# Fiscal and Monetary Policy under Imperfect Commitment

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*To Anna and Gustavo, Amalia and Dario*

*Cosa vale il denaro ... ?*

# Contents

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# Introduction

The purpose of this thesis is to analyze how fiscal and monetary policies should be designed in a context where policymakers have credibility problems. We develop methodologies and applications to show how different degrees of policymakers' credibility affect the determination of policy choices, such as taxes or monetary instruments, and more generally the economic outcomes.

In particular, I try to understand how the economy behaves when building fully credible institutions may not be possible. There are several reasons why policymakers may not follow their announced plans. This may be because particular events arise (like wars, political pressures, etc.), because policymakers are replaced by others, or because the policy goals change over time.

The present thesis contains three essays that deal with optimal policy problems under imperfect commitment. They have been written in collaboration with Ricardo Nunes. In chapter 1 - *Loose Commitment* -, we develop a new methodology to solve optimal policy problems taking into account that policymakers may not fulfill their promises, and analyze the effects of policymakers commitment on capital and labor taxation. In chapter 2 - *Political Disagreement, Lack of Commitment and the Level of Debt* - we consider a case where commitment is limited by the fact that policymakers with different objectives alternate in office. In particular, we study how lack of commitment and political turnover affect the level of public debt. Finally, in chapter 3 - *The Macroeconomic Effects of Unstable Monetary Policy Objectives* - we study how the possibility of changes in policy objectives influences monetary policy choices.

The importance of credibility in policy choices has its roots in the famous 1976

Lucas' critique. This critique pointed out the need of considering the interactions between government policies and private agents' expectations when performing policy evaluations. In particular, it was emphasized that rational agents' decisions are based on their expectations about future policies. For example, the agents' choices about consumption and savings depend on their expectations about future taxes and monetary policy instruments. This raised the need to develop economic models where these types of strategic interactions between agents and policymakers are explicitly taken into account. In this context, the papers of Kydland and Prescott (1977), Calvo (1978) and Barro and Gordon (1983a), analyzed government decision problems where agents' decisions are based on rational expectations about future policies and clarified the importance of policymakers' credibility.

These authors considered a rational, forward looking policymaker, who makes policy plans in order to maximize the well-being of its citizens and show that the resulting policy decisions are typically time-inconsistent. This means that if given an opportunity to change its plans at a later date, such benevolent policymaker will generally do so. The announced plans will not be credible, unless the government is able to tie his hands in advance, making binding commitments that prevent him to change his policies at a future date. When policy plans are not credible rational agents will realize that future policies will not necessarily coincide with the announced plans, and adjust their decisions accordingly. As explained by Kydland and Prescott (1977), in a case where governments cannot credibly commit (no-commitment case) social welfare is lower than in the case where the government can commit (full-commitment case). This clarifies how policymakers credibility affect economic outcomes and, in particular, why building credibility is important.

In the macroeconomic literature, optimal policy problems are typically analyzed under the assumptions either of no-commitment or of full-commitment. Models' prediction under these two alternatives can differ significantly, thus leaving us without a clear answer.

Both the no-commitment and full-commitment assumptions are extreme depictions of the reality. On the one hand, following the theoretical developments mentioned above, there has been a considerable effort to build credible institutions to mitigate the time-inconsistency problem. Nowadays, in many countries central banks are independent from the government and have specific goals in terms of inflation and output stabilization. To a minor extent, there has been an effort to build credibility also in fiscal policy. For example, the adoption of some fiscal agreements like the European Stability and Growth Pact, has limited the discretionality of fiscal policy authorities. These examples suggest the limits of the no-commitment assumption.

On the other hand, despite these efforts to increase policymakers' credibility, it is difficult to think that institutions are fully credible. This can be for several reasons. When particular events arises (e.g. financial crisis, wars, etc), even benevolent policymakers maybe subject to particular pressures to change their announced plans. More importantly, policymakers can be replaced. The chairman and the board of advisors of a central bank are in office for a finite period of time and governments are subject to political turnover as result of elections. In these case, the credibility problem is connected to the possibility that policy objectives can change. In such circumstances, the plans made by the predecessors can be irrelevant for the newly appointed policymakers.

A challenging issue in research was whether the time-consistency problem could be resolved. One possibility comes from the theory of repeated games. Under some conditions, as suggested by Backus and Driffill (1985) and Barro and Gordon (1983b), it is possible to sustain equilibria close to the full-commitment solution through reputation mechanisms and trigger strategies. Alternatively, in some cases the careful management of policy instruments may solve the time-consistency problem. In a model with public debt, Lucas and Stokey (1983) showed how the incentive to renege on previous plans can be eliminated by an appropriate choice of the debt maturity structure.

Here we take instead a different approach. We consider that, for the reasons explained above (particular events, policymakers turnover, etc.), the mechanisms to sustain the full-commitment equilibrium may not always work in practice. This leaves open the possibility that policymakers can disregard the announced plans. In this thesis, we develop methodologies and applications to analyze policy design in all the situations where institutions and policymakers, as it happens in the real world, sometimes fulfill their promises, but sometimes they do not. This is what we call *loose commitment* settings.

We show how allocations and social welfare change with the degree of credibility of policymakers. This can help understand the desirability of building credibility in all the cases where reaching full-credibility may not be possible. For example, in chapter 1, we study the implications of limited commitment for capital and labor taxation. We find that average allocations seem to be closer to the no-commitment solution. This may suggest that there is limited scope for building credibility in fiscal policy, when full credibility cannot be reached.

We also analyze the importance of credibility in a context where policy objectives can change over time. In chapter 2, we analyze a fiscal policy problem, taking into account that policymakers with different policy objectives alternate in office. This part of our work is related to the political economy literature, as recently summarized in the books of Drazen (2000) and Persson and Tabellini (2000). For our purposes, it is worth emphasizing that, when considering optimal policy problems in the presence of political turnover, it is necessary to relax the full-commitment assumption. It is indeed implausible that a policymaker is able to make credible commitments on behalf of a successor with possibly different policy goals. In the political economy literature, this kind of problems has been typically analyzed under the no-commitment assumption. This implies that policy plans are never credible, no matter whether the government who announced these plans stays in power or not.[1] Institutions can be credible even in the presence of political turnover. In our work, we assume that the incumbent government can credibly commit regarding the policies prevailing when they are reappointed, while if a party with different objectives is elected all previous promises are disregarded.

In this respect, our contribution is to integrate the analysis about the time-inconsistency of optimal policy choices, typical of the dynamic macroeconomic literature, into a political economy model. This allows us to disentangle and quantify the effects of imperfect commitment and political turnover. In chapter 2, we study

[1] Alternatively, the literature has analyzed political turnover in cases where the time-inconsistency problem does not arise, by assuming that future policies do not affect agents' current decisions.

how these two factors affect fiscal policy choices and in particular the level of public debt. We also discuss the welfare implications of building credible institutions in the presence of political turnover. Finally, in chapter 3, we study how the possibility of changes in policy objectives affects monetary policy choices.

Finally, our contribution is to some extent methodological. We indeed show how optimal policy problems under limited commitment can be formulated recursively and solved with dynamic programming techniques. After the work of Kydland and Prescott (1977), it was thought that dynamic programming was inapplicable to government policy design problems. For the same reasons giving rise to the time-inconsistency problem, these kind of problems are not recursive. Indeed, the fact that current private decisions depend on future government policies, suggest that the government decision rules cannot be expressed as time-invariant functions of natural state variables. However, the subsequent contributions of Kydland and Prescott (1980) and Marcet and Marimon (1998), showed that these problems can have a recursive formulation by expanding the state of the economy to include a Lagrange multiplier as a *co-state* variable. This multiplier indicates the cost of keeping the promises the government made in the past. In the first period, when there are no promises inherited from previous periods, this cost is zero. These methods assume that government is committed to follow the optimal plan. Otherwise, the government would always be willing to discard previous promises, which is equivalent to reinitialize the value of the multiplier to zero. This precisely signals the time-inconsistency of such plans.

In our work, we integrate in a single framework the recursive methods of Marcet and Marimon (1998), suitable to find the optimal plans under full-commitment, with the methods developed by Klein and Rios-Rull (2003) and Klein et al. (2004) to find the Markov-Perfect solutions to policy problems where governments cannot commit. By doing so, we show how many policy problem with limited commitment can be written recursively, and be cast into a dynamic programming framework. Importantly, the possible applications of these techniques are not restricted to optimal policy problems. They can indeed be applied to a wide set of dynamic economic problems, where the specific assumption about commitment matters. For example, possible applications are to the principal-agent type of problems, in the decision problem of firms' with long-run relationship with their customers or in a context of risk-sharing among heterogenous agents.

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# Chapter 1

# Loose Commitment

Due to time-inconsistency or political turnover, policymakers' promises are not always fulfilled and plans are revised periodically. This fact is not accounted for in the commitment or the discretion approach. We consider two settings where the planner occasionally defaults on past promises. In the first setting, a default may occur in any period with a given probability. In the second, we make the likelihood of default a function of endogenous variables. We formulate these problems recursively, and provide techniques that can be applied to a general class of models. Our method can be used to analyze the plausibility and the importance of commitment and to characterize optimal policy in a more realistic environment. We illustrate the method and results in a fiscal policy application.

## 1.1 Introduction

In a general class of macroeconomic models, households' behavior depends on expectations of future variables. Characterizing optimal policy in such circumstances is intricate. A planner influences households' expectations through its actions, and households' expectations influence the actions of the planner. Following the seminal papers by Kydland and Prescott (1977) and Barro and Gordon (1983a), the literature has taken two different approaches to tackle this problem - commitment and discretion.

Both the commitment and discretion approaches are to some extent unrealistic. The commitment approach does not match the observation that governments have defaulted on past promises. The discretion approach rules out the possibility that the government achieves the benefits of making and keeping a promise, even if there is an *a posteriori* incentive to default. It seems more reasonable to assume that institutions and planners sometimes fulfill their promises and sometimes do not.

This paper characterizes optimal policy in two frameworks where some promises are kept while others are not. We first consider a setting where current promises will be fulfilled with a given probability. This setting can easily be extended to one where promises are only kept during a finite tenure. Lastly, we make the likelihood of default a function of endogenous variables.

There may be several interpretations for the *loose commitment* settings just described. A political economy interpretation is that governments fulfill their own promises, but it is possible that a new government is elected and the previous government's promises are not considered. Another interpretation is that a government commits to future plans, but defaulting becomes inevitable if particular events arise, such as wars or political instability.[1] Another interpretation of these settings is that policymakers are often required to reevaluate policy. Hence, the optimal current policy recommendation should already take into account that policy will be revised in the future.

The contribution of this paper is in part methodological. Our methods can indeed be used to analyze *loose commitment* problems in a wide class of models. In particular, our main contributions with respect to the existing literature (see Roberds (1987), and Schaumburg and Tambalotti (2007)) are the following. First, we develop methods that can be applied to a general class of non-linear models. On the contrary, the methods used in Schaumburg and Tambalotti (2007) can only be applied to linear quadratic frameworks. At a first glance, it may be thought that linear quadratic methods can provide an accurate approximation of non-linear models. However, in

[1] As it is common in the discretion literature, we consider that a default on past promises occurs whenever a reoptimization takes place. For the purposes of this paper it is indifferent whether the reoptimization is undertaken by the same planner or by a newly appointed one.

the case of *loose commitment* settings, there is an important feature to be taken into account. As discussed later, in order to have a correct linear quadratic approximation, it is often required to impose the so called "timeless perspective" assumption, which is in sharp contrast with the *loose commitment* settings under consideration. Second, we extend the *loose commitment* approach to models with endogenous state variables. Beside other considerations, this is of particular interest when studying commitment problems. Indeed, it is precisely through the choice of a state variables that policymakers can affect strategically the actions of their successors. This issue raises interesting questions and poses methodological challenges. Third, we show a setting where the current policymaker can affect the likelihood of fulfilling promises.

As an illustration of what can be learned, we provide an application to fiscal policy. When the probability of keeping promises is decreased from 1 to 0.75, most variables move substantially towards discretion. This suggests that the full commitment assumption would lead to an inaccurate depiction of the real world, where promises are not always fulfilled. We then discuss the effects of a default on economic variables. The main effect of reneging on past promises is an increase in the capital tax. Policy instruments are also found to change relatively more than private allocations during a default. We then discuss our default and commitment cycles in the spirit of the political business cycle literature. We characterize how the welfare gains change as a function of the probability to commit or the implied average time period before a default. In the endogenous probability model, we find that since the probability of commitment depends on endogenous state variables, the planner actively manipulates these state variables in order to enhance commitment.

### 1.1.1 Methodology

In an important contribution, Roberds (1987) considers that promises may not always be kept and proposes the probabilistic model also analyzed here. The author's model and assumptions are very specific, and his method is not generalizable to other applications. In another important contribution, Schaumburg and Tambalotti (2007) extended Roberds work to linear quadratic models and apply their methods to a

Barro-Gordon type of monetary model without endogenous state variables. These methods can be applied to models that are exactly linear quadratic and, as opposed to our methods, cannot be applied to non-linear models. Importantly, in the context of *loose commitment*, linear quadratic methods can not be used to approximate most non-linear models. As shown by Curdia et al. (2006), Debortoli and Nunes (2006) and Benigno and Woodfoord (2006), a correct linear-quadratic approximation can in general be derived if one imposes the timeless perspective assumption. This is a necessary assumption in all the optimal policy models where the first-best allocation cannot be attained, for instance due to the presence of distortionary taxes. The timeless perspective assumes that the problem is initialized at the full commitment steady state and that promises are always fulfilled. This is clearly in contrast with the *loose commitment* assumption.

The tools for the analysis of time-inconsistent and time-consistent policy are recent. The key reference for solving time-inconsistent models is Marcet and Marimon (1998). Klein and Rios-Rull (2003) show how to solve for the time-consistent policy with linear quadratic techniques. Klein et al. (2004) recognize that the techniques proposed in Klein and Rios-Rull (2003) do not deliver controlled accuracy and propose a technique based on generalized Euler equations and a steady state local analysis. Judd (2004) proposes global approximation methods instead of steady state local analysis. In the solution procedure, we use a global method and generalized Euler equations, taking the recent contributions of Judd (2004) and Klein et al. (2004). Besides the points presented in Judd (2004), employing a global method is specially important here. For the solution to be accurate, one needs to perform a good approximation both in commitment and default states. These two states and the corresponding policy functions are not necessarily similar for one to be *a priori* certain that a local approximation would suffice.

Finally, we prove the recursiveness of the solution using the tools of Marcet and Marimon (1998). We show how to solve for linear and non-linear models, without and with endogenous state variables, relying only on one fixed point. As a by-product, our methodology can be used as a homotopy method to obtain the time-consistent solution.

### 1.1.2 Literature review

Unlike the reputational equilibria literature, as in Backus and Driffill (1985), we are not aiming at building setups where a planner of a certain type resembles another type. We aim at characterizing the solution of planners that can make credible promises, but the commitment technology may become inoperative when it is time to fulfill them.

Barro and Gordon (1983b) address the time-inconsistency issue with trigger strategies. Our paper is not aimed at building equilibria where private agents try to enforce a given equilibrium. Such strategies are quite intricate and raise enforcement and coordination issues.[2] Hence, one can think that the planner may not always be forced to fulfill its promises, as in the *loose commitment* setting.

Flood and Isard (1989) consider a central bank commitment to a rule with escape clauses. The rule does not incorporate some important shocks affecting the economy. When such shocks hit the economy, it may be better to abandon the rule. One can interpret that our probability of default is their probability of anomalous shocks. Another interpretation is that we consider policymakers who are more rational, and do not leave important shocks outside the commitment rule. In such interpretation, the rule is always better and the planner only defaults if the commitment technology becomes inoperative.

Persson et al. (2006), elaborating on an earlier proposal of Lucas and Stokey (1983), suggest a mechanism that makes the commitment solution to be time-consistent. Each government should leave its successor with a carefully chosen maturity of nominal and indexed debt for each contingent state of nature and at all maturities. Even though such strategies do eliminate the time-consistency problem, this structure of debt is not observed in reality. The view of this paper is that at certain points in time the commitment solution may be enforced, but in some contingencies discretion is unavoidable. This paper will consider a model with endogenous public good. Rogers

[2]There are several issues on the enforcement and coordination of trigger strategies. Firstly, agents may not be able to learn such strategies because the punishment never occurs in equilibrium. Secondly, many atomistic private agents would need to develop and coordinate on highly sophisticated expectations mechanisms. Thirdly, if the punishment occurred, it is not clear that the economy would not renegotiate and enforce a better equilibrium.

(1989) showed that in such case debt restructuring can not enforce the commitment solution.

More importantly, most of the macroeconomics literature has either followed a commitment or discretion approach. This paper presents a general method that can potentially fill this gap. This paper can characterize policy under the more realistic description that some promises are fulfilled while others are not. These methods can be directly applied to the dynamic political economy literature, where different governments alternate in office, as in Alesina and Tabellini (1990). Due to technical reasons that this paper overcomes, such literature had always assumed a discretion approach or avoided time-inconsistency issues.

The paper is organized as follows: section 1.2 describes the model, section 1.3 analyzes the probabilistic setting, section 1.4 considers an extension with endogenous probabilities and section 1.5 concludes.

## 1.2 The model

The methods and frameworks described in this paper can be applied to a wide set of dynamic optimization problems. Instead of discussing the methods in an abstract way, we will show an application to a fiscal policy problem. The model we are going to use has been described for instance in Martin (2007).

A representative household derives utility from private consumption ($c$), public consumption ($g$) and leisure ($1-l$). The household has 1 unit of time each period that he can allocate between leisure and labor ($l$). He rents capital ($k$) and supplies labor ($l$) to a firm. Labor and capital earnings are taxed at a rate ($\tau^l$) and ($\tau^k$) respectively.

Following Greenwood et al. (1988), the household can also decide on the capital utilization rate $v$. Therefore, the amount of capital rented to firms will be $vk$. We are also going to assume that the depreciation rate of capital is a function of its utilization rate, ($\delta(v_t)$). In this model, we are going to abstract from public debt.[3]

[3] Dealing with debt and commitment issues is a topic that requires extensive treatment on itself and is beyond the scope of this paper. This case is addressed in chapter 2.

For given capital taxes ($\tau_t^k$), labor taxes ($\tau_t^l$), wages ($w_t$), and interest rate ($r_t$), the household problem is:

$$\max_{\{k_{t+1},c_t,l_t,v_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t, l_t) \tag{1.1}$$
$$s.t : c_t + k_{t+1} = (1 - \tau_t^k) r_t v_t k_t + (1 - \tau_t^l) w_t l_t + (1 - \delta(v_t)) k_t$$

where $\beta$ denotes the discount factor. There is uncertainty in this economy because it is not know in advance whether the planner will fulfill past promises or not. The households' first order conditions (FOCs) are:

$$u_{c,t} - \beta E_t\{u_{c,t+1}[(1 - \tau_{t+1}^k) r_{t+1} v_{t+1} + 1 - \delta(v_{t+1})]\} = 0 \tag{1.2}$$
$$u_{c,t}(1 - \tau_t^l) w_t + u_{l,t} = 0 \tag{1.3}$$
$$(1 - \tau_t^k) r_t - \delta_{v,t} = 0 \tag{1.4}$$

In this problem, the source of time-inconsistency appears in equation (1.2). Today's household decisions depend on the expectation of future variables. In particular, the contemporaneous capital accumulation decision depends on future returns on capital. It is important to note that equation (1.4) is stating that the current capital tax is distortionary. If the planner would raise capital taxes, households could choose a lower capital utilization rate.[4] This feature is important for our model, because if the planner does not keep his past promises and the capital utilization is fixed, then the capital tax could be set at an extremely high and implausible value. Martin (2007) showed that with fixed capital utilization, and for plausible calibrations, an equilibrium under discretion does not exist. There may be other reasons that inhibit the planner to choose an extremely high capital rate when it defaults, we chose this specification that guarantees the model to have a well defined solution for the commitment and discretion case.

[4]As discussed in Martin (2007), it is important that at least some depreciation is not tax deductible, as assumed for instance in Greenwood et al. (1998). If this is not the case, the current capital tax is not viewed by the current government as distortionary and an equilibrium in such economy may not exist. In some developed economies, there is a tax allowance for accounting depreciation, which differs from the actual depreciation. If there is excess depreciation due to a high capacity utilization such depreciation would still not be tax deductible.

Total output $y_t$ is produced according to the function $F(k_t, v_t, l_t) = (k_t v_t)^\theta l_t^{1-\theta}$. Firms operate in perfectly competitive markets. Hence, wages and interest rates are given by:

$$r_t = F_{kv,t} \tag{1.5}$$

$$w_t = F_{l,t} \tag{1.6}$$

The planner provides the public good $g$, sets taxes $\tau^k$ and $\tau^l$, satisfying the balanced budget constraint:

$$g_t = \tau_t^k r_t v_t k_t + \tau_t^l w_t l_t \tag{1.7}$$

Combining the households and governments' budget constraint one obtains the feasibility constraint:

$$y_t = c_t + g_t + k_{t+1} - (1 - \delta(v_t))k_t \tag{1.8}$$

To make our problem simpler, we can proceed with a number of simplifications. Equation (1.4) can be used to express the capital utilization as a function of other variables:

$$v_t = v(k_t, l_t, \tau_t^k) \tag{1.9}$$

Similarly, using the household and government budget constraints, private and public consumption can be expressed as functions:

$$c_t = c(k_{t+1}, k_t, l_t, \tau_t^k, \tau_t^l) \tag{1.10}$$

$$g_t = g(k_t, l_t, \tau_t^k, \tau_t^l) \tag{1.11}$$

Hence, the FOCs in equations (1.2, 1.3) can be written in a more compact form:

$$b_1(x_t(\omega^t), k_t(\omega^t)) + \beta E_t b_2(x_{t+1}(\omega^{t+1}), k_{t+1}(\omega^{t+1})) = 0 \tag{1.12}$$

The vector of functions $b_1, b_2$ depends on several variables, where $x_t \equiv (k_{t+1}, l_t, \tau_t^k, \tau_t^l)$ is the vector of contemporaneous control variables, $k_t$ is the state variable and $\omega^t$ is the history of events up to $t$.[5] Note that $v_t, c_t$ and $g_t$ have already been substituted in equation (1.12).

[5]The class of models that our methodology is able to handle is fairly general and has the same requirements of Marcet and Marimon (1998). The separability in equations (1.12) is not necessary and the terms $k_t, k_{t+1}, x_t, x_{t+1}$ can all interact in a multiplicative way. Our methodology is also able to handle participation constraints and other infinite horizon constraints, as also described in Marcet and Marimon (1998).

## 1.3 The probabilistic model

We will consider a model where a planner is not sure whether his promises will be kept or not. As explained above, this uncertainty can be due to several factors. For simplicity, we assume that these events are exogenous and that in any period the economy will experience default or commitment with given exogenous probabilities. In section 1.4, we relax this assumption. Since it is indifferent whether it is the same or a new planner who defaults and reoptimizes, we use the terms "reelection", "new planner" and "default" interchangeably.

To make matters simple, we abstract from any shock other than the random variable $s_t$ describing default ($D$) or commitment ($ND$) in period $t$. It is a straightforward generalization to include other sources of uncertainty, but the notation would be harder to follow. More formally, suppose the occurrence of Default or No Default is driven by a Markov stochastic process $\{s_t\}_{t=1}^{\infty}$ with possible realizations $\bar{s}_t \in \Phi \equiv \{D, ND\}$, and let $\Omega^t$ be the set of possible histories up to time $t$:

$$\Omega^t \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : \bar{s}_j \in \Phi, \forall j = 1, ..., t\} \tag{1.13}$$

We only consider the histories $\omega^t = \{D, \bar{s}_1, \bar{s}_2, ..., \bar{s}_t\}$, i.e. histories that start with a default on past promises. This is because in the initial period there are no promises to be fulfilled or equivalently the current government has just been settled. Before turning to the planner's problem, we describe the problem of individual agents.

### 1.3.1 Individual agents and constraints

In equation (1.12) we wrote the households' FOCs. These equations depend on future variables and hence households need to form rational expectations using available information. Given our institutional setting, households believe the promises of the current planner, but consider that if a different planner comes into play, then different policies will be implemented and past promises will not be kept. As it is common in the time-consistency literature, economic agents will take future controls that can not be committed upon as functions of the state variable, i.e. $x_{t+1}(\{\omega^t, D\}) = \Psi(k_{t+1}(\{\omega^t, D\}))$ where we use the short notation $\{\omega^t, D\}$ to denote $\{\omega^t, \bar{s}_{t+1} = D\}$.

The function $\Psi(.)$ denotes the vector of policy functions that rational agents anticipate to be implemented in future periods.[6] Therefore, the constraint in equation (1.12) becomes:

$$b_1(x_t(\omega^t), k_t(\omega^t)) + \beta Prob(\{\omega^t, ND\}|\omega^t) b_2(x_{t+1}(\{\omega^t, ND\}), k_{t+1}(\{\omega^t, ND\})) \quad (1.14)$$
$$+\beta Prob(\{\omega^t, D\}|\omega^t) b_2(\Psi(k_{t+1}(\{\omega^t, D\})), k_{t+1}(\{\omega^t, D\})) = 0,$$

where we use the short notation

$$Prob(\{\omega^t, ND\}|\omega^t) \equiv Prob[\{s_j\}_{j=0}^{t+1} = \{\omega^t, ND\}|\{s_j\}_{j=0}^{t} = \omega^t].$$

Note that $k$ is a state variable and hence it is understood that $k_{t+1}(\{\omega^t, ND\}) = k_{t+1}(\{\omega^t, D\})$, $\forall \omega^t$.

### 1.3.2 The planner

When default occurs, a new planner is appointed and it will be taking decisions from that point onwards. Therefore, it is convenient to separate all histories $\omega^t$ with respect to the first time when default occurs. This is because we want to know which histories correspond to which planner. We now define the subset of $\Omega^t$ of histories where only commitment has occurred up to time $t$ as:

$$\Omega_{ND}^t \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : \bar{s}_j = ND, \forall j = 1, ..., t\} \quad (1.15)$$

and the subsets of histories where the first default occurs in period $i$,

$$\Omega_{D,i}^t \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : (\bar{s}_i = D) \wedge (\bar{s}_j = ND), \forall j = 1, ..., i-1\}, \text{ if } i \leq t \quad (1.16)$$
$$\Omega_{D,i}^t \equiv \oslash, \text{ if } i > t$$

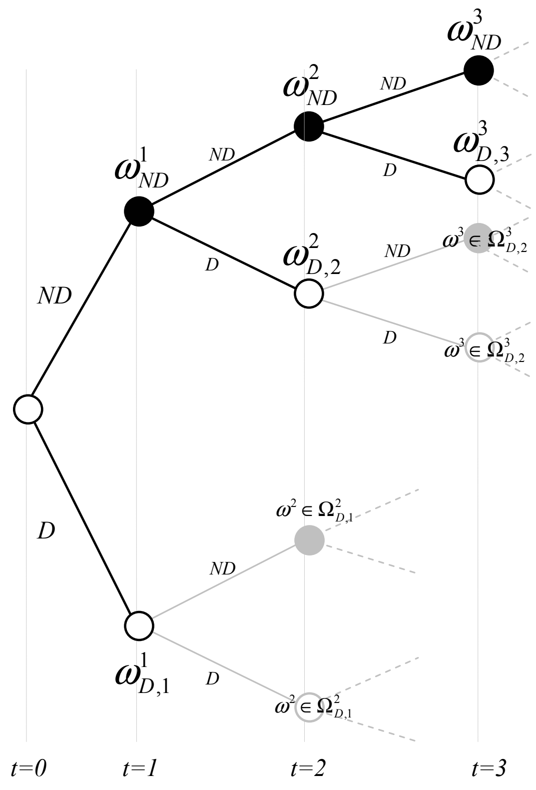
By construction note that $\{\Omega_{ND}^t, \Omega_{D,1}^t, ..., \Omega_{D,t}^t\}$ is a partition of the set $\Omega^t$. Moreover, it can be seen that the sets $\Omega_{ND}^t$ and $\Omega_{D,i}^i$ are singletons.[7] Therefore, in order to avoid confusion between histories and sets of histories, we will refer to these singleton sets as $\omega_{ND}^t$ and $\omega_{D,i}^i$ respectively.

[6] For further discussions on this issue see Klein et al. (2004).

[7] $\Omega_{ND}^t$ only contains the history $\{D, \bar{s}_1 = ND, \bar{s}_2 = ND, ..., \bar{s}_t = ND\}$ and similarly the set $\Omega_{D,i}^i$ only contains the history $\{D, \bar{s}_1 = ND, \bar{s}_2 = ND, ..., \bar{s}_{i-1} = ND, \bar{s}_i = D\}$.

In figure 1.1, we show a more intuitive representation of the particular partition of histories specified above, where we use the name of the unique history ending in a given node to denote the node itself. White nodes indicate when a new planner is settled (default has occurred), while black nodes indicate the cases where the first planner is still in power (no default has occurred). We can see that in any period $t$ there is only one history $\omega^t_{ND}$ such that commitment has always occurred in the past. Moreover, there is also only one history $\omega^i_{D,i} = \left\{\omega^{i-1}_{ND}, D\right\}$, meaning that the first default occurred in period $i$. In our institutional setting, a new planner is then settled from the node $\omega^i_{D,i}$ onward and it will make its choices over all the possible histories passing through the node $\omega^i_{D,i}$, that is the sets $\Omega^t_{D,i}, \forall t \geq i$.

**Figure 1.1:** Diagram of the possible histories




We will now write the problem of the current planner where to simplify notation, and without loss of generality, we abstract from the presence of constraints in the maximization problem:

$$W(k_0) = \max_{\substack{\{x_t(\omega^t)\}_{t=0}^{\infty} \\ \omega^t \in \Omega^t}} [\sum_{t=0}^{\infty} \sum_{\omega_{ND}^t} \beta^t \{Prob(\omega^t) u(x_t(\omega^t), k_t(\omega^t))\} \tag{1.17}$$
$$+ \max_{\substack{\{x_t(\omega^t)\}_{t=1}^{\infty} \\ \omega^t \in \Omega_{D,1}^t}} \left\{ \sum_{t=1}^{\infty} \sum_{\omega^t \in \Omega_{D,1}^t} \beta^t \{Prob(\omega^t) u(x_t(\omega^t), k_t(\omega^t))\} \right\}$$
$$+ \max_{\substack{\{x_t(\omega^t)\}_{t=2}^{\infty} \\ \omega^t \in \Omega_{D,2}^t}} \left\{ \sum_{t=2}^{\infty} \sum_{\omega^t \in \Omega_{D,2}^t} \beta^t \{Prob(\omega^t) u(x_t(\omega^t), k_t(\omega^t))\} \right\}$$
$$+ ...]$$

where we are using the short notation $Prob(\omega^t) = Prob(\{s_j\}_{j=0}^t = \omega^t)$. Equation (1.17) makes it explicit that inside the maximization problem of the current government there are other planners maximizing welfare during their tenures. Given that $\{\Omega_{ND}^t, \Omega_{D,1}^t, ..., \Omega_{D,t}^t\}$ is a partition of the set $\Omega^t$, all the histories are contemplated in our formulation. Since $\forall\, t > i$, $\Omega_{D,i}^t = \{\omega_{D,i}^i, \{\bar{s}_j\}_{j=i}^t\}$, we can rewrite the probabilities for $\omega^t \in \Omega_{D,i}^t$ in the following way:

$$Prob(\omega^t) = Prob(\omega_{D,i}^i \wedge \omega^t) = Prob(\omega^t | \omega_{D,i}^i) Prob(\omega_{D,i}^i), \forall\, \omega^t \in \Omega_{D,i}^t,\ t \geq i. \tag{1.18}$$

Substituting for these expressions into equation (1.17) and collecting the common term in the summation, we obtain:

$$W(k_0) = \max_{\substack{\{x_t(\omega^t)\}_{t=0}^{\infty} \\ \omega^t \in \Omega^t}} \left\{ \sum_{t=0}^{\infty} \sum_{\omega_{ND}^t} \beta^t \{Prob(\omega^t) u(x_t(\omega^t), k_t(\omega^t))\} \right. \tag{1.19}$$
$$\left. + \sum_{i=1}^{\infty} \beta^i Prob(\omega_{D,i}^i) \left[ \max_{\substack{\{x_t(\omega^t)\}_{t=i}^{\infty} \\ \omega^t \in \Omega_{D,i}^t}} \sum_{t=i}^{\infty} \sum_{\omega^t \in \Omega_{D,i}^t} \beta^{t-i} \{Prob(\omega^t | \omega_{D,i}^i) u(x_t(\omega^t), k_t(\omega^t))\} \right] \right\}$$

Since we are assuming that any future planner is also maximizing we can define the value functions:

$$\xi_i(k_i(\omega_{D,i}^i)) \equiv \max_{\substack{\{x_t(\omega^t)\}_{t=i}^{\infty} \\ \omega^t \in \Omega_{D,i}^t}} \sum_{t=i}^{\infty} \sum_{\omega^t \in \Omega_{D,i}^t} \beta^{t-i} \{Prob(\omega^t | \omega_{D,i}^i) u(x_t(\omega^t), k_t(\omega^t))\} \tag{1.20}$$

where it was made explicit that each planner assigns probability one to its initial node. The value functions $\xi_i(k_i)$ summarize the happenings after the node $\omega^i_{D,i}$. Since $\Omega^t_{D,i} \cap \Omega^t_{D,j} = \oslash$ for $i \neq j$, the choices of future planners are independent between themselves. This formulation is very general since one can assume several institutional settings that the future planners will face. For example, one can assume that some future planners have full commitment while others do not. For simplicity we assume that all future planners face the same institutional settings which at this stage we do not specify, thus implying that $\xi_i(k_i) = \xi(k_i)\ \forall i$.[8] Since all the histories $\{\Omega^t_{D,1}, ..., \Omega^t_{D,t}\}$ are already being maximized by other planners, it is equivalent to consider that the initial planner maximizes over the single history $\{\omega^t : \omega^t \in \Omega^t_{ND}\} \equiv \omega^t_{ND}$ instead of $\omega^t \in \Omega^t$. We can therefore rewrite the problem at period $t = 0$ as:

$$W(k_0) = \max_{\{x_t(\omega^t_{ND})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \Big\{\beta^t \{Prob(\omega^t_{ND}) u(x_t(\omega^t_{ND}), k_t(\omega^t_{ND}))\} + \sum_{i=1}^{\infty} \beta^i Prob(\omega^i_{D,i}) \xi(k_i(\omega^i_{D,i})) \Big\} \tag{1.21}$$

To further simplify the problem, we assume that the random variable $s_t$ is i.i.d..[9] Also to simplify notation denote $Prob(\{\omega^t, ND\}|\omega^t) = \pi$ and $Prob(\{\omega^t, D\}|\omega^t) = 1 - \pi$, which implies that:

$$Prob(\omega^t_{ND}) = \pi^t \tag{1.22}$$

$$Prob(\omega^t_{D,t}) = \pi^{t-1}(1 - \pi). \tag{1.23}$$

With this formulation at hand, we are ready to show that our problem can be written as a saddle point functional equation (SPFE), and that the optimal policy functions of the planner are time-invariant and depend on a finite set of states.

[8] In chapter 2 we relax this assumption and focus on political disagreement issues.

[9] It is straightforward to generalize our formulation to Markov processes.

### The recursive formulation

Collecting results from the previous section, the problem of the current planner is:

$$\max_{\{x_t(\omega_{ND}^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta\pi)^t \{u(x_t(\omega_{ND}^t), k_t(\omega_{ND}^t)) + \beta(1-\pi)\xi(k_{t+1}(\omega_{D,t+1}^{t+1}))\} \tag{1.24}$$

$$s.t : b_1(x_t(\omega_{ND}^t), k_t(\omega_{ND}^t)) + \beta(1-\pi) b_2(\Psi(k_{t+1}(\{\omega_{ND}^t, D\})), k_{t+1}(\{\omega_{ND}^t, D\})) + \beta\pi b_2(x_{t+1}(\omega_{ND}^{t+1}), k_{t+1}(\omega_{ND}^{t+1})) = 0$$

Due to the fact that we do have future controls in the constraints through the term $\beta\pi b_2(x_{t+1}(\omega_{ND}^{t+1}), k_{t+1}(\omega_{ND}^{t+1}))$, the usual Bellman equation is not satisfied.[10] Building on the results of Marcet and Marimon (1998), we show that problems of this type can be rewritten as a SPFE that generalizes the usual Bellman equation. This result is summarized in proposition 1.

**Proposition 1** *Problem (1.24) can be written as saddle point functional equation as:*

$$W(k,\gamma) = \min_{\lambda \geq 0} \max_{x} \{H(x,k,\lambda,\gamma) + \beta(1-\pi)\xi(k') + \beta\pi W(k',\gamma')\} \tag{1.25}$$

$$s.t : \gamma' = \lambda, \ \ \gamma_0 = 0$$

*where*

$$H(x,k,\lambda,\gamma) = u(x,k) + \lambda g_1(x,k) + \gamma g_2(x,k) \tag{1.26}$$

$$g_1(x,k) = b_1(x,k) + \beta(1-\pi) b_2(\Psi(k'), k') \tag{1.27}$$

$$g_2(x,k) = b_2(x,k) \tag{1.28}$$

**Proof.** See the appendix. ■

Proposition 1 makes it clear that the current planner maximizes utility of the representative agent subject to the constraints $g_1(x,k) + \beta\pi g_2(x',k') = 0$, where the latter is incorporated in $H$. If there is no commitment, the continuation of the problem is $\xi(k')$. If the current promises will be fulfilled, then the continuation of

[10] For details see Stokey et al. (1989).

the problem is $W(k', \gamma')$, and promises are summarized in the co-state variable $\gamma'$. The co-state variable is not a physical variable and the policymaker always faces the temptation to set it to zero. Also note that in our problem only the first constraint contained in equation (1.12) contains future control variables. Therefore, only the first element of the vector $\lambda$ needs to be included as a co-state variable. The optimal policy functions of such problem are time invariant and depend on a finite number of states, as proposition 2 describes.[11]

**Proposition 2** *The solution of problem (1.24) is a time invariant function with state variables* $(k_t, \gamma_t)$*, that is to say:*

$$\psi(k, \gamma) \in \arg\min_{\lambda \geq 0} \max_{x} \{H(x, k, \lambda, \gamma) + \beta(1 - \pi)\xi(k') + \beta\pi W(k', \gamma')\} \tag{1.29}$$
$$s.t : \gamma' = \lambda, \ \gamma_0 = 0$$

**Proof.** See the appendix. ■

### 1.3.3 Equilibrium

In the institutional setting built in equation (1.24), we only assume that all planners from period 1 onward will face the same problems. From now on, we also assume that all future planners face the same institutional setting as we specify in period 0. In other words, we specify their problems in the same way as the problem of the planner in period 0. Thus we can use the following definition of equilibrium.

**Definition 1** *A Markov Perfect Equilibrium where each planner faces the same institutional setting must satisfy the following conditions.*

1. Given $\Psi(k)$ and $\xi(k)$, the sequence $\{x_t\}$ solves problem (1.24);

2. The value function $W(k, \gamma)$ is such that $\xi(k) = W(k, 0) \equiv W(k)$;

[11] As it is common in the time-consistent literature and also in the optimal taxation literature we do not prove that the optimal policy function is unique. Nevertheless, we found no evidence of multiple solutions.

3. The policy functions $\psi(k,\gamma)$ solving problem (1.24) are such that $\Psi(k) = \psi(k,0)$.

The second part of the definition imposes directly that the problem of the initial and future planners must be equal. When a planner comes to office, he has not previously made any promise and therefore the co-state variable is reset to zero. While a planner is in office, he makes promises, and faces the temptation to deviate and reoptimize. In other words, the multiplier encoding the planner's promises is not a physical state variable and could always be put to zero. We are assuming that such a deviation only occurs with probability $1-\pi$. The third part of the definition imposes a consistency requirement in the constraints. More precisely, we require the policy functions $\Psi(k)$ that agents expect to be implemented under default to be consistent with the optimal policy function. We refer to the notion of Markov Perfect Equilibrium because the function $\Psi$ only depends on the natural state variables $k$. Also, in this equilibrium neither the planner nor individual agents desire to change behavior. Individual agents are maximizing and their beliefs are correct. The planner, taking as given $\Psi$ and $\xi = W$, is also maximizing.

### 1.3.4 Solution strategy

The previous propositions showed that the problem can be written recursively. But at first sight, solving this problem looks complex. Two main features are worth noting. First, a *loose commitment* setting raises both the difficulties of problems with commitment and those of problems with discretion. More precisely, as in the problems with commitment, making the problem recursive requires to include the Lagrange multiplier as a co-state variable. Moreover, as in the problems with discretion, the derivatives of the (unknown) policy functions enter in the FOCs of the problem. Second, the policy functions ($\Psi(k)$) and the value function ($\xi(k)$) appear in the constraints and in the objective function. These functions are taken as given by the current planner. One could try to guess such functions, solve the problem, update the guessed policy and value functions, and iterate until convergence. But, such procedure would imply solving three fixed points, namely one for the problem itself and two for $\Psi(k)$ and $\xi(k)$.

In this section, we discuss how to solve the problem in an easier way. We use the FOCs of the associated lagrangian formulation. Our generic problem is:

$$W(k_0) = \max_{\{x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta\pi)^t \left[u\left(x_t, k_t\right) + \beta\left(1-\pi\right)\xi\left(k_{t+1}\right)\right] \tag{1.30}$$

$$s.t.\ g_1\left(x_t, k_t\right) + \beta\pi g_2\left(x_{t+1}, k_{t+1}\right) = 0 \qquad \forall t = 0, ..., \infty$$

where $g_1$ and $g_2$ are defined by equations (1.27) and (1.28) respectively.

Details on the FOCs can be found in the appendix. The term $\xi_{k,t+1}$ appears in the FOCs, because the current planner tries to influence future planners. The value function $\xi\left(k_{t+1}\right)$ summarizes the welfare that agents will achieve with a planner appointed at $t+1$. From the perspective of the planner appointed at $t+1$, the state variables $k_{t+1}$ can not be changed. Nevertheless, from the perspective of the current planner, who is in charge at period $t$, $k_{t+1}$ is not given and can be set strategically.[12] Note that, in a model without state variables, as in Schaumburg and Tambalotti (2007), these interactions between current and future planners are not present, thus significantly simplyfing the problem.

The FOCs expressed in eqs. (A-8), (A-9) and (A-10) allows us to solve for the optimal policy. As described in Definition 1, we consider the case where future planners face the same problem as the current planner, thus implying $\xi\left(k_t\right) = W(k_t)$ and hence $\xi_{k,t+1} = W_{k,t+1}$. We will show a solution method that only relies on solving one fixed point. To obtain the derivative $W_{k,t+1}$ we can use envelope results, which are summarized in result 1.

**Result 1** *Using envelope results it follows that:*

$$\frac{\partial W\left(k_t\right)}{\partial k_t} = \frac{\partial u(x_t\left(k_t\right), k_t)}{\partial k_t} + \lambda_t g_{1,k_t,t} \tag{1.31}$$

*where all variables are evaluated using the optimal policy of a planner appointed in period $t$, given the state $k_t$.*

[12]Note that, when default occurs, the lagrange multiplier is set to zero and cannot be used to influence incoming planners.

Result 1 uses the fact that all the planners are maximizing the same function, which allows the use of envelope principles.[13] It is important to note that in equation (1.31) all the variables are evaluated with the optimal policy of a newly elected government.

By Definition 1, the policy functions that the current and future planners implement are equal. If we use the envelope result to substitute $\xi_{k,t+1} = W_{k,t+1}$, the FOCs only depend on the functions $\psi(k_t, \lambda_{t-1})$ and $\Psi(k)$. Using Definition 1 and Proposition 1 we know that $\Psi(k) = \psi(k, 0), \forall k$, which also considerably simplifies the problem. We use a collocation method to solve for the optimal policy functions. Hence, using Result 1, Proposition 1 and 2, we can solve the problem relying on only one fixed point.

We want to stress that in our framework the global solution methods proposed in Judd (1992) and Judd (2004) are more appropriate than local approximations. Besides the arguments presented by Judd, there are other reasons specific to our problem. The value function derivative, the levels and derivative of policy under default are present in the FOCs. The allocations under default and commitment are not likely to be similar. Demanding for a local approximation to deliver a good approximation in distant points to levels, derivatives and value functions is quite demanding.

The linear quadratic approximation proposed in Benigno and Woodfoord (2006) is only valid under the timeless perspective assumption. This requires to assume that initial commitments are equal to the steady-state commitment. There are several reasons that make the timeless perspective approach inappropriate in our framework. Firstly, we consider that commitments may be broken and analyze the subsequent transition dynamics. Secondly, our model does not have a deterministic steady state point around which one can take an approximation. Indeed, shutting down uncertainty completely changes the problem. Thirdly, under discretion the allocations can be very far from the commitment steady-state. Our method is more suitable and it is also simpler. Even for an exactly linear quadratic model, Schaumburg and Tam-

[13]A proof of this envelope result is available upon request.

balotti (2007) only solved a model with no endogenous state variables, and suggested a procedure to handle endogenous state variables relying on three fixed points. The method presented here relies on only one fixed point in policy functions.

Besides these considerations, there is a crucial drawback of applying the linear-quadratic timeless perspective approach to study problems with *loose commitment* settings, as suggested by Schaumburg and Tambalotti (2007). This point was already discussed in the introduction and its methodological discussion.

### 1.3.5 Results

In order to proceed to the numerical solution, we specify a per-period utility function:

$$u(c_t, g_t, l_t) = (1 - \phi_g)\left[\phi_c \log(c_t) + (1 - \phi_c)\log(1 - l_t)\right] + \phi_g \log(g_t) \tag{1.32}$$

and a depreciation function:

$$\delta(v_t) = \frac{\chi_0}{\chi_1} v_t^{\chi_1} \tag{1.33}$$

We use a standard calibration for an annual model of the US economy. Table 1.1 summarizes the values used for the parameters. The parameters $\chi_0$ and $\chi_1$ imply that in steady state the capital utilization rate ($v$) is about 0.8, and the depreciation rate $\delta(v)$ is about 0.08.

**Table 1.1:** Parameter values

| Parameter | Value | Description |
|---|---|---|
| $\beta$ | .96 | Discount factor |
| $\phi_c$ | .285 | Weight of consumption vs. leisure |
| $\phi_g$ | .119 | Weight of public vs. private consumption |
| $\theta$ | .36 | Capital share |
| $\chi_0$ | .171 | Depreciation function parameter |
| $\chi_1$ | 1.521 | Depreciation function parameter |

Table 1.2 presents the long run average for several variables, and across different parameterizations of $\pi$. The columns with $\pi = 1$ and $\pi = 0$ correspond to the full commitment and the full discretion cases, respectively. In the full commitment

**Table 1.2:** Average values

| | **1.00** | **0.75** | **0.50** | **0.25** | **0.00** |
|---|---|---|---|---|---|
| **k** | 1.122 | 0.947 | 0.899 | 0.880 | 0.870 |
| $\lambda$ | -0.536 | -0.177 | -0.080 | -0.030 | 0.000 |
| **g** | 0.093 | 0.076 | 0.072 | 0.070 | 0.069 |
| **c** | 0.196 | 0.216 | 0.220 | 0.222 | 0.224 |
| **y** | 0.378 | 0.368 | 0.364 | 0.363 | 0.362 |
| $\tau^k$ | 0.000 | 0.131 | 0.163 | 0.178 | 0.187 |
| $\tau^l$ | 0.384 | 0.251 | 0.218 | 0.203 | 0.191 |
| **l** | 0.233 | 0.245 | 0.248 | 0.250 | 0.250 |
| **u** | 0.798 | 0.799 | 0.801 | 0.800 | 0.799 |

Note: The values refer to long run averages.

model, the capital tax is zero, a result common in the optimal taxation literature with full commitment, e.g. as in Chamley (1986). With full discretion, the capital tax is roughly 19%. In the discretion model, once capital has been accumulated, the government has a temptation to tax it, since it constitutes a relatively inelastic tax base. However, due to the possibility of changing the capital utilization rate, capital is not an entirely fixed factor of production that can be heavily taxed. As we will discuss later, if private agents are surprised with higher than expected capital taxes, then the capital utilization rate is lowered.

There is another reason why capital is not heavily taxed when promises are reneged. In this model, governments cannot issue debt and have to balance their budgets every period. In Chamley (1986), there is a big incentive to tax capital very highly in earlier periods to obtain large amounts of assets and eliminate distortionary taxation in later periods.[14] By imposing a balanced budget, higher capital taxation revenues have to be used immediately. Even though in our model the incentives to tax capital under discretion are mitigated, it is still the case that capital taxes under discretion are higher than under commitment. For an example where the reverse can happen the reader is referred to Benhabib and Rustichini (1997) and Klein et al.

[14]The zero long run tax on capital still holds for a variety of cases (including balanced budget) as shown by Judd (1985).

(2004).

Since capital taxes are lower under commitment, the level of capital is higher. As a direct consequence of no capital tax revenues, labor taxes need to be higher. Higher labor taxes induce households to work less. Private consumption is lower in the full commitment economy, while public consumption is higher. Obviously, the allocations in terms of leisure, private and public consumption are more efficient in the full commitment economy.

We now turn into commenting the *loose commitment* settings. This means to consider the effects of changes in the probability of commitment $\pi$. The main purpose of this paper is to provide a theoretical basis for this kind of models. Providing a definitive answer on the probability $\pi$ or empirically estimating these models is beyond the scope of this paper. Nevertheless, we can provide some interpretation of the probability $\pi$. A probability of reelection $\pi$ implies an expected tenure of $1/(1-\pi)$. For example, a value of 0.75 implies that a planner is expected to be in power for 4 years on average. A calibration based on the political history of the US implies a value of 0.8, while the political history of Italy would imply a calibration around 0.3.[15] The numbers above are excluding the possibility of broken promises during the tenure, therefore a value of 0.75 might be considered an upper bound. Rallings (1987) tried to obtain a measure of how many manifesto pledges were actually implemented. Some of these pledges often reveal political options, such as the composition of expenditures, and may not always be related to time-inconsistency issues. The average number reported by the author is 0.63 and 0.71 for Britain and Canada respectively. Arguably, these estimates may be considered higher than the actual $\pi$. Ideological promises are easier to fulfill than promises where a temptation to default actually exists. While Rallings (1987) estimates includes both types of promises, the measure $\pi$ only refers to the latter. Obviously, the measure $\pi$ depends on the country being studied and the specific policy and institutional settings involved.

In our analysis, all variables seem to be much closer to the discretion solution rather than to the commitment solution. If the probability of committing is 0.75,

[15] In Italy, if we consider a change in government when the same prime minister is in power but the coalition changes, then this number drops to 0.

**Table 1.3:** Relative difference from full commitment

| | **1.00** | **0.75** | **0.5** | **0.25** | **0.00** |
|---|---|---|---|---|---|
| **k** | 1.0 | 0.306 | 0.115 | 0.040 | 0.0 |
| $\lambda$ | 1.0 | 0.330 | 0.149 | 0.056 | 0.0 |
| **g** | 1.0 | 0.292 | 0.125 | 0.042 | 0.0 |
| **c** | 1.0 | 0.286 | 0.143 | 0.071 | 0.0 |
| **y** | 1.0 | 0.375 | 0.125 | 0.063 | 0.0 |
| $\tau^k$ | 1.0 | 0.299 | 0.128 | 0.048 | 0.0 |
| $\tau^l$ | 1.0 | 0.311 | 0.140 | 0.062 | 0.0 |
| **l** | 1.0 | 0.294 | 0.118 | 0.000 | 0.0 |

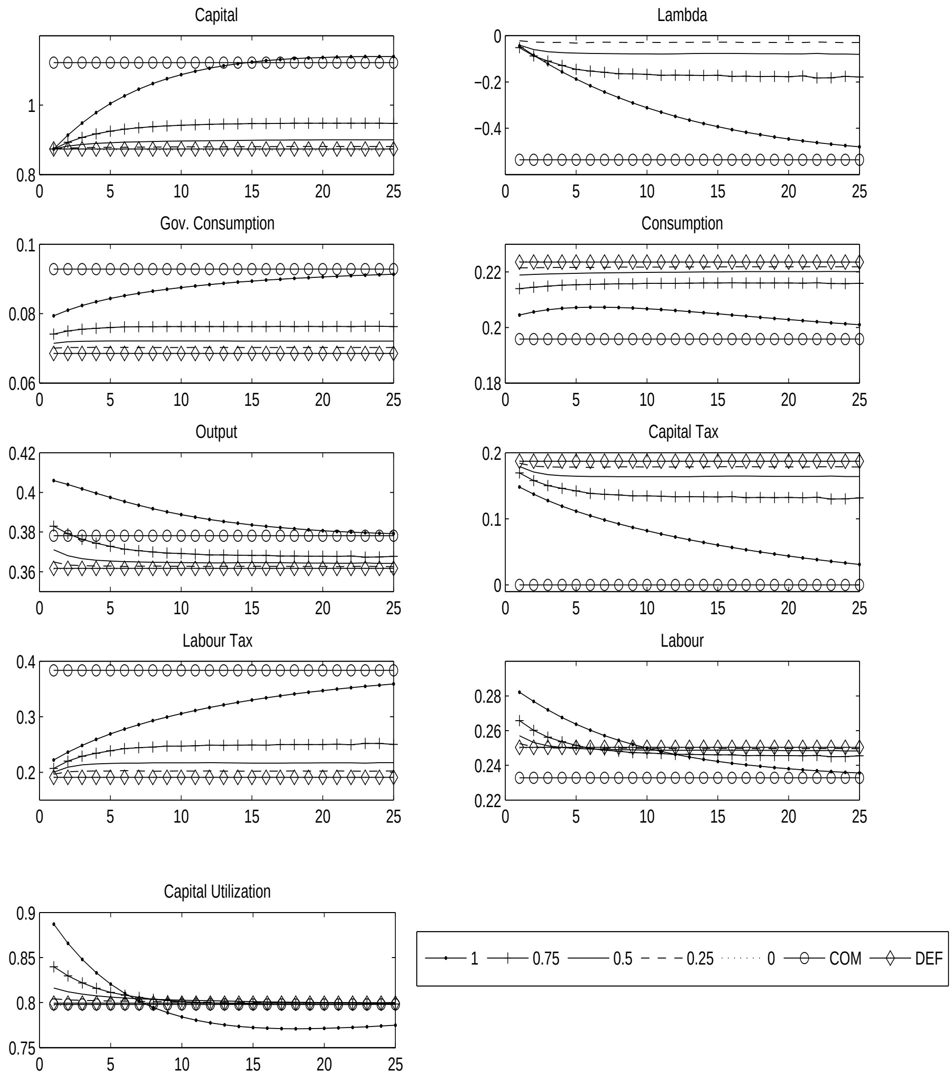
Note: The values are computed with the formula: $(x_\pi - x_{\pi=0})/(x_{\pi=1} - x_{\pi=0})$.

average allocations only move about 31% of the distance from discretion to commitment. Table 1.3 computes this value for all the allocations. The other way to interpret our results is that small reductions in the probability $\pi$ from the full commitment solution have dramatic effects. When $\pi$ is decreased from 1 to 0.75, the absolute drop in capital is 69% of the difference between full commitment and discretion. This suggests that the full commitment assumption would lead to an inaccurate depiction of the real world, where promises are not always fulfilled.

We now describe the transition dynamics. Figure 1.2 plots the average path during the first 25 quarters, where we have initialized capital at its steady state value under discretion and considered that no promises had been made (i.e. $\lambda_{-1} = 0$). Since the economy starts with a relatively low level of capital, the utilization rate of capital is relatively high. This occurs despite the fact that capital taxes are high in early periods, as commitment starts to build. In initial periods, since capital taxes are relatively high, labor taxes are lower and labor is higher. As time evolves, the picture confirms the results of table 1.2. Indeed, at later periods all the variables are relatively closer to the discretion path.

We should finally comment on the path of the lagrange multiplier. This variable may not have economic interest per se, since it is unobservable. We should nevertheless mention that as all other variables, the lagrange multiplier is also much closer to the discretion steady state of 0. This suggests that a local approximation around

**Figure 1.2:** Average allocations



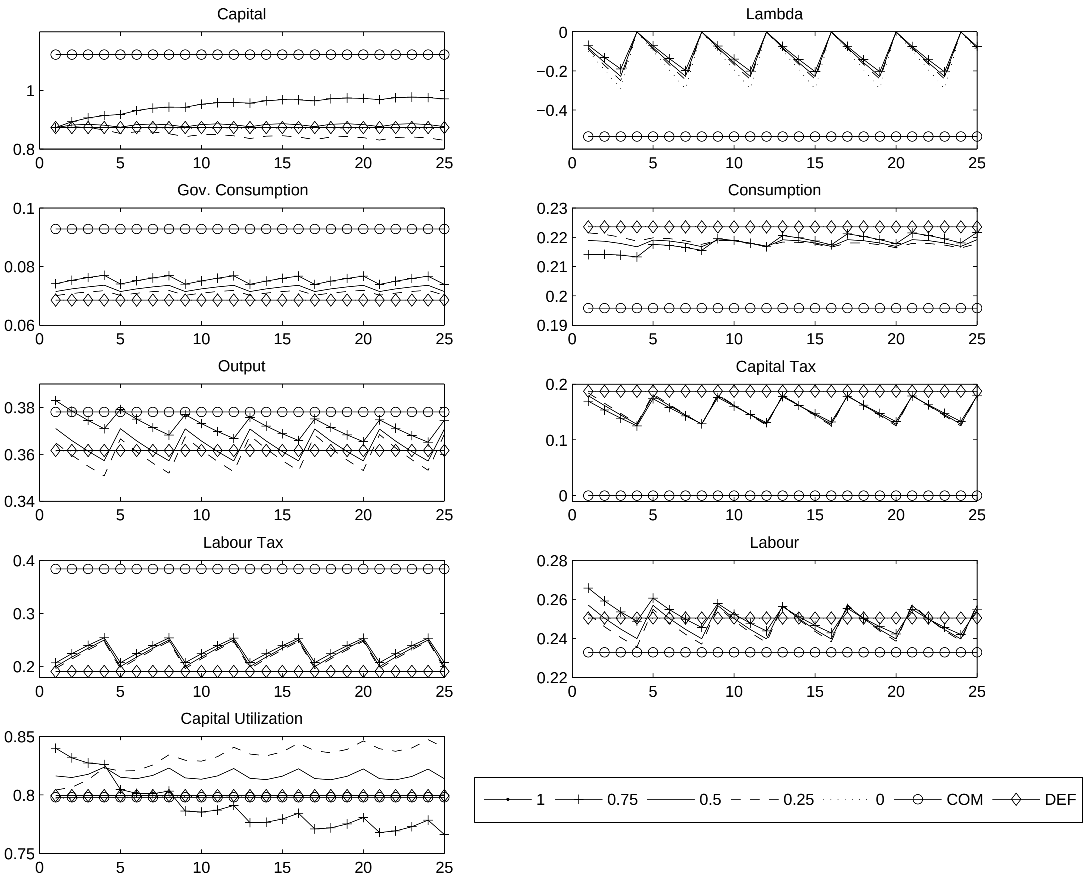

Note: The figure plots for several values of $\pi$ the average path across realizations. Capital is initialized at the discretion steady state. The lagrange multiplier is initialized at zero, considering that there were no previous promises in the first period.

the full commitment steady state, as performed in the timeless perspective approach may be a poor approximation for *loose commitment* frameworks. Also our analysis suggests that characterizing allocations in a full commitment analysis may be less realistic than in a full discretion approach.

So far the analysis has only referred to average paths. We now analyze what happens in the specific periods when governments renege on their past promises. For this purpose, in figure 1.3 we plot the paths followed in a given history. We consider the history where by chance a new planner is reappointed every four years. When default occurs, the planner breaks the promise of a low capital tax. Capital is a relatively inelastic tax base and hence capital taxes are increased. Capital accumulation is almost unchanged but households immediately react by decreasing the capital utilization rate. Since there are more capital tax revenues, labor taxes are cut, leading to more labor. Overall, output increases mainly fueled by the increase in labor input.

Our analysis can be related to the literature on political cycles, as described for instance in Drazen (2000). The empirical analysis of political cycles mentions that aggregate output and private consumption are not affected by the political cycle. In our model, both output and private consumption are not found to move much in relative terms. The literature has argued that policy instruments are more affected by the political cycle. Our model has the same prediction. Public consumption, capital taxes and labor taxes are the policy instruments and these face greater variations in relative terms. This empirical literature has also found that at the end of a tenure output and private consumption are not higher than average, a feature also predicted in our model. It is also found in the data that at the end of the tenure there are not significant tax cuts leading to less tax revenues, which also conforms with our model. There is some empirical evidence that public expenditure is increased before the elections, being the evidence stronger for government transfers. This feature is also replicated in our model, public consumption is higher as the tenure evolves. It is not our aim to match political cycles, because electoral competition is absent from the model. But, our simple model does not contradict the empirical evidence found in that literature.

**Figure 1.3:** Particular history: default every 4 periods




Note: The figure plots for several values of $\pi$ a particular history realization. In this history a default occurs every four periods. Capital is initialized at the discretion steady state. The lagrange multiplier is initialized at zero, considering that there were no previous promises in the first period.

### 1.3.6 Welfare calculations

In this section, we turn to measure the welfare implications of building commitment. In our framework, this means considering the welfare gains of increasing $\pi$. In order to have an appropriate welfare measure, consider two regimes, a benchmark regime (B) and an alternative regime (A). The life-time utility $W$ of the representative agent in both regimes is given by:

$$W^i(k_{-1}, 0) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, l_t^i, g_t^i) \qquad i = A, B \tag{1.34}$$

where $\{c_t^i, l_t^i, g_t^i\}_{t=0}^{\infty}$ is the optimal allocation sequence in regime $i$, and where expectations are taken with respect to the shock driving commitment and default. We define $\varpi$ as the increase in private consumption in the benchmark regime that makes households indifferent between the benchmark and an alternative regime. More formally $\varpi$ is implicitly defined as:

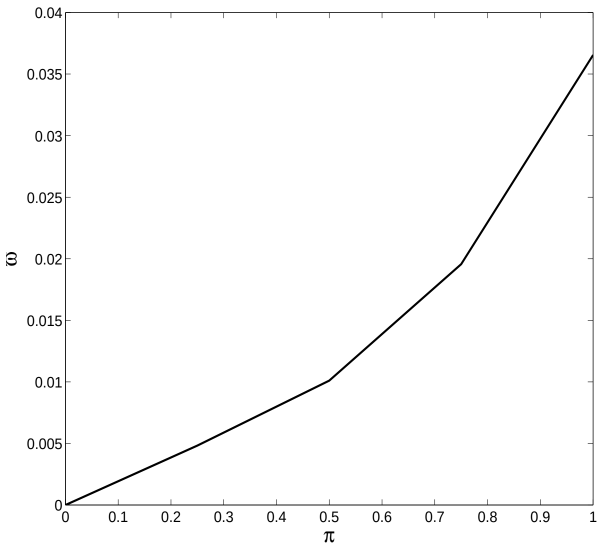
$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^A, l_t^A, g_t^A) = E_0 \sum_{t=0}^{\infty} \beta^t u((1+\varpi) c_t^B, l_t^B, g_t^B) \tag{1.35}$$

For the calculations that follow we considered the benchmark regime to be the full discretion case and we initialized capital at the steady state prevailing when $\pi = 0.5$.[16] The welfare improvement from discretion to commitment is equivalent to an increase in private consumption of 3.65%. Figure 1.4 shows the welfare gains for different probabilities of commitment. When the probability of default increases from 0 to 0.25, only 13% of the benefits of commitment are achieved. We plot the relative welfare gains as a function of $\pi$ in figure 1.4. The function is convex suggesting that increasing $\pi$ from low to intermediate levels results in relative small welfare gains. Most of the gains from enhancing commitment can only be achieved when $\pi$ is already high. In figure 1.5, we plot the relative welfare gains as a function of the

[16] We also considered initializing capital at other steady states or expressing $\varpi$ in consumption units of the alternative regime. This means computing $\varpi$ as $E_0 \sum_{t=0}^{\infty} \beta^t u((1-\varpi) c_t^A, l_t^A, g_t^A) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^B, l_t^B, g_t^B)$. The results remain unchanged.

expected time before a default occurs (i.e. $1/(1-\pi)$). In this metric the welfare gains function is concave. The welfare gains per unit of time of moving from 1 to 2 years are higher than the gains of moving from 2 to 3 years. This result may seem more intuitive, since as the expected commitment period increases there are less welfare gains to be achieved.

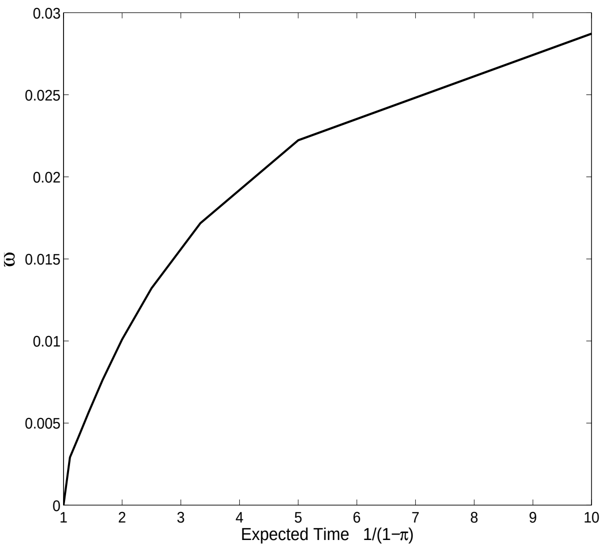
**Figure 1.4:** Welfare gains on $\pi$ axis




In a related work on optimal monetary policy, Schaumburg and Tambalotti (2007) found qualitatively different results. First, allocations move linearly in the probability $\pi$. For instance, when $\pi$ moves from 1 to 0.75 inflation goes 25% of the distance towards discretion. Secondly, most of the welfare gains are achieved at low levels of commitment. In other words, the welfare is always concave regardless of the metric used. When $\pi$ is 0.75, about 90% of the welfare gains from commitment are obtained. Comparing absolute welfare measures in our model and theirs is unclear, but the welfare gains of moving from discretion to commitment are also much higher in their model.[17]

[17]In Barro-Gordon models the welfare loss penalizes quadratically deviations of inflation from

**Figure 1.5:** Welfare gains on expected time axis




These considerations suggest that there may be different effects of building commitment in monetary or in fiscal policy. We have investigated potential sources of differences between our results and those in Schaumburg and Tambalotti (2007). First, as figure 1.5 suggests, some differences could be due to the choice of the model time period. The original calibration used in Schaumburg and Tambalotti (2007) is quarterly. We also tried an annual calibration of their model, and the results do not change qualitatively. Another potential difference is related to the role played by endogenous state variables. An endogenous state variable, provides the planner with an instrument to influence his sucessors. Therefore, it may be expected that in a model with endogenous state variables, the benefit of adding commitment is smaller. While our model has one endogenous state variable their model has none. To test this hypothesis, we also considered their model with an hybrid Phillips curve as in Galí

zero and deviations of the output gap from a target level. The inflation and output gap under commitment are nearly zero. Under discretion the inflation is quite high and the output gap is still zero. Since standard calibrations give a much higher weight to inflation deviations in the loss function, the gains from commitment are substantial.

and Gertler (1999). The results do move slightly in the direction that we predict, but a significant difference between their results and ours remains. We can therefore conclude that building commitment in fiscal policy may have different effects than in monetary policy.

### 1.3.7 Discussion on monetary and fiscal policy commitment

In the early 70's, economists and policymakers were still not aware of the importance of commitment and its implications for policy design. The commitment in both fiscal and monetary policy were low at that time. After the seminal contributions in the late 70's and early 80's regarding time-inconsistency, there has been a concern to increase institutional commitment. The reforms to increase monetary policy commitment were far more intense than the ones relative to fiscal policy. In fact, we observed a pattern of inflation that is consistent with low commitment in the 70's and high commitment today. In the 70's, the inflation level was much closer to the steady state level of discretion than the one of commitment. Nowadays, the opposite is true.

The patterns in fiscal policy are somehow different. We have not observed institutional reforms aimed at increasing commitment comparable to the ones in monetary policy. Also, as discussed in Klein and Rios-Rull (2003), the level of taxes both in the 70's and today are much closer to the full discretion prediction than to the commitment one. This evidence suggests that fiscal policy commitment was low in the 70's and is still low today, while monetary policy commitment was low in the 70's and is high today.

Combining our results with those of Schaumburg and Tambalotti (2007), may shed light on this issue. It seems that when commitment is low, the welfare gains of more commitment are higher in monetary policy than in fiscal policy. Also, it may be very hard to implement the full commitment solution, since defaults are always possible and flexibility is necessary. While most welfare improvements can be achieved at intermediate levels of commitment in monetary policy, the same is not true for fiscal policy.

It may also be argued that it may be easier to achieve high levels of commitment in monetary policy than in fiscal policy. The turnover in government's presidents is higher than in central bank's governors. More importantly, it may be difficult to establish a fiscal authority with full commitment, because such an institution would have to rule out democratic choices as they violate the commitment plan. Our results and the intuition that higher commitment levels are easier to attain in monetary policy than in fiscal policy, may also help explaining the lower effort devoted to fiscal commitment than monetary commitment.

## 1.4 Endogenous probability model

In this section, we consider an extension where the probability of defaulting depends on the states of the economy. Since capital is the only natural state variable in the economy, we will consider that the probability of defaulting today depends on the current capital stock. The planner and households will consider that the probability of commitment in the next period is given by a function $P(k_{t+1})$, instead of $\pi$. Following the steps described in the previous sections, the objective function of the planner becomes:

$$\sum_{t=0}^{\infty} \beta^t \frac{\Pi_{j=0}^{t}(P(k_j))}{P(k_0)}(u(x_t, k_t) + \beta(1 - P(k_{t+1}))W(k_{t+1})) \tag{1.36}$$

As before, the probability of being in charge in the first period is 1.[18] It seems more reasonable to assume that the commitment probability depends on aggregate capital, while one household can only decide on his own single capital. This means that each household is atomistic and takes the aggregate capital stock as given. Therefore, as it seems reasonable, the individual household capital accumulation decision does not incorporate the effect in the commitment technology.[19] Hence, the constraints that

[18]The special term $P(k_0)$ in the objective function does not induce any time-inconsistency problem because $k_0$ is predetermined. If the probability function depended on a non-predetermined variable, then the FOCs would be different when a planner starts and when it is already in power. This would introduce another source of time inconsistency.

[19]More formally, one could model a continuum of agents on a real interval between 0 and 1. All agents would be equal, and therefore their decisions would be equivalent to a representative agent, who takes aggregate capital as given.

the planner faces are:

$$b_1(x_t(\omega_{ND}^t), k_t(\omega_{ND}^t)) + \beta(1 - P(k_{t+1}))b_2(\Psi(k_{t+1}(\{\omega_{ND}^t, D\})), k_{t+1}(\{\omega_{ND}^t, D\})) \\ + \beta P(k_{t+1})b_2(x_{t+1}(\omega_{ND}^{t+1}), k_{t+1}(\omega_{ND}^{t+1})) = 0 \tag{1.37}$$

We need to prove that this setting can also be written as a SPFE. This result is done in Proposition 3, and details are available in the appendix.

**Proposition 3** *The problem of a planner maximizing Eq. (1.36) subject to Eq. (1.37) can be written as saddle point functional equation.*

**Proof.** See the appendix. ■

With proposition 3 at hand, it then follows that the solution to the problem is a time-invariant function. In the appendix, we also describe the FOCs and some simplifications that are very useful for computational work. In comparison with the exogenous probability case, the FOCs with respect to all variables except to capital remain unchanged. In the FOC with respect to capital, some new terms appear reflecting that the commitment probability can be influenced. Unlike households, the planner does not take aggregate capital as given. One extra term refers to the appearance of $P(k_{t+1})$ in the constraints of households. The other term refers to the expected change in utility, induced by the change in the commitment probability. This is captured by the term $P_{k_{t+1}}\beta(W(k_{t+1}, \lambda_t) - W(k_{t+1}, 0))$. If capital is increased, the commitment probability is increased by $P_{k_{t+1}}$. This increases the chances of the current planner to obtain tomorrow's continuation value under commitment $W(k_{t+1}, \lambda_t)$. Nevertheless, it decreases the chances of obtaining tomorrow's continuation value under discretion $W(k_{t+1}, 0)$.

This model raises an extra difficulty in terms of computational work. As explained above, not only the derivative of the value function appears in the FOCs, but also its level. This means that an envelope result would not suffice to eliminate the value function from the problem, and that such value function needs to be approximated.

As a consequence, while the exogenous probability model only required one fixed point, the endogenous probability model requires two fixed points.

In what follows, we will consider a probability function such that when capital is higher there is a higher probability of commitment. This assumption could be justified on political economy grounds. More capital implies more output and a higher probability of reelection. We will consider the following probability function:

$$P(k_t) = 1 - \frac{1}{(\frac{k_t}{\tilde{k}})^\rho + 1} \tag{1.38}$$

The parameter $\tilde{k}$ is a normalization such that $P(\tilde{k}) = 0.5$. The higher is $\rho$, the easier it is for the planner to influence its reelection probability. In the case of $\rho = 0$, the probability is always constant. We can use a homotopy from the model in section 1.3 to this model by changing $\rho$ from 0 to the desired value. We chose $\rho = 30$ and $\tilde{k}$ to be equal to the average capital allocation when $\pi = 0.5$. Our normalization of $\tilde{k}$ allows us to directly compare the results with the probabilistic model when $\pi = 0.5$.

Results are presented in table 1.4. In the endogenous probability model, capital is now higher. In this context, more capital implies a higher probability of commitment and more commitment is welfare improving. As a consequence, the planner lowers capital income taxes in order to foster households' capital accumulation.[20] Finally, since more commitment is achieved, average allocations move towards the full-commitment equilibrium.

In the endogenous probability model, the welfare gain relative to discretion is 2.6%. This value is higher than the welfare gains obtained in the benchmark case of $\pi = 0.5$. One reason is that the commitment probability is higher. The other reason is that the welfare gains function is convex in $\pi$. A varying probability around a mean may therefore induce some additional welfare gains. In a political economy interpretation, our model would suggest that governments accumulate more capital to increase the probability of being reelected. This policy improves social welfare, since it reduces political turnover thus increasing the commitment probability.

[20] As explained above, households themselves do not internalize the positive effects of capital on the commitment probability.

**Table 1.4:** Endogenous probability - average values

| | $\pi = 0.5$ | **End. Prob.** |
|---|---|---|
| $\mathbf{k}$ | 0.899 | 0.932 |
| $\lambda$ | -0.080 | -0.064 |
| $\mathbf{g}$ | 0.072 | 0.082 |
| $\mathbf{c}$ | 0.220 | 0.209 |
| $\mathbf{y}$ | 0.364 | 0.365 |
| $\tau^k$ | 0.163 | 0.153 |
| $\tau^l$ | 0.218 | 0.268 |
| $\mathbf{l}$ | 0.248 | 0.246 |
| $\mathbf{u}$ | 0.801 | 0.790 |
| $\bar{\pi}$ | 0.500 | 0.738 |

## 1.5 Conclusions

The commitment and discretion approaches are to some extent unrealistic. This paper tried to characterize optimal policy in a setting where some promises are fulfilled while others are not. One interpretation of such setting is based on political turnover, where governments make promises but may be out of power when it is time to fulfill them. Alternatively, governments may make promises but when certain events arise the commitment technology breaks and such promises are reneged. This framework can also be thought as providing an optimal policy prescription knowing that at a later date policy is going to be revised.

From the methodological point of view, our contribution is to show a solution technique for problems of *loose commitment* with the following main features. First, it can be applied to a wide class of non-linear models, with or without endogenous state-variables. While there were other works on similar *loose commitment* settings, such methods could not be used in the standard non-linear macro models, like the fiscal policy problem described here. Second, building on the results of Marcet and Marimon (1998), we proved that the solution to our problem is recursive. Third, we implemented an algorithm which is relatively inexpensive, and makes use of global approximation techniques which are pointed out in the literature as more reliable. Finally, as a by-product, our procedure can be used as a homotopy method to find the time-consistent solution.

We show that in the optimal taxation model under *loose commitment*, average allocations seem to be closer to the time-consistent solution. We also characterize the economic consequences of reneging on promises. Our results suggest that when promises are not fulfilled capital taxes will be raised and labor taxes will be lowered. As a consequence, capital utilization rate drops and labor input increases. Several features of the model are in accordance with some empirical results on political cycles.

We then consider that the probability of committing is a function of the endogenous state variables. In such model, the government creates incentives for the agents to accumulate such state variables. Intuitively, the government tries to increase commitment because more commitment is welfare improving.

Regarding welfare, we find that for an upper bound of the probability of commitment around 0.75, most of the gains from commitment are not achieved. While the welfare gains are a concave function of the expected time before a default, they are a convex function of the probability of commitment. These results are different from those obtained in the literature regarding monetary policy. This may explain the observation that more interest has been devoted to building commitment in monetary rather than fiscal policy.

The methods provided in this paper are general and can be applied to a variety of macroeconomic problems. The setups formulated here can be easily brought into the dynamic political economy literature. This literature has commonly abstracted from the presence of time-inconsistency or assumed a discretion approach. Our paper allows to consider the more realistic setting that governments can fulfill promises when they are reelected. This paper also sets up the base for addressing problems where different governments do not face the same institutional settings or disagree on policy objectives. Finally, the applications of our methodology are not restricted to optimal policy problems. Indeed, it can be used in many dynamic problems where commitment plays an important role, like the relationship between firms and its customers and shareholders, or in other principle-agent type of problems.

# Appendix

## Proofs

**Proof.** of Proposition 1

Drop history dependence and define:

$$r(x_t, k_t) \equiv u(x_t, k_t) + \beta(1-\pi)\xi(k_{t+1}))$$
$$g_1(x_t, k_t) \equiv b_1(x_t, k_t) + \beta(1-\pi)b_2(\Psi(k_{t+1}), k_{t+1})$$
$$g_2(x_{t+1}, k_{t+1}) \equiv b_2(x_{t+1}, k_{t+1})$$

Our problem is thus:

$$\max_{\substack{\{x_t(\omega^t)\}_{t=0}^{\infty} \\ \omega^t=\omega^t_{ND}}} \sum_{t=0}^{\infty} (\beta\pi)^t \{r(x_t, k_t)\} \tag{A-1}$$
$$s.t: \; g_1(x_t, k_t) + \beta\pi g_2(x_{t+1}, k_{t+1}) = 0$$

which fits the definition of Program 1 in Marcet and Marimon (1998). To see this more clearly note that our discount factor is $\beta\pi$ and we have no uncertainty. Since $\omega^t_{ND}$ is a singleton, we have previously transformed our stochastic problem into a non-stochastic problem. Therefore, we can write the problem as a saddle point functional equation in the sense that there exists a unique function satisfying

$$W(k, \gamma) = \min_{\lambda \geq 0} \max_{x} \{h(x, k, \gamma, \lambda) + \beta\pi W(k', \gamma')\} \tag{A-2}$$
$$s.t: \; \gamma' = \lambda, \; \gamma_0 = 0$$

where

$$h(x, k, \lambda, \gamma) = r(x, k) + \lambda g_1(x, k) + \gamma g_2(x, k) \tag{A-3}$$

or in a more intuitive formulation define:

$$H(x, k, \lambda, \gamma) = u(x, k) + \lambda g_1(x, k) + \gamma g_2(x, k) \tag{A-4}$$

and the saddle point functional equation is:

$$W(k, \gamma) = \min_{\lambda \geq 0} \max_{x} \{H(x, k, \lambda, \gamma) + \beta(1-\pi)\xi(k') + \beta\pi W(k', \gamma')\} \tag{A-5}$$
$$s.t: \gamma' = \lambda, \; \gamma_0 = 0$$

■

**Proof.** of Proposition 2: Using Proposition 1, this proof follows trivially from the results of Marcet and Marimon (1998). ■

**Proof.** of Proposition 3

Define an additional variable $\eta$, The law of motion for $\eta$ is:

$$\eta_{t+1} = \eta_t P(k_{t+1}) \tag{A-6}$$

with $\eta_0 = 1$. The problem of the planner can then be rewritten as:

$$\begin{aligned} \min_{\{\lambda_t,\varphi_t\}_{t=0}^{\infty}} \max_{\{x_t\}_{t=0}^{\infty}} \mathscr{L} = \sum_{t=0}^{\infty} \beta^t \eta_t (u\left(x_t, k_t\right) + \beta\left(1 - P(k_{t+1})\right)\xi\left(k_{t+1}\right)) \\ + \lambda_t\left(g_1\left(x_t, k_t\right) + \beta P(k_{t+1}) g_2\left(x_{t+1}, k_{t+1}\right)\right) \\ + \varphi_t(\eta_{t+1} - \eta_t P(k_{t+1})) \end{aligned} \tag{A-7}$$

Using similar redefinitions as in the proof of proposition 1 the result follows. The condition $\eta_0 = 1$ signals that a new planner is in charge. Equation (A-6) is still valid because it only refers to the evolution of $\eta$ through commitment states. Finally, note that in terms of notation $\Psi(k) \equiv \Psi(k, \lambda = 0, \eta = 1)$.[21] ■

## First-order conditions

### Probabilistic Model

To solve the problem set up the Lagrangian:

$$\begin{aligned} \min_{\{\lambda_t\}_{t=0}^{\infty}} \max_{\{x_t\}_{t=0}^{\infty}} \mathscr{L} = \sum_{t=0}^{\infty} (\beta\pi)^t \left(u\left(x_t, k_t\right) + \beta\left(1 - \pi\right) W\left(k_{t+1}\right)\right) \\ + \lambda_t\left(g_1\left(x_t, k_t\right) + \beta\pi g_2\left(x_{t+1}, k_{t+1}\right)\right) \end{aligned}$$

[21] For the purpose of this proof one has to include $\eta_t$ as a state variable. This is only convenient for this proof and as discussed later is not necessary for the numerical work.

The FOCs are[22]:

$$\frac{\partial \mathscr{L}}{\partial z_t} : u_{z_t,t} + \lambda_t g_{1,z_t,t} + \lambda_{t-1} g_{2,z_t,t} = 0 \tag{A-8}$$

$$\frac{\partial \mathscr{L}}{\partial k_{t+1}} : u_{k_{t+1},t} + \beta\left(1-\pi\right) W_{k_{t+1},t+1} + \lambda_t (g_{1,k_{t+1},t} + \beta\pi g_{2,k_{t+1},t+1}) \tag{A-9}$$

$$+ \beta\pi\left(u_{k_{t+1},t+1} + \lambda_{t+1} g_{1,k_{t+1},t+1}\right) - \lambda_{t-1} g_{2,k_{t+1},t} = 0$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_t} : g_1\left(x_t, k_t\right) + \beta\pi g_2\left(x_{t+1}, k_{t+1}\right) = 0 \tag{A-10}$$

$$\forall t = 0, ..., \infty \qquad \lambda_{-1} = 0$$

where $z_t \equiv (l_t, \tau_t^k, \tau_t^l)$, and using equations (1.27) and (1.28) it follows that $g_{1,x_t,t} = b_{1,x_t,t}$, $g_{2,x_t,t} = b_{2,x,t}$, $g_{1,k_t,t} = b_{1,k_t,t}$, $g_{2,k_t,t} = b_{2,k_t,t}$, $g_{2,k_{t+1},t} = b_{2,k_{t+1},t}$ and

$$g_{1,k_{t+1},t} = b_{1,k_{t+1},t} + \beta\left(1-\pi\right)\left[\left(b_{2,x_{t+1},t+1}\Psi_{k_{t+1}} + b_{2,k_{t+1},t+1}\right)\right]$$

**Endogenous Probability Model**

To simplify the problem, it is useful to multiply the second constraint by $\eta_t$, which does not change the solution. Set up the Lagrangian:

$$\min_{\{\lambda_t,\varphi_t\}_{t=0}^{\infty}} \max_{\{x_t\}_{t=0}^{\infty}} \mathscr{L} = \sum_{t=0}^{\infty} \beta^t \{\eta_t(u\left(x_t, k_t\right) + \beta\left(1 - P(k_{t+1})\right) W\left(k_{t+1}\right)) \tag{A-11}$$

$$+ \lambda_t \eta_t \left(g_1\left(x_t, k_t\right) + \beta P(k_{t+1}) g_2\left(x_{t+1}, k_{t+1}\right)\right)$$

$$+ \varphi_t(\eta_{t+1} - \eta_t P(k_{t+1}))\}$$

The FOCs are:

[22]The symbol $f_{x_t,i}$ denotes the partial derivative of the function $f(m_i)$ with respect to $x_t$. We suppressed the arguments of the functions for readability purposes.

$$\frac{\partial \mathscr{L}}{\partial z_t} : u_{z_t,t} + \lambda_t g_{1,z_t,t} + \lambda_{t-1} g_{2,z_t,t} = 0 \tag{A-12}$$

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial k_{t+1}} &: u_{k_{t+1},t} + \beta \left(1 - P(k_{t+1})\right) W_{k_{t+1},t+1} + \lambda_t (g_{1,k_{t+1},t} + \beta P(k_{t+1}) g_{2,k_{t+1},t+1}) \\ &+ \beta P(k_{t+1}) \left(u_{k_{t+1},t+1} + \lambda_{t+1} g_{1,k_{t+1},t+1}\right) - \lambda_{t-1} g_{2,k_{t+1},t} \\ &- \beta P_{k_{t+1}}(k_{t+1}) W(k_{t+1}) - \varphi_t P_{k_{t+1}}(k_{t+1}) \\ &+ \lambda_t P_{k_{t+1}}(k_{t+1})(b_2(x_{t+1}, k_{t+1}) - b_2(\Psi(k_{t+1}), k_{t+1})) = 0\end{aligned} \tag{A-13}$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_t} : g_1\left(x_t, k_t\right) + \beta P(k_{t+1}) g_2\left(x_{t+1}, k_{t+1}\right) = 0 \tag{A-14}$$

$$\frac{\partial \mathscr{L}}{\partial \varphi_t} : \eta_{t+1} - \eta_t P(k_{t+1}) = 0 \tag{A-15}$$

$$\frac{\partial \mathscr{L}}{\partial \eta_{t+1}} : \beta \left(u_{t+1} + \beta \left(1 - P(k_{t+2})\right) W\left(k_{t+2}\right)\right) + \varphi_t - \beta \varphi_{t+1} P(k_{t+2}) = 0 \tag{A-16}$$

$$\forall t = 0, ..., \infty \qquad \lambda_{-1} = 0$$

To obtain equations (A-12) and (A-13) one has to divide the original FOC by $\beta^t \eta_t$, and equation use (A-15).[23] One can solve equation (A-16) forward and obtain:

$$-\varphi_t = \beta \sum_{i=t+1}^{\infty} \beta^{i-1} \frac{\Pi_{j=0}^{i}(P(k_j))}{P(k_{t+1})} (u(x_i, k_i) + \beta(1 - P(k_{i+1}))W(k_{i+1})) \tag{A-17}$$

This equation states that $-\varphi_t$ is equal to $\beta$ times the value function starting at period t+1. Note that it represents the value function when past promises are kept, because all the terms considered refer to commitment states. Simplifying notation:

$$-\varphi_t = \beta W(k_{t+1}, \lambda_t) \tag{A-18}$$

One can use the equation above to eliminate $\varphi_t$ in equation (A-13).

For the numerical work all the simplifications above are convenient to reduce the number of equations in the system. Since $\eta_t$ is eliminated from the problem, this variable is not a state variable necessary for the numerical approximation. Intuitively,

[23]Note that in performing such operation the term $\eta_{t-1}$ multiplies $\lambda_{t-1}$. If there is a default that term disappears. If there is no default it is still the case that $\eta_t = \eta_{t-1} P(k_t)$. Therefore, the original FOC and the changed FOC in (A-12, A-13) are equivalent.

at each point in time the planner that is in charge only needs to know the current promises summarized by $\lambda_{t-1}$, and the capital stock $k_t$. The probability of committing between $t-1$ and $t$ is a bygone. The probability of committing between $t$ and $t+1$, $P(k_{t+1})$, is not predetermined.

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# Chapter 2

# Political Disagreement, Lack of Commitment and the Level of Debt

In this chapter, we analyze how public debt evolves when successive policymakers have different policy goals and cannot make credible commitments about their future policies. We consider several cases to be able to disentangle and quantify the respective effects of imperfect commitment and political disagreement. Absent political turnover, imperfect commitment drives the long-run level of debt to zero. With political disagreement, debt is a sizeable fraction of GDP and increasing in the degree of polarization among parties, no matter the degree of commitment. The frequency of political turnover does not produce quantitatively relevant effects. These results are consistent with much of the existing empirical evidence. Finally, we find that in the presence of political disagreement the welfare gains of building commitment are lower.

## 2.1 Introduction

In the fiscal policy literature, there is not a clear theoretical understanding of the forces driving the observed patterns of public debt. This paper explores how debt evolves when governments cannot make credible commitments about future policies and when policymakers with different policy goals alternate in office. We consider several cases to be able to disentangle and quantify the respective effects of imperfect commitment and political disagreement.

As it is well known, the evolution of debt matters in a world where the provision of public goods has to be financed by raising distortionary taxes.[1] In this context, as shown e.g. in the works of Barro (1979), Lucas and Stokey (1983) and Aiyagari et al. (2002), debt is used to smooth over time the deadweight losses associated with such distortions. These models do not provide a complete explanation of some basic and stylized facts, like why public debt is a sizeable fraction of GDP in many developed countries and why there is a substantial variation in the debt/GDP ratio both over time and across countries with similar economic conditions.[2]

In macroeconomic models, the optimal (second-best) allocations are usually characterized as the solution to a Ramsey problem. It is assumed that the same planner is always in charge and that he can commit to future policies, maximizing the welfare of a representative agent.[3] Under these assumptions and with complete financial markets, as shown by Lucas and Stokey (1983), the long-run level of debt crucially depends on the initial conditions.[4] Countries starting with high debt will have high debt forever, and countries with low debt will have low debt forever. Since initial conditions are exogenous to the model and empirically difficult to determine, such a theory can not explain what induces countries to accumulate debt.

Policymaking in practice departs from the idealized environment described in Lucas and Stokey (1983) in many dimensions. In this work, we investigate how imperfect commitment and disagreement among successive policymakers can provide an incentive to accumulate debt, and analyze their interactions. There are important reasons to think that these two forces may considerably affect the behavior of debt.

[1] When lump-sum taxes are available, the debt policy is irrelevant, since the so-called Ricardian equivalence holds, see e.g. Barro (1979).

[2] In the appendix we report the values of the debt/GDP ratio for many OECD countries.

[3] For a study about policy choices made by self-interested policymakers see e.g. Persson and Tabellini (2000).

[4] Lucas and Stokey (1983), as we do here, analyzed an economy with complete financial markets. Removing this assumption, as shown by Aiyagari et al. (2002), it is optimal to accumulate assets.

First, the role of commitment is related to the time-inconsistency problem in optimal policy choices, as illustrated in the seminal works of Kydland and Prescott (1977) and Barro and Gordon (1983). In our context, the solution under full-commitment is time inconsistent because a planner, at a given point in time, is willing to abandon his previous plans to manipulate the interest rate. For example, if the planner needs to issue debt, he has an incentive to reduce the interest rate. Hence, the planner is willing to lower current taxes, in order to foster current consumption. Indeed, because of a smoothing motive, this leads to an increase in the demand for savings and thus to a reduction in the interest rate. As a consequence, because of the lower tax revenues, in a one-time deviation from the full-commitment solution, the planner runs deficits and accumulates debt.[5] Therefore, it seems worth exploring how debt evolves when the planner cannot make credible commitments about his future policies.[6] We thus check whether a positive long-run level of debt may be the outcome of the optimal policy under the no-commitment assumption and other imperfect commitment settings.

Second, some studies in the political economy literature (see e.g. Alesina and Tabellini (1990) and Persson and Svensson (1989)), have emphasized how the presence of political disagreement provides incentives to accumulate an inefficiently high level of debt. Indeed in such cases, debt may be used strategically to influence the choices of the successors. In comparison with the no-disagreement case, more resources are used for current purposes and less resources, i.e higher debt, are left to the successors to pursue their objectives. In a world characterized by political disagreement, the assumption of full-commitment seems unrealistic. However, it would still be reasonable to assume that governments may have commitment during their tenures, but cannot commit on behalf of their successors, who have different objectives. In this paper, we consider a framework with political disagreement among successive policymakers, where commitment plays an important role in the strategic

[5]This happens unless the initial level of debt is sufficiently high. In the latter case, the improvement in the interest rate, since applied to a larger base, can be sufficient to finance the initial tax cut.

[6]As in Lucas and Stokey (1983), we assume that there is still commitment to honor the debt payments. On the contrary, the absence of commitment is referred to future policy actions. For a further discussion on this issue see Niepelt (2006).

game between policymakers and private agents.[7] In this context, the incumbent policymaker makes different choices depending on his ability to commit while staying in office. This allows us to explicitly analyze the effects of commitment in a world with political disagreement among successive policymakers.

### 2.1.1 Main findings

We build on the simple Lucas and Stokey (1983) model, introducing endogenous government expenditure, which has to be financed by raising a proportional income tax and/or by issuing debt. We analyze optimal policy choices under different assumptions about commitment and disagreement among successive policymakers.

We develop a framework that allows us to disentangle and quantify the effects of imperfect commitment, frequency of turnover and political disagreement in a dynamic context. In this respect, our contribution is methodological. Our framework can indeed be used to analyze the effects of commitment in a wide set of infinite-horizon optimal policy problems, where policymakers with different objectives alternate in office. In other words, the methodology developed here allows us to integrate the analysis about the time-inconsistency of optimal policy choices, typical of the dynamic macroeconomic literature, into a political economy model. By doing so, we are able to measure the implications of building commitment in the presence of political disagreement.

From an economic point of view, the main contribution and findings of our analysis are the following. First, abstracting from political disagreement, we study the optimal fiscal policy under the no-commitment assumption. Under a wide set of initial conditions and parameterizations, we find that debt goes to zero in the long-run. Perhaps surprisingly, this means that there is a striking difference in the behavior of debt in a one-time deviation from commitment and in the no-commitment (time-consistent) solution. As we will discuss later, reducing debt over time is the only way

[7] The dynamic political economy literature has been limited to frameworks where private agents' current choices do not depend on future policy, see e.g. Azzimonti-Renzo (2004). Also note that Alesina and Tabellini (1990) by setting the initial level of debt to zero, restrict their analysis to a case where the time inconsistency problem does not play any role.

the planner with no-commitment can favorably affect the interest rate.

Second, we study the behavior of debt in cases where the planner has access to a commitment technology, but under some circumstances, say because of political pressures, big shocks etc., he may renege on his past promises. This is what we call the *loose commitment* setting. Because of the striking difference in the behavior of debt between the full-commitment and the no-commitment cases, it seems worth checking how debt evolves under *loose commitment*. We find that even in the latter case the level of debt converges to zero in the long-run. This suggests that the steady state dependency on initial conditions found in Lucas and Stokey (1983) is not robust to small deviations from the full-commitment case. In addition, our results suggest that departing from the full-commitment assumption cannot help explaining why the level of debt is a sizeable fraction of GDP.

Third, we also find that debt is generally increased in periods when the planner reneges on his past promises and reduced over the periods of commitment. This result is interesting since it suggests that the simple expectation that the planner may surprise the economy at a future date induces him to commit to reduce debt over time.

Fourth, we investigate one case where the imperfect commitment assumption is natural, i.e. when successive planners have different policy goals. We find that in the presence of political disagreement, debt is a sizeable fraction of GDP, regardless of the assumptions made about commitment. In our numerical exercises, political disagreement seems to be the main driving force for accumulating deficits. On the contrary, the effects of imperfect commitment and political turnover have a small impact on the level of debt. Our predictions are consistent with most of the existing empirical evidence. Indeed, while there is a large consensus on the positive relationship between the degree of political polarization and debt accumulation, the empirical findings about the effects of the frequency of political turnover are less clear-cut. More importantly, our results suggest that when testing empirically the effects of political instability on the level of debt, it is important to control both for measures of polarization among parties and measures of political turnover, rather than using any of them as a generic indicator of political instability.

Finally, when analyzing welfare implications, we find that the gains from commitment are lower in the presence of political disagreement than in a no-disagreement case. From an intuitive point of view, this happens because in the absence of political disagreement governments with more commitment will maximize overall social welfare. On the contrary, with political disagreement a better commitment technology can be used by each party to maximize specific groups' welfare.

### 2.1.2 Related literature

Krusell et al. (2006) analyze the time-consistent solution of the otherwise standard Lucas and Stokey (1983) model, where government expenditures are exogenous. The authors find as a solution a multiplicity of steady states and discontinuous policy functions, where debt adjusts for one or two periods and then remains constant. Their main finding is that under no-commitment the equilibrium is close to the solution under commitment. In our paper, we also build on the Lucas and Stokey (1983) model, but consider the case where government expenditure is endogenous. The presence of this additional instrument in the hands of the policymaker widens the set of his feasible choices. In section 2.3, we extensively discuss how this make a difference. We obtain continuous policy functions and we find that in the absence of commitment debt goes to zero. This result is surprising because it is usually the case that in a one-time deviation from commitment debt increases.

In the literature, several papers have analyzed the effects of lack of commitment on debt in monetary economies. When nominal debt is present, the monetary authority usually has an incentive to raise the price level to reduce the real value of the outstanding debt. The first period of the full-commitment solution reveals such incentives, since debt is eroded in real terms. Martin (2006) and Diaz-Gimenez et al. (2006) confirm that in a monetary economy under discretion the features of a one-time deviation from commitment are obtained in every period.[8] They find that the steady-state level of debt can be positive, negative or zero depending on the parametrization

[8]If debt is real, their models do not have a time-inconsistency problem and real debt remains constant. The authors show that this result is also obtained for a monetary economy with inflation and consumption taxes.

of the utility function. If it is easy (difficult) for households to substitute cash goods then government holds assets (debt).[9] As in Krusell et al. (2006) we focus on a real economy. Since in most countries central banks are independent and committed to price stability, we believe that focusing on a real economy is a reasonable assumption. Our result that debt converges to zero is not due to the presence of nominal bonds nor it is achieved with surprise inflation.

Some studies in the political economy literature have analyzed how policy decisions are formulated when policymakers with different political views alternate in office. The work of Alesina and Tabellini (1990) shows how political uncertainty and turnover lead to an inefficiently high steady state level of debt and public expenditure and how these political factors can account for the differences in debt management across countries and over time. Azzimonti-Renzo (2004), as we do here, extends the previous works to an infinite horizon problem, but in a context where commitment about future policy does not affect private agents' choices. The author considers a fiscal policy model with balanced budget, and public but no private capital. Instead, we focus on the effects of political disagreement on the level of government debt. Our main contribution with respect to this literature is to study optimal policy where commitment plays a role in the strategic interactions between agents and the policymakers. Moreover, we solve the problem under different commitment settings. We indeed consider the case where parties cannot commit at all, but we also assume that parties can credibly commit for the future, in case they are reappointed in office. This allows to disentangle and quantify the effects of imperfect commitment, political disagreement and frequency of political turnover on the level of debt. Finally, it allows to measure the welfare gains from commitment in the presence of political disagreement.

In recent work, Song et al. (2006) and Battaglini and Coate (2008) study the evolution of debt in a dynamic political economy framework, and provide an explanation for the presence of a long-run positive level of debt. They consider models with political conflicts over public goods redistribution, either across generations or across

[9] Ellison and Rankin (2007) examine the case of indexed debt building on Nicolini (1998)

geographical districts. In these works, however, the interest rate is exogenous and the commitment problem arises because of repeated voting. In our work we instead study an infinite horizon problem, where the disagreement is about the composition of a public good. More importantly, we consider a case where policy choices are time-inconsistent because of the policymaker's incentive to manipulate the interest rate, which is present even in the absence of repeated voting or political turnover. In addition, we consider the strategic interactions between policymakers with different objectives alternating in office. This setting allows to integrate the analysis about the time-inconsistency of optimal policy choices, typical of the dynamic macroeconomic literature, into a political economy model.

The paper is organized as follows: in section 2.2 we introduce the model and, as benchmark for our analysis, we recover the solution under full-commitment. In section 2.3, we describe the solution under no-commitment, i.e. the time-consistent solution. In section 2.4, we illustrate the behavior of debt under the less extreme assumption of *loose commitment*. In section 2.5, we study the joint implications of political disagreement and imperfect commitment and we compare our findings with the existing empirical literature. Finally we discuss welfare implications. Section 2.6 concludes.

## 2.2 The model

We build our analysis on a simple model, as in Lucas and Stokey (1983), where time-inconsistency issues arise, and analyze the solution under the assumptions of full-commitment, full discretion and *loose commitment*. For the time being, we abstract from uncertainty and political disagreement between successive governments.[10] The latter feature is introduced in section 2.5.

We consider an economy where labor is the only factor of production, and technology is linear,

$$c_t + g_t = 1 - x_t \tag{2.1}$$

[10]The absence of uncertainty is for notational convenience. In the presence of exogenous shocks, many of our considerations are still valid under the assumption of complete financial markets.

for $t = 0, 1, 2, ...$, where $c_t$, $g_t$ and $x_t$ denote private consumption, public consumption and leisure, respectively.

The representative household derives utility from leisure and the consumption of private and public goods. The latter are provided by a benevolent government and financed through a proportional tax ($\tau$) on labor income and/or by issuing a one-period bond $b_t^G$ with price $p_t$. At any point in time, the government budget constraint is

$$g_t + b_{t-1}^G = \tau_t(1 - x_t) + p_t b_t^G. \tag{2.2}$$

In a decentralized equilibrium, given taxes, prices and the quantities of public expenditure, the representative household chooses consumption, savings and leisure by solving the following problem:

$$\max_{\{c_t, x_t, b_t^P\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, x_t, g_t)$$
$$s.t. \qquad c_t + p_t b_t^P = (1 - x_t)(1 - \tau_t) + b_{t-1}^P, \qquad \forall t = 0, 1, 2, ... \tag{2.3}$$

where $p_t$ is the price at time $t$ of private bond holdings ($b_t^P$), paying one unit of consumption at time t+1.

The household's first order conditions are

$$\frac{u_{x,t}}{u_{c,t}} = (1 - \tau_t) \tag{2.4}$$
$$p_t = \beta \frac{u_{c,t+1}}{u_{c,t}}, \tag{2.5}$$

together with the budget constraint (2.3). Equation (2.4) and (2.5) represent the equilibrium condition in the labor market and the bond market, respectively.

In what follows, we analyze the problem of the government and characterize its solution under the assumption of full-commitment. This will serve as a benchmark for our discussion in subsequent sections.

### 2.2.1 The case of full-commitment

If the government has full-commitment, for a given initial level of debt ($b_{-1}$), it solves the following problem

$$\max_{\{c_t,g_t,b_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1-c_t-g_t, g_t)$$

$$s.t. \qquad c_t u_{c,t} + \beta u_{c,t+1} b_t = (c_t + g_t) u_{x,t} + b_{t-1} u_{c,t}, \qquad \forall t = 0,1,2,... \tag{2.6}$$

where we made use of the household's optimality conditions (2.3) and (2.5), the resource constraint (2.1) and the market clearing condition $b_t^P + b_t^G = 0$, to substitute for taxes, public expenditure, leisure and government debt.

We also rule out Ponzi schemes, by imposing the transversality condition

$$\lim_{T\to\infty} \beta^T u_{c,T} b_T = 0. \tag{2.7}$$

For our purposes it is worth recalling some features of the resulting equilibrium. As discussed in Lucas and Stokey (1983), in the full-commitment case after an initial jump, all the allocations, including the amount of debt, reach their (deterministic) steady-state level, and remain constant from then on.[11] This is because, apart from $t = 0$, all the periods look identical and the government is willing to smooth private and public consumption over time. However, the steady state allocations depend on the initial condition $b_{-1}$. In other words, countries starting with high debt will have high debt forever, and countries with low debt will have low debt forever. Because of this dependency on initial conditions, which are exogenous to the model and empirically difficult to determine, this theory cannot explain why countries accumulate debt to start with. Moreover, it cannot explain why the level of debt is so different across countries with similar economic conditions.

As anticipated, allocations in the first-period are different. This is related to the time-inconsistency problem typical of this setting. The government, when making its plans at period $t = 0$, would like to use taxes and public expenditure to manipulate the bond price. This is because of the following. For a generic $t > 0$, current

[11] See the optimality conditions in the appendix.

consumption influences both $p_t$ and $p_{t-1}$. As a consequence, if the government uses taxes and public expenditure to increase the price of the bond $p_t$, other things equal, it also decreases $p_{t-1}$. At an optimum, it turns out that the costs of such a procedure offset the benefits. However, at $t = 0$ things are different. Since previous prices ($p_{-1}$) are given, if the initial level of debt is positive, the government can benefit from an increase in the price of the bond, without incurring any additional cost. For example, by setting its policies such that current consumption is higher than in the future, the government is able to foster the demand for savings, thus selling bonds at a more convenient price.[12] These incentives to increase initial consumption prevail whenever the government is allowed to make a new plan. This is why the solution to this problem is in general time-inconsistent.

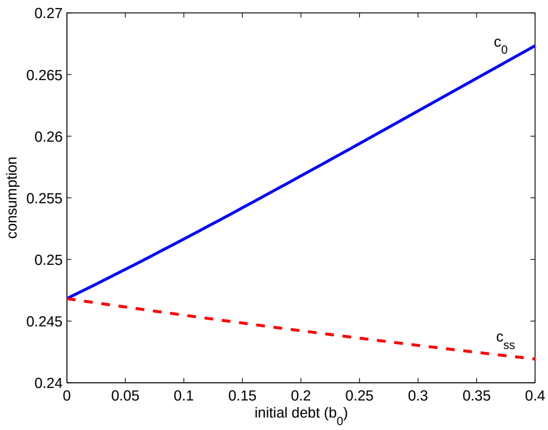
To explain better the mechanism described above, in figure 2.2.1 we plot the level of consumption at $t = 0$ ($c_0$) and the steady-state level of consumption ($c_{ss}$), for a given positive initial level of debt ($b_{-1} \geq 0$), under the full-commitment assumption.[13] We can see that the higher is debt, the bigger is the difference between current and future consumption, and thus the higher is the drop in the interest rate. This happens because the higher is debt the larger is the base on which the improved interest rate is applied. As a consequence, the higher is the inherited level of debt, the greater is the willingness to manipulate the interest rate.

Now we can look at the behavior of debt in the first period, by looking at the government budget constraint in equation (2.2). On the one hand, the tax cut necessary to foster initial consumption reduces the tax revenues of the government. On the other hand, the resulting lower interest rate allows the government to sell bonds at a higher price. Whether $b_0 > b_{-1}$ depends on the composite effect of these two forces. In figure 2.2, we plot the level of debt chosen in the first period (and thus the steady state level of debt), as a function of $b_{-1}$. For low levels of $b_{-1}$, the reduction of tax revenues is bigger than the benefits derived from higher bond prices. The government thus runs a deficit in the first period and accumulates debt. However, if the initial

[12]The opposite happens when $b_{-1} < 0$.

[13]The picture is based on the calibration of the next sections. See Lucas and Stokey (1983) for the analytical solution of the model in the case of a quadratic utility function.

**Figure 2.1:** Consumption under full-commitment




Note: The figure plots, for different level of initial debt, the level of consumption in the first period (solid line) and the steady state level of consumption (dashed line). The reported values correspond to the calibration specified in table A-4.

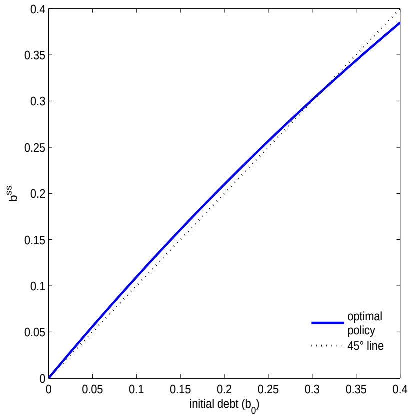
level of debt is large enough, the increase in bond prices applies to a larger base. As a consequence the tax cut can be self-financed and the level of debt can also decrease. This can be seen in figure 2.2, at a level at which the continuous line intersects the 45-degrees line.

To summarize, the long-run level of debt under the full-commitment assumption depends on initial conditions. In the first-period and, consequently, in a one-time deviation from commitment there is a tendency to run deficits and increase debt. In order to understand if the absence of commitment can lead to public debt accumulation, we now relax the full-commitment assumption.

## 2.3 The time-consistent solution

In this section, we analyze the problem of a benevolent planner which, as opposed to the case of the previous section, does not have access to a commitment technology. More precisely, we consider the case in which the current planner cannot make credible promises about his future actions. We keep the assumption that the planner can

**Figure 2.2:** Debt dynamics under full-commitment




Note: The figure plots the steady state level of debt ($b^{ss}$), that is the level of debt prevailing from the first period on, as a function of the initial debt ($b_0$). The reported values correspond to the parametrization specified in table A-4.

credibly commit to repay his loans.[14] In what follows, we also assume that reputation mechanisms are not operative, focusing only on Markov-Perfect equilibria, as defined for instance in Klein et al. (2004).

In this case the problem of the planner is

$$V(b_{t-1}) = \max_{\{c_t, g_t, b_t\}} u(c_t, 1 - c_t - g_t, g_t) + \beta V(b_t) \tag{2.8}$$

$$s.t. \qquad c_t u_{c,t} + \beta u_c(\Psi(b_t))b_t = (c_t + g_t)u_{x,t} + b_{t-1}u_{c,t}. \tag{2.9}$$

It is important to highlight the presence of the function $\Psi(b_t)$ in the constraint (2.9). This determines the quantity of consumption the consumer expects for period $t+1$ as a function of the outstanding level of debt $b$ at the beginning of next period. This represents the main difference with respect to the full-commitment case. Since the current planner cannot make credible commitments about his future actions, the future stream of consumption is not under his direct control. By taking as given the policy $\Psi(b_t)$ of his successor (or himself in the next period), the current planner can

[14]This may be justified by thinking that if the planner would be allowed to default on his outstanding debt at no cost, then consumers would anticipate it, and there would be no trading.

only influence future consumption through his current debt policy. Being the function $\Psi(b_t)$ unknown, the solution of this problem relies on solving a fixed point problem in $\Psi(b_t)$.[15]

We can now look at the first order conditions of the associated Lagrangian, and in particular at the so called generalized Euler equation (GEE)

$$\gamma_t(u_{cc,t+1}\Psi_{b,t}b_t + u_{c,t+1}) = u_{c,t+1}\gamma_{t+1},$$

where $\gamma_t$ indicates the Lagrange multiplier attached to constraint (2.9).[16,17] The inspection of the previous equation allows us to describe the behavior of the economy in a (deterministic) steady state. In particular, for the GEE to be satisfied in steady state, it must be that

$$\gamma u_{cc}\Psi_b b = 0. \tag{2.10}$$

We can identify three different cases in which such relationship holds, as illustrated in figure 2.3. This picture, together with the steady states implied by eq. (2.10), gives a qualitative representation of the transition dynamics obtained in our numerical experiments.[18]

First, we have the case in which $\gamma = 0$. This means that constraint (2.9) is not binding, and we are at an unconstrained optimum. From an economic point of view, this is saying that the planner can avoid to raise distortionary taxes and can finance his public expenditure through the interest payments received on his outstanding assets. This represents the first-best solution.[19] As discussed later, this steady state can only be reached if the planner initially holds a sufficiently high level of assets (roughly 5 times GDP under our calibration).

[15] See Klein et al. (2004) and Judd (2004) for a detailed discussion on this topic.

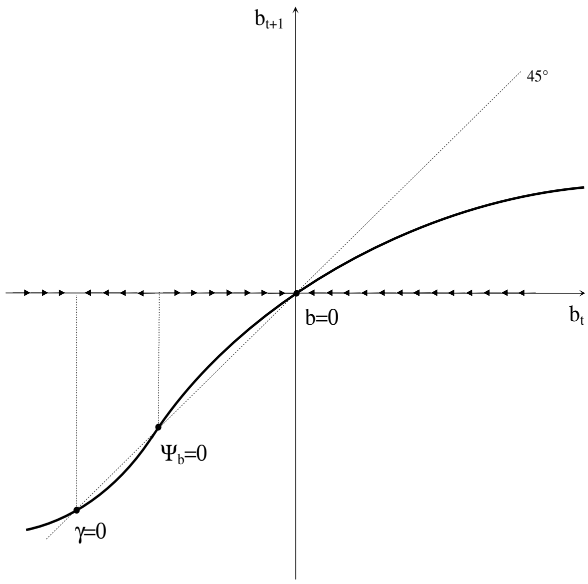
[16] By doing so, we are implicitly assuming differentiability of the function $\Psi(b_t)$. We do not have a formal proof about the existence and/or uniqueness of this solution. However, in our numerical exercises we do find a continuous and differentiable solution.

[17] In the present framework, the generalized Euler equation is the derivative of the Lagrangian associated with the problem (2.8) w.r.t. $b_t$.

[18] Results do not change qualitatively for any of the parametrization we used.

[19] In this case, the level of government debt should be $b = -g^\star/(1-\beta)$, where $g^\star$ is the first-best level of public consumption.

**Figure 2.3:** Debt dynamics in the time-consistent Case




Note: The figure is a qualitative representation of debt equilibrium dynamics resulting from our numerical experiments.

Second, we have the case in which $\Psi_b = 0$. This can happen when a marginal change in the level of debt does not induce any change in the equilibrium level of private consumption. This case cannot be ruled out. However, given the presence of distortionary taxation, this is not due to Ricardian equivalence. On the contrary, when a planner inherits a higher level of debt, he has to raise more distortionary taxes. Because of the bigger distortions created, by a substitution effect, this will reduce hours worked and private consumption. An increase in debt also creates a wealth effect that decreases hours worked and increases private consumption.

Both the wealth and substitution effects lead to a reduction in hours worked as debt increases. The composite effect on private consumption can be understood by examining the aggregate resource constraint. By differentiating equation (2.1) with respect to debt ($b$) we can see that in equilibrium we must have

$$\frac{\partial c}{\partial b} + \frac{\partial g}{\partial b} = -\frac{\partial x}{\partial b}. \tag{2.11}$$

In a model where public expenditure is exogenous, the effects on consumption must be equal to the ones on hours worked. As a consequence, in such case, $\Psi_b$ cannot be zero. But in our framework, there is another way for the planner to cope with the higher burden created by the higher debt. That is, by reducing the amount of public good provision. As a result it is possible that a marginal change in the level of debt does not produce any effect on the level of equilibrium consumption (i.e. $\Psi_b = 0$) as long as the effects on leisure ($x$) and public expenditure ($g$) exactly offset each other. In our calibration, we find that this second type of steady state is at a point where the level of assets is roughly two times GDP. Moreover, for the reasons explained below, this steady state is found to be unstable.[20]

Finally, we have a steady state, associated with a level of debt equal to zero. As illustrated in figure 2.3, this is the steady state to which the economy will converge in the (more relevant) cases in which the government initially holds a positive amount of debt or relatively small amount of assets. The intuition for such a result goes as follows. As explained for the full-commitment case, whenever a government inherits a positive amount of debt, it has the incentive to use the instruments at its disposal to reduce the interest rate payments or, equivalently, to increase the selling price of bonds, as given by (2.5). To do so, the demand for savings should increase, which will happen if current consumption increases more than future consumption. A government with full-commitment could promise the desired level of future consumption regardless of the debt level. In the no-commitment case this is no longer true. The government can only influence future actions through the states variables, which in our case is debt. The higher the inherited debt, the higher will be the incentive in the next period to increase consumption again, in order to manipulate bond prices. Therefore, to face favorable bond prices, the current government needs to leave a lower debt to its successor. If it does not do so, the successor will raise consumption even more, and the anticipated positive consumption growth would harm the current bond price. It follows that debt is reduced until a level of zero debt is reached. At

[20] The position of this point depends on the marginal rate of substitution between private and public consumption and between consumption and leisure. Moreover, a priori, there may be more than one level of debt for which this can happen. For all the utility specifications and the calibrations used in our numerical exercises, we found a unique steady state with $\Psi_b = 0$.

this point, the incentive to manipulate the interest rate vanishes.

To provide a more concrete description of the behavior of our economy, we solve the model numerically by assuming the following functional form for the utility function:[21]

$$u(c,x,g) = (1-\phi_g)\left[\phi_c \frac{c^{1-\sigma_c}-1}{1-\sigma_c} + (1-\phi_c)\frac{x^{1-\sigma_x}-1}{1-\sigma_x}\right] + \phi_g \frac{g^{1-\sigma_g}-1}{1-\sigma_g},$$

where $\phi_c$ and $\phi_g$ denote the preference weights on private and public consumption.

We use a standard calibration for an annualized model of the US economy in order to match long-run ratios of our variables. Table A-4 summarizes the parameter values.[22]

The evolution of the allocations over time is illustrated in figures 2.4 and 2.5 where, for comparison, we also display the solution under full-commitment. For a given level of initial debt, we can observe a decreasing pattern of private consumption and an increasing interest rate.[23] This is achieved by lowering taxation and increasing public consumption over time.

It is important to highlight that in the initial period, in the no-commitment case taxes are higher and public consumption is lower than in the full-commitment case. Such policies allow not only to foster private consumption in the desired way, but also to run a surplus. As a result debt decreases over time. As the level of debt and interest payments are reduced, public consumption is raised and taxes are reduced. This will make consumers work more and consume less over time.

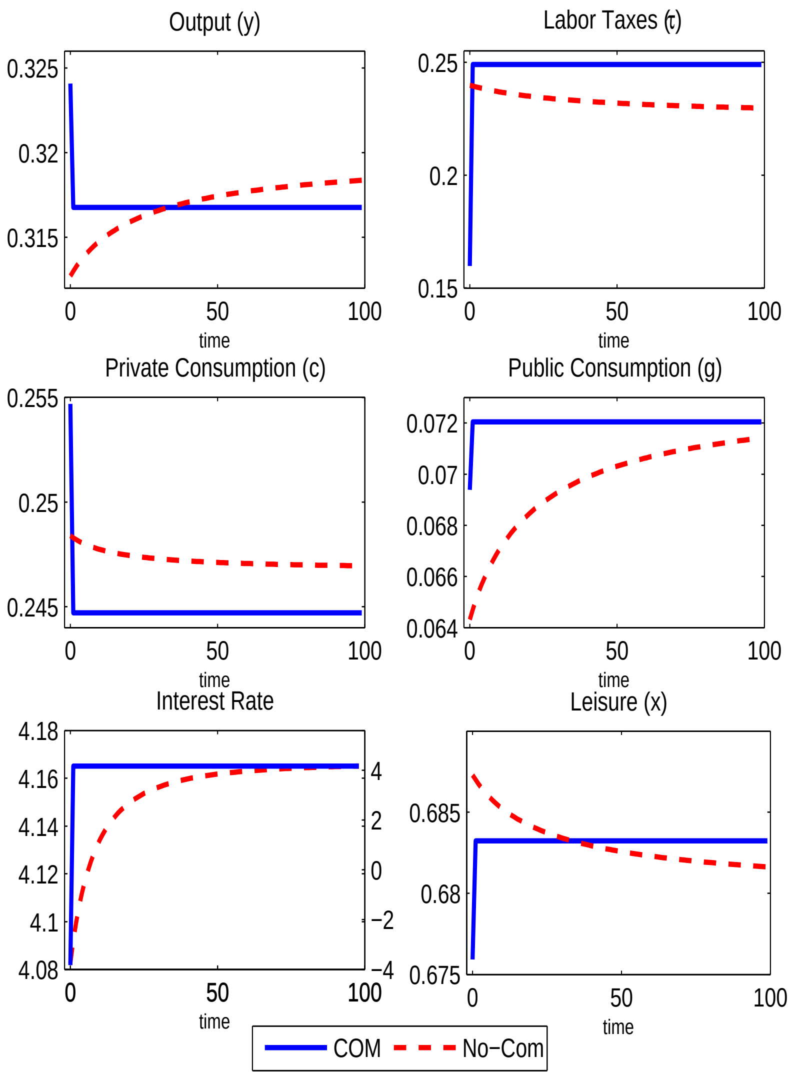
As discussed above, it is feasible to have lower taxes and lower levels of private consumption only if the level of public consumption is increased. In a model where public expenditure is exogenously determined, for example, it will not be possible to have lower taxes and lower consumption at the same time. In that context, for an exogenously given amount of public expenditure, lower taxes will imply a higher

[21] We assume separability as it is convenient for our analysis in section 2.5.

[22] The ratios that we match are $c/g$, $c/y$, income taxes ($\tau$), the fraction of time devoted to leisure ($x$) and the interest rate. We have tried many parameter specifications to check that results do not change qualitatively.

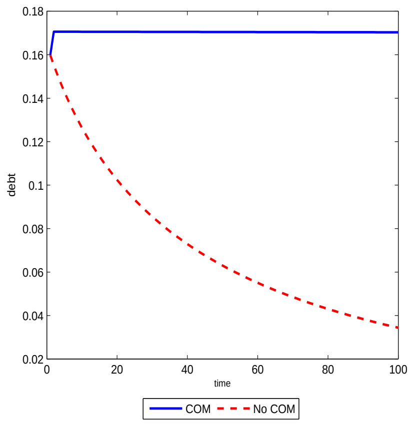
[23] Here we initialize debt at approximately 50% of steady state GDP under commitment. Even though the steady state under commitment depends on initial conditions, long-run GDP is almost insensitive to variations of debt.

**Figure 2.4:** Commitment vs. no-commitment: time pattern of allocations




Note: The figure plots the equilibrium allocations over time, giving an initial condition of $b = .16$ which is roughly 50% of GDP under our parametrization. The interest rate (lower-left panel) for the full-commitment case (continous line) has to be referred to the right-hand scale.

**Figure 2.5:** Commitment vs. no-commitment: time pattern of debt




Note: The figure plots the evolution of debt over time, giving an initial condition of $b = .16$ which is roughly 50% of GDP under our parametrization. The solid line corresponds to the full-commitment case, while the dashed line corresponds to the no-commitment case.

amount of hours worked and thus, by the resource constraint, higher consumption.[24] This prevents having a decreasing pattern of consumption and reducing debt at the same time.

Our results suggests that with no-commitment the exposure of the government in terms of debt/assets will be lower than in the case of full-commitment. This result may seem counterintuitive when compared with our discussion about the temptation to deviate from full-commitment (see section 2.2.1). In general, however, there is no reason why behavior with no-commitment should mimic the policy implemented in a one-time deviation from full-commitment. In the commitment case, the planner can benefit from the interest rate manipulation simply by taxing less today, and promising that future consumption will be lower, regardless of the level of debt. In the case of no-commitment, the government realizes that in order to conveniently manipulate the interest rate, it has to leave a lower debt to its successor. Thus debt decreases over time.

[24]In this reasoning, we are considering that we are in the upward-sloping part of the Laffer curve.

So far, we have explained why with a positive initial level of debt the planner will run surpluses until the zero level is reached. By a symmetric argument, we have that if the planner starts with assets, he is willing to induce an increasing pattern of consumption, in order to increase the interest payments he is receiving. Unless the initial level of assets is so high (i.e. in figure 2.3 as long as the initial condition is to the right of the point where $\Psi_b = 0$), the planner achieves this goal by increasing taxes over time, reducing public expenditure and accumulating deficits until the level of outstanding assets is zero.

In case the planner initially holds a sufficiently high amount of assets (i.e. in figure 2.3 the initial condition is to the left of the point where $\Psi_b = 0$) an increasing path of consumption is instead obtained by reducing taxes and increasing public expenditure. Indeed, in such circumstances, public expenditure is already so high that moving taxes will have a bigger effect on private consumption. To induce an increasing path of consumption it is therefore convenient to lower taxation, slightly increase public expenditure and accumulate assets over time. This process will continue until the point where a level of zero taxation is reached and public expenditure can be financed only through the interest payments on the asset holdings.

To summarize, we find that debt dynamics are very different depending on the specific assumptions about commitment. In the absence of commitment, debt converges to zero in the long-run, unless the planner initially holds a substantially high amount of assets. In the subsequent analysis, as it seems more reasonable, we will ignore that case. From a positive point of view, both the full commitment and the full discretion case are unappealing. In the former case, the level of debt crucially depends on initial conditions, while in the latter case the implication of the model of zero long-run debt is clearly at odds with the empirical evidence.

## 2.4 Loose commitment

As shown in the previous sections, the evolution of debt changes dramatically depending on whether we assume full-commitment or no-commitment. Both cases are clearly extreme depictions of reality. As it seems more realistic, in this section we analyze the case where a benevolent policymaker has the ability to commit but, under some circumstances (like wars, political pressures, etc.), it may renege on its past promises. We refer to this case as *loose commitment*. This allows us to check whether in such circumstances it is possible to have a steady-state with a positive level of debt, independently of the initial condition. In what follows, we show that this is not the case. Unless there is full-commitment, debt converges to zero in the long-run.

We introduce *loose commitment* into the basic model of the previous sections, following the methodology developed in chapter 1.[25] We consider an institutional setting where the ability to commit is driven by an exogenous shock $s_t \in \{0,1\}$.[26] In particular, we assume that at any point in time $t$, each government faces a probability $\pi$ of being reappointed ($s_{t+1} = 1$) in the following period, while with probability $1-\pi$ another government will come into power ($s_{t+1} = 0$). There is an alternative interpretation for parameter $\pi$. Since the average duration of a tenure is $1/(1-\pi)$, a higher $\pi$ implies a larger horizon over which the current government is expected to commit.

In this section, we assume that successive governments share the same objectives (i.e. there is no political disagreement).[27] A government can credibly commit to its own future policies. However, when a new government is appointed, a reoptimization occurs and previous promises are then discarded.

Taking into account that next period either the current government will be in charge or a new one is elected, the implementability condition (2.6) can be written as

$$c_t u_{c,t} + \beta\pi u_{c,t+1} b_t + \beta(1-\pi) u_c(\Psi(b_t)) b_t = (c_t + g_t) u_{x,t} + b_{t-1} u_{c,t}. \tag{2.12}$$

This is obtained by expanding the term $\beta u_{c,t+1}$ in the Euler equation (2.6). With probability $\pi$, the current government will stay in power for another period. In that case, we are assuming that a commitment technology is operational and future

[25] Schaumburg and Tambalotti (2007) developed a similar methodology than can be applied only to linear-quadratic problems. Our problem is not linear-quadratic and, more importantly, the non-linearity of the policy functions is crucial to determine the level of debt.

[26] For simplicity, we will abstract from other shocks hitting the economy.

[27] In this context, it is equivalent to say that a new government is appointed or that the same government defaults on its past promises. This assumption is relaxed in the next section

variables can be directly controlled by the government. With probability $1-\pi$, a new government is elected. In that case, it is anticipated that the new government will disregard previous promises and implement new policies, which are a function of next period's state variable, $b_t$. It is thus anticipated that a level of consumption $c_{t+1} = \Psi(b_t)$ will prevail under a newly elected government. The functions $\Psi(\cdot)$ will be specified later.

It can be shown that the problem of a government, in the first period of its tenure, can be written as

$$V(b_{-1}) = \max_{\{c_t, g_t, b_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta\pi)^t \{u(c_t, 1-c_t-g_t, g_t) + \beta(1-\pi)\xi(b_t)\}, \tag{2.13}$$

subject to the sequence of constraint (2.12) for $t = 0, 1, 2, ....$[28]

The objective function contains two parts. First, the current government can make its own plans for the cases in which it will be in charge. This is represented by the first term in the summation. Uncertainty about being in office in the future makes the government to discount next periods' utilities at the rate $\beta\pi$. Second, with probability $1-\pi$ a new government is elected. The current government can only influence the decisions of its successors through the state variable $b$. This effect is captured through the function $\xi(b_t)$. This represents the value that the current government obtains if a reoptimization occurs at $t+1$.

Our formulation (2.13) is quite general in the sense that it nests as special cases the full-commitment ($\pi = 1$) and no-commitment solution ($\pi = 0$), and the continuum between such extremes ($0 < \pi < 1$). In chapter 1 we prove that such kind of problems can be cast into the framework of Marcet and Marimon (1998). By doing so, one can prove that the problem is recursive and that the policy functions are time-invariant and only depend on a finite set of state variables.[29] For current purposes, it is worth mentioning that constraint (2.12) is associated, in a Lagrangian formulation, with a Lagrange multiplier, which we denote as $\gamma$. Marcet and Marimon (1998) show that such a Lagrange multiplier measures the values of past commitments. In our

[28]Such formulation encompasses all the possible histories of commitment/no-commitment, as shown in chapter 1 in a more general framework.

[29]As before, we are only focusing on Markov Perfect equilibria.

formulation, when a new government is appointed, the Lagrange multiplier($\gamma$) is set to zero since past commitments do not need to be fulfilled.

It must also be emphasized that the policy function $\Psi(b)$ and the value function $\xi(b)$ are unknown functions, taken as given by the current government. As a consequence, such functions need to be found as a solution of a fixed point problem. However, when successive planners share the same objective function $\xi(b){=}V(b)$. This allows the use of an envelope result to get rid of the value function $\xi(b)$. We solve the problem numerically, by a collocation method on the first-order conditions of problem (2.13).[30]

In figure 2.6 we show the average value of debt for several degrees of commitment, as measured by the parameter $\pi$.[31] We find that even a relatively small departure from the full-commitment assumption makes the economy behave very similarly to the no-commitment case. If at period $t = 0$ the government holds debt (assets), it accumulates surpluses (deficits), until the level of zero debt is reached. Hence, the property that the steady state level of debt is determined by the initial conditions is not robust to small deviations from the full-commitment case.

In table 2.1, we show long-run average allocations with a degree of commitment of $\pi = .75$, together with the full-commitment and no-commitment cases.[32] Unless there is full-commitment, the debt/GDP ratio is zero, due to the reasons explained above. All the other ratios are substantially unchanged, apart from a small reduction in taxes. In this type of models, the steady state interest rate is only minimally affected by the outstanding level of debt.[33] As a consequence, a higher level of debt only affects the base to which the interest rate is applied. This explains why lower debt implies lower taxation, and only slightly affects the other allocations.

To gain a deeper understanding of the transition dynamics, it is useful to look at

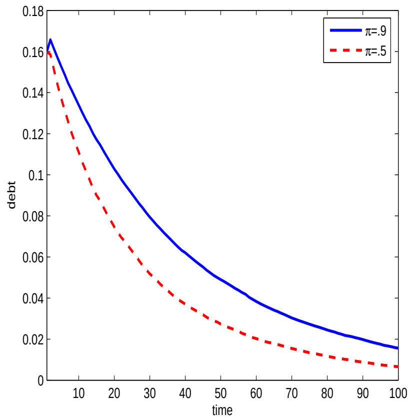
[30] The policy functions are approximated either by splines or by Chebyshev polynomials.

[31] Averages are taken with respect to the realizations of the history of the shock $\{s_t\}_{t=0}^{\infty}$.

[32] As before, we set as the initial condition for the simulation a level of $b_0$ equal to 50% of GDP. Such initial condition only matters for the values of the full-commitment case.

[33] The steady state interest rate in the full-commitment and no-commitment cases, is $\beta^{-1}$, thus totally independent of debt. In the *loose commitment* settings, the interest rate slightly oscillates even in the long-run, due to the presence of the shock $s_t$. In contrast, the level of debt would affect the long-run interest rate, for example, in a model where there is a concern about the default on outstanding debt. However analyzing such case is out of the scope of this study.

**Figure 2.6:** Loose commitment: time pattern of debt




Note: The figure plots the evolution of debt over time, under different values of parameter $\pi$. The solid line corresponds to the case of $\pi = .9$, while the dashed line corresponds to the case of $\pi = .5$. We take average across simulations of the histories of the shock $\{s_t\}_{t=0}^{\infty}$. The initial condition is $b = .16$ (roughly 50% of GDP).

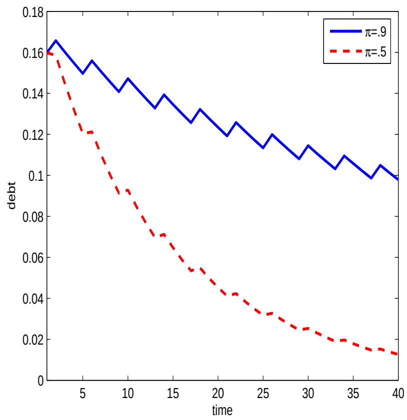
**Table 2.1:** Average long-run allocations under loose commitment

| | COM | $\pi = .75$ | No COM |
|---|---|---|---|
| $b/y$ | 0.52 | 0.0 | 0.0 |
| $g/y$ | 0.26 | 0.26 | 0.26 |
| $c/y$ | 0.74 | 0.74 | 0.74 |
| $\tau$ | 0.28 | 0.26 | 0.26 |

figure 2.7. Here we consider a particular realization of the shocks $\{s_t\}_{t=0}^{\infty}$, where a reoptimization occurs every 4 periods, independently of the probability $\pi$.

It is interesting to observe that in the *loose commitment* framework, debt is increased when a reoptimization occurs and past promises are not kept. On the other hand, debt is decreased when promises are fulfilled. This is in contrast with the no-commitment solution, where debt is always reduced. This occurs because in the no-commitment solution the planner knows that he can conveniently affect the interest rate only if debt is reduced. In the *loose commitment* setting this is no longer true. With probability $1 - \pi$ the planner will be replaced, and promises will not be

**Figure 2.7:** Loose commitment: reoptimization every 4 periods




Note: The figure plots the evolution of debt over time, in the particular history of the shock $\{s_t\}_{t=0}^{\infty}$ such that a reoptimization occurs every four periods. The solid line corresponds to the case of $\pi = .9$, while the dashed line corresponds to the case of $\pi = .5$. The initial condition is $b = .16$ (roughly 50% of GDP).

kept. In that case, the level of debt is key to determine the policy of the successor and thus the interest rate. But with probability $\pi$, promises will be fulfilled, and such promises will determine the interest rate independently of the level of debt. In a *loose commitment* setting, the planner can thus afford to increase debt when reoptimizing, and conveniently manipulate the interest rate, as long as he promises to reduce debt if he stays in office in the following periods. Finally, we note that if $\pi$ is near zero debt is always reduced, no matter whether promises are kept or not. In that case, debt is reduced at a higher pace when promises are kept.

To summarize, in our *loose commitment* setting, we find that unless the government has full-commitment, debt goes to zero in the long-run and that debt is reduced mainly in the periods over which the government has commitment. This is saying that the behavior of the economy resembles the one that would prevail under no-commitment. Moreover, our considerations about the transition dynamics suggests that the simple possibility that in a future period a reoptimization will occur is enough to make it optimal to reach the zero level of debt.

## 2.5 Political disagreement

In this section, we extend our analysis to take into account political disagreement among successive planners alternating in office. There are two reasons why we believe this case is interesting.

First, this is a case where the assumption of imperfect commitment is natural. In the presence of political turnover, the party currently in office cannot make credible commitments about the choices of a successor, who in general has different objectives. Therefore, there is also the issue of imperfect commitment. As explained in the previous section, this feature, *per se*, drives debt to zero.

Second, as discussed in Alesina and Tabellini (1990), the presence of political disagreement and political uncertainty provides an incentive to accumulate an excessive level of debt with respect to the standard (Ramsey) case. In particular, they show that disagreement on the composition of the public good and more frequent political turnovers lead to a higher steady state level of debt. These two channels may explain the different debt levels across countries and over time.

In the dynamic political economy literature, it has been typically assumed that in the presence of political turnover, the assumption about commitment of policymakers is irrelevant to determine private agents' choices. For example, this happens whenever the current choices of the agents (e.g. their savings/consumptions) do not depend on the expectations about future policy choices (e.g., future taxes, public expenditure, etc.).

Our contributions with respect to this literature are the following. First, we consider a framework where the time-inconsistency of policy choices do play a role in the strategic interactions between private agents and policymakers. In our case, as discussed above, the time-inconsistency problem arises because of the governments' incentives to manipulate the interest rate once the amount of private savings is given. Second, we combine imperfect commitment with political disagreement into a unique framework. We first solve the problem assuming that governments cannot make credible commitments about their future actions, no matter whether they are reappointed in office or not. In such circumstances, the probability of re-election is totally unrelated to the probability of being committed in the future. Moreover, we also consider the case that governments can commit for the case they are reappointed, but cannot commit on behalf of their successors. By doing so, we are able to distinguish and quantify the effects of imperfect commitment and political disagreement, which in principle would seem to drive debt in opposite directions. Finally, it allows to measure the welfare gains from commitment in the presence of political disagreement.

We introduce political disagreement in a relatively stylized way. Let's consider an economy where there are two types of public goods $g^1$ and $g^2$. As a consequence the aggregate resource constraint (2.1) takes now the form

$$c_t + g_t^1 + g_t^2 = 1 - x_t. \tag{2.14}$$

Two political parties, $A$ and $B$, with different preferences about the composition of a public good alternate in office. Parties derive utility from leisure ($x$) and consumption of a pr ivate good ($c$) and the two types of public goods, $g^1$ and $g^2$. However, party $A$ has a preference for public goods of type 1, while party $B$ prefers the public good $g^2$. More formally, we assume that period utility, for party $i = A, B$, is given by

$$u(c_t, l_t, g_t^i) = (1-\phi_g)\left[\phi_c \frac{c_t^{1-\sigma_c}-1}{1-\sigma_c} + (1-\phi_c)\frac{x_t^{1-\sigma_x}-1}{1-\sigma_x}\right] + \phi_g \frac{(g_t^i)^{1-\sigma_g}-1}{1-\sigma_g}, \tag{2.15}$$

where $\phi_c$ and $\phi_g$ denote the preference weights on private and public consumption.

Preferences for the composite public good $g_t^i$ are given by

$$g_t^A = g_t^1 + \alpha g_t^2 \tag{2.16}$$

$$g_t^B = g_t^2 + \alpha g_t^1, \tag{2.17}$$

where differences between public goods can be interpreted as differences in their nature (e.g. education vs. military expenses), or in specific characteristics of the same good (e.g. location, producers, type of production, etc.). The latter can better rationalize our assumption of perfect substitution between the two types of goods. The parameter $\alpha \leq 1$ measures the degree of disagreement between the two parties. In the limiting cases, a value of $\alpha = 1$ means no disagreement, while $\alpha = 0$ indicates complete disagreement.[34]

[34]We do not consider the possibility that utility is decreasing in the public good preferred by the other party, thus setting $\alpha \geq 0$.

Another feature regarding our specification of political disagreement is also worth noting. The intuition that political disagreement provides incentives to excessively accumulate debt works as long as, for any given level of $g$, the current government perceives to spend better than its successor an additional unit of public expenditure. Otherwise, it would be convenient for the current government to spend less in the current period and leave more resources for its successor, even if it has different preferences. Given our preferences specification, as in equations (2.15), (2.16) and (2.17), this is guaranteed as long as we assume that the parameter $\sigma_g < 1$.[35]

It can be shown that the problem of a government of type $i = A, B$, at the beginning of its tenure, can be written as

$$V(b_{-1}) = \max_{\left\{c_t, b_t, g_t^1, g_t^2\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta\pi)^t \{u(c_t, 1 - c_t - g_t^1 - g_t^2, g_t^i) + \beta(1-\pi)\xi(b_t)\} \quad (2.18)$$

subject to the sequence of constraint (2.12) for $t = 0, 1, 2, ...$, and the specification of $g^i$ as in (2.16) and (2.17).

The previous formulation is similar to the one of problem (2.13). The main difference is that, when a reoptimization occurs, choices are taken by a government with different preferences. As a consequence, in the objective function of party $i$, the function $\xi(b_t)$ is the lifetime utility that party $i$ obtains if the other party is elected at $t + 1$.

Since the problem faced by the two government types is fully symmetric, in a given state of the world they will always choose the same level of debt, private and public consumption. Only the composition of the public good will be different. Moreover, perfect substitutability between the two public goods implies that, at an optimum, party $A$ only chooses public good of type 1, while party $B$ only chooses the type 2 good. Since one and only one of the public goods is produced in each period, we can make the simplification that $g_t$ denotes $g_t^1$ or $g_t^2$. This symmetry allows us to define

[35] This assumption will not be necessary under different specifications of the utility function, e.g. removing the perfect substitutability between the two types of public good. However, this will complicate our analysis, since in this case both types of public goods will be provided by each type of government.

the lifetime utility derived by a party $i$ when the other party is in charge $\xi(\cdot)$ as

$$\xi(b_{-1}) = \sum_{t=0}^{\infty} (\beta\pi)^t \left[u(c_t^*, 1 - c_t^* - g_t^*, \alpha g_t^*) + \beta (1 - \pi) V(b_{t+1}^*)\right] \tag{2.19}$$

where stars denote variables evaluated with the policy functions solving problem (2.18). In the case there is no disagreement ($\alpha = 1$), we have that $\xi(\cdot) = V(\cdot)$.

Our formulation (2.18) allows to study all the combinations of degree of commitment and political disagreement. For example, if $\pi = 1$ and $\alpha = 1$ we have full commitment and no disagreement among planners, as in the standard Ramsey formulation of section 2.2.1. On the other extreme, when $\pi = 0$ and $\alpha < 1$ we have political disagreement with no-commitment. By changing the values of the parameters $\pi$ and $\alpha$, we are therefore able to disentangle the effects of these two sources of inefficiency.

We will next define our concept of equilibrium, which we restrict to be within the class of Markov equilibria.

**Definition 1** *A Markov Perfect Equilibrium with Imperfect Commitment and Political Disagreement is an allocation* $\{c_t, g_t, b_t\}_{t=0}^{\infty}$ *satisfying the following conditions:*

1. Given $\Psi(b)$ and $\xi(b)$, the allocation $\{c_t, g_t, b_t\}_{t=0}^{\infty}$ solves problem (2.18);

2. The value function $\xi(b)$ is described by Eq.(2.19) and $V(b)$ is the maximum of problem (2.18);

3. The policy function of consumption $\psi(b, \gamma)$ solving problem (2.18) is such that $\Psi(b) = \psi(b, 0)$.

The first part of the definition is a simple optimality requirement. The second part states that the functions $\xi$ and $V$ need to be consistent between themselves. The third part of the definition, states that the functions that the future government is expected to implement are optimal. Since the current and future governments face the same problem, the functions that the current government and future governments implement are equal. In particular, it is worth recalling that the public good provided only changes in nature, but not in level. Nevertheless, as stated previously, when a new government is elected the Lagrange multiplier ($\gamma$) is set to zero. Marcet and Marimon (1998) show that such Lagrange multiplier measures the values of past commitments. In our formulation, when a new government is appointed, the Lagrange multiplier ($\gamma$) is set to zero, since past commitments do not need to be fulfilled.[36]

As explained in section 2.4, $\Psi(b)$ and the value function $\xi(b)$ are unknown and need to be found as a solution of a fixed point problem. In the current case, however, the fact that $\xi(b)$ and $V(b)$ are not equal does not allow us to apply envelope results as can be done in absence of disagreement.

In order to understand the role played by the degree of commitment and political disagreement in shaping debt dynamics, we solve the optimal policy problem under different regimes. As explained above, the effects of commitment and political disagreement can be analyzed and disentangled by solving our model under different values of the parameters $\pi$ and $\alpha$.

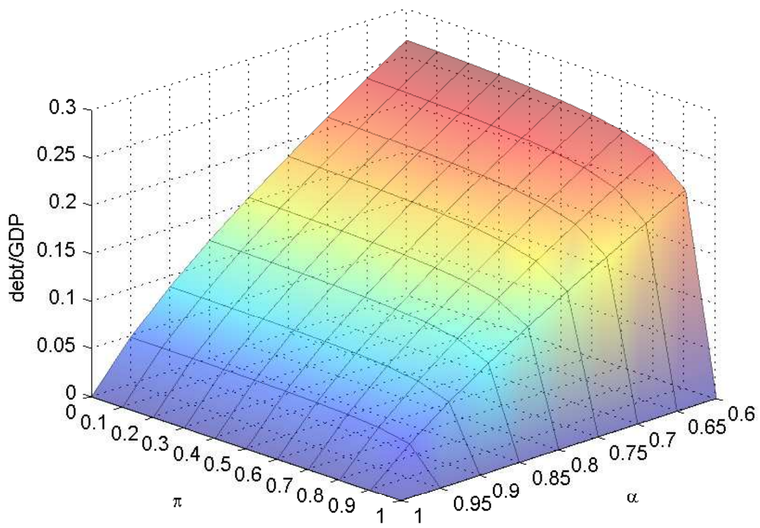
### 2.5.1 The effects of political disagreement and no-commitment

We start by considering the effect of political disagreement, abstracting from commitment issues. In other words, as in Alesina and Tabellini (1990), we keep the extreme assumption that governments can never commit, no matter if they are reelected or not, and act with full discretion. It is worth emphasizing that in this case the parameter $\pi$ is unrelated to the degree of commitment and only measures the probability of being reelected.

Figure 2.8 shows the long-run level of debt for different values of $\alpha$ and $\pi$. First, we can see that once we add a bit of political disagreement between successive planners, debt converges to a positive level in the long-run. Second, we can observe that a higher degree of disagreement (i.e. lower $\alpha$) and more frequent turnover (lower value of $\pi$), imply a higher level of debt. However, while the effects of different degrees of disagreement are relevant, those of the frequency of turnover seems quantitatively less important. For example, when $\alpha = .9$ the effect on the level of debt between having $\pi = .75$ and $\pi = 0$ is less than 5% of GDP.

[36] See the discussion in chapter 1.

**Figure 2.8:** Long-run debt with political disagreement without commitment



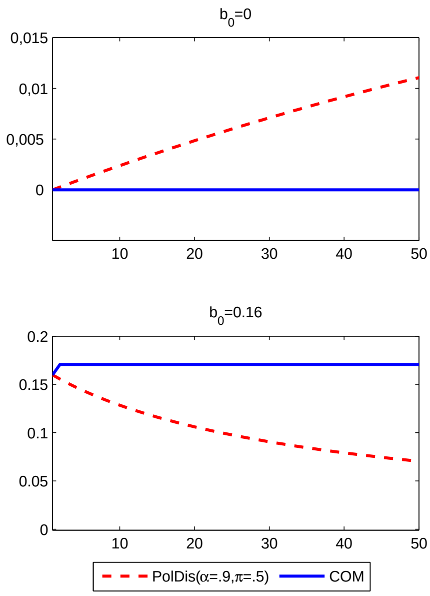

Note: The figure plots the long-run level of debt, for different degree of disagreement ($\alpha$) and frequency of turnover ($\pi$). Governments do not have commitment, regardless of the probability ($\pi$).

In figure 2.9 we plot the transition dynamics for the case of $\alpha = .9$ and $\pi = .5$ for two different initial conditions.[37] In the upper panel we set the initial level of debt to zero. Note that while in the full-commitment/no-disagreement case debt does not change, under political disagreement debt is accumulated over time. This result is consistent with the intuition that under political disagreement an inefficiently high level of debt is accumulated. However, the picture changes if we set the level of debt above the steady state under political disagreement. As we can see in the upper panel of figure 2.9, in that case debt under disagreement is inefficiently low.

From our analysis we can conclude that no-commitment, political disagreement and political turnover can explain why debt is (and remains) a sizable fraction of GDP in most countries. Moreover, countries with more polarized political forces should experience a higher level of debt than more homogenous countries. If governments

[37]The qualitative message does not change for other values of these parameters

**Figure 2.9:** Evolution of debt under political disagreement without commitment




Note: The figure plots the evolution of debt under full-commitment and no-disagreement, as indicated by the solid line, and a case with political disagreement ($\alpha = .9$) and political turnover ($\pi = .75$), as indicated by the dashed line. In the upper panel the initial condition is $b = .16$, while in the lower panel the initial condition is $b = .16$.

dislike the public services provided by their potential successors, they are willing to spend more in the current period and leave less resources (i.e. more debt) as inheritance. The predictions of the model suggest that it is the different degree of polarization among the political forces, rather than the frequency of political turnover and the degree of commitment, what really matters in explaining the differences in the debt level across countries and over time.

### 2.5.2 The effects of political disagreement with commitment

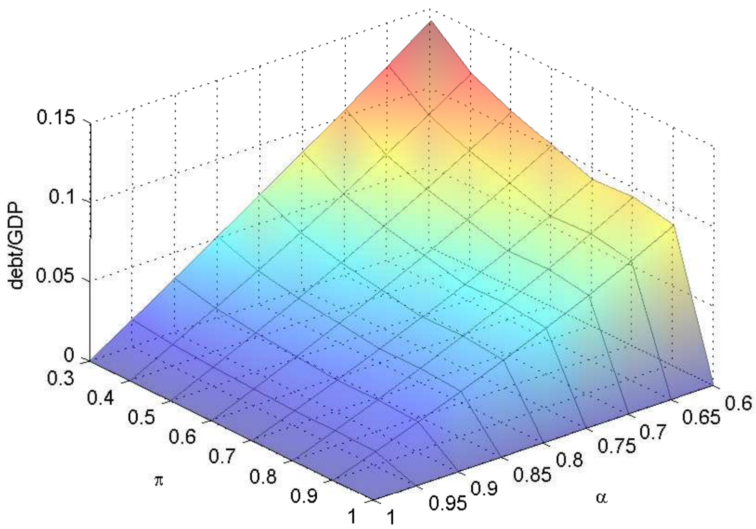
We now investigate the case where a government does commit over its tenure, but cannot commit on behalf of its successors. Besides being a more realistic depiction of reality, there are three main reasons to investigate this case. First, from a static point of view, to see the implications of political disagreement without removing completely the commitment assumption. Second, from a dynamic perspective, allowing for commitment over the tenure in the presence of political disagreement generates volatility in the variables as a consequence of political cycles, even in a fully symmetric model as ours. Indeed, because of the possibility to commit, choices differ depending on whether the government is new in office or not. Third, we can investigate the gains from commitment in a world characterized by political disagreement.

We should first note that, in this context, a higher political turnover also necessarily implies a lower degree of commitment. The lower is the probability of being re-elected, the shorter is the horizon over which the government is expected to commit. In other words, there are now two effects related to the parameter $\pi$. A higher $\pi$ first implies less frequent turnover which leads, *ceteris paribus* to slightly lower debt, according to our analysis in section 2.5.1. But it also means a higher degree of commitment, as shown in section 2.4. Exploring the composite effects of these two forces seems therefore worthwhile.

In figure 2.10, we show the average long-run level of debt in the case of political disagreement and commitment over the tenure.[38] As in the previous case, the level of debt is considerably increasing in the degree of disagreement and the effects of

[38] Averages are taken with respect to the realizations of the shock driving the election process.

**Figure 2.10:** Long-run debt with political disagreement and commitment over the tenure




Note: The figure plots the average long-run level of debt for different degree of disagreement ($\alpha$) and frequency of turnover($\pi$). Averages are taken across realizations of the shock $s_T$, where $T = 1000$.

changes in parameter $\pi$ are quite small. However, as parameter $\pi$ changes, debt now changes non-monotonically. A higher (lower) $\pi$ means a longer (shorter) tenure, on average. On the one hand, the commitment horizon is longer (shorter). This means that debt can be increased by more (less) when a reoptimization occurs. On the other hand, it is less (more) likely that the other party comes into power, so the incentives to accumulate debt are smaller (bigger). The composite effect of changing $\pi$ depends on the relative strength of these two forces, which are difficult to disentangle. Finally, it is worth emphasizing that the effects of a marginal change in the frequency of political turnover depends on the degree of disagreement. This suggests that, from an empirical point of view, it is important to consider the interactions among the two forces when testing the relationship between debt policies and political instability.

By comparing figure 2.8 and figure 2.10, we can also see that the long-run level of debt is lower when there is commitment within the tenure than if governments cannot commit. This result can be explained as follows. As emphasized in section 2.4, in the *loose commitment* setting debt is reduced during the commitment periods and increased whenever past plans are abandoned. In the present context, when governments can commit over their tenure, debt is lower on average because of the longer commitment horizon of alternating governments.

To summarize, according to our model, differences in the frequency of political turnover cannot account for the differences in the level of debt across countries and over time. There are mainly two reasons for this result. First, from a qualitative point of view, the relationship between frequency of turnover and the level of debt seems unclear. This crucially depends on the relative importance of the opposite effects of having a longer (shorter) tenure versus those of having a longer (shorter) commitment horizon. Second, from a quantitative point of view, such effects seems of minor importance.

### 2.5.3 Relationship with the empirical evidence

There is a large body of empirical studies about the effects of political polarization and frequency of turnover on deficits and debt accumulation. However, in most of these studies, polarization and turnover are not analyzed together, since they are usually considered as alternative proxies of political instability.

There are many studies analyzing the effects of political polarization on public debt and deficits. Different works have measured polarization in different ways, but it is generally found that a larger degree of polarization increases debt. Roubini and Sachs (1989) find that coalition governments (interpreted as polarization) are more likely to run deficits.[39] Volkerink and de Haan (2001) and Huber et al. (2003) find that the fragmentation of governments (in terms of size or political ideology) is a source for relatively higher deficits. Alt and Lassen (2006) find that fiscal transparency and

[39]The authors do not present a regression with the average tenure and the evidence regarding this variable is only suggestive. The finding that coalition governments tend to accumulate more deficits has been challenged for instance by Haan and Sturm (1997). See also Alesina et al. (1997) for some evidence supporting Roubini and Sachs.

less polarization reduce debt. Woo (2003) finds that countries with high polarization, measured as income inequality, have bigger fiscal deficits.

There is also a large empirical literature examining the effects of the average tenure or the re-election probability. In this case, however, results are controversial. Alt and Lassen (2006), in contradiction with the theory, find that shorter tenures reduce debt. Skilling and Zeckhauser (2002) also find that political competition decreases debt. Lambertini (2004) and Franzese (2001) find that the incumbent's probability of being voted out of office can not explain budget deficits. Grilli et al. (1991) find mixed results regarding the effects of the average tenure. de Haan and Sturm (1994) find that the frequency of government changes is positively correlated to budget deficits.

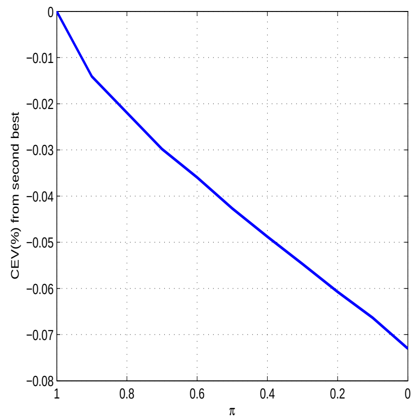
The overview of the empirical literature shows that there is some consensus that polarization is translated into more debt or deficits. On the contrary, the findings on the re-election probability are quite mixed. Our paper can help understand these results. We find that both polarization and the probability of election matters, but the effect of the second variable is small. Different degrees of commitment and small changes in the economic structure among countries may blur the effects of the re-election probability. On the other hand, our model suggests that the effects of polarization are quite strong and easy to detect, as the data confirms. Overall, from an empirical point of view, the more important implication of our analysis is that when testing the relationship between debt and political instability rather than a generical indicator of the latter feature, it is important to consider both the degree of polarization and the frequency of political turnover.

### 2.5.4 Welfare implications

We now turn our attention to the welfare implications of imperfect commitment and political disagreement. For all the settings analyzed in the previous sections, we computed the consumption equivalent variation from the second-best full-commitment and no-disagreement case.

In figure 2.11, we show the welfare costs of imperfect commitment. Moving from full-commitment ($\pi = 1$) to no-commitment ($\pi = 0$), the implied loss is equivalent to

**Figure 2.11:** The welfare costs of losing commitment




Note: The figure plots the welfare losses from reducing commitment (as measured by $\pi$) in a world without political disagreement, expressed in percentage consumption-equivalent variation (CEV) from the benchmark case of full-commitment.

a per-period reduction in consumption of 0.07%. This means that while the degree of commitment has a big impact on the level of debt, it has less striking welfare implications. However, it should be remarked that in our model the level of debt affects welfare relevant allocations because of the interest rate payments. Since the long-run interest rate is only minimally affected by the level of debt, the latter can only have limited effects on consumers' welfare. This will not be the case in a world where there are other links between the level of debt and the interest rate, like introducing the possibility that the government defaults on its outstanding debt. Analyzing such cases is certainly interesting, but is out of the scope of the present analysis. Our work should serve as a reference for countries with small risk of default and which can easily refinance their debt.

We now consider the welfare implications in a world with political disagreement. In this context, building commitment is not necessarily welfare improving. Indeed, in this context commitment is used to pursue partisan objectives, and it can thus be detrimental for agents having different preferences from the government in office. In what follows, we use as measure of social welfare the average of the lifetime utility of our two types of agents, and then compute the consumption equivalent variation from the second-best solution.[40]

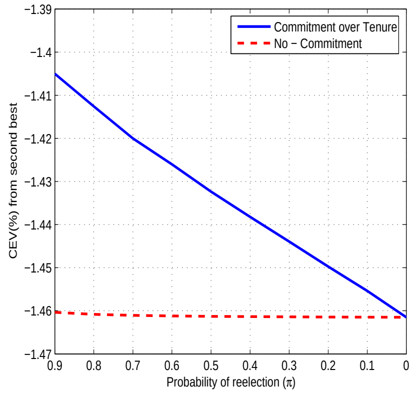
First, we analyze the welfare implications of less frequent political turnover (i.e. higher $\pi$). For this purpose, as shown in figure 2.12, we compute the consumption equivalent variation for different values of parameter $\pi$, for a given degree of disagreement ($\alpha = .9$). The dashed line refers to the case where governments cannot commit, while the continous line indicates the case where governments commit over their tenures. When governments do not have commitment over the tenure, welfare is almost constant in $\pi$, which says that the frequency of turnover, *per se*, is quantitatively irrelevant in terms of welfare. On the contrary, when governments have commitment over their tenure, the lower is the frequency of turnover the higher is welfare. In this case, a higher $\pi$ means not only less frequent turnover but also a longer commitment horizon. This last component seems therefore the one with more important welfare implications. In figure 2.13, we analyze the welfare consequences of increasing $\pi$ in a world where governments have commitment over the tenure and there is political disagreement. The figure plots the welfare gains achieved by increasing $\pi$ from zero to another value. As before, we find that the higher is disagreement (i.e. the lower is $\alpha$) the lower are the welfare gains for an increase in $\pi$. This result is interesting since it suggests that, according to our analysis, building commitment is less important in a country with more polarization among political parties.

Second, in a world characterized by political disagreement, we compute the welfare implications of having commitment over the tenure. For this purpose, in figure 2.14, we show the difference in welfare (in consumption equivalent variation from second-best) between having commitment over the tenure and not having commitment (regardless of $\pi$). We plot such differences for several values of $\alpha$ and for values of $\pi = 0.75$ and $\pi = 0.5$.[41] We observe that the difference in welfare is positive. Interestingly this is saying that having governments committing over their tenures is

[40] Since the problem is fully symmetric, the difference in the utilities of the two types of agents is only due to the type of party starting in office.

[41] A value of $\pi = .75$ implies that a government is replaced on average every four periods (i.e. four years under our calibration). This is why we take it is as benchmark.

**Figure 2.12:** The welfare costs of political turnover ($\alpha = .9$)



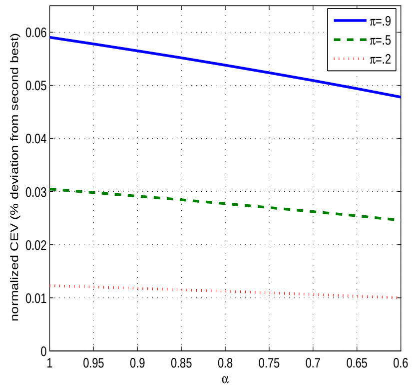

Note: The figure plots the welfare gains from less frequent turnover (as measured by $\pi$) in a world with political disagreement ($\alpha = .9$). The continous line refers to the case where governments can commit over their tenure, while the dashed line refers to the case where there is not commitment over the tenure (regardless of $\pi$). Values are expressed in percentage consumption-equivalent variation (CEV) from the benchmark case of full-commitment and no-disagreement.

welfare improving also in a world with political disagreement. We also notice that such gains from commitment are decreasing in $\alpha$. This suggests once more that building commitment is less important the higher is the degree of polarization (i.e. the lower is $\alpha$).

In a world with political disagreement, the fact that a government has commitment may also be detrimental for agents having different preferences from the policymaker. This is shown in figure 2.15, where we compare the welfare implications of commitment depending on which of the two parties starts in office. Interestingly, we can see that building commitment is welfare improving if the favorite party starts in office (continous line). On the contrary, it is detrimental if the adverse party, i.e. the party providing a public good that the agent value less, starts (dashed line) in office. Overall, our analysis therefore suggests that building commitment is welfare improving even in a world with political disagreement and that building commitment is less important the higher is the polarization among political forces.

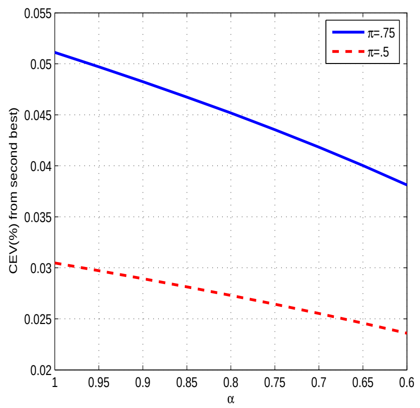
We finally check if there is any circumstance under which an agent prefers that

**Figure 2.13:** The welfare gains of less frequent political turnover



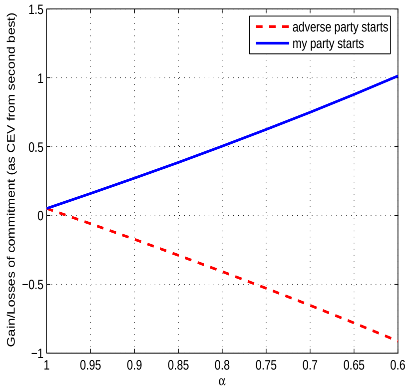

Note: The figure plots the consequences of political disagreement (measured by $\alpha$) to the welfare gains of increasing $\pi$. We first computed welfare for each $\pi$ measured as percentage consumption-equivalent variation (CEV) from the benchmark case of full-commitment and no-disagreement. In the figure, each line plots the welfare differences from $\pi = 0$ to another level of $\pi$.

**Figure 2.14:** The welfare gains from Commitment in the presence of political disagreement




Note: The figure plots, for several degrees of disagreement (measured by $\alpha$) the difference in welfare (in percentage consumption-equivalent variation (CEV) from second-best), between the case where governments have commitment over their tenure and the case where there is not commitment over the tenure (regardless of $\pi$).

**Figure 2.15:** Welfare implications of commitment in the presence of political disagreement




Note: The figure plots, for several degrees of disagreement (measured by $\alpha$) the difference in welfare (in percentage consumption-equivalent variation (CEV) from second-best), between the case where governments have commitment over their tenure and the case where there is not commitment over the tenure (regardless of $\pi$). The continuous line refers to the case where the favorite party starts in office, while the dashed line indicates the case where the adverse party starts. The frequency of turnover is $\pi = .75$.

the adverse party always stays in power. This may indeed happen when the gain from having political stability outweighs the cost associated with a provision of a less favorable public good. For this purpose, we compute the welfare that the agent would obtain under political turnover (measured by $\pi$) against the welfare he will obtain if the opposite party will always be in charge. As shown in table 2.2, welfare is higher in the latter case only for a very small level of disagreement. Indeed, when $\alpha = .95$ having political turnover (as implied by $\pi = .75$) still gives a welfare level that is higher by approximately 1% of per-period second-best consumption. This is because in our model, the gains from commitment are much smaller than the gains from having less political disagreement.

## 2.6 Conclusions

As it has been documented in the literature, imperfect commitment, political disagreement and political uncertainty may be important sources of inefficient fiscal

**Table 2.2:** Welfare implications of political stability (as CEV (%) from second-best)

| $\alpha$ | With Turnover | | With Adverse Party |
|---|---|---|---|
| | $\pi = .75$ | $\pi = .5$ | |
| 1 | -0.02 | -0.04 | 0.00 |
| 0.95 | -0.70 | -0.72 | -1.36 |
| 0.9 | -1.41 | -1.43 | -2.74 |
| 0.8 | -2.91 | -2.93 | -5.62 |
| 0.6 | -6.36 | -6.38 | -11.93 |
| 0.4 | -10.75 | -10.77 | -19.39 |
| 0.1 | -22.61 | -22.62 | -36.88 |

Note: In the first column, we report the welfare implications of having political turnover ($\pi = .75$ and $\pi = .5$) between parties with different objectives (disagreement measured by $\alpha$). In the second column we report the welfare that an agent would get if the party providing the less preferred good would always stay in office.

policies. Our work provides an analysis to distinguish and quantify the effects of each of these forces on the level of debt in a dynamic context.

On the methodological side, our contribution is to develop a framework to analyze the interactions between commitment and political disagreement that can be applied to a wide set of optimal policy problems. In other words, this allows us to integrate the analysis about the time-inconsistency of optimal policy choices, typical of the dynamic macroeconomic literature, into a political economy model.

On the economic side, we show that imperfect commitment drives the long-run level of debt to zero. Thus, the dependency of debt on initial conditions found in Lucas and Stokey (1983) is shown not to be robust to small departures from the full-commitment assumption. Interestingly, for debt to be driven to zero, it is enough that both the agents and the government anticipate the possibility of the temptation to surprise the economy and to manipulate the interest rate, even if this ultimately never occurs. We find that debt is increased when the government reneges on past promises and is driven to zero when past promises are fulfilled.

Our framework allows the incorporation of political disagreement and imperfect commitment into dynamic macro models. We find that debt is increasing in the level of disagreement among political parties. On the other hand, the frequency of turnover and the degree of commitment do not seem to produce quantitatively important effects. These results are consistent with most of the existing empirical literature. Altogether, our analysis suggests the importance of distinguishing between the degree of polarization among parties and the frequency of political turnover when analyzing the effects of political instability on debt policies.

From a normative point of view, we show that according to our model, the welfare gains from commitment are lower when successive planners disagree about their goals than in the no-disagreement case. With political disagreement, a better commitment technology will not be used to maximize overall welfare but to pursue partisan goals. We plan to pursue this line of research further both from a theoretical and an empirical point of view.

There are many interesting aspects which deserve further explorations. Among them, we have abstracted from the possibility of default on outstanding debt. This feature is important since it will generate an additional link between the level of debt and the interest rate. This may have an important impact on the interest rate manipulation mechanism, which is crucial in our analysis. Moreover, there will be bigger welfare implications of having different levels of debt.

More generally, our framework would allow us to address several interesting questions. For example, it would be interesting to address some normative implications of this line of research, especially in relationship with fiscal discipline, like limits on deficits and debt holdings, currently imposed on many developed countries by supranational authorities. In this context, considering other forms of political conflicts, voting mechanisms and shocks affecting the economy is an interesting line for future research.

# Appendix

## The full-commitment case - optimality conditions

Following Lucas and Stokey (1983), under the assumption of complete financial markets, the problem of a planner with full-commitment can be written as

$$V(b_{-1}) \max_{\{c_t,g_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1-c_t-g_t, g_t)$$

$$s.t. \qquad \sum_{t=0}^{\infty} \beta^t [c_t u_{c,t} - (c_t+g_t)u_{x,t}] = b_{-1}u_{c,0}. \tag{A-1}$$

The planner is thus facing a unique intertemporal constraint (A-1). Taking derivatives with respect to $\{c_t, g_t\}_{t=0}^{\infty}$ we have

$$\begin{aligned}
c_0 :& \qquad (u_{c,0} - u_{x,0})(1-\Lambda) = \Lambda[c_0(u_{cc,0} - u_{xx,0}) - u_{cc,0}b_{-1}] \\
c_t :& \qquad (u_{c,t} - u_{x,t})(1-\Lambda) = \Lambda c_t(u_{cc,t} - u_{xx,t}) \qquad t = 1, 2, ... \\
g_t :& \qquad u_{g,t} - u_{x,t}(1-\Lambda) = -\Lambda u_{xx,t} \qquad t = 0, 1, 2, ...
\end{aligned}$$

where $\Lambda$ denotes the Lagrange multiplier associated with constraint (A-1). For notational convenience only, we assumed separability in the utility function, thus implying $u_{cg} = u_{xc} = u_{xg} = 0$.

It is worth noting that, for $t = 1, 2, ...$, such conditions are static and identical, thus implying that from period $t = 1$ onward all the allocations are constant. In $t = 0$, unless $b_{-1} = 0$, the first-order condition with respect to consumption is different, as can be seen by the presence of the last term on the right-hand side. This reflects the incentive to manipulate the interest rate at $t = 0$, which is the source of the time-inconsistency problem in the present framework.

## Data and calibration

**Table A-3:** Debt in the OECD countries in 2006

| | gross | net | | gross | net |
|---|---|---|---|---|---|
| Australia | 15.0 | -2.8 | Korea | 27.9 | -30.2 |
| Austria | 69.1 | 41.8 | Luxembourg | 6.6 | . |
| Belgium | 91.2 | 76.8 | Netherlands | 59.4 | 33.7 |
| Canada | 68.0 | 27.6 | New Zealand | 29.8 | -3.5 |
| Czech Republic | 39.3 | 2.8 | Norway | 48.1 | -149.3 |
| Denmark | 39.7 | 6.9 | Poland | 51.7 | 16.6 |
| Finland | 48.2 | -60.6 | Portugal | 74.3 | 46.6 |
| France | 75.3 | 43.0 | Slovak Republic | 38.4 | -11.7 |
| Germany | 71.3 | 51.9 | Spain | 46.8 | 26.7 |
| Greece | 120.6 | 86.9 | Sweden | 56.0 | -15.0 |
| Hungary | 68.8 | 43.9 | Switzerland | 54.2 | 21.0 |
| Iceland | 24.5 | 8.5 | United Kingdom | 47.9 | 41.7 |
| Ireland | 32.5 | 4.9 | United States | 60.9 | 42.8 |
| Italy | 120.8 | 95.4 | Euro Area | 76.8 | 51.3 |
| Japan | 176.2 | 89.5 | Total OECD | 76.9 | 44.4 |

General government financial liabilities (percent of nominal GDP). Source: OECD Economic Outlook

**Table A-4:** Parameter values

| Parameter | Value | Description |
|---|---|---|
| $\beta$ | .96 | discount factor |
| $\phi_c$ | .2 | weight of consumption (priv. + publ.) vs. leisure |
| $\phi_g$ | .2 | weight of public vs. private consumption |
| $\sigma_x$ | 3 | Elasticity of leisure |
| $\sigma_c$ | 2 | Elasticity of private consumption |
| $\sigma_g$ | .95 | Elasticity of public consumption |

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# Chapter 3

# The Macroeconomic Effects of Unstable Monetary Policy Objectives

Central banks, whether independent or not, may occasionally be subject to external pressures to change policy objectives. We analyze the optimal response of central banks to such pressures and the resulting macroeconomic consequences. We consider several alternative scenarios regarding policy objectives, the degree of commitment and the timing of external pressures. The possibility to adopt "more liberal" objectives in the future increases current inflation through an accommodation effect. Simultaneously, the central bank tries to anchor inflation by promising to be even "more conservative" in the future. The immediate effect is an output contraction, the opposite of what the pressures to adopt "more liberal" objectives may be aiming. We also discuss the opposite case, where objectives may become "more conservative" in the future, which may be the relevant case for countries considering the adoption of inflation targeting.

## 3.1 Introduction

In the last decades, there has been a big effort to build monetary institutions that are independent from political forces and other types of external pressures. In most cases, this process has implied the definition of specific goals to be assigned to central banks, typically in terms of inflation and output stabilization. However, it has often been the case that central banks are subject to some pressures to change their policy goals. For instance, the European Central Bank has been recently criticized for adopting policies that do not properly take into account output growth. In other cases, as it is currently happening in the United States, the adoption of inflation targeting or other institutional reforms are being discussed. Even if there is not an immediate reaction to such pressures, the central bank and the public may attach some probability that monetary policy may change its objectives in the future. Changes in policy objectives may also simply be due to the the appointment of a new central bank chairman or staff, who may have different views from the predecessors.

The aim of this paper is to study how the possibility of changes in future policy goals affects current economic outcomes. It is widely accepted in the modern macroeconomic literature that firms and households, when setting current prices and wages, take into account their expectations about future inflation. In this context, when a policy that generates a higher inflation bias (henceforth, a "liberal policy") is expected in the future, inflation expectations increase and will be reflected in current prices and wages. As a consequence, given the policy chosen by the central bank, the possibility of a future change in policy objectives affects the economic variables already in the current period. In this paper, we analyze the optimal response of a central bank in such situation and its macroeconomic effects.

Optimal monetary policy is typically characterized under the assumptions that a central bank is in charge for an infinite horizon, it can credibly commit to future policy plans, and its policy goals do not change. In this paper, we relax these assumptions. First, we need to consider that current and future policy objectives may not necessarily coincide. Second, when considering the possibility of changes in policy objectives, the assumption about the credibility of the central bank needs to be rethought. For instance, if the central bank has received some external pressures to change its policy goals, it will adopt the best possible policy to fulfill its new objectives, and the plans made when priorities were different will become irrelevant.

We consider several alternative scenarios regarding policy objectives, the degree of commitment and the timing of external pressures. The main part of our analysis refers to the case where an independent central bank has been appointed, possibly one with more conservative objectives than society.[1] The central bank is not completely insulated from the opinions prevailing in the economic environment, and may occasionally be subject to some external pressures to adopt more liberal objectives in the future. This typically happens when countries face adverse economic conditions, like recessions or financial crisis. In all the settings taken into consideration, the possibility that liberal policies may be implemented implies an increase in current inflation through an accommodation effect. At the same time, the central bank tries to anchor inflation expectations by promising to be even more conservative in the future. The immediate effect is an output contraction, which is the opposite of what the pressures to implement liberal policies may be aiming. These effects are stronger the higher is the probability that objectives change. In this respect, adopting inflation targeting may be better than appointing a conservative central banker. In the former case, changing objectives usually requires costly institutional reform. In the latter, it just requires that the central bank chairman and advisors, at the end of their term of office, are substituted by ones with different policy objectives.

We also analyze the opposite case, where objectives may become more conservative in the future. This is the relevant scenario for countries discussing the adoption of more stringent inflation objectives, like a specific low target for inflation. In this context, the possibility of more conservative policy in the future creates a positive externality for the liberal central bank.

Our paper is related to the literature about political economy in monetary policy, following the seminal contribution of Alesina (1987). We want to emphasize that our goal is not to provide a partisan analysis of monetary policy, where one attributes

[1] As shown by Rogoff (1985), appointing a central banker with higher aversion towards inflation than the overall society allows to reduce the inefficiencies associated with time-inconsistency.

economic cycles to political parties. According to the empirical literature, it seems difficult to match timely and sistematically certain parties with effective changes in monetary policy.[2] It seems implausible that all governments have always successfully influenced central bank's objectives, and that they did so as soon as they were elected. However, there have been frequent cases where central banks have been subject to external pressures. Our goal is to understand what are the effects of certain types of pressures, and what is the optimal response of a central bank in such situation.

We carry out our analysis in the framework commonly used in the literature on monetary policy design. As it has become standard, we do not assume a New Classical Phillips curve and consider instead its New Keynesian counterpart. The features of modern science of monetary policy considered here have important consequences, and induce different mechanisms than the ones modeled in Alesina (1987) and other "partisan models" of monetary policy. In the New Keynesian framework and unlike its New Classical counterpart, the possibility of a future change in policy objectives implies a different level of inflation and output in earlier periods. This is because firms and households, when setting current prices and wages, take into account future economic conditions. With respect to the partisan theory of economic fluctuations, our analysis suggests that it is difficult to find a link between economic outcomes and the objectives of certain policymakers.

We also study the importance of central bank credibility when policy objectives can change. To do so, we relax the discretion assumption, typical of the political economy literature. Instead, and following the recent contributions of Schaumburg and Tambalotti (2007) and our methods shown in chapter 1, we model the central bank as possessing a loose commitment technology. In this context, in order to lower inflation expectations created by the possibility to adopt liberal policies, a conservative central bank optimally promises to be even more conservative in the

[2] Alesina et al. (1997) point out several empirical successes of the Alesina (1987) model, while Sheffrin (1989) and Drazen (2000) point out some empirical failures. Chappell et al. (1993) conclude that political influences on the Federal Reserve are not clearly connected with a party tenure, since they occur indirectly through the appointments of FOMC members and thus different ideologies overlap in the committee. Faust and Irons (1999) conclude that partisan effects in US macroeconomic data is fragile, and that there is little evidence that the partisan effects on the economy operate through changes in monetary policy.

future. If the conservative central bank does not possess a commitment technology, as in Alesina (1987) or Rogoff (1985), such policy is not possible and the effects of future liberal objectives are stronger. Our analysis suggests that, besides other considerations, credibility or commitment is important to counteract the negative externalities generated by the pressures for liberal policies.

Finally, we model the case where the central bank has complete insulation from external pressures in the short-run, but understands that objectives may change in the future. This can be due, for example, to the fact that the current chairman, at the end of his tenure he can then be replaced by one with different views about the conduct of monetary policy. This model allows us to better understand the key effects arising from possible changes in future objectives.

The paper is organized as follows. Section 3.2, examines a simple model with an analytical solution. In that framework, we discuss the importance of commitment and inflation targeting. In section 3.3, we solve a model where external pressures do not materialize immediately. Section 3.4 examines several alternative scenarios where the basic mechanisms identified are still present. It considers different types of changes in the policy objectives, the strategic use of lagged inflation in a hybrid Phillips curve, and a case with full-commitment and no disagreement. Section 3.5 concludes.

## 3.2 The model

We base our analysis on a simple monetary model. Inflation dynamics are described by a New Keynesian Phillips curve (NKPC). As well known, the NKPC is a reduced form approximation of the relationship between inflation and output in an economy with monopolistic competition and staggered price setting.[3] The NKPC takes the form:

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} \tag{3.1}$$

[3]The theoretical framework underlying such relationship is described in Woodford (2003) and Galí (2008). This specification of the NKPC holds in a neighborhood of a zero inflation steady state. Throughout our analysis, we abstract from the changes that may derive from having a different steady state level of inflation.

where $\pi_t$ denotes price inflation and $y_t$ measures the output-gap, i.e. the difference between current output and the output level that would prevail under flexible prices.

As it is standard in the optimal monetary policy literature, we assume that the central bank controls inflation directly.[4] The objectives of the monetary authority are characterized by a period quadratic loss function taking the following form:

$$U = \frac{1}{2}\left[\pi_t^2 + w(y_t - \widetilde{y}_t)^2\right] \tag{3.2}$$

The monetary policy authority aims at minimizing a weighted average of deviations of inflation and output-gap from their respective targets. The parameter $w$ measures the relative importance of output vs. inflation stabilization. The inflation target is normalized to zero, while $\widetilde{y}_t \geq 0$ represents the (exogenously given) output target. The latter variable indicates that monetary policy aims at correcting some inefficiencies in the economy, like the presence of monopolistic power, distortionary taxes, frictions in the labor market, etc. The parameter $\widetilde{y}_t \geq 0$ can therefore be interpreted as the difference between the efficient level of output and the output that would prevail under flexible prices. This difference generates a trade-off between output and inflation stabilization around their respective targets.

In what follows, we consider that the objectives of the central bank depending on the values assigned to $\widetilde{y}$ and $w$, can be either "liberal" ($\ell$) or "conservative" ($c$). In particular, we use the term "liberal central bank" to refer to a case where the output target and the relative weight to output stabilization are higher than a "conservative central bank", that is $\widetilde{y}^\ell > \widetilde{y}^c$, $w^\ell > w^c$, or both.

### 3.2.1 Modeling policy changes

To keep the problem as simple as possible, we start by considering a case where a conservative central bank is in charge. At any a given point in time, the monetary authority can adopt more liberal objectives. In this simple model, we can derive analytical results that illustrate the main points of our analysis. In section 3.3, we

[4]The interest rate $i_t$ required to implement the desired inflation level can be obtained from the demand side of the economy, not modeled here

consider the case where the central bank faces some insulation from external pressures, and hence policy objectives can change in the future but not immediately.

Consider that the objectives of a conservative central bank are characterized by a relative weight for output $w^c$, and a target for the output gap $\widetilde{y}^c > 0$.[5] At any point in time, with probability $0 < q < 1$ the objectives of the central bank will remain unchanged, while with probability $1 - q$ the central bank will be subject to external pressures and more liberal objectives will be adopted, i.e. setting $\widetilde{y}^\ell > \widetilde{y}^c$, $w^\ell > w^c$ or both.[6]

Moreover, we assume that the central bank can only make credible commitments about future policy while objectives remain unchanged. If objectives do change, the more liberal central bank will set a new policy, and previous commitments will be disregarded. This assumption can be justified on the grounds that if the central bank has received some external pressures to change its policy goals, it will adopt the best possible policy to fulfill its new objectives, and thus disregards the plans made when priorities were different.[7]

In this context, it can be shown that the central bank makes its policy plans to solve the following problem:

$$V^{c,c} = \max_{\{\pi_t, y_t\}_{t=0}^{\infty}} -\frac{1}{2}\sum_{t=0}^{\infty} (\beta q)^t \left\{ \left[\pi_t^2 + w^c (y_t - \widetilde{y}^c)^2\right] + \beta(1-q)V^{c,\ell} \right\} \quad (3.3)$$

$$s.t. \quad \pi_t = \kappa y_t + \beta q \pi_{t+1} + \beta (1-q) \pi_0^\ell \qquad \forall t = 0, 1, .... \quad (3.4)$$

The objective function is given by an infinite sum, discounted at the rate $\beta q$. This reflects the positive probability that objectives will remain unchanged over the infinite future. Each term in the summation is composed by two parts. The first part, in the square brackets, is the period loss function. The second part is the value function $V^{c,\ell}$, summarizing the utility the central bank gets if next period policy objectives change.[8]

[5] The simple case where $\widetilde{y}^c = 0$ is discussed in section 3.2.3.

[6] Our analysis can be easily extended to analyze the symmetric case where a liberal central bank faces the possibility of adopting more conservative objectives. We discuss this case in section 3.4.4.

[7] In section 3.4.3, we analyze the case where the central bank can make credible commitments contingent on its policy objectives.

[8] In the present framework, the function $V^{c,\ell}$ is just a constant, and is irrelevant for the maxi-

**Table 3.1:** Possible settings

| | Same objectives ($w^\ell = w^c$ and $\widetilde{y}^c = \widetilde{y}^\ell$) | Different objectives ($w^\ell > w^c$ or $\widetilde{y}^c > \widetilde{y}^\ell$) |
|---|---|---|
| $q = 1$ | Full Commitment (Objectives never change) | |
| $0 < q < 1$ | Loose Commitment | Probabilistic changes commitment only if no changes |
| $q = 0$ | Discretion | Objectives change in every period commitment not possible |

The central bank faces a sequence of constraints represented by the NKPC, where for any period $t$ inflation expectations are an average between two terms. The first term, with weight $q$, is the inflation that would prevail under the current (conservative) central bank ($\pi_{t+1}$) and upon which there is credible commitment. The second term, with weight $(1-q)$, is the inflation that would be implemented under liberal objectives ($\pi_0^\ell$), which is taken as given by the current central bank. In equilibrium, such level of inflation is determined by solving a symmetric problem to the one described above, using as preferences those of a more liberal central bank.

This framework nests many possible alternative settings, as summarized in table 3.1. For example, if we set $\widetilde{y}^c = \widetilde{y}^\ell$ and $w^\ell = w^c$ we have a typical problem of a central bank, without changes in policy goals. In addition, setting $q = 1$ we have the standard full-commitment case, while by setting $q = 0$, we have the problem of a central bank operating under discretion.

mization problem. The current policymaker does not have a state variable to influence the choices when policy objectives change. This analysis changes in a model with endogenous state variables, like the one considered in section 3.4.2.

### 3.2.2 Effects of changes in the output target $\widetilde{y}$

We start analyzing the case where a change in policy objectives only implies a change in the output target, while keeping unchanged the relative weight to output stabilization ($w^\ell = w^c = w$). While the current conservative objective function prevails, output gap and inflation evolve according to

$$\begin{aligned} \pi_t &= \gamma_2^{-t}\pi_0 && (3.5) \\ y_t - \widetilde{y}^c &= -\frac{\kappa}{w}\frac{1-\gamma_2^{-(t+1)}}{1-\gamma_2^{-1}}\pi_0, && (3.6) \end{aligned}$$

where

$$\pi_0 = \frac{1}{\gamma_2\left(1-\gamma_1\right)}\left(\kappa\widetilde{y}^c + \beta\left(1-q\right)\pi_0^\ell\right) \tag{3.7}$$

and $0 < \gamma_1 < 1 < \gamma_2$ are coefficients satisfying $\gamma_1\gamma_2 = \beta q$ and $\gamma_1 + \gamma_2 = (1 + \beta q + \frac{\kappa^2}{w})$, which are thus independent of the policy objectives.[9] Such coefficients can also be shown to be strictly increasing in $q$.[10]

Equation (3.6) implies that the output gap is always below its target and that such difference increases over time. Inflation is always positive, declines over time and converges asymptotically to zero. In the initial period, the central bank can use inflation to expand output towards its efficient level. To do so, it also needs to keep inflation expectations at a low level by promising to reduce inflation in the subsequent periods. As time passes by, inflation is then reduced and output converges to a permanently and inefficiently low level. The policy plan prescribed by (3.5) and (3.6) is in general time-inconsistent. Whenever the central bank is allowed to review its policies, it has an incentive to surprise the economy and to implement a higher inflation than expected.

From these equations we can observe that, at any point in time, the distance between output gap and inflation from their respective targets can be expressed as increasing functions of the initial level of inflation $\pi_0$. The latter variable, as we

[9] In the definition of $\gamma_1$ and $\gamma_2$ (see equations (A-5) and (A-6) in the appendix), the only parameter related to the policy objectives is the relative weight of output stability $w$, which is assumed to be constant. This assumption is relaxed in section 3.4.1.

[10] See the appendix for the derivation of these results.

can see in (3.7), depends on two factors. First, on the output target of the current policymaker $\widetilde{y}^c$, which is a measure of the amplitude of the distortions in the economy. Second, it depends on the inflation that is implemented if liberal objectives are adopted, $\pi_0^\ell$. This reflects the externality that the possibility of changing the policy objectives generates on the current central bank. In this respect, it is worth noticing the presence of an inflation bias even when $\widetilde{y}^c = 0$, that is in the absence, according to the current objectives, of any trade-off between output and inflation stabilization. This particular case is discussed in details in section 3.2.3. Moreover, we must emphasize that equation (3.5) holds independently of the specific institutional setting (in terms of credibility of the central bank, duration of the tenure, etc.) prevailing once the liberal objectives are adopted. The only factor that matters is the initial level of inflation $\pi_0^\ell$ implemented in such circumstance. For convenience, we assume that once the change in objectives occurs, the central bank faces a symmetric problem to the one described above.[11]

We first study the effects of limited commitment alone, assuming that policy goals cannot be changed, but that the monetary authority is not fully credible. In particular, we consider that at any given point in time a reoptimization may occur with probability $0 < (1-q) < 1$, but the objectives of the central bank will be the same, i.e. $\widetilde{y}^c = \widetilde{y}^\ell$. As shown by Schaumburg and Tambalotti (2007) the optimal path of inflation under limited commitment is given by:[12]

$$\pi_t^{LC} = \left(\frac{\bar{\gamma}_2}{\gamma_2}\right)^t \frac{\bar{\gamma}_2 - \beta}{\gamma_2 - \beta}\bar{\pi}_t, \tag{3.8}$$

where $\bar{\pi}_t$ is the level of inflation prevailing under the assumption of full commitment and no uncertainty about policy objectives, as derived in the appendix. Since $\gamma_2 < \bar{\gamma}_2$ we have that in any period, $\pi_t^{LC} \geq \bar{\pi}_t$. This means that limited commitment introduces an inflation bias. Since $\frac{\partial \gamma_2}{\partial q} > 0$, such bias is decreasing in the probability of commitment q.

We can now turn to the main part of our analysis, to study the effects of the possibility that policy goals may change. In this context, at any point in time and

[11]The previous assumption can be easily relaxed without affecting qualitatively our results.
[12]See the appendix for the derivation of these results.

with probability $1 - q$, the central bank may adopt more liberal objectives, thus setting $\widetilde{y}^{\ell} > \widetilde{y}^{c}$. It can be shown that in that case

$$\pi_t = \underbrace{\left(\frac{\bar{\gamma}_2}{\gamma_2}\right)^t \frac{\bar{\gamma}_2 - \beta}{\gamma_2 - \beta}}_{\substack{\text{Limited Commitment} \\ > 1}} \quad \underbrace{\frac{\widetilde{y}^c + \Phi \widetilde{y}^{\ell}}{\widetilde{y}^c (1 + \Phi)}}_{\substack{\text{Liberal Objectives} \\ > 1}} \bar{\pi}_t \tag{3.9}$$

where $\Phi \equiv \frac{\beta(1-q)}{\gamma_2(1-\gamma_1)} = \frac{\beta - \beta q}{\gamma_2 - \beta q} < 1$ and $\frac{\partial \Phi}{\partial q} < 0$.

When more liberal objectives may be adopted in the future, two different forces affect current inflation, independently of whether such changes ultimately occur or not. The first is related to the reduction in credibility associated with the possibility of a change in policy objectives. This is represented by the first fraction on the left-hand side of equation (3.9), which is the same as in (3.8). The second force is related to the adoption of more liberal objectives, as represented by the second fraction on the left-hand side. We can thus conclude that the possibility that policy objectives become more liberal introduces an additional inflation bias with respect to the limited commitment case. Such additional bias can be shown to be increasing in the difference between $\widetilde{y}^c$ and $\widetilde{y}^{\ell}$ and in the probability of policy changes $(1 - q)$.[13] In other words, the more liberal the alternative objectives are, the more likely is the change, the higher is current inflation. Moreover, higher inflation is associated with a reduction in output. This can be clearly seen in equation (3.6), a higher initial inflation ($\pi_0$) lowers output in all periods.

The main message of our analysis is twofold. First, consider an economy experiencing a low productivity level or a recession. In that case, discussing the adoption of more liberal monetary policy objectives is detrimental for the economy. It generates inflationary pressures and it exacerbates the recession. Second, when there is the possibility that more liberal objectives are adopted, even the most conservative central bank should accommodate the inflationary pressures to avoid a stronger recession. In other words, the fact that inflation is higher in a given period should not be necessarily interpreted as a signal that a more liberal monetary policy is being implemented.

[13] For the derivation of these results see the appendix.

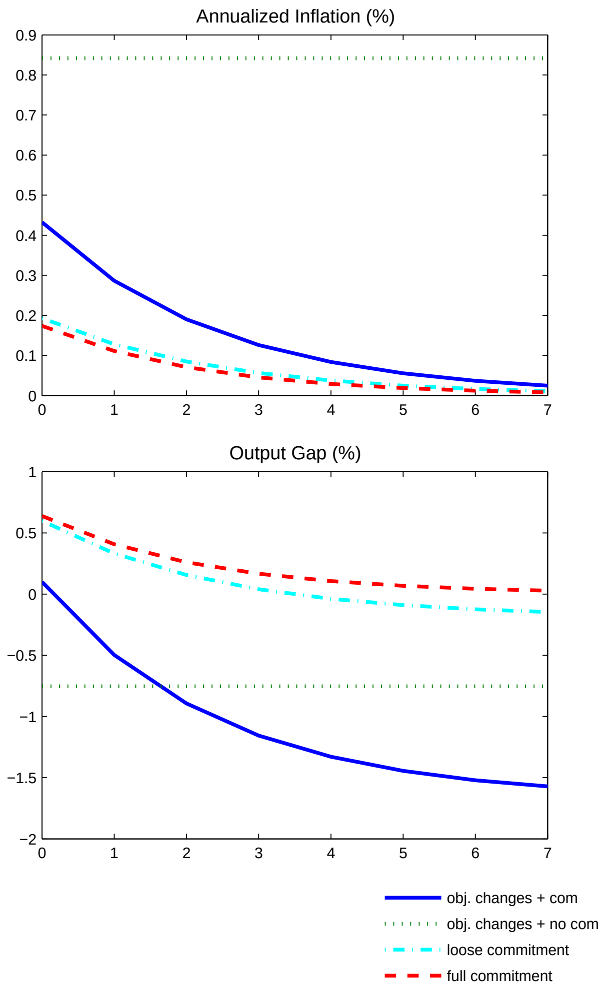
It can be just the optimal reaction of a conservative central bank to the externalities generated by the possibility that liberal policies prevail in the future.

In order to give a quantification of the effects related to the uncertainty about policy objectives, in figure 3.1 we plot the optimal path of inflation and output gap while conservative objectives prevail. We assume that the central bank has an output target $\widetilde{y}^c = 0.01$ and that with probability $1 - q = 10\%$ it will adopt more liberal objectives, i.e. setting $\widetilde{y}^\ell = .1$. For the remaining parameters, we used a standard quarterly calibration of the model, with $\beta = .99$, $\kappa = .1$, and $w = .048$. For comparison, we also plot the allocations prevailing under full-commitment and under limited commitment. When there is the possibility that more liberal objectives are adopted (solid line), in the first period there is a 0.25% increase in (annualized) inflation and a .5% reduction in output with respect to the standard case with full-commitment and no possibility of policy changes (dashed line). More importantly, the reduction in output is even bigger at a longer horizon, reaching a level of about 1.5%. From the picture, it is also clear that most of the effects are due to the changes in objectives, rather than to the associated loss of credibility, measured by the difference between the full-commitment (dashed line) and the limited commitment case (dashed-dotted line).

We can conclude that the mere possibility that more liberal policies can be adopted in the future, even if this ultimately never occurs, induces an increase in inflation in the short-run and a significant and permanent reduction in output. The intuition for this result goes as follows. Other things equal, the possibility of having more liberal policies in the future increases inflation expectations. As a consequence, on the one hand the central bank increases current inflation, thus accommodating the higher inflation expectations in the effort to limit the reduction in output. On the other hand, the central bank promises to reduce inflation in the future, for the case the conservative objectives still prevail.[14] In this way, it reduces the inflation expectations, limiting the negative externality generated by the possibility that policy objectives

[14]These two effects, namely the accommodation effect and the anchoring expectations effect, can be better disentangled in a framework where policy objectives can change only occasionally, say every $T$ periods, as the one considered in section 3.3.1.

**Figure 3.1:** Adoption of liberal objectives: Optimal Inflation and Output under a conservative central bank



become more liberal. This means that as time passes by, inflation will not be used to foster production, and thus output will be lower.

Finally, we want to understand how the credibility of the central bank matters when policy objectives can change. To do so, we compare two alternative settings. The first is the one described above, where the central bank can make credible promises regarding the policies that will be implemented if the objectives remain unchanged. The second is a setting where, by assumption, the central bank is not credible at all, no matter whether the policy objectives may change or not. The latter is the standard approach followed in the political economy literature, like e.g. in Alesina (1987). In that case, we have that inflation ($\pi^{NC}$) and output ($y^{NC}$) are constant over time and given by

$$\pi^{NC} = \frac{1}{1+\frac{\kappa^2}{w}-\beta q}\left[\kappa \widetilde{y}^c + \beta\left(1-q\right)\pi_0^{\ell}\right] \tag{3.10}$$

$$y^{NC} - \widetilde{y}^c = -\frac{\kappa}{w}\pi^{NC} \tag{3.11}$$

In order to compare the two alternative settings, we combine (3.5) and (3.7) and divide the resulting expression by (3.10), considering that the inflation implemented with liberal objectives ($\pi^{\ell}$) is the same in the two cases. We then obtain that

$$\frac{\pi_t}{\pi^{NC}} = \gamma_2^{-t}\frac{1+\frac{\kappa^2}{w}-\beta q}{\gamma_2-\beta q} < 1 \tag{3.12}$$

where the last inequality follows from the fact that $\gamma_2 > 1+\frac{\kappa^2}{w}$. Moreover, dividing (3.6) by (3.11) we have

$$\frac{y_t-\widetilde{y}^c}{y^{NC}-\widetilde{y}^c} = \frac{1-\gamma_2^{-(t+1)}}{1-\gamma_2^{-1}}\frac{\pi_t}{\pi^{NC}}, \tag{3.13}$$

which, at least for $t=0$, is smaller than 1, implying $y_0 > y^{NC}$.[15]

The difference between the two alternative settings is also shown in figure 3.1. In a context where policy goals can be changed, when the central bank can commit (solid line) inflation in the first period ($t=0$) is approximately half of the value prevailing when commitment is ruled out (dotted line).

[15]To see this result notice that, from 3.11, $y^{NC}-\widetilde{y}^c$¡0.

Our analysis suggests that when policy objectives can change, having credibility is important. It allows to implement a lower inflation and, at least in the short-run, a higher output. This is because a credible central bank can keep inflation expectations relatively low by promising to lower inflation in case liberal objectives are not adopted. In other words, a credible institution can counteract more efficiently, the inflationary pressures arising from the possibility of adopting more liberal objectives.

To summarize, the possibility of adopting more liberal objectives generates the following effects on the conservative central bank:

- In any period, it generates an inflation bias and a lower output, with respect to the case with full-commitment and constant objectives.

- The previous effects are due in part to the loss of credibility, and in part to the change in objectives, as described in (3.9). The latter seems quantitatively more important.

- The more liberal are the policies (i.e. the higher is $\widetilde{y}^{\ell}$) and the more likely is the change in objectives (lower $q$), the higher is the inflation bias.

- The higher is the inflation bias, the bigger is the associated reduction in output, as described by (3.6).

- When there is the possibility policy goals can be changed, credibility of the central bank allows to keep inflation and output gap closer to their targets, as described in equation (3.12) and (3.13).

### 3.2.3 Discussion: commitment, the conservative central banker and inflation targeting

The optimal monetary policy literature has proposed many ways to limit the time-inconsistency problem. In a remarkable contribution, Rogoff (1985) suggested to appoint a conservative central bank that is more averse than society towards inflation.[16] Rogoff shows that the conservative central bank, even operating under discretion, can implement the policy of a benevolent planner under full commitment, i.e. the best possible policy. As a result, it may be concluded that the credibility of a central bank may be of little importance, as long as its degree of inflation aversion is high enough.

We have shown previously, that when there is the possibility that objectives may change, if the conservative central bank does have some commitment then it can achieve more favorable allocations. We can further illustrate this point in the following example. Consider an (extreme) case where the conservative central bank has a target for the output gap $\widetilde{y}^c = 0$.[17] In this case, the central bank perceives that there are no distortions in the economy to be corrected through the use of inflation. As a consequence, if policy objectives cannot be changed, the central bank implements the allocation $\pi_t = 0$ and $y_t = 0$ in all periods, no matter what is the degree of commitment. In this context, appointing such a conservative central bank is equivalent to the adoption of a strict inflation targeting policy. The interesting question is to understand what happens if such a conservative central bank faces the possibility that future objectives can change to $\widetilde{y}^\ell > 0$.

As suggested from our discussion in the previous section, and as can be seen in figure 3.2, the conservative central bank now implements a positive inflation and a negative output gap.[18] The figure also plots the policy where the conservative can make credible commitments (solid line), and the policy where credible commitments are ruled out (dotted line). The difference between the two lines indicates that introducing credible commitments helps significantly in keeping inflation and output close to their targets.

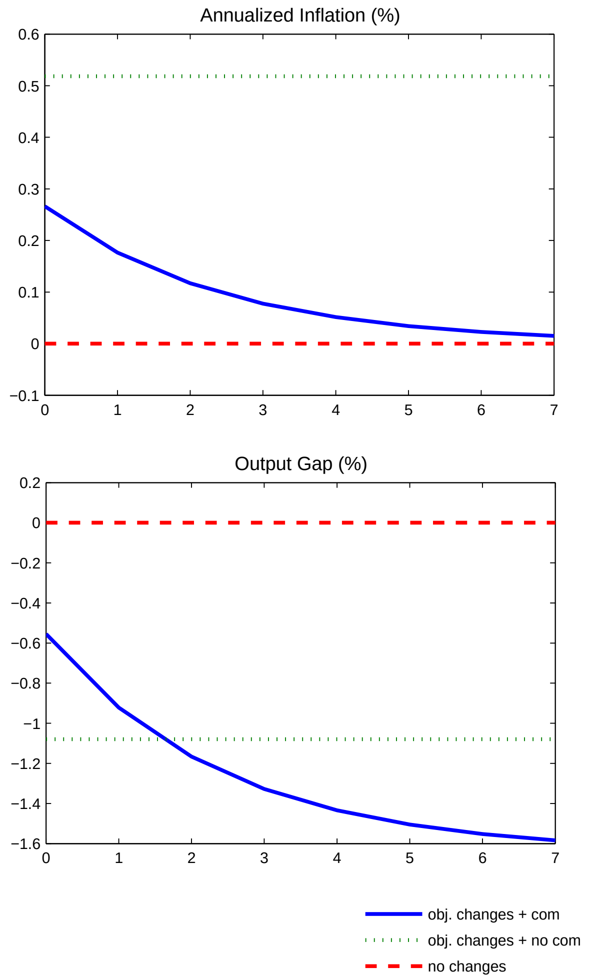
Even independent and conservative central banks, may be subject to some external pressures. Our work emphasizes that even a Rogoff conservative central banker, or

[16] Walsh (1995) and Svensson (1997) also suggested alternative ways to solve the time-inconsistency problem.

[17] Our result is still valid if the conservative central banker is one assigning zero weight on output stabilization ($w = 0$) as long as cost-push shocks are added to the NKPC.

[18] Figure 3.2 plots inflation and output gap for the case of $\widetilde{y}^c = 0$ and $\widetilde{y}^\ell = 0.1$, while keeping all the other parameters as in the previous section.

**Figure 3.2:** Changes in objectives and Inflation targeting: Optimal Inflation and Output



a central bank adopting an inflation targeting policy, can benefit from commitment. Besides other considerations, commitment is important to counteract the effects of the possibility of changes in policy objectives.

Another interesting point in our framework, is that the probability that objectives change in the future matters. In this respect, we can emphasize the difference between appointing a conservative central bank and implementing an inflation targeting regime. A conservative central bank puts more weight on inflation, and therefore can be equivalent to an inflation targeting regime, as in the example just shown. However, if an inflation targeting regime has been implemented, changing policy objectives is likely to be harder. It requires institutional reforms that are usually costly and lengthy. When a conservative central bank is in office, without a clearly specified target, changing policy objectives is easier. It is enough that the next chairman is not as conservative as the current one. Therefore, implementing an inflation targeting regime or appointing a conservative central bank is not equivalent. The two cases imply different probabilities that objectives change in the future, which immediately affects current outcomes.

## 3.3 A model with periodic objective changes

There are several examples of interference in central bank policy in OECD countries. Nevertheless, it may not be entirely plausible to assume that external pressures occur continuously, or that the objectives of the central bank can be changed immediately. In this section, we add several features that make the model more realistic. We consider the case where the conservative central bank knows that it will decide the monetary policy course for at least $T$ periods. A possible interpretation of this setting is that the central bank is independent and the chairman or the members of the board of advisors are in charge for $T$ periods. At the end of their tenure, policy objectives can change as long as officers with different views are appointed.

Both the conservative central bank and private agents know that in $T$ periods the current conservative objectives can persist (with probability $q$) or give place to liberal objectives (with probability $1 - q$). In the latter case, for simplicity, we assume the central bank will face a symmetric problem, where liberal objectives are unaltered for $T$ periods, and then can change with probability $1-q$. The model in this section, besides being more realistic, allows us to better understand how the conduct of monetary policy is affected.

As before, the central bank can commit to a plan as long as the objectives remain unchanged. In this setting, analytical solutions are not available, and one needs to use the tools of shown in the previous chapters to solve the model numerically.[19] In the present framework, the problem of the conservative ($c$) or liberal ($\ell$) central bank is the following. Taking as given the sequence of policy $\{\pi^j_{(m+1)T-1}\}_{m=0}^{\infty}$ and the value function $V^{ij}$, with $j \neq i$, the problem of the central bank can be written as:

$$V^i = \max_{\{\pi_t, y_t\}_{t=0}^{\infty}} E_0 \sum_{m=0}^{\infty} \left(\beta^T q\right)^m \left[ -\frac{1}{2} \sum_{t=0}^{T-1} \beta^t \left[\pi^2_{m+t} + w^i (y_{m+t} - \widetilde{y}^i)^2\right] + \beta^T (1-q) V^{ij} \right] \tag{3.14}$$

$$s.t. \;\; \pi_{mT+t} = \kappa y_{mT+t} + \beta E_{mT+t}(\pi_{mT+t+1}) \qquad t = 0, 1, ..., T-2 \tag{3.15}$$

$$\pi_{mT+t} = \kappa y_{mT+t} + (1-q)\beta E_{mT+t}(\pi^j_{mT+t+1}) + q\beta E_{mT+t}(\pi^i_{mT+t+1}) \quad t = T-1 \tag{3.16}$$

$$\forall \; m = 0, ..., \infty$$

where $m$ indexes the number of tenures, each lasting for $T$ periods. The objective function reflects the institutional setting the central bank faces. At the end of any tenure (T periods where objectives can not change), current objectives ($i$) can remain unaltered with probability $q$. This history is summarized in the outer summation. Within each tenure, plans are made for $T$ periods, as indicated in the inner summation. Finally, the central bank ($i$) internalizes that with some probability $(1-q)$, at the end of tenure the objectives will be of type ($j$). In this case, central bank ($i$) will

[19] The present work combines the tools developed in chapter 1 for the probabilistic model and the T-periods model. The probabilistic model was also addressed in Roberds (1987) and Schaumburg and Tambalotti (2007). We also use features of chapter 2, where we considered disagreement among successive policymakers.

get the loss function $V^{ij}$. More formally, define the sequence $\{\pi_t^i, y_t^i\}_{t=0}^{\infty} \equiv \arg\max V^i$ $\forall i = \ell, c$. The value function $V^{ij}$, $\forall i = \ell, c$ and $j \neq i$ is given by

$$V^{ij} \equiv E_0 \sum_{m=0}^{\infty} \left(\beta^T q\right)^m \left[ -\frac{1}{2} \sum_{t=0}^{T-1} \beta^t \left[ {\pi_{m+t}^j}^2 + w^i (y_{m+t}^j - \widetilde{y}^i)^2 \right] + \beta^T (1-q) V^{ji} \right] \quad (3.17)$$

The sequence of constraints (3.15) and (3.16) also reflects the institutional setting. Within any tenure $m$, we can divide the constraints into two groups, depending on how inflation expectations are formed. In the periods $t = 0, ..., T-2$, inflation expectations internalize that in the next period objectives do not change. In the last period of the tenure $(T-1)$, agents recognize that with some probability $(q)$ objectives do not change, while with probability $(1-q)$ objectives do change. We employ the following definition of equilibrium:

**Definition 1** *A Markov Perfect Equilibrium with objective changes must satisfy the following condition. For any $i = \ell, c$ and $j \neq i$, given the sequence $\{\pi_{mT+t}^j, y_{mT+t}^j\}_{t=0}^{\infty}$, $\forall m = 0, ...\infty$*

1. The value function $V^{ij}$ satisfies equation (3.17).

2. The sequence $\{\pi_{mT+t}^i, y_{mT+t}^i\}_{t=0}^{\infty}, \forall m = 0, ...\infty$ solves (3.14) subject to (3.15) and (3.16).

In order to solve problem (3.14), we first write its recursive formulation. To do so we apply the technique of Marcet and Marimon (1998), and we write the problem as a saddle point functional equation that generalizes the usual Bellman equation. The proof of that result requires considering each tenure as one big period, and then applying the results of chapter 1 to address the probabilistic switch at the end of each tenure. Proposition 1 in the appendix proves this result in detail. As stated in Proposition 2 in the appendix, we can then characterize the policy functions solving our problem as tenure invariant functions of the Lagrange multipliers associated with the constraints (3.15) and (3.16). We are not claiming that the policy functions are time-invariant. Indeed, the policy functions change in the different periods within a tenure.

In order to solve our problem we have to find, for both central banks ($i = \ell, c$), the policy functions satisfying the equilibrium conditions stated above. In particular, for each central bank, we need to find as many policy functions as the number of periods within each tenure ($T$). Moreover, as can be seen in (3.16), the policies of central bank $j$ enter the problem of central bank $i$ (and viceversa). This implies that we have to solve a fixed point problem in such policy functions. In addition, the implied value functions $V^{ij}$ and $V^{ji}$ also enter the problem and need to be solved for endogenously.[20] We also have to take into account the possibility of default on past promises, an event that occurs whenever there is a change in objectives. This significantly complicates the analysis, since both the levels and the derivatives of the policy functions enter the first-order conditions of the problem.

Regarding the number of periods where objectives remain unchanged ($T$), and the probability that objectives change, we could have considered several calibrations. We considered that central bank objectives remain unchanged with certainty for four periods ($T = 4$), at that point there is a probability of that objectives change equal to $q = 0.5$. This calibration is convenient for illustrative purposes.[21]

### 3.3.1 Results

We first consider the case of disagreement in the output gap target level. Figure 3.3 plots the optimal policy functions in the case where the conservative central bank may be temporarily replaced by a liberal one. One can easily translate the policy functions to certain realizations of events. The conservative central bank implements the policy functions shown in the figure until policy objectives are changed. When such change occurs, the liberal policy is implemented until objectives become conservative again. In this model, the possibility that objectives change is only present every four periods - those periods are signaled in the pictures with continuous vertical lines. Therefore, once it is know that the liberal policies are not implemented, the conservative central

[20]Note that $V^{ij}$ and $V^{ji}$ are value functions under disagreement, therefore unlike in chapter 1 one can not use an envelope result.

[21]This calibration implies that objective changes on average every 8 periods, roughly the same as in the case considered in the previous section.

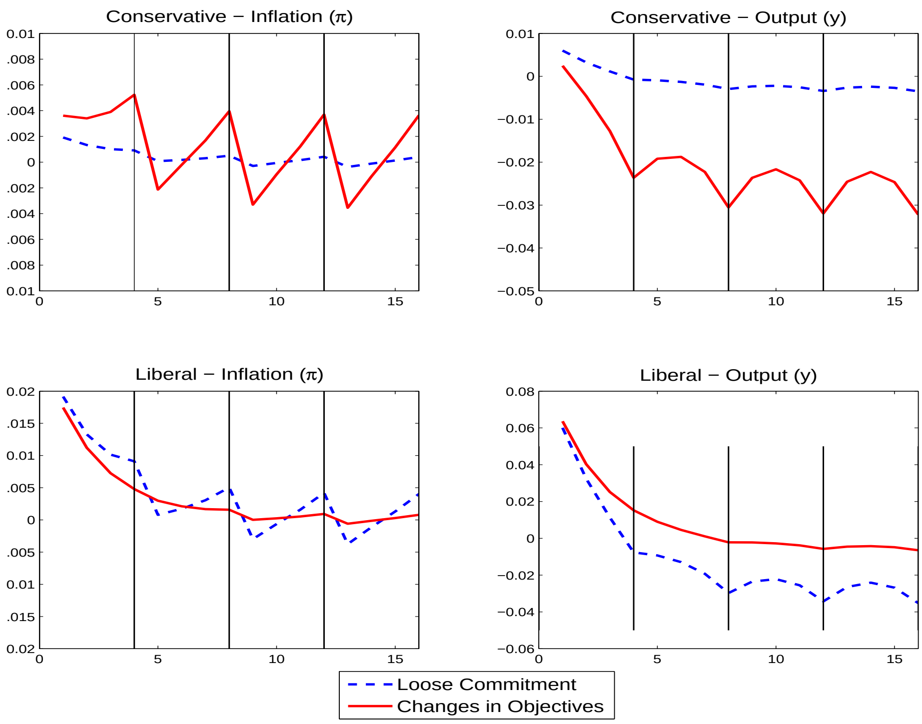
bank is insulated from any pressure for four periods. This feature of the model tries to capture the fact that in reality political pressures may not be able to change objectives immediately. The upper panel plots the policy functions of the conservative central bank for inflation and output, the lower panel refers to the liberal policy functions.

In figure 3.3, we also plot the policy functions that occur in a loose commitment setting, but where objectives do not change. Loose commitment refers to the fact that the central bank occasionally disregards previous commitments and makes a new plan. In such case, the policy functions with the same objectives are implemented from the beginning until the next reoptimization occurs. The differences between the policy functions with loose commitment and with changes in objectives are only due to the possibility that objectives change. In both cases, there is a common loose commitment feature.

The possibility that a liberal central banker be appointed in the future affects the optimal policy functions of the conservative central bank in several ways. When the conservative central bank starts (periods 1 to 4), inflation is now higher and output is lower. This is due to an accommodation effect. The possibility that the liberal policies with high inflation are implemented in the future affects current outcomes through inflation expectations. High inflation expectations either imply higher current inflation or lower output. The optimal policy of the conservative implies a combination of higher inflation and lower output.

We also observe that the conservative central bank implements a low inflation level immediately after knowing that the pressures to adopt liberal policies have dissipated (periods 5, 9, 13,...), and that objectives will not change in the following four periods. The rationale of this policy is to anchor inflation expectations. When it is known that in the next period a liberal policy may be implemented, inflation expectations increase. In order to keep inflation expectations low, the conservative central bank finds it optimal to promise to reduce inflation if objectives remain unchanged. This promise regarding future policy affects beneficially the current period through inflation expectations. Note that this promise is extremely time-inconsistent, if there is not a change in objectives, the conservative central bank needs to fulfil its promise of implementing a very low inflation level. This case exemplifies how commitment is

**Figure 3.3:** Model with Occasional Changes in Objectives




used to balance distortions across time and states of nature.

Finally, note that the first period of the conservative central bank (period 1) is fundamentally different from any first period after being confirmed in office (periods 5, 9, 13,...). As we had explained before, periods 5, 9, 13 are characterized by low inflation that was promised in the previous period. In the current setting, we are assuming that past promises are not binding in period 1 because the conservative central bank was not in office before. This is the reason why inflation is relatively high in period 1, when compared, for instance, with period 5.

Summarizing our results, we have found that the possibility that liberal policies are implemented creates three effects in the policies of the conservative. First, the conservative central bank needs to raise inflation to accommodate inflation expectations - the accommodation effect. The accommodation effect is higher the closer is the period where liberal policies may take over. Second, to counteract high inflation expectations, the conservative promises to implement a low inflation if the objectives do not change - the anchoring effect. The anchoring effect only materializes when liberal objectives are not adopted. The combination of the accommodation and anchoring effect explains why the conservative starts with low inflation and then increases it. In the model with one period tenure ($T = 1$), the strength of the anchoring and accommodation effect did not change over time. And in that case, we proved analytically that the possibility that liberal objectives be adopted induced the conservative to implement higher inflation in every period. Third, the conservative experiences lower output due to the possibility that liberal policies are implemented. This result may come as surprising. If a liberal sector of the economy thinks that current output is too low, and makes pressures for changing the central bank policy, then the outcome is that the economy experiences an even lower output. If the liberal policies are implemented in a later period, then the economy will experience an expansion. However, as long as the objectives do not change, the economy experiences a recession, the opposite outcome of what the pressures for liberal policies may be aiming.

Table 3.2 reports welfare and the average allocations. The average inflation that the conservative implements is higher relative to the loose commitment case where the conservative is never substituted by a liberal policy. As expected, the overall output average when conservative and liberal objectives coexist is higher than the case where the conservative objectives are unchallenged. But, as we cautioned before, the conservative central bank experiences a lower output due to the pressures of liberal policies. Hence, as long as the central bank's objectives do not change, there is a negative effect on output. In terms of welfare, pressures to adopt liberal objectives create a negative externality on the conservative. Even though, overall output is slightly higher, inflation is further away from target.

In a partisan theory of output and inflation, Alesina (1987) considered a classical Phillips curve where current inflation surprises affect current outcomes. In that context, the possibility of a future change in policy does not affect current outcomes.

**Table 3.2:** Inflation and Output in a Model with Occasional Changes in Objectives

| | Changes in Objectives | | | Loose Commitment | |
|---|---|---|---|---|---|
| | Average with $c$ | Average with $\ell$ | Overall | $c$ | $\ell$ |
| $\pi$ | 0.2254 | 0.57 | 0.3968 | 0.0723 | 0.7226 |
| y | -1.6888 | 1.7822 | 0.0382 | 0.0083 | 0.083 |
| welfare | -0.0036 | -0.0272 | . | -0.0003 | -0.0302 |

Note: The table reports the average allocations across different simulations of the model.

For instance, the possibility that a liberal policy is implemented in period 4, 8, 12 has no consequences on the economy and the optimal policy functions in any other periods. Here, we consider instead a (standard) New Keynesian Phillips curve, where future conditions also influence current outcomes. Therefore, our work has very different mechanisms from the analysis considered in Alesina, where for instance the accommodation effect is simply absent. In addition, Alesina assumed that the central bank acts with discretion in every period. Given the developments in central bank commitment, we are assuming that there is commitment to policies aimed at maximizing the same objectives. Since the anchoring effect found in our model is based on a commitment, that effect is also not present in Alesina's model.

## 3.4 Alternative scenarios

In this section, we consider different alternative scenarios where a liberal and conservative objectives coexist. We consider different objectives on inflation stabilization, a hybrid Phillips curve, and a full commitment cooperative setting. In all these cases, the main intuition presented in the baseline case still holds. We finally discuss the reverse scenarios that we examined, the case where future policy objectives may become more conservative.

### 3.4.1 Changes on the relative weight of inflation stabilization

Consider the case where both conservative and liberal objectives agree on the output target level, but disagree on the importance of inflation stabilization. In particular, we assume $\widetilde{y}^c = \widetilde{y}^\ell = \widetilde{y}$ and $w^c < w^\ell$. In this context, if policy objectives can change in every period ($T = 1$) with probability $1 - q$, we can show that inflation evolves according to

$$\pi_t = \underbrace{\frac{(1+\Phi^\ell)(1-\Phi^c)}{1-\Phi^c\Phi^\ell}}_{\substack{\text{Liberal Objectives} \\ > 1}} \underbrace{\left(\frac{\bar{\gamma}_2}{\gamma_2}\right)^t \frac{\bar{\gamma}_2 - \beta}{\gamma_2 - \beta}}_{\substack{\text{Limited Commitment} \\ > 1}} \bar{\pi}_t,$$

where $\Phi^c \equiv \frac{\beta(1-q)}{\gamma_2^c\left(1-\gamma_1^c\right)} = \frac{\beta-\beta q}{\gamma_2^c-\beta q} < \Phi^\ell \equiv \frac{\beta(1-q)}{\gamma_2^\ell\left(1-\gamma_1^\ell\right)} = \frac{\beta-\beta q}{\gamma_2^\ell-\beta q} < 1$.

Consistent with our analysis in section 3.2.2, we can conclude that pressures to increase the relative weight of output stabilization generates an inflation bias. This bias increases with the probability of a policy change $(1 - q)$.

We provide a quantitative example, for the case where the tenure of the central bank lasts for $T = 4$ periods. More specifically, we consider that $\widetilde{y}^c = \widetilde{y}^\ell = 0.03$, $w^c = 0.048$ and $w^\ell = 0.48$. This case is also plausible since it has been argued that the full-commitment microfounded calibration of $w$ is much lower than what policy makers often implement in practice. Figure 3.4 presents the policy functions and Table 3.3 presents the average allocations in the economy. All the results and intuition that we mentioned previously remain unaltered.

### 3.4.2 Hybrid Phillips curve

In this section, following Galí and Gertler (1999), we consider the possibility that the Phillips curve may also include a backward looking term. The hybrid Phillips curve takes the form

$$\pi_t = \kappa y_t + (1-\alpha)\beta E_t \pi_{t+1} + \alpha\beta\pi_{t-1}. \tag{3.18}$$

**Figure 3.4:** Alternative Scenario: different weights of inflation stabilization

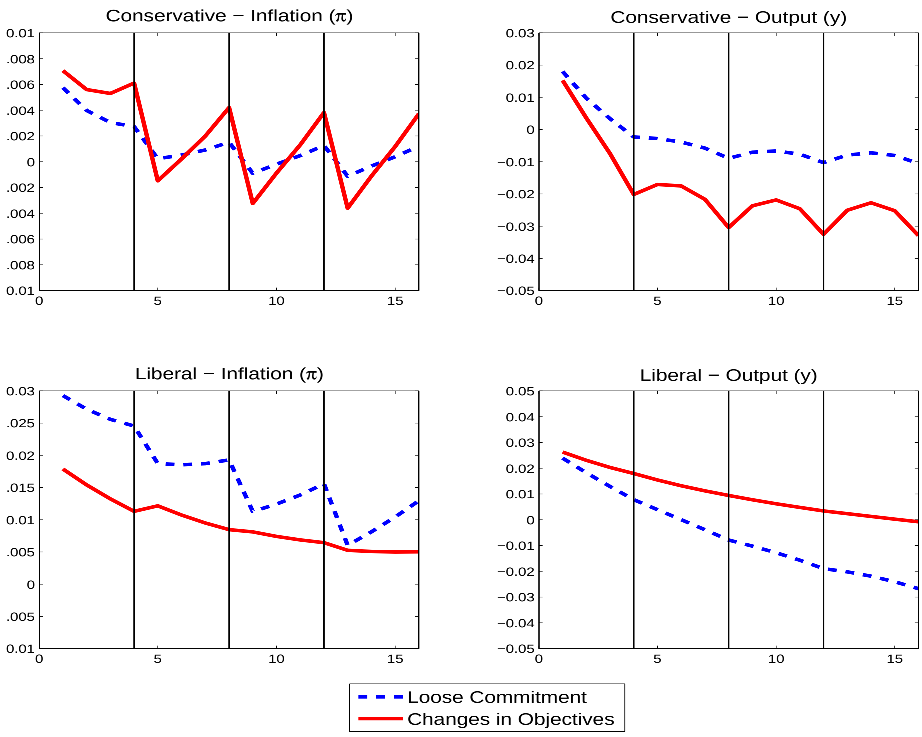


**Table 3.3:** Inflation and Output - Changes in w

| | Changes in Objectives | | | Loose Commitment | |
|---|---|---|---|---|---|
| | Average with $c$ | Average with $\ell$ | Overall | $c$ | $\ell$ |
| $\pi$ | 0.336 | 1.1208 | 0.7265 | 0.2168 | 2.0627 |
| y | -1.3006 | 1.4567 | 0.0713 | 0.0249 | 0.2137 |
| welfare | -0.007 | -0.0329 | . | -0.0027 | -0.0483 |

Note: The table reports the average allocations across different simulations of the model.

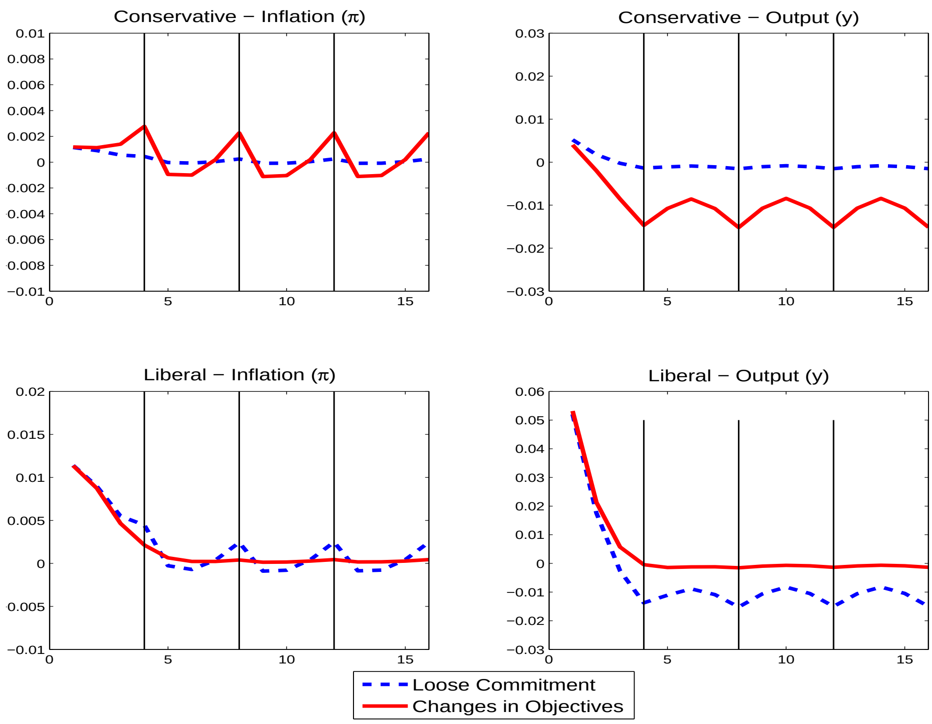
The presence of lagged inflation introduces a state variable in the model.[22] The presence of such state variable is relevant for our analysis, since it allows the central bank to influence strategically the future decisions. Whether there is a change in objectives or not, current policy will be affected by the past inflation level. Therefore, the central bank can strategically choose an inflation to influence next period decisions, even if objectives change.

We do not have the goal to characterize policy with indexation, where the functional forms of policy objectives may differ from the ones considered here. For direct comparability purposes, we leave the the central banks' objective functions as in the baseline scenario of section 3.3. This allows us to understand whether the presence of inflation as a state variable changes the interaction patterns between the conservative and liberal policies. Another important point is that the model with the hybrid Phillips curve actually alleviates the time-inconsistency problem. This is because the weight on expected future inflation is smaller. Since a reoptimization will now imply an inflation level that is not as high as before, the conservative central bank necessity to accommodate inflation expectations is not so strong.

In accordance with the empirical evidence in Galí and Gertler (1999), we set $\alpha$ to be 0.3. Figure 3.5 plots the policy functions.[23] The qualitative features of the effects of the possibility of adopting liberal objectives are the same as in the baseline case. Table 3.4 reports the relevant statistics. The table mainly confirms the results

[22]Under this specification, propositions 1 and 2 in the appendix also apply.
[23]We have set initial inflation to be 0. Considering other values does not change the results.

**Figure 3.5:** Alternative Scenario: Hybrid Phillips Curve



explained previously.

**Table 3.4:** Inflation and Output - Hybrid Phillips Curve

| | Changes in Objectives | | | Loose Commitment | |
|---|---|---|---|---|---|
| | Average with $c$ | Average with $\ell$ | Overall | $c$ | $\ell$ |
| $\pi$ | 0.0878 | 0.3567 | 0.2216 | 0.0404 | 0.4038 |
| y | -0.8503 | 0.9003 | 0.0207 | 0.0045 | 0.0449 |
| welfare | -0.0017 | -0.0253 | . | -0.0002 | -0.0264 |

Note: The table reports the average allocations across different simulations of the model.

### 3.4.3 A full commitment solution

For the sake of realism, we assumed an intermediate level of commitment where the central bank could make binding promises as long as external pressures do not alter policy objectives. This feature reflects the inherent disagreement on different policy objectives. In this section, we want to analyze a setting where the central bank has full commitment, even though the policy objectives may change over time. In this framework, it does not make sense to consider that there is disagreement about policy objectives. Instead, the central bank realizes that policy objectives may change over time, but does not disagree with those objectives when they change. One interpretation of this framework is that the structural parameters of the economy, like the degree of nominal rigidities and the degree of firms' monopolistic power, evolve stochastically, thus changing the magnitude of the distortion the central bank aims to correct and the effectiveness of its policy.[24] Another interpretation of this setting is that the central bank itself is subject to preferences shocks.

In the baseline setting described in section 3.3, when external pressures succeed in altering the policy objectives policy is reset to achieve the new objectives disregarding

[24]This interpretation of the model is partly related to the literature on robust control of Hansen and Sargent (2007), and to the literature about optimal monetary policy in the presence of noisy indicators as in Aoki (2003). In our analysis, however, we focus on the effects of evolving objectives, and assume the structural relationships describing the economy and the exogenous shocks are known and common knowledge.

previous objectives. In contrast, in the model of this section, policies are chosen to maximize the overall welfare. In fact, since there is no disagreement, we can think that there is a unique policymaker taking decisions.

More specifically, we assume the central bank's policy goals can be $\widetilde{y}^{\ell} = 0.1$ or $\widetilde{y}^{c} = 0.01$. For convenience, we still refer to the terminology central bank $\ell$ or $c$ when the current output gap is respectively $\widetilde{y}^{\ell}$ or $\widetilde{y}^{c}$. Every four periods ($T = 4$) the objectives remain unchanged with probability $q = 0.5$, while with probability $1 - q$ the objectives do change. The problem of the central bank can be written as:

$$V(\widetilde{y}_0) = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + w(y_t - \widetilde{y}_t)^2\right]$$

$$s.t. \quad \pi_t = \kappa y_t + \beta E_t \pi_{t+1}$$

where expectations are taken with respect to the variable $\widetilde{y}$. This problem can be written recursively by considering a tenure as a unique period and then applying standard dynamic programming techniques.

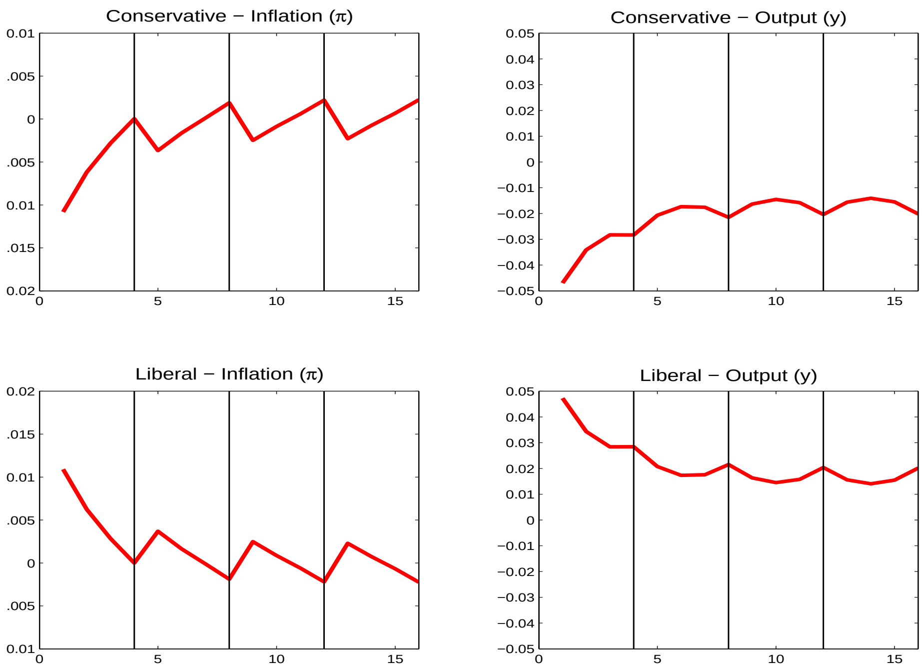
Figure 3.6 plots the policy functions for both types of objectives.[25] The policy functions of the conservative are extremely similar to the other cases considered previously. There is an accommodation effect, which becomes more visible the closer is the period where the liberal policies may emerge (periods 4, 8, 12 in the graph). In addition, whenever the objectives do not change, and thus the conservative objectives are kept (periods 5, 9, 13), inflation is reduced due to the anchoring effect. Also, as before, the conservative central bank experiences lower output due to the presence of the liberal.[26]

The main difference in the conservative policy between this model and the baseline one in section 3.3 is in the initial period. The main reason is that in this model past

[25] In the model of section 3.3.1, when policy objectives are changed the lagrange multiplier is reset to zero. Afterwards, the policy functions depend on the evolution of the lagrange multiplier. There, we plotted the policy functions depending on the time spent in office, because there is a unique mapping between the evolution of the lagrange multiplier and the time spent in office. In the setup of this section, the lagrange multiplier is never set to zero. Therefore, even when objectives change, policy functions depend on the entire history that occurred previously. Nevertheless, we found that the qualitative features of the policy functions are not affected by the previous history of events.

[26] The analog of this model with no changes in objectives, is the usual full commitment solution where output converges to zero.

**Figure 3.6:** Alternative Scenario: Full Commitment and objective changes



plans are always fulfilled. Therefore, the anchoring effect is also present in the initial period. The other interesting feature is that this anchoring effect is much stronger when the conservative is newly appointed (period 1 in the graph) rather than when it is reappointed (periods 5, 9, 13). The reason is that the when objectives are conservative there is no disagreement with the liberal objectives, and therefore there is cooperation. The anchoring effect reduces inflation expectations, which allows to increase output for a given inflation rate. When current objectives are liberal, it is specially important to achieve a high output level. Therefore, a liberal central bank promises that inflation will be at a particularly low level when conservative objectives prevail.

The next table plots the average allocations in the economy. As before, under a conservative central bank, the possibility that liberal policies may be adopted lowers output. The anchoring effect is stronger and is present more frequently in this economy.[27] In fact, when policy objectives may become more liberal, a conservative central bank implement a lower average inflation, which comes at the expense of a deeper recession.[28] This model confirms our findings in previous sections. The possibility that future policy objectives may become less conservative affects considerably the current optimal policy of a conservative central bank. We still observe an anchoring and accommodation effect, while output is lowered.

### 3.4.4 The effects of adopting conservative objectives

We have solved our model where both the liberal and conservative policies are set optimally. We have mainly described the effects that a liberal policy has on a conservative central bank. This may seem the most reasonable case in the OECD economies, where politicians occasionally exert some influence for more expansionary policies. Nevertheless, our model yields implications for the opposite case, when a

[27] In the baseline model of section 3, the anchoring effect is not present in the initial 4 periods. Note that the policies of the conservative from period 1 to 4 are more likely to be implemented than the policies from period 5 to 8, which in turn are more likely to be implemented than subsequent policies.

[28] We do not compare welfare between this case and the full-commitment benchmark, because the utility functions are different, and such comparisons are meaningless.

**Table 3.5:** Inflation and Output - Full commitment case.

| | Changes in Objectives | | | Full Commitment | |
|---|---|---|---|---|---|
| | Average with $c$ | Average with $\ell$ | Overall | $c$ | $\ell$ |
| $\pi$ | -0.2429 | 0.2507 | 0.0027 | 0.0005 | 0.0048 |
| y | -2.5179 | 2.5466 | 0.002 | 0.0018 | 0.0176 |

Note: The table reports the average allocations across different simulations of the model.

central bank expects that more conservative objectives may be adopted in the future. This may be the relevant case for economies where the adoption of more stringent inflation objectives, like a specific low inflation target, is being discussed. Arguably, and subject to interpretation, this may be what it is currently happening in the United States.

In the baseline case, the possible adoption of more objectives makes the liberal to implement a lower inflation rate. In addition, inflation expectations become lower, which allows the liberal central bank to achieve higher output. Both these effects make the liberal to achieve a better welfare outcome. In this sense, the possibility that more conservative objectives are adopted constitutes a positive externality for a liberal central bank. All these conclusions are mainly robust to the other scenarios that we have considered.[29]

## 3.5 Conclusions

Many countries frequently discuss whether central bank's objectives should be changed. These discussions may result in institutional reforms, or may influence in a particular direction the appointment of a new chairman or board members of the central bank. This paper analyzes the macroeconomic effects induced by the fact that

[29]The only exception, is the model with full commitment and no disagreement, as the one considered in section 3.4.3. There, liberal policies implement an even higher inflation level, and then reduce it over time achieving a specially high output. This policy is only possible because the conservative policies are cooperative, and anchor inflation expectations very firmly by promising a very low inflation level. The main feature that an expansion is obtained still holds in that case.

central bank objectives may change in the future. We analyze optimal policy in such situation and its economic implications.

The paper is not aimed at providing a theoretical basis for partisan economic fluctuations, as for instance in Alesina (1987) and Drazen (2000). In practice, it may be hard to match directly political parties with systematic and successful changes in central bank policy. In this respect, the novelty of our analysis is to show how the possibility of future policy changes already produce effects in earlier periods. Following the recent literature on monetary policy, we model inflation dynamics with a New Keynesian Phillips curve, where expectations about future economic conditions affect current outcomes. Our analysis thus clarifies, the theoretical difficulties to find a clear relationship between economic outcomes and policymakers' objectives. We indeed show that if liberal objectives can be adopted in the future, high inflation may be the optimal response of a conservative central bank. We can thus observe a high level of inflation no matter whether liberal objectives are eventually adopted or not.

The most common case in reality is the one where a conservative central bank faces pressures to pursue more expansionary policies. In this circumstances, the optimal response of a conservative central bank is to increase current inflation through an accommodation effect. Simultaneously, the central bank tries to anchor inflation expectations by promising to be even more conservative in the future. Overall, we find that the possibility that policy objectives may become more liberal generates a negative externality for the conservative central bank. More interestingly, we also find that they lead to a contraction in current output, which is precisely the opposite of what pressures to adopt liberal policies may be aiming. The more likely is the adoption of liberal objectives, the stronger are these effects. Along this dimension, the adoption of an inflation targeting regime seems to be preferable to a conservative central bank. An inflation targeting regime insulates more the central bank from external pressures. Changing policy objectives indeed requires an institutional reform, rather than simply appointing a chairman or advisors with different views.

We have also discussed to which extent credibility matters, in a context where policy objectives can be changed. This is done by taking into account several commitment settings, following the recent contributions of Schaumburg and Tambalotti

(2007) and our methods developed in chapter 1. In particular, we show how credible institutions are able to partially counteract the bad externalities generated by the possibility that policy objectives may become more liberal. This result is interesting since it clarifies that having a central bank with sufficient aversion towards inflation, as suggested by Rogoff (1985), or equivalently adopting an inflation targeting policy, does not eliminate the scope for having credible institutions.

Finally, our paper draws conclusions about the reverse case, where the current central bank may perceive that policy objectives may become more conservative in the future. This case may be relevant for countries that are discussing the adoption of inflation targeting regimes, which is for instance the case of the United States. In this case, the possibility of more conservative policy in the future creates a positive externality for the liberal central bank. Also, inflation expectations become lower, which is translated into lower current inflation and higher current output.

# Appendix

## Optimal monetary policy with changes in policy objectives

We derive the solution to the optimal policy problem described in section 3.2. To do so, we write the first-order necessary conditions of the planner's problem described in (3.3), given by

$$\pi_t : \quad -\pi_t - \lambda_t + \lambda_{t-1} = 0 \tag{A-1}$$

$$y_t : \quad -w^i(y_t - \widetilde{y}^c) + \lambda_t \kappa = 0 \tag{A-2}$$

$$\lambda_t : \quad \pi_t = \kappa y_t + \beta q \pi_{t+1} + \beta\left(1-q\right)\pi_0^{\ell} \tag{A-3}$$

where $\lambda_t$ is the Lagrange multiplier associated with the NKPC. Rearranging equations (A-1)-(A-3) we obtain the second-order difference equation

$$\left[\beta q L^{-2} - \left(1 + \beta q + \frac{\kappa^2}{w^c} + 1\right) L^{-1} + 1\right] \lambda_{t-1} = \kappa \widetilde{y}^c + \beta\left(1-q\right)\pi_0^{\ell}$$

whose solution is given by

$$\left(1 - \gamma_1^c L^{-1}\right)\left(1 - \gamma_2^c L^{-1}\right)\lambda_{t-1} = \kappa \widetilde{y}^c + \beta\left(1-q\right)\pi_0^{j} \tag{A-4}$$

where,

$$\gamma_1 = \frac{\left(1 + \beta q + \frac{\kappa^2}{w^i}\right) - \sqrt{\left(1 + \beta q + \frac{\kappa^2}{w^i}\right)^2 - 4\beta q}}{2} \tag{A-5}$$

$$\gamma_2 = \frac{(1 + \beta q + \frac{\kappa^2}{w^i}) + \sqrt{(1 + \beta q + \frac{\kappa^2}{w^i})^2 - 4\beta q}}{2}. \tag{A-6}$$

It is convenient to emphasize that $\gamma_1\gamma_2 = \beta q$ and $\gamma_1 + \gamma_2 = \left(1 + \beta q + \frac{\kappa^2}{w}\right)$ and $0 < \gamma_1 < 1 < \gamma_2$. Moreover,

$$\frac{\partial \gamma_2}{\partial q} = \frac{\beta}{2}\left(1 + \frac{(\gamma_1 + \gamma_2) - 2}{\gamma_2 - \gamma_1}\right) = \beta\left(\frac{\gamma_2 - 1}{\gamma_2 - \gamma_1}\right) > 0.$$

The unique stable solution to (A-4) is given by the expression

$$\lambda_t = \frac{1}{\gamma_2^c}\lambda_{t-1} - \frac{1}{\gamma_2^c(1-\gamma_1^c)}\left(\kappa\widetilde{y}^c + \beta(1-q)\pi_0^\ell\right).$$

Solving backward an imposing the initial condition $\lambda_{-1} = 0$, we obtain

$$\lambda_t = \frac{1-(\gamma_2^c)^{-(t+1)}}{(1-\gamma_1^c)(1-\gamma_2^c)}\left[\kappa\widetilde{y}^c + \beta(1-q)\pi_0^\ell\right].$$

Using (A-1) we obtain the following expression for the evolution of inflation and output

$$\pi_0 = -\lambda_0 = \frac{1}{\gamma_2^c(1-\gamma_1^c)}\left[\kappa\widetilde{y}^c + \beta(1-q)\pi_0^\ell\right]$$

and

$$\begin{aligned}\pi_t &= (\gamma_2^c)^{-t}\pi_0 \\ y_t - \widetilde{y}^c &= -\frac{\kappa}{w^c}\frac{1-(\gamma_2^c)^{-t+1}}{1-(\gamma_2^c)^{-1}}\pi_0\end{aligned}$$

which corresponds to equations (3.5)-(3.7).

For later convenience, we notice that since the liberal central bank is facing a problem that is symmetric with the one described above, using a symmetric expression to (3.7) we have

$$\pi_0^\ell = \frac{1}{\gamma_2^\ell\left(1-\gamma_1^\ell\right)}\left[\kappa\widetilde{y}^\ell + \beta(1-q)\pi_0^c\right] \tag{A-7}$$

## The case of full-commitment and constant policy objectives

The standard case of full-commitment and no uncertainty about policy objectives is a special case of the problem described above where $\widetilde{y}^c = \widetilde{y}^\ell \equiv \widetilde{y}$, $w^c = w^\ell \equiv w$ and $q = 1$. In this case, we have that $\pi_0^\ell = \pi_0^c \equiv \bar{\pi}_0$. Defining $\bar{\gamma}_2$ as the value taken by (A-6 when $q = 1$ we have

$$\bar{\pi}_0 = \frac{1}{\bar{\gamma}_2 - \beta}\kappa\widetilde{y}^c \tag{A-8}$$

where $\bar{\gamma}_2$ is the value taken by $\gamma_2$ when $q = 1$. Moreover, from (3.5) and (3.6)

$$\bar{\pi}_t = \frac{\bar{\gamma}_2^{-t}}{\bar{\gamma}_2 - \beta}\kappa\widetilde{y}^c \tag{A-9}$$

$$\bar{y}_t - \widetilde{y}^c = -\frac{\kappa}{w}\frac{1-\bar{\gamma}_2^{-(t+1)}}{1-\bar{\gamma}_2^{-1}}\frac{1}{\bar{\gamma}_2 - \beta}\kappa\widetilde{y}^c = -(1-\bar{\gamma}_2^{-1})\widetilde{y}^c \tag{A-10}$$

## The case of limited commitment

The case of limited commitment is one where the objectives of policymakers coincides. However, the monetary authority is not fully credible because at any point in time there is a probability $(1-q)$ that its previous promises are disregarded and that a reoptimization occurs. This case correspond to one where $\widetilde{y}^c = \widetilde{y}^\ell \equiv \widetilde{y}$, $w^c = w^\ell \equiv w$ and $0 < q < 1$. As in the full-commitment case, we have that $\pi_0^\ell = \pi_0^c \equiv \bar{\pi}_0$. The resulting allocations are given by similar expressions to (A-8)-(A-10), substituting the value of $\gamma_2$ as given by (A-6), instead of $\bar{\gamma}_2$. As a consequence, we have that inflation and output are given by

$$\begin{aligned} \pi_t^{LC} &= \frac{\gamma_2^{-t}}{\gamma_2 - \beta}\kappa\widetilde{y}^c \\ y_t^{LC} - \widetilde{y}^c &= -\frac{\kappa}{w}\frac{1-\gamma_2^{-(t+1)}}{1-\gamma_2^{-1}}\frac{1}{\gamma_2-\beta}\kappa\widetilde{y}^c. \end{aligned}$$

Since $\frac{\partial\gamma_2}{\partial q} > 0$ we have that the higher is the probability of commitment, the lower are inflation and output. Finally, equation (3.8) is obtained dividing the above expressions by (A-9).

## The case of changes in the output target

We now analyze the case where there is uncertainty about the output target, i.e. where the current conservative target can be replaced by $\widetilde{y}^\ell > \widetilde{y}^c$, but keeping unchanged the weight on output stabilization, $w^c = w^\ell = w$. Substituting this into (3.7), and using the fact that in this case, being the output target the only difference among the two types of policymakers, $\gamma_1^c = \gamma_2^\ell \equiv \gamma_1$ and $\gamma_2^c = \gamma_2^\ell \equiv \gamma_2$ we obtain

$$\pi_0^c\left(1 - \frac{\beta^2(1-q)^2}{[\gamma_2(1-\gamma_1)]^2}\right) = \frac{1}{\gamma_2(1-\gamma_1)}\left[\kappa\widetilde{y}^c + \frac{\beta(1-q)}{\gamma_2(1-\gamma_1)}\kappa\widetilde{y}^\ell\right].$$

For convenience, we define $\Phi \equiv \frac{\beta(1-q)}{\gamma_2(1-\gamma_1)} = \frac{\beta-\beta q}{\gamma_2-\beta q} < 1$ and notice that $\frac{\partial\Phi}{\partial q} < 0$.

We thus have

$$\pi_0 = \frac{\Phi}{(1-\Phi)}\frac{\kappa}{\beta(1-q)}\frac{\widetilde{y}^c + \Phi\widetilde{y}^\ell}{(1+\Phi)}$$

Using this expression to substitute for $\pi_0$ in (3.5) and then dividing everything for (A-9) one obtains

$$\pi_t = \underbrace{\frac{\widetilde{y}^c + \Phi\widetilde{y}^\ell}{\widetilde{y}^c\,(1+\Phi)}}_{\substack{\text{Liberal Objectives}\\ >1}} \quad \underbrace{\left(\frac{\bar{\gamma}_2}{\gamma_2}\right)^t \frac{\bar{\gamma}_2-\beta}{\gamma_2-\beta}}_{\substack{\text{Limited Commitment}\\ >1}} \bar{\pi}_t,$$

which is the same as (3.9).

Finally, it easy to see that $\pi_t$ is increasing in the difference between $\widetilde{y}^\ell$ and $\widetilde{y}^c$. We can also show that it is strictly decreasing in $q$, indeed

$$\frac{\partial \pi_t}{\partial q} = \frac{\widetilde{y}^c + \Phi\widetilde{y}^\ell}{\widetilde{y}^c\,(1+\Phi)}\frac{\partial \pi_t^{LC}}{\partial q} + \frac{\partial \frac{\widetilde{y}^c+\Phi\widetilde{y}^\ell}{\widetilde{y}^c(1+\Phi)}}{\partial q}\pi_t^{LC} < 0,$$

since both terms of the sum are negative. The first term is negative because of our result in the previous section, while the second term can be written as

$$\frac{\partial \frac{\widetilde{y}^c+\Phi\widetilde{y}^\ell}{\widetilde{y}^c(1+\Phi)}}{\partial q}\pi_t^{LC} = \frac{\partial \frac{\widetilde{y}^c+\Phi\widetilde{y}^\ell}{\widetilde{y}^c(1+\Phi)}}{\partial \Phi}\frac{\partial \Phi}{\partial q}\pi_t^{LC} = \underbrace{\frac{\widetilde{y}^\ell - \widetilde{y}^c}{\widetilde{y}^c(1+\Phi)^2}}_{>0}\underbrace{\frac{\partial \Phi}{\partial q}}_{<0} < 0.$$

### The case of changes the relative weight of output $w$

When there is uncertainty about the output weight, we have that the current conservative weight can be replaced by $w^\ell > w^c$, while keeping unchanged the output target $\widetilde{y}^c = \widetilde{y}^\ell = \widetilde{y}$. Substituting (A-7) into (3.5) one obtains

$$\pi_t = \frac{(\gamma_2^c)^{-t}}{\gamma_2^c\,(1-\gamma_1^c)}\frac{\left(1+\Phi^\ell\right)}{\left(1-\Phi^c\Phi^\ell\right)}\kappa\widetilde{y}$$

which divided by (A-9) delivers the expression in section 3.4.1.

## Recursive formulation of the problem of section 3.3

For notational convenience only, we abstract from the presence of uncertainty other than the one regarding the policy objective changes.

**Proposition 1** *Being $\lambda$ the vector of lagrange multipliers associated with the constraints 3.15 and 3.16, problem (3.14) can be written as a saddle point functional equation (SPFE) as follows:*

$$W(\gamma) = \min_{\lambda \geq 0} \max_{\{\pi_t, y_t\}_{t=0}^{T-1}} \{h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma)\} + \beta(1-q)V^{ij} + \beta q W(\gamma')\}$$
$$s.t. \quad \gamma' = \lambda, \quad \gamma_0 = 0$$
$$V^{ij}, \quad \pi_T^j \quad given$$

*where*

$$h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma) \equiv \ell(\{\pi_t, y_t\}_{t=0}^{T-1}) + \lambda g_1(\{\pi_t, y_t\}_{t=0}^{T-1}) + \gamma g_2(\{\pi_t, y_t\}_{t=0}^{T-1})$$
$$\ell(\{\pi_t, y_t\}_{t=0}^{T-1}) \equiv \sum_{t=0}^{T-1} \beta^t \left[{\pi_t}^2 + w^i(y_t - \widetilde{y})^2\right]$$

$$g_1(\{\pi_t, y_t\}_{t=0}^{T-1}) \equiv \begin{bmatrix} \pi_0 - \kappa y_0 - \beta\pi_1 \\ \vdots \\ \pi_{T-2} - \kappa y_{T-2} - \beta\pi_{T-1} \\ \pi_{T-1} - \kappa y_{T-1} - \beta(1-q)\pi_T^j \end{bmatrix}$$
$$g_2(\{\pi_t, y_t\}_{t=0}^{T-1}) \equiv \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \pi_0^i \end{bmatrix}$$

**Proof.** of Proposition 1 Let's define the real valued function $r(\cdot)$ as follows:

$$r(\{\pi_t, y_t\}_{t=0}^{T-1}) \equiv \sum_{t=0}^{T-1} \beta^t \left[\pi_{m+t}^2 + w^i(y_{m+t} - \widetilde{y}^i)^2\right] + \beta^T(1-q)V^{ij}$$
$$V^{ij} \quad given$$

Moreover, $g_1(\cdot)$ and $g_2(\cdot)$ are defined as in the second part of the proposition.

Problem 3.14 is therefore equivalent to:

$$V^i = \max_{\pi_t, y_t} -\frac{1}{2} E_0 \sum_{m=0}^{\infty} (\beta^T q)^m r(\{\pi_{mT+t}, y_{mT+t}\}_{t=0}^{T-1})$$

$$s.t. \qquad g_1(\{\pi_{mT+t}, y_{mT+t}\}_{t=0}^{T-1}) + g_2(\{\pi_{(m+1)T+t}, y_{(m+1)T+t}\}_{t=0}^{T-1}) \geq 0$$

This formulation fits the definition of Program 1 in Marcet and Marimon (1998). We can therefore write the problem as a saddle point functional equation in the sense that there exists a unique function satisfying:

$$W(\gamma) = \min_{\lambda \geq 0} \max_{\{\pi_t, y_t\}_{t=0}^{T-1}} h(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma) + \beta q W(\gamma')\}$$

$$s.t. \quad \gamma' = \lambda, \quad \gamma_0 = 0$$

$$V^{ij}, \quad \pi_T^j \quad given$$

where:

$$h(c, k, \lambda, \gamma) = r(\{\pi_t, y_t\}_{t=0}^{T-1}) + \lambda g_1(\{\pi_{mT+t}, y_{mT+t}\}_{t=0}^{T-1}) + \gamma g_2(\{\pi_t, y_t\}_{t=0}^{T-1})$$

or in a more intuitive formulation define:

$$h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma) \equiv \ell(\{\pi_t, y_t\}_{t=0}^{T-1}) + \lambda g_1(\{\pi_t, y_t\}_{t=0}^{T-1}) + \gamma g_2(\{\pi_t, y_t\}_{t=0}^{T-1})$$

$$\ell(\{\pi_t, y_t\}_{t=0}^{T-1}) \equiv \sum_{t=0}^{T-1} \beta^t \left[{\pi_t}^2 + w^i (y_t - \widetilde{y})^2\right]$$

and the saddle point functional equation is:

$$W(\gamma) = \min_{\lambda \geq 0} \max_{\{\pi_t, y_t\}_{t=0}^{T-1}} \{h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma)\} + \beta(1-q) V^{ij} + \beta q W(\gamma')\}$$

$$s.t. \quad \gamma' = \lambda, \quad \gamma_0 = 0$$

$$V^{ij}, \quad \pi_T^j \quad given$$

■

**Proposition 2** *For any type of monetary regime $i = \ell, c$ the solution of problem (3.14) is a tenure invariant function $\psi(\gamma)$, such that:*

$$\psi(\gamma) = \arg\min_{\lambda \geq 0} \max_{\{\pi_t, y_t\}_{t=0}^{T-1}} \{h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma)\} + \beta(1-q)V^{ij} + \beta q W(\gamma')\}$$

$$\gamma' = \lambda, \ \gamma_0 = 0$$

**Proof.** of Proposition 2: using Proposition 1, this proof follows directly from the results of Marcet and Marimon (1998). ■

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