of the effect of government size on life satisfaction around the world. Public Choice, v. 130, p. 267-292.

Berg, A.; Portillo, R.; Yang, S.-C. S.; Zanna, L.-F. (2013). "Public Investment in Resource-Abundant Developing Countries", IMF Economic Review, vol. 61(1), 92–129.

Bjerkholt, O. (2002). "Fiscal Rule Suggestions for Economies with Non-Renewable Resources", paper prepared for the conference *Rules-Based Fiscal Policy in Emerging Market Economies*, Oaxaca, Mexico.

Blanchard, O. J.; and Galí, J. (2007). Real Wage Rigidities and the New Keynesian Model. Journal of Money, Credit and Banking, v. 39, s1, 35–65.

Blanchard, O. J.; and Kahn, C. M. (1980).The solution of linear difference models under rational expectations. Econometrica, v. 48, nº 5, 1305-1311.

Bouakez, H.; Guillard, M.; and Roulleau-Pasdeloup, J. (2017a). Public Investment, Time to Build, and the Zero Lower Bound. Review of Economic Dynamics, 23, 60–79.

Bouakez, H.; Guillard, M.; and Roulleau-Pasdeloup, J. (2017b). The optimal composition of public spending in a deep recession. Mimeo.

Bouakez, H.; Guillard, M.; and Roulleau-Pasdeloup, J. (2019). The optimal composition of public spending in a deep recession. Journal of Monetary Economics, In Press.

Caceres, C.; Cevik, S.; Fenochietto, R.; and Gracia, B. (2015). "The Day After Tomorrow: Designing an Optimal Fiscal Strategy for Libya", Journal of Banking and Financial Economics, vol. 2(4), 32–50.

Carboni, O.A.; and Medda, G. (2011). Government spending and growth in a neoclassical model. Mathematics and Financial Economics, v. 4, 269-285.

Carcillo, S.; Leigh, D.; and Villafuerte, M. (2007). "Catch-Up Growth, Habits, Oil Depletion, and Fiscal Policy: Lessons from the Republic of Congo", International Monetary Fund Working Paper, WP/07/80.

Carroll, C. (2006). "The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems", Economics Letters, vol. 91(3), 312–320.

Carvalho, F. A.; Valli, M. (2011). Fiscal Policy in Brazil through the lens of an estimated DSGE model. Brazilian Central Bank, Working Paper Series, nº 240.

Cavalcanti, M. A. F. H.; Vereda, L. (2011). Propriedades dinâmicas de um modelo DSGE com parametrizações alternativas para o Brasil. Instituto de Pesquisa Econômica Aplicada, Textos para Discussão, nº 1588.

Chalk, N. (1998). "Fiscal Sustainability with Non-Renewable Resources", International Monetary Fund Working Paper, WP/98/26.

Chari, V. V.; Christiano, L. J.; and Kehoe, P. J. (1994). Optimal fiscal policy in a business cycle model. Journal of Political Economy, v. 102, 3, 617-652.

Chari, V. V.; Christiano, L. J.; and Kehoe, P. J. (1995). Policy analysis in business cycle models. In: Cooley, T. F. (Ed.) Frontiers of Business Cycle Research, Princeton: Princeton University Press.

Chatterjee, S.; Hadi, A. S. (2012). Regression Analysis by Example. Wiley, 5th edition.

Chen, B.-L. (2006). Economic growth with an optimal public spending composition. Oxford Economic Papers, v. 58, 123–36.

Cherif, R.; Hasanov, F. (2013). "Oil Exporters' Dilemma: how much to save and how much to invest", World Development, vol. 52, 120–31.

Christiano, L.; Eichenbaum, M.; and Rebelo, S. (2011). When is the government spending multiplier large? Journal of Political Economy, v. 119, 1, 78 – 121.

Christiano, L.; Fisher, J.M. (2000) Algorithms for solving dynamic models with occasionally binding constraints. Journal of Economic Dynamics and Control, v. 24, 8, pp. 1179-1232.

Cogan, J. F.; Cwik, T.; Taylor, J. B.; and Wieland, V. (2010). New Keynesian versus Old Keynesian Government Spending Multipliers. Journal of Economic Dynamics and Control, v. 34, 3, 281-295.

Corden, M.; Neary, P. (1982). "Booming sector and de-industrialization in a small open economy", Economic Journal, 92(368), 825–848.

den Haan, W.; Marcet, A. (1990). "Solving a Simple Growth Model by Parameterizing Expectation", Journal of Business and Economic Statistics, 31–34.

Devajaran, S., Swaroop, V., and Zou, H. (1996). The composition of public expenditure and economic growth. Journal of Monetary Economics, v. 37, 313–44.

Diamond, P.A. (1965). National debt in a neoclassical growth model. American Economic Review v. 55, 1126-1150.

Di Tella, R.; MacCulloch, R. J.; Oswald, A. J. (2003). The macroeconomics of happiness. Review of Economics and Statistics, v. 85, n. 4, p. 809-827.

Donald and Lang (2007). Inference with difference-in-differences and other panel data. Review of Economics and Statistics, v. 89(2), p. 221-233.

Easterly, W.; and Rebelo, S. (1993). Fiscal policy and economic growth: an empirical investigation. Journal of Monetary Economics, v. 32, 417-458.

Engel, E.; Valdés, R. O. (2000). "Optimal Fiscal Strategy for Oil Exporting Countries", International Monetary Fund Working Paper, WP/00/118.

Evans, P.; and Karras, G. (1994). Are government activities productive? Evidence from a panel of US states. Review of Economics and Statistics, LXXVI, 1–11.

Faraglia, E.; Marcet, A.; Oikonomou, R; and Scott, A. (2014). "Optimal Fiscal Policy Problems Under Complete and Incomplete Financial Markets: A Numerical Toolkit", London Business School mimeo.

Fisher, W. H.; and Turnovsky, S. J. (1998). Public investment, congestion, and private capital accumulation. The Economic Journal, v. 108, 447, 399–413.

Flavin, P. (2019). State government public goods spending and citizens' quality of life. Social Science Research, v. 78, p. 28-40.

Flavin, P.; Pacek, A. C.; Radcliff, B. (2011). State intervention and subjective well-being in advanced industrial democracies. Politics and Policy, v. 39(2), p. 251-269.

Flavin, P.; Pacek, A. C.; Radcliff, B. (2014). Assessing the impact of the size and scope of government on human well-being. Social Forces, v. 92(4), p. 1241-1258.

Friedman, M. (1957). "A Theory of the Consumption Function", Princeton: Princeton University Press.

Galí, J.; Lopez-Salido, J. D.; and Vallés, J. (2007). Understanding the Effects of Government Spending on Consumption. Journal of the European Economic Association, v. 5, 1, 227-270.

Galí, J.; and Monacelli, T. (2008). Optimal monetary and fiscal policy in a currency union. Journal of International Economics, v. 76, 1, 116-132.

Garcia-Milà, T.; and McGuire, T. J. (1992). The contribution of publicly provided inputs to States' economies. Regional Science and Urban Economics, 1992, v. 22, 2, 229-241.

Glomm, G.; and Ravikumar, B. (1997). Productive government expenditures and long-run growth. Journal of Economic Dynamics and Control, v. 21, 1, 183-204.

Ghosh, S.; and Gregoriou, A. (2008). The composition of government spending and growth: is current or capital spending better? Oxford Economic Papers, v. 60, 3, 484–516.

Gobetti, S. W. (2009). "Política fiscal e pré-sal: como gerir as rendas do petróleo e sustentar o equilíbrio macrofiscal do Brasil", XIV Brazilian National Treasury Award.

Grier, K.; and Tullock, G. (1989). An empirical analysis of cross-national economic growth, 1951-1980. Journal of Monetary Economics, v. 24, 259-276.

Gupta, S.; Clements, B.; Baldacci, E.; and Mulas-Granados, C. (2005). Fiscal policy, expenditure composition, and growth in low-income countries. Journal of International Money and Finance, v. 24, 441–63.

Hall, R. (1980). Labor Supply and Aggregate Fluctuations. Carnegie-Rochester Conference Series on Public Policy, 12, 7-33.

Harding, T.; and van der Ploeg, F. (2013). "Official forecasts and management of oil windfalls", International Tax and Public Finance, vol. 20(5), 827–866.

Hessami, Z. (2010). The size and composition of government spending in Europe and its impact on well?being. Kyklos, v. 63(3), p. 346-382.

Holtz-Eakin, D. (1994). Public-sector capital and the productivity puzzle. Review of Economics and Statistics, LXXVI, 12–21.

Iacono, R. (2017). "A comparison of fiscal rules for resource-rich economies", Economic Analysis and Policy, v. 55, C, 179–193. .

International Monetary Fund (2015). "Fiscal Monitor - The Commodities Roller Coaster: a fiscal framework for uncertain times", Background Notes, Washington.

Kacapyr, E. (2008). Cross-country determinants of satisfaction with life. International Journal of Social Economics, v. 35, p. 400-416.

Kahaneman, D.; Thaler, R. (1991). Economic analysis and the psychology of utility: applications to compensation policy. American Economic Review, 81(2), p. 341-346.

Kasmaoui, K.; Bourhaba, O. (2017): Happiness and public expenditure: evidence from a panel analysis. MPRA Paper 79339, University Library of Munich, Germany.

Kiyia, K. (2012). Life satisfaction and public finance: empirical analysis using U.S. micro data. Department of Economics, University of Washington, Seattle.

Knoll, B.; Pitlik, H. (2016). Who benefits from big government? A life satisfaction approach. Empirica, v. 43, p. 533-557.

Kormendi, R. C.; and Meguire, P. G. (1985). Macroeconomic determinants of growth: cross-country evidence. Journal of Monetary Economics, v. 16, 141-164.

Kotakorpi, K.; Laamanen, J.-P. (2010). Welfare state and life satisfaction: evidence from public health care. Economica, v. 77(307), p. 565-583.

Kydland, F. E.; and Prescott, E. C. (1982). Time to build and aggregate fluctuations. Econometrica, v. 50, nº. 6, 1345-1370.

Landau, D. (1983). Government expenditure and economic growth: a cross-country study, Southern Economic Journal, v. 49, 783-792.

Lansing, K. J. (1998). Optimal fiscal policy in a business cycle model with public capital. The Canadian Journal of Economics, v. 31, 2, 337-364.

Leduc, S.; and Wilson, D. (2013). Roads to prosperity or bridges to nowhere? Theory and evidence on the impact of public infrastructure investment. NBER Macroeconomics Annual, 27, 1, 89-142.

Leeper, E. M.; Walker, T. B.; and Yang, S.-C. S. (2010). Government investment and fiscal stimulus. Journal of Monetary Economics, v. 57, 8, 1000–1012.

Leigh, D.; and Olters, J.-P. (2006). “Natural-Resource Depletion, Habit Formation, and Sustainable Fiscal Policy: Lessons from Gabon”, International Monetary Fund Working Paper, WP/06/193.

Liang, K.-Y.; Zeger, S. L. (1986). Longitudinal data analysis using generalized linear models. Biometrika, v. 73(1), p. 13-22.

Liberini, F.; Redoano, M.; Proto, E. (2017). Happy Voters. Journal of Public Economics, v. 146, p. 41-57.

Livani, T.; Graham, C. (2019). Do social protection programs improve life satisfaction? Evidence from Iraq. World Development Perspectives, v. 16.

Lucas Jr., R. E. (1988). On the mechanics of economic development. Journal of Monetary Economics, v. 22, 3-42.

Lucas Jr., R. E.; and Stokey, N. L. (1983). “Optimal Fiscal and Monetary Policy in an Economy without Capital”, Journal of Monetary Economics, 12(1), 55–93.

Maliszewski, W. (2009). “Fiscal policy rules for oil producing countries: a welfare-based assessment”, International Monetary Fund Working Paper, WP/09/126.

Marcet, A.; Lorenzoni, G. (1999). “The parameterized expectation approach: some practical issues", In R. Marimon and A. Scott (eds.). **Computational Methods for Study of Dynamic Economies**, 143–171, Oxford University Press, New York.

Medas, P.; and Zakharova, D. (2009). "A Primer on Fiscal Analysis in Oil-Producing Countries", International Monetary Fund Working Paper, WP/09/56.

Moulton, B. R. (1986). Random group effects and the precision of regression estimates. Journal of Econometrics, v. 32(3), p. 385-397.

Munnell, A. H.; and Cook, L. M. (1990). How Does Public Infrastructure Affect Regional Economic Performance? New England Economic Review, 11-33.

Mussolini, C. C.; Teles, V. K. (2012). Ciclos reais e política fiscal no Brasil. Estudos Econômicos, v. 42, nº 1, 75-96.

Nakata, T. (2017). Optimal government spending at the zero lower bound: a non-Ricardian analysis. Review of Economic Dynamics, v. 23, 150-169.

Nordheim, O.; and Martinussem, P. E. (2020). Happiness and the role of social protection: how types of social spending affected individuals? life satisfaction in OECD countries, 1980-2012. Journal of International and Comparative Social Policy, v. 36(1), p. 1-24.

O'Connor, K. J. (2017). Happiness and welfare state policy around the world. Review of Behavioral Economics, v. 4(4), p. 397-420.

Oliveira, P. P.; Rocha, F.; Duarte, J.; Pereira, L. F. V. N. P.; Gadelha, S. R. B. (2014). Mais recursos ou mais eficiência? Uma análise de oferta e de demanda por serviços de saúde no Brasil. Anais do XL Encontro Nacional de Economia, ANPEC, 2014.

Olters, J.-P. (2007). "Old Curses, New Approaches? Fiscal Benchmarks for Oil-Producing Countries in Sub-Saharan Africa", International Monetary Fund Working Paper, WP/07/107.

Ono, H.; and Lee, K. S. (2013). Welfare states and the redistribution of happiness. Social Forces, v. 92(2), p. 789-814.

Ott, J.C. (2010). Good governance and happiness in nations: technical quality precedes democracy and quality beats size. Journal of Happiness Studies, v. 11(3), p. 353-368.

Ott, J.C. (2011). Government and happiness in 130 nations: good governance fosters higher level and more equality of happiness. Social Indicators Research, v. 102(1), p. 3-22.

Ouweneel, P. (2002). Social security and well-being of the unemployed in 42 nations. Journal of Happiness Studies, v. 3(2), p. 167-192.

Ovaska, T.; Takashima, R. (2006). Economic policy and the level of self-perceived well-being: an international comparison. The Journal of Socio-Economics, v. 35(2), p. 308-325.

Pacek, A.; Radcliff, B. (2008). Assessing the welfare state: the politics of happiness. Perspectives on Politics, v. 6(2), p. 267-277.

Perovic, L. M.; Golem, S. (2010). Investigating macroeconomic determinants of happiness in transition countries: how Important Is government expenditure? Eastern European Economics, v. 48(4), p. 59-75.

Pieschacón A. (2009). "Implementable Fiscal Rules for an oil-exporting Small Open Economy Facing Depletion", OxCarre Research Paper 44, University of Oxford.

Price, R.W.R.; Dang, T.-T.; and Botev, J. (2015). Adjusting fiscal balances for the business cycle: new tax and expenditure elasticity estimates for OECD countries. OECD Economics Department Working Papers, n. 1275.

Rabin, M. (1998). "Psychology and Economics". Journal of Economic Literature, 36, 11-46.

Ram, R. (2009). "Government spending and happiness of the population: additional evidence from large cross-country samples". Public Choice, 138: 483-490.

Rocha, F.; Oliveira, P. P.; Duarte, J.; Gadelha, S. R. B.; Pereira, L. F. V. N. (2017). Can education targets be met without increasing public spending? An Analysis for Brazilian Municipalities. Economics Bulletin, v. 37(1), p. 391-401.

Rodríguez-Pose, A.; Maslauskaite, K. (2012). Can policy make us happier? Individual characteristics, socio-economic factors and life satisfaction in Central and Eastern Europe. Cambridge Journal of Regions, Economy and Society, v. 5(1), p. 77-96.

Romer, P. (1986). Increasing returns and long-run growth. Journal of Political

Economy, v. 94, 1002-1037.

Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. Review of Economic Studies, v. 49, 4, 517–31.

Schmidt, S. (2013). Optimal monetary and fiscal policy with a zero bound on nominal interest rates. Journal of Money, Credit and Banking, v. 45, 7, 1335–1350.

Schmitt-Grohé, S.; and Uribe, M. (2005). Optimal fiscal and monetary policy in a medium-scale macroeconomic model. NBER Macroeconomics Annual, 20, 383-425.

Schmitt-Grohé, S.; and Uribe, M. (2007). Optimal simple and implementable monetary and fiscal rules. Journal of Monetary Economics, v. 54, 6, 1702-1725.

Segal, P. (2012). "How to spend it: resource wealth and the distribution of resource rents", The Oxford Institute for Energy Studies, SP 25.

Segura, A. (2006). "Management of Oil Wealth Under the Permanent Income Hypothesis: The Case of Sao Tome and Principe", International Monetary Fund Working Paper, WP/06/183.

Sims, E.; and Wolff, J. (2018). The Output and Welfare Effects of Government Spending Shocks over the Business Cycle. International Economic Review, v. 59, 3, 1403-1435.

Smets, F.; and Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the Euro Area. Journal of the European Economic Association, v. 1, issue 5, 1123-1175.

Smets, F.; and Wouters, R. (2007). Shocks and frictions in US business cycles: a Bayesian DSGE approach. American Economic Review, v. 97, 3, 586-606.

Solow, R. M. (1956). A contribution to the theory of economic growth. Quarterly Journal of Economics, v. 70, 1, 65-94.

Solow, R. M. (1957). Technical change and the aggregate production function. Review of Economics and Statistics, v. 39, 3, 312–320.

Sturm, M.; Gurtner, F.; and Alegre, J. G. (2009). "Fiscal Policy Challenges in oil-exporting countries: a review of key issues", European Central Bank Occasional Paper Series, n. 104.

Swan, T. W. (1956). Economic growth and capital accumulation. Economic Record, v. 32, 2, 334-361.

van den Bremer; T. S.; van der Ploeg, F. (2013). “Managing and Harnessing Volatile Oil Windfalls”, IMF Economic Review, vol. 61(1), 130–167.

van der Ploeg, F., (2010), “Aggressive Oil Extraction and Precautionary Saving: coping with volatility”, Journal of Public Economics, vol. 94(5-6), 421–433.

van der Ploeg, F., (2014). “Guidelines for exploiting natural resource wealth”, Oxford Review of Economic Policy, vol. 30(1), 145–169.

van der Ploeg, F.; and Venables, A. J. (2011). “Harnessing windfall revenues: optimal policies for developing economies”, The Economic Journal, vol. 121(551), 1–30.

van der Ploeg, F.; and Venables, A. J. (2012). “Natural Resource Wealth: the challenge of managing a windfall”, Annual Review of Economics, vol. 4, 315–337.

Veenhoven, R. (2000). Well-being in the welfare state: level not higher, distribution not more equitable. Journal of Comparative Policy Analysis, v. 2(1), p. 91-125.

Vereda, L.; Cavalcanti, M. A. F. H. (2010). Modelo dinâmico estocástico de equilíbrio geral (DSGE) para a economia brasileira: versão 1. Instituto de Pesquisa Econômica Aplicada, Textos para Discussão, nº 1479.

Woodford, M. (2011). Simple analytics of the government expenditure multiplier. American Economic Journal: Macroeconomics, v. 3, 1, 1-35.

Yamamura, E. (2011). The influence of government size on economic growth and life satisfaction. Japanese Economy, v. 38(4), p. 28-64.