ESSAYS ON MONETARY AND FISCAL POLICY

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The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "**Essays on Monetary and Fiscal Policy**" by **Andrea Pescatori** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy**.

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To my Family

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Abstract

The thesis is focused on the policy implications of financial markets imperfections for the business cycle. The view that the financial structure and the performance of credit markets may be important to understand macroeconomic facts dates back at least to Gurley and Shaw (1955). However, many results were found in a (static) partial equilibrium setup.

In the recent years a big effort has been posed in incorporating credit market imperfections in a dynamic general equilibrium framework. Within this strand of literature, most prominently, the works of Bernanke and Gertler (1998), Carlstrom and Fuerst (1998) and Kyiotaki and Moore (1997) show how the existence of financial frictions and credit markets imperfections can amplify and propagate the effects of aggregate shocks hitting the economy.

The present thesis studies - in a dynamic stochastic generale equilibrium framework - how monetary and/or fiscal policy can mitigate the macroeconomic volatility and improve on social welfare when we are in presence of different forms of imperfections in credit. While the previous works mainly concentrate on the firms sector the present thesis is focused on the household sector.

In the first chapter I study the implications of market incompleteness for the household sector. This is milder type of credit friction (the absence or no access to state contingent markets) which becomes interesting only when households are heterogeneous.

I calculate the social welfare loss implied by the households' inability of hedging against interest rate and inflation risks. In a world where some households

are highly indebted I show that optimal monetary policy reaction (through interest rate) to inflationary pressure should be 'milder' than it is usually prescribed.

The second chapter is based on the KM framework. This is a stronger type of credit market imperfection where households are not allowed to borrow more then a fraction of the value of their collateral. The collateral used is the housing stock held by households.

In the chapter I address a debated question: should asset prices - and in particular housing prices - be a separated target in a simple implementable rule for a monetary authority? Or, in other words, should monetary policy react counter-cyclically during an housing price boom?

Finally in the third chapter I consider a different dimension at which credit friction may operate: a fraction of households does not have access at all to financial markets while the other does. This reintroduces features that were common to a more traditional literature: constrained agents cannot smooth their consumption over time so they are only indirectly affected by changes in interest rate. On the other hand the way of financing government spending is not anymore irrelevant even in presence of lump sum taxes. The analysis is carried on in an open economy-monetary union framework. In particular I analyze the short-run and long-run spillovers on foreign country generated by a fiscal shock in the home country. I also study how different fiscal rule can reduce inflation and output volatility and help the centralize central bank to stabilize inflation.

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Introduction

> *... Theoretical writers are too apt, in their calculations, to overlook these intervals; but eight or ten years, recurring not unfrequently, are serious spaces in human life. In prosperous times the mercantile classes often realizes fortunes, which goes far towards securing them against the future; but unfortunately the working classes, though they share in the general prosperity, do not share in it so largely in the general adversity ... To them fluctuations must always bring more evil than good.*

Malthus, *Principle of Political Economy*, 2d ed., pg.437 (1837).

The thesis is divided into three chapters.

In the first chapter I challenge a widespread result in monetary policy literature: the price level should be stabilized and, as corollary, the nominal interest rate should vary with the *Wicksellian* determinants of the real interest rate.

I study how this result is altered when the representative agent assumption is abandoned and financial wealth heterogeneity across households is introduced. I derive a welfare-based loss function for the policy maker which includes an additional *target* related to the cross-sectional distribution of household debt.

My results differ from standard ones in two respects. First, thanks to its ability to affect interest payments volatility, monetary policy has real effects even in a flexible-price cashless-limit environment. Second, in a setup with nominal rigidities, price stability is no longer optimal. The extent of deviation from price stability depends on the initial level of debt dispersion.

I use US micro data to calibrate the model and I find that the departure from price stability is still relatively small under the baseline calibration.

Finally, I also study the design of an optimal simple implementable rule. I find that *superinertial* rules that also include a separate target on debt dispersion outperforms standard Taylor rules.

In the second chapter (co-authored with Caterina Medicino) I assess the role of housing price movements in influencing the optimal design of monetary policy. Even though the relationship between liquidity constraints and consumption behavior is well documented in the empirical and theoretical literature, little attention has been paid to credit frictions at the household level in the monetary business cycle literature.

This chapter represents the first attempt to evaluate welfare-based monetary policy using a model with heterogeneous agents and credit constraints at the household level. In evaluating optimal monetary policy, I take advantage of recent advances in computational economics, by adopting the approach of Schmitt-Grohe and Uribe (2003).

Our results indicate that under an optimally designed simple monetary policy rule, housing price movements should not be a separate target variable in addition to inflation. Furthermore, the welfare loss arising from targeting housing prices becomes quantitatively more significant the higher the degree of access to the credit market.

In the last chapter (co-authored with Massimiliano Pisani) I analyze the effects of fiscal policy in a currency area. I develop a two-region model with sticky prices, a common monetary authority and regional fiscal policies. I break the Ricardian equivalence and allow for Keynesian effects of public expenditure introducing rule-of-thumb agents in each region.

Main results are the following. First, consistently with the empirical evidence, after a public spending shock in one region private agents demand for imports increases and the terms of trade appreciates. Second, a countercyclical fiscal rule can restore the Taylor principle and the uniqueness of the equilibrium. Finally, a countercyclical fiscal rule contributes to reduce macroeconomic volatility.

Chapter 1

Incomplete Markets, Idiosyncratic Shocks and Optimal Monetary Policy

Since the end of ’80s many countries have witnessed a sharp increase in households’ debt, a phenomenon which has drawn the attention of policy makers and economists. This phenomenon is even more dramatic at a disaggregate level: aggregate data on the indebtedness of the household sector conceal substantial variation in the distribution of the debt across individual households. For example, in the United States, in 2001, around 45% of households had mortgage debt, while around one quarter of households held no debt at all.

In such an environment, monetary policy is likely to have stronger effects on the real sector.[1] In particular it may play a substantial redistributive role on households’ wealth affecting their balance sheet.

In the present paper we assess whether households financial imbalances should be a (quantitatively) relevant source of concern for the monetary authority and ask how, in this scenario, monetary policy should be optimally designed.

Despite the relevance of this issue, the economic literature, so far, has not

[1] see Debelle (2004) for example.

provided a clear-cut answer. A strand of the literature has studied the macroeconomic implications of household debt by introducing heterogeneous agents. Many works, however, lack welfare analysis and thus cannot provide any normative guidance. Barnes and Young (2003), for example, find that interest rate shocks contribute importantly to changes in household debt. Iacoviello (2005) shows that, in presence of borrowing constraints, a rise in income inequality could lead to an increase in debt and debt dispersion. Den Haan (1997) asks whether the cross-sectional distribution of asset holdings has a quantitative role in the determination of the real interest rate. In a recent work, Doepke and Schneider (2005) show that, a moderate inflation episode can lead to a high redistribution of wealth because of changes in the value of nominal assets.

Other papers, instead, do perform welfare analysis but lack business cycle considerations. Albanesi (2005), for example, studies optimal monetary and fiscal policy with heterogeneous holdings of money balances. In this case distributional considerations may determine a departure from the Friedman rule. On similar lines, Akyol (2003) finds that, in a model with a liquid and illiquid asset, a positive inflation can improve risk sharing, and therefore, welfare.

Hence, the above mentioned literature misses to put together welfare analysis and business cycle fluctuations. Moreover, there is no role for monetary policy coming from nominal rigidities as in the recent monetary business cycle literature.[2] On the contrary, this second strand of literature, assuming a representative agent, has been widely focused on normative issues regarding the role of monetary policy in stabilizing the economic cycle - e.g. King, Khan and Wolman (2003) and Rotemberg and Woodford (1997). A distinctive conclusion,

[2]Exceptions can be found in Mendicino and Pescatori (2004)

recurrent in this framework, may be illustrated by the recent work of Schmitt-Grohe Uribe (2005). They show that, even in a rich medium-scale model with a large variety of frictions, price stability remains quantitatively a central goal for monetary policy.

However to address questions regarding households financial imbalances it seems crucial to depart from a complete market/representative agent hypothesis.

This paper tries to link the two strands of literatures: I introduce heterogenous households in a tractable sticky price model - e.g. Gali (2001). In particular I relax the complete market assumption - only nominal riskless bonds are available - and I assume that households may differ in their asset holdings. This is tantamount to a model where agents hold heterogeneous portfolios with different exposure to interest rate risk.

I show that, for this setup, the welfare-based loss function for the policy maker includes an extra *target variable* in addition to the ones typically found in the literature (inflation and output gap). In other words, the introduction of heterogenous nominal bond holdings entails that the central bank tries to minimize also a measure of consumption dispersion across households - which, in turn, is strictly related to the cross-sectional distribution of household-debt.

This implies a departure from standard results of the literature in two aspects. First, thanks to its ability to affect interest payments volatility, monetary policy has real effects even in a flexible-price cashless-limit environment. Second, in a setup with nominal rigidities, price stability - the standard goal of monetary policy in that case - is no longer optimal.

In other words, the introduction of debt-burdened households creates a trade-off between interest rate reactions meant to stabilize prices and the ones that stabilize the debt service volatility. In fact, the volatility of interest payments introduces a source of idiosyncratic uncertainty at household level - which, in turn, is welfare reducing.

Finally, we also show that a measure of debt dispersion would be an important separate target for an optimally designed simple implementable rule. More precisely, rules that also include a separate target on debt dispersion outperform standard rules which only target inflation and output gap.

The extent of deviation from price stability depends on the economy's *initial* level of debt dispersion. In order to calibrate the initial debt dispersion I use micro data from the US Board of Governors' Survey of Consumers Finances for the year 2001. Under the baseline calibration our model suggests that the policy prescriptions of its representative agent counterpart (i.e. the equivalent model with symmetric asset positions) may constitute a reasonable approximation: the magnitude of deviation from zero inflation we get is small. However, unlike in the representative agent model, the initial response of nominal interest rate to disturbances is much smaller.

As last remark, we observe that a high dispersion in the initial net-debt positions does call into question the price stability goal. In this case, aggregate shocks affecting the *natural* rate of the economy would imply a large and persistent deviation from zero inflation.

1.1 The Model

The baseline model is a cashless limit dynamic sticky price model with common factor markets and no capital accumulation (Clarida et al., 1999, Gali, 2001; Rotemberg and Woodford, 1997, 1999). I depart from the baseline model in two aspects: markets are incomplete and the initial distribution of nominal debt across households is not degenerate.[3]

There are two sources of aggregate uncertainty: the level of total factor productivity, A, and the level of real government purchases, G, which are assumed to be financed with lump-sum taxes. Aggregate shocks may have an idiosyncratic impact on households budget constraint.

The government can finance the exogenous stream of public consumption with lump sum taxes T^G. In period-0 the government is also able to implement a redistributive transfers scheme, $\bar{\tau}$, to favor wealth equality. However it is not allowed to change it thereafter.

The monetary authority controls the short term nominal interest, R, takes the fiscal redistributive scheme as given and can commit to a state-dependent rule. This last one allows the monetary authority to respond to all of the relevant state variables of the economy.

In this section, I describe a recursive equilibrium, with households and firms solving dynamic optimization problems for given fiscal and monetary policy rule.

1.1.1 Households

I assume a continuum of households indexed by $h \in [0, 1]$ maximizing the following utility

$$U_0^h = E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t^h) - v(N_t^h) \right]$$

[3]Using the US Board of Governors Survey of Consumer Finances for the year 2001 I find that the net nominal credit position substantially differ across households (see calibration section for further details). The first 10% of the distribution holds a stock of net-debt higher than 120,000USD; while the last 10% (the 90th percentile) holds a stock of net-credit of about 880,000USD. The median is approximately zero.
From a modeling point of view we could generate a non-degenerate distribution of assets across agents introducing idiosyncratic income or preference shocks at household level. However, for tractability reasons and because they are irrelevant for the exposition of the main arguments, we do not need to introduce them.

E_0 denotes the expectation operator conditional on the information set at date-0 and β is the inter-temporal discount factor, with $0 < \beta < 1$. Households get utility from consumption and disutility from working. Both functions are strictly increasing and twice differentiable, however $v(.) : [0, \bar{N}^{+}) \to \mathbb{R}$ is strictly convex while $u(.) : \mathbb{R}_{+} \to \mathbb{R}$ is strictly concave in the consumption index C. This is defined as a Dixit-Stiglitz aggregator of different goods produced in the economy with constant elasticity $\theta > 1$:[4]

$$C_t^h = \Big(\int_0^1 c^h(z)^{\frac{\theta-1}{\theta}} dz \Big)^{\frac{\theta}{\theta-1}}$$

Let P_t represent the aggregate price index such that

$$P_t^{1-\theta} = \int_0^1 P_t(z)^{1-\theta} dz$$

where $P_t(z)$ denotes the price of good-z. Then, for each household, the optimal allocation of a given amount of expenditures among the different goods generates the good-z demand schedules

$$c_t^h(z) = \Big(\frac{P_t(z)}{P_t} \Big)^{-\theta} C_t^h \tag{1.1.1}$$

Each household-h earns a nominal wage W_t per hour worked and can buy or issue a nominal riskless bond B_t^h (IOUs) - its market price $1/R_t$ is taken as given. The variable X_t^h collects terms which are rebated to households in lump sum fashion: it summarizes a lump sum government tax (transfer) T_t^h and lump sum profits from firms F_t^h. So the budget constraint takes the following form:

[4]In a representative agent economy having no upper bound for hours worked do not represent a serious concern. However, when there is a continuum of heterogenous agents, the possibility of supplying an unbound amount of hours, having the wage unaffected, is not realistic and would pose no lower bound for the natural debt limit.

$$P_tC_t^h + B_t^h/R_t = B_{t-1}^h + W_tN_t^h + P_tX_t^h \tag{1.1.2}$$

where

$$X_t^h = T_t^h + F_t^h \tag{1.1.3}$$

In period-0 firms shares are equally split across households and are not subsequently traded.[5] In other words we can write $F_t^h = F_t$ where F_t is the total amount of profits made in the economy. The government tax (transfer) can be divided into an aggregate tax T_t^G - needed to finance current government spending G_t - and a household specific constant transfer $\bar{\tau}^h$. So the additive component X_t^h of the budget constraint can be written as $X_t^h = \bar{\tau}^h - T_t^G + F_t$.

I now turn to households necessary conditions for optimality. For each household-h the intra-temporal consumption-leisure choice reads (I write the real wage as $W_t^r \equiv W_t/P_t$)

$$W_t^r = v_n(N_t^h)/u_c(C_t^h) \tag{1.1.4}$$

while the inter-temporal optimality condition is given by the Euler equation

$$\beta R_t E_t \frac{u_c(C_{t+1}^h)}{P_{t+1}} = \frac{u_c(C_t^h)}{P_t} \tag{1.1.5}$$

Savers will purchase debt issued by borrowers only if they know that they can be repaid *almost surely*, I thus introduce a *natural debt limit*

$$B_t^h/P_t \geq -\phi_b^h \tag{1.1.6}$$

[5]The trading restriction imposed here on stocks may not be innocuous given the absence of complete financial markets. However more than one concern has prevented us to introduce this additional feature. Mainly I believe that a sticky price model is not well suited to describe firms' profits behavior over the business cycle - see for example Christiano et.al. 1997.

The value of ϕ_b^h is the maximum level of debt a household is able to repay satisfying the consumption plan $\{C_t^h\}_{t=0}^{\infty}$ to be a non-negative random sequence (for a derivation of the natural debt limit in this economy see Appendix section G).

1.1.2 Firms

I assume a continuum of firms, each producing a differentiated good with a technology

$$y_t(z) = A_t N_t(z) \tag{1.1.7}$$

where (log) productivity $a_t = \log(A_t)$ follows a Markov-stationary exogenous stochastic process.

I will also assume that employment is subsidized at a constant subsidy rate $1 - \tau_\mu$. Hence, all firms face a common real marginal cost, which in equilibrium is given by

$$mc_t = \frac{W_t^r}{A_t}\tau_\mu \tag{1.1.8}$$

The government has the same consumption aggregator as the private sector and it demands the same fraction, τ_t^G, of the output of each produced good $g_t(z) = \tau_t^G y_t(z)$.

Recalling the private sector static-optimality condition - equation (1.1.1) - I define the aggregate private sector demand for a good-z by summing up individual households' demands: $c_t(z) \equiv \int_0^1 c_t^h(z)dh$.[6]

Hence the total demand function for each differentiated good is

[6]For notational convenience I will introduce the distribution of agents over variables only when strictly necessary.

$$y_t^d(z) \equiv c_t(z) + g_t(z) = \Big(\frac{P_t(z)}{P_t}\Big)^{-\theta} Y_t \qquad (1.1.9)$$

where

$$Y_t = \Big(\int_0^1 y(z)^{\frac{\theta-1}{\theta}} dz\Big)^{\frac{\theta}{\theta-1}} \quad \text{and} \quad G_t = \tau_t^G Y_t$$

denotes the aggregate (demanded) output and the aggregate government spending, respectively, such that [7]

$$Y_t = C_t + G_t$$

Firms are monopolistic competitors and are allowed to change prices with a Calvo probability $1-\psi$. Each household-shareholder h would like to have firms maximize discounted profits using its own stochastic discount factor $\Lambda_{t,t+k}^h$. The pricing-policy that a shareholder-h would like to see implemented in firm-z is:[8]

$$\sum_{k=0}^{\infty} (\psi\beta)^k E_t \Lambda_{t,t+k}^h P_{t+k}^{\theta} Y_{t+k} [P_t^{h,\star}(z)/P_{t+k} - \frac{\theta}{\theta-1} mc_{t+k}] = 0 \qquad (1.1.10)$$

If managers have been delegated a linear rule then, under the assumption of zero steady state inflation, shareholder-h would like to see implemented

$$\log P_t^{h,\star}(z) = \frac{\theta}{\theta-1} + (1-\psi\beta) \sum_{k=0}^{\infty} (\psi\beta)^k E_t \Big[\log(mc_{t+k} P_{t+k})\Big] \qquad (1.1.11)$$

From above expression we see that the equilibrium choice of the relative price $P_t^{h,\star}(z)$ is the same for all resetting firms and across all shareholders. In other words, whatever is the distribution of voting rights across households, to a first order approximation, they would like to see implemented the same pricing rule.

[7] To derive the

[8] For a derivation and interpretation of the firms' optimality condition see Woodford 2003 or Gali 2001, among others

This means that the losses in which each shareholder incurs from deviating from *his* optimal rule are of second order.

The pricing rule has a simple interpretation: firms set prices at a level such that a (suitable) weighted average of anticipated future markups matches the optimal frictionless markup $\theta/(\theta-1)$.

1.1.3 The Government

The government lump sum tax/subsidy, T_t^h is household specific. However it can be split into two components: an aggregate component, T_t^G, and a constant redistributive component, $\bar{\tau}^h$. The latter captures a constant redistribution scheme chosen at time-0 before any shock realization. I assume it has zero mean: $\int_0^1 \bar{\tau}^h dh = 0$. The former component, T_t^G, is instead the same for each household and is aimed to finance current government expenditure G_t such that the government runs a balanced budget deficit in each period. At all times it must hold

$$T_t = -\int_0^1 T_t^h dh = T_t^G - \int_0^1 \bar{\tau}^h dh = T_t^G = G_t \qquad (1.1.12)$$

The availability of lump sum taxes, and the absence of transaction frictions, renders the way government finances its current deficit irrelevant also in an heterogenous agent model - which is not necessarily true when money balances are not a dominated asset. [9] At the same time the availability of lump sum taxes imply that there is no need of using inflation as absorber of unexpected

[9] Akyiol (2004) studies a heterogenous-agents endowment economy with lump sum taxes and open-market operations. An open market operation involves a transfer to agents holding government debt. Given that there is a non-degenerate distribution of agents with respect to bond holdings, there is a different level of transfer to each agent which is not the case when the government makes a lump sum transfer to each agent. This does not happen in a representative agent models where lump sum transfers of money (i.e. "helicopter drop") and retiring existing debt through open market operations are equivalent.

adverse fiscal shocks - as often studied in the public finance literature.[10] Thus, the structure imposed to the government behavior allows us to focus the analysis on the household liabilities only.

I define a fixed redistributive transfer scheme to be a measure (or the cumulative marginal distribution) of households, Φ^{τ}, over transfers $\bar{\tau}$, satisfying $\int_{-\infty}^{\infty} \tau d\Phi^{\tau}(\tau) = 0$. The government has to choose Φ^{τ} once and for all at time-0. Let Φ_t be the distribution of households over the beginning of period bond holdings. I assume that the choice of Φ^{τ} must be made prior to any shock realization. This entails that the government information set of time-0 is simply given by the initial measure of households Φ_{-1} over bond holdings.[11]

For the role and interpretation of the transfer scheme and also for an alternative setup without transfers see section (1.3).

1.1.4 Monetary Authority

I abstract from monetary frictions and I assume that the central bank can control the riskless short-term gross nominal interest rate R_t.[12]

The zero lower bound on nominal interest rate is assumed to be never binding under the optimal policy regime. Finally, I also assume that the central bank has full information in setting its instrument.

The time-t available information is captured by the all relevant time-t state of the economy. In particular, as it will be clear shortly, I allow the monetary authority to respond to an exogenous state vector Z_t, to an endogenous aggregate state vector S_t - defined in the next paragraph - and to a third set of co-states denoted $\mathcal{L}_t$. As a matter of notation I write $\omega_t = (Z_t, S_t, \mathcal{L}_t)$ and $R_t = R(\omega_t)$.

[10] See, among others, Chari, Christiano and Kehoe (1994)

[11] In fact the government has more detailed information, it know the asset position of each household.

[12] See Woodford 2003 Ch2 for a discussion about a "cashless" limit economy.

1.1.5 Recursive Equilibrium

Let $Z_t = (A_t, \tau_t^G)$ be the vector of exogenous economy-wide stochastic processes and Φ_t be the measure (cumulative distribution) of households over asset holdings at time-t. The law of motion concerning Φ_t is described by the function $f(.)$ such that $\Phi_t = f(\Phi_{t-1}, Z_t)$.

Let also

$$\Delta_{p,t} = \int_0^1 \Big(\frac{P_t(z)}{P_t}\Big)^{-\theta} dz \tag{1.1.13}$$

represent the price dispersion in the economy. In the case of unfrequent possibilities of readjusting prices $\Delta_{p,t-1}$ becomes a state for our economy.

Having defined $S_t = [\Phi_t, \Delta_{p,t-1}]$ I can now introduce the aggregate state vector for this economy $\omega_t = (Z_t, S_t, \mathcal{L}_t)$ and the vector of state relevant for each individual household $s_t^h = (b_{t-1}^h, X_t^h, \omega_t)$ where $b_t \equiv B_t/P_t$. The role of the aggregate state is to allow agents to predict future prices and monetary authority actions. The household's problem can be recast in the following recursive form

$$V(s,\omega) = \max\Big[u(C) - v(N) + \beta E\, V(s', \omega')\Big] \tag{1.1.14}$$

$$s.t.$$

$$c + b'/R(\omega) = b/\Pi(\omega) + w(\omega)N + X(s,\omega)$$

$$\Phi' = f(\Phi, Z, Z')$$

$$b \geq -\phi_b$$

The policy function for asset investment is $b' = b(s)$.

For given monetary policy and transfer scheme $\Big(R(\omega), \Phi^\tau\Big)$ and an initial condition ω_0 a *recursive imperfectly competitive equilibrium* is a law of motion $f(.)$, value and policy functions V and b, pricing functions $\Big(w(\omega), \Pi(\omega), (p(z))(\omega)_{z\in[0,1]})\Big)$ such that i)V and b solve (1.1.14). ii) The pricing functions, together with a law of motion for the price level, solve the optimal price setting firm problem. iii) There is consistency between aggregate variables and summing up of agents optimal choices - i.e. Φ generates bond market clearing $\int_0^1 b' d\Phi = 0$ and labor market clears.[13]

1.2 Idiosyncratic Interest Payments Risk

This section is preliminary to the welfare analysis. Here I describe how portfolios heterogeneity coupled with incomplete markets may affect the aggregate equilibrium allocation.

For the rest of the paper I will use the following utility functional form: $u(.)$ is in the CRRA class such that $-u_{cc}C/u_c \equiv \sigma$ is a constant, while $v(.)$ is such that, given some $\delta > 0$, $\varphi \equiv v_{nn}N/v_n$ is "at least approximately constant" for $N \in I(\bar{N}, \delta)$ - where φ is the inverse of the Frisch labor elasticity.

1.2.1 Effects on Aggregate Labor Supply

Using the consumption-leisure relation we observe that the individual labor supplies are shifted by the different levels of consumption - which in turn are related to individual wealth. For example, a relatively "poor" household has its labor supply shifted downward: it will work relatively more, given the wage.

Incomplete Markets I now want to see the impact of this shift on the *aggregate* labor supply schedule. We can write the consumption-leisure choice as

$$C_t^h = W_t^{r\,1/\sigma} N_t^{h\,-\varphi/\sigma} \tag{1.2.1}$$

[13] A formal proof of the existence of an equilibrium for an economy very similar ours can be found in Miao 2005.

Integrating up the above equation with respect to households we recover a relation between aggregate consumption C_t and aggregate labor N_t:[14]

$$C_t = (W_t^r)^{1/\sigma} N_t^{-\varphi/\sigma} \int_0^1 (N_t^h/N_t)^{-\varphi/\sigma} \tag{1.2.2}$$

Let

$$\Delta_{n,t} \equiv \int_0^1 (N_t^h/N_t)^{-\varphi/\sigma} \tag{1.2.3}$$

denote the labor supply distortion - ultimately linked to wealth dispersion. Taking a log-transformation and using *hats* for *logs* we can reformulate the above expression:

$$\hat{W}_t^r = \varphi \hat{N}_t + \sigma \hat{C}_t - \sigma \hat{\Delta}_{n,t} \tag{1.2.4}$$

By Jensen inequality we realize that for all $\sigma > 0$ and $\varphi > 0$ we have $\log \Delta_{n,t} > 0$.[15] This means that, for a given aggregate consumption, the aggregate labor supply is pushed rightward by an amount proportional to a *measure* eventually related to the economy wide debt dispersion.[16] This creates a time-varying wedge, at aggregate level, between the factor price of labor and the "aggregate" marginal rate of substitution. To understand whether this wedge or its volatility involve an inefficiency we have to introduce a concept of efficiency.

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[14] I have simply defined aggregate consumption as $C_t \equiv \int_0^1 C_t^h dh$ and aggregate labor supply as $N_t \equiv \int_0^1 N_t^h dh$

[15] We can think of $X \equiv N^h/N$ as a positive random variable with mean equal to one. While $f(u) = u^{-\varphi/\sigma}$ is a strictly convex function $\forall \sigma > 0$ and $\varphi > 0$. This means that $E_h[f(X)] > f(E_h(X)) = f(1) = 1$

[16] If we set $\hat{C}_t = \hat{A}_t - \hat{g}_t + \hat{N}_t$ - as it will be clear later - we can write a proper labor supply schedule $\hat{W}_t^r = \sigma \hat{A}_t - \sigma \hat{g}_t + (\sigma + \varphi)\hat{N}_t - \sigma \hat{\Delta}_{n,t}$.

Complete Markets Under the assumption of complete markets households can perfectly insure against interest rate risk.[17] Changes in the prevailing interest rate and inflation would affect each households' budget constraint differently depending on their nominal bonds asset position. However if a full set of state-contingent claims on consumption is available at time-0 then - regardless of the initial asset position - the consumption of each household is perfectly correlated with aggregate consumption. This also means that each household will consume as much as the average consumption times a constant of proportionality

$$C_t^h = \delta(h)C_t; \;\; \forall h \in [0,1] \tag{1.2.5}$$

The function $\delta : [0,1] \rightarrow \mathbb{R}^+$, satisfying $\int_0^1 \delta(h)dh = 1$, is time invariant and reflects wealth differences across households. It is possible to determine $\delta(h)$ from Φ_{-1} and $\{\tau^h\}_{h\in[0,1]}$.

As before we can write the following equation

$$\hat{W}_t^r = \varphi \hat{N}_t + \sigma \hat{C}_t - \sigma \hat{\Delta}_{n,t} \tag{1.2.6}$$

However now

$$\Delta_{n,t} = \Big(\int_0^1 \delta(h)^{-\sigma/\varphi} dh\Big)^{\varphi/\sigma} \tag{1.2.7}$$

This means that $\Delta_{n,t}$ is a constant across time. We will refer to it simply as Δ_n.

1.2.2 Efficient allocation vs Flexible Price Equilibrium

To stress the role played by incomplete markets, I first shut off the distortion stemming from price stickiness, I will reintroduce it at the end of this section.

[17]This is true under our specified functional form for the households utility.

In a environment without nominal rigidities the price decision rule reduces to a constant mark-up μ over the real marginal cost regardless of household sector. Let the employment subsidy exactly offset the monopolistic distortion, i.e. $\mu\tau_\mu = 1$, thus a symmetric equilibrium implies that the real wage $\hat{W}^r_t = \hat{A}_t$ and $\hat{N}_t = \hat{Y}_t - \hat{A}_t$. Using the resource constraint, $\hat{C}_t = \hat{Y}_t - \hat{g}_t$ where $\hat{g}_t \equiv -\log(1 - \tau^G_t)$, together with production function, I substitute out aggregate consumption and aggregate labor from equation (1.2.6). Hence I am able to write the flexible price (*natural*) level of output Y^f_t as:

$$\hat{Y}^f_t \equiv \frac{\sigma}{\sigma+\varphi}\hat{g}_t + \frac{1+\varphi}{\sigma+\varphi}\hat{A}_t + \frac{\sigma}{\sigma+\varphi}\hat{\Delta}_{n,t} \tag{1.2.8}$$

In the case of complete markets we have shown that Δ_n is a constant. I call the associated level of output as *efficient* output Y^e.[18][19] Using logs we have

$$\hat{Y}^e_t = \frac{\sigma}{\sigma+\varphi}\hat{g}_t + \frac{1+\varphi}{\sigma+\varphi}\hat{A}_t + \frac{\sigma}{\sigma+\varphi}\hat{\Delta}_n \tag{1.2.9}$$

This is the equilibrium allocation that would be obtained under flexible prices, perfect competition, no distortionary taxation plus complete markets.

So from the previous equation we can find an exact relation between the output prevailing in the flexible-prices environment and the *efficient* level of output

$$\hat{Y}^f_t - \hat{Y}^e_t = \frac{\sigma}{\sigma+\varphi}(\hat{\Delta}_{n,t} - \hat{\Delta}_n) \tag{1.2.10}$$

Thus deviations of $\Delta_{n,t}$ from Δ_n introduce a real imperfection in the economy

[18] When $\Delta_n = 1$ the flexible price allocation is equivalent to the one usually found in the literature.

[19] Aggregating individual Euler equations we can also define the *efficient rate of interest* as $r^e_t \equiv \sigma E_t \Delta \hat{C}^e_{t+1} = \sigma E_t \Delta \hat{Y}^e_{t+1} - \sigma E_t \Delta \hat{g}_{t+1}$.

that creates a wedge between the *natural* and the *efficient* level of output. [20] [21]

We also notice that not only Y_t^f does not deliver the efficient allocation but it is also not independent of monetary policy, to the extent that the latter can affect $\Delta_{n,t}$.

We now turn to the sticky price model. Using equation (1.1.9) we write the total hours demanded by firms

$$N_t = \int_0^1 N_t(z)dz = \frac{Y_t}{A_t}\Delta_{p,t} \tag{1.2.11}$$

where

$$\Delta_{p,t} = \int_0^1 \Big(\frac{P_t(z)}{P_t}\Big)^{-\theta} dz \tag{1.2.12}$$

is the usual measure of price dispersion - which, in turn, is the source of welfare losses from inflation or deflation.

We now establish an exact relation between the sticky price output Y_t and its efficient level Y_t^e, which allow us to define an output gap measure $x_t \equiv \hat{Y}_t - \hat{Y}_t^e$ and to disclose the role played by the two sources of distortion.

Using the household first order conditions we can find an exact relation that expresses the marginal costs as function of the deviation of output from the *efficient* level of output and we write (see Appendix-B for details)

$$\hat{mc}_t = (\sigma + \varphi)x_t + \varphi\hat{\Delta}_{p,t} - \sigma(\hat{\Delta}_{n,t} - \hat{\Delta}_n) \tag{1.2.13}$$

price distortion $\Delta_{p,t}$ and the labor supply distortion $\Delta_{n,t}$ affects the output gap.

[20] It is also worth noting that a higher dispersion of hours worked, shifting the labor supply downward, generates *overproduction* pushing aggregate output over its efficient level. This result hinges on a strictly decreasing marginal utility of consumption: a one-unit reduction of consumption is more ‘painful’ in absolute terms then the benefit of a one-unit increase in consumption. Hence a reduction in consumption, for a given wage, has a stronger effect, in absolute terms, on labor supply than an increase has. Thus, *ceteris paribus*, the higher the wealth inequality the higher the aggregate labor supply.

[21] For a related concept, although introduced in a different environment, see also Blanchard and Gali 2005

Hence, we have determined an additional source of deviation from the efficient level of output which a benevolent policy maker would like to offset. Generally speaking we can identify two different dimensions at which we could analyze how heterogeneity may generate welfare concerns.

In a *static* dimension the level around which $\Delta_{n,t}$ oscillates - which will be Δ_n - may reduce social welfare given that it represents the long-run differences in consumption and leisure across households. However any policy action meant to change Δ_n would not be a pareto improvement but would depend on the way we express social preferences and we care about wealth inequality.

In a *dynamic* dimension instead, taken as given the level Δ_n, reducing the volatility of $\Delta_{n,t}$ represents a strictly pareto improvement. The volatility is in facts a consequence of households' impossibility to hedge perfectly against aggregate shocks that can affect interest rate and inflation an so interest rate payments.

In the next section I analyze in further details the role played by fiscal and monetary policy. Prominently, I show that the monetary authority does *not* have necessarily to deal with inequality - the static dimension of the problem which should be more a fiscal policy concern. However, even in this case, it will be clear that a central bank still plays a crucial role in offsetting the redistributive impact that *aggregate* shocks have on households' budget constraints - the dynamic dimension of the problem. Moreover, I will clarify why the stock of debt/assets accumulated by households becomes a source of idiosyncratic uncertainty at household level - which, in turn, is the source of volatility for our distortion $\Delta_{n,t}$.

1.3 Welfare Analysis

In this section we lay out the problem of a benevolent policy maker reacting to aggregate exogenous disturbances when the economy is populated by a continuum of households which show a non-degenerate distribution over nominal asset holdings. The standard stabilization prescription of replicating the flexible price equilibrium allocation is challenged. With an incomplete market structure and portfolio heterogeneity featuring in the economy we must now face also the *redistributive* character of standard policy recommendations and the implied distortion.

The policy objective of a benevolent policy maker is maximizing a welfare function W which aggregates agents' utilities $\mathrm{W} : \mathcal{U} \rightarrow \mathbb{R}$.[22]

$$\mathrm{W}_t = E_t \sum_{k=0}^{\infty} \beta^k \int_0^1 \eta(h)[u(C_t^h) - v(N_t^h)]dh \tag{1.3.1}$$

where $\eta(h) : [0,1] \rightarrow \mathbb{R}^+$ represents a time-invariant weighting function.

Transfers Scheme Approach When transfers are optimally chosen (see next section) our economy oscillates around the *efficient* and *socially desirable allocation* - for any arbitrary initial asset distribution.[23] This is a convenient condition for the derivation of a quadratic welfare-based loss function.

For the case $\eta(h) = 1$ every household is weighted the same: the above welfare criterion, given strictly concave utility functions, strictly prefers consumption (wealth) equality. In this case transfers would be chosen in order to restore - in absence of any shock realization - wealth equality.

[22] Qualitatively, our results do not depend from the welfare criterion chosen, in fact the less utilitarian is the welfare function the stronger are our results.

[23] For a definition of efficient allocation in our economy see section 1.2

Unequal Pareto Weights Approach Without any transfer scheme ($\tau^h = 0 \ \forall h \in [0,1]$) wealth would be unequally distributed. Creditors would be rich and debtors poor. However we can always find a positive weighting function $\eta(h)$ such that - in absence of any shock realization - the welfare criterion is maximized. This is to say that the welfare criterion would relatively overweight rich households. It turns out that such a weighting function would be the one that makes a social planner recover the complete markets solution discussed in paragraph 1.2.1. The weights would be given by the inverse of each initial households marginal utility. They can be normalized such that we can use the steady state consumption - i.e. $\eta(h) = 1/u'(\bar{C}^h)$.[24]

Both approaches would make the central bank accept the initial (and long run) wealth inequality. Loosely speaking this is equivalent to a monetary authority that accepts the wealth distribution *in statu quo nunc*.

In the appendix (D) I show that the two approaches give the same results. In what follows I will take equal weights $\eta(h) = 1$ and transfers chosen to deliver a socially desirable steady state from which the monetary authority does not have incentive to deviate - i.e. wealth equality.

1.3.1 Optimal Policy

We assume that the optimal policy honors commitments made in the past. This form of policy commitment has been referred to as optimal from a *timeless perspective* (see Woodford). The difference with respect to a standard Ramsey problem is that we will be looking for policy functions that are time invariant.

[24]Let $\tilde{\eta}(h) = 1/u'(C_0^h)$. We use the following normalization:

$$\eta(h) \equiv \tilde{\eta}(h)\frac{u'(C_0^h)}{u'(\bar{C}^h)} = \tilde{\eta}(h)\frac{u'(C_0)u'(\delta(h))}{u'(\bar{C})u'(\delta(h))} = \tilde{\eta}(h)\frac{u'(C_0)}{u'(\bar{C})}$$

In other words the monetary or fiscal authority cannot exploit any advantage at time-0.[25]

The optimal fiscal and monetary policy is a rule for $\{R_t\}_{t\geq 0}$ and a feasible fixed transfer system $(\bar{\tau}^h)_{h\in[0,1]}$ which are consistent with the imperfectly competitive equilibrium (CE) defined in section (1.1.5) and maximize the welfare function as defined in equation (1.3.1) given exogenous processes Z_t, initial conditions S_{-1} and s_{-1}, and values for a set of Lagrange multipliers $\mathcal{L}$ associated with the constraints introduced for satisfying CE-conditions dated $t < 0$.

Thus we have now determined the extra state variables $\mathcal{L}_t$ to which the monetary authority was viewed as responding to in section (1.1.4.).

In carrying out our analysis we will not have to determine the value functions for the private sector behavior but we will simply focus on the first order conditions. In order to do it we will take a local approximation of the model which means we need to find a reasonable point around which to perform the approximation (for a discussion on the approximation procedure see appendix). A natural candidate is the steady state of the deterministic version of our model where aggregate shocks have been shut off. However even in this case fiscal and monetary authority can affect the steady state values of the endogenous variables of the system. Because we want to keep staying close to those values - when starting from initial conditions close enough to them and for small enough exogenous disturbances - then we have to characterize the optimal long run steady state.

In other words the presumption is that the optimal policy which guides the economy through business cycle fluctuations will be oscillating around the

[25] In a closely related setup Khan et al. (2003) introduce in the standard (unconstrained) Ramsey problem lagged Lagrange multiplier corresponding to the forward-looking constraints in the initial period making the problem stationary. The initial values are chosen to be the steady state values. For a discussion see also Benigno-Woodford 2005

optimal *long-run* policy - and not about a generic steady state. In the next paragraph we specify what we mean for optimal long run policy.

Optimal Steady State

We characterize the solution to our policy problem only for initial conditions near certain steady-state values, allowing us to use local approximations when we characterize optimal policy.[26] Hence our local characterization describes policy that is optimal from a timeless perspective in the event of small disturbances.

For any given initial distribution of debt across households Φ_{-1} we wish to find an initial degree of price dispersion $\Delta_{p,-1}$ and a transfer system $\Phi^{\tau} = H(\Phi_{-1}, \Delta_{p,-1})$ - implemented before any shock realizations - such that the solution of the deterministic problem involves a constant policy in each period and where $\bar{\Delta} = \Delta_{p,-1}$ and $\bar{\Phi} = \Phi_{-1}$.

We state the following proposition (for a proof see Appendix-C)

Proposition 1.3.1. *In the deterministic equivalent model where all sources of uncertainty are shut off but for the Calvo signal, the long run optimal monetary policy entails no price dispersion*

$$\bar{\Pi} = \bar{\Delta}_p = 1 \tag{1.3.2}$$

and any given initial distribution of households over debt, Φ_{-1}, induces an optimal constant transfer system $\Phi^{\tau}(\bar{\tau}) \propto \Phi_{-1}$ such that for each household we have

$$\bar{\tau}^h = -\bar{b}^h(1/\bar{\Pi} - 1/\bar{R}) = -\bar{b}^h(1-\beta) \tag{1.3.3}$$

We next characterize the optimal steady state. By proposition (1.3.1) the steady state inflation rate is zero hence the steady state nominal gross interest

[26] In a representative agent model it can be usually shown that these steady-state values have the property that if one starts from initial conditions close enough to the steady state, and exogenous disturbances thereafter are small enough, the optimal policy subject to the initial commitments remains forever near the steady state (see Benigno and Woodford (2005)). This does not necessarily entail the stronger result of convergence. Indeed, in many fiscal policy setup the deterministic model shows a unit root - if the stochastic version has a quasi-random walk - in government bonds.

rate is equal to the inverse of the subjective discount factor $R = 1/\beta$. We can write the steady state budget constraint for a generic household-h as:

$$\bar{C}^h = \bar{b}^h(1-\beta) + \bar{W}\bar{N}^h + \bar{F} - \bar{G} + \bar{\tau}^h \tag{1.3.4}$$

where $\bar{b}^h$ is the initial period bond holdings of household-h. Because the government sets a constant transfer $\bar{\tau}^h = -\bar{b}^h(1-\beta)$ for each household-h in steady state we have

$$\bar{C}^h = \bar{C} \text{ and } \bar{N}^h = \bar{N} \ \forall h \in [0,1] \tag{1.3.5}$$

Firms optimal price decision rule implies that the constant markup over marginal costs must be equal to one. Hence we can finally write

$$\bar{N} = \bar{A}^{\frac{1-\sigma}{\sigma+\varphi}}(1-\bar{\tau}^g)^{-\frac{\sigma}{\sigma+\varphi}} \ \forall h \in [0,1] \tag{1.3.6}$$

and

$$\bar{C}^h = \bar{C} = \bar{Y} - \bar{G} = \bar{A}^{\frac{1+\varphi}{\sigma+\varphi}}(1-\bar{\tau}^g)^{\frac{\varphi}{\sigma+\varphi}} \ \forall h \in [0,1] \tag{1.3.7}$$

The most important consideration to be made is that, thanks to the transfers scheme, even in presence of debt dispersion the steady state found is *non-distorted* and *socially desirable*: marginal rate of substitutions equal marginal rate of transformations and the consumption allocation maximizes the welfare criterion chosen.[27] The fact that I will analyze the economy oscillating about its efficient level of output is crucial for the derivation of a purely quadratic objective function for the policy maker.

[27] I recall that we have chosen $\eta(h) = 1$.

1.3.2 The Economy under the Optimal Transfers Scheme

I define $\tilde{b}_t^h \equiv b_t^h - \bar{b}^h$ and, exploiting that $\bar{\tau}^h = -\bar{b}^h(1-\beta)$, I re-formulate the agent-h budget constraint:

$$C_t^h + \tilde{b}_t^h/R_t = \tilde{b}_{t-1}^h/\Pi_t + W_t^r N_t^h + F_t - G_t + \bar{b}^h(\frac{\beta R_t - 1}{R_t} - \frac{\Pi_t - 1}{\Pi_t}) \quad (1.3.8)$$

From this expression we see that heterogenous debt holdings introduce a source of idiosyncratic uncertainty at household level, which is captured by $\bar{b}^h(\frac{\beta R_t - 1}{R_t} - \frac{\Pi_t - 1}{\Pi_t})$.

If the economy oscillates close enough to its efficient level then the value of $\tilde{b}_t^h$ is relatively small compared to $\bar{b}^h$. This means that $\bar{b}^h(\frac{\beta R_t - 1}{R_t} - \frac{\Pi_t - 1}{\Pi_t})$ represents the main component of the impact on households balance sheet of fluctuations in interest payments (or interest income).

Intuitively, anticipating results, a monetary authority who is willing to shut off the debt servicing volatility should set the above term to zero:

$$\frac{\beta R_t - 1}{R_t} - \frac{\Pi_t - 1}{\Pi_t} = 0 \quad (1.3.9)$$

which implies

$$R_t^o = \frac{1 + \pi_t}{\beta - (1-\beta)\pi_t} \quad (1.3.10)$$

The economic intuition is that, ceteris paribus, if we want to leave households asset position unchanged $\tilde{b}_t^h = 0$ than the nominal interest rate must mildly react to inflationary (deflationary) pressures. On the other hand any reaction of the nominal rate different from R_t^o leads to what we call an *arbitrary redistribution*: it generates a wealth effects that adds to consumption volatility.

A Log-Linear Representation

We develop further the previous idea recasting the system in deviations from average quantities (for a discussion on the approximation method see Appendix 6.6). We first define the consumption and employment cross-sectional gap

$$\tilde{C}_t^h \equiv \log C_t^h/C_t = \hat{C}_t^h - \hat{C}_t$$
$$\tilde{N}_t^h \equiv \log N_t^h/N_t = \hat{N}_t^h - \hat{N}_t$$

The resource constraint for this economy without capital accumulation - which is simply $C_t = (1-\tau_t^G)Y_t$ - can be written as

$$C_t = W_t^r N_t + F_t - G_t \tag{1.3.11}$$

So we can re-write the household-h budget constraint of equation (1.3.8) subtracting it the above resource constraint (1.3.11)

$$C_t^h - C_t + \tilde{b}_{t-1}^h/R_t = \tilde{b}_{t-1}^h/\Pi_t + W_t^r(N_t^h - N_t) + \bar{b}^h\left(\frac{\beta R_t - 1}{R_t} - \frac{\Pi_t - 1}{\Pi_t}\right) \tag{1.3.12}$$

Taking a linear expansion of this equation around the steady state of the deterministic model we have

$$\tilde{C}_t^h + \beta\tilde{b}_t^h = \tilde{b}_{t-1}^h + \bar{W}^r\bar{N}\tilde{N}_t^h + \bar{b}^h(\beta\hat{R}_t - \pi_t) \tag{1.3.13}$$

Recalling the result of section (1.2)

$$C_t = W_t^{r\,1/\sigma} N_t^{-\varphi/\sigma}\Delta_{n,t} \tag{1.3.14}$$

we can find how $\tilde{N}_t^h$ is related to $\tilde{C}_t^h$:

$$\frac{\varphi}{\sigma}\tilde{N}_t^h = -\tilde{C}_t^h + \hat{\Delta}_{n,t} \tag{1.3.15}$$

In case of small enough exogenous disturbance the term $\Delta_{n,t}$ will be either small or constant for raking welfare. In fact we have $\Delta_{n,t} \simeq .5\frac{\varphi}{\sigma}Var_h\tilde{N}_t^h$. So we can substitute $\tilde{C}^h$ for $\tilde{N}^h$ in equation (1.3.13) using expression (1.3.15) [28] to get

$$\kappa_c\tilde{C}_t^h = \tilde{b}_{t-1}^h - \beta\tilde{b}_t^h + \bar{b}^h(\beta\hat{R}_t - \pi_t) \tag{1.3.16}$$

where $\kappa_c = 1 + \frac{\sigma}{\varphi}$.

This equation deserves attention. We have rewritten the saving decision of each household, $\tilde{b}_t^h$, as choosing how much to deviate from the steady state level of savings $\bar{b}^h$. This means that, in a local approximation and at any degree, the impact of interest rate and inflation in the budget constraint *through* $\tilde{b}_t^h$ is negligible. The all impact is instead captured by $\bar{b}^h(\beta\hat{R}_t - \pi_t)$ which represents debt servicing deviations from their steady state level for a household that enters the world with a stock of debt equal to $-\bar{b}^h$.

We conclude the description of the system at individual-level by taking a log-linear expansion of the households Euler equation in deviation from aggregate levels.

$$E_t\Delta\tilde{C}_{t+1}^h = -\varphi_b\tilde{b}_t^h \tag{1.3.17}$$

We have introduced the term $\varphi_b\tilde{b}_t^h$. If $\varphi_b = 0$ the approximated system would have an exact unit root in individual asset holdings. However, this is not the behavior of the non-linear model - which show a quasi-random walk given the existence of a natural borrowing limit - but is the result of our approximation. In

[28] We recall that in steady state we have offset the monopolistic distortion such that $\bar{W}^r\bar{N} = \bar{A}\frac{\tau_\mu}{\mu}\bar{Y}/\bar{A} = \bar{Y}$. We further normalize the output to one $\bar{Y} = 1$.

our analysis we choose to capture the quasi-random walk behavior by assuming a very small but strictly positive value for φ_b.[29]

1.3.3 A Linear Quadratic Approach - Loss Function

We derive a second order approximation of the policy objective, equation (1.3.1), about the deterministic Ramsey steady state derived above. Details of the derivation can be found in the appendix here we simply claim the result:

$$\mathrm{W}_t \simeq E_t \sum_{k=0}^{\infty} \beta^k L_t \tag{1.3.18}$$

where

$$L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_c \int_0^1 (\tilde{C}_t^h)^2 + o(\|S_{t-1}\|^2) \tag{1.3.19}$$

The approximation error is strictly related to the deviations of our variables from their steady state values and $\|S_{t-1}\|$ represents a bound on the amplitude to exogenous shocks and to the deviations of the time-t state.[30]

The presence of staggered prices brings in gains from minimizing relative price fluctuations. The relative weight between inflation and output gap, is standard in the literature: $\lambda_x \equiv \frac{\kappa}{\theta}$, where κ is the Phillips curve parameter.[31]

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[29]This can be micro-funded by introducing small quadratic adjustment costs on debt transactions. For a related discussion see also Schimd-Grohe Uribe (2003). See also Kim Kim Kollman 2005 on barrier methods to convert an *optimization problem with borrowing constraints as inequalities into a problem with equality constraints* and then solving the converted model using a local approximation

[30]In fact it is not always the case that imposing a bound on the amplitude of exogenous disturbance is enough to guarantee that the system will oscillate about the steady state. An explosive system is clearly a counter example, but even stable systems with important amplification mechanism are likely to spend many periods very far away from the point about which the model is approximated. In our case the only variable that is likely to deviate persistently from the steady state is b_t^h. A second problem is that the approximation is taken around the deterministic steady state. The system is more likely to oscillate about the mean of its stationary distributions (i.e. about the stochastic steady state). Aggregate shocks affect the moments of the household wealth distribution which in turn affects prices and hence quantities. Here, again, we have implicitly disregarded this contribution to the oscillation of aggregate and individual variables

[31]In this case is given by $\kappa \equiv \frac{(\sigma+\varphi)(1-\psi)(1-\beta\psi)}{(1+\varphi\theta)\psi}$

However in our case an additional term is affecting the country welfare: the cross-sectional consumption dispersion. It enters with the following relative weight:

$$\lambda_c \equiv \frac{(1-\psi)(1-\beta\psi)}{(1+\varphi\theta)\psi}\frac{\sigma}{\theta}(1-\bar{\tau}^G+\sigma/\varphi) \tag{1.3.20}$$

The crucial parameters for understanding the conflicts between price stability and debt dispersion are λ_x and the relative risk aversion σ. The higher the distortion generated by nominal rigidities (i.e. the higher the stickiness ψ or the CES elasticity θ) the lower will be λ_x. On the other hand a higher curvature of the households' utility function would clearly imply - for any given consumption dispersion - a relatively higher social benefit from consumption equality.

The steady state level of government consumption lowers the weight simply because it reduces the steady state level of private consumption. However any $\bar{\tau}_t^G < 1$ gives still a strictly positive weight. The term σ/φ at first sight could be misleading. It is true that the higher the labor elasticity the higher the weight but at the same time the lower will be the consumption dispersion (see also the definition of κ_c in equation (1.3.16)).[32]

From the reformulated household budget constraint (equation (1.3.16)) we see that our new target $var_h(\tilde{C}_t^h)$ is mainly associated with the source of idiosyncratic uncertainty: the debt servicing (interest income) $\bar{b}^h(\beta\hat{R}_t - \pi_t)$. As we will see in the next section a central bank has enough instruments to soften the impact of aggregate shocks on the households' balance sheet stemming from

[32] I will clarify it with an example. In the extreme case $\varphi = 0$ (labor supply perfectly elastic) $\sigma/\varphi \to \infty$. However it is also true, see equation (1.2.1), that $C_t^h = C_t \ \forall h \in [0,1]$ such that consumption dispersion is zero and the loss function is no longer well defined. For such a case it would be useful to rewrite the loss in term of labor dispersion. After some algebra we can find a relation between the consumptions dispersion and the measure of hours worked dispersion: $\int_0^1 \tilde{C}_t^{h^2} \simeq 2\frac{\varphi}{\sigma}\hat{\Delta}_{n,t}$. If we substitute it in the loss we can define the weight on $\hat{\Delta}_{n,t}$ as$\lambda_\Delta = 2\lambda_x(1-\frac{\sigma\tau^G}{\sigma+\varphi})$, if there is no government consumption this boils down to $\lambda_\Delta = 2\lambda_x$

debt servicing volatility.

1.3.4 Calibration

As common in the business cycle literature we let the relative risk aversion and the inverse of the Frisch elasticity parameters take values in the following range: $\sigma \in [1, 5]$ and $\varphi \in [0, 3]$.

The time is meant to be a quarter and we assign a value of 0.9902 to the subjective discount factor $\beta = .99$, which is consistent with an annual real rate of interest of 4 percent (see Prescott 1986)

We set the steady state share of government purchases $\bar{\tau}^G = 20\%$ matching the US historical experience in postwar period. Following Sbordone (2002) and Gali and Gertler (1999), we assign a value of 2/3 to ψ, the fraction of firms that cannot change their price in any given quarter. This value implies that on average firms change prices every 3 quarters. The price elasticity of the demand θ is set to 11 such that the steady state markup is 10%.

For the driving processes I follow Schmitt-Grohe Uribe 2005 and I set the persistence parameters $\rho_a = .86$ and $\rho_g = .87$, for the productivity and government spending respectively. While the standard deviations of the correspondent innovations are $\sigma_a = .0064$ and $\sigma_a = .0160$. The two processes are assumed to be uncorrelated.

We calibrate the debt dispersion parameter $\zeta_b^2 \equiv \int_0^1 \bar{b}_{-1}^{h^2} dh$ [33] using micro data from the Federal Reserve Board's Survey of Consumers Finances (SCF) for the year 2001. We calculate the net debt position for each household in the survey. We calculate a gross credit position by summing up the following variables: saving accounts, money market account, investment in money market

[33] Also defined by the distribution $\zeta_b^2 = \int_{-\infty}^{\infty} u^2 d\Phi_{-1}(u)$

Parameter	Value	Description
β	.9902	Subjective discount factor (quarterly)
σ	2	Relative risk aversion
φ	.1	Frisch elasticity
θ	11	Price-elasticity of demand for a specific good variety
μ	.10	Firms markup
ψ	.75	Fraction of non-resetter firms
$\bar{\tau}^G$	.25	Steady state value of government consumption over GDP
ζ_b	2.96	Fixed-Income asset dispersion
ρ_A	.86	Serial correlation of (log) of technology process
ρ_G	.87	Serial correlation of (log) of government spending process
σ_A	.0064	Std. dev. innovation to (log) of technology
σ_G	.0160	Std. dev. innovation to (log) of government consumption

Table 1.1: Structural Parameters

funds, CDs, total bonds.[34] On the other hand we proxy a debit position by summing up: mortgage debt, other lines of credit, residential debt, checking account debt, installation loans and other debt. The net debt is given by the algebraic difference between the credit and debit gross positions.

Because in our model in steady state everybody earns the same wage and financial income we divide the net-debit position by the total household income, then we calculate the variance of the sample. The value we find for the year 2001, calibrated for our quarterly model, is $\zeta_b = 2.96$.[35] In table 1.1 we give a summary of the all parameters just described.

[34]By that we mean: US saving bonds, Federal government bonds other than U.S. saving bonds, bonds issued by state and local governments, corporate bonds, mortgage-backed bonds and other types of bonds.

[35]We calibrate the model normalizing total steady state output to unity.

1.4 Optimal Monetary Policy

In this section we examine, in a formal manner, the design of optimal monetary policy when debt servicing stabilization (i.e. consumption dispersion) is a policy goal.

We first show the relations that must hold among aggregate variables. Once we have linearized the households' Euler equations aggregation is straightforward and delivers the same aggregate system usually found in the sticky price literature.

Using households' Euler equations and the output gap definition:

$$\sigma E_t \Delta x_{t+1} = \hat{R}_t - E_t \pi_{t+1} - r_t^e \tag{1.4.1}$$

where we have defined the *efficient interest rate* r_t^e as the real interest rate prevailing in the flexible price model without wealth dispersion.

From firms' optimal condition and exploiting the marginal cost relation with the output gap, we also can state the New Philips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{1.4.2}$$

where $\kappa \equiv (\sigma + \varphi)(1 - \psi)(1 - \psi\beta)/\psi$.

In order to close the system we only need to know the monetary policy behavior. In the next paragraphs we will find the optimal interest rate rule that a benevolent central bank would be willing to implement.

1.4.1 Flexible-Price Environment

In order to have a better understanding of the channel through which the redistributive effect of monetary policy has an impact on social welfare, we first analyze the case of fully flexible prices - for which we have an analytical solution.

If prices are perfectly flexible the Phillips curve (1.4.2) is no longer well defined and simply tells us that marginal costs are constant.[36] We recall the output gap in the flexible price case: $x_t = \hat{Y}_t^F - \hat{Y}_t^N = \frac{\sigma}{\sigma+\varphi}\hat{\Delta}_{n,t}$.[37] In first order approximation this is driven only by exogenous aggregate shocks.[38] Hence we can rewrite the IS relation previously found (1.4.1) replacing the output gaps with the natural interest rate r_t^e [39]

$$\hat{R}_t - E_t\pi_{t+1} = r_t^e \tag{1.4.3}$$

Inflation creates no distortion, in fact we have $\Delta_{p,t} = 0$ at all times and x_t^2 is of order higher than the second - it drops out from the loss function. Hence the period by period loss function simplifies to

$$L_t = \lambda_c \int_0^1 \tilde{C}_t^{h^2} \tag{1.4.4}$$

Monetary authority seeks the state-contingent path for inflation $\{\pi_t\}_{t=0}^{\infty}$ that minimizes expected discounted sum of losses conditioning upon the initial state of the world in period-0 and subject to the constraint that this evolution represents a possible rational-expectations equilibrium.

The monetary authority faces the following problem:

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \tilde{C}_t^{h^2}$$

$$s.t.\ \ (1.3.16), (1.3.17), (1.4.3),$$

$$(\tilde{b}_{t-1}^h)_{h\in[0,1]} \ \text{ given}$$

[36]This is the limiting case of our model in which $\psi = 0$ and $1/\kappa = 0$.

[37]This is always true in log-deviations from the steady state, not only when $\hat{\Delta}_n = 0$.

[38]We have that $\frac{\sigma}{\sigma+\varphi}\hat{\Delta}_{n,t} = \frac{\varphi}{\sigma+\varphi}\int_0^1 \tilde{N}_t^{h^2}$

[39]This is a linear function of the exogenous aggregate stochastic processes $\hat{A}_t$ and $\hat{g}_t$ as standard in the literature. See Gali 2001

Together with the above constraints the optimality conditions take the form (see appendix for further details):

$$\tilde{C}^h: \quad \lambda_c \tilde{C}_t^h = \kappa_c \lambda_{1,t}^h - \lambda_{2,t}^h + \beta^{-1}\lambda_{2,t-1}^h \ \ \forall h \in [0,1] \tag{1.4.5}$$

$$\tilde{b}^h: \quad \beta(E_t \lambda_{1,t+1}^h - \lambda_{1,t}^h) = \varphi_b \lambda_{2,t}^h \qquad \forall h \in [0,1] \tag{1.4.6}$$

$$\pi: \quad \int_0^1 \bar{b}^h \lambda_{1,t}^h = \lambda_{3,t-1}/\beta \tag{1.4.7}$$

$$\hat{R}: \quad \beta \int_0^1 \bar{b}^h \lambda_{1,t}^h = \lambda_{3,t} \tag{1.4.8}$$

We have a collection of lagrange multipliers associated to the previous constraints $\mathcal{L} = \{\mathcal{L}_t\}_{t\geq 0}$ where $\mathcal{L}_t = ((\lambda_{1,t}^h)_{h\in[0,1]}, (\lambda_{2,t}^h)_{h\in[0,1]}, \lambda_{3,t})$ and new set of initial conditions $\mathcal{L}_{-1}$. The above system involves a continuum of equations difficult to handle directly. In order to find the optimal state-contingent path for inflation we introduce new aggregate variables. Recalling that $\bar{b}^h$ is the steady state household-h bond position, we define the covariances (or dispersions) among some key variables:

- the multipliers-debt covariance

$$\lambda_{i,t} \equiv \int_0^1 \bar{b}^h \lambda_{i,t}^h dh \tag{1.4.9}$$

- the consumption-debt covariance

$$w_t \equiv \int_0^1 \bar{b}^h \tilde{C}_t^h dh \tag{1.4.10}$$

- and the debt dispersion[40]

$$z_t \equiv \int_0^1 \bar{b}^h \tilde{b}_t^h dh \tag{1.4.11}$$

[40] Notice that, from our definitions, we can also write $z_t/\zeta_b^2 = \int_0^1 \bar{b}^h b_t^h / \int_0^1 \bar{b}^{h^2} - 1$. In a simple 2-agents economy example say agent-1 has an initial debt of 10£, $\bar{b}^1 = -10$£, in this case $\zeta_b^2 = 50£^2$. If in the next periods he increases the debt of 10£ up to -11£ then we have $z/\zeta_b^2 = 1.1 - 1 = 10\% > 0$

We will make use of the already defined steady state debt-dispersion parameter $\zeta_b^2 \equiv \int_0^1 \bar{b}^{h^2}$.

First of all we notice that if the steady state household distribution over debt is uncorrelated with the household joint distribution over consumption and debt then we have $\lambda_{i,t} = z_t = w_t = 0$. In this case, because of a clear lack of instruments, monetary policy has nothing to say about how to reduce wealth dispersion even if its implied distortion $\Delta_{n,t}$ is not necessarily zero.

However this does not mean that monetary policy has no welfare impact. Even in a flexible price environment, the way the central bank reacts to inflation can be welfare reducing. We now state the following proposition (a formal proof is given in the appendix):

Proposition 1.4.1. *In a flexible price environment, with the only distortion created by wealth dispersion, optimal monetary policy is given by a state-contingent path for inflation*

$$\pi_t = \beta E_t \sum_{j=0}^{\infty} \beta^j r^e_{t+j} + \frac{z_{t-1}}{\zeta_b^2}, \ \ \forall\, t \geq 0 \qquad (1.4.12)$$

which implies a targeting rule [41]

$$\hat{R}_t = \frac{\pi_t}{\beta} + \frac{z_t - z_{t-1}/\beta}{\zeta_b^2}, \ \ \forall\, t \geq 0 \qquad (1.4.13)$$

The optimal interest rate reaction is a function of inflation, π_t, and the debt dispersion, z_t. The coefficient on inflation, being of order 1.01, satisfies the Taylor principle but is much smaller than the standard Taylor prescription. The reason is that a stronger reaction to inflation (deflation) would entail a higher (lower) real cost (revenue) for debt servicing with respect to some long run average that hurts households who have accumulated a big stock of debt (credit).

[41] *For a definition of targeting rules see Svensson 2003, Giannoni-Woodford, or Woodford Ch7*

The second term in the central bank rule reflects a measure of aggregate households financial imbalances. Whenever z_t is different from zero the central bank has something to do about the distortion created by the existence of a non-degenerate distribution of households over nominal asset holdings. A positive value for z_{t-1} says that there is a positive correlation between those households who are worst off (higher marginal utility than the average) and households who are over-accumulating debt (i.e. that have accumulated a stock of debt higher than their long run average). In this case, ceteris paribus, the central bank should have a looser monetary policy and restore the second best optimum (i.e. $w_t = z_t = 0$).[42]

We can see that, in absence of any nominal distortion, the central bank, under the optimal policy, can always achieve such a goal. Rewriting the households budget constraint (1.3.16) in term of covariances and using the optimal policy we have that

$$\kappa_c w_t = -\beta z_t + z_{t-1} + \zeta_b^2(\beta \hat{R}_t - \pi_t) = 0, \ \forall t \geq 0 \tag{1.4.14}$$

which in turn implies $z_t = 0 \ \forall t \geq 0$. In other words, in equilibrium, for any given initial debt dispersion z_{-1}, the optimal monetary policy rule reads $\hat{R}_t = \pi_t/\beta$ $\forall t \geq 1$ delivering what we can call a second best allocation.

This result tells us also, as corollary, that monetary policy has nothing to exploit from time-0 absence of commitment, and timeless perspective and standard Ramsey deliver the same problem. In fact, as shown in the appendix, if consumption-debt covariance w_t and debt dispersion z_t are zero for all $t \geq 0$

[42] Recalling the definition of z_t the steady state dispersion ζ_b^2 in the policy rule can be interpreted as a scaling parameter. An important check for the accuracy of our approximation can be indeed found in the ratio z_t/ζ_b^2. Whenever this ratio is higher than 1 the steady state debt dispersion is lower than the current deviation-from-steady-state debt dispersion; which means that the model is surely drifting away

then it must be the case that

$$\lambda_{1,t} = \lambda_{1,t-1} = \lambda_{1,-1} = 0 \tag{1.4.15}$$

which entails $\lambda_{2,-1} = \lambda_{3,-1} = 0$. However, this does not imply that for each multiplier-h we have $\lambda^h_{i,-1} = 0$[43] but it only says that there is no need of any inflation surprise at time zero given that the central bank is unable to target each individual household.

This means that a central bank is reacting to aggregate imbalances which are favoring either the group creditors or the group of debtors. What monetary can do for social welfare is realizing if, because of changes in the real interest rate, some groups in the economy are more affected than others.

1.4.2 Sticky-Price Environment

We now characterize the optimal responses to shocks in the case that prices are sticky ($\psi > 0$).

The central bank problem is to choose processes $\{\pi_t, \hat{R}_t, (\tilde{C}^h_t)_{h\in[0,1]}, (\tilde{b}^h_t)_{h\in[0,1]}\}_{t\geq 0}$ to minimize (1.3.18) subject to the constraint (1.3.16), (1.3.17), (1.4.1), (1.4.2) for every $t \geq 0$, [44] given initial conditions $(\tilde{b}^h_{-1})_{h\in[0,1]}$ and the evolution of the exogenous shocks $\{A_t, \hat{g}_t\}_{t\geq 0}$.

Hence we have:

[43] in general it will not be the case if $b^h_{-1} \neq \bar{b}^h$ for some $h \in [0,1]$

[44] Together also with initial constraints of the form $\pi_0 = \bar{\pi}_0$ $x_0 = \bar{x}_0$ and $(\tilde{C}^h_0 = \bar{C}^h_0)_{h\in[0,1]}$ which ensure the commitment from a timeless perspective.

$$\begin{aligned}
\min \quad & E_0 \sum_{t=0}^{\infty} \beta^t \Big(\pi_t^2 + \lambda_x x_t^2 + \lambda_c \int_0^1 \tilde{C}_t^{h^2} \Big) \\
s.t. \quad & \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \\
& \sigma E_t \Delta x_{t+1} = \hat{R}_t - E_t \pi_{t+1} - r_t^e \\
& \kappa_c \tilde{C}_t^h = -\beta \tilde{b}_t^h + \tilde{b}_{t-1}^h + \bar{b}^h (\beta \hat{R}_t - \pi_t), \ \ \forall h \in [0,1] \\
& E_t \Delta \tilde{C}_{t+1}^h = -\varphi_b \tilde{b}_t^h, \ \ \forall h \in [0,1] \\
& (\tilde{b}_{-1}^h)_{h \in [0,1]} \ \ \text{given}
\end{aligned} \tag{1.4.16}$$

Necessary conditions read (lagrange multipliers associated to the constraints are μ_1, μ_2 λ_1^h λ_2^h):[45]

$$\lambda_x x_t + \kappa \mu_{1,t} + \sigma \mu_{2,t} - \sigma \beta^{-1} \mu_{2,t-1} = 0 \tag{1.4.17}$$

$$\pi_t + \mu_{1,t-1} - \mu_{1,t} - \beta^{-1} \mu_{2,t-1} - \lambda_{1,t} = 0 \tag{1.4.18}$$

$$\mu_{2,t} + \beta \lambda_{1,t} = 0 \tag{1.4.19}$$

$$\lambda_c \tilde{C}_t^h = \kappa_c \lambda_{1,t}^h - \lambda_{2,t}^h + \beta^{-1} \lambda_{2,t-1}^h \tag{1.4.20}$$

$$\beta (E_t \lambda_{1,t+1}^h - \lambda_{1,t}^h) = \tilde{\varphi}_b \lambda_{2,t}^h \tag{1.4.21}$$

Where again we use previous section definitions. The final system is:

[45] see appendix for details on necessary conditions

$$\lambda_x x_t + \kappa\mu_{1,t} + \sigma\mu_{2,t} - \sigma\beta^{-1}\mu_{2,t-1} = 0 \tag{1.4.22}$$

$$\pi_t + \mu_{1,t-1} - \mu_{1,t} - \beta^{-1}\mu_{2,t-1} - \lambda_{1,t} = 0 \tag{1.4.23}$$

$$\mu_{2,t} + \beta\lambda_{1,t} = 0 \tag{1.4.24}$$

$$\lambda_c w_t = \kappa_c\lambda_{1,t} - \lambda_{2,t} + \beta^{-1}\lambda_{2,t-1} \tag{1.4.25}$$

$$\beta(E_t\lambda_{1,t+1} - \lambda_{1,t}) = \tilde{\varphi}_b\lambda_{2,t} \tag{1.4.26}$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t \tag{1.4.27}$$

$$\sigma E_t\Delta x_{t+1} = R_t - E_t\pi_{t+1} - r_t^e \tag{1.4.28}$$

$$\kappa_c w_t = -\beta z_t + z_{t-1} + \zeta_b^2(\beta\hat{R}_t - \pi_t) \tag{1.4.29}$$

$$E_t w_{t+1} = w_t - \tilde{\varphi}_b z_t \tag{1.4.30}$$

We now analyze the optimal response of inflation and output gap to a transitory disturbance to the level of productivity and government spending and to what we have called a financial shock.

We perform the exercise for different values of the debt dispersion parameter. We show the results for our calibrated value, $\zeta_b = 2.96$, but also for a lower and higher value, respectively 2 and 3.87 (see fig.1).

I analyze the case when a (transitory) productivity and/or government spending shocks give rise to an increase in the *efficient* rate of interest, r_t^e. In this case, *ceteris paribus*, households who are net creditors in the economy would enjoy higher returns on their bond holdings. This can be seen also from the evolution of the consumption-debt covariance w_t which is positive: higher returns on assets allow those households to have, in average, a higher consumption than the aggregate per capita consumption. At the same time indebted households must decrease their consumption and, to service their debt, they are accumulating an even higher stock of debt $z_t < 0$.

Hence monetary policy faces a clear trade-off between dwindling the redistributive impact of an increase in the natural rate and shutting off nominal distortion pursuing price stability. In the baseline model without asset dispersion the stabilization policy would be straightforward: tracking the natural rate and closing all the gaps. However, as we can see in fig1 - the higher the debt dispersion ζ_b the bigger the deviation from price stability. At the time of the impact of the shock the nominal interest rate does not move together with the natural rate (what we would have found in the baseline model). On the contrary the reaction is much smaller and for $\zeta_b = 3.87$ we even have an inversion of sign: the nominal rate decreases at the time of the shock. This results depends on the persistence of the exogenous shock: in order to reduce its idiosyncratic wealth effect on household the optimal monetary policy erodes part of the stock of debt; as a consequence it reduces the change in interest payments.

Repeating the analysis for higher values of ζ_b does not alter the main conclusion. However, from figure 3, we can see that the same shock now implies a stronger deviation from price stability: the higher the economy is indebted the higher will be the deviation from zero inflation.[46].

Summarizing the results for the two calibrated exercises, the deviation from price stability is still relatively small. Deviations are of the order of 0.2% for a 1.0% change in the efficient rate.

As the steady state debt dispersion increases, the deviation of the nominal interest rate from the natural rate increases further. In our calibrated example, for a $\zeta_b = 10.36$, a negative productivity shock (for example) which gives the

[46] A crucial preference parameter for the persistence of the shocks is given by the intertemporal rate of substitution $1/\sigma$. The lower it is the higher the persistence. Our results are for $\sigma = 2$, for $\sigma = 10$ the persistence of deviation from price stability has is much more hump-shaped and long lasting.

efficient rate a deviation of about .4%, now implies an inflation rate stimulus of about .08% (5 times more than before).

We want to find a simple rule for our model that approximate the global optimal monetary policy. Hence we simulate our model for 9000 periods (we discard the first 1000 observations) and we estimate the following simple rule (ERS):

$$\hat{R}_t = \eta_r \hat{R}_{t-1} + \eta_p \pi_t + \eta_{z_0} z_t + \eta_{z_1} z_{t-1} \tag{1.4.31}$$

We find that under our calibration [47] the weights on current and past debt-dispersion, η_{z_0} and η_{z_1} respectively, are significantly non-zero (see Table 1.2). Recalling the flexible price solution we see that, also for the sticky price model, an approximated optimal policy has a negative reaction on the beginning of period debt dispersion. The rule is also showing *superinertia*: the coefficient on lagged interest rate is in fact greater than one. The coefficient on inflation is still very high meaning that the welfare loss stemming from price dispersion is still dominating. However, the higher the steady state parameter ζ_b the lower the weight given to inflation. In figure 4 I perform the same exercise of figure 1 but for using the ERS (rule) instead of the optimal one. The responses are quite similar to the optimal, however is worth noting how the inflation deviations, relatively small, are quite persistent.

Finally, having derived the central bank loss function we can easily compare different policy rules in terms of their welfare score. I rank alternative rules on the basis of the *unconditional* expected welfare. To compute it, I simulate 1200 paths for the endogenous variables over 600 quarters and then compute the average loss per period across all simulations. For the initial distribution of the

[47] For this exercise we set $\sigma = 5$. Because under the baseline calibration we still have quantitatively small deviations from price stability, the estimates for η_p are not very precise.

state variables I run the simulation for 200 quarters prior to the evaluation of welfare. In order to calculate the consumption dispersion I use 100 households: I draw from the US CFS 100 net-bond positions which stand for $\bar{b}^h$. Table 1.3 gives a definition of the rule used and table 1.4 ranks all rules according to their welfare score.

Under the baseline calibration the optimal rule (GOMP) and the estimated simple rule (ERS) gives almost the same result: the percentage loss - expressed in steady state consumption - of the estimated rule with respect to the optimal is only 0.065%. This means that a simple rule can still be a very good approximation of the optimal one.

In the baseline framework without debt dispersion targeting zero inflation (IT) would be optimal. However, once debt dispersion is introduce, the IT rule results in a 7.6% higher loss than the GOMP. The sub-optimality of the IT rule can also be seen from figure 5 in which we compare it with the ESR (which is, as we said, a good approximation of the GOMP rule). As we can see, inflation and output gap are stabilized at the cost a much larger variation in w_t and z_t which in turn represent the $var_h(\tilde{C}_t)$ and so the welfare loss from consumption dispersion.

To see the importance of the debt-covariance, z_t and z_{t-1}, as a separate target we also estimate a simple rule only with inflation and interest rate as targets (ERSbis). As we can see from table 1.4 it delivers a welfare loss very close to the pure IT rule and, again, it turns out in almost an 8% loss with respect to ERS.

Finally we also analyze a quite standard Taylor rule (TR) and a Taylor rule augmented with the debt covariance z_t and z_{t-1} as targets. We observe a huge loss of order 10 times higher than the GOMP rule. This is not surprising given that it comes mainly from inflation losses, however we can see how the ATR outperforms the standard TR by more than 9 times. Moreover, targeting z_t and z_{t-1} not only reduces the losses stemming from consumption dispersion but reduces also the output gap and inflation volatility.

1.5 Conclusion

Most of the results in the recent monetary policy literature have been derived under the assumption of a representative household.[48] The present paper relaxes that assumption. We have introduce an effect on households' balance sheet stemming from variations in servicing (in the returns from) the stock of net-debt (credit) - variations that, in turn, are related to economy-wide aggregate disturbances. Those variations imply an *arbitrary* redistributive pattern in the economy and thus, potentially, a greater dispersion of consumption and hours worked.

To determine what a central bank could do, we first introduce a transfers scheme. This leaves aside problems related to "long run" wealth inequality.[49]

The first result is that even in a flexible-price environment monetary policy has real effects through its ability to affect interest payment (income) volatility.

The second important result is found when price stickiness is introduced. In the baseline sticky price model this entail that price stability is the prominent policy goal. The direct corollary is that the nominal rate should track closely the *natural* rate.[50] However this is in clear contrast with the objective of stabilizing interest rate income (repayments) which would imply the inflation rate (and not the interest) to track the *natural* real rate.

[48] Or assuming complete markets which, after all, is the same thing.

[49] For which a central bank does not have enough policy instruments, especially if does not want to exploit surprise inflation.

[50] which, being exogenous, could be very high volatile.

Quantitatively, in our calibrated exercise, we find that the magnitude of deviation from zero inflation is still relatively small.

However, patterns of household debt dispersion should be monitored by any monetary authority who is willing to keep the price stability goal as credibly central.

Finally, designing a simple implementable rule, we find that *superinertial* rules, that incorporates also a measure of debt dispersion as separate target, outperforms standard Taylor rules.

Appendix

A Some Results

Some results used.

In a second order approximation, for any variable $x \in \mathbb{R}_+$ and $\bar{x} \in \mathbb{R}_+$

$$\frac{x-\bar{x}}{\bar{x}} \simeq \hat{x} + .5\hat{x}^2 \tag{A-1}$$

$$(\frac{x-\bar{x}}{\bar{x}})^2 \simeq \hat{x}^2 \tag{A-2}$$

Where $\hat{x} = \log(x/\bar{x})$.

Given a function of the following kind $f(x,y) = xg(y)$, with $y \in \mathbb{R}_+$, $g(.)$ twice differentiable and $\bar{x} = 0$, we have that

$$f_y(\bar{x}, y) = \bar{x}g'(y) = 0,$$

$$f_{yy}(\bar{x}, y) = \bar{x}g''(y) = 0,$$

This means that if take the 2-order approximation of f about $(\bar{x}, \bar{y})$ $\forall \bar{y}$ we find that

$$f(x,y) \simeq g(\bar{y})x + g'(\bar{y})x(y-\bar{y}) = g(\bar{y})x + \bar{y}g'(\bar{y})x\hat{y} \tag{A-3}$$

In order to calculate the effect of price and output dispersion on overall output we use the following result for a household or firms variable, say $x(h)$, in deviation from its average value, $x \equiv \int_0^1 x(h); dh$:

$$\int_0^1 \log(x_t(h)/x_t) \simeq -0.5 \int_0^1 \Big(\frac{x_t(h) - x_t}{x_t}\Big)^2 \tag{A-4}$$

This also means that

$$\int_0^1 \hat{x}_t(h) - \hat{x}_t \simeq -0.5 \int_0^1 (\hat{x}_t(h) - \hat{x}_t)^2 \tag{A-5}$$

We note that the first order effect is zero.

In relation with the previous result, if we let $x_t(h) = X_t(h)/X_t$ and we have $\bar{x} = 1$ and $\int_0^1 x_t(h) = 1$ then $\Delta_{x,t} = \int_0^1 x_t^{\alpha}(h)$ can be approximated as

$$\hat{\Delta}_{x,t} = \log \Delta_{x,t} \simeq -0.5\alpha \int_0^1 \hat{x}_t^2(h) \tag{A-6}$$

B Output Gap

We have defined the efficient rate of output Y^e as the one prevailing with complete markets equal initial wealth distribution and flexible prices. In this case it is easy to show that

$$\hat{Y}_t^e = \frac{\sigma}{\sigma + \varphi}\hat{g}_t + \frac{1+\varphi}{\sigma + \varphi}\hat{A}_t + \frac{\sigma}{\sigma + \varphi}\hat{\Delta}_n \tag{A-7}$$

The introduction of nominal rigidities does not alter any fundamental relation but for the markup determination. So we still have that $\hat{mc}_t = \hat{W}_t^r - \hat{A}_t$, from the consumption/leisure choice $\hat{W}_t^r = \sigma\hat{C}_t + \varphi\hat{N}_t$, from the resource constraint $\hat{Y}_t - \hat{g}_t = \hat{C}_t$. However it does alter output aggregation of the production functions $Y_t = A_t N_t/\Delta_{p,t}$ such that consumption (in *logs*) is given by

$$\hat{C}_t = \hat{A}_t + \hat{N}_t + \hat{g}_t - \hat{\Delta}_{p,t} \tag{A-8}$$

So we can write

$$\hat{mc}_t = (\sigma + \varphi)x_t + \varphi\hat{\Delta}_{p,t} \tag{A-9}$$

For our market structure we cannot exploit the aggregate consumption/leisure relation directly. However, even in the sticky price case, the aggregate consumption-labor relation found in section (1.2) must hold:

$$\hat{W}^r_t = \varphi\hat{N}_t + \sigma\hat{C}_t - \sigma\hat{\Delta}_{n,t} \tag{A-10}$$

Moreover it is always true that $\hat{W}^r_t = \hat{mc}_t + \hat{A}_t$ and that aggregate consumption is related to output as above in equation (A-8). Combining those two relations with (A-10) and using the output gap definition $x_t \equiv \hat{Y}_t - \hat{Y}^e_t$ we get

$$\hat{mc}_t = (\sigma + \varphi)x_t + \varphi\hat{\Delta}_{p,t} - \sigma(\hat{\Delta}_{n,t} - \hat{\Delta}_n) \tag{A-11}$$

as in equation (1.2.13) of the text.

C The Optimal Deterministic Steady State

Here we show the existence of an optimal steady state, i.e., of a solution to the recursive policy problem defined in section (2.5), that involves (under appropriate initial conditions) constant values for all variables, in the case of no stochastic disturbances: $A_t \equiv \bar{A}$ and (without loss of generality) $G_t \equiv \bar{G} = 0$.

To prove the result we split the problem in two stages. In the first stage the government sets and commits to a redistributive policy Φ^τ taking as given inflation, price dispersion and total production - i.e. $R_t = R^\star$, $\Pi_t = \Pi^\star$, $Y_t =$

$Y^\star \; w_t = w^\star$. Using the consumption-leisure condition we can write $N_t^h = v_n^{-1}(w^\star/u_c(C_t^h))$. We accordingly redefine the momentary utility

$$u(C_t^h) - v(N_t^h) = \tilde{u}(C_t^h) \tag{A-12}$$

and the wage income

$$w_t N_t^h = g(C_t^h) \tag{A-13}$$

We can now formulate the deterministic version of the Ramsey problem for a given (and at the moment arbitrarily) initial distribution of households over debt Φ_{-1}

$$\max \textstyle\sum_{t=0}^{\infty} \beta^t \int_0^1 \tilde{u}(C_t^h) dh \tag{A-14}$$

$s.t.$

$$\begin{gathered} C_t^h + b_t^h/R^\star = b_{t-1}^h/\Pi^\star + \bar{\tau}^h + g(C_t^h) \; \forall h \in [0,1] \\ \int_0^1 \bar{\tau}^h dh = 0 \\ \int_0^1 C_t^h dh = Y^\star \end{gathered} \tag{A-15}$$

We denote the associate set of lagrange multiplier $\{(\varphi_t^h)_{h\in[0,1]}, \varphi_{1,t}, \varphi_{2,t}\}$. The FOC for optimal consumption allocation reads

$$\tilde{u}_c(C_t^h) = \varphi_t^h g'(C_t^h) + \varphi_{2,t} \; \forall h \in [0,1] \tag{A-16}$$

On the other hand we have the relation $\varphi_t^h = \varphi_{1,t}$. Putting together the two equations we realize that individual consumption must be equalized

$$C_t^h = \bar{C}_t \; \forall h \in [0,1] \tag{A-17}$$

The intuition is straightforward, for a given amount of available resources and (strictly) concave utilities the previous solution is a necessary and sufficient conditions which tells us that a social planner will (strictly) prefer to equate marginal utilities of consumption across agents.

The induced transfer system - denoted $\Phi^{\tau^\star}(\bar{\tau})$ - can be recovered from the intertemporal households budget constraint and will be proportional to the initial debt dispersion $\Phi^{\tau^\star}(\bar{\tau}) \propto \Phi_{-1}$ with the constant of proportionality function of $R^\star$ and $\Pi^\star$. In fact for each household we have:

$$\bar{\tau}^h = \bar{b}^h_{-1}(1/\Pi^\star - 1/R^\star)\ \forall h \in [0,1] \tag{A-18}$$

In the second stage we take $\Phi^{\tau^\star}(\bar{\tau})$ as given and we wish to find an initial degree of price dispersion Δ_{-1} such that the recursive problem involves a constant policy.

However, under the optimal transfer scheme we have shown that households consumes and work the same, this means that our second stage boils down to the same problem solved in Benigno-Woodford 2005 which show that zero price dispersion (i.e. zero inflation) is the optimal long-run monetary policy. Given no price dispersion $\bar{\Delta}_p = 0$ we have

$$1 = \bar{p}(z) = \mu \bar{mc} = \mu \bar{w}/\bar{A} \tag{A-19}$$

Because the employment is subsidized at a rate τ_μ which exactly offset the monopolistic distortion we have

$$\bar{W}^r = \frac{\bar{A}}{\mu \tau_\mu} = \bar{A} \tag{A-20}$$

So output is at its efficient level:

$$\bar{Y} = \bar{A}^{\frac{1+\varphi}{\sigma+\varphi}}(1-\bar{\tau}^{g})^{-\frac{\sigma}{\sigma+\varphi}} \tag{A-21}$$

D Loss Function

We recall that the resource constraint implies at all time that $C_t = Y_t - G_t = Y_t(1-\tau_t^G)$. We start from the utility coming from consumption (we recall that we name $\widehat{C_t^h/C_t} \equiv \tilde{C}_t^h$)

$$u(C_t^h) = u(\frac{C_t^h}{C_t}(Y_t - G_t)) \simeq \tag{A-22}$$

$$\simeq \bar{u} + u_c(\bar{Y}-\bar{G})(\tilde{C}_t^h + .5\tilde{C}_t^{h^2}) + u_c\bar{Y}(\hat{Y}_t + .5\hat{Y}_t^2) + .5u_{cc}\bar{Y}^2[(1-\bar{\tau})^2\tilde{C}_t^{h^2} + \hat{Y}_t^2]$$

$$+(u_c\bar{Y} + u_{cc}\bar{Y}(\bar{Y}-\bar{G}))\tilde{C}_t^h\hat{Y}_t - u_{cc}\bar{Y}\bar{G}\hat{Y}_t\hat{G}_t - [u_c + (\bar{Y}-\bar{G})u_{cc}]\bar{G}\hat{G}_t\tilde{C}_t^h + t.i.p$$

Rearranging and integrating with respect households and using the fact that $\int_0^1 \tilde{C}_t^h = -.5\int_0^1 \tilde{C}_t^{h^2} + h.s.o.$ we get:[51]

$$\int_0^1 u(C_t^h) = \tag{A-23}$$

$$= t.i.p. + u_c\bar{Y}\hat{Y}_t - .5u_c\bar{Y}[\sigma(1-\bar{\tau}^G)\int_0^1 \tilde{C}_t^{h^2} - (1-\sigma)\hat{Y}_t^2] + u_c\bar{Y}\sigma\bar{\tau}^G\hat{Y}_t\hat{G}_t =$$

$$= t.i.p. + u_c\bar{Y}[\hat{Y}_t + (1-\sigma)\hat{Y}_t^2 + \sigma\bar{\tau}^G\hat{Y}_t\hat{G}_t - .5\sigma(1-\bar{\tau}^G)\int_0^1 \tilde{C}_t^{h^2}]$$

We turn to the disutility from working. We define $\tilde{N}_t^h \equiv \hat{N}_t^h - \hat{N}_t$ and we will make use of the two following facts: $\int_0^1 \tilde{N}_t^h \simeq -.5\int_0^1 \tilde{N}_t^{h^2}$ and from the labor supply conditions we realize that in a second order approximation it must be that $\tilde{N}_t^{h^2} \simeq \frac{\sigma^2}{\varphi^2}\tilde{C}_t^{h^2}$.

We now turn to the quadratic approximation of the disutility of labor.

[51] In the text we have made use labor supply dispersion $\Delta_{n,t}$. However in the derivation of the loss function we will prefer to work with $\tilde{C}_t^h$. It is nonetheless not difficult to see how $\Delta_{n,t}$ would enter in the loss function derivation: just note that we can write $C_t^h = (Y_t - G_t)(N_t^h/N_t)^{-\varphi/\sigma}/\Delta_{n,t}$

$$\int_0^1 v(N_t^h)dh = \tag{A-24}$$

$$= \int_0^1 v(\frac{N_t^h}{N_t}N_t)dh = \int_0^1 v(\frac{N_t^h}{N_t}\frac{1}{A_t}\int_0^1 y(z)dz)dh \simeq t.i.p +$$

$$+ v_n\frac{\bar{Y}}{\bar{A}}[\int_0^1 \hat{y}_t(z)dz + .5(1+\varphi)\int_0^1 \hat{y}_t^2(z)dz - (1+\varphi)\hat{A}_t\int_0^1 \hat{y}_t(z)dz + \frac{\sigma^2}{\varphi}\int_0^1 \tilde{C}_t^{h^2}dh] = t.i.p +$$

$$+ v_n\frac{\bar{Y}}{\bar{A}}[E_z\hat{y}_t(z) + .5(1+\varphi)[(E_z\hat{y}_t(z))^2 + V_z\hat{y}_t(z)] - (1+\varphi)\hat{A}_tE_z\hat{y}_t(z) + \frac{\sigma^2}{\varphi}\int_0^1 \tilde{C}_t^{h^2}]$$

Having defined $\varphi \equiv \varphi/\bar{A}$ and used $E_z[\hat{y}_t(z)^2] = (E_z\hat{y}_t(z))^2 + V_h\hat{y}_t(z)$. We will make use of the fact that $\hat{Y}_t = E_z\hat{y}_t(z) + .5(1-1/\theta)V_z\hat{y}_t(z)$ and $(E_z\hat{y}_t(z))^2 = \hat{Y}_t^2$ and also that $\hat{A}_tE_z\hat{y}_t(z) = \hat{A}_t\hat{Y}_t$ (being the other terms of order higher than the second).

We can write:

$$\int_0^1 v(N_t^h)dh \simeq$$

$$t.i.p + v_n\frac{\bar{Y}}{\bar{A}}[\hat{Y}_t + .5(1+\varphi)\hat{Y}_t^2 + .5(1/\theta+\varphi)V_z\hat{y}_t(z) - (1+\varphi)\hat{A}_t\hat{Y}_t + \frac{\sigma^2}{\varphi}\int_0^1 \tilde{C}_t^{h^2}]$$

Using the steady state relation $u_c = v_n/\bar{A}$ we can put together both expressions we have found (up to a multiplicative constant) to define the loss function we were looking for:

$$L_t = \tag{A-25}$$

$$= (\sigma+\varphi)\hat{Y}_t^2 - 2(\sigma+\varphi)\hat{Y}_t\hat{Y}_t^N + (1/\theta+\varphi)V_z\hat{y}_t(z) + \sigma(1-\bar{\tau}^G+\sigma/\varphi)\int_0^1 \tilde{C}_t^{h^2} =$$

$$= (\sigma+\varphi)x_t^2 + (1/\theta+\varphi)V_h\hat{y}_t(h) + \sigma(1-\bar{\tau}^G+\sigma/\varphi)\int_0^1 \tilde{C}_t^{h2}$$

We have made use of the fact that $(\sigma+\varphi)\hat{Y}_t^N = (1+\varphi)\hat{A}_t + \sigma\bar{\tau}^G\hat{G}_t$ and of the output gap definition $x_t \equiv \hat{Y}_t - \hat{Y}_t^N$. Then knowing that $V_z\hat{y}_t(z) = \theta^2V_z\hat{p}_t(z)$ and following Woodford (Ch3) we write

$$L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_c \int_0^1 \tilde{C}_t^{h^2} \tag{A-26}$$

where $\kappa \equiv (1-\psi)(1-\beta\psi)(\sigma+\varphi)/\psi/(1+\varphi\theta)$ is the Phillips Curve parameter, while $\lambda_x \equiv \frac{\kappa}{\theta}$ and $\lambda_c \equiv \frac{(1-\psi)(1-\beta\psi)}{(1+\varphi\theta)\psi}\sigma(1-\bar{\tau}^G+\sigma/\varphi)$.

If we had to use $\eta(h)$ we simply observe that when $\eta(h) = 1/u_c(C^h)$ we have that

$$\eta(h)u_c(C^h) = 1$$

and

$$\eta(h)v_n(N^h)/A = \eta(h)u_c(C^h) = 1$$

Hence all the results would hold (up to a multiplicative constant).

E Optimal Monetary policy. Flex Case

For convenience we restate proposition (1.4.1).

In a flexible price environment with the only distortion created by wealth dispersion globally optimal monetary policy is given by a state-contingent path for inflation

$$\pi_t = \beta E_t \sum_{j=0}^{\infty} \beta^j r_{t+j}^n + \frac{z_{t-1}}{\zeta_b}, \quad \forall\, t \geq 0 \tag{A-27}$$

which implies

$$\hat{R}_t = \frac{\pi_t}{\beta} + \frac{z_t - z_{t-1}/\beta}{\zeta_b}, \quad \forall\, t \geq 0 \tag{A-28}$$

Proof.

We write the Lagrangian for the policy problem

$$\mathcal{LG} = E_0 \textstyle\sum_{t=0}^{\infty} \beta^t \lambda_c \int_0^1 \tilde{C}_t^{h^2} + \int_0^1 \lambda_{1,t}^h \Big(\kappa_c \tilde{C}_t^h - \tilde{b}_{t-1}^h + \beta \tilde{b}_t^h - \bar{b}^h(\beta \hat{R}_t - \pi_t)\Big) +$$
$$+ \textstyle\int_0^1 \lambda_{2,t}^h \Big(\Delta \tilde{C}_{t+1}^h + \varphi_b \tilde{b}_t^h\Big) + \lambda_{3,t}\Big(\hat{R}_t - \pi_{t+1} - r_t^n\Big) + \int_0^1 \lambda_{2,-1}^h \tilde{C}_0^h/\beta - \lambda_{3,-1}\pi_0/\beta$$

Necessary conditions read (we substitute out the interest rate $\hat{R}_t$):

$$\kappa_c \tilde{C}_t^h - \tilde{b}_{t-1}^h + \beta \tilde{b}_t^h - \bar{b}^h(\beta r_t^n + \beta E_t \pi_{t+1} - \pi_t) = 0 \forall h \in [0,1] \quad \text{(A-29)}$$

$$\Delta \tilde{C}_{t+1}^h + \varphi_b \tilde{b}_t^h = 0 \forall h \in [0,1] \quad \text{(A-30)}$$

$$\lambda_c \tilde{C}_t^h = \kappa_c \lambda_{1,t}^h - \lambda_{2,t}^h + \beta^{-1} \lambda_{2,t-1}^h \quad \forall h \in [0,1] \quad \text{(A-31)}$$

$$\beta(E_t \lambda_{1,t+1}^h - \lambda_{1,t}^h) = \varphi_b \lambda_{2,t}^h \qquad \forall h \in [0,1] \quad \text{(A-32)}$$

$$\int_0^1 \bar{b}^h \lambda_{1,t}^h = \lambda_{3,t-1}/\beta \quad \text{(A-33)}$$

$$\beta \int_0^1 \bar{b}^h \lambda_{1,t}^h = \lambda_{3,t} \quad \text{(A-34)}$$

We multiply the first four equations by $\bar{b}^h$ and we integrate up with respect to agent-h (we use the definitions given in the text).

$$\kappa_c w_t - z_{t-1}^h + \beta z_t^h - \bar{b}^h(\beta \hat{R}_t - \pi_t) = 0 \quad \text{(A-35)}$$

$$E_t \Delta w_{t+1} + \varphi_b z_t = 0 \quad \text{(A-36)}$$

$$\lambda_c w_t = \kappa_c \lambda_{1,t} - \lambda_{2,t} + \beta^{-1} \lambda_{2,t-1} \quad \text{(A-37)}$$

$$E_t \lambda_{1,t+1} - \lambda_{1,t} = \varphi_b \lambda_{2,t}/\beta \quad \text{(A-38)}$$

$$\lambda_{1,t} = \lambda_{1,t-1} \quad \text{(A-39)}$$

If our solution is right then $\zeta_b(\beta \hat{R}_t - \pi_t) = \beta z_t - z_{t-1}$. So equation A-35 can be written (for every $t \geq 0$) as $\kappa_c w_t = -\beta z_t + z_{t-1} + \beta z_t - z_{t-1} = 0$. Given that $w_t = 0$ $\forall t \geq 0$ then from equation (A-36) we also have $z_t = 0$ $\forall t \geq 0$.

From equation (A-39) we have that the first multiplier must be constant $\lambda_{1,t} = \lambda_{1,-1}$ and $\lambda_{3,t} = \beta \lambda_{1,-1}$. Using equation (A-38) this means that $\lambda_{2,t} = 0$ $\forall t \geq 0$.

Hence, by the last unused equation (A-37) we must also have that for all $t \geq 1$

$$0 = \lambda_c w_t = \kappa_c \lambda_{1,-1} - \lambda_{2,t} + \lambda_{2,t-1}/\beta = \kappa_c \lambda_{1,-1}$$

which implies $\lambda_{1,-1} = 0$. Now it is straightforward to see that also $\lambda_{2,-1} = 0$. So the system is satisfied and the initial values of the cross-lagrange multiplier consistent with our solution are exactly $\lambda_{i,-1} = 0$. □

For clearness we write the system under optimal policy

$$\hat{R}_t - E_t \pi_{t+1} = r_t^n \quad \text{(A-40)}$$

$$\hat{R}_t = \pi_t/\beta + \frac{z_t - z_{t-1}/\beta}{\zeta_b} \quad \text{(A-41)}$$

$$z_t = 0 \ \forall t \geq 0, \quad z_{-1} \ \text{given} \quad \text{(A-42)}$$

which also can be written as

$$\pi_t = \beta E_t \sum_{j=0}^{\infty} r_{t+j}^n + z_{t-1}/\zeta_b, \quad z_{t-1} \ \text{given} \quad \text{(A-43)}$$

F Discussion on Aggregation

Krusell and Smith (1998) shows that in an economy with incomplete market, idiosyncratic income shocks and an asset (capital) available for partial self-insurance an *approximate* aggregation result holds. In their words "...all aggregate variables - consumption, the capital stock and relative prices - can be almost perfectly described as a function of two simple statistics: the mean of the wealth distribution and the aggregate productivity shock". Moreover, the marginal propensity to save out of current wealth is almost completely independent of the levels of wealth and labor income (even with leisure choice).

Den Haan (1997), in a setup similar to ours, shows that, without tight borrowing constraints, policy functions are almost-linear and the effects of changes of asset distribution on prices are much smaller than the ones implied by aggregate shocks. For example even if the stationary level of the interest is shifted by wealth heterogeneity (as also shown in Hugget 1993 in relation with the low-riskfree puzzle), the percentage changes during business cycle fluctuations are mainly driven by aggregate shocks.

The previous results suggested my conjecture that variations in the cross-sectional distribution of assets do not affect are of minor order with respect to variations in the other endogenous state variables. In this model, in fact, the first moment of the asset distribution - which is a "sufficient statistics" in Krusell and Smith - is constant by construction. Second and higher moments do affect endogenous variable but mainly their stationary levels (as shown by Hugget for example) rather than their oscillations around those levels - that is what I care for my welfare analysis.

G The Natural Debt Limit

Imposing $C_t^h \geq 0$ and $N_t^h \leq \bar{N}^+$ implies the emergence of what Aiyagari, in a slightly simpler context, calls a *natural debt limit.* We iterate forward the household budget constraint (A-50) (we drop the index-h)

$$-b_{t-1} \leq \Pi_t \sum_{j=0}^{\infty} \mathbf{R}_{t,t+j} \left(W_{t+j}^r \bar{N}^+ - T_{t+j} + F_{t+j} + \bar{\tau}^h \right) \tag{A-44}$$

where we have defined

$$\mathbf{R}_{t,t+j} \equiv \frac{\Pi_{t+1+j} \cdots \Pi_{t+1}}{R_{t+j} \cdots R_t} \text{ and } \mathbf{R}_{t,t} \equiv 1 \tag{A-45}$$

The above equation must hold a.s. and $\forall t \geq 0$.

We introduce the following notation. Over all possible realizations we have:

$$\underline{y} \equiv \min W_t^r \bar{N}^+ - T_t^G + F_t$$

,

$$\frac{1}{1-\underline{\beta}} \equiv \min \sum_{j=0}^{\infty} \mathbf{R}_{t,t+j} \text{ and } \underline{\pi} = \min \Pi_t$$

We define the natural debt limit ϕ_b^h as

$$\phi_b^h \equiv \frac{\underline{\pi}}{1-\underline{\beta}}(\underline{y} + \bar{\tau}^h) \tag{A-46}$$

and (for each household-h)

$$b_t^h \geq -\phi_b^h \tag{A-47}$$

We can go further recalling that $\bar{\tau}^h = -\bar{b}^h(1-\beta)$ and $\tilde{b}_t^h = b_t^h - \bar{b}^h$. Hence we can write

$$\tilde{b}_{t-1}^h \geq -\frac{\underline{\pi}}{1-\underline{\beta}}\underline{y} - \bar{b}^h\Big(1 - \underline{\pi}\frac{1-\beta}{1-\underline{\beta}}\Big) \equiv \tilde{\phi}_b^h \tag{A-48}$$

Hence differences between debt limits written in debt deviation $\tilde{b}$ across households are relatively small. We will take an *ad hoc* borrowing limit

$$\phi_b = \min_h \tilde{\phi}_b^h$$

such that, for each household, we can write

$$\tilde{b}_t^h \geq -\phi_b \tag{A-49}$$

H The Complete Markets Case

I assume a continuum of households indexed by $h \in [0,1]$ maximizing the following utility

$$U_0^h = E_0 \sum_{t=0}^{\infty} \beta^t \Big[u(C_t^h) - v(N_t^h)\Big]$$

the budget constraint takes the following form:

$$P_t C_t^h + E_t B_t^h Q_{t,t+1} = B_{t-1}^h + W_t N_t^h + P_t X_t^h \tag{A-50}$$

Where now B_t is a set of state-contingent securities that pays 1 dollar. While $Q_{t,t+1}$ is the pricing kernel.

From the Euler equations we have that

$$\frac{C_{t+1}^h}{C_t^h} = \frac{C_{t+1}^{h^o}}{C_{t+1}^{h^o}},\ \forall (h, h^o) \in [0,1]^2 \tag{A-51}$$

In the next proposition we claim that there exists an average household.

Proposition .1. *For any continuous initial distribution of wealth* $\exists h^o \in [0,1]$ *such that* $C_t^{h^o} = C_t\ \forall t \geq 0$

Proof. Given any continuous initial distribution of wealth $\exists h^o \in [0,1]$ $C_0^{h^o} = \int_0^1 C_0^h dh$.

From the Euler then we have that

$$C_t^{h^o} = \frac{C_0^{h^o}}{C_0^h} C_t^h = \frac{C_0}{C_0^h} C_t^h \tag{A-52}$$

So

$$C_t^{h^o} \int_0^1 C_0^h dh = C_0 \int_0^1 C_t^h dh \tag{A-53}$$

which shows the above proposition. □

So we can now introduce a metric for the deviations of consumptions from the average consumption:

$$C_t^h = \frac{C_0^h}{C_0} C_t = \delta(h) C_t \tag{A-54}$$

$$\Delta_{n,t}^{CM} = \left(\int_0^1 \delta(h)^{-\varphi/\sigma} \right)^{\varphi/\sigma} \tag{A-55}$$

So under complete markets $\Delta_{n,t}$ is constant.

To determine the value of this constant we have to specify the initial wealth - so the transfer scheme.

We can always find a transfer scheme such that $\delta(h) = 1 \ \forall h \in [0,1]$.

This would also be the optimal scheme that a benevolent government would implement weighting households the same.

To find this transfer scheme we write the inter-temporal budget constraint and we impose that $C_t^h = C_t \ \forall h \in [0,1], t \geq 0$. Then inter-temporal budget constraint is:

$$B_{-1}^h = \sum_{t=0}^{\infty} E_0 Q_{0,t}[W_t N_t^h + P_t \bar{\tau}^h - P_t C_t^h] \tag{A-56}$$

Given that $C_t^h = C_t$ then it must also be that $N_t^h = N_t$ so that (considering that the profits equals the taxes for subsidies[52]) we have that $C_t = W_n N_t$. So the budget constraint reduces to

$$B_{-1}^h = \bar{\tau}^h \sum_{t=0}^{\infty} E_0 Q_{0,t} P_t \tag{A-57}$$

or

$$\bar{\tau}^h = -\frac{b_{-1}^h/\Pi_0}{\sum_{t=0}^{\infty} E_0 Q_{0,t} P_t/P_0} = -\frac{b_{-1}^h/\Pi_0}{\sum_{t=0}^{\infty} \beta^t E_0 u_{c,t}/u_{c,0}} \tag{A-58}$$

If we call $1 - \beta^* = \sum_{t=0}^{\infty} \beta^t E_0 u_{c,t}/u_{c,0}$ we can write

$$\bar{\tau}^h = -\frac{b_{-1}^h}{\Pi_0}(1 - \beta^*) \tag{A-59}$$

Given that $\Pi_0 = 1$ is optimal in case of no initial relative price distortion we set:

$$\bar{\tau}^h = -b_{-1}^h(1 - \beta^*) \tag{A-60}$$

[52]subsidy rate is constant but total subsidies are not and are always equal to profits

Figures and Tables

Parameters	mean	standard deviation
η_r	1.0325	0.0076
η_p	64.1613	6.5743
η_{z_0}	1.4852	0.1832
η_{z_1}	-0.8381	0.0809

Table 1.2: Estimated Simple Rule.
We simulate our model for 9000 periods (we discard the first 1000 observations) and we estimate the following simple rule(ERS): $\hat{R}_t = \eta_r \hat{R}_{t-1} + \eta_p \pi_t + \eta_{z_0} z_t + \eta_{z_1} z_{t-1}$.

Rule Code	η_r	η_p	η_x	η_{z0}	η_{z1}	
GOMP	-	-	-	-	-	Optimal Policy Rule
ESR	1.033	64.16	0	1.485	-0.838	
ESRbis	1.033	64.16	0	0	0	
ATR	0	3	.5	1.485	-0.838	
TR	0	3	.5	0	0	
IT	-	∞	-	-	-	$\pi_t = 0$

Table 1.3: Monetary Policy Rules
Rules used for welfare comparison. For ESR, ESRbis, ATR and TR the functional form is: $\hat{R}_t = \eta_r \hat{R}_{t-1} + \eta_p \pi_t + \eta_x x_t + \eta_{z_0} z_t + \eta_{z_1} z_{t-1}$.

Losses	GOMP	ESR	ESRbis	ATR	TR	IT
Levels	6.28e-5	6.29e-5	6.76e-5	3.88e-4	3.51e-3	6.76e-5
Inflation	1.17e-5	1.18e-7	5.58e-6	2.46e-4	3.45e-3	0
Output gap	1.12e-5	3.94e-6	2.27e-6	1.12e-4	2.48e-5	0
Cons. Disp.	3.99e-5	5.88e-5	5.98e-5	3.05e-5	3.81e-5	6.76e-5

Table 1.4: Welfare Comparison
The welfare loss is expressed in steady state consumption ('Levels').Thisis also split by targets (loss stemming from: inflation, output gap and consumption dispersion)

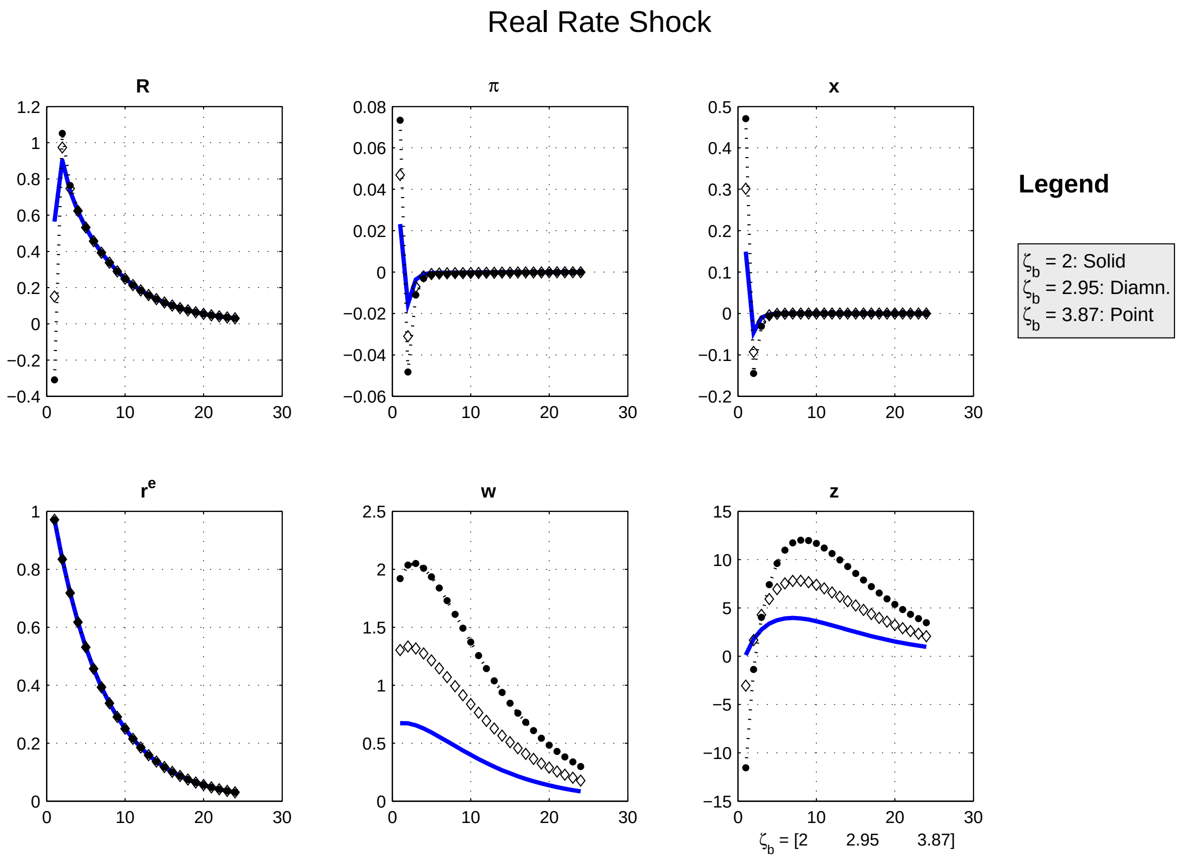

Figure 1.1: IRF: Low-Medium Steady State Debt Dispersion

Impulse response functions to a positive 1% shock to the *efficient* real rate of interest r_t^e (the one prevailing in a flex-complete market environment). The debt standard deviation across households, ζ_b, takes values: 2.00, 2.95 and 3.87 represented by a solid line, diamonds and points, respectively.

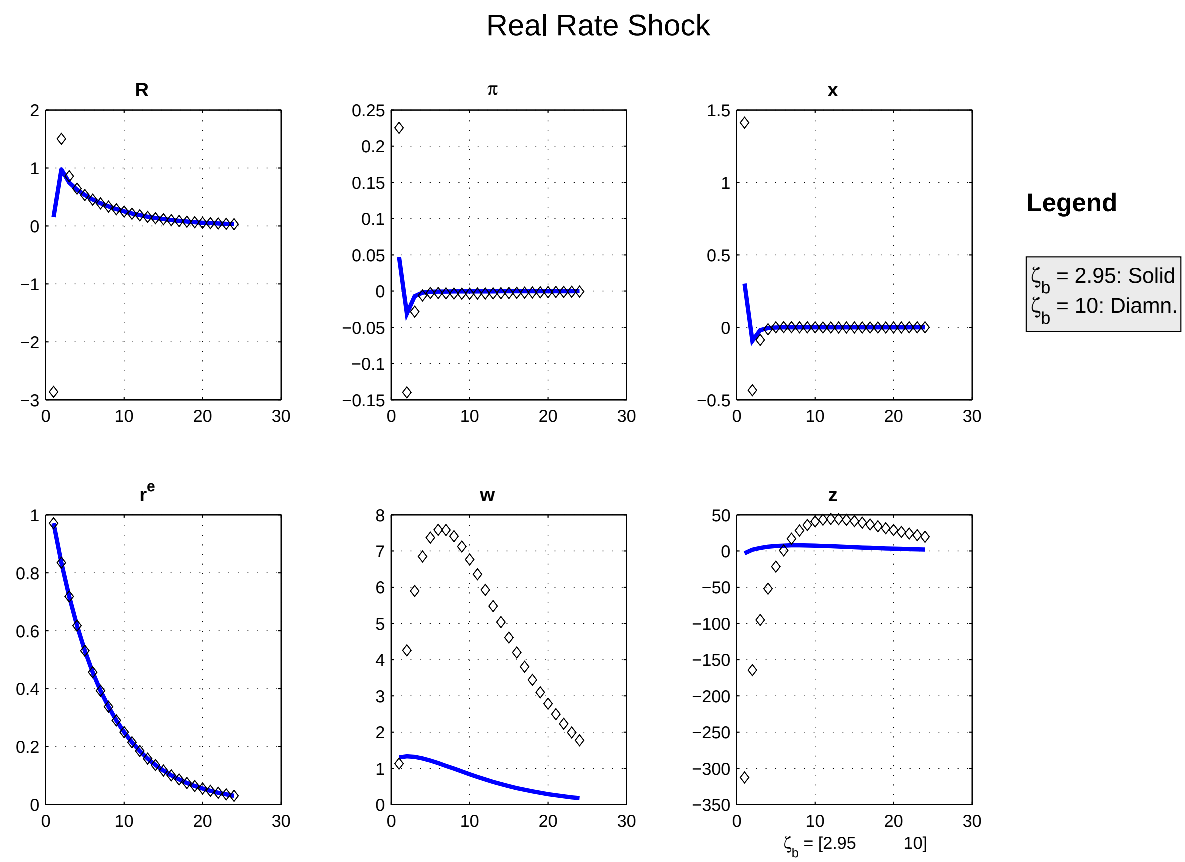

Figure 1.2: IRF: Medium-High Steady State Debt Dispersion

Impulse response functions to a positive 1% shock to the *efficient* real rate of interest r_t^e (the one prevailing in a flex-complete market environment). The debt standard deviation across households, ζ_b, takes values: 2.95 and 10.0 represented by a solid line and diamonds, respectively.

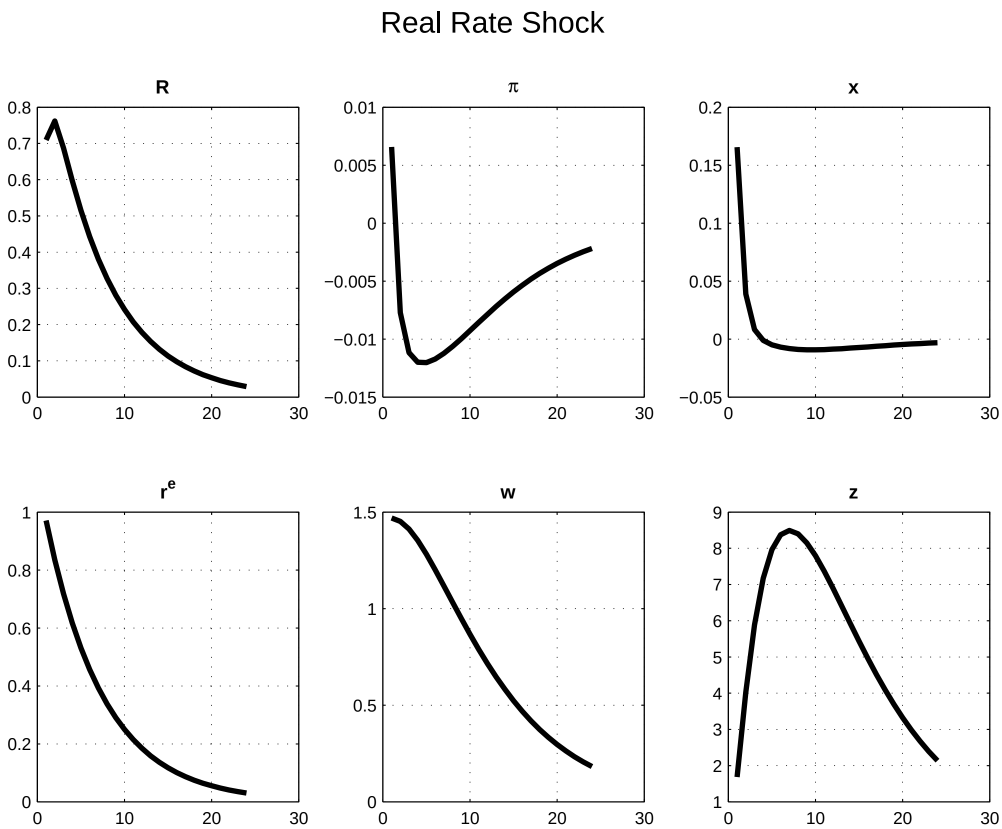

Figure 1.3: IRF: Estimated Policy Rule

Impulse response functions to a positive 1% shock to the *efficient* real rate of interest r_t^e (the one prevailing in a flex-complete market environment). Policy Rule adopted: $\hat{R}_t = 1.033\hat{R}_{t-1} + 64.16\pi_t + 1.485z_t - .838z_{t-1}$ (ESR) - see Table 1.3

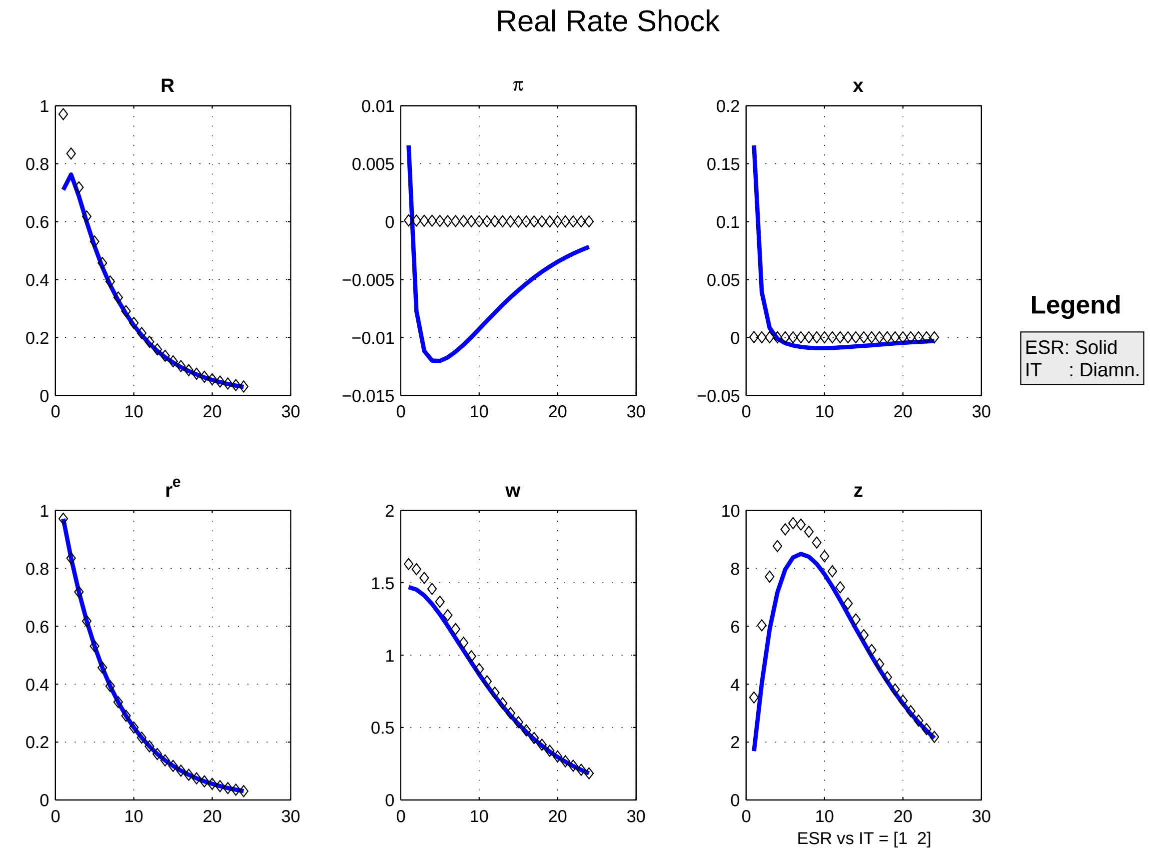

Figure 1.4: IRF: Estimated Policy Rule vs Targeting Zero Inflation

Impulse response functions to a positive 1% shock to the *efficient* real rate of interest r_t^e (the one prevailing in a flex-complete market environment). Comparing Policy Rules:
$\hat{R}_t = 1.033\hat{R}_{t-1} + 64.16\pi_t + 1.485z_t - .838z_{t-1}$ (ESR) vs $\pi_t = 0$ (IT) - see Table 1.3. ESR solid line, IT diamonds.

Chapter 2

Credit Frictions, Housing Prices and Optimal Monetary Policy Rules*

2.1 Introduction

The recent rise in housing prices in most OECD countries has attracted the attention of policy makers and academics and raised concern as to its macroeconomic implications.[1] Should asset prices be taken into account for monetary policy purposes? Because credit frictions can amplify and propagate shocks (and so distortion) this is not a trivial question even in a bubble free setup.

This paper assesses the relevance of housing prices when formulating monetary policy rules. Since residential property represents the largest share of household assets, and since many bank loans are secured by real estate collateral, it is worth investigating whether housing prices have any role, distinct from that of other asset prices, in a model incorporating credit market frictions at the

*This Chapter is part of a joint work with Caterina Mendicino - Department of Economics, Stockholm, Sweden.

[1]See, for example, Borio and McGuire (2004) regarding the relationship between housing and equity prices, Iacoviello (2004) regarding the relevance of housing prices and credit constraints to the business cycle, Girouard-Blndal (2001) regarding the role of housing prices in sustaining consumption spending in the recent downturn of the world economy, and Case-Quiugley-Shiller (2001) for empirical evidence of the housing wealth effect.

household level. Even though the relevance of liquidity constraints to consumption behavior is well documented in the empirical and theoretical literature (see Zeldes (1997), Jappelli and Pagano (1997)) little attention has been payed to credit frictions at the household level in the monetary business cycle literature. This paper represents the first attempt to evaluate welfare-based monetary policy using a model that incorporates heterogeneous agents and credit constraints at the household level.

The model is based on that of Kiyotaki and Moore (1997). To account for the existence of credit flows, we assume two types of agents, differing in terms of discount factors. Consequently, impatient agents, that have a higher propensity to consume out of wealth, are borrowers. Physical assets are used as collateral for borrowing. As in Iacoviello (2004), we depart from Kiyotaki and Moore's framework in two ways. First, unlike Kiyotaki and Moore, we focus on the household sector. In fact, Kiyotaki and Moore's agents are entrepreneurs that produce and consume the same type of goods using a physical asset. Moreover, agents are risk neutral and represent two different sectors of the economy, borrowers being "farmers" and lenders being "gatherers". In contrast, in our framework, households are risk adverse and, in addition to consumption and leisure, also consider house holdings as a separate argument of their utility function. Housing services are assumed to be proportional to the real amount of housing stock held. Second, we extend the model to include nominal price rigidities and thus a role for monetary policy. To summarize, our model economy is characterized by three types of distortions. First, monopolistic competition in the goods market allows for price setting above the marginal cost (average markup distortion). Second, nominal price rigidities, modeled as a quadratic adjustment cost on goods' market price setting are adopted as a source of monetary non neutrality. Third, creditors cannot force debtors to repay unless debts are secured by collateral, thus generating a role for housing prices and monetary policy.

In evaluating optimal monetary policy we take advantage of recent advances in computational economics, by adopting the approach of Schmitt-Grohe and Uribe (2003). Our results indicate that optimally designed simple monetary policy rules should not take into account current housing price movements. In fact, under normal circumstances, we find that explicitly aiming for housing price stability is not welfare improving, relative to a strict overall price stability policy. The remainder of the paper is organized as follows. Section 2 briefly reviews the relevant literature, while section 3 describes the role of housing as collateral. Section 4 lays out the model and derives the equilibrium conditions. Section 5 examines model calibration, section 6 analyzes the steady state, while section 7 describes the welfare measure used and the method for evaluating the optimal design of monetary policy. Finally, section 8 comments on the results.

2.2 Related Literature

Asset prices and monetary policy: welfare-based evaluation vs inflation-output volatility criterion. A number of papers have tried to understand to which extent asset price movements should be relevant for monetary policy[2]. Cecchetti et al. (2000, 2002) demonstrate that reacting to asset prices reduces the likelihood of bubble formation. On the other hand, Bernanke and Gertler (2001), among others, conclude that inflation-targeting central banks should not respond to asset prices in particular. In fact, conditional on a strong response to inflation, the gain engendered by a response to asset prices is negligible. These

[2] See e.g. Goodhart and Hoffman, Batini and Nelson (2000), Bernanke and Gertler (1999, 2001), Cecchetti, Genberg, Lipsky and Wadhwani (2000), Cecchetti, Genberg and Wadhwani(2003), Taylor(2001), Kontonikas and Montagnoli (2003), Faia and Monacelli (2004).

studies employ a financial accelerator framework that allows for credit market frictions and exogenous asset price bubbles. The method adopted for evaluating the performance of different monetary policy rules is based on the implied volatility of output and inflation. Different conclusions as to the desirability of including asset prices as an additional argument in monetary policy rules, depend mainly on different assumptions concerning the stochastic nature of the model, i.e., the shocks considered.

Iacoviello's analysis (2004) directly relates to housing prices. He shows the relevance of housing prices to the transmission and amplification of shocks to the real sector. Nevertheless, when computing the inflation-output volatility frontiers, it turns out that responding to housing prices does not produce significant gains in terms of output and inflation stabilization.[3]

The main shortcoming of all this literature is the absence of welfare considerations in evaluating optimal monetary policy. However, mixed results are also presented by more recent studies that rely on a welfare-based approach. In fact, while Faia and Monacelli (2004) - in a financial-accelerator setup - demonstrate that reacting to asset prices is not optimal, Dupor (2005) demonstrates that monetary policy should react to asset price fluctuations when they are driven by irrational expectation shocks to the future returns to capital.

Optimal monetary policy in economies with nominal rigidities: distorted vs non-distorted equilibrium. More broadly, our paper is related

[3] Iacoviello (2004) does not distinguish between residential and commercial properties. Thus, houses are not only a source of direct utility but also an input of production and the asset used in the credit market to secure both firms' and households' debts. Iacoviello (2004), as Faia and Monacelli (2004), adds collateral constraints tied to firms' real estate holdings (housing) to Bernanke, Gertler and Gilchrist (2000) model. Moreover, he also introduces collateral constraints in the household sector. These modeling choices are consistent with the aim of showing the importance of financial factors for macroeconomic fluctuation. Instead, being interested in the role of housing prices for the optimal design of monetary policy, we restrict our attention to the household sector.

to the considerable amount of literature treating optimal monetary policy in economies with nominal rigidities[4]. This literature assumes that the central bank is a benevolent policy maker that strives to maximize consumer welfare.[5] Most of the models consider a dynamic system centered on an efficient non-distorted equilibrium. In practice, the policy maker neutralizes any source of inefficiency present in the economy and unrelated to the existence of nominal rigidities. Thus, the only task left for monetary policy is to oset the distortions associated with price rigidities, so as to replicate the flexible price equilibrium allocation. The motivation behind this modeling choice is purely technical; in fact, it is sufficient for a first-order approximation of the equilibrium conditions to approximate welfare only up to the second order.[6] Following a method introduced by Rotemberg and Woodford (1997), in these kinds of models it is possible to derive a discounted quadratic loss function from the quadratic approximation of the utility function, and compute optimal policy using a simple linear-quadratic method, as in traditional monetary policy theory.

An alternative approach considers optimal monetary and fiscal policy in models evolving around equilibria that remain distorted.[7] In such models different types of distortions, besides price rigidities, provide a rationale for the conduct of monetary policy. To obtain a welfare measure that is accurate to the second order, it is necessary to use a higher-order approximation of the model's equilibrium conditions. In fact, up to first-order accuracy, the agents' discounted utility function equals its non-stochastic steady-state value. Since commonly considered monetary policy rules do not affect the non-stochastic steady state, it is not

[4]See, for example, Rotemberg and Woodford (1997), Clarida, Gali and Gertler (1999), King and Wolman (1999), Erceg, Henderson and Levin (2000),

[5]The literature is divided into two streams on the basis of the fundamental assumption regarding the deterministic equilibrium around which the model economy evolves.

[6]See Woodford (2003).

[7]See Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2004).

possible to rank different rules on the basis of a first-order approximation. Benigno and Woodford (2003) propose an extension of Rotemberg and Woodford's method. Based on computing a second-order approximation of the model's structural equations, it is possible to substitute out the linear terms from the Taylor approximation of the expected utility, obtaining a purely quadratic approximation of the welfare function (containing no linear terms). Once a quadratic function is derived, optimal monetary policy can then be evaluated, using the first-order approximation of the model's equations as constraints. Thus, the linear-quadratic method is introduced again. The method used in this paper is instead the one suggested by Schmitt-Grohe and Uribe (2003). They show that given the first-order terms of the Taylor expansions of the functions expressing the model's solution, the second-order terms can be identified by solving a linear system of equations, the terms of which are first-order terms and derivatives up to the second order of the equilibrium conditions evaluated at the non-stochastic steady state.

2.3 Housing Prices and Borrowing Constraint

Why should housing prices be relevant for monetary policy in a bubble-free model? Our main hypothesis is that housing is used as a collateral in the loan market housing prices are related to consumption and economic activity through both a traditional wealth effect and a credit channel. Increased housing prices contribute to a rise in the value of the collateral, which allows households to borrow and consume more. Consequently, increased household indebtedness could increase the sensitivity of households to changes in interest rates and sudden decreases in housing prices themselves. Thus, housing price movements are relevant to the assessment of how private consumption evolves and of the ability of households to smooth the effect of shocks.

We consider a modified version of the standard business cycle model in which households derive utility from owning houses and use them as collateral in the loan market. We depart from the representative agent framework by assuming two groups of agents, borrowers and lenders. The borrowing constraint is not derived endogenously but is consistent with standard lending criteria used in the mortgage and consumer loan markets. The borrowing constraint is introduced through the assumption that households cannot borrow more than a fraction of the value of their houses. The household borrows (B_{it}) against the value of his housing wealth as follows:

$$B_{it} \leq \gamma E_t[Q_{t+1} h_{it}] \tag{2.3.1}$$

where Q_{t+1} is the housing price and h_{it} is the end of the period stock of housing. Mortgage loans refinancing takes place every period and the household repays each new loan after one period. The overall value of the loan cannot be higher than a fraction of the expected value of the collateral. This fraction γ, referred to as *loan to value ratio*, should not exceed one. This can be explained with reference to the overall judicial costs that a creditor incurs in case of debtor default. Since housing prices affect the collateral value of the houses, price fluctuations play a large role in determining borrowing conditions at the household level. Borrowing against a higher-valued house is used to finance both consumption and investment in housing.

2.4 The Model

Consider a sticky price economy populated by a monopolistic competitive goods-producing firm, a monetary authority, and two types of households. To impose the existence of credit flows in this economy, we assume ex ante heterogeneity at the household level, i.e., agents differing in terms of the subjective discount factor. We assume a continuum of households of mass 1: n *Impatient Households* (lower discount rate) that borrow in equilibrium and (1-n) *Patient Households* (higher discount rate) that lend in equilibrium.

2.4.1 Households

The households derive utility from a flow of consumption and services from house holding - assumed to be proportional to the real amount of housing stock held - and disutility from labor:

$$\max_{\{c_{it},h_{it},L_{it}\}} E_t \sum_{t=0}^{\infty} \beta_i^t U(c_{it}, h_{it}, L_{it})$$

with $i = 1, 2$ and $\beta_1 > \beta_2$ s.t. a *budget constraint*

$$c_{it} + q_t(h_{it} - h_{it-1}) + \frac{b_{it-1}}{\pi_t} = \frac{b_{it}}{R_t} + w_t L_{it} + f_{it} - T_{it} \tag{2.4.1}$$

and a *borrowing constraint*

$$b_{it} \leq \gamma E_t[q_{t+1}\pi_{t+1}h_{it}] \tag{2.4.2}$$

Except for the gross nominal interest rate, R, all the variables are expressed in real terms; π_t is the gross inflation (P_t/P_{t-1}) and q_t is the price of housing in real terms (Q_t/P_t). The household can borrow (b_t) using as collateral the next period's expected value of real estate holdings (the stock of housing). This borrowing constraint is relevant only for the impatient households since the patient ones lend in equilibrium. In the budget constraint T_{it} represents lump sum taxes imposed by the fiscal authority, and f_{it} represents dividends distributed from firms (we assume that only the patient households own the firms). Thus,

$f_{1t} = \frac{1}{(1-N)}(D_t/p_t)$ where D_t represents the dividends of the representative firm while $f_{2t} = 0$.

The agents optimal choices are characterized by:

labor supply

$$-U_{L_{it}} = U_{c_{it}} w_t \tag{2.4.3}$$

borrowing condition

$$\frac{U_{c_{i,t}}}{R_t} \geq \beta_i E_t \frac{U_{c_{i,t+1}}}{\pi_{t+1}} \tag{2.4.4}$$

housing demand

$$U_{c_{i,t}} q_t - \beta_i E_t U_{c_{i,t+1}} q_{t+1} \geq U_{h_{i,t}} \tag{2.4.5}$$

The second equation relates the marginal benefit of borrowing to its marginal cost. The third equation states that the opportunity cost of holding one unit of housing, $\left[U_{c_{i,t}} q_t - \beta_i E_t U_{c_{i,t+1}} q_{t+1}\right]$, is greater than or equal to the marginal utility of housing services. The above equations hold with equality for patient households. Since the patient households' borrowing constraint is not binding in the neighborhood of the steady state, these households face a standard problem, the only exception being the existence of housing services in the utility function.

Impatient Households

We can show that impatient households borrow up to the maximum in the neighborhood of the deterministic steady state. If fact, if we consider the Euler equation of the impatient household

$$\mu_2 = (\beta_1 - \beta_2) U_{c_2} > 0 \tag{2.4.6}$$

where μ_{2t} is the lagrange multiplier associated to the borrowing constraint[8]. Thus, the borrowing constraint holds with equality in a neighborhood of the steady state

$$b_{2t} = \gamma E_t[q_{t+1}\pi_{t+1}h_{2t}] \tag{2.4.7}$$

And we get the following optimal choices for labor, borrowing and housing services

$$-U_{L_{2t}} = U_{c_{2t}}w_t \tag{2.4.8}$$

$$\frac{U_{c_{2t}}}{R_t} - \mu_t = \beta_2 E_t U_{c_{2t+1}} \frac{1}{\pi_{t+1}} \tag{2.4.9}$$

For constrained agents, the marginal benefits of borrowing are always greater than the marginal cost:

$$U_{h_{2t}} + \beta_2 E_t U_{c_{2t+1}} q_{t+1} + \mu_t \gamma E_t q_{t+1}\pi_{t+1} = U_{c_{2t}} q_t \tag{2.4.10}$$

Moreover, the marginal benefit of holding one unit of housing arises not only its marginal utility, but also from the marginal benefit of being allowed to borrow more.

2.4.2 Firms

The final good producing firms

Perfectly competitive firms produce an homogenous final good y_t using $y_t(i)$ units of each intermediate good $i \in (0, 1)$ adopting a constant return to scale, diminishing marginal product and constant elasticity of substitution technology:

[8]Once we assume the existence of different discount factors with $\beta_1 > \beta_2$, in the deterministic steady state impatient households are willing to borrow up to the maximum.

$$y_t \leq \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$

with $\theta > 1$

The price of the intermediate good $y_t(i)$ is denoted by $P_t(i)$ and taken as given by the competitive final good producing firm. Solving for cost minimization[9] yields the following constant price elasticity (θ) demand function for type of good i which is homogeneous of degree one in the total final output:

$$y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} y_t$$

By Combining the demand function with the production function is possible to derive the price index for intermediate goods:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{1/(1-\theta)}$$

The intermediate sector

In the wholesale sector there is a continuum of firms indexed by $i \in (0, 1)$ and owned by consumers. Intermediate producing firms act on a monopolistic market and produce $y_t(i)$ units of a differentiated type of goods i using $L_t(i)$ units of labor according to the following constant return to scale technology

$$Z_t L_t(i) \geq y_t(i) \tag{2.4.11}$$

[9] Costs minimization implies

$$\min_{\{y_t(i)\}} \int_0^1 P_t(i) y_t(i) di$$
$$s.t.\ \ y_t \leq \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$

where Z_t is the aggregate productivity shock which follows the following autoregressive process

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \varepsilon_{Zt}, \quad \varepsilon_{Zt} \backsim^{iid} N(0, \sigma_{\varepsilon_Z}), \quad 0 < \rho_Z < 1$$

Cost Minimization Monopolistic competitive firms hire labor from households in a competitive market on period by period basis. Cost minimization implies the following nominal marginal cost s_t^n:

$$\frac{W_t}{Z_t} = s_t^n(i) \tag{2.4.12}$$

and thus the total cost could be expressed as follows:[10]

$$W_t L_t(i) = s_t^n(i) y_t(i)$$

Price Setting Assume now that intermediate firms set the price of their differentiated goods every period, but face a quadratic cost of adjusting the price between periods.[11] The cost is measured in terms of the final good[12]

[10]In equilibrium the firm chooses input such that the marginal product equals the markup times the factor price. In fact, in terms of gross markup $(1+\eta_t) = \frac{1}{s_t}$:

$$\frac{\bar{y}_t(i)}{L_t^*(i)} = (1+\eta_t) W_t$$

[11]The Calvo setting (most commonly used) and the price-adjustment cost setting deliver the same linearized system of necessary conditions up to re-parametrization. For a second-order approximation this is not true: the second-order term in the resource constraint and in the firms' FOC does not allow a one-to-one mapping between the two models.

The second-order terms in the Calvo setting are ultimately related to the second-order moments of the price distribution, while in the other case they are simply related to the chosen adjustment-cost functional form. However, given the demanding assumptions of the re-setting process in a Calvo-type framework, it is hard to tell which of the two set-ups is quantitatively more reliable.

To save computing time, we have preferred to use the price adjustment cost framework.

[12]See Kim JME 1995

$$\frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2$$

where $\phi_p > 0$ represents the degree of nominal rigidity and π is the gross steady state inflation.

Each firm faces the following problem:

$$\max_{\{P_t(i)\}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}\left[\frac{D_t(i)}{P_t}\right]$$
$$s.t.$$
$$y_t(i)=\left[\frac{P_t(i)}{P_t}\right]^{-\theta} y_t$$

where $\Lambda_{t,t+j}=\beta_1^j \frac{U_{c1t+j}}{U_{c1t}}$is the *relevant discount factor*. The firm's profits in real terms are given by :

$$\frac{D_t(i)}{P_t}=\frac{P_t(i)}{P_t}y_t(i)-s_t(i)y_t(i)-\frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2$$

Using the results from the cost minimization problem, we replaced the real total costs , $w_tL_t(i)$, with a function of real marginal costs and total output.[13] Thus, substituting for the total costs and the firm's production, the profits maximization problem becomes:[14]

$$\max_{\{P_t(i)\}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}\left\{y_t\left[\left(\frac{P_t(i)}{P_t}\right)^{1-\theta}-s_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\theta}\right]-\frac{\phi_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)^2\right\} \tag{2.4.13}$$

[13]
$$w_tL_t(i)=s_t(i)y_t(i)=\frac{w_t}{Z_t}y_t(i)$$

[14]The derivative with respect to the firm's price, multiplied for the price level P_t, yields:

$$0=E_t\Lambda_{t,t+1}\left[\phi_p\frac{P_t}{\pi}\frac{P_{t+1}(i)}{P_t(i)^2}\left(\frac{P_{t+1}(i)}{\pi P_t(i)}-1\right)\right]+$$
$$+y_t\left[(1-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta}+\theta s_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\theta-1}\right]-\phi_p\frac{P_t}{\pi P_{t-1}(i)}\left(\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right)$$

2.4.3 The Fiscal Authority

We assume:

$$G_t = T_t$$

where G_t is government consumption of the final good and T_t represents lump sum taxes/transfers, where $T_t = (1-n)T_{1t} + nT_{2t}$. Government consumption evolves according to the following exogenous process:

$$(\ln G_t - \ln G) = \rho_G (\ln G_{t-1} - \ln G) + \varepsilon_{Gt} \quad \text{where} \quad \varepsilon_{Gt} \backsim^{iid} N(0, \sigma_{\varepsilon_G}), \quad 0 < \rho_G < 1 \tag{2.4.14}$$

where G is the steady state share of government consumption.

2.4.4 Equilibrium and Aggregation

In a symmetric equilibrium all firms make identical decisions so that:

$$y_t(i) = Y_t \qquad P_t(i) = P_t \qquad L(i) = L_t$$

Consequently, total production becomes

$$Y_t = Z_t L_t \tag{2.4.15}$$

while price setting becomes

$$0 = E_t U_{c1t+1} \left[\phi_p \frac{\pi_{t+1}}{\pi} \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \right] + U_{c1t} \left\{ y_t \left[\theta \left(s_t - \frac{\theta - 1}{\theta} \right) \right] - \phi_p \frac{\pi_t}{\pi} \left(\frac{\pi_t}{\pi} - 1 \right) \right\} \tag{2.4.16}$$

The market clearing conditions are as follows:

$$(1-n)L_{1t} + nL_{2t} = L_t \qquad (1-n)c_{1t} + nc_{2t} = C_t$$
$$(1-n)b_{1t} + nb_{2t} = 0 \qquad (1-n)h_{1t} + nh_{2t} = 1$$
$$T_t = (1-n)T_{1t} + nT_{2t} \qquad G_t = T_t$$

where H_t is in fixed supply, normalized to 1. The resource constraint is as follows:

$$Y_t = C_t + \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 + G_t \tag{2.4.17}$$

The production of the final sector needs to be allocated according to price adjustment costs and to consumption by households and government

2.5 Parameters Values

We set the parameters of the model on the basis of quarterly evidence. The household discount factors are $(\beta_1, \beta_2) = (0.99, 0.98)$. The patient household discount factor implies an average annual rate of return of approximately 4%. Previous estimates of discount factors for poor or young households[15] have been used as a reference in the calibration of β_2. We assume a separable utility function as follows:

$$U(c_{it}, h_{it}, L_{it}) = \frac{c_{it}^{1-\varphi_c}}{1-\varphi_c} + \nu_h \ln h_{it} - \nu_L \frac{L_{it}^{1+\varphi_L}}{1+\varphi_L}$$

As a benchmark case, we assume the log utility of consumption to be $\varphi_c = 1$ (risk aversion), and we set $\varphi_L = 2$ (inverse of the Frisch labor supply elasticity). The weight on labor disutility, ν_L, equals 30, such that labor supply is approximately 1/3. The weight on housing is $\nu_h = 0.019$. This last parameter implies a

[15] In fact, Lawrance (1991) and Samwick (1998) estimate discount factors, respectively, for poor and young households in the range (0.97, 0.98).

Model's Parameters

		Preferences		
$\beta_1 = 0.99$		$\varphi_c = 1$		$\nu_h = 0.019$
$\beta_2 = 0.98$		$\varphi_L = 0.01$		$\nu_L = 1$
	Technology		**BOC**	
	$\theta = 11$		$\gamma = 0.5$	
	$\phi_p = 161$			
		Shocks		
	$\rho_Z = 0.82$		$\sigma_Z = 0.0056$	
	$\rho_G = 0.9$		$\sigma_G = 0.0074$	

Table 2.1: Structural Parameters.

steady state value of real estate over annual output of 140%. In line with the literature on nominal rigidities, we set the elasticity of substitution, θ, to 11 which gives a steady-state markup of 10%, in line with empirical evidence. The baseline choice for the loan to value ratio[16], γ, is 50% and the fraction of borrowed constraint population is set to 50%. We calibrate the steady state government consumption value to be 20% of total output. In accordance with Schmitt-Grohe and Uribe (2004) we calibrate the technology and government spending shocks according to standard values in the real business cycle literature.[17] Table 2.1 summarizes the calibrated parameters.

2.6 Understanding the Model

2.6.1 A look at the Deterministic Steady State

To find the deterministic steady state of the model we solve a *nonlinear rootfinding problem.*[18]

[16]Using US data from 1974 to 2003, Iacoviello (2004) estimates the households' loan to valio ratio equal to 0.55.

[17]For the technology shock see, Cooley & Prescott (1995, chapter 1 in Cooley's book), or Prescott 1986.

[18]In the *nonlinear rootfinding problem*, a function f mapping $\mathbb{R}^n$ to $\mathbb{R}^n$ is given and one must compute an n-vector x, called a *root* of f, that satisfies $f(x) = 0$. In our problem the $f(x)$ is represented by ss. We can write the system as a $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ function where L_1 and L_2

From the euler equations we have that $R = \Pi/\beta_1$. Given that in the deterministic steady state, the wage equals the marginal costs $(\theta - 1)/\theta$ we find c_i as functions of L_i:

$$c_i^{\sigma} \nu_L L_i^{\varphi} = w \qquad (2.6.1)$$

Defining $\tilde{\beta} = \beta_2 + \gamma(\beta_1 - \beta_2)$ we can also find the value of the housing stock for impatient agents:

$$(qh_2)_{ss} = \nu_H c_2^{\sigma}/(1 - \tilde{\beta}) \qquad (2.6.2)$$

The borrowing constraint is

$$b_2/\Pi = \gamma(qh_2)_{ss} = \gamma \nu_H c_2^{\sigma}/(1 - \tilde{\beta}) \qquad (2.6.3)$$

The systems is closed by the resource constraint and one of the two budget constraints. [19] The model shows *superneutrality*. In fact, we can write the bond holdings as:

$$b_2/\Pi - b_2/R = \gamma(qh_2)_{ss}(1 - \beta_1) = \frac{\gamma \nu_H c_2^{\sigma}}{(1 - \tilde{\beta})}(1 - \beta_1) \qquad (2.6.4)$$

Thus, the system does not include any nominal variable; in fact, in the steady state the cost of adjusting prices is nil and the inflation rate does not affect any real variable.[20]

We now analyze how the non-stochastic steady state varies under different parameterizations of the model.

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are unknowns and easily implement a numerical algorithms for solving the system quickly and accurately.

[19]For convenience we choose the one of the impatient.

[20]This clearly depends on how we wrote down the price adjustment cost function.

First, let us observe how the degree of friction in the credit market affects the deterministic steady state of the model. Figure 2.1 indicates that housing prices are increasing in terms of γ. A higher value of γ implies easier access to the credit market, and consequently, increased demand for housing on the part of borrowers. For the housing market to clear, lenders should be willing to sell part of their housing holdings. Thus, the user cost of housing has to increase. Lower credit friction means that lenders are able to postpone more consumption until the future, while, in contrast, borrowers can consume more in the present. Thus, in the long run, the consumption path of the lenders is increasing in terms of γ, while decreasing for borrowers. Lower consumption implies less leisure for borrowers (i.e., greater labor supply) and more leisure for lenders. Over all, there is an increase in the total labor supply, and thus an increase in total production.

Figure 2.2 and 2.3 also presents the effect of the gap in discount factors, defined as $\beta_1 - \beta_2$. As far as $\tilde{\beta} < \beta_1$, the value of housing stock as a proportion of total output is increasing in terms of γ. On the other hand, housing prices are inversely related to the gap in discount factors when $\gamma < 1$. We can consider $\tilde{\beta}$ to be the true discount factor, modified for the credit constraint, that impatient households use to evaluate the decision whether or not to own housing. When households can fully finance their own investment, i.e., $\gamma = 1$, we have that $\tilde{\beta} = \beta_1$ and the gap in discount factors plays no role in determining the level of any variable. For values of $\gamma > 1$ the logic is inverted. Generally speaking, higher housing prices imply higher gross or net debt gearing (whichever is measured) for the household sector. In particular, with our calibration, at values of γ close to unity total household debt outstanding is close to 100% of total disposable income.[21]

Figure 2.4 compares our model with an equivalent representative agent model.[22] It is interesting to note that the existence of debtors in the economy creates *overproduction* in the sense that the output is greater than the one delivered by the representative agent model. However, the existence of heterogeneous agents is not in itself sufficient to generate a level of production greater than that implied by representative agents not subject to the monopolistic competition distortion.[23]

Comparing the steady-state level of housing prices, our model implies a lower housing price level than the representative agent model does. For any $\gamma < 1$, in our economy the stock of housing held by patient agents is always greater than the average holdings; in the representative agent model, however, the housing stock of patient agents is clearly constructed to be identical to the average holdings. So, while in the representative agent model the stock of housing held by the agent always equals unity, in our model $h_1 > 1$ and decreases with γ. This means that q_{ss} is always lower than in the representative agent economy as far as $\gamma < 1$. Moreover, the difference in terms of housing prices is always positive, though decreasing in terms of γ. At the same time, the wider the discount factor gap, the less the difference in prices.

[21] Which, for some anglo-saxon countries, is not at all far away from actual data.

[22]

$$Y_{representative} - Y_{heterogeous}$$

[23] Also note that the closer competition is to perfect competition the smaller the difference between the representative agent output and ours. This is mainly, but not solely, because the profits are rebated only to the patient agents. So in our model, greater monopoly power exacerbates the wealth inequality.

2.6.2 Responses to Shocks

Let us now examine how exogenous shocks are propagated in this economy. To close the model, we assume that the central bank follows a simple rule of this form:

$$\hat{R}_t = \bar{\alpha}\pi_t \tag{2.6.5}$$

where $\bar{\alpha} = 1.5$. Figure 2.5 displays the reaction to a positive productivity shock. As expected, a positive productivity shock leads to a reduction in the marginal cost, which implies decreased inflation. Thus the effect of a positive technology shock has a reduced impact on total production. Since it is costly to change prices, inflation decreases less then it should, so consumption rises less than needed, implying reduced total employment. Impatient households smooth the effect of the shock on consumption by increasing their investment in housing. Thus, for the housing market to clear, the price of the asset increases, and consequently, the level of current indebtedness rises.

Let us write the budget constraint on borrowers as a first-order approximation, as follows:

$$\begin{aligned}\hat{c}_{2,t} + \frac{qh_2}{c_2}\hat{h}_{2,t}(1-\gamma\beta_1) = \frac{qh_2}{c_2}\hat{h}_{2,t}[\gamma\beta_1(\hat{\gamma}_t + E_t q_{t+1} - rr_t) + \\ +\gamma\pi_t + \hat{h}_{2,t-1} - \gamma\hat{b}_{2,t-1}] + \frac{wL_2}{c_2}(\hat{w}_t + \hat{L}_{2,t}) - \frac{G}{Yc_2}g_t\end{aligned} \tag{2.6.6}$$

Where we define the ex ante real rate as $rr_t = \hat{R}_t - E_t\pi_{t+1}$. First, we notice that in a first-order approximation and close to the steady state, the down payment equals $1-\gamma\beta_1$, recalling that here β_1 is the price of a unit of debt in steady state. This means that households are able to fully finance their housing investment using debt only if $\gamma = 1/\beta_1 > 1$. We can identify three main channels through which the wealth effect operates. 1) A *collateral effect*: an increase in borrowing ability can be driven either by higher housing price expectations or by higher γ. The magnitude of their impact is given by the discounted steady-state debt-consumption ratio, $\beta_1 b_2/c_2$. 2) An *interest rate channel* that operates through rr_t: a higher real rate means higher interest payments on the stock of debt, redistributing wealth from debtors to creditors. 3) A *nominal debt effect* that operates through inflation: since debt is not indexed, unexpectedly higher inflation today redistributes wealth from creditors to debtors. We also notice that in this model, house prices have no traditional wealth effect. More precisely, given that housing investment is zero in the steady state, the direct impact of housing prices subject to the budget constraint is always zero, whatever the approximation order taken.

Figure 2.6 displays the effect of increased government expenditure. As a result of this increase, labor demand also increases and thus production as well. The consequent increase in marginal costs raises current in.ation. Due to higher taxes, individual consumption decreases. This, coupled with the increased real interest rate, induces borrowers to reduce their debt levels by reducing their housing holdings; consequently, housing prices decline.

2.7 Computation and Welfare Measure

2.7.1 Computation

Ever since Kydland and Prescott (1982) published their findings[24] the first-order approximation approach has been the most popular numerical approximation method for solving models too complex to produce exact solutions. However,

[24]They applied to a real business cycle model a special case of the method of linear approximation around deterministic steady states developed in Magill (1977).

first-order approximations may produce clearly misleading results[25]. To compare the welfare effects of implementable policy rules that have no first-order effects on a model's deterministic steady state, we need to rely on higher-order approximation methods. As shown by Kim and Kim (2003)[26], in this context first-order approximation methods are not locally accurate. In general, a second-order accurate approximation of the welfare function re- quires a second-order expansion to the model.s equilibrium conditions. The first-order approximation solution is not always accurate enough, due to the certainty equivalence property, i.e., the coincidence of the first-order approximation of the unconditional means of endogenous variables with their non- stochastic steady-state values. This ignores important effects of uncertainty on the average level of household welfare. A first-order approximation of the policy functions would give an incorrect second-order approximation of the welfare function.[27]

To overcome this limitation and obtain a second-order accurate approximation, we have adopted a perturbation technique introduced by Fleming (1971), applied to various types of economic models by Judd and various coauthors, [28] and recently generalized by Schmitt-Grohe and Uribe (2002).[29] Second order

[25]See for example Tesar (1992) for a case where completing asset markets makes all agents worse off, Kim and Kim (2003) for stressing the same results in a two agents stochastic model.

[26]They show that a welfare comparison based on the linear approximation of the policy functions of a simple two-country economy, may yield the odd result of welfare being greater under autarky than under a condition of full risk sharing.

[27]See Woodford (2002) for a discussion of situations in which second-order accurate welfare evaluations can be obtained using first-order approximations of the policy functions.

[28]See Judd and Guu (1993, 1997) for applications to deterministic and stochastic, continuous- and discrete-time growth models in one state variable, Gaspar and Judd (1997) for multidimensional stochastic models in continuous time approximated up to the fourth order, Judd (1998) for a presentation of the general method, and Jin and Judd (2001) for an extension of these methods to more general rational-expectations models.

[29]They derive a second-order approximation of the policy function of a general class of dynamic, discrete-time rational-expectation models. They show that in a second-order expansion of the policy functions, the coefficients of the linear and quadratic terms of the state vector are independent of the volatility of the exogenous shocks. Thus, only the constant term is affected by uncertainty.

approximations are quite convenient to implement since, even when capturing the effects of uncertainty, do not suffer from the "curse of dimensionality".[30] In fact, in accordance with Schmitt-Grohe and Uribe, given the first-order terms of the Taylor expansions of the functions expressing the model's solution, the second-order terms can be identified by solving a linear system of equations the terms of which are the first-order terms and derivatives up to the second order of the equilibrium conditions evaluated at the non-stochastic steady state.

2.8 Welfare Measure and Optimal Rules

How should monetary policy be in a world economy with credit frictions at the household level? To answer this question, we rely on utility-based welfare calculations, assuming that the benevolent monetary authority maximizes the utility of households, subject to the model's equilibrium conditions. Formally, the optimal monetary policy maximizes lifetime household utility:

$$V_t \equiv E_t\left[\sum_{i=1}^{2}\eta_i\sum_{j=0}^{\infty}\beta_i^j U(c_{i,t+j},h_{i,t+j},L_{i,t+j})\right]$$

where η_i represent the weights on households' utilities. We choose η_1=(1-β_1) and η_2=(1-β_2) such that given a constant consumption stream the two agents reach the same level of utility.

We measure welfare as the conditional expectation at time zero ($t = 0$), the time at which all the state variables of the economy assume their steady state values. Since different policy regimes, even those not affecting the non-stochastic steady state, are associated with different stochastic steady states, so as not to neglect the welfare effects occurring during the transition from one steady state to another, we use a conditional welfare criterion. Thus, we evaluate welfare

[30] Models with many state variables can be solved without much computational effort.

conditional on the initial state being the non-stochastic steady state[31].

We evaluate the optimal setting of monetary policy in the constrained class of simple interest rate rules. Thus, we assume that the central bank follows an interest rate rule of the form

$$R_t = \Theta(X) \tag{2.8.1}$$

Where X represents easily observable macroeconomic indicators tested as possible arguments of the rule

$$X = \left[R_{t-1}, \frac{\pi_{t-s}}{\pi_{ss}}, \frac{y_{t-s}}{y_{ss}}, \frac{q_{t-s}}{q_{ss}} \right]$$

with s=$\{0, 1\}$.

As an implementability condition, the policies are required to deliver local uniqueness of the rational expectations equilibrium. The configuration of parameters satisfying the requirements and yielding the greatest welfare gives the optimal implementable rule. In characterizing optimal policy, we search over a grid considering different ranges of the parameters. Then, we compute the welfare level V_0^* associated with the optimal rule:

$$V_0^* \equiv E_0 \left[\sum_{i=1}^{2} \eta_i \sum_{j=0}^{\infty} \beta_i^j U(c_{i,j}^*, h_{i,j}^*, L_{i,j}^*) \right] \tag{2.8.2}$$

where $c_{i,j}^*, h_{i,j}^*$ and $L_{i,j}^*$ denote the contingent planes for consumption, housing and labor, respectively, under the optimal policy regime.

To compare different rules, we relate the deviations of the welfare associated with the different rules from the deterministic steady-state welfare.

[31] An alternative to making the evaluation conditional on a particular initial state could be to make it conditional on a distribution of values of the initial state. In any case, when there is a time-inconsistency problem, the optimality of the rule may depend on the initial conditions. One way to overcome this problem could be to find the rule that would prevail under the commitment of a "timeless perspective" (see Giannoni and Woodford 2002).

Optimal Simple Rule			
$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_t + (1-\alpha_R)\alpha_y \hat{y}_t + (1-\alpha_R)\alpha_q \hat{q}_t$			
$\alpha_R = 0$	$\alpha_\pi = 3$	$\alpha_y = 0$	$\alpha_q = 0$
$Welfare\ Loss = 0.00937003$			

Table 2.2: Optimal Simple Rule
The welfare loss represent the loss in terms of consumption with respect to the steady state's welfare.

2.9 Optimal Simple Rules

To investigate how monetary policy should optimally be designed in a model that incorporates housing prices, we maximize households' total welfare with respect to the coefficients of a simple monetary policy rule. As in the monetary business cycle literature, we allow the nominal interest rate to respond to inflation, output, and the lagged interest rate. In accordance with the literature on asset prices and monetary policy, we also consider the optimality of responding to current housing price movements. Thus, we search for the optimum value of the coefficient of an implicit interest rate rule - α_π, α_y, α_R and α_q - using the following grid: $\alpha_\pi \in [1,3]$, $\alpha_R \in [0,0.9]$, $\alpha_y \in [0,2]$ and $\alpha_q \in [0,2]$.[32] Table 2.2 summarizes the main findings; we express the welfare loss with respect to the steady state's welfare.

Optimization over this simple rule indicates that the central bank should not take into account variations of housing prices from the steady-state level. This means that the housing price is not an appropriate variable to consider for the optimal design of simple monetary policy rules in this economy. Optimal policy is instead characterized by a strong response to inflation deviations from the target. In fact, α_π equals the upper limit of its parameter space. In contrast, it is not optimal to react to output. These results are consistent with those obtained by

[32]We consider 25 linearly spaced points for each coefficient.

Schmitt-Grohe and Uribe (2003). They also show that it is optimal to respond to deviations of output from potential output, but not to output variations per se. While the concept of "output gap" is well understood in models characterized only by inefficiencies related to price stickiness, the definition of potential output in our economy is not clear. Interest rate smoothing also turns out not to be optimal. Given that our model economy is cashless, in the absence of capital, the only motive for smoothing the interest rate would come from the existence of credit friction. However, it turns out that targeting the lagged interest rate is also not optimal.

Figure 2.7 and 2.8 show how the two different monetary policy regimes affect the reaction to shocks in this economy. We consider the case of a central bank following the optimal simple rule or, alternately, also targeting housing prices. Thus, we vary the weight on housing prices in the monetary rule, α_q, from zero to one. The first thing to notice is the reaction of inflation. Under both shocks, the analyzed inflation is destabilized when the monetary authority targets housing prices. At the same time, real housing prices gain little in terms of stabilization, because, as they are mainly driven by exogenous variables (i.e., the technology and government spending shocks), they are closely related to fundamentals. This even applies if the credit constraint amplifies the fluctuations of household variables such as consumption, housing stock, and borrowing. On the other hand, in a first-order approximation, aggregate variables such as output and total labor supply remain mainly unaffected by change imposed by the specification of monetary policy rule.[33] Not only inflation, but also the nominal and real interest rates are much more volatile than those prevailing under the optimal rule. Thus,

[33] In the IRF of the system approximated to the second order (not shown here), we instead see how targeting housing prices entails lower value for long-term aggregate output and non-zero long-term inflation.

Lagged Interest Rate Rule			
$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_{t-1} + (1-\alpha_R)\alpha_y \hat{y}_{t-1} + (1-\alpha_R)\alpha_q \hat{q}_{t-1}$			
$\alpha_R = 0$	$\alpha_\pi = 3$	$\alpha_y = 0$	$\alpha_q = 0$
$Welfare\ Loss = 0.00937858$			

Table 2.3: Lagged Interest Rate Rule
The welfare loss represent the loss in terms of consumption with respect to the steady state's welfare.

targeting housing prices could in fact even increase the redistribution of wealth among households through the *interest rate-inflation channel*. To summarize, targeting housing prices seems not to be very effective at reducing the wealth effect stemming from the credit friction mechanism; at the same time, it amplifies the redistributive wealth channel that operates via inflation and interest rates, and increases the relative price dispersion losses. As a result, there is a reduction in the overall welfare of the economy.

It is often argued in the monetary policy literature that implicit rules cannot be implemented in practice. For this reason, we have adopted a simple rule according to which the nominal interest rate reacts to the last period's inflation, output, and housing prices (see Table 2.3). The result turns out to be the same: targeting housing prices is not optimal.

Table 2.4 compares the optimal implicit simple rule with a number of different ad hoc rules using the welfare-based approach. As explained in section 2.8, to compare different rules, we must relate the deviations of the welfare associated with the different rules from the steady-state welfare.

Our results clearly indicate that targeting housing prices tends to reduce welfare. In fact, a unitary response to current housing prices implies a 1% welfare loss with respect to the optimal rule.

It is worth noting that responding to output also tends to reduce welfare. The higher the percentage decline in welfare, the lower the response to inflation;

Rule	**Welfare Loss**	**% Loss relative to optimal**
No Interest Rate Smoothing		
$\hat{\mathbf{R}}_t = \alpha_\pi \hat{\pi}_t + \alpha_y \hat{\mathbf{y}}_t$		
$\alpha_\pi = 3 \quad \alpha_y = .5$	0.13115	0.1218
$\alpha_\pi = 1.5 \quad \alpha_y = .5$	1.07945	1.0701
$\hat{\mathbf{R}}_t = \alpha_\pi \hat{\pi}_t + \alpha_q \hat{\mathbf{q}}_t$		
$\alpha_\pi = 3 \quad \alpha_q = 1$	0.98708	0.9777
$\hat{\mathbf{R}}_t = \alpha_\pi \hat{\pi}_t + \alpha_y \hat{\mathbf{y}}_t + \alpha_q \hat{\mathbf{q}}_t$		
$\alpha_\pi = 3 \quad \alpha_y = .5 \quad \alpha_q = 1$	1.47957	1.4702
$\alpha_\pi = 2 \quad \alpha_y = .5 \quad \alpha_q = 1$	5.20375	5.1944
Interest Rate Smoothing		
$\hat{\mathbf{R}}_t = \alpha_R \hat{\mathbf{R}}_{t-1} + (\mathbf{1} - \alpha_\mathbf{R})\alpha_\pi \hat{\pi}_t + (\mathbf{1} - \alpha_\mathbf{R})\alpha_y \hat{\mathbf{y}}_t$		
$\alpha_R = .9 \quad \alpha_\pi = 3$	0.01551353	0.0061
$\alpha_R = .6 \quad \alpha_\pi = 3$	0.00967056	0.0003
$\alpha_R = .9 \quad \alpha_\pi = 1.5$	0.08176999	0.0724
$\hat{\mathbf{R}}_t = \alpha_R \hat{\mathbf{R}}_{t-1} + (\mathbf{1} - \alpha_\mathbf{R})\alpha_\pi \hat{\pi}_t + (\mathbf{1} - \alpha_\mathbf{R})\alpha_y \hat{\mathbf{y}}_t$		
$\alpha_R = .9 \quad \alpha_\pi = 3 \quad \alpha_y = .5$	0.16378627	0.1544
$\alpha_R = .9 \quad \alpha_\pi = 1.5 \quad \alpha_y = .5$	1.04395526	1.0346
$\hat{\mathbf{R}}_t = \alpha_R \hat{\mathbf{R}}_{t-1} + (\mathbf{1} - \alpha_\mathbf{R})\alpha_\pi \hat{\pi}_t + (\mathbf{1} - \alpha_\mathbf{R})\alpha_q \hat{\mathbf{q}}_t$		
$\alpha_R = .9 \quad \alpha_\pi = 3 \quad \alpha_q = 1$	2.10236651	2.0930
$\hat{\mathbf{R}}_t = \alpha_R \hat{\mathbf{R}}_{t-1} + (\mathbf{1} - \alpha_\mathbf{R})\alpha_\pi \hat{\pi}_t + (\mathbf{1} - \alpha_\mathbf{R})\alpha_y \hat{\mathbf{y}}_t + (\mathbf{1} - \alpha_\mathbf{R})\alpha_q \hat{\mathbf{q}}_t$		
$\alpha_R = .9 \quad \alpha_\pi = 3 \quad \alpha_y = .5 \quad \alpha_q = 1$	2.10296440	2.0936
$\alpha_R = .9 \quad \alpha_\pi = 2 \quad \alpha_y = .5 \quad \alpha_q = 1$	8.95578571	8.9464

Table 2.4: Different Policy Rules: Welfare Comparison
The welfare loss represent the loss in terms of consumption with respect to the steady state's welfare. The % Loss is the welfare loss with respect to the optimal rule

Volatility (γ =0.6)

	q	π
$\hat{\mathbf{R}}_t = \alpha_\pi \hat{\pi}_t + \alpha_y \hat{\mathbf{y}}_t + \alpha_q \hat{\mathbf{q}}_t$		
$\alpha_\pi = 3 \quad \alpha_q = 0$	2.015	0.044
$\alpha_\pi = 3 \quad \alpha_q = 1$	2.104	1.082
$\alpha_\pi = 3 \quad \alpha_y = .5 \quad \alpha_q = 1$	2.089	1.401
$\alpha_\pi = 1.5 \quad \alpha_y = .5$	2.128	1.356

Table 2.5: Policy Rules Volatility
Percent Standard deviation

in fact, a 0.5 response to output reduces welfare by approximately 0.1% when the response to inflation is 3 and by approximately 1% when α_π is set to 1.5. Even worse is the case in which the interest rate also responds to housing prices. The welfare loss is 1.5% in the first case and 5% in the second in the absence of interest rate smoothing, and approximately 2% and 8%, respectively, in the presence of a target for lagged interest rate in addition to inflation, housing prices, and output. Positive interest rate smoothing worsens the welfare performance of the simple rules considered here.

Finally, we demonstrate how inflation and housing price volatility varies depending on different simple rules (see Table 2.5). As expected, the lowest inflation volatility is achieved under the optimal rule, and the same holds for housing prices. In fact, all the other rules imply higher volatility for both variables.

2.9.1 Credit rationing and optimal monetary policy

Now we check the robustness of the results under different degrees of access to the credit market. In the baseline model we assume that households can borrow up to 60% of the expected next-period value of their houses.[34] Independently of the value of γ the optimal result remains unchanged (see Table 2.6). Thus,

[34]In Italy for instance, until the mid-80 a maximum loan to value ratio of 50% was imposed by regulation. Following deregulation this ratio was increased to 75% in 1986 and to 100% in 1995

Optimal Simple Rules

rule	$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_t + (1-\alpha_R)\alpha_y \hat{y}_t + (1-\alpha_R)\alpha_q \hat{q}_t$			
optimal weights	$\alpha_R = 0$	$\alpha_\pi = 3$	$\alpha_q = 0$	$\alpha_y = 0$
γ	$\gamma = .001$	$\gamma = .3$	$\gamma = .4$	$\gamma = .6$
Welfare Loss	0.00978993	0.00962163	0.00951627	0.00917090

Table 2.6: Optimal Simple Rules at Different L2V Ratios
The welfare loss represent the loss in terms of consumption with respect to the steady state's welfare

the degree of access to the credit market doesn't affect the design of optimal monetary policy. The welfare loss with respect to the steady state's welfare decreases with γ. The lower the collateral requirement the higher the welfare level.

However, as Table 2.7 shows, the welfare cost of deviating from the optimal rule, increases with γ. In fact, the welfare cost of introducing housing prices, lagged interest rate or output target in addition to an inflation target is higher the higher the degree of access to the credit market. As often found in theoretical and empirical contributions that analyze the effect of liberalizing, increasing competitiveness or opening financial markets also in this case there are perils when the degree of access to credit markets increase. The bigger the credit market the higher the resources invested in it and the greater it is its relevance in the monetary policy transmission mechanism; hence, in this case, the impact of 'bad' policies may be amplified and propagated relatively more and consequently generate higher welfare losses.

We now look to the volatility of inflation under different rules (see Table 2.8). As expected the optimal rule, independently of γ, implies the lowest volatility. If more variables than simply inflation are targeted, the volatility of inflation increases. Applying a housing price target reduces the effectiveness of the target

Deviating From Optimality

γ	$\gamma = 0.001$	$\gamma = 0.3$	$\gamma = 0.4$	$\gamma = 0.6$
weights	$\alpha_\pi = 3$	$\alpha_q = 1$		
welfare loss	0.93544628	0.95492812	0.96848473	1.01134530
% Loss relative	0.9257	0.9453	0.9590	1.0022
weights	$\alpha_\pi = 3$	$\alpha_q = 1$	$\alpha_y = 0.5$	
welfare loss	1.39750869	1.42845289	1.44974618	1.52065058
% Loss relative	1.3877	1.4188	1.4402	1.5115
weights	$\alpha_\pi = 3$	$\alpha_R = 0.9$		
welfare loss	0.01464221	0.01505223	0.01525402	0.01586302
% Loss relative	0.0049	0.0054	0.0057	0.0067

Table 2.7: Simple Rules: Deviating from Optimality.
The welfare loss represent the loss in terms of consumption with respect to the steady state's welfare. The % Loss is the welfare loss with respect to the optimal rule

Simple Rules and Inflation Volatility

$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_t + (1-\alpha_R)\alpha_y \hat{y}_t + (1-\alpha_R)\alpha_q \hat{q}_t$

γ	$\gamma = 0.001$	$\gamma = 0.3$	$\gamma = 0.6$	$\gamma = 0.75$
$\alpha_\pi = 3$ (*optimal simple rule*)	0.0513	0.0493	0.0447	0.0406
$\alpha_\pi = 3, \alpha_q = 1$	1.0603	1.0656	1.0801	1.0823
$\alpha_\pi = 3, \alpha_R = 0.9$	0.0790	0.0798	0.0804	0.0793
$\alpha_\pi = 1.5, \alpha_y = 0.5$	1.3410	1.3505	1.3561	1.3568
$\alpha_\pi = 3, \alpha_q = 1, \alpha_y = 0.5$	1.3702	1.3787	1.4056	1.4178
$\alpha_R = 0.9, \alpha_\pi = 3, \alpha_q = 1, \alpha_y = 0.5$	1.4451	1.4866	1.6121	1.8225

Table 2.8: Simple Rules and Inflation Volatility.
Percent Standard deviation

on inflation, and the same holds for an output target. Targeting the lagged interest rate, in contrast, has a negligible effect on inflation volatility. Consistently with the results concerning the cost of deviating from the optimal rule, over the different rules considered, inflation volatility increases slightly with γ, the only exception being the optimal rule case. In fact, when the degree of credit market access is higher, monetary policy is more effective. Thus, unless the central bank follows the optimal rule, increasing credit market access, and thus reducing inefficiency, implies an increase in the volatility of inflation.

2.10 Conclusions

We have examined optimal monetary policy rules in an economy that contains credit market frictions at the household level and heterogeneous agents. To assess the potential role of housing price considerations in designing monetary policy, we rely on a model based on that of Kiyotaki andMoore (1997). Thus, two types of agents, differing in terms of their discount factors, are assumed and physical assets must be used as loan collateral.

As a result, housing price movements should not be a separate target variable in addition to inflation in an optimally designed simple monetary policy regime. In fact, making housing price stability an explicit objective is welfare reducing with respect to a strict price stability policy. Our results are in line with the idea that under normal circumstances, asset prices should not be considered to be targets of monetary policy, as has previously been stressed by Svensson (2004).[35]

Moreover, the welfare loss engendered by targeting housing prices be- comes quantitatively more significant the greater the access to the credit market. Reducing credit market imperfections implies decreasing inflation volatility and improving welfare if and only if the central bank adheres to an optimally designed simple rule.

Appendix

A Steady State

The real wage in steady state equals the real marginal cost:

[35]Svensson argues that performing a *flexible inflation targeting* there is no need for the ECB to take asset prices movements into account.

$$w = s = \frac{\theta - 1}{\theta} \tag{ss.1}$$

Given β_1 and assuming $\pi_{ss} = 1$, we find the following steady state value for the interest rate:

$$R = \frac{1}{\beta_1} \tag{ss.2}$$

Since the deterministic steady state for the other variables is not solvable analytically, a *nonlinear rootfinding problem* arises. In a nonlinear rootfinding problem, a function f mapping $\mathbb{R}^n$ to $\mathbb{R}^n$ is given and one must compute an n-vector x, called a *root* of f, that satisfies $f(x) = 0$. In our problem the $f(x)$ is represented by the following equations:

$$-U_{L_1} = U_{c_1} w \qquad -U_{L_2} = U_{c_2} w$$

$$\tfrac{U_{h_1}}{q} = U_{c_1} (1 - \beta_1) \qquad \tfrac{U_{h_2}}{q} = U_{c_2} (1 - \beta_2) - \gamma\mu$$

$$\mu = U_{c_2} (\beta_1 - \beta_2)$$

$$c_2 = b_2 \left(\frac{1}{R} - 1 \right) + wL_2$$

$$b_2 = \gamma q h_2 \qquad b_1 = \tfrac{n b_2}{(1-n)}$$

$$qh = q(1-n)h_{1t} + nh_{2t}$$

$$h_1 = \tfrac{qh_1}{q} \qquad h_1 = \tfrac{qh_2}{q}$$

$$h = 1$$

$$c = (1-n)c_1 + nc_2 \qquad L = (1-n)L_1 + nL_2$$

$$y = c \qquad c = L$$

Where

$$U_{c_i} = c_i^{-\varphi_c} \qquad U_{L_i} = -\nu_L L_i^{+\varphi_L} \qquad U_{h_i} = \tfrac{\nu_h}{h_i}$$

We implement a numerical algorithms for solving the system quickly and accurately.

$$U(c_{it}, h_{it}, L_{it}) = \frac{c_{it}^{1-\varphi_c}}{1-\varphi_c} + \nu_h \ln h_{it} - \nu_L \frac{L_{it}^{1+\varphi_L}}{1+\varphi_L}$$

B Solution Method

The set of equilibrium conditions and the welfare function of the model can be written as:

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0$$

where E_t is the expectation operator,y_t is the vector of non-predetermined variable and x_t of predetermined variables. This last vector consists of x_t^1 endogenous predetermined state variables and x_t^2 exogenous state variables. In the baseline case of our model we have:

$$y_t = [\pi_t, q_t, w_{t,} y_t, L_t, c_t, s_t, V_{1t}, V_{2t}]'$$

$$x_t^1 = [b_{2t}, h_{2t}, R_t]' \quad x_t^2 = [Z_t, G_t]'$$

The welfare function is given by the conditional expectation of lifetime utility as of time zero: $V_{it} \equiv \max E_t \left[\sum_{j=0}^{\infty} \beta_i^j U(c_{i,t+j}, h_{i,t+j}, L_{i,t+j})\right]$ .Thus, in the optimum it will be: $V_{it} = U(c_{i,t}, h_{i,t}, L_{i,t}) + \beta_i E_t V_{it+1}$.We add to the system of equilibrium conditions, two equations in two unknowns: V_{1t} and V_{2t}.

The vector of exogenous state variables follows a stochastic process:

$$x_{t+1}^2 = \Delta x_t^2 + \eta \varepsilon_{t+1} \qquad \varepsilon_t \sim iidN(0, \Sigma)$$

where η a matrix of known parameters[36].

[36]In our model, since the shocks ar uncorrelated, η is a vector.

The solution of the model is given by the policy function and the transition function:

$y_t = g(x_t, \sigma) \qquad x_t = h(x_t, \sigma) + \eta\varepsilon_{t+1}$ where σ^2 is the variance of the shocks.

Following Schmitt-Grohe and Uribe (2003), we compute numerically the second order approximation of the functions g and h around the non-stochastic steady state $x_t = x$ and $\sigma = 0$. The solution of the system gives an evolution of the original variables of the form

$$y_t = \alpha_1 x_t^1 + \alpha_2 x_t^2 + \alpha_3 \left(x_t^1\right)^2 + \alpha_4 \left(x_t^2\right)^2 + \alpha_5 x_t^1 x_t^2 + \eta\sigma^2$$

where all the variables are expressed in log deviations. The solution also depends on the variance of the shocks.

Since we evaluate the welfare functions conditional on having at t=0 all the variables of the economy equal to their steady state values, the second order approximate solution for the welfare functions is given by[37]:

$$V_{it} = \eta_{V_i}\sigma^2$$

where η_{V_i} is a vector of known parameters that depends on the monetary policy used and σ^2 is the variance of the shocks

[37]Since in the system all the variables are in log-deviation from their steady state values, they equals zero.

C First Order Approximation

The system can be represented by 13 equations in 13 variables, plus exogenous shocks.

$$\hat{y}_t = (1-n)\frac{c_1}{y}\hat{c}_{1,t} + n\frac{c_2}{y}\hat{c}_{2,t} - g_s s g_t \tag{A-1}$$

$$\hat{y}_t = \hat{Z}_t + \hat{L}_t \tag{A-2}$$

$$\hat{L}_t = (1-n)\frac{L_1}{L}\hat{L}_{1,t} + n\frac{L_2}{L}\hat{L}_{2,t} \tag{A-3}$$

$$(1-n)h_1\hat{h}_{1,t} + nh_2\hat{h}_{2,t} = 0 \tag{A-4}$$

$$\varphi_c\hat{c}_{i,t} + \varphi_c\hat{L}_{i,t} = u_t \tag{A-5}$$

$$\hat{b}_{2,t} = \gamma_t + E_t\hat{q}_{t+1} + E_t\pi_{t+1} + \hat{h}_{2,t} \tag{A-6}$$

$$c_2\hat{c}_{2,t} + qh_2\Delta\hat{h}_{2,t} = \gamma q h_2(\beta_1\hat{b}_{2,t} - \hat{b}_{2,t-1} - \beta_1\hat{R}_t + \pi_t) + wL_2(\hat{w}_t + \hat{L}_{2,t}) - \frac{G}{Y}g_t \tag{A-7}$$

$$\varphi_c E_t\Delta\hat{c}_{1,t+1} = \hat{R}_t - E_t\pi_{t+1} \tag{A-8}$$

$$\varphi_c(\beta_2 E_t\hat{c}_{2,t+1} - \beta_1\hat{c}_{2,t}) = \beta_1\hat{R}_t - \beta_2 E_t\pi_{t+1} + (\beta_1-\beta_2)\hat{\mu}_t \tag{A-9}$$

$$(1-\beta_1)(\hat{h}_{1,t} - \nu_{h,t}) + \varphi_c(\beta_1 E_t\hat{c}_{1,t+1} - \hat{c}_{1,t}) = \beta_1 E_t\hat{q}_{t+1} - \hat{q}_t \tag{A-10}$$

$$(1-\tilde{\beta})(\hat{h}_{2,t} - \nu_{h,t}) + \varphi_c(\beta_2 E_t\hat{c}_{2,t+1} - \hat{c}_{2,t}) = \tag{A-11}$$

$$= \tilde{\beta}E_t\hat{q}_{t+1} - \hat{q}_t + \gamma(\beta_1-\beta_2)(\hat{\mu}_t + \gamma_t + \pi_{t+1})$$

$$\pi_t = \beta_1\pi_{t+1} + \frac{\theta-1}{\phi_p}\hat{mc}_t \tag{A-12}$$

$$\hat{mc}_t = \hat{w}_t - \hat{Z}_t \tag{A-13}$$

$$\hat{R}_t = f(\Omega_t) \tag{A-14}$$

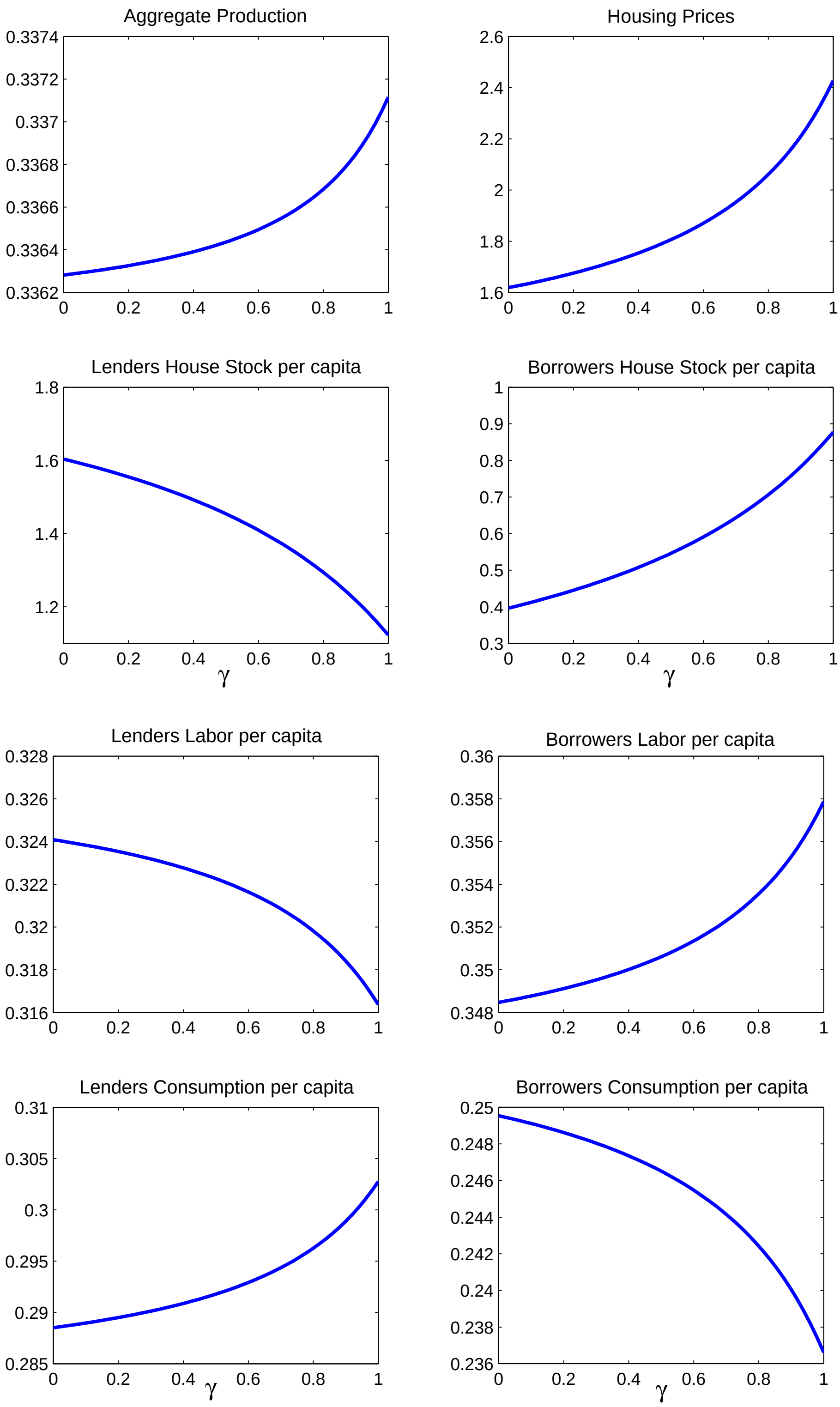


Figure 2.1: Steady State Analysis. Per capita variables. Changing the L2V ratio γ. All other parameters at their baseline values.

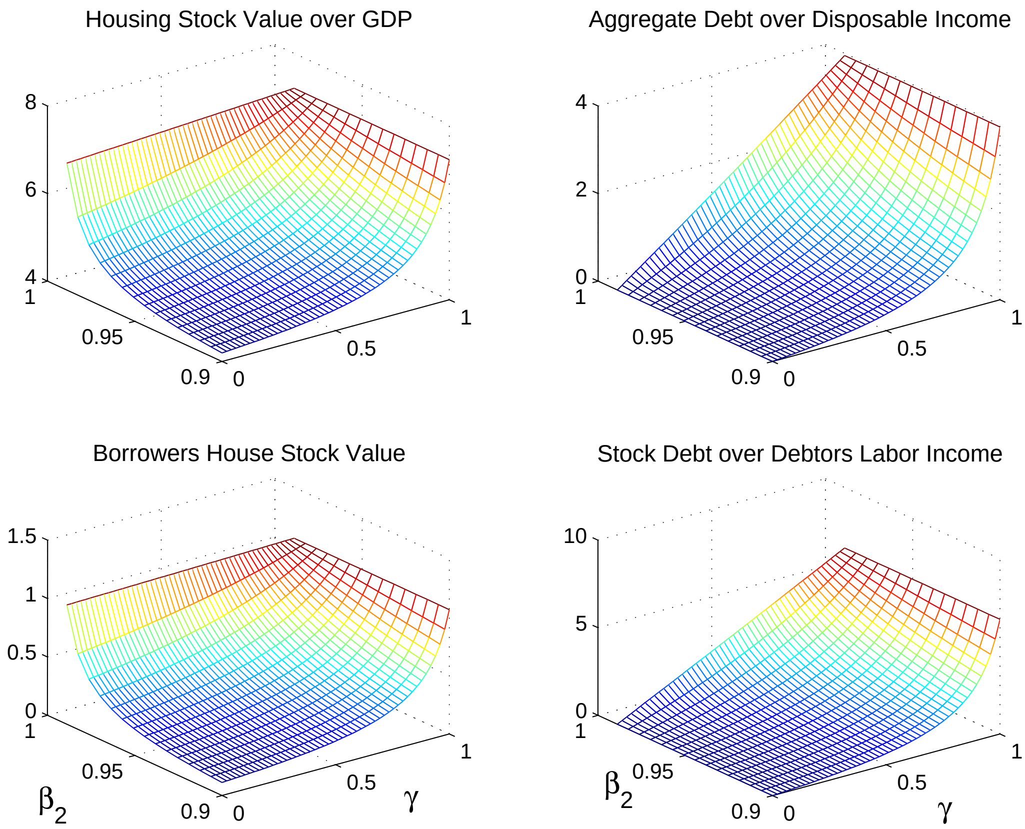


Figure 2.2: Steady State Analysis. Aggregate stocks. γ and β_2.

Changing the L2V ratio γ and borrowers discount factor β_2. All other parameters at their baseline vales.

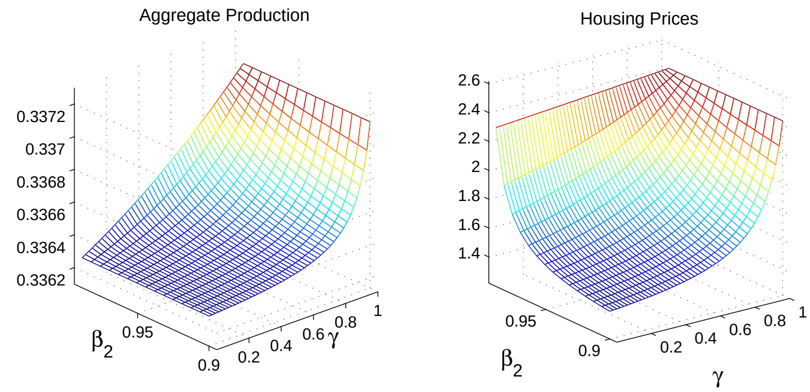


Figure 2.3: Steady State Analysis. Production and housing prices. γ and β_2.

Changing the L2V ratio γ and impatient discount factor β_2. All other parameters at their baseline vales.

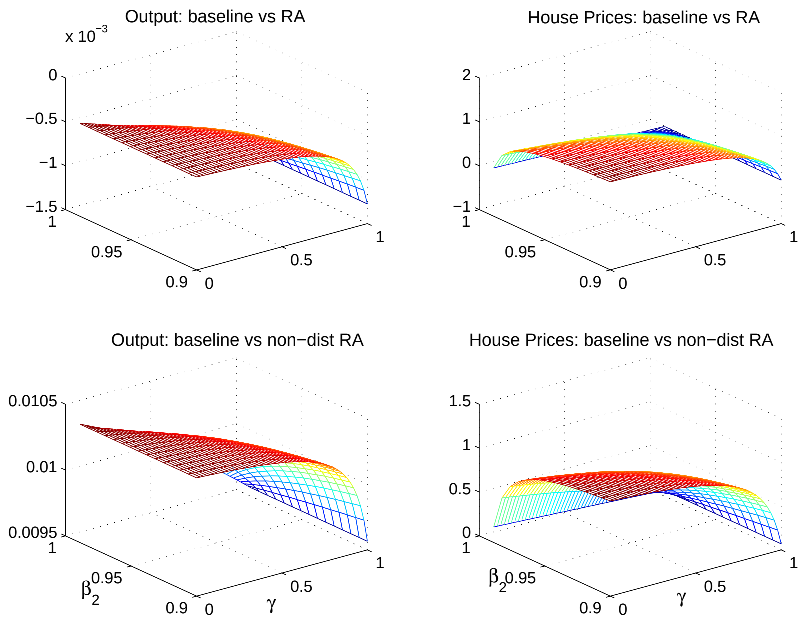


Figure 2.4: Steady State Analysis. Output and housing prices. Comparison.

We compare the (distorted and non-distorted) steady state of the representative agent equivalent model. Changing the L2V ratio γ and impatient discount factor β_2. All other parameters at their baseline vales.

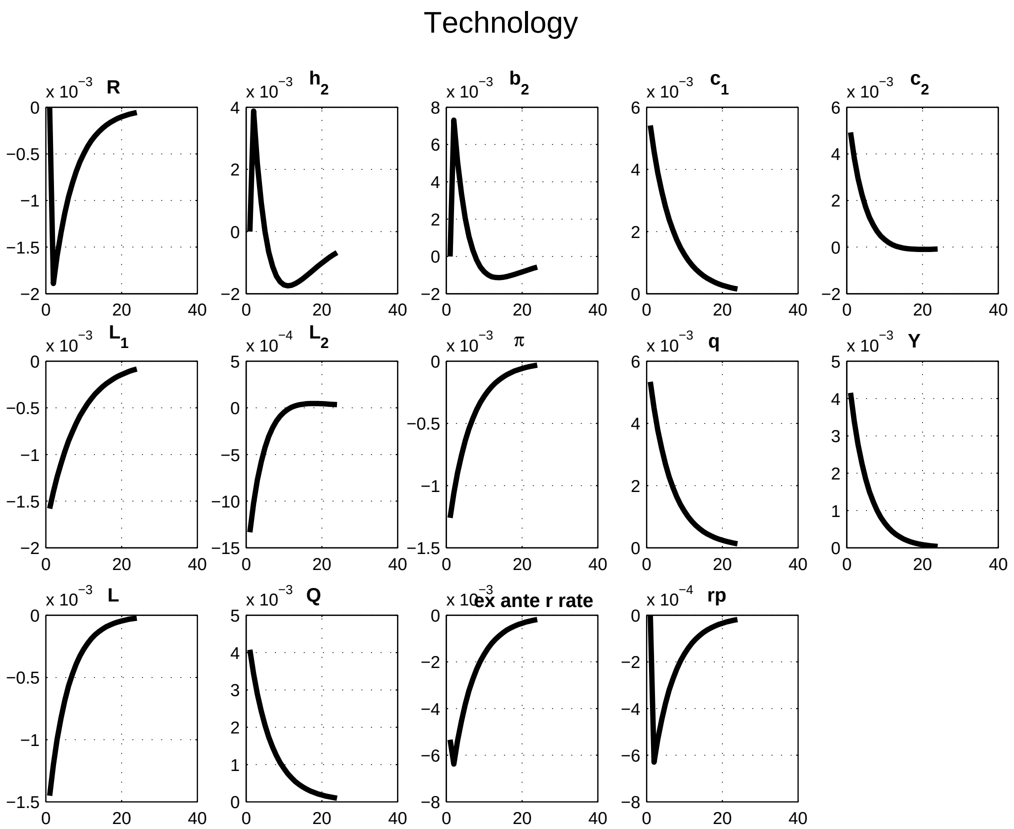


Figure 2.5: Impulse Response Function. Technology shock. Baseline.

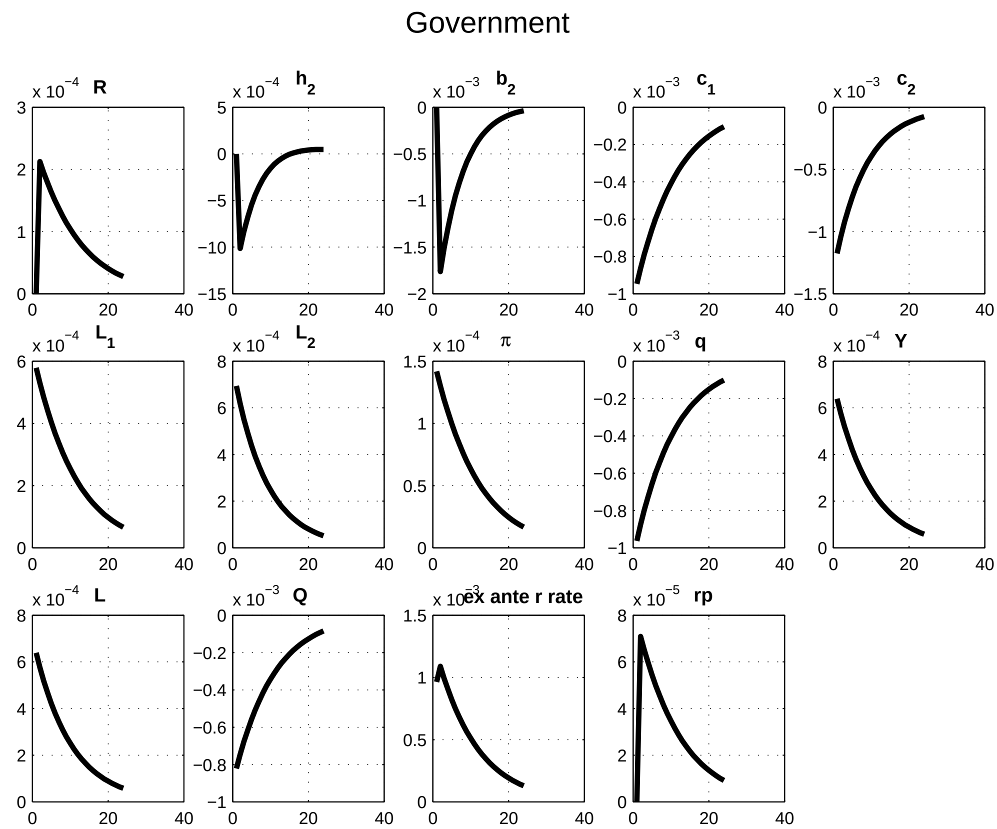


Figure 2.6: Impulse Response Function. Government spending shock. Baseline.

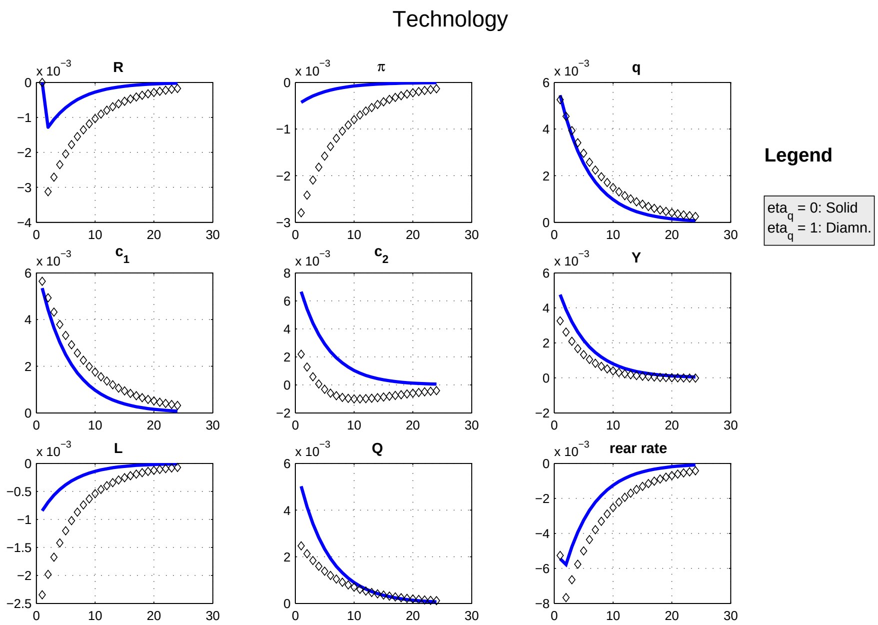


Figure 2.7: Impulse Response Function. Technology shock. Reaction to House Prices

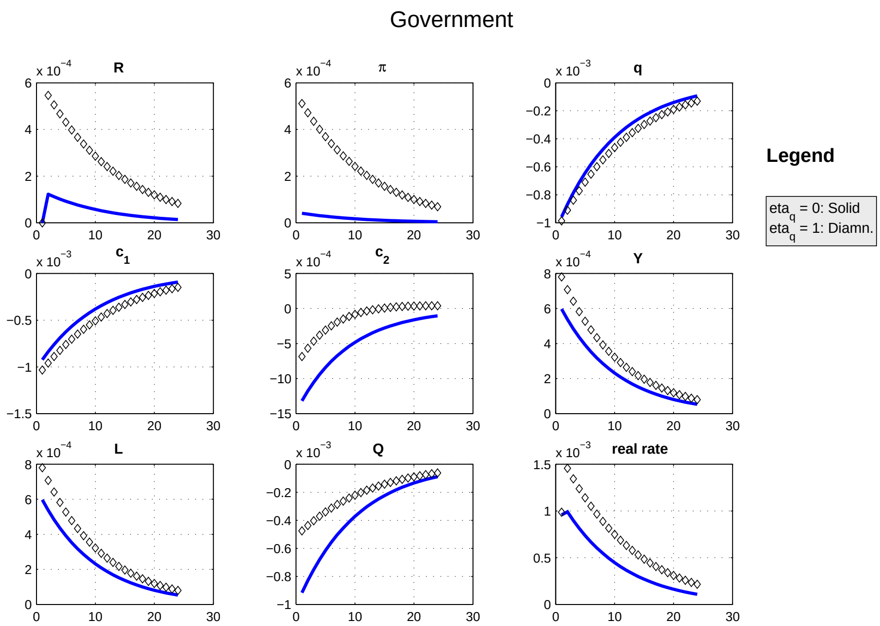


Figure 2.8: Impulse Response Function. Government spending shock. Reaction to House Prices

Chapter 3

Fiscal Policy and Macroeconomic Stability in a Monetary Union*

The fiscal rules that EMU country members are requested to respect have stimulated and renewed a long-standing debate about the spillovers of nationally-driven fiscal policies in a monetary union.[1]

International fiscal spillovers are particularly relevant in a highly integrated area. Changes in the level of public expenditure and taxation in one country are able to affect the economic performance of its partners. The related change in relative national welfare levels raises the question about the opportunity of some form of coordination or centralization of national fiscal policies.[2]

Recently, several contributions have investigated empirically the spillovers of fiscal policy. Giuliodori and Beetsma (2004) use a VAR to explore cross-border effects of fiscal policy. Main result is that a fiscal expansion in Germany, France and Italy - the three major countries in the euro area - leads to significant

*This Chapter is part of a joint work with Massimiliano Pisani - Bank of Italy, Research Department. Usual disclaimers hold.

[1]Among the first contributes, see for istance Buiter, Corsetti and Roubini (1993), De grauwe (1998), Eichengreen and Wyplosz (1998) and Giavazzi and Pagano (1990).

[2]See among the others Canzoneri and Henderson (1991), Canzoneri et al. (2004), Lombardo and Sutherland (2003).

higher imports from other countries belonging the European union by stimulating domestic activity. By contrast, any direct spillover caused by government purchases of foreign goods seems to be unimportant. In the same spirit, Fatàs and Mihov (2001a,b) estimate a VAR using OECD data. They find that a positive discretionary change in public expenditure leads to an increase in output and, most importantly, private consumption and imports.[3]

In this paper we analyze the short-run effects of fiscal policy in a monetary union. Following Benigno (2003), we develop a two-country setup with centralized monetary policy and sticky prices. Consistently with the above reported evidence, we allow for positive effects of public expenditure on private consumption. Following Gali et al.(2003, 2004), we assume there are two types of agents in each region of the union: Ricardian agents that have access to financial markets and smooth consumption over time; rule-of-thumb agents that do not save and in each period consume all their available income.[4] Thanks to rule-of-thumb agents, in each region aggregate consumption may increase after an exogenous positive shock in public expenditure, with the consequence of further stimulating not only the domestic economic activity but also that of the other region belonging to the union. Public expenditure can be financed through public debt and/or lump sum taxation. The latter is set according to a systematic simple tax rule. Given that rule-of-thumb agents break the ricardian equivalence, we can compare alternative tax rules in terms of their macroeconomic stabilization properties.

[3]Blanchard and Perotti (2002) estimate a VAR using U.S. data. They also find that private consumption and domestic activity increase after a positive fiscal shock. For other empirical studies on fiscal policy effects, see Canova and Pappa (2004), Perotti (1999), Mountford and Hulig (2004), Ramey and Shapiro (1998), Hemming et al. (2002).

[4]See Campbell and Mankiw (1989).

To maintain simplicity and tractability, we abstract from a number of issues. First, we only look at the short run effects of fiscal policy. We therefore concentrate on spillovers induced by changes in terms of trade, the amount of imports and the area-wide interest rate. We do not consider issues such as the long-run sustainability of public debt. Second, we do not specify the preferences over public goods and mainly look at the impact of spending changes on aggregate demand. Third, we (realistically) confine our analysis to public goods that are supplied domestically. Fourth, we do not perform a micro-founded welfare analysis.

We conduct several exercises. We initially analyze whether the Taylor principle - which says that in closed economy an active reaction of the monetary policy to inflation guarantees a unique equilibrium - holds in a currency union featuring non-Ricardian agents. Then we perform an impulse response analysis of the public spending spillovers. Finally, we compare alternative fiscal rules in terms of their ability of affecting the cyclical properties of inflation and output.

The main results are as follows.

First, according to the determinacy analysis, the Taylor principle does not hold when the share of rule-of-thumb agents and the degree of price stickiness are sufficiently high: aggregate private demand becomes more sensitive to current available income than to the real interest rate.

Second, setting the parameters to values commonly used in the literature, we are able to replicate the increase of the private demands for imports following a domestic public spending shock. The positive spillovers on the other member of the area - generated by changes in relative prices, the common nominal interest rate and the amount of traded goods - are stronger the higher are the share of rule-of-thumb, the lower the home bias, the higher elasticity of substitution between domestic and imported goods, the bigger the relative size of the region in which the shock originates.

Third, more 'flexible' regional fiscal rules - i.e. ones that countercyclically react to domestic output - reduce the macroeconomic volatility.

Our paper contributes to the recent theoretical literature of fiscal policy in a monetary union. Corsetti and Pesenti (2001) analyze the spillovers induced by public spending through changes in the terms of trade. Duarte and Wolman (2002) analyze the impact of national public expenditure on inflation differentials across regions of a currency union. However, these authors do not consider Keynesian effects of public spending. Coenen and Straub (2005) revisit the effects of government spending shocks on private consumption within an estimated New-Keynesian DSGE model of the euro area featuring rule-of-thumb households. Their setup, differently from ours, is based on a closed economy; hence, they do not consider cross-regional spillovers. Finally, Canzoneri et al (2004) develop a framework close to ours, based on a two-region currency union and rule-of-thumb agents. They focus on effects of public expenditure on regional inflation differential. Differently from us, they do not perform a systematic analysis neither of the equilibrium determinacy nor of the stabilization properties of alternative fiscal rules.

The paper is structured as follows. Next section illustrates the setup. Section three describes the results. Section four illustrates the conclusions.

3.1 The setup

We develop a two-region model with sticky prices, a common central bank and two fiscal authorities. Each fiscal authority has sovereignty over only one region. The two regions are labelled H (Home) and F (Foreign)and may have different size. The whole area is populated by a continuum of households on the interval $[0,1]$. The population on the segment $[0,n)$ belongs to region H $(0<n<1)$, while the population on the segment $[n,1]$ belongs to F. There is no possibility of migration across regions. We assume that in each region a fraction of the households does not have access to financial markets and hence consumes all the available income. We call them rule of thumb agents. In region H the share of rule-of-thumb agents over its population is equal to λ (with $0\leq\lambda<1$), in region F to λ^* (again $0\leq\lambda^*<1$). Remaining households, to the contrary, have access to a complete set of internationally traded state contingent securities. We call them optimizing or Ricardian agents. In what follows, we indicate variables relative to the region F with a star (*).

3.1.1 Preferences

The preferences of the generic household belonging to region H can be expressed as (equivalently for the other region):

$$U_t = E_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}\left[\frac{C_s^{1-\sigma}}{1-\sigma}-\frac{N_s^{1+\varphi}}{1+\varphi}\right]\right\} \tag{3.1.1}$$

E_t denotes the expectation conditional on the information set at date t, while β is the intertemporal discount factor $(0<\beta<1)$. Agents obtain utility from consumption C, while they receive disutility from supplying labor N. The utility function is separable in these two factors. The elasticity of intertemporal substitution is $1/\sigma$ $(\sigma>0)$, while $1/\varphi$ is the Frisch labor elasticity $(\varphi>0)$. The index C is defined as:

$$C_t \equiv \left[\gamma_H^{\frac{1}{\theta}}C_{H,t}{}^{\frac{\theta-1}{\theta}}+(1-\gamma_H)^{\frac{1}{\theta}}\,C_{F,t}{}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{3.1.2}$$

where C_H and C_F are indexes of consumption across the continuum of differentiated goods produced respectively in region H and F:

$$C_{H,t} \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n c(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad C_{F,t} \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 c_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.1.3}$$

The elasticity of substitution between the bundles C_H and C_F is θ ($\theta > 0$), while the elasticity of substitution across goods produced within a country is ε ($\varepsilon > 1$).

Similarly for region F we have

$$C_t^* \equiv \left[(1-\gamma_F)^{\frac{1}{\theta}} C_{H,t}^{*\ \frac{\theta-1}{\theta}} + \gamma_F^{\frac{1}{\theta}} C_{F,t}^{*\ \frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{3.1.4}$$

The bundles C_H^* and C_F^* are similar to their counterparts in region H. The parameter γ_H and γ_F are the weights on domestic bundles. They measure the degree of home bias. If $n = .5$ and $.5 < \gamma_H = \gamma_F < 1$ we have an identical home bias across countries. If $\gamma_H > n$ and $\gamma_F > (1-n)$ there is home-bias.

We assume that in each region the composition of the investment basket is the same as that of the corresponding consumption bundle. Hence, in the region H the index of investment I is defined as:

$$I_t \equiv \left[\gamma_H^{\frac{1}{\theta}} I_{H,t}^{\frac{\theta-1}{\theta}} + (1-\gamma_H)^{\frac{1}{\theta}} I_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{3.1.5}$$

where the indexes I_H and I_F are defined as:

$$I_{H,t} \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n I_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad I_{F,t} \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 I_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.1.6}$$

In region F, the following investment index holds:

$$I_t^* \equiv \left[(1-\gamma_F)^{\frac{1}{\theta}} I_{H,t}^{*\frac{\theta-1}{\theta}} + \gamma_F^{\frac{1}{\theta}} I_{F,t}^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{3.1.7}$$

The bundles I_H and I_F are similar to their counterparts in region H.

We assume that in each region public expenditure is completely biased towards domestically produced goods and that the aggregator is similar to that of private agents; hence, we can define public expenditure bundle G in the region H and G^* in region F as:

$$G_t \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n g(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad G_t^* \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 g_t^*(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.1.8}$$

We derive the price indexes from the above described bundles. The price index P is the minimum expenditure in region H required to purchase goods resulting in the index C, such that $C = 1$. It is equal to:

$$P_t = \left[\gamma_H P_{H,t}^{1-\theta} + (1-\gamma_H) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{3.1.9}$$

where P_H and P_F are equal to:[5]

$$P_{H,t} = \left[\left(\frac{1}{n} \right) \int_0^n p(h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}, \quad P_{F,t} = \left[\left(\frac{1}{1-n} \right) \int_n^1 p_t(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}} \tag{3.1.10}$$

The price indexes in region F are similarly defined. The consumer price index is equal to:

$$P_t^* = \left[(1-\gamma_F) P_{H,t}^{*1-\theta} + \gamma_F P_{F,t}^{*1-\theta} \right]^{\frac{1}{1-\theta}} \tag{3.1.11}$$

where P_H^* and P_F^* are defined as their counterparts in region H:

$$P_{H,t}^* = \left[\left(\frac{1}{n} \right) \int_0^n p^*(h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}, \quad P_{F,t}^* = \left[\left(\frac{1}{1-n} \right) \int_n^1 p_t^*(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}} \tag{3.1.12}$$

We assume that firms set prices considering the whole area as a common market and that there are no transaction costs in transporting goods across regions. It follows that the law of one price holds:

$$p_t(h) = p_t^*(h), \quad p_t(f) = p_t^*(f) \tag{3.1.13}$$

[5] The index P_H is the minimum expenditure in region H required to purchase goods resulting in the index C_H, such that $C_H = 1$. A similar definition applies to the index P_F, P_H^*, P_F^*.

Given the structure of consumption and investment bundles, the law of one price implies that:

$$P_{H,t} = P^*_{H,t}, \quad P_{F,t} = P^*_{F,t} \tag{3.1.14}$$

We define the terms of trade T of the region H as the ratio of the price of the bundle of goods imported from region F relative to the price of the bundle domestically produced:

$$T_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{3.1.15}$$

The real exchange rate of the region H is defined as the ratio of the consumer price index of region F relative to that of region H:[6]

$$RS_t \equiv \frac{P^*_t}{P_t} \tag{3.1.16}$$

3.1.2 Intratemporal Allocation

Given a decision on C, household in region H optimally allocates the expenditure on C_H and C_F by minimizing the total expenditure PC under the constraint given by (3.1.2). The resulting demands are:

$$C_{H,t} = \gamma_H \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t, \quad C_{F,t} = (1-\gamma_H)\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} C_t \tag{3.1.17}$$

Then, given the decisions on C_H and C_F, the household allocates the expenditure among the differentiated goods by minimizing expenditures $P_H C_H$ and $P_F C_F$ under the constraints given by (3.1.3). The demands of a generic good h and f are:

$$C_t\left(h\right) = \frac{1}{n}\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}, \quad C_t\left(f\right) = \frac{1}{1-n}\left(\frac{p_t(f)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{3.1.18}$$

[6]The real exchange rate is not constant because of the home bias assumption. For the purchasing power parity condition to hold, in fact, the assumptions of international law of one price, tradeability of goods and symmetric preferences should be satisfied. Home bias implies that preferences are not symmetric, but mirror symmetric.

Similar equations hold for the investment goods and for bundles of agents belonging to region F. Total demands of good h and f are:

$$\begin{aligned} y_t(h) &= \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\varepsilon}\left(C_{H,t}+I_{H,t}+C^*_{H,t}+I^*_{H,}+G_t\right) \\ y_t(f) &= \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\varepsilon}\left(C_{F,t}+I_{F,t}+C^*_{F,t}+I^*_{F,t}+G^*_t\right) \end{aligned} \tag{3.1.19}$$

Finally, we compute aggregate demand in both regions by using the appropriate Dixit-Stiglitz aggregators:

$$Y_{H,t} \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}}\int_0^n y(h)^{\frac{\varepsilon-1}{\varepsilon}}dh\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad Y_{F,t} \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}}\int_n^1 y_t(f)^{\frac{\varepsilon-1}{\varepsilon}}df\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.1.20}$$

We apply equation (3.1.20) to (3.1.19) and obtain:

$$Y_{H,t} = \gamma_H\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}(C_t+I_t)+(1-\gamma_F)\left(\frac{P_{H,t}}{P^*_t}\right)^{-\theta}(C^*_t+I^*_t)+G_t \tag{3.1.21}$$

$$Y_{F,t} = (1-\gamma_H)\left(\frac{P_{F,t}}{P_t}\right)^{-\theta}(C_t+I_t)+\gamma_F\left(\frac{P_{F,t}}{P^*_t}\right)^{-\theta}(C^*_t+I^*_t)+G^*_t \tag{3.1.22}$$

3.1.3 Ricardian Agents

We assume that the Ricardian agents, differently from the rule-of-thumb agents, have access to financial complete markets, at domestic and international level. Having the same preferences it is possible to show that exists a representative Ricardian agent in each country. We will use the superscript o to label his variables. In period-t each Ricardian agent in region-H chooses consumption C^o_t, capital K^o_{t+1}, investment I^o_t, a portfolio of nominal state-contingent securities A^o_{t+1} with a pricing kernel $\Lambda_{t,t+1}$ and a nominal one-period risk-less bond B^o_t issued by the national government[7]. This is in order to maximize the utility

[7]At the time of the portfolio decision, A^o_{t+1} is a random variable, whose value will depend upon the state of the world in period $t+1$. However the household chooses the complete

function (3.1.1) subject to (Ricardian agents in the Foreign region face a similar constraint):

$$\begin{aligned} & E_t \left\{ \Lambda_{t,t+1} A^o_{t+1} \right\} - A^o_t + B^o_t / R_t - B^o_{t-1} \\ \leq \; & W_t N^o_t + R^k_t P_t K^o_{t-1} \\ & + D^o_t - P_t T^o_t - P_t C^o_t - P_t I^o_t \end{aligned} \tag{3.1.23}$$

where W_t is the nominal wage, $R^K_t P_t K^o_{t-1}$ are nominal revenues from renting physical capital holdings K^o_{t-1} to firms at the real rental cost R^K_t. D^o_t are nominal profits from owning shares of one or more domestic firms. T^o_t are lump-sum taxes (or transfer, if negative) paid by Ricardian agents. The capital is accumulated according to the following law:

$$K^o_t = (1-\delta) K^o_{t-1} + \phi \left(\frac{I^o_t}{K^o_{t-1}} \right) K^o_{t-1} \tag{3.1.24}$$

where the term $\phi \left(\frac{I^o_t}{K^o_{t-1}} \right) K^o_{t-1}$ represents the capital adjustment costs. We assume $\phi' > 0$ and $\phi'' \leq 0$, with $\phi'(\delta) = 1$, and $\phi(\delta) = \delta$.

Given the international financial markets structure, idiosyncratic risk is shared across households that have access to financial markets. At the margin consumption utilities, weighted by the real exchange rate, must be equated in every state of nature. Intuitively, a benevolent social planner would allocate consumption across Ricardian agents such that the marginal benefits from an extra unit of foreign consumption equal its marginal costs, given by the Home marginal utility of consumption times the real exchange rate $RER_t = \frac{P^*_t}{P_t}$, i.e., the relative price

specification of this random variable, its value in every possible state. The absence of arbitrage opportunities (a necessary requirement for equilibrium) then requires that there exist a (unique) stochastic discount factor (or asset pricing kernel),$\Lambda_{t,t+1}$ with the property that the price in period t of any portfolio with random value A_{t+1} in the following period is given by $E_t \Lambda_{t,t+1} A^{o,*}_{t+1}$

of C_t^o in terms of $C_t^{o\star}$.[8] Hence we have the following international risk-sharing condition that holds between Ricardian households:

$$\frac{(C_t^o)^{-\sigma}}{P_t} = \xi_0 \frac{\left(C_t^{o,\star}\right)^{-\sigma}}{P^\star} \tag{3.1.25}$$

The constant ξ_0 represents the initial wealth distribution. Given that we are not interested in levels but only in deviation from the non-stochastic steady state the value of constant ξ_0 does not play any role for our results.

Government bonds, B_t^o, are redundant assets so, by no arbitrage condition, their price must satisfy

$$1/R_t = E_t \Lambda_{t,t+1} \tag{3.1.26}$$

where

$$\Lambda_{t,t+1} = \beta \frac{\left(C_{t+1}^o\right)^{-\sigma} P_t}{\left(C_t^o\right)^{-\sigma} P_{t+1}}$$

The first order conditions for investment and capital are respectively given by the following two equations:

$$Q_t = \frac{1}{\phi'\left(\frac{I_t^o}{K_{t-1}^o}\right)} \tag{3.1.27}$$

$$\begin{aligned} P_t Q_t &= E_t \left\{ \beta \left(\frac{C_{t+k}^o}{C_t^o} \right)^{-\sigma} R_{t+1}^K \right\} \\ &+ E_t \left\{ \beta \left(\frac{C_{t+k}^o}{C_t^o} \right)^{-\sigma} Q_{t+1} \left[1 - \delta + \phi_{t+1} - \left(\frac{I_{t+1}^o}{K_{t+1}^o} \right) \phi'_{t+1} \right] \right\} \end{aligned} \tag{3.1.28}$$

Equation (3.1.27) defines the (real) shadow value of capital in place (the Tobin's Q). Given the assumptions on ϕ, the elasticity of the investment-capital ratio

[8] If we define the probability of being in the state of the world $s' \in S'$ conditional to the present state s as $\pi^{prob}(s'/s)$ the following equation $Q^{kernel}(s'/s) = \beta \frac{u'(C(s'))}{u_c(C(s))} \pi^{prob}(s'/s)$ must hold $\forall s' \in S'$ for all union-wide Ricardian agents - i.e. for all union-wide agents that have access to financial markets

with respect to Q is $\eta \equiv -\frac{1}{\phi''(\delta)\delta}$. Equation (3.1.28) states that the value of capital in place must be equated across time periods. At the optimum, the shadow price of capital must equal the next period's sum of capital's marginal product, shadow value and the capital stock contribution to lower installation costs.

We do not report the intratemporal efficiency condition linking the consumer's marginal rate of substitution and real wage. We follow Gali et al. (2006) and assume that the wage is set by a union, hours are determined by firms' labor demand. We refer the reader to Section (3.1.5) below and Appendix (A) for a detailed description of the labor market.

3.1.4 Rule-of-Thumb Agents

Home rule-of-thumb households do not borrow or save, because of lack of access to financial markets. Hence, they cannot smooth their consumption path. Given that they have same preferences and face the same budget constraint, there exists a representative rule-of-thumb agent in each region. The budget constraint of the rule-of-thumb agent in region H is:[9]

$$P_t C_t^r = W_t N_t^r - P_t T_t^r \qquad (3.1.29)$$

As in the case of optimizing households, hours N_t^r are determined by firms' labor demand and are not chosen optimally by each household given the wage W_t.[10] Taxes T_t^r (or transfer, if negative) are paid (received) in lump-sum fashion.

Rule-of-thumb agents are the key feature of the model: they break the Ricardian equivalence (that holds only for Ricardian agents) and hence allow for

[9] A similar equation holds for rule-of-thumb agents in the region F.

[10] As emphasized by Gali et al. (2004), under a perfectly competitive labor market, hours and consumption of rule-thub agents would move in opposite directions in response to movements in real wages. This is not plausible. Under the alternative framework illustrated below, the three variables are positively correlated.

positive effects of public spending on private demand.

3.1.5 Aggregation

Aggregate consumption is given by a weighted average of the consumption variables for each type of household. So, it is equal to:

$$C_t \equiv n\lambda C_t^r + n\left(1-\lambda\right)C_t^o \tag{3.1.30}$$

while aggregate labor is:

$$N_t \equiv n\lambda N_t^r + n\left(1-\lambda\right)N_t^o \tag{3.1.31}$$

Similarly, aggregate investment and capital are respectively:

$$I_t \equiv n\left(1-\lambda\right)I_t^o \tag{3.1.32}$$

$$K_t \equiv n\left(1-\lambda\right)K_t^o \tag{3.1.33}$$

We assume that wages are determined according to the following schedule:

$$\frac{W_t}{P_t} = L(C_t, N_t) \tag{3.1.34}$$

where the function L is increasing in both arguments, capturing both convex marginal disutility of labor and wealth effects. This function can be interpreted as a generalized wage schedule consistent with a variety of models of wage determination. Given the wage, each firm decides how much labor to hire, and allocates its labor demand uniformly across households, independently of their type. Accordingly:

$$N_t^r = N_t^o \tag{3.1.35}$$

for every t; as a consequence, we get:

$$N_t \equiv n\lambda N_t^r + n\left(1-\lambda\right)N_t^o = nN_t^r = nN_t^o \tag{3.1.36}$$

We assume that the resulting wage markup is sufficiently high (and fluctuations sufficiently small) that the following inequalities are satisfied at all times:

$$L(C_t, N_t) \quad > \quad \left(C_t^o\right)^\sigma N_t^\varphi \tag{3.1.37}$$

$$L(C_t, N_t) \quad > \quad \left(C_t^r\right)^\sigma N_t^\varphi \tag{3.1.38}$$

Both conditions, and their analogues in the region F, guarantee that in each country both types of households will meet firms' labor demand at the prevailing wage (see the Appendix for more details).

3.1.6 Firms

Region H and F have a continuum of monopolistic firms of mass n and $(1-n)$, respectively. Firms solve two problems: a static cost minimization and an intertemporal profit maximization problem. Here, we consider only the problem solved by firms belonging to region H Firms in region F solve a similar problem.

The cost minimization problem

In each period, the generic firm h hires capital and labor from agents belonging to its region and combine them according to a common Cobb-Douglas technology:

$$y(h) = Z_t K_t(h)^\alpha N_t(h)^{1-\alpha} \tag{3.1.39}$$

The result of the cost minimization problem is the marginal cost equation:

$$\frac{MC_t(h)}{P_t} = \frac{\left(R_t^k\right)^\alpha (W_t/P_t)^{1-\alpha}}{Z_t \alpha^\alpha (1-\alpha)^{1-\alpha}} \tag{3.1.40}$$

where MC_t is the nominal marginal cost which is common across firms of the same region. The following two first order conditions, with respect to labor and capital respectively, hold:

$$\frac{W_t N_t(h)}{P_t} = (1-\alpha)\frac{MC_t y(h)}{P_t} \tag{3.1.41}$$

$$R_t^k K_t(h) = \alpha \frac{MC_t y(h)}{P_t} \tag{3.1.42}$$

The price setting problem

Firms set prices in a staggered fashion, as in Calvo (1983): each firm resets its price with probability $1-\vartheta$ each period, independently of the time elapsed since last adjustment. As a consequence, each period a fraction $1-\vartheta$ of producers reset their price, while a fraction ϑ keep their prices unchanged.

A firm resetting its price in period t will maximize:

$$\max_{p_t^{new}(h)} E_t \sum_{k=0}^{\infty} \vartheta^k \left\{\Lambda_{t,t+k} \left(p_t^{new}(h) - MC_{t+k}\right) y_{t+k}\left(h\right)\right\} \tag{3.1.43}$$

subject to the sequence of demand constraints:

$$y_{t+k}\left(h\right) = \left(\frac{p_t^{new}(h)}{P_{H,t+k}}\right)^{-\varepsilon} \left(C_{H,t+k} + I_{H,t+k} + C^*_{H,t+k} + I^*_{H,t+k} + G_{t+k}\right) \tag{3.1.44}$$

where $p_t^{new}(h)$ represents the price chosen by firms resetting prices at time t. The first order condition for the above problem is:

$$E_t \sum_{k=0}^{\infty} \vartheta^k \left\{\Lambda_{t,t+k} \left(p_t^{new}(h) - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k}\right) y_{t+k}\left(h\right)\right\} \tag{3.1.45}$$

At the optimum, firms equate expected discounted marginal revenues to expected discounted marginal costs. Profits are rebated lump-sum to domestic Ricardian households.

Finally, the equation describing the dynamics for the price level of the composite good produced in region H is:

$$P_{H,t} = \left[\vartheta P_{H,t-1}^{1-\varepsilon} + (1-\vartheta)\, p_t^{new}\left(h\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{3.1.46}$$

3.1.7 Fiscal policy

The Home government budget constraint is (in what follows similar relations hold true for the foreign country):

$$\frac{B_{G,t}}{R_t} - B_{G,t-1} = P_{H,t}G_t + P_t T_{X,t} \tag{3.1.47}$$

B_G is the negative of a riskless one-period nominal bond domestically sold (government debt). T_X are total lump-sum taxes (in consumption units) paid by the households. We assume that Ricardian and rule-of-thumb agents are equally taxed ($T^r = T^o$); hence the following equation holds for the total amount of collected taxes:

$$T_{X,t} = (1-\lambda)\, nT_t^o + n\lambda T_t^o = nT_t^o = nT_t^r \tag{3.1.48}$$

We assume that taxes are set accordingly to the following tax rule:

$$\log T_{X,t} = \phi_b(B_{G,t-1}/P_t) + \phi_g \log(P_{H,t}G_t/P_t) + \phi_{yx}\log(Y_{H,t}) + \text{const} \tag{3.1.49}$$

Taxes react to public debt, public expenditure and (possibly) also to output volumes when $\phi_{yx} \neq 0$. To guarantee stability of the government budget constraint we assume $\phi_b > 0$. The higher the parameter the faster the government debt returns to its steady state value. The parameter ϕ_g, instead, determines how the government consumption is initially financed - at the extremes with only taxes $\phi_g = 1$ or with deficit spending $\phi = 0$. The constant term is determined accordingly with steady state values. In most of the sections we will assume that the governments have to finance a stream of public consumption G which evolves exogenously according to the following first order autoregressive process:

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \tag{3.1.50}$$

where $0 < \rho < 1$ and $\varepsilon_{g,t}$ represents an i.i.d shock with constant variance $\sigma^2_{\varepsilon_g}$. We define g_t as $\left(\frac{G_t - \bar{G}}{\bar{Y}}\right)$ where $\bar{G}$ and $\bar{Y}$ are the steady-state level respectively of public expenditure and output.

Below we also analyze a fiscal rule composed by the previous tax rule plus a public spending rule (government consumption now is endogenous):

$$g_t = \rho_g g_{t-1} - \phi_{yg} \log(Y_{H,t}/\bar{Y}) + \varepsilon_{g,t} \tag{3.1.51}$$

When $\phi_{yg} = 0$ we go back to the previous case.

3.1.8 Monetary Policy

We assume that the central bank of the union set nominal interest R_t every period according to the following interest rule:

$$\log R_t = \phi_\pi \log \pi_t^U + \phi_y \log Y_t^U + \text{const} \tag{3.1.52}$$

The rule reacts to the union-wide inflation rate $\pi_t^U \equiv \left(P_t^U / P_{t-1}^U\right)$ and to union-wide output Y_t^U. The constant term is determined accordingly with steady state values. We define P_t^U as:

$$P_t^U = P_t^n P_t^{*1-n}$$

while Y_t^U is defined as:

$$Y_t^U = Y_{H,t}^n Y_{F,t}^{1-n}$$

3.1.9 The market clearing conditions

The following market clearing conditions hold in the region H:

- labor market

$$\int_0^n N_t\left(h\right) dh = nN_t \tag{3.1.53}$$

- capital market

$$\int_0^n K_t\left(h\right) dh = n\left(1-\lambda\right) K_{t-1} \tag{3.1.54}$$

- public sector's bond

$$B_{G,t} = n\,(1-\lambda)\,B_t^o \tag{3.1.55}$$

Similar market clearing conditions hold in the region F.

- The following resource constraints holds, respectively for the Home and Foreign good:

$$Y_{H,t} = \gamma_H \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} (C_t + I_t) + (1-\gamma_F) \left(\frac{P_{H,t}}{P_t^*}\right)^{-\theta} (C_t^* + I_t^*) + G_t \tag{3.1.56}$$

$$Y_{F,t} = (1-\gamma_H) \left(\frac{P_{F,t}}{P_t}\right)^{-\theta} (C_t + I_t) + \gamma_F \left(\frac{P_{F,t}}{P_t^*}\right)^{-\theta} (C_t^* + I_t^*) + G_t^* \tag{3.1.57}$$

3.1.10 The shocks

The model features three sources of uncertainty (in every region).

As we described in the fiscal policy section government spending has an exogenous stochastic component.

$$g_t = \rho_g g_{t-1} + \phi_{yg} \log(Y_{H,t}/\bar{Y}) + \varepsilon_{g,t}$$

when $\phi_{yg} = 0$ government spending is a standard exogenous AR(1) process. If $\phi_{yg} \neq 0$ then it becomes a spending rule with a an exogenous disturbance $\varepsilon_{g,t}$. In both cases we set the autoregressive parameter $\rho_g = 0.87$ in line with most of the empirical evidence on government spending processes.

Total factor productivity, Z_t, is assumed to be a stationary AR(1) process:

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_{z,t}$$

where $0 < \rho_z < 1$ and $\varepsilon_{z,t}$ represents an i.i.d shock with constant variance $\sigma^2_{\varepsilon_z}$.

Finally we introduce a markup shock to the firms' price equation.

$$\log u_t = \rho_u \log u_{t-1} + \varepsilon_{u,t}$$

where $0 < \rho_u < 1$ and $\varepsilon_{u,t}$ represents an i.i.d shock with constant variance $\sigma^2_{\varepsilon_u}$.

In all the following analyzes we will assume a diagonal variance-covariance matrix between the 6 shocks (3 in each country) that characterize the monetary union.

3.1.11 Equilibrium

The equilibrium of the model follows by combining the aggregate demand block with the aggregate supply. It is a sequence of allocations and prices such that - given initial conditions K_{-1}, B_{-1}, B^G_{-1}, $P_{H,t-1}$ and their foreign counterparts - the following conditions hold in the region H (correspondent conditions hold in the region F):

- the representative Ricardian agent satisfies consumption intratemporal conditions (3.1.17) and (3.1.18) and their investment analogues, the risk-sharing condition (3.1.25), the capital accumulation law (3.1.24), the labor market equation (3.1.34), the labor market's participation constraint (3.1.37), the intertemporal conditions (3.1.26)-(3.1.28)

- the representative rule-of-thumb agent satisfies consumption intratemporal conditions (3.1.17), (3.1.18), the budget constraint (3.1.29), the labor market equation (3.1.34) and the labor market's participation constraint (3.1.38)

- the public sector budget constraint (3.1.47) and the tax rule (3.2.1)
- the monetary policy policy rule (3.1.52)
- the clearing conditions (3.1.53)
- the law of motion of exogenous shock (3.1.50)

The model equilibrium is not solvable in a closed form solution. We log-linearize it around a non-stochastic steady state equilibrium. We report both the steady state and the log-linearized equations in the appendix.

3.1.12 Calibration of the Model

We calibrate the model at quarterly frequency as in Gali et al. (2006). The baseline calibration is reported in Table (3.2).

We assume the two regions having equal size ($n = 0.5$) and the share of rule-of-thumb agents in each region, λ and λ^*, are set equal to 0.5. This is still within the range of estimated values in the literature of the weight of the rule-of- thumb behavior (see Gali (2006) and Mankiw(2000)).

We set the capital share, α, in both countries to .35, the depreciation rate of capital, δ, to 0.025 and identical total factor productivity in the two countries. This implies a steady-state investment-to-GDP ratio of about 22% in both countries. Following King and Watson (1996) the elasticity of investment with respect to Tobin's q, η, is set to 1. The elasticity of substitution across brands produced in the same country, ε, is set equal to 6, which means a steady state mark-up of 20 percent. The probability that firms do not adjusting prices, ϑ, is the same across regions and equal to 0.75 (this value corresponds to an average duration of one year).

The elasticity of substitution between goods produced in region H and F, θ, is set equal to 1.5. In the baseline model, for better interpretability of the results we assume no-home-bias - i.e. we set the home-bias parameters, γ_H and γ_F, equal to 0.5 - which means imports are about 40% of the GDP. However we will also use a (more realistic) value for those parameters such as .75 that would imply an import-over-GDP ratio roughly equal to 22%, in line with national accounting values for European countries.
The elasticity of marginal utility with respect to consumption, σ, is set equal to 1. The elasticity of wages with respect to hours, φ, is set equal to 0.2. This is not in line with micro studies but it is still a widely used value in the business cycle literature (see Cooley- Prescott (1995)).

In the baseline calibration we set the weight on inflation in the monetary reaction function,ϕ_π , equal to 1.5; the weight on output, ϕ_Y, equal to 0. In the tax rule, the weight on public expenditure, ϕ_G, is equal to 0.12, the one on public debt, ϕ_b, is equal to 0.3 while ϕ_x, the weight on domestic output is zero (i.e. government consumption is exogenous). The autoregressive coefficient in the law of motion of public spending, ρ_g, is equal to 0.87. Those values correspond to the ones estimated in Gali et. al. (2006). Regarding the other two processes we set the technology shock autoregressive parameter $\rho_z = .90$ and the innovation standard deviation $\sigma_z = 0.057$ while, for the markup shock we have $\rho_u = .30$ and $\sigma_u = 0.010$

As shown in the appendix the share of rot consumers λ and λ^* does not affect the steady state values of aggregate variables and prices: the same result obtained in a closed economy holds (see Gali et. al 2006).
Under the baseline calibration the terms of trade and the RER are equal to one. In Table 3.2 we show prominent steady state and parameters values.

The government spending over GDP ratio is set equal to .20 in both regions.

Variable	Value		Parameter	Value
Aggregate Consumption cap.	1.287		λ, λ^* (RoT Share)	0
Real GDP cap.	2.173		n (Country-H Size)	.5
Investment	0.266		θ (CES home-foreign)	1.5
Capital output ratio	8.309		γ_h, γ_f (home-bias)	.75
Investment-GDP ratio	0.207		ϵ_h, ϵ_f (CES goods)	6
Imports	0.217		σ (Rel Risk Aver.)	2
Imports-GDP ratio	0.200		φ (Inv. Frisch Lab.El.)	0.2
Aggregate Labor	0.347		α_h, α_f (Capital Share)	.35
Real Wage	1.694			
Govt. Spending-GDP ratio	.200			

Table 3.1: Steady State Values (left) relative to the Baseline Calibration (right). "cap." stands for "per capita". Given perfect symmetry values are for both the home and foreign country.

In Table (3.2) we show how long run increase in government spending has a negative spillover to the neighbor country. In particular a 10% increase in the region-H public spending has a positive effect on the terms of trade which appreciates. Higher government consumption crowds private consumption out and investment in. The overall impact on imports in negative. Hence, spillovers on the region-F are negative: foreign output decreases and so does consumption and investment. In the next paragraphs we will focus on the short-run effects of transitory changes in government spending which could revert the previous result.

3.2 Results

In this section we initially analyze under which conditions the model has a unique equilibrium. Then, we investigate the domestic and international effects of a public expenditure shock through an impulse response analysis. Finally, we consider how and if different fiscal rules can help in reducing macroeconomic volatility.

Home	Value		Foreign	Value
Terms of Trade	0.996			
Real Exchange Rate	0.998			
Aggr. Cons. cap.	-1.58		Aggr. Cons. cap.	-0.13
Real GDP cap.	1.86		Real GDP cap.	-0.13
Investment	1.86		Investment	-0.13
Capital output ratio	0.11		Capital output ratio	-0.11
Investment-GDP ratio	0.23		Investment GDP ratio	-0.22
Imports	-0.52		Imports	-0.29
Imports-GDP ratio	-2.34		Imports GDP ratio	-0.17
Aggregate Labor	1.69		Aggregate Labor	0.04
Real Wage	0.17		Real Wage	-0.17
Govt. Spending-GDP ratio	0.10		Govt. Spending GDP ratio	0

Table 3.2: Steady state percentage difference relative to baseline calibration
For the baseline calibration see (Table 3.2). Values are relative to a long run 10% increase in the home country government spending-GDP ratio. Home (left) and foreign (right) country. 'cap.' stands for 'per capita'.

3.2.1 Indeterminacy, rule-of thumb agents and the Taylor principle in a monetary union

As emphasized by Woodford (2001), a monetary rule that reacts to inflation adjusting the nominal interest rate more than one-to-one is a sufficient condition for the existence of a unique equilibrium (Taylor principle). This result holds in a closed economy with full access to financial markets and no capital accumulation. Dupor (2005) finds that a similar result also holds when capital accumulation is added, Gali et al. (2005) when the share of rule-of-thumb agents is sufficiently low.

Here we study the existence of a unique equilibrium in a currency union model with partial access to financial instruments.

In Figure 3.1 (panel A) we explore the existence of a unique equilibrium as a function of the degree of price stickiness (indexed by parameter ϑ) and the weight of rule-of-thumb households (indexed by parameter λ) - keeping values symmetric in both countries. The arrows represent the movement of the indeterminacy region, (the dark one), when the value of the coefficient of relative risk adversion σ increases from its baseline value, equal to one, to two. Remaining parameters are set to their baseline values.

The main result is that the combination of a high degree of price stickiness with a large share of rule-of-thumb agents generates indeterminacy. The intuition of this results can be illustrated with the following example. Let's consider a transitory but persistent increase in the region H's production due to a non fundamental shock. Sluggish price adjustment induces a decline in the markups which allows real wages to go up - even if labor productivity declines given the higher employment. Higher real wages generate a boom in rule-of-thumb consumption. Hence when the share of those agents is high enough their increase in consumption more then offset the decrease in Ricardian consumption and investment (the latter is generated by a monetary rule that reacts with a coefficient bigger than one on inflation). On the other hand, exports to the foreign country are very sensitive to changes in the relative prices. Under the baseline calibration the increase in region H output generates a positive spillover on the neighbor country stimulating its output, employment and so foreign rule-of-thumb consumption. When the terms of trade does not appreciate enough foreign imports (home exports) barely decreases at impact and then increases. A higher share of foreign rule-of-thumb agents further mitigates this effect. This means that aggregate demand for output produced in region H increases, making possible to sustain the persistent boom in output the was originally anticipated by agents. This result is similar to that found by Gali et al. (2005) for a closed economy.

The Figure also shows that a higher σ, to which corresponds a lower intertemporal elasticity of substitution $1/\sigma$, increases the indeterminacy area shifting the frontier towards the origin. When $1/\sigma$ is low, the Ricardian consumption is less sensitive to real interest rate. In terms of the previous example, it decreases less when real interest rate increases; hence, now a lower share of rule-of-thumb agents is able to offset the drop of Ricardian demand.

Figure (3.1) panel B shows results for several configurations of λ (with $\lambda = \lambda^*$) and the parameter measuring the reaction of monetary authority to inflation, ϕ_π. As expected, equilibrium determinacy necessitates a relatively low share of rule-of-thumb households and a relatively high ϕ_π. When λ is relatively high, the inverted Taylor principle holds: to have a unique equilibrium, monetary policy should increase nominal interest rate less than one-for-one when inflation increases. The size of indeterminacy shrinks as ϕ_x, the parameter measuring the reaction of fiscal authority to domestic output, increases.[11] This result is also quantitatively interesting because a positive small reaction of taxes to output (it is $\phi_x = 0.1$ in the figure) entails a big reduction in the indeterminacy (dark) area.

Panel A in Figure (3.2) reports the equilibrium properties for all configurations of λ and λ^*. Remaining parameters are set to their baseline values. As anticipated in the previous example, a combination of high large shares of rule-of-thumb in both regions generates indeterminacy. The size of the indeterminacy region shrink gradually as the size of φ, the inverse of the Frisch labor elasticity, increases (while keeping other parameters constant). If the country-size is not 0.50 the two axes should be weighted by the respective sizes (n and $1 - n$).

Once λ is weighted by n, the country-size does not affect the results as far as there is no home bias - which means in this case that $\gamma_H = n$ and $\gamma_F = 1 - n$.

When the two regions are perfectly symmetric we could always construct a

[11] This is also true when monetary policy itself reacts to union output.

sunspot where the two regional outputs are perfectly positively correlated and treat the currency union as a closed economy.

Instead the slightest deviation from this symmetry would make impossible to assume the two outputs perfectly correlated.[12]

In this case and differently from the closed economy, higher values of σ could reintroduce determinacy (see panel B in Figure 3.2). Then a standard monetary policy rule can still guarantee the determinacy of the equilibrium. Using the previous example, given a lower intertemporal elasticity of substitution the increase in region H output generates a sudden drop in the terms of trade which causes foreign marginal costs and real wages to drop. In presence of a high share of rule-of-thumb agents in region F a strong drop in the real wages implies a dramatic fall in their imports (region H exports). This is able to revert the region H increase in consumption making aggregate demand for output H to decrease. This is not only related to the intertemporal elasticity of substitution but whenever the system absorbs exogenous shocks through changes in relative prices rather then quantities - i.e. more specifically when σ, φ are high and θ is low.[13]

3.2.2 Impulse response analysis

Purpose of this section is to analyze the spillovers of an exogenous increase in Home public expenditure having stimulating effects on private consumption and activity. The crucial parameters are three: λ, the share of rule-of-thumb agents; γ_H, the parameter regulating the degree of home bias in consumption and investment; θ, the elasticity of substitution between Home and Foreign goods in the consumption and investment bundles. In what follows, the three

[12]Notice that in Figure3.2 Panel B the two shares of rule-of-thumb consumers are not exactly the same. For the plot we set $\lambda^* = \lambda + 10^{-4}$.

[13]This cannot be analyze in a closed economy-one sector model.

parameters are changed each in turns, while the remaining are set equal to their respective baseline values, as illustrated in the previous section. Finally, we will also analyze how fiscal rules can dampen government spending effects.

Baseline vs No Rule-of-Thumb Agents

We compare the impulse responses of the benchmark model ($\lambda = 0.5$) to those obtained assuming that there are no rule-of-thumb agents ($\lambda = 0$) - see figure 3.3 from panel A to panel C.

After a positive public expenditure shock in the region H, in presence of rule-of-thumb agents the domestic private aggregate consumption C increases: the consumption of the rule-of-thumb agents increases, given the positive income effect associated to the increase in real wages; this increase more than compensate the decrease of Ricardian agent's consumption.[14] The labor increase associated to the higher consumption allows for a strong increase in the domestic output (the public expenditure multiplier is greater than one). Consumer price inflation increases, as well as the price inflation of the domestically produced good. The domestic investment, given our monetary rule, is crowded out but not enough to compensate the consumption boom. In the case of no rule-of-thumb-agents, private consumption drops, output increases but less than the public expenditure does (hence the public expenditure multiplier is smaller than one), inflation raises but to a lower extent than in the previous case so that investment is also crowded out to a less extent.

Finally under our fiscal rule the higher public consumption is financed mainly with public debt: the public debt initially increases, taxes increases only slowly to repay the debt. As we will see later the way of financing crucially determines the magnitude of the crowding in effect for aggregate consumption.

[14]The increase of real wages is determined by the assumptions of flexible wages and sticky prices.

The sources of spillovers are three: the terms of trade, the amount of imports and the nominal interest rate set by the central bank of the union.

The terms of trade appreciate: government is completely biased towards domestic goods. A positive shock to government spending is a direct demand shock for goods produced domestically that generates an increase in their prices.[15] On impact, the terms of trade appreciation is stronger in presence of rule-of-thumb agents, given the higher increase in aggregate demand. The appreciation induces a positive substitution effect towards the good produced in region F. This effect, coupled with the positive income effect of higher wages in region H, stimulates imports of region H, inducing higher production, labor effort and also higher production-prices in region F. When there are only Ricardian agents, the amount of imports decreases, contributing to lower output and labor effort in region F.

The central bank rises the nominal interest rate, given the higher union-wide inflation. The higher interest rate implies a decrease in the consumption and investment of region F. Aggregate consumption in region F decreases: the higher consumption of rule-of-thumb agents, favoured by the increase of labor effort, is not sufficient for compensating the lower consumption of Ricardian agents. In absence of rule-of-thumb agents, the nominal interest rate increases by less, given the lower increase of inflation at the union level.

The bottom line is that, under the baseline calibration, the introduction of rule-of-thumb agents revert the sign of the response of domestic imports and foreign output volume from negative to positive.

Sensitivity to the Home Bias

In Figure 3.4 we now compare the impulse responses of the benchmark model

[15]Given the assumption of no home bias, the real exchange rate stays constant at its steady state level.

(no home bias, i.e. $\gamma_H = 0.5$) to those of the model having mirror home bias ($\gamma_H = \gamma_F = 0.75$): in each region agents have a stronger preference for the domestically produced good.

In presence of home bias, the reaction of the terms of trade is stronger; this induces a positive income effect on domestic agents; in particular, the real wage increase is stronger, inducing an increase of the rule-of-thumb agents' consumption; aggregate consumption in region H increases by more, implying higher output and inflation rate.

Spillovers are as follows. Imports are higher in presence of home bias, given that the increase of the consumption and substitution effect are higher.

The terms of trade and real exchange rate appreciate by a greater extent. This induces a negative income effect on agents belonging to region F: notwithstanding higher imports from region H, the amount of output and labor effort decrease in region F: the domestic real wage decrease is stronger, depressing aggregate consumption and demand.

The home bias does not affect the size of the interest rate increase, given that the central bank faces the same rise of union-wide inflation rate. The higher interest rate reduces the consumption and investment of Ricardian agents in region F.

Hence, home bias does not affect union-wide variables; however, it increases diverges across the two regions. In particular, following a public spending shock in region H, private consumption, labor effort and output volume in region F decrease, while they increase in region H.

Sensitivity to the Elasticity of Demand

In Figure 3.5 we increase the elasticity of substitution between goods produced in the two regions, θ, from the baseline value equal to 1.5 to a value equal to 5.

For a given increase in the relative price of the good produced in the region H, private agents are more willing to substitute for the good produced in region F in correspondence of a higher θ. Hence, the positive income effect on agents belonging to region H is lower; private consumption and output increase to a lower amount.

The lower income effect in region H implies that the increase in imports is lower; the terms of trade appreciates to a lower extent.

The interest rate increases, given that inflation rate increases at the union level. All the union-wide variables are not affected by the change in the elasticity of substitution. Labor effort and output increase in region F: agents substitute the good produced in the region H with the good they produce; the relative income effect positively affects the consumption of rule-of-thumb agents; its increase more than compensate the decrease of Ricardian agents' demand and stimulate economic activity in the region F.

Overall, higher elasticity of substitution implies a convergence of the output and inflation variations across the two regions. In particular, in the region F private consumption and labor effort increase, as well as in region H.

Sensitivity to the Country Size

In Figure 3.6 we compare our benchmark, characterized by the two regions having equal size ($n = 0.5$) to the case where the region H is relatively big ($n = 0.95$).

Qualitatively, variables in the region H are not affected by the change in the size. At union level, the interest rate, inflation rate and output increase, in particular, the union-wide inflation rate and output closely mimic their correspondent variables in the region H, given the big size of the latter. Spillovers to the region F are particularly strong: the big size of region H implies not only an increase in the amount of the good produced in region F, but also, differently from the previously considered cases, in the relative price (a depreciation of the region H's terms of trade and of the real exchange rate). The related positive income effect stimulates the consumption of rule-of-thumb agents in the region F, which induces an increase in aggregate consumption. The big region government spending multiplier for the small region output is greater than one.

Sensitivity to Fiscal rules

In Figure 3.7 we now modify the benchmark fiscal rule ($\phi_b = 0.3$, $\phi_g = 0.12$) by allowing a stronger reaction to public expenditure ($\phi_g = 1$). Hence, the new rule does not permit deficit spending, given that public expenditure variation are entirely financed by taxation. The direct implication of the new rule is that the multiplier of the public expenditure becomes lower than one. The higher burden of taxes make the consumption of rule-of-thumb decrease. Given the decrease in private demand, imports from region H decrease. The interest rate increase is lower, given the lower increase of inflation. In the region F - given the lack of a spillover stimulating the economic activity - output, investment and consumption decrease.

3.2.3 Policy Frontier

Shocks that generate a negative correlation between output and inflation force the central bank to face a trade-off between the variability of output and that of inflation at the union level. We investigate whether fiscal rules contribute to alleviate this trade-off. We compare the stabilization property of alternative fiscal rules, assumed to be symmetric across countries: a 'rigid' tax rule, calibrated using the baseline values, that mainly focuses on stabilizing the level of domestic public debt; a 'flexible' tax rule, that limits the movements of domestic output; a 'flexible' public spending rule, that, given the rigid tax rule, limits the movements of domestic output. To compare alternative tax rules based on lump-sum taxation is not trivial in our case, given that we break Ricardian equivalence through the rule-of-thumb agents.

If the government itself is the main source of uncertainty in the economy then a rule which allows to exploit the limited financial markets participation of some agents would only generate higher macro-volatility. In this case financing government consumption with taxes, $\phi_g = 1$, kills the 'Keynesian' effect and improve macro-stability (see Figure 3.7). Hence, in order to evaluate how fiscal policy could help to reduce macroeconomic volatility we shut off the shock associated to government expenditure $\sigma_{\varepsilon^g} = 0$ and we focus on how different simple fiscal rules could make the system better absorb a technology and a markup shock.

In the 'flexible' tax rule we keep $\phi_b = .30$ and $\phi_g = .12$ at their baseline values while the parameter measuring the reaction of taxation to output is set equal to $\phi_{yx} = 1$ (symmetrically in region F):

$$\log T_{X,t} = .30(B_{G,t-1}/P_t) + .12\log(P_{H,t}G_t/P_t) + \log(Y_{H,t}) + \text{const} \qquad (3.2.1)$$

In the 'flexible' public spending rule we keep $\rho_g = .87$ at its baseline value while the parameter measuring the reaction of government expenditure to output is set equal to $\phi_{yg} = 1$ (symmetrically in region F):

$$g_t = .87g_{t-1} - \log(Y_{H,t}/\bar{Y}) \qquad (3.2.2)$$

We compute the inflation-output volatility frontiers for alternative parameterizations of the interest rate rule.[16] Specifically, we minimize the weighted unconditional variances of output and inflation at different relative preferences of the monetary authority for inflation versus output variance. We consider only values of the parameters that generate a unique equilibrium.

We assume a symmetric markup and technology shock, following the process reported in section (3.1.10) (for their calibration see section 3.1.12).

Panel A in Figure 3.8 shows the union-wide inflation-output frontier in correspondence of each tax rule. Government spending is held constant at its steady state value for both regions. The flexible tax rule contributes to reduce the trade-off between stabilizing the inflation and output: for most of plotted space, given the variance of one variable the variance of the other decreases. However, when the monetary policy is mainly focused on stabilizing one of the two variables, the difference between the two fiscal rules decreases. In particular, when the central bank mainly minimizes the variance of the inflation, the curves intersect and the flexible rule has a lower capability of stabilizing economy than the rigid rule.

Panel B in Figure 3.8 shows what happens to the variances of output and inflation of region H. The diagram is similar to that of the union, given that we assume a symmetric structure of the shocks. For the same reason, the frontier computed using variables of region F is similar. To save on space, we do not report it.

Finally, we analyze the stabilization property of a public expenditure rule. It is known that this kind of rule stabilizes the economy of the currency union

[16] See Levin et al. (1999).

also in absence of rule-of thumb agents.[17] The parameters of the tax rule are set to their baseline values - i.e. $\phi_b = .30$ $\phi_g = .12$ and $\phi_{yx} = 0$. Panel A in Figure 3.9 reports the results obtained for the union-wide inflation and output. The spending rules, offsetting the variation in the private spending, stabilizes the economy of the union. The frontier of the monetary authority shifts towards the origin. As in the case of alternative tax rules, the two frontiers will eventually intersect when the central bank mainly stabilizes inflation. As shown in Panel B, this property holds also at regional level (to save on space, we report only the results obtained in the case of the region H, those relative to region F are similar).

3.3 Conclusions

In this paper we have analyzed the effects of fiscal policy in a currency area using a DSGE model with sticky prices. We have allowed for the positive effects of public expenditure on private consumption and a nontrivial role of lump-sum taxes by introducing rule-of-thumb agents. We have assumed that fiscal policy is managed at regional level in a systematic way having the stabilization of domestic variables as exclusive target. We have done the opposite assumption for the common monetary policy which reacts to the union-wide variables only.

Given this framework, we have explored which characteristics the monetary and fiscal rules should have to guarantee a unique stable equilibrium. We have analyzed the domestic and cross-regional effects of a given regional public expenditure shock. Finally, we have analyzed the capability of decentralized (at

[17] See Beetsma and Jensen (2005). We have computed the shift of the frontier in correspondence of the public expenditure rule and no rule-of-thumb agents. Results are not qualitatively different from those reported in the text. To save on space, we do not report them.

regional level) simple tax and expenditure rules to stabilize the union-wide variables.

The main results are two: we have been able to reproduce, consistently with empirical evidence on countries belonging to the European Union, the increase in private demand for imported goods that follows a domestic public spending shock; second, the presence of rule-of-thumb agents, that allows to reproduce the quoted stylized fact, does not dramatically affect the capability of regional fiscal policy to contribute to the stability and equilibrium determinacy of the union-wide economy, at least compared to the case of closed economy model.

There are various directions in which the main point of this paper can be further developed. First, we have assumed that fiscal policy is exclusively conducted at regional level; this assumption can be relaxed by assuming that there are some fiscal transfers across regions or that there is a central fiscal authority (at union level). Second, we have assumed that, differently from rule-of-thumb agents, the Ricardian agents can share idiosyncratic risk; we can relax this assumption and assume that also Ricardian agents face some financial frictions; an alternative assumption, with implications for the dynamics of the model, is that Ricardian agents have access only to a riskless bond traded domestically and across regions. Finally, we can add distortive taxation, that directly affect relative prices, and analyze how the union-wide implications of fiscal policy change.

Appendix

A Labor Market Structure

We assume that in each country labor markets are not perfectly competitive. Following GLV 2006 we assume an aggregate labor supply of the following form:

$$\mu_w \Big(\frac{C_t^i}{n_i}\Big)^{\sigma} \Big(\frac{N_t^i}{n_i}\Big)^{\varphi} = w_t^i \tag{A-1}$$

The coefficient μ_w represents a wage markup. The previous equation can be rationalize introducing a union. In each region, say region H, firms have a demand for different types of labor:

$$N_t(j) = \frac{1}{n}\Big(\frac{w_t(j)}{w_t}\Big)^{-\epsilon_n} N_t$$

Different types of labors are randomly distributed across households (disregardful of household type). It follows that in aggregate we will have $N_t^r = N_t^o = N_t/n$

The union sets wages in order to maximize the objective function (with a slight abuse of notation n_j denotes both the mass of households of labor type-j and its set):

$$\int_{j\in n_j} c_t(j)^{-\sigma} w_t(j) N_t(j) - \frac{N_t^{1+\varphi}(j)}{1+\varphi} dj$$

subject to a labor demand schedule

$$N_t(j) = \frac{1}{n}\Big(\frac{w_t(j)}{w_t}\Big)^{-\epsilon_n} N_t$$

first order conditions for this problem gives (we define $\mu_w \equiv \epsilon_n/(\epsilon_n - 1)$):

$$w_t = \mu_n \Big(\frac{C_t}{n}\Big)^{\sigma} \Big(\frac{N_t}{n}\Big)^{\varphi} \int_{j\in n_j} \Big(\frac{1}{n_j}\Big)^{\sigma} \Big(\frac{C_t(j)}{C_t}\Big)^{-\sigma} dj$$

The same holds symmetrically true for the other country.

We assume that the union is not able to observe the marginal utility of each single households but only aggregate consumption. We take $n_z = n$ and using a marginal utility of average consumption in the objective function cancels the integral in the above condition delivering the same result as postulated in the beginning[18].

B Non-Stochastic Steady State

The steady state has a closed form solution for $ToT = 1$. This is not the case when countries are not symmetric - e.g. different government spending ratios. In this case there is no closed form solution but the main relations among variables keep the same functional form.

The ToT is our unknown to be found.

$$\left(\frac{P_H}{P}\right)^{\theta-1} = \gamma_h + (1-\gamma_h)TOT^{1-\theta} \tag{A-2}$$

$$\left(\frac{P_F}{P^*}\right)^{\theta-1} = \gamma_f + (1-\gamma_f)TOT^{\theta-1} \tag{A-3}$$

$$RER = \frac{P^*}{P_F}\frac{P_H}{P}TOT \tag{A-4}$$

From the FOC on capital we find return on capital which is the same in both countries is $r^k = 1/\beta - 1 + \delta$.

The steady state markup is μ and μ^* in the home and foreign country, respectively. It implies:

$$mc = \frac{1}{\mu}\frac{P_H}{P}$$

$$mc^* = \frac{1}{\mu^*}\frac{P_F}{P^*}$$

[18] We calibrate the model such that the participation constraint for each households is almost always satisfied.

using marginal costs we can find the great ratios for both countries (we assume zero steady state price dispersion).

$$\frac{K}{Y} = \alpha mc/r^k$$
$$\frac{Y}{N} = Z^{\frac{1}{1-\alpha}}\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}}$$

Using the output-labor ratio we can find the real wage

$$w = \frac{Y}{N}(1-\alpha)mc \qquad \text{(A-5)}$$

symmetrically for the other country.

We will impose the trade balance equal to zero. This means the value of imports must be equal to the value of export or, in other words, that the value of the output produced in one country (the GDP) must be equal to the country total aggregate demand

$$P(C+I) + P_H G = P_H Y_H$$

which means (we assume $G/Y_H = g$)

$$\frac{C}{Y_H} = \frac{P_H}{P} - \delta\frac{K}{Y_H} - \frac{P_H}{P}g$$

from the labor supply equation we can finally determine the outputs' levels (we introduce μ_w the wage markup).

$$\mu_w\left(\frac{C}{Y_H}\right)^{\sigma}\left(\frac{N}{Y_H}\right)^{\varphi}{Y_H}^{\sigma+\varphi} = n^{\sigma+\varphi}w$$

symmetrically for the other country.

We are know able, using the great ratios and the output levels, to calculate also the consumption and investment levels.

We close the system imposing that the trade balance must be actually equal to zero.

$$TB = (1-\gamma_f)\frac{P_H}{P}\Big(\frac{P_H}{P^*}\Big)^{-\theta}(C^* + I^*) - (1-\gamma_h)\Big(\frac{P_F}{P}\Big)^{-\theta}\frac{P_F}{P}(C+I) = 0$$

It is interesting to note that, as for a closed economy, aggregate steady state variables are not affected by the RoT share λ. This result is in fact not really surprising given that the main restriction imposed to the RoT is to smooth consumption *intertemporally*. On the other hand, as we have shown, technology and market distortions determine the great ratios of the economy and so also the aggregate labor income making the number of capital holders affect only the wealth distribution of the economy but not the production side.

From the previous equations we can find the RoT consumption from their budget constraint (given that $N^o = N^r = N/n$, $T_x = nT^r$ and we assume $T_x/Y_H = G/Y_H = g$)

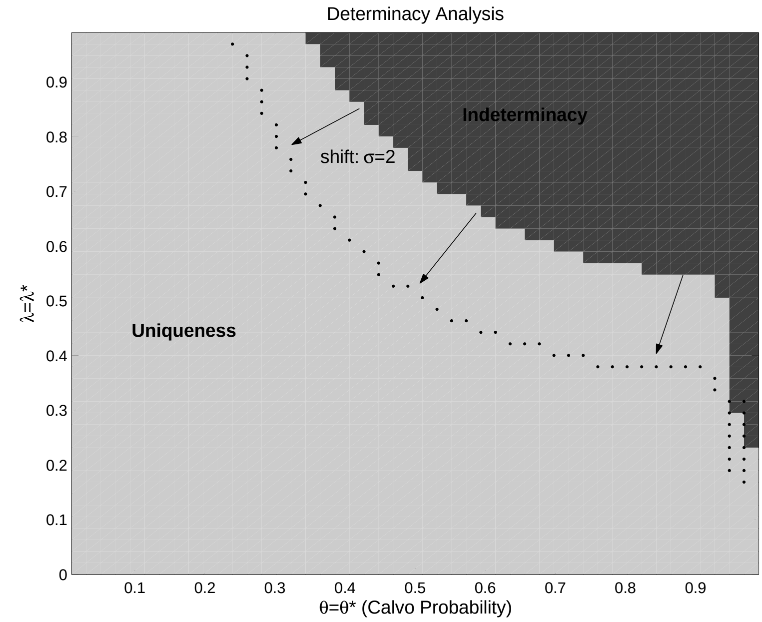
$$\frac{nC^r}{Y} = (1-\alpha)mc - g \tag{A-6}$$

or in terms of aggregate RoT consumption

$$\frac{\lambda nC^r}{Y} = \lambda[(1-\alpha)mc - g] \tag{A-7}$$

This means that the aggregate steady state RoT consumption depends linearly on their share. Optimizers consumption is determined residually.

Panel A



Panel B

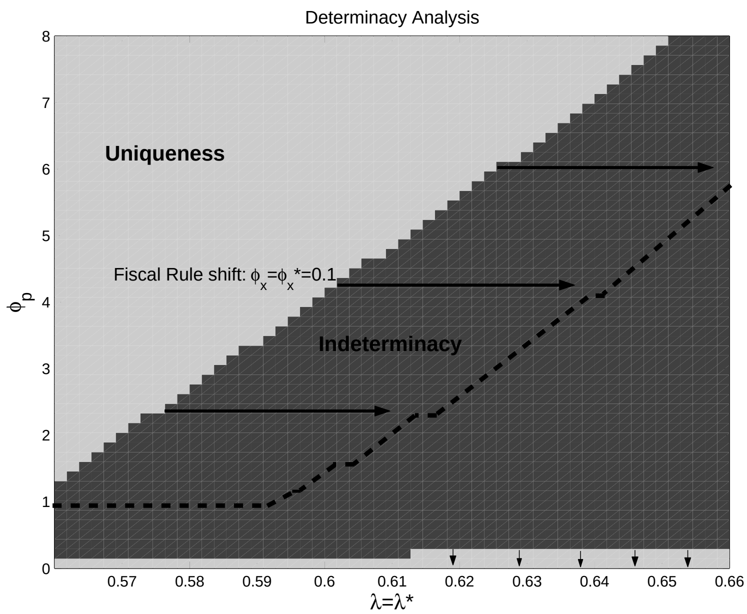
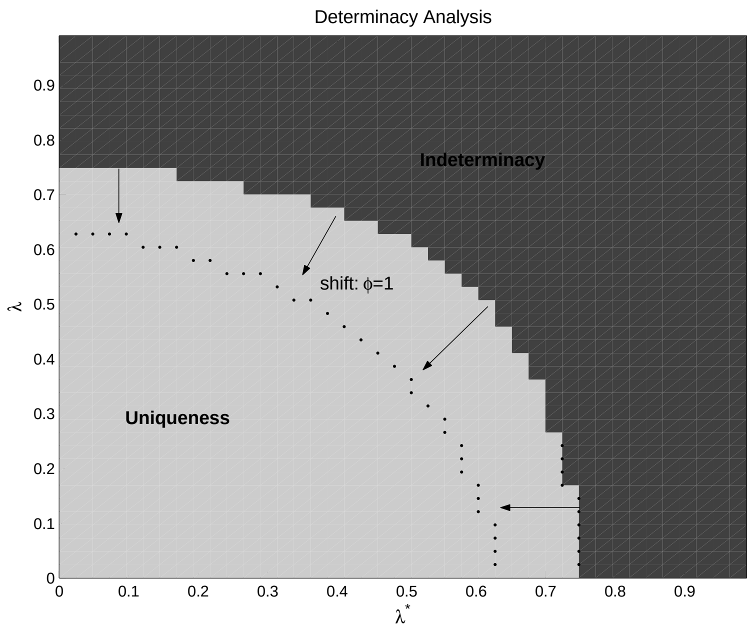


Figure 3.1: Indeterminacy Regions. First.
All other parameters at their baseline values. Dotted lines show the shift in the indeterminacy region frontier.

Panel A



Panel B

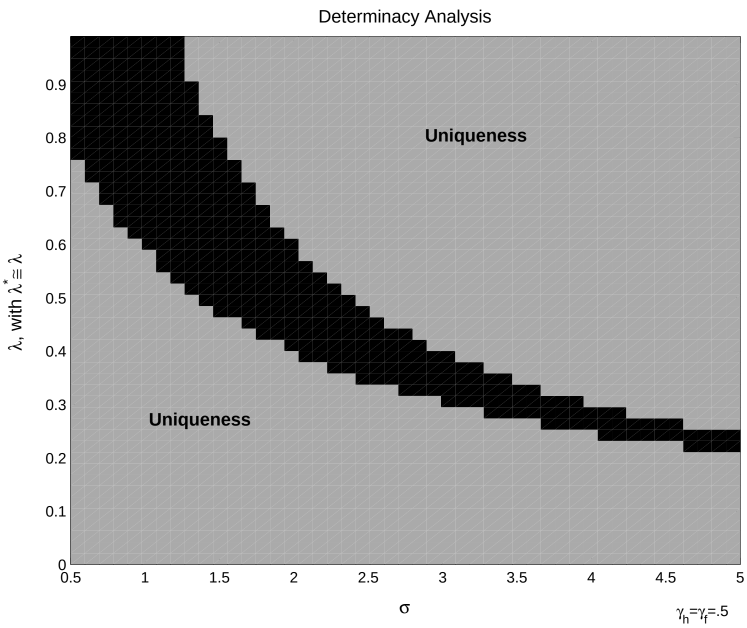
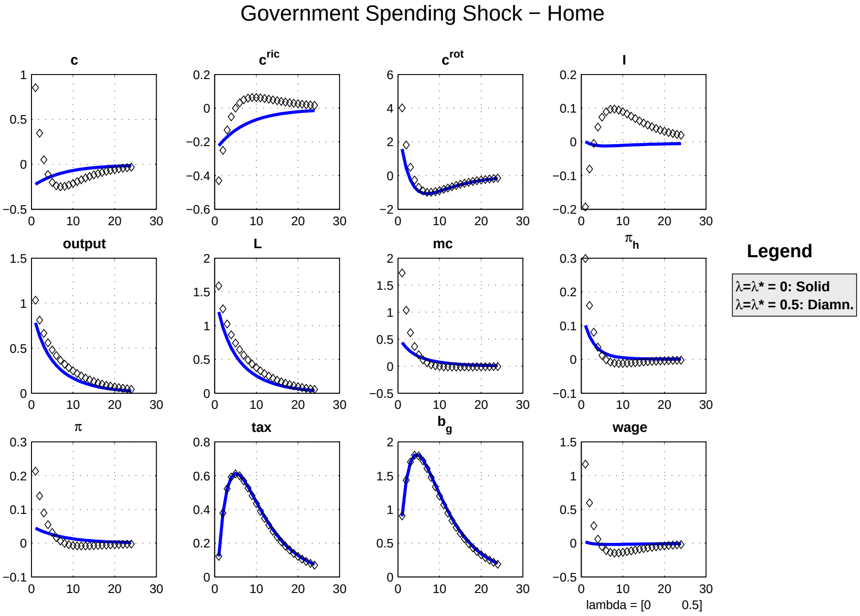


Figure 3.2: Indeterminacy Regions. Second.
All other parameters at their baseline values. Dotted lines, when plotted, show the shift in the indeterminacy region frontier.

Panel A



Panel B

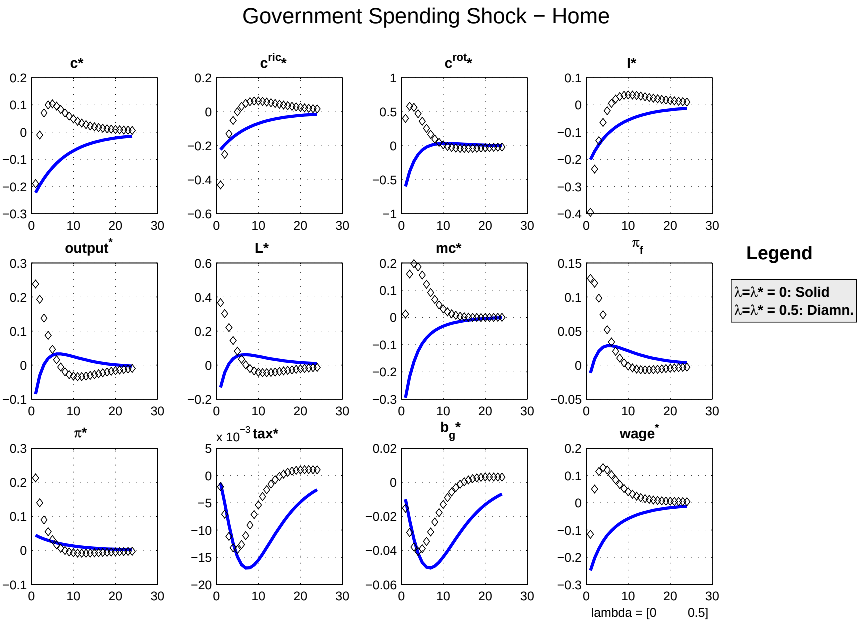


Figure 3.3: Impulse response functions

Impulse response function to a 1% increase in the region-H government spending. All other parameters at their baseline values. Horizontal axis: time (quarters). Vertical axis: deviation from steady state.

Panel C

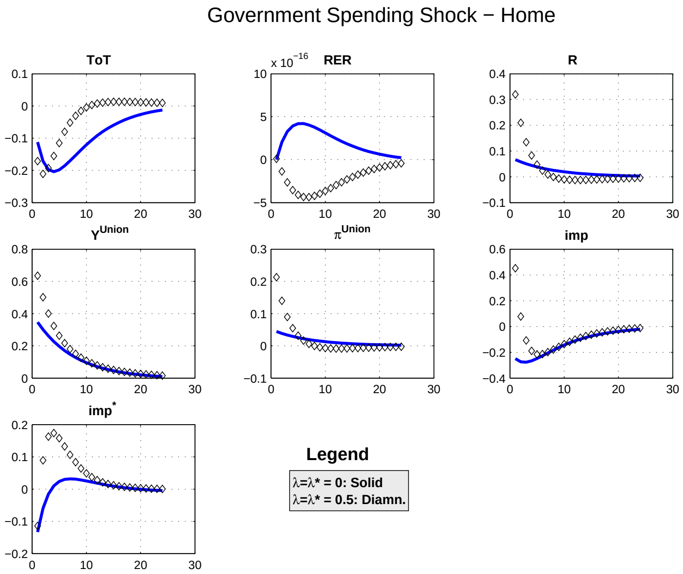
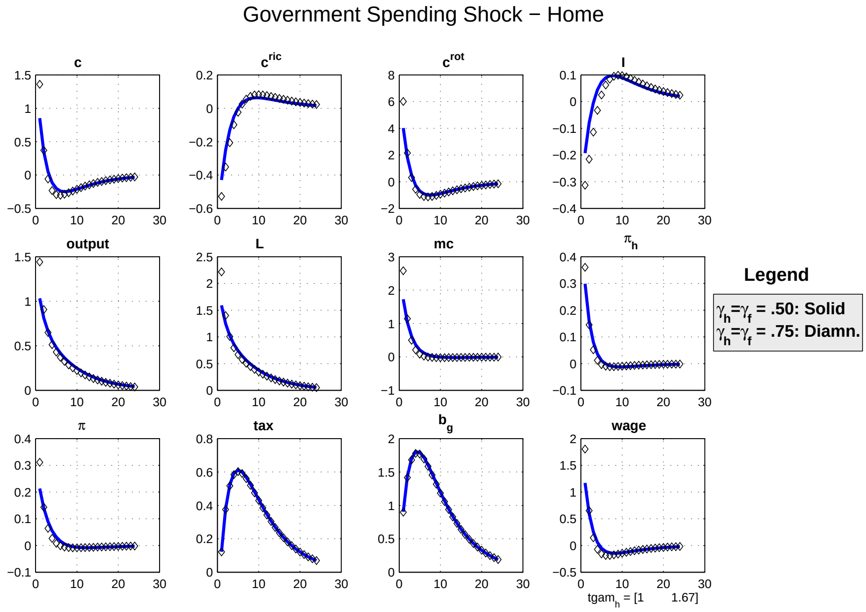


Figure 3.3: Impulse response functions
Impulse response function to a 1% increase in the region-H government spending. All other parameters at their baseline values. Horizontal axis: time (quarters). Vertical axis: deviation from steady state.

Panel A



Panel B

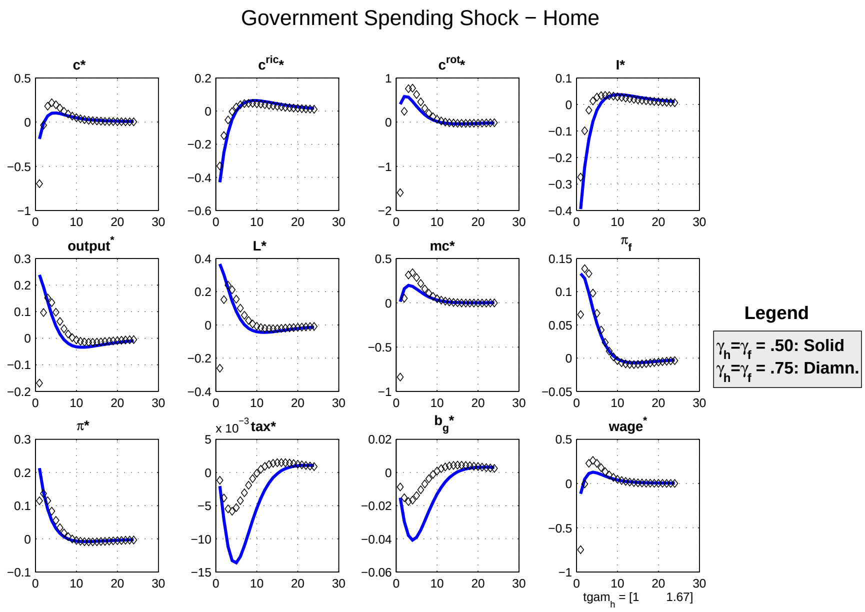


Figure 3.4: Impulse response function. Home bias.
Impulse response function to a 1% increase in the region-H government spending. All other parameters at their baseline values. Horizontal axis: time (quarters). Vertical axis: deviation from steady state.

Panel C

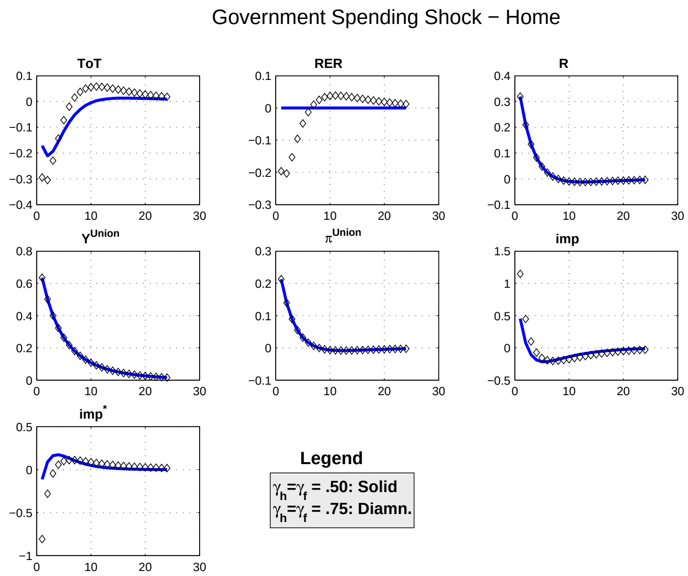
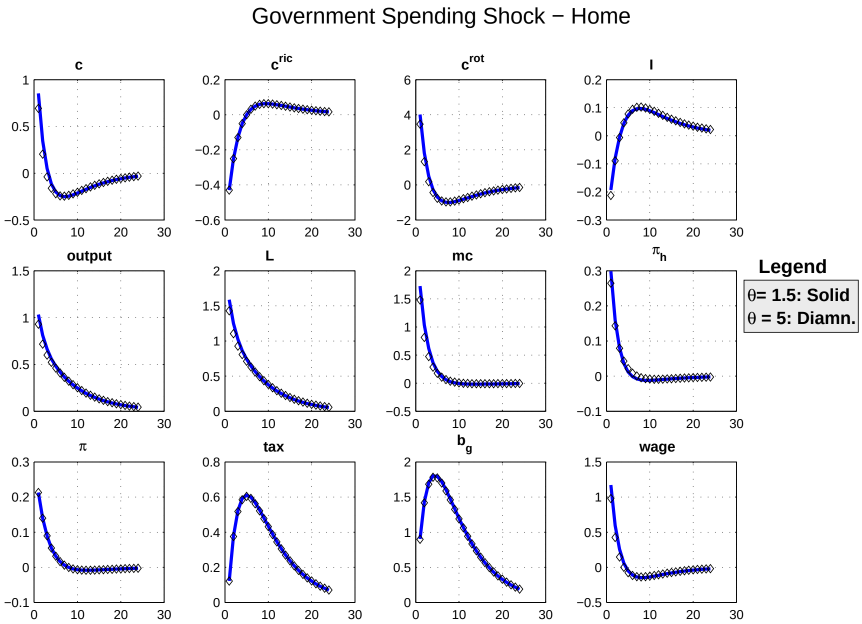


Figure 3.4: Impulse response functions. Home bias.
Impulse response function to a 1% increase in the region-H government spending. All other parameters at their baseline values. Horizontal axis: time (quarters). Vertical axis: deviation from steady state.

Panel A



Panel B

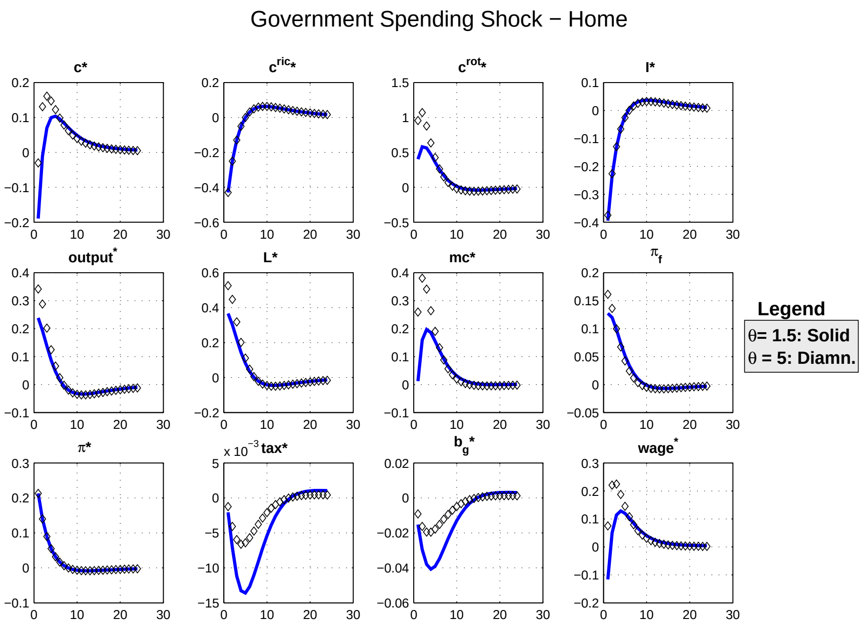


Figure 3.5: Impulse response functions. Elasticity of Substitution
Impulse response function to a 1% increase in the region-H government spending. All other parameters at their baseline values. Horizontal axis: time (quarters). Vertical axis: deviation from steady state.

Panel C

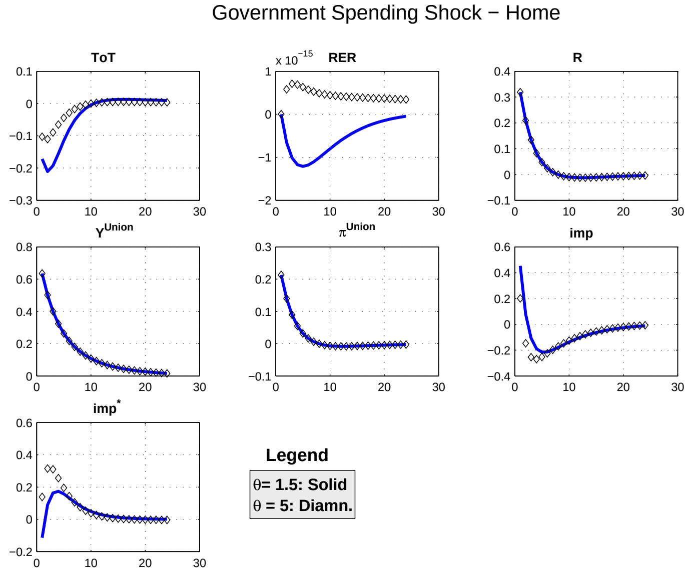
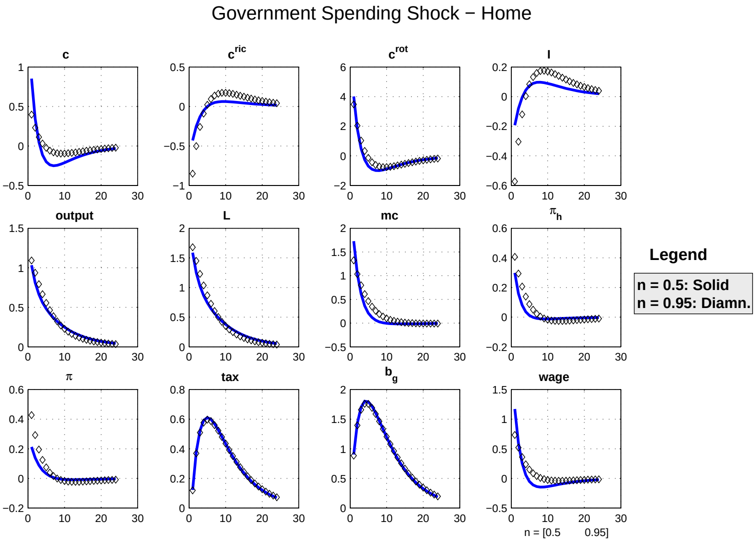


Figure 3.5: Impulse response functions. Elasticity of substitution.
Impulse response function to a 1% increase in the region-H government spending. All other parameters at their baseline values. Horizontal axis: time (quarters). Vertical axis: deviation from steady state.

Panel A



Panel B

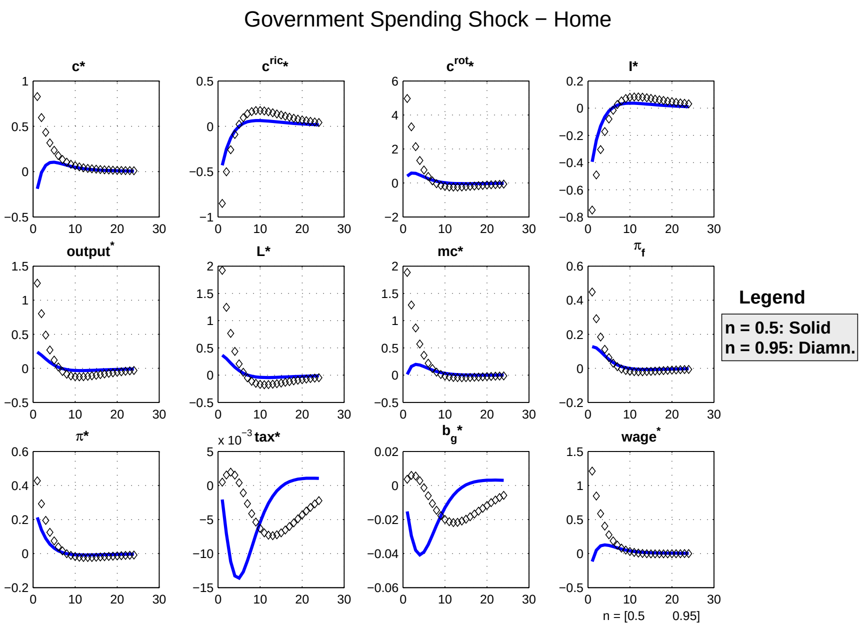


Figure 3.6: Impulse response functions. Size.
Impulse response function to a 1% increase in the region-H government spending. All other parameters at their baseline values. Horizontal axis: time (quarters). Vertical axis: deviation from steady state.

Panel C

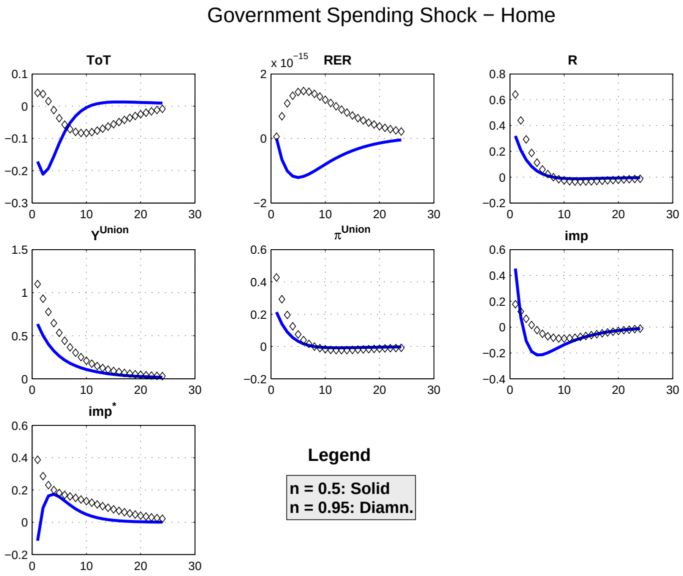
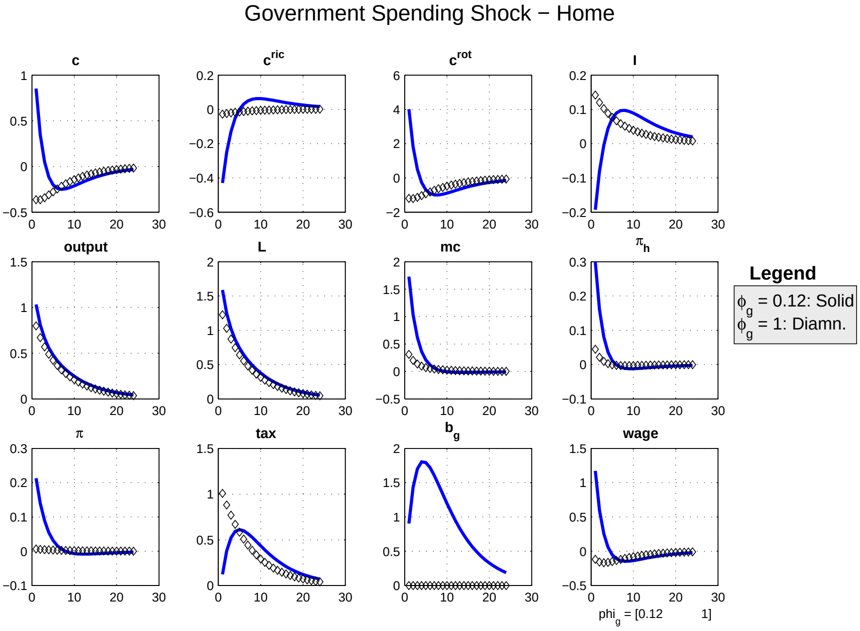


Figure 3.6: Impulse response functions. Size.
Impulse response function to a 1% increase in the region-H government spending. All other parameters at their baseline values. Horizontal axis: time (quarters). Vertical axis: deviation from steady state.

Panel A



Panel B

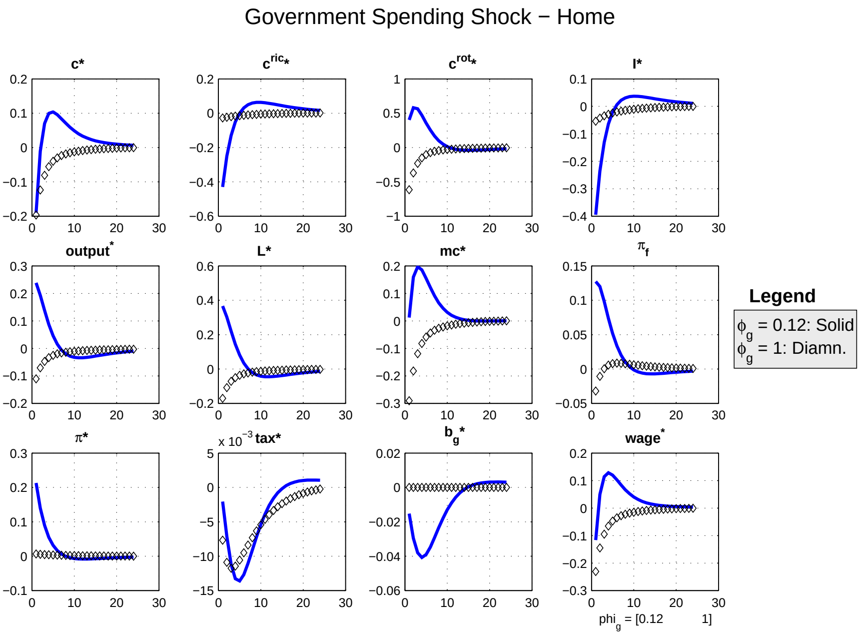


Figure 3.7: Impulse response functions. Fiscal Rules
Impulse response function to a 1% increase in the region-H government spending. All other parameters at their baseline values. Horizontal axis: time (quarters). Vertical axis: deviation from steady state.

Panel C

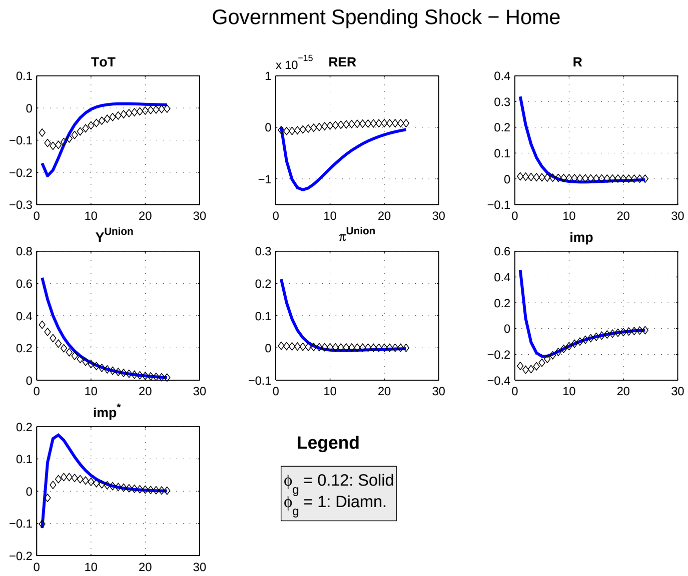
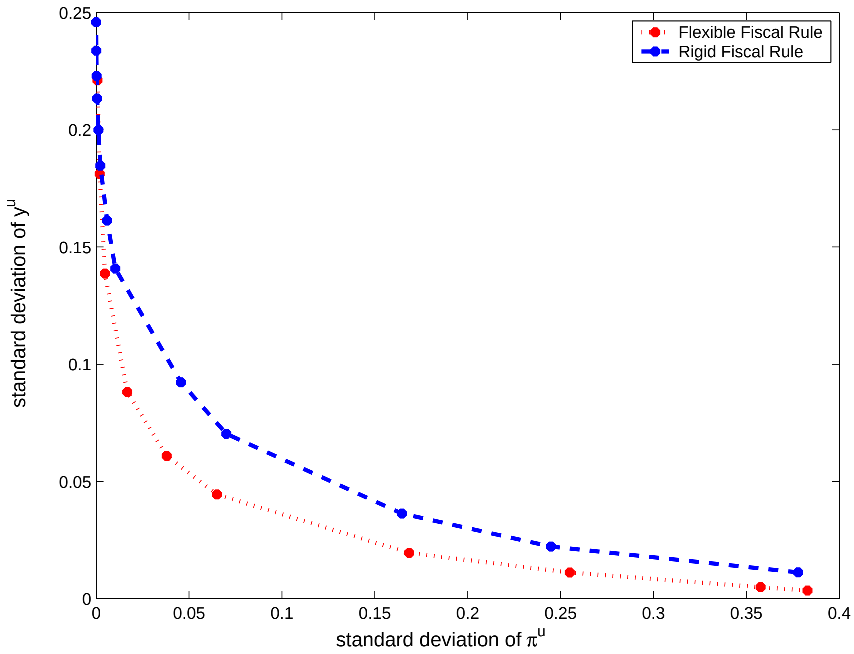


Figure 3.7: Impulse response functions. Fiscal Rules
Impulse response function to a 1% increase in the region-H government spending. All other parameters at their baseline values. Horizontal axis: time (quarters). Vertical axis: deviation from steady state.

Panel A



Panel B

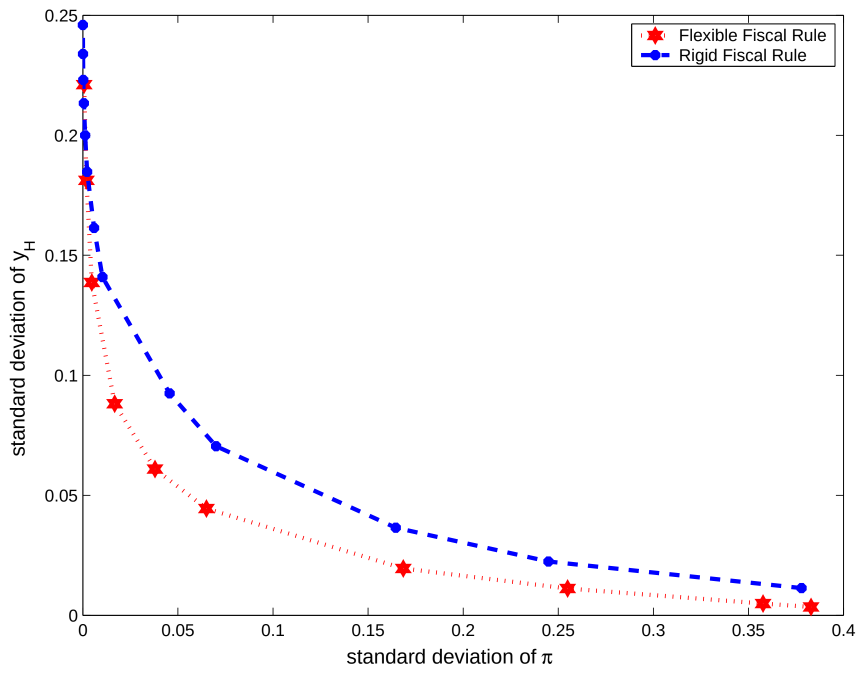
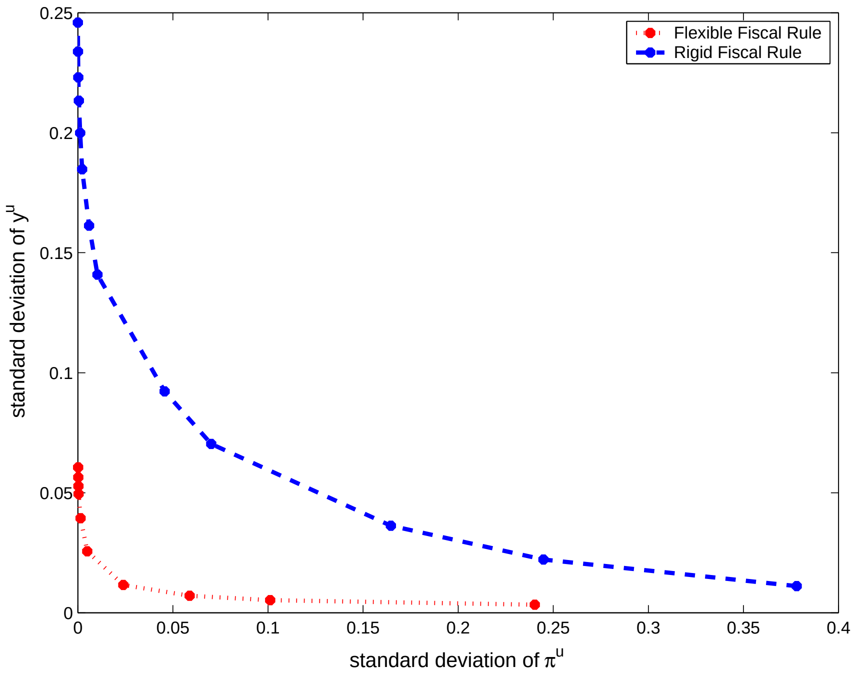


Figure 3.8: Monetary policy frontier.

Monetary policy frontier. Panel A Union-wide output and inflation volatility. Panel B Region-F output and inflation volatility. Two different fiscal rule. Baseline calibration.

Panel A



Panel B

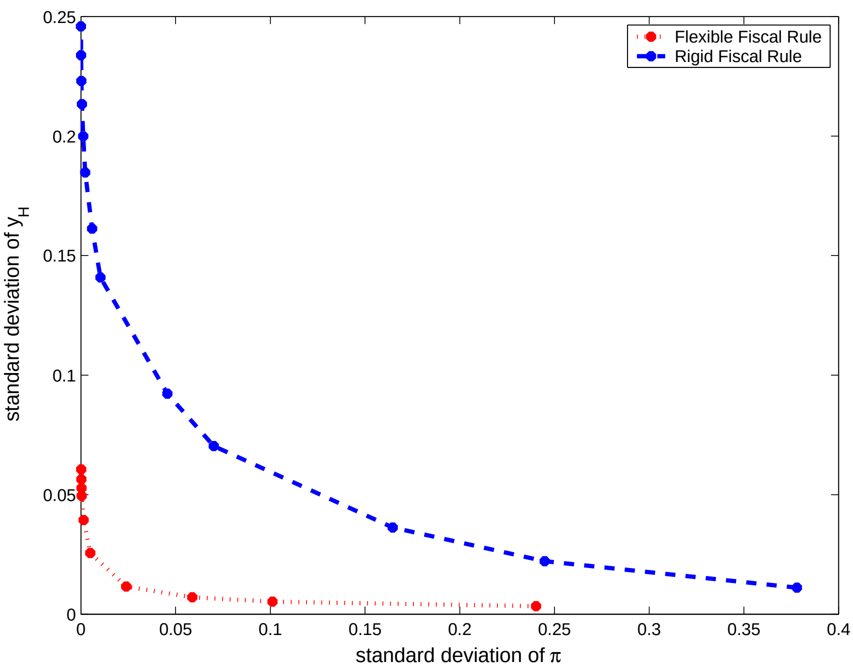


Figure 3.9: Monetary policy frontier.

Monetary policy frontier. Panel A and B Union-wide output and inflation volatility. Panel A no rule-of-thumb consumers, Panle B $\lambda = \lambda^{\star} = 0$. Two different fiscal rule. The rest of the parameters at the baseline calibration.

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