

DOCTORAL THESIS

“Essays on Monetary Policy, Wage Bargaining and
Fiscal Policy”

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Introduction

General equilibrium models with monopolistic competition and nominal rigidities have become a workhorse in the design of the optimal monetary and fiscal policy response to shocks over the business cycle. This literature pays scant attention to the role of strategic interaction among private agents and policy makers. We identify two interesting economic problems that can hardly abstract from the issue: the monetary policy implications of unionized labor markets; the coordination problems arising when monetary and fiscal policy do not necessarily coordinate or agree on the optimal stabilization policy mix. Those questions are analyzed in the context of a New-Keynesian framework.

The first chapter generalizes the baseline New-Keynesian model to allow for a unionized labor force, a distinctive feature of labor markets in most of OECD countries. The presence of large wage setters affects the transmission channel of monetary policy. Big unions internalize the effects of their wage policy on inflation, by anticipating that a rise in the wage produces inflationary pressures and a consequent reduction of labor demand through monetary policy tightening. Tougher inflation stabilization policies punish wage increases with a harsher contraction of aggregate labor demand, giving unions the incentive to restrain real wages. In this context, the central bank can raise long-run employment by implementing more aggressive stabilization policies. Strategic interaction creates a transmission mechanism of monetary policy acting via labor supply, rather than aggregate demand. The effectiveness of this channel increases in wage setting centralization, as bigger unions internalize to a greater extent the impact of their wage policy on inflation. As a consequence, depending on the labor market structure, policy makers have an additional reason to stabilize inflation, other than the usual concerns about relative price dispersion. This fact may be important in that it is likely to alter the policy trade-off in favor of more conservative policies. Such a question is the object of Chapter 2.

Chapter 2 designs optimal monetary policy rules in a New-Keynesian model featuring the presence of non-atomistic unions. The central bank faces an additional trade-off with respect to the one traditionally considered in the literature. In fact, steady state efficiency can be enhanced only by increasing aggressiveness in stabilizing inflation, or equivalently only by accepting a higher volatility of the output gap. The more is centralized the wage bargaining process, the higher is the marginal gain of stabilizing inflation in terms of steady state efficiency, as the effectiveness of the strategic interaction channel increases in labor market concentration. Consequently, the optimal monetary policy stance is tighter. It turns out that concentrated labor markets call for more aggressive stabilization policies. Finally, it is computed the cost of deviating from optimal policy. Such a cost is measured as the fraction of consumption that agents are willing to give up to be indifferent between the optimal policy and a given alternative regime. The welfare cost is decomposed in order to disentangle steady state and stabilization effects of policy. The welfare analysis shows that most of the cost can be accounted for by the steady state component. The result confirms the intuition that in the presence of concentrated labor markets it is optimal to tighten monetary policy, in order to exploit strategic interaction so as to increase long-run employment.

Chapter 3 evaluates the effects of fiscal discretion in a currency area, where a common and independent monetary authority commits to optimally set the union-wide nominal interest rate. National governments implement fiscal policy by choosing government expenditure without coordinating with the central bank. The assumption of fiscal policy coordination across countries is retained in order to evaluate the costs exclusively due to discretion, leaving aside the free-riding problems stemming from non-cooperation. In such a context, nominal rigidities potentially generate a stabilization role for fiscal policy, in addition to the one of ensuring efficient provision of public goods. However, it is showed that, under discretion, aggregate fiscal policy stance is inefficiently loose and the volatility of government expenditure is higher than

optimal. As an implication, the optimal monetary policy rule involves the targeting of union-wide fiscal stance, on top of inflation and output gap. The result questions the welfare enhancing role of government expenditure, as the proper instrument for stabilizing asymmetric shocks. In fact, discretion entails significant welfare costs, the magnitude depending on the stochastic properties of the shocks and, for plausible parameter values, it is not optimal to use fiscal policy as a stabilization tool.

Chapter 1

Non-Atomistic Wage Setters and Monetary Policy in a New-Keynesian Framework

1.1 Introduction

New-Keynesian (NK) models have been extensively used in recent years to analyze the impact of monetary policy on business cycle fluctuations and to provide guidelines in the design of optimal monetary policy rules.

NK literature commonly disregards potential strategic interaction between policy makers and large wage setters, by assuming atomistic private agents. Yet, collective wage bargaining is a distinctive feature of labor markets in most of OECD countries. Figure 1.1 plots union density, measured as the fraction of workers affiliated to some union, against bargaining coverage, defined as the fraction of workers covered by union-negotiated terms and conditions of employment. Despite the historically low union membership rates, a large fraction of wage contracts is negotiated in the context of collective agreements: the average coverage level is twice as high as the density level (60 versus 34 percent). In continental Europe, at least two out of three workers are covered by bargained wage setting with the exception of Switzerland and Eastern Europe. The

divergence between density and coverage is due to the widespread practice of extending by law the collective contract to the non-unionized work force as well. Tables 1.1 and 1.2 show that several OECD countries feature highly centralized wage bargaining. In fact, negotiations are delegated to few large unions whose decisions have a considerable impact on the aggregate level of wages, which is in turn one of the main forces driving the real cost of labor and, as a consequence, of inflation. In such an environment, strategic interaction between wage setters and the monetary policymaker is an issue.

Although quite recent, the idea of studying how the presence of large wage setters affects the monetary policy transmission channel is not new. Bratsiotis and Martin (1999), Iversen and Soskice (2000) and Lippi (2002, 2003) among others¹ show that, in the presence of a unionized labor force, the systematic behavior of the central bank has an impact on labor supply decisions and, as a consequence, on the long-run equilibrium level of employment and production. These models are static and deterministic and they hardly relate central bank's targets to the microeconomic structure of the model economy. As a consequence, they are silent on the optimal monetary policy response to shocks over the cycle. Nevertheless, they identify a source of monetary non neutrality that may well alter the traditional monetary policy results delivered by NK models. Therefore, it may be a fruitful improvement upon the state of the art to merge these two strands of the literature.

This chapter generalizes the baseline NK model to allow for a unionized labor force. It is shown that, once the presence of large wage setters is taken into account, an additional channel of transmission of monetary policy, other than the conventional demand side channel, is created. The degree of wage setting centralization and the aggressiveness of the central bank in stabilizing inflation jointly affect the equilibrium

¹See also Cukierman and Lippi (1999) and Coricelli, Cukierman and Dalmazzo (2006). Holden (2005) took this literature a step forward by considering the effects of the monetary regime on wage setters' incentives to coordinate their decisions. Zanetti (2005) develops a NK model to study the monetary policy implications of unionized labor markets. His model however differs from the one outlined here, since atomistic unions are assumed.

level of real economic activity in the long-run. The classical neutrality result is not challenged: a temporary shock to the policy instrument dies off in the long-run. A change in the policy rule, however, has a permanent real effect since it alters the steady state equilibrium level of employment. Two assumptions are key for the result: wage setters have positive mass and they internalize the consequences of their actions. Since wage setters are non-atomistic, they are able to influence the aggregate wage index. In addition, if unions understand that firms set the price at a mark-up over the marginal cost, they also realize that a variation in the aggregate wage index has an impact on inflation, triggering the reaction of the central bank. Then, wage inflationary pressures will induce the monetary authority to contract aggregate demand and, as a consequence, aggregate labor demand. The higher central bank's inflation aversion, the stronger the response of the nominal interest rate and the more severe the contraction of aggregate labor demand. Therefore, tougher inflation stabilization policies raise the steady state level of employment by giving unions the incentive to restrain wages. Because of strategic interaction, the central bank can push output towards Pareto efficiency without creating inflation.

In this context, price stability is consistent with the elimination of any deviation of real economic activity from Pareto efficiency and it is, as a consequence, the optimal policy. The outcome distinguishes the model outlined here from the standard NK, where, without the proper fiscal policy, zero inflation under full commitment is still optimal, but it can be reached only at the cost of a suboptimal production level. Price stability as the optimal policy follows from the fact that price stickiness is the only source of dynamic inefficiency. The introduction of other dynamic distortions would introduce a tension between inflation and output gap stabilization and would then undermine the policy implication. However, the main message would survive: concentrated labor markets provide an additional reason to stabilize inflation fluctuations other than the usual concerns about relative price dispersion. This fact may be

important in that it is likely to alter the trade-off traditionally considered in favor of more conservative policies. Such a question is the object of Chapter 2.

The Chapter is organized as follows. Section 2 describes the model economy, the main results and the policy implications. Section 3 concludes.

1.2 The Model

The model economy consists of a continuum of households and firms and a finite number of unions. Households and firms are modelled as in the baseline NK model with goods prices staggered à la Calvo (1983)². The main differences with respect to the standard framework are in the structure of the labor market. Households indeed delegate wage setting decisions to unions and, for given wage, they are willing to supply whatever quantity of labor is required to clear the markets.

The central bank sets the nominal interest rate, reacting to endogenous variations in inflation according to the following policy rule

$$i_t = \rho + \gamma_\pi \pi_t \tag{1.1}$$

where i_t is the log of the nominal interest rate factor, ρ is the steady state level of i_t , inflation is defined as $\pi_t = \log P_t - \log P_{t-1}$ and $\gamma_\pi > 1$.

It is assumed that the fiscal policy is responsible for offsetting the static distortions arising because of imperfectly competitive goods markets, while, differently from the baseline model, the inefficiency arising in labor markets is not corrected for. Lump-sum transfers and taxes are available and they are free to adjust in order to balance the government budget constraint at all times.

²For derivations of the baseline model I refer to Calvo (1983), Clarida, Galí and Gertler (1999), Galí (2003), Walsh (2003) and Woodford (2003)

1.2.1 Households

The economy is populated by a continuum of infinitely lived households indexed by i on the unit interval $[0,1]$, each of them consumes a continuum of differentiated goods and supplies a differentiated labor type. Households have preferences defined over consumption and hours worked described by the utility function³

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_{t,i} - \frac{L_{t,i}^{1+\phi}}{1+\phi} \right] \quad (1.2)$$

where C is aggregate consumption, obtained aggregating in the Dixit-Stiglitz form the quantities consumed of each variety $f \in [0, 1]$

$$C_{t,i} = \left[\int_0^1 C_{t,i}(f)^{\frac{\theta_p-1}{\theta_p}} df \right]^{\frac{\theta_p}{\theta_p-1}} \quad (1.3)$$

and the parameter $\theta_p > 1$ is representing the elasticity of substitution among varieties. Defining the aggregate price index⁴ as

$$P_t = \left[\int_0^1 P_t(f)^{1-\theta_p} df \right]^{\frac{1}{1-\theta_p}} \quad (1.4)$$

optimal allocation of expenditure among varieties implies

$$C_{t,i}^*(f) = \left[\frac{P_t(f)}{P_t} \right]^{-\theta_p} C_{ti} \quad (1.5)$$

The budget constraint faced by households in each period is

$$C_{t,i} + \delta_{t,t+1} B_{t,i} \leq B_{t-1,i} + \frac{W_{t,i}}{P_t} L_{t,i} + T_{t,i} + Div_{t,i} \quad (1.6)$$

$\delta_{t,t+1}$ is the price vector of a state contingent asset paying one unit of consumption in a particular state of nature in period $t+1$, B_t is the vector of the corresponding

³The analysis is restricted to the case of log utility. In this case not only the model is more tractable, but the policy analysis is particularly intuitive and transparent. However, it is possible to show that all results derived here continue to hold in the more general case of a CRRA utility function.

⁴The price index has the property that the minimum cost of a consumption bundle C_t is $P_t C_t$

state contingent claims purchased by the household and B_{t-1} the value of the claims for the current realization of the state of nature. $\frac{W_{t,i}}{P_t}L_{t,i}$ represents real labor income. Finally, each consumer receives a share $Div_{t,i}$ of the aggregate profits and lump-sum government transfers $T_{t,i}$. Households maximize their lifetime utility (1.2) subject to the budget constraint (1.6) choosing state contingent paths of consumption and assets. Optimal allocation of consumption over time implies the standard Euler equation

$$C_t^{-1} = E_t[\beta(1 + R_t)C_{t+1}^{-1}] = E_t[\beta(1 + I_t)\frac{P_t}{P_{t+1}}C_{t+1}^{-1}] \quad (1.7)$$

R_t , the risk-free real interest rate, is the rate of return of an asset that pays one unit of consumption in every state of nature at time $t+1$ and the risk-free nominal interest rate, I_t , is the rate of return of an asset that yields one unit of currency in every state of nature at time $t+1$. Integrating (1.5) across households, total demand of variety f is

$$C_t^*(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\theta_p} C_t; \quad C_t = \int_0^1 C_{t,i} di \quad (1.8)$$

Let aggregate output Y_t be defined by

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\theta_p-1}{\theta_p}} df\right]^{\frac{\theta_p}{\theta_p-1}} \quad (1.9)$$

then the clearing of all goods markets

$$Y_{t,f} = C_{t,f} \quad (1.10)$$

implies

$$Y_t = C_t \quad (1.11)$$

Combining the Euler equation with the monetary policy rule, after imposing (1.11), yields

$$Y_t = \Pi_t^{-\gamma\pi} [E_t \{\Pi_{t+1}^{-1} Y_{t+1}^{-1}\}]^{-1} \quad (1.12)$$

where Π_t is the gross inflation rate, defined as

$$\Pi_t \equiv \frac{P_t}{P_{t-1}} \quad (1.13)$$

Equation (1.12) fully describes the aggregate demand block of the model: it relates aggregate output demand to inflation, conditionally on expectations about future variables. Note that the reaction of output to inflation depends on central bank's aggressiveness in stabilizing inflation.

1.2.2 Firms

Consider a continuum of monopolistically competitive firms, indexed by f on the interval $[0, 1]$, each producing a differentiated good using a continuum of labor types according to the following constant return to scale technology

$$Y_t(f) = A_t L_{t,f} \tag{1.14}$$

Productivity (TFP), denoted by A_t , follows an autoregressive process represented by

$$\log A_{t+1} = \rho_a \log A_t + \varepsilon_{t+1,a} \tag{1.15}$$

where ε_t is white noise with standard deviation $\sigma_{\varepsilon,a}$. The effective labor input is obtained aggregating in the Dixit-Stiglitz form the quantities hired of each differentiated labor type

$$L_{t,f} = \left[\int_0^1 L_{t,f}(i)^{\frac{\theta_w-1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w-1}}$$

The parameter $\theta_w > 1$ is representing the elasticity of substitution among labor types. Firms do not have market power in the labor market, then they take wages as given. Defining the aggregate wage⁵ as

$$W_t = \left[\int_0^1 W_t(i)^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}} \tag{1.16}$$

⁵As for the price index, aggregate wage has the property that the minimum cost of a unit of composite labor input L_t is $W_t L_t$

cost minimization implies

$$L_{t,f}^*(i) = \left[\frac{W_t(i)}{W_t} \right]^{-\theta_w} L_{t,f} \quad (1.17)$$

Firms set the price in order to maximize profits, subject to the constraint that demand must be satisfied at the posted price, according to equation (1.8). Prices are set in staggered contracts with random duration as in Calvo (1983): in any period each firm faces a constant probability $1 - \alpha$ to reoptimize and charge a new price. A subsidy is used by the fiscal authority to undo the steady state distortion induced by firms' market power in the goods markets. The definition of the price index and profit maximization imply

$$\left[\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} \right]^{\frac{1}{1 - \theta_p}} = \frac{E_t \sum_{j=0}^{\infty} (\alpha \beta)^j MC_{t+j} \Pi_{t,t+j}^{\theta_p}}{E_t \sum_{j=0}^{\infty} (\alpha \beta)^j \Pi_{t,t+j}^{\theta_p - 1}} \quad (1.18)$$

where $\Pi_{t,t+j} \equiv \frac{P_{t+j}}{P_t}$ and the real marginal cost is identical across firms and equal to

$$MC_t = \frac{W_t}{P_t A_t} \quad (1.19)$$

Integrating (1.17) across firms yields total demand of labor faced by household i

$$L_t^*(i) = \left[\frac{W_t(i)}{W_t} \right]^{-\theta_w} L_t; \quad L_t = \int_0^1 L_{t,f} df \quad (1.20)$$

It is convenient to rewrite (1.18) in the form

$$\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} = \left(\frac{K_t}{F_t} \right)^{1 - \theta_p} \quad (1.21)$$

defining K and F

$$K_t \equiv E_t \sum_{j=0}^{\infty} (\alpha \beta)^j MC_{t+j} \Pi_{t,t+j}^{\theta_p} \quad (1.22)$$

$$F_t \equiv E_t \sum_{j=0}^{\infty} (\alpha \beta)^j \Pi_{t,t+j}^{\theta_p - 1} \quad (1.23)$$

Note that (1.22) and (1.23) can be expressed recursively as

$$K_t = MC_t + \alpha\beta E_t \{(\Pi_{t+1})^{\theta_p} K_{t+1}\} \quad (1.24)$$

$$F_t = 1 + \alpha\beta E_t \{(\Pi_{t+1})^{\theta_p-1} F_{t+1}\} \quad (1.25)$$

Equation (1.21) fully describes the aggregate supply block of the model: it relates aggregate output supply to inflation, conditionally on expectations about future variables.

Finally, it can be easily shown that the aggregate production function is given by

$$Y_t \Delta_t = A_t L_t \quad (1.26)$$

where Δ_t ⁶ is defined as

$$\Delta_t = \int_0^1 \frac{Y_t(f)}{Y_t} df \quad (1.27)$$

and represents a measure of relative price dispersion, evolving according to the law

$$\Delta_t = (1 - \alpha) \left(\frac{1 - \alpha \Pi_t^{\theta_p-1}}{1 - \alpha} \right)^{\frac{\theta_p}{\theta_p-1}} + \alpha \Pi_t^{\theta_p} \Delta_{t-1} \quad (1.28)$$

1.2.3 Unions

The economy is populated by a finite number of unions indexed by j , where $j \in \{1, \dots, n\}$, $n \geq 2$. All workers are unionized and they split equally among unions so that each union has mass n^{-1} . The mass can be interpreted as the degree of wage setting centralization (CWS) as well as unions' ability to internalize the consequences of their actions. As a matter of fact, the higher is the number of unions the lower

⁶It can be proved that $\log(\Delta)$ is a function of the cross sectional variance of relative prices and it is of second order.

is their mass and then the lower the impact of union's j wage policy on aggregate variables.

It is assumed that wages are fully flexible and any possibility of pre-commitment to future wage policies is ruled out. Each union j sets the real wage on behalf of her members to maximize their lifetime utility function (1.2) subject to the budget constraint⁷ (1.6) and labor demand (1.20) for all members $i \in j$. Unions set wages simultaneously and each of them takes other unions' real wages as given.

The assumption that wage setters have positive mass is key for the outcome of the model. Since unions are non-atomistic, they internalize the impact of their wage policy on the aggregate wage. Then they also realize that an increase in union's j wage creates inflationary pressures via the price setting rule of firms, inducing the central bank to contract aggregate demand, and then aggregate labor demand. Formally, the aggregate wage index (1.16), aggregate demand (1.12), the production function (1.26) and the short run aggregate supply (1.21) are internalized on top of the budget constraint (1.6) and labor demand (1.20). It follows that aggregate labor demand is a function of $\frac{W_{j,t}}{P_t}$ through the monetary policy rule. The elasticity of aggregate labor demand to changes in the wage is⁸

$$\Sigma_L = \gamma_\pi \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \quad (1.29)$$

implying the following elasticity of labor demand perceived by the j -th union for each of her members

$$\eta = \theta_w \left(1 - \frac{1}{n}\right) + \frac{1}{n} \Sigma_L \quad (1.30)$$

This is a weighted average of the elasticity of substitution among labor types and the

⁷Fiscal policy and dividends are taken as given, as it is usually assumed in the literature. See Lippi (2002, 2003)

⁸For the derivation of Σ_L see Appendix A. Note that Σ_L is not constant over time. However, as it is shown in the appendix, for empirically relevant values of the parameters and for the calibrations considered below, elasticity fluctuations do not generate quantitatively significant variation out of the steady state at a first-order accuracy. Then it is assumed in the rest of the chapter that elasticity is constantly equal to its steady state value.

elasticity of aggregate labor demand, which is in turn an increasing function of γ_π . This is because the more restrictive the policy stance, the harsher will be the contraction of aggregate demand as a reaction to inflation variability, with the consequence of making labor demand more sensitive to a variation in the wage. The effect is increasing in the mass of the union as larger unions internalize more the impact of their wage policy on aggregate variables. Note that price stickiness enters negatively through the elasticity of aggregate labor demand. Indeed, when price stickiness raises, the fraction of firms re-optimizing in each period is lower. Therefore, also the impact of a change in the real wage on inflation, and then on aggregate output through central bank's reaction, has to be lower.

The solution to unions' problem implies the following relation

$$\frac{W_t}{P_t} = \frac{\eta}{\eta - 1} L_t^\phi C_t \quad (1.31)$$

Index j has been dropped because of symmetry. The first order condition for unions has the same form as in the standard case with atomistic wage setters. The real wage in fact is set at a mark-up over the marginal rate of substitution. However, the mark-up depends not only on the elasticity of substitution among labor types, but also on the number of unions and on central bank's aggressiveness in stabilizing inflation. Tough inflation stabilization policies discourage wage pressures by punishing a wage increase with a contraction of aggregate demand. Note finally that unions have been modelled in such a way that the case of non-atomistic wage setters nests the two limiting cases of monopolistically competitive and perfectly competitive labor markets. When the number of unions tends to infinity, the wage mark-up becomes $\frac{\theta_w}{\theta_w - 1}$. Alternatively, if the elasticity of substitution between labor types tends to infinity, the wage collapses to the competitive level.

1.2.4 The Sticky Price Equilibrium

Given Δ_{-1} , the exogenous stochastic process A_t and a value for the policy parameter γ_π , the rational expectation equilibrium for the sticky price economy is a process $\{Y_t, \Pi_t, F_t, K_t, \Delta_t\}_{t=0}^\infty$ satisfying the following system of equations

$$Y_t^{-1} = \Pi_t^{\gamma_\pi} E_t \{ \Pi_{t+1}^{-1} Y_{t+1}^{-1} \}$$

$$\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} = \left(\frac{K_t}{F_t} \right)^{1 - \theta_p}$$

$$K_t = \frac{\eta}{\eta - 1} \left(\frac{Y_t}{A_t} \right)^{1 + \phi} \Delta_t^\phi + \alpha \beta E_t \{ (\Pi_{t+1})^{\theta_p} K_{t+1} \}$$

$$F_t = 1 + \alpha \beta E_t \{ (\Pi_{t+1})^{\theta_p - 1} F_{t+1} \}$$

$$\Delta_t = (1 - \alpha) \left(\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} \right)^{\frac{\theta_p}{\theta_p - 1}} + \alpha \Pi_t^{\theta_p} \Delta_{t-1}$$

$$\eta = \theta_w \left(1 - \frac{1}{n} \right) + \frac{1}{n} \gamma_\pi \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha}$$

which can be easily obtained using equations (1.11), (1.12), (1.19), (1.21), (1.24), (1.25), (1.26), (1.28), (1.30) and (1.31).

1.2.5 The Pareto Optimum

For the subsequent analysis it is useful to derive the Pareto efficient level of output, consumption and labor. Pareto efficiency requires that the marginal rate of substitution between consumption and leisure equalizes the corresponding marginal rate of transformation

$$A_t = L_t^\phi C_t \quad (1.32)$$

The goods market clearing condition (1.11) and the production function (1.26), can be used to get the Pareto efficient values of output

$$Y_t^* = A_t \quad (1.33)$$

and employment

$$L_t^* = 1 \quad (1.34)$$

Hence, at the non-stochastic steady state

$$Y^* = C^* = L^* = 1$$

1.2.6 The Steady State

The non-stochastic steady state of the model is derived setting the shocks to their mean value. It is straightforward to prove that the steady state level of the gross inflation rate and price dispersion are equal to one, using aggregate demand and the law of motion for price dispersion. Moreover, from the short run aggregate supply and the definition of the auxiliary variables K_t and F_t , we can obtain the steady state value of output, employment and consumption

$$Y = L = C = \left[1 - \frac{1}{\eta}\right]^{\frac{1}{1+\phi}} \quad (1.35)$$

The result can be easily compared with the two benchmark cases usually studied in the literature, monopolistic competition and perfectly competitive labor markets, that can in turn be seen as particular cases of the non-atomistic wage setters framework.

Letting the number of unions tend to infinity, employment, consumption and output are back to the monopolistic competition levels

$$\lim_{n \rightarrow \infty} L = \lim_{n \rightarrow \infty} \left[1 - \frac{1}{\eta}\right]^{\frac{1}{1+\phi}} = \left[\frac{\theta_w - 1}{\theta_w}\right]^{\frac{1}{1+\phi}} \quad (1.36)$$

When indeed there are infinitely many unions, their mass tends to zero and they do not internalize the effect of their actions on the aggregate variables. As a consequence, the strategic interaction channel is shut down and the degree of Pareto inefficiency depends only on the degree of substitutability among labor types in the production process.

The perfect competition result arises instead when perfect substitutability among labor types is assumed

$$\lim_{\theta_w \rightarrow \infty} L = \lim_{\theta_w \rightarrow \infty} \left[1 - \frac{1}{\eta} \right]^{\frac{1}{1+\phi}} = 1 \quad (1.37)$$

as labor demand becomes perfectly elastic.

Some conclusions can be drawn looking at the steady state level of employment (1.35).

First, recall from (1.34) that the efficient level of employment is $L_t^* = 1$. Hence, the steady state is not Pareto efficient: imperfect substitutability of labor types and the presence of unions drive a wedge between the marginal productivity of labor and the marginal rate of substitution, determining a suboptimal employment equilibrium level. As market power on the goods markets is offset by fiscal policy, the steady state distortion is coming exclusively from the labor market side.

Second, the steady state is not independent of the monetary policy rule. This is because the central bank is able to induce wage restraint by implementing tougher stabilization policies. Then the steady state level of employment, output and consumption are increasing functions of the coefficient entering the Taylor rule. The outcome of the model does not challenge the conventional neutrality result: a transitory shock to the nominal interest rate dies off in the long run and leaves the steady state unaffected. The way in which the central bank systematically behaves, however, has an impact on real economic activity.

Moreover, the labor market structure interacts with monetary policy in determining

the long-run equilibrium values of the real variables. In fact, the way in which a change in the degree of wage setting centralization affects equilibrium depends on the monetary policy stance: a less unionized labor market enhances welfare, provided that monetary policy is not too aggressive in stabilizing inflation. To prove it, it is sufficient to look at the elasticity of labor demand perceived by the j -th union (1.30). As it is a weighted average between θ_w and Σ_L , where the weights are respectively $1 - 1/n$ and $1/n$, η increases in n if and only if $\theta_w > \Sigma_L$. This is equivalent to say that η increases in n if and only if

$$\gamma_\pi \leq \bar{\gamma}_\pi \tag{1.38}$$

where

$$\bar{\gamma}_\pi \equiv \theta_w \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)} \tag{1.39}$$

As the steady state is in turn increasing in η , a decentralization of wage setting raises long-run employment only when (1.38) is satisfied. This result is quite intuitive. The presence of unions has two opposite effects: on one side the higher market power tends to depress employment; on the other hand, the strategic interaction channel of monetary policy tends to increase employment restraining real wage demands. The second effect decreases with the number of unions, because unions with a lower mass internalize less the consequences of their actions on aggregate variables. When the central bank is aggressive, the wage restraint induced by monetary policy is very important and it may be excessively costly to reduce the degree of wage setting centralization. When (1.38) is satisfied, the argument is reversed and the lower is the mass of the unions, the higher is welfare. For a sensible calibration of parameters, the threshold value of γ_π is much higher than the one empirically observed⁹. Then, for empirically plausible values of parameters, a decentralization in the wage bargaining process is welfare enhancing. This seems to be in contrast with some contributions pointing towards a hump-shaped

⁹For the calibration considered below and displayed in Table 3 the threshold value is equal to 128.1553

relation between centralization of wage setting and employment¹⁰. This is because the model is well defined only for $n \geq 2$. A single encompassing union would act as a planner and would behave so as to attain Pareto efficiency, independently of monetary policy.

Finally, in the case of $\gamma_\pi \rightarrow \infty$, efficiency is restored

$$\lim_{\gamma_\pi \rightarrow \infty} L = \lim_{\gamma_\pi \rightarrow \infty} \left[1 - \frac{1}{\eta} \right]^{\frac{1}{1+\phi}} = 1 \quad (1.40)$$

This case is known in the literature as strict inflation targeting. When the coefficient entering the Taylor rule tends to infinity, inflation is on target not only on average, but also period by period. Since the target inflation rate implied by the specified Taylor rule is zero, strict inflation targeting allows the central bank to achieve price stability also outside the steady state. The model predicts that strict inflation targeting implements Pareto efficiency in the long run, through the strategic interaction channel. This result introduces an additional reason to penalize deviations from price stability.

1.2.7 The Dynamics

Log-linearizing the model around the non-stochastic steady state allows to fully characterize the equilibrium dynamics at a first order accuracy by

$$\hat{x}_t = E_t \hat{x}_{t+1} - (i_t - E_t \pi_{t+1} - r_t^*) \quad (1.41)$$

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} (1+\phi) \hat{x}_t \quad (1.42)$$

(1.41) and (1.42) are respectively the conventional IS equation and the New-Keynesian Phillips curve (NKPC) and r_t^* is an exogenous disturbance defined as

$$r_t^* = -(1-\rho_a)a_t + \rho$$

¹⁰For instance, this is the case made by Calmfors and Driffill (1988). However, the empirical and the literature are far from having reached a consensus in this respect. For a survey of the issue, see Calmfors (2001).

Note that the output gap

$$\hat{x}_t \equiv \hat{y}_t - \hat{y}_t^* \tag{1.43}$$

refers to output deviations from Pareto efficiency rather than from the flexible price equilibrium. In fact, the flexible price equilibrium does not need to be efficient, as fiscal policy is not assumed to offset the static distortion arising from imperfectly competitive labor markets. Therefore, it is more insightful to relate inflationary pressures to a welfare relevant variable such as the gap between actual and efficient output.

From the Phillips curve, it is immediate to see that strict inflation targeting allows to fully stabilize the output gap. The steady state value of the gap depends on the monetary policy stance and, as it has been previously showed, it is zero under a strict inflation targeting policy. Hence, price stability is consistent with the elimination of any deviation of real economic activity from Pareto efficiency and it is, as a consequence, the optimal policy. This is the outcome of strategic interaction: the central bank affects the equilibrium level of output not only through aggregate demand, but also through labor supply. Then, it is possible to push the economy towards Pareto efficiency without creating inflation. This result distinguishes the model outlined here from the standard NK, where, without the proper fiscal policy, zero inflation under full commitment is still optimal, but can be reached only at the cost of a suboptimal production level.

1.3 Conclusions

This Chapter builds a model where the presence of large wage setters creates a new monetary policy transmission channel. Two main differences with respect to the baseline model should be highlighted.

First, the policy rule has a permanent effect on real economic activity, while in the standard framework changes in the policy rule do not have any effect on the steady state value of real variables.

Moreover, when the strategic interaction channel is at work, the central bank can always push output towards Pareto efficiency by being tougher in stabilizing inflation. Hence, strict inflation targeting allows to simultaneously stabilize inflation and output around its efficient value.

Price stability as the optimal policy clearly emerges as a consequence of the fact that price stickiness is the only source of dynamic inefficiency. The introduction of other dynamic distortions would create a policy trade-off between inflation and output gap stabilization. It would be interesting to allow for a non trivial policy problem and characterize the optimal monetary policy. This is the topic of Chapter 2.

Figure 1.1: Union density versus union coverage in OECD countries, 2000.
 Source: OECD Employment Outlook 2004. Percentage of wage earners.

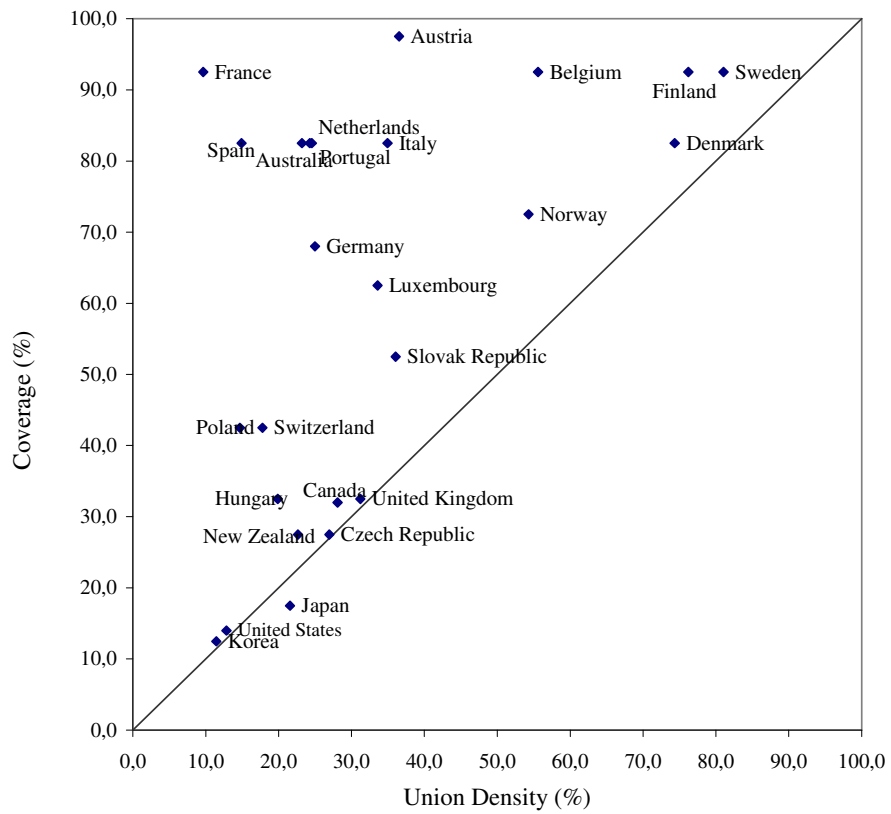


Table 1.1: **Centralization of Collective Bargaining in OECD countries for the period 1995-2000. Source: OECD Employment Outlook 2004.** 1 = Company and plant level predominant; 2 = Combination of industry and company/plant level; 3 = Industry level is predominant; 4 = Predominantly industrial bargaining, but also recurrent central-level agreements; 5 = Central-level agreements of overriding importance.

Country	Centralization
Austria	3
Belgium	3
Czech Republic	1
Denmark	2
Finland	5
France	2
Germany	3
Hungary	1
Ireland	4
Italy	2
Netherlands	3
Poland	1
Portugal	4
Slovak Republic	2
Spain	3
Sweden	3
United Kingdom	1
Switzerland	2
Australia	2
Canada	1
Japan	1
United States	1

Table 1.2: **Coordination of Collective Bargaining in OECD countries for the period 1995-2000. Source: OECD Employment Outlook 2004.** (a) The degree of co-ordination includes both union and employer co-ordination. Each characteristic has been assigned a value between 1 (little or no co-ordination by upper-level associations) and 5 (co-ordination of industry- level bargaining by encompassing union confederation or co-ordinated bargaining by peak confederations or government imposition of wage schedule/freeze, with a peace obligation).

Country	Coordination
Austria	4
Belgium	4.5
Czech Republic	1
Denmark	4
Finland	5
France	2
Germany	4
Hungary	1
Ireland	4
Italy	4
Netherlands	4
Poland	1
Portugal	4
Slovak Republic	2
Spain	3
Sweden	3
United Kingdom	1
Switzerland	4
Australia	2
Canada	1
Japan	4
United States	1

Chapter 2

Optimal Simple Monetary Policy Rules and Non-Atomistic Wage Setters

2.1 Introduction

Chapter 1 extends a baseline DSGE model with sticky prices to the case of a unionized labor force. The goal of the present Chapter¹ is to allow for a non trivial policy trade-off in a NK model augmented with unions and to study how such a trade-off is modified depending on the labor market structure, in order to characterize the optimal monetary policy. As a first step the analysis is restricted to the case of simple rules, while the case of the fully optimal policy is left to future research. The design of the optimal policy rule is performed by using the methodology introduced by Rotemberg and Woodford (1997) and further developed by Benigno and Woodford (2005). The method resorts to a second order approximation to households' lifetime utility as an approximate welfare measure. Because of the long-run non-neutrality of the rule, the welfare measure is decomposed in such a way to disentangle the steady state and the stabilization effects of policy.

¹The first version of the paper on which the chapter is based has been circulated as ECB Working Paper No.690, October 2006, <http://ecb.int/pub/pdf/scpwps/ecbwp690.pdf>

We show that the presence of large wage setters creates an additional dimension of the policy trade-off with respect to the one traditionally considered by the literature. This is because in a model with unions, being more aggressive in stabilizing inflation allows to reduce steady state distortion by inducing wage restraint. But, as tougher inflation stabilization policies amplify output gap volatility, the policy maker has to trade-off steady state efficiency against dynamic efficiency. Two are the forces underlying the policy dilemma: wage setting centralization and the volatility of the cost push shock. Highly centralized labor markets are associated to high gains of aggressiveness in terms of average distortion. In fact, larger unions internalize more the impact of their wage policy on inflation, making more effective the strategic interaction channel of monetary policy. On the other hand, the more volatile is the cost push shock, the more costly is price stability in terms of gap fluctuations. This implies a high cost of reducing average distortion.

The two forces interact resolving the policy trade-off and determining the following optimal policy results.

If the volatility of the cost push shock is sufficiently low and the concentration of the labor market is high enough, strict inflation targeting is optimal. A high volatility of the cost push shock induces the policy maker to accept some volatility of inflation. However, optimal aggressiveness increases in labor market concentration. Finally, a decomposition of the approximate welfare measure allows to compute the cost of deviating from the optimal policy and to decompose the total effect in steady state cost and stabilization cost. It is showed that the steady state cost, as a fraction of the total, decreases with the standard deviation of the cost push shock and increases with wage setting centralization.

Section 2 describes the model economy, Section 3 derives and gives an economic interpretation of the welfare criterion, Section 4 computes the optimal simple interest rate rule. Section 5 concludes.

2.2 The Model: Sticky Prices, Unionized Labor Markets and Wage Mark-up Shocks

Consider the same economy as in Chapter 1. The model outlined there shares with the baseline NK model an unpleasant feature: the lack of a non-trivial policy trade-off, which is perceived to be as an empirically relevant problem by any central banker. It is needed to create a tension between inflation and output gap stabilization. Therefore, it is assumed from now on that the wage mark-up is fluctuating exogenously around its mean value². The first order condition is modified accordingly to include a random shock

$$\frac{W_t}{P_t} = \exp\{\mu_t^w\} \frac{\eta}{\eta - 1} L_t^\phi C_t \quad (2.1)$$

μ_t^w follows an autoregressive process represented by

$$\mu_{t+1}^w = \rho_u \mu_t^w + \varepsilon_{t+1,u} \quad (2.2)$$

where $\varepsilon_{t,u}$ is white noise with standard deviation denoted by $\sigma_{\varepsilon,u}$ ³.

Whenever the economy is hit by wage mark-up shocks, it is not feasible to attain the Pareto efficient outcome by stabilizing inflation. In fact, complete inflation stabilization replicates the flexible price equilibrium, which is not efficient because of stochastic deviations of the real wage from the marginal rate of substitutions. As a consequence, the central bank has to trade-off inflation fluctuations against output deviations from Pareto efficiency.

²This can be seen as a shortcut to include other forms of nominal rigidities, such as wage stickiness. See also Clarida et al. (1999), Galí (2003) and Woodford (2003)

³As before Σ_L is not constant over time. Again, you can show that, for empirically relevant values of the parameters and for the calibrations considered below, elasticity fluctuations do not generate quantitatively significant variation out of the steady state at a second-order accuracy. To this purpose the model has been approximated to second order and simulated using the method developed by Schmitt-Grohé and Uribe (2004b). Then it is assumed in the rest of the paper that elasticity is constantly equal to its steady state value. This is inconsequential also for the results obtained in the welfare analysis.

As the dynamics of the model is now driven by an additional shock, the sticky price allocation can be redefined as it follows. Let $x_t = (Y_t, \Pi_t, \Delta_t)$ and $X_t = (F_t, K_t)$. Given Δ_{-1} , exogenous stochastic processes A_t and μ_t^w and given a value for the policy parameter γ_π , the rational expectation equilibrium for the sticky price economy is a process $\{x_t, X_t\}_{t=0}^\infty$ that satisfies the following system of equations

$$\begin{aligned}
Y_t^{-1} &= \Pi_t^{\gamma_\pi} E_t \{ \Pi_{t+1}^{-1} Y_{t+1}^{-1} \} \\
\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} &= \left(\frac{K_t}{F_t} \right)^{1 - \theta_p} \\
K_t &= \frac{\eta}{\eta - 1} \exp\{\mu_t^w\} \left(\frac{Y_t}{A_t} \right)^{1 + \phi} \Delta_t^\phi + \alpha \beta E_t \{ (\Pi_{t+1})^{\theta_p} K_{t+1} \} \\
F_t &= 1 + \alpha \beta E_t \{ (\Pi_{t+1})^{\theta_p - 1} F_{t+1} \} \\
\Delta_t &= (1 - \alpha) \left(\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} \right)^{\frac{\theta_p}{\theta_p - 1}} + \alpha \Pi_t^{\theta_p} \Delta_{t-1} \\
\eta &= \theta_w \left(1 - \frac{1}{n} \right) + \frac{1}{n} \gamma_\pi \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha}
\end{aligned}$$

While changing the dynamics, the presence of cost push shocks does not alter the steady state of the economy. Hence, the analysis performed in Chapter 1 continues to hold.

Before introducing the policy problem, it is convenient to define a measure of average distortion. A reasonable candidate is the wedge between marginal productivity and the marginal rate of substitution. While the efficient steady state implies the following marginal rate of substitution

$$mrs^* = (L^*)^\phi C = 1$$

at the actual steady state

$$mrs = L^\phi C = 1 - \eta^{-1}$$

so that $\Phi \equiv \eta^{-1}$ can be defined as a measure of steady state inefficiency.

2.3 The Policy Problem

The previous section defines the private sector equilibrium when the central bank credibly commits to a monetary policy rule. The policy problem faced by the central bank can then be described as the choice of the coefficients entering the rule, taking into account the reaction of the agents to the policy commitment.

I wish to find the optimal monetary policy rule within a class of simple and implementable rules of the kind described by equation (1.1). A rule is said to be implementable if it brings about a locally unique rational expectation equilibrium in a neighborhood of the non-stochastic steady state, under the assumption of sufficiently tightly bounded exogenous processes. An implementable rule is optimal, within the particular family of policies taken into consideration, if it yields the highest value for a suitably defined welfare criterion.

The definition of such a criterion and the analysis of its implications for the monetary policy problem are the objects of the section. The issue is addressed using the linear-quadratic approach introduced by Rotemberg and Woodford (1997) and further developed by Benigno and Woodford (2005). Because of the long-run non-neutrality of the rule, the welfare measure is decomposed in such a way to disentangle the steady state and the stabilization effects of policy.

Optimality is judged from a timeless perspective. For a policy to be optimal in this sense, it is sufficient to limit central bank's ability to exploit the expectations already in place at the time the commitment is chosen.

2.3.1 The Welfare Criterion

The conditional expectation of lifetime utility as of time zero is

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{L_t^{1+\phi}}{1+\phi} \right] \quad (2.3)$$

It might seem natural to define the optimal policy rule at time zero as the one that maximizes (2.3) subject to the constraints imposed by the behavior of the private sector. However, the use of (2.3) leads to a time inconsistent selection of the rule. This is because the optimal choice correctly takes into account the effects of policy on future expectations, but not on the expectations formed prior to time zero. Past expectations about current outcomes are in fact given at the time of policy selection. As a consequence, should the policy be reconsidered at a later period, the new commitment would not be a continuation of the original plan: the policy maker has the incentive to fool the agents whenever she has the possibility of revising her commitments. To overcome the time inconsistency problem, I closely follow Benigno and Woodford (2005) who propose to penalize the rules exploiting the expectations already in place at the time the commitment is chosen. According to their method the welfare criterion can be defined in three steps. The intuition of the procedure is described below while I refer to the appendix for the technical details.

First, one needs to characterize the unconstrained timelessly optimal policy. The term unconstrained here refers to the fact that the optimal policy does not necessarily need to be implemented by a simple policy rule of the kind described by equation (1.1). Note also that, differently from the case studied by Benigno and Woodford (2005), average distortion is controlled by the monetary authority.

Second, it is computed the gain of fooling the agents, that is the value of choosing a policy that does not validate past expectations about current equilibrium outcomes. This is equivalent to compute the gain of deviating from the timelessly optimal plan.

Finally, the welfare criterion is constructed by subtracting from U_0 the gain of

fooling the agents, $\Psi(\mu^{w,0})$, associated to the policy under scrutiny

$$\hat{U}_0 = U_0 - \Psi(\mu^{w,0})$$

Since $\Psi(\mu^{w,0})$ is a function of the whole history of cost push disturbances up to time zero, it is computed the unconditional expected value of the modified welfare criterion, integrating over all possible histories of shocks. A second order approximation to \hat{U}_0 yields the purely quadratic approximate welfare measure

$$\begin{aligned} \hat{W}_0 = & \frac{\bar{U}(\Phi)}{1-\beta} - \frac{1}{2} \frac{\Phi(1-\Phi)}{1+\phi} E \sum_{t=0}^{\infty} \beta^t (\mu_t^w)^2 + \\ & - \frac{1}{2} \frac{\theta_p}{\lambda} E \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + (1+\phi) \frac{\lambda}{\theta_p} \hat{x}_t^2 \right] - E\Psi(\mu^{w,0}) \end{aligned} \quad (2.4)$$

where $\lambda = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$ and \bar{U} is the steady state level of utility. All variables are expressed in log deviations from the non-stochastic steady state and the welfare relevant output gap

$$\hat{x}_t \equiv \hat{y}_t - \hat{y}_t^*$$

is defined as the output deviation from a properly defined target

$$\hat{y}_t^* \equiv a_t - \frac{\Phi}{1+\phi} \mu_t^w$$

The welfare criterion can be used not only to determine the rule that is optimal within a given class, but also to compute the cost of deviating from the optimized rule. Consider two policy regimes, R (reference) and A (alternative), respectively characterized by the induced allocations $(\{C_t^R, L_t^R\}_{t=0}^{\infty})$ and $(\{C_t^A, L_t^A\}_{t=0}^{\infty})$. Then the associated welfare is

$$U^R = U(\{C_t^R, L_t^R\}_{t=0}^{\infty}) \text{ and } U^A = U(\{C_t^A, L_t^A\}_{t=0}^{\infty})$$

Let the cost of regime A be denoted by γ . I measure γ as the fraction of regime R's consumption that households would be willing to give up in order to be as well off as under regime A. Formally it is implicitly defined by

$$U(\{(1 - \gamma)C_t^R, L^R\}_{t=0}^\infty) = U(\{C_t^A, L^A\}_{t=0}^\infty)$$

It can be easily shown that, given the functional form of the utility function

$$\gamma = 1 - \exp\{(1 - \beta)(U^A - U^R)\} \quad (2.5)$$

2.3.2 Average Distortion, Inflation Stabilization and Welfare

A well defined approximate welfare measure allows to analyze what are the objectives of a benevolent central bank willing to choose the state-contingent path of the economic variables preferred by the private sector. It turns out that, differently from a standard NK framework, the evaluation of alternative policies cannot disregard possible effects stemming from the policy rule non-neutrality due to the presence of unionized labor markets.

In fact, the welfare function can be decomposed into two parts: a stabilization component measuring the welfare effects of fluctuations around the non-stochastic steady state

$$W_0^{Stab} = -\frac{1}{2} \frac{\theta_p}{\lambda} E \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + (1 + \phi) \frac{\lambda}{\theta_p} \hat{x}_t^2 \right] - \Psi(\mu^{w,0}) \quad (2.6)$$

and a steady state component measuring the welfare effects due to a change in the average distortion of the economy

$$W_0^{StSt} = \frac{\bar{U}(\Phi)}{1 - \beta} - \frac{1}{2} \frac{\Phi(1 - \Phi)}{1 + \phi} E \sum_{t=0}^{\infty} \beta^t (\mu_t^w)^2 \quad (2.7)$$

The stabilization component provides a rationale for minimizing inflation and output gap deviations from properly defined targets. Inflation fluctuations are penalized in that they create unnecessary variability in the relative price dispersion. The target level of inflation is zero, because only complete price stability would remove any dispersion in relative prices. Fluctuations in the output gap are also costly. This is because price stickiness implies inefficient changes in the average mark-up charged by firms. As in the case of atomistic agents studied by Benigno and Woodford (2005), the output target is a linear combination of the natural and the efficient output

$$\hat{y}_t^* = \Phi \hat{y}_t^n + (1 - \Phi) \hat{y}_t^{FB}$$

where \hat{y}_t^n

$$\hat{y}_t^n \equiv a_t - \frac{1}{1 + \phi} \mu_t^w \tag{2.8}$$

is the natural output and the efficient output is

$$\hat{y}_t^{FB} = a_t \tag{2.9}$$

The case of non-atomistic agents exhibits however an interesting additional feature. For the policy rule has permanent real effects, steady state distortion, which is commonly disregarded as independent of policy, cannot be taken as given in a model featuring the presence of large wage setters. In particular, when alternative policy rules are evaluated on welfare theoretical grounds, one cannot abstract from the contribution of the steady state component W_0^{StSt} . Looking at (2.7), two are the channels through which average distortion affects welfare. The first one is represented by the term

$$\frac{\bar{U}(\Phi)}{1 - \beta}$$

This is the discounted steady state level of utility, which is a decreasing function of Φ . Recall that Φ is the wedge between the marginal rate of substitution and the marginal rate of transformation. As long as Φ is positive, the agents are willing to give up leisure in exchange for consumption at a rate that is on average higher than the one implied by the technological constraints. Hence, they would be better off consuming less leisure and more goods. Tougher stabilization policies induce unions to restrain wages, increasing the steady state level of employment and then enhancing efficiency and welfare. The second component

$$-\frac{1}{2} \frac{\Phi(1-\Phi)}{1+\phi} E \sum_{t=0}^{\infty} \beta^t (\mu_t^w)^2$$

isolates the negative effect of inefficient wage mark-up fluctuations. When the steady state is non distorted, this term disappears and wage mark-up fluctuations do not matter per se but only to the extent they create output gap variability. Only when the steady state is distorted, changes in the mark-up directly and negatively affect welfare. The result is quite intuitive: though transitory, inefficient fluctuations add on top of a positive and permanent level of average distortion, then it would be welfare improving to smooth them over the cycle. It can be proved that the steady state component is strictly decreasing in average distortion⁴.

The analytical expression of the welfare criterion allows to get the intuition of how the policy problem is affected by the strategic interaction channel of monetary policy. Big players in the labor markets internalize the consequences of their actions on aggregate variables. This gives the monetary authority a chance of controlling average distortion that in turn reduces welfare through the two channels described above. As a consequence, the central bank has an additional reason to stabilize inflation other

⁴There exists a threshold value for the variance of the cost push shock such that, above that threshold, steady state welfare is not monotone decreasing in average distortion. However, for those values the approximation would not be second order accurate, so that the analysis disregards this case.

than the usual concern about relative price dispersion: the policy maker has to face an additional dilemma.

2.3.3 The Trade-Off: an Additional Dimension

Being the welfare criterion purely quadratic, it is sufficient to approximate the structural equations to first order, to obtain an approximation to the optimal policy at a first order accuracy. Hence, the policy problem consists in selecting the inflation coefficient entering the policy rule in order to maximize \hat{W}_0 subject to the following log-linear constraints

$$\hat{x}_t = E_t \hat{x}_{t+1} - (i_t - E_t \pi_{t+1} - r_t^*) \quad (2.10)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + (1 - \Phi) \lambda \mu_t^w \quad (2.11)$$

where (2.10) is the IS equation and (2.11) is the New-Keynesian Phillips curve (NKPC). r_t^* is a composite disturbance defined as follows

$$r_t^* = -(1 - \rho_a) a_t + (1 - \rho_u) \frac{\Phi}{1 + \phi} \mu_t^w + \rho$$

Looking at the policy problem, it is possible to isolate an additional dimension of the trade-off with respect to the one traditionally studied in the literature.

Because of the cost push disturbance, it is not feasible to fully stabilize inflation and output gap simultaneously: it is possible to reduce inflation volatility only at the cost of increasing gap volatility. This is the classical trade-off between inflation and output gap stabilization. In an economy populated by atomistic agents, its solution determines optimal fluctuations and provides a complete description of optimal monetary policy. In a model with unions, however, this is not the end of the story. It may be optimal to

deviate from those optimal fluctuations in exchange for less average distortion. But the only way to reduce average distortion is by being more aggressive in stabilizing inflation. Therefore, static efficiency can be enhanced only at the cost of more volatility in the output gap. In other words, static efficiency is costly in terms of dynamic efficiency: this is the additional dilemma faced by the policy maker.

The economic intuition suggests that the key forces underlying the new policy trade-off are the standard deviation of the cost push shock relative to the TFP shock, as in the baseline NK model, and wage setting centralization. The higher the relative standard deviation of the cost push shock (RS), the higher the cost of price stability relative to gap stability. Then, also the cost of reducing average distortion has to be higher in terms of dynamic efficiency. On the other hand, the more the labor market is concentrated, the bigger are unions and then the stronger is the strategic interaction channel of monetary policy. This implies that being tough in stabilizing inflation pays more in terms of average distortion, so that the additional dimension of the trade-off gains importance relatively to the traditional stabilization concerns.

2.4 Optimal Simple Policy Rules

I turn now to the design of the optimal simple rule which is subsequently used as a benchmark to evaluate the performance of alternative suboptimal rules. The welfare criterion is computed analytically. However, welfare maximization is performed numerically over a grid since first order conditions do not have a closed form solution. Before stating the optimal monetary policy results, it is useful to study the behavior of the welfare function.

It has been established so far that, under a timeless perspective, a benevolent policy

maker is choosing the rule in order to maximize

$$\begin{aligned}\hat{W}_0 &= \frac{\bar{U}(\Phi)}{1-\beta} - \frac{1}{2} \frac{\Phi(1-\Phi)}{1+\phi} E \sum_{t=0}^{\infty} \beta^t (\mu_t^w)^2 + \\ &\quad - \frac{1}{2} \frac{\theta_p}{\lambda} E \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + (1+\phi) \frac{\lambda}{\theta_p} \hat{x}_t^2 \right] - E\Psi(\mu^{w,0})\end{aligned}$$

subject to the constraints imposed by private agents' behavior

$$\hat{x}_t = E_t \hat{x}_{t+1} - (i_t - E_t \pi_{t+1} - r_t^*)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + (1-\Phi) \lambda \mu_t^w$$

Using the IS equation, the Phillips curve and the policy rule the equilibrium dynamics can be represented by a system of stochastic difference equations

$$\begin{bmatrix} \hat{x}_t \\ \pi_t \end{bmatrix} = A E_t \begin{bmatrix} \hat{x}_{t+1} \\ \pi_{t+1} \end{bmatrix} + B(r_t^* - \rho) + C \lambda (1-\Phi) \mu_t^w \quad (2.12)$$

where

$$\Omega = \frac{1}{1 + \kappa \gamma_\pi}$$

$$A = \Omega \begin{bmatrix} 1 & 1 - \beta \gamma_\pi \\ \kappa & \kappa + \beta \end{bmatrix}$$

$$B = \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

$$C = \Omega \begin{bmatrix} -\gamma_\pi \\ 1 \end{bmatrix}$$

The system has a unique solution and the state-contingent evolution of inflation and output gap is

$$\pi_t = f_{\pi,a} a_t + f_{\pi,u} \mu_t^w \quad (2.13)$$

$$\hat{x}_t = f_{x,a}a_t + f_{x,u}\mu_t^w \quad (2.14)$$

where $f_{\pi,a}$, $f_{\pi,u}$, $f_{x,a}$ and $f_{x,u}$ are a function of structural parameters and of the coefficients entering the policy rule. The solution of inflation and output gap are used in the welfare criterion to solve for expectations. Finally, (2.4) can be related to the monetary policy stance.

$$\begin{aligned} E\hat{W}_0 = & \frac{\bar{U}(\Phi)}{1-\beta} - \frac{1}{2} \frac{\Phi(1-\Phi)}{1+\phi} \frac{\sigma_u^2}{1-\beta} - \frac{1}{2} \frac{\sigma_a^2}{1-\beta} \frac{\theta_p}{\lambda} (f_{\pi,a}^2 + \tilde{\lambda}f_{x,a}^2) + \\ & - \frac{1}{2} \frac{\sigma_u^2}{1-\beta} \frac{\theta_p}{\lambda} (f_{\pi,u}^2 + \tilde{\lambda}f_{x,u}^2) + f_{\pi,u}\lambda\Gamma \end{aligned} \quad (2.15)$$

The Appendix shows how to recover coefficients $f_{\pi,a}$, $f_{\pi,u}$, $f_{x,a}$ and $f_{x,u}$ and function (2.15). Γ and $\tilde{\lambda}$ are convolutions of parameters defined in the Appendix.

Before computing the optimal monetary policy, it is instructive to look at the shape of the welfare criterion and to study how it changes when CWS and RS vary. In order to plot the welfare function it is considered a range of values for the monetary policy stance, chosen from an equally spaced grid on the interval [1.25,125]. The length of each subinterval is fixed to 0.25. Given the very high value of the upper bound of the grid, a policy setting $\gamma_\pi = 125$ is referred to as strict inflation targeting. Parameters are calibrated as it is reported in Table 2.1. These values are conventionally used in the NK literature. It has been checked that results are robust to alternative plausible calibrations. Concerning the cost push shock, autocorrelation is set to zero while alternative calibrations of $\sigma_{\varepsilon,u}$ are considered in order to match different values of the relative standard deviation, as it is displayed in Table 2.2. It is labelled as high, medium or low a cost push shock standard deviation that is respectively twenty, ten or five times TFP standard deviation. These are the three representative cases commented below. Note that in general the values considered for the standard deviation of the cost push shock are quite high. Hence the calibration is relatively conservative in the sense that results are biased against the argument that unionized labor markets matter for optimal monetary policy.

Two are the main results suggested by the numerical analysis.

First, given wage setting centralization and the chosen bounds for aggressiveness in inflation stabilization, you can find a value of the relative standard deviation, RS^* , such that if $RS < RS^*$ strict inflation targeting performs better than any other policy considered within the bounds. If relative standard deviation is higher, the welfare function has a maximum within the bounds⁵. This is because a high relative standard deviation implies high marginal costs of over-stabilizing inflation relatively to marginal gains in terms of average distortion: the stabilization dimension of the trade-off dominates the second one. The intuition is confirmed looking at the graphs.

The left hand panel of Figure 2.1 displays the welfare criterion for an economy with three unions and low RS . The function is strictly increasing in the inflation coefficient, hence strict inflation targeting is the optimal policy. The right hand panel shows the welfare cost of deviating from the optimized value. To grasp some insight, total welfare is decomposed in steady state and stabilization component in Figures 2.2 and 2.3 respectively. In both charts the solid line represents actual welfare while the dotted line is the value that corresponds to the inflation coefficient maximizing total welfare. Looking at Figures 2.2 and 2.3, it is immediate to see that in the optimal policy the steady state component is maximized while the stabilization component is not. Hence, given the degree of concentration in the labor markets a low RS resolves the trade-off between stabilization and average distortion in favor of the latter. The opposite is observed in the case of a high RS . Figure 2.4 again displays total welfare for an economy with three unions. Now the function has a maximum. If the effect of policy is decomposed, as in Figure 2.5 and 2.6, it is evident that the stabilization part is maximized while steady state welfare is not. The additional dilemma is dominated

⁵The apparent discontinuity is induced only by the fact that the welfare function is evaluated numerically over a grid. The most plausible conjecture, however, is that it can always be found a maximum if the upper bound of the grid is sufficiently high. Moreover, the results considered altogether do not suggest any discontinuity: high CWS always calls for higher γ_π and high RS always requires lower γ_π .

by the traditional concerns about stabilization.

Then, it can be inferred that the higher is the relative standard deviation of the cost push shock the less labor market unionization matters in terms of optimal monetary policy.

The second result is that RS^* is increasing with the centralization of wage setting: it is more likely to prefer strict inflation targeting when labor markets are concentrated. The intuition is that high CWS implies high steady state marginal gains from inflation stabilization. Once again it is insightful to have a look at the plots.

Consider the case of three unions and low, high or medium RS as depicted in Figures 2.1, 2.4 and 2.7 respectively. It can be easily seen that $RS^*=MEDIUM$, i.e. if the relative standard deviation of the cost push shock is higher than or equal to the medium value, then strict inflation targeting is not optimal. However, if you consider the case with two unions as in figures 2.8 and 2.9, it is clear that $RS^*=HIGH$. This means that when the degree of CWS increases it is needed a higher volatility of the cost push shock to rule out strict inflation targeting as the optimal policy.

With a clear intuition of how the welfare criterion is affected by the key forces underlying the policy trade-off, it is straightforward to interpret the optimal monetary policy results.

Optimal monetary policy is defined by the inflation coefficient entering the Taylor rule that maximizes the welfare criterion over the grid. Table 2.3 shows the value of γ_π as a function of the degree of centralization of wage setting and of the relative standard deviation of the cost push shock. The main result is that the optimal stance is always increasing in the centralization of the wage bargaining process. Interestingly, if the volatility of the cost push shock is sufficiently low and the concentration of the labor market is high enough, then strict inflation targeting is optimal even in the presence of inefficient fluctuations of output. This is the case of low RS and 2, 3 or 5 unions. On the other hand, for high values of the volatility of the cost push shock, the policy

maker accepts some volatility of inflation as in the standard NK model. However, the more the labor market is concentrated, the higher is the optimal aggressiveness.

Then, it can be concluded that the optimal policy is significantly affected by the labor market structure.

Welfare analysis allows to assess more closely the relevance of the changes induced in the policy prescriptions by the presence of a unionized labor force. Tables 2.4 and 2.5 display the welfare cost of adopting an ad-hoc Taylor rule with a coefficient $\gamma_\pi = 1.5$ instead of the optimal one. The two extreme cases of high and low RS are considered for an economy characterized by 2, 3 or 15 unions.

If RS is high, welfare costs are almost entirely accounted for by the stabilization component that is however implausibly high (always more than three percentage points). In the case of $N = 2$ the steady state cost is not negligible (0.2473 percentage points of consumption) while it is not significant for $N = 3$ and $N = 15$ (less than a hundredth of a percentage point). On the other hand, if RS is low and the labor market is highly concentrated (as in the case of $N = 2$ or $N = 3$), not only the steady state component is not negligible, it is also the most important part of the welfare cost. Finally, if the wage bargaining process is sufficiently decentralized, as for $N = 15$, the steady state component is again negligible as in the case of high RS .

Hence, welfare analysis suggests that both the total and the steady state cost of deviating from the optimal policy are increasing in the centralization of wage setting. In particular, the steady state cost as a fraction of the total increases with CWS and decreases with the relative standard deviation of the cost push shock.

We can conclude that, unless implausibly high values for the standard deviation of the cost push shock are assumed, it is costly to disregard the labor market structure as a determinant of the optimal monetary policy. This is because the central bank can induce wage restraint and then reduce average distortion through aggressive inflation stabilization. The gains stemming from aggressiveness are greater than the costs asso-

ciated to a higher variability of the output gap. The fact that that most of the cost is coming from the steady state component is in line with the economic intuition.

2.5 Conclusion

It has been studied whether and how the labor market structure affects the monetary policy problem in a model with nominal rigidities and non-atomistic unions. In particular, it is computed the optimal simple interest rate rule as a function of the degree of wage setting centralization.

The main finding is that the optimal aggressiveness in stabilizing inflation is increasing in wage setting centralization. Moreover, the relevance of policy prescriptions is assessed resorting to welfare analysis. It turns out that it is significantly costly to disregard possible inefficiencies stemming from high degrees of centralization of the bargaining process.

Figure 2.1: Low RS , 3 Unions

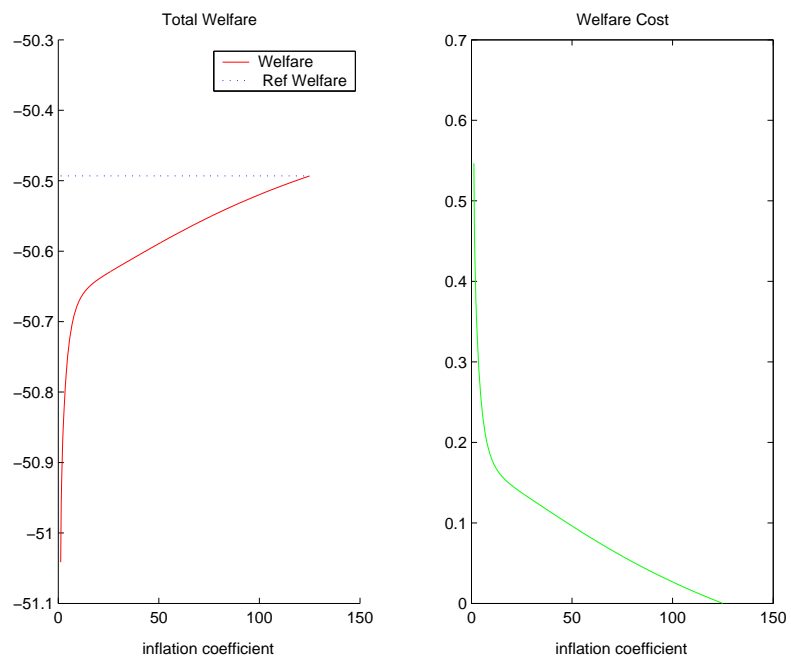


Figure 2.2: Low RS , 3 Unions

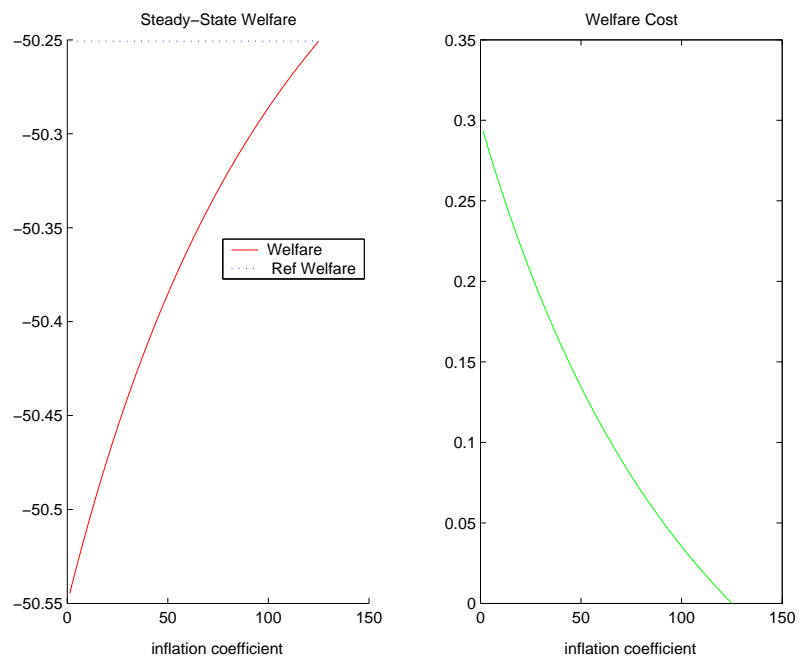


Figure 2.3: Low RS , 3 Unions

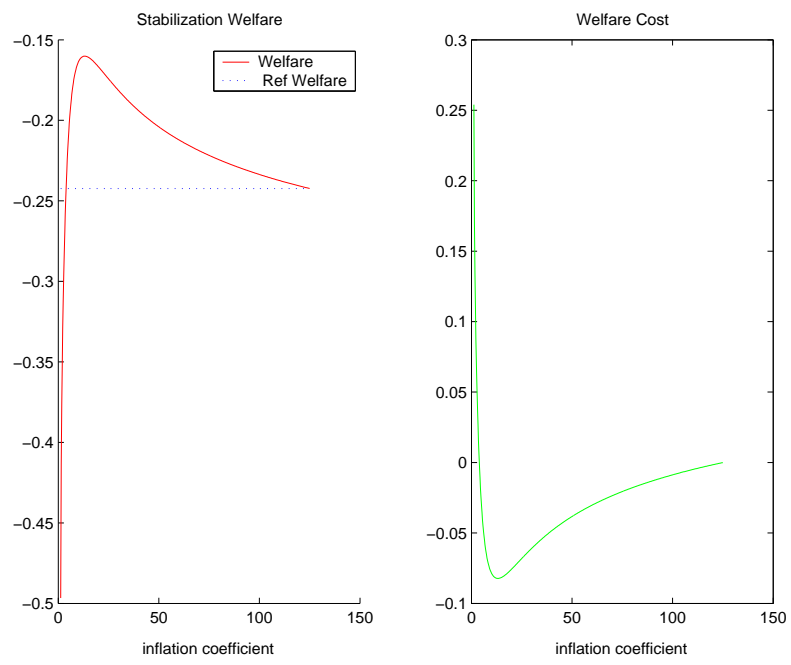


Figure 2.4: High RS , 3 Unions

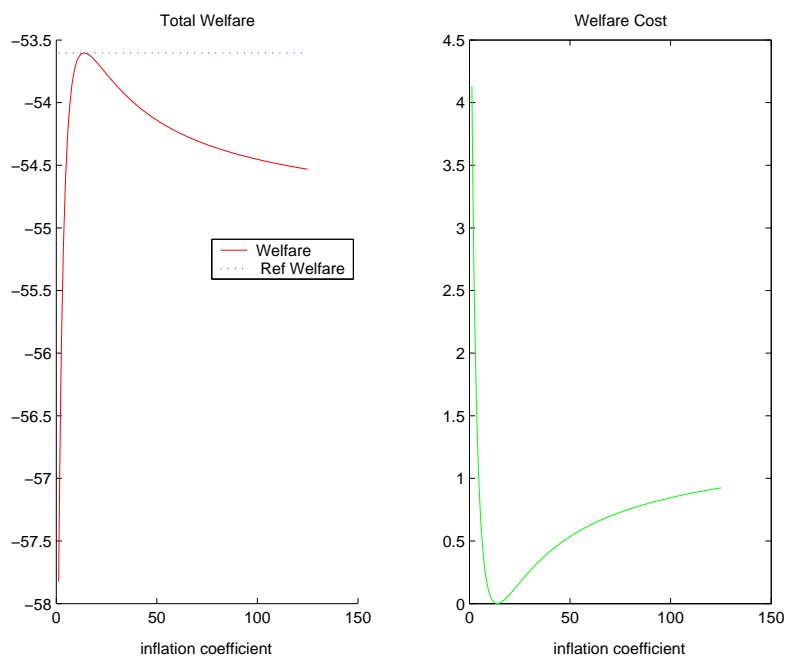


Figure 2.5: High RS , 3 Unions

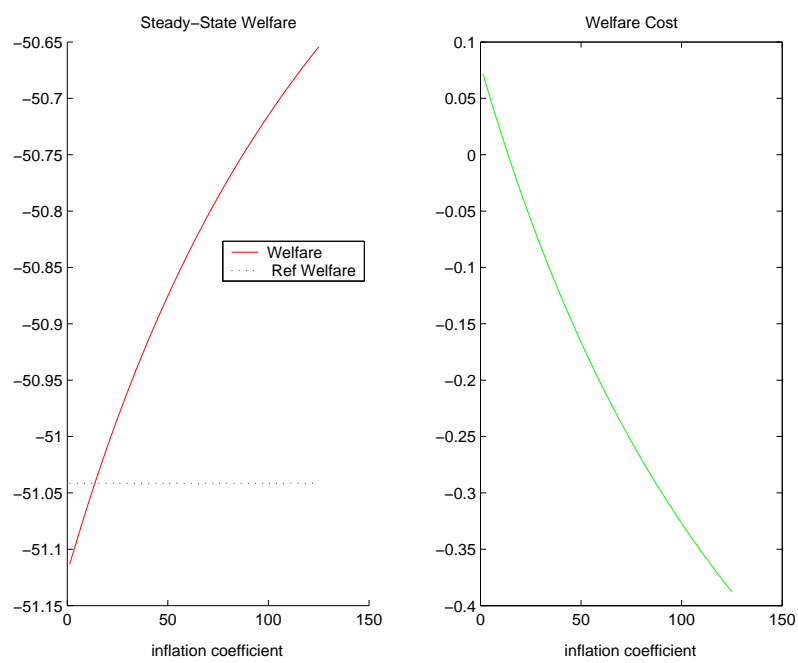


Figure 2.6: High RS , 3 Unions

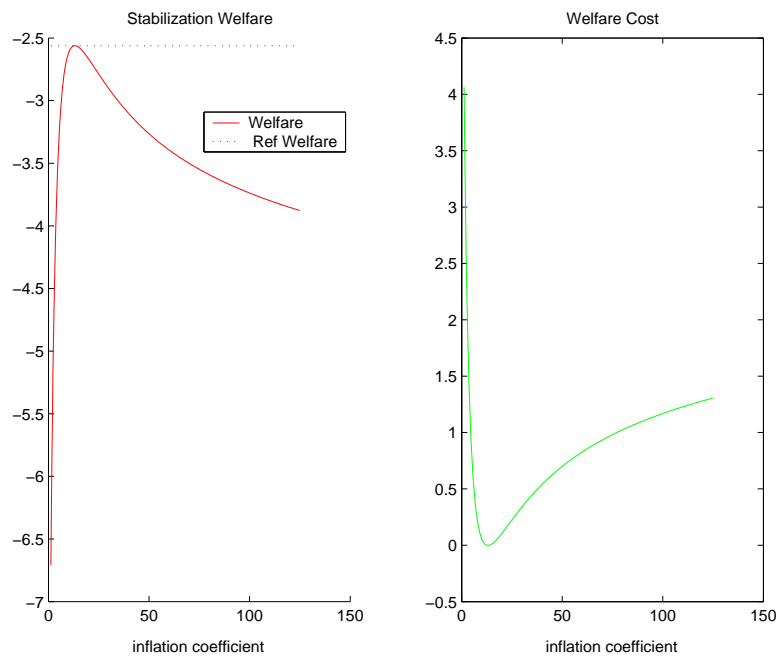


Figure 2.7: Medium RS , 3 Unions

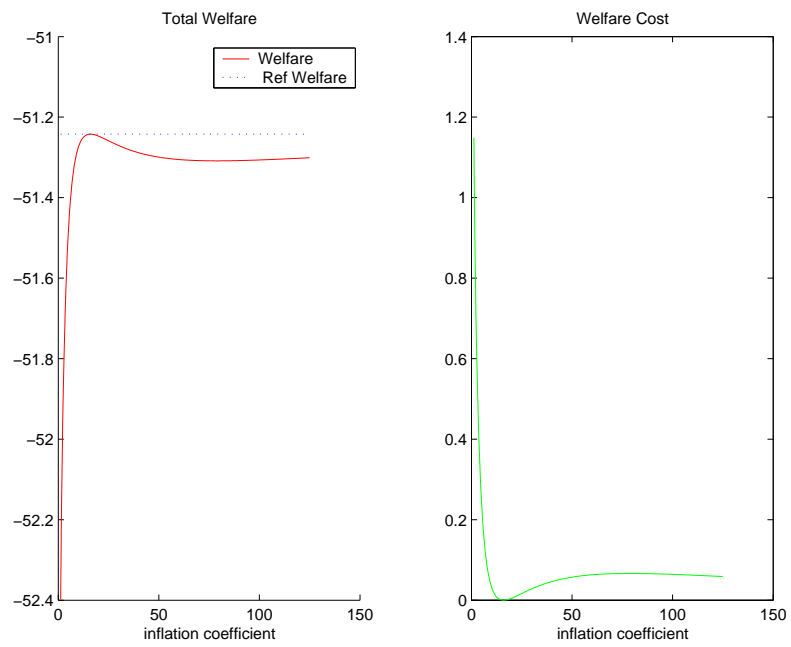


Figure 2.8: Medium RS , 2 Unions

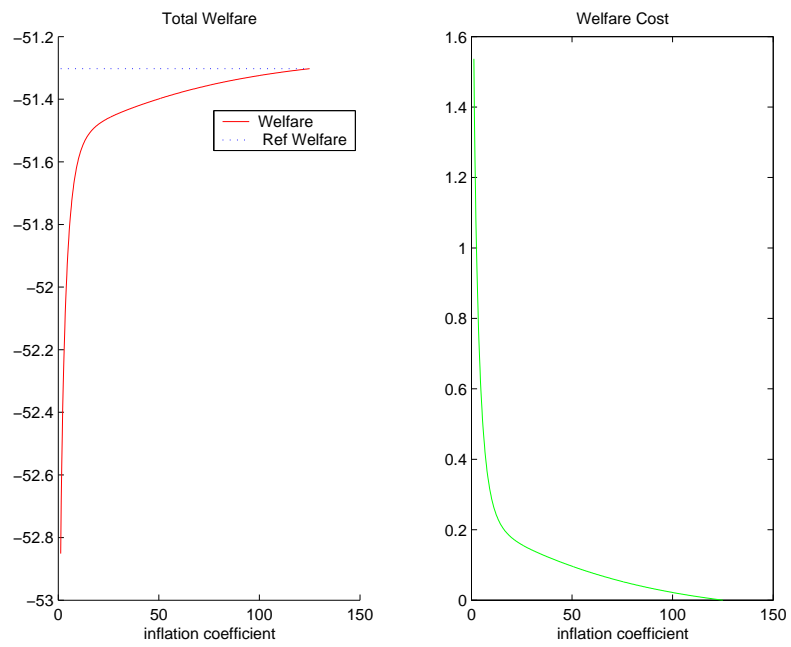


Figure 2.9: High RS , 2 Unions

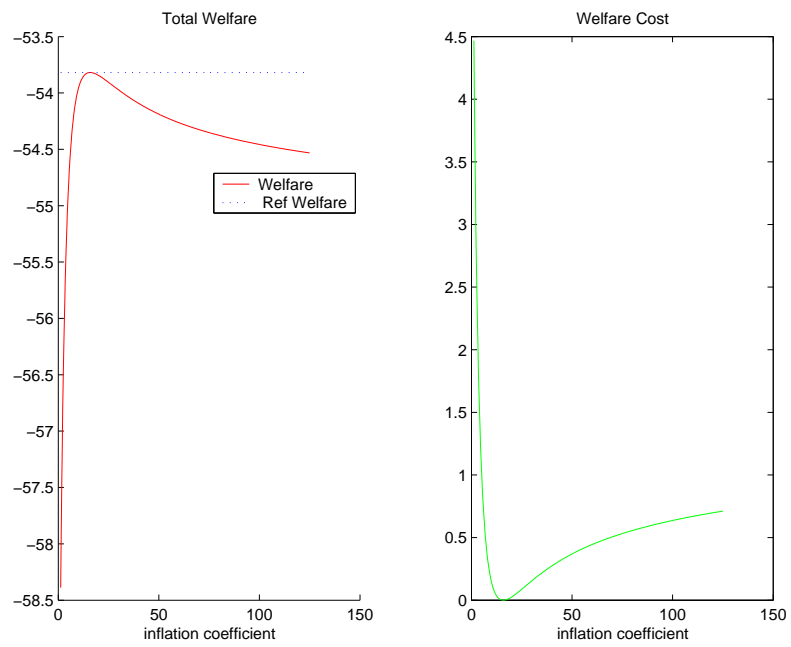


Table 2.1: **Baseline Calibration**

Parameter	Value
Price Stickiness	0.75
Discount Factor	0.99
Elast. Subst. Goods	11
Elast. Subst. Labor Types	11
Elast. Marginal Disutility Labor	1
TFP autocorrelation	0.95
TFP Std. Dev. Innovation	0.0071

Table 2.2: Cost Push Shock Calibration

Std. Dev. Innovation	Relative Std. Dev.
0.0227	1
0.0455	2
0.1137	5 (LOW)
0.2274	10 (MEDIUM)
0.4548	20 (HIGH)

Table 2.3: Optimal Monetary Policy

Std. Dev. CP	Number of Unions	Optimal Stance
High	2	16
High	3	13.75
High	5	13.25
High	10	13.00
High	15	12.75
Low	2-3-5	Strict Inflation Targeting
Low	10	14.00
Low	15	13.50

Table 2.4: **Welfare Costs of Deviation from Optimal Policy: High Cost Push Shock Standard Deviation.** The cost is measured relatively to the optimized rule. It is expressed as the percentage decrease in the output process associated to the optimal policy necessary to make welfare under the ad-hoc rule as high as under the optimized rule.

N	$\frac{\text{Optimal Stance}}{\text{Ad-hoc Rule}}$	Total Cost	Steady-State Cost	Stabilization Cost
2	$\frac{16}{1.5}$	3.9858	0.2473	3.7477
3	$\frac{13.75}{1.5}$	3.6646	0.0703	3.5968
15	$\frac{12.75}{1.5}$	3.4470	0.0058	3.4415

Table 2.5: **Welfare Costs of Deviation from Optimal Policy: Low Cost Push Shock Standard Deviation.** The cost is measured relative to the optimized rule. It is expressed as the percentage decrease in the output process associated to the optimal policy necessary to make welfare under the ad-hoc rule as high as under the optimized rule.

N	$\frac{\text{Optimal Stance}}{\text{Ad-hoc Rule}}$	Total Cost	Steady-State Cost	Stabilization Cost
2	$\frac{125}{1.5}$	0.8802	0.7118	0.1696
3	$\frac{125}{1.5}$	0.4627	0.2925	0.1708
15	$\frac{13.5}{1.5}$	0.2462	0.0036	0.2426

Chapter 3

Discretionary Fiscal Policy and Optimal Monetary Policy in a Currency Area

3.1 Introduction

Literature on optimal monetary and fiscal policy in currency areas is rapidly growing. In this context, the nominal exchange rate as a shock absorbing device is not available and, in the presence of nominal rigidities, fiscal policy is regarded as a potentially alternative instrument to deal with asymmetric shocks. Beetsma and Jensen (2004, 2005) and Galí and Monacelli (2005) characterize the optimal policy mix, under the assumptions of commitment of both monetary and fiscal policy, cooperation of fiscal authorities across countries and perfect coordination between the central bank and national governments. Two are the main results. Monetary policy stabilizes union average inflation and output gap. Fiscal policy only takes care of asymmetric shocks: average government expenditure is set at its efficient level and then is not used to stabilize the currency area as a whole.

These results constitute a useful benchmark. However, it is hard to believe that fiscal policy is set under commitment, cooperatively across countries and coordinating

with the monetary authority. A strand of the literature tests the robustness of optimal policy results to the assumption of fiscal cooperation. We merely focus on the issue of fiscal discretion. Credibility and transparency have recently become the key guidelines in the practice of central banking and they certainly are the criteria inspiring the design of European monetary policy institutions. It is according to those principles that the European Central Bank has been assigned by statute the primary objective to maintain price stability. In contrast, even if within the limits imposed by the Stability and Growth Pact, fiscal policy is conducted in a discretionary fashion by national governments, whose tenure is limited in time and whose unique mildly binding constraints are represented by electoral promises.

Such an asymmetry poses some questions of particular interest. First, it is relevant to assess the effects of fiscal discretion, to investigate whether they undermine the achievement of the stabilization goals pursued by the monetary authority and to study the optimal response of monetary policy to potential misbehavior of national governments. While those issues concern a closed economy as well as a currency area, the evaluation of welfare costs stemming from fiscal discretion features some peculiarities that are specific to the case of a monetary union. In fact, those costs could offset the benefits of using fiscal policy as a stabilization tool, as an alternative to the nominal exchange rate. To answer these questions, we modify the framework built by Galí and Monacelli (2005) to allow for a policy game, where the central bank commits to the optimal monetary policy plan, taking into account that fiscal policy is acting under discretion.

This Chapter¹ shows that discretionary governments generate an inefficiently loose aggregate fiscal stance, as long as the central bank faces a short-run trade-off. This is because the central bank and the government do not agree on the costs and benefits

¹The first version of the paper on which the chapter is based has been circulated as Working Paper No. 602 of Economics Department of Bologna University, September 2007, <http://www2.dse.unibo.it/wp/602.pdf>

associated to monetary policy actions. In particular, governments evaluate monetary policy tightening as more recessionary. As a consequence, they have the incentive to deviate from the full commitment solution and to generate a public spending over-expansion in the case of a negative output gap. This leads to higher than optimal volatility of government expenditure and, through aggregate demand, to higher than optimal volatility either of aggregate inflation or aggregate output gap. Hence, fiscal discretion exacerbates the stabilization trade-off, making harder the job of the central bank in dampening union-wide fluctuations. If the monetary authority internalizes government misbehavior, the optimal policy rule involves the targeting of union-wide fiscal stance, on top of inflation and output gap.

Finally, we perform welfare analysis, resorting to second order approximation to households' lifetime utility as a welfare criterion. Not surprisingly fiscal discretion entails welfare costs, the magnitude depending on the stochastic properties of the shocks. In particular, for some plausible parameter values, the cost is higher than the benefit of addressing asymmetric shocks. Therefore, the model casts some doubts on the desirability of using fiscal policy as a stabilization tool, or at least opens the question of designing suitable institutional arrangements to cope with the problem of discretionary governments.

3.2 Literature Review

Several papers study monetary and fiscal interaction, both in closed and open economy.

Dixit and Lambertini (2003b) consider the case of a model where output is sub-optimally low, as inefficiencies arising from monopolistically competitive goods markets are not corrected by any production subsidy. The fiscal policy objective function is assumed to be social welfare, while monetary policy is delegated to a central bank with an inflation target more conservative than society, in the spirit of the proposal by Rogoff (1985). It is showed that the constrained efficient outcome can be implemented

by assigning identical objectives to policy makers, being the output target the social optimum and the inflation target appropriately conservative. Dixit and Lambertini (2003a) assume policy makers to have the same inflation and output targets, but not necessarily the same weights, according to an ad-hoc quadratic objective function. It is showed that output and inflation goals can be achieved without the need for fiscal coordination across countries and without the need for monetary commitment, irrespectively of which authority moves first. Finally, Dixit and Lambertini (2001) show that under the more general case of different goals and weights, the conflict of objectives prevents both authorities to implement the desired outcomes. Our work differs from those contributions in two respects. First, we assume the central bank and the government to be benevolent, while they differ in their ability to commit to future policies. Second, the desired outcome is not implementable, as we allow for the presence of short-run stabilization trade-offs.

Faia (2005) studies the policy game arising in a currency area, where national governments independently choose domestic public spending and nominal debt. The Ramsey outcome is compared with a regime where all policy makers act under discretion and the common central bank is assumed to move after observing national governments' choices. In such a context, each government realizes both that the monetary authority has the incentive to deflate debt by loosening monetary policy and that the resulting inflation costs are shared among all area members. This generates a free riding problem, leading to an equilibrium characterized by higher than optimal debt, public spending and inflation. This contribution differs from ours, as the results abstract from the presence of nominal rigidities. In addition, monetary policy is not set optimally, because of the lack of commitment on the part of the central bank.

Adam and Billi (2007) investigate non-cooperative monetary and fiscal policy games in a closed economy featuring steady-state distortions under the assumption that policymakers cannot commit to future policies. In this environment, inflation and public

spending upward biases emerge as the optimal response to the static distortions. The paper shows that appointing a central banker more conservative than society in terms of inflation targets improves steady-state welfare, at the small cost of generating some stabilization biases, arising because of departures from the assumption of benevolent policy makers. The authors also compute the optimal inflation rate, defined as the one that would be chosen by a Ramsey planner internalizing that fiscal variables are chosen in a discretionary fashion. That optimal inflation rate is conceptually the same as the one derived in our model. However, only its steady-state value is computed while we are interested in characterizing its state-contingent path. This is because we want to focus on the optimal monetary policy response to shocks under fiscal discretion. Moreover, our analysis is performed within a linear-quadratic framework without steady-state distortions. This allows to derive an explicit targeting rule specifying how the objectives of the central bank optimally relate to each other.

Finally, contributions by Sargent and Wallace (1981), Leeper (1991) and Woodford (2001) gave rise to an important strand of the literature that characterizes the optimal monetary and fiscal policy mix leaving aside coordination problems, by assuming that all policy variables are chosen by a unique authority. A representative, though not exhaustive, sample includes Ferrero (2005), Lambertini (2006), Leith and Wren-Lewis (2006), Pappa and Vassilatos (2007) and Schmitt-Grohé and Uribe (2004a, 2007) together with the authors cited above, Beetsma and Jensen (2004, 2005) and Galí and Monacelli (2005).

3.3 The Private Sector Equilibrium

The currency union is represented by a continuum of infinitely many countries indexed by i on the unit interval $[0,1]$. Each country is a small open economy whereas the union as a whole is assumed to be a closed economic system. The members of the currency area have symmetric preferences and are ex-ante identical in terms of technology and

market structure, but they are subject to asymmetric shocks. Each economy is populated by infinitely many households and firms interacting on goods, labor and asset markets. Goods markets are imperfectly competitive and prices are set in staggered contracts with random duration. Labor markets are monopolistically competitive and labor mobility across countries is ruled out. Moreover, the wage mark-up is assumed to fluctuate exogenously around its mean value in order to create a meaningful policy trade-off at the union-wide level. Financial markets are complete and the law of one price is assumed to hold.

Monetary policy is in charge to set the union-wide nominal interest rate, while fiscal policy is responsible for choosing government expenditure and taxes. It is assumed for simplicity that the static distortion due to imperfect competition on goods and labor markets is undone by means of subsidies, while lump-sum taxes and transfers are available and they adjust so as to balance the government budget constraint at all times.

It is described next the private sector equilibrium as a function of monetary and fiscal policy.

3.3.1 Households

Each household in country i consumes a continuum of private and public goods and sells differentiated labor services to firms. Preferences are described by a utility function defined over private consumption, public expenditure and leisure

$$U_0^i = \sum_{t=0}^{\infty} \beta^t \left((1 - \chi) \log C_t^i + \chi \log G_t^i - \frac{(N_t^i)^{1+\varphi}}{1 + \varphi} \right) \quad (3.1)$$

C_t^i is a composite consumption good defined as

$$C_t^i = \frac{(C_{i,t}^i)^{1-\alpha} (C_{F,t}^i)^\alpha}{(1 - \alpha)^{(1-\alpha)} \alpha^\alpha} \quad (3.2)$$

where $C_{i,t}^i$ is a CES aggregator of domestically produced varieties

$$C_{i,t}^i = \left[\int_0^1 C_{i,t}^i(j)^{\frac{\epsilon_p-1}{\epsilon_p}} dj \right]^{\frac{\epsilon_p}{\epsilon_p-1}} \quad (3.3)$$

$C_{i,t}^i(j)$ denotes the quantity consumed of variety j produced in country i and ϵ_p is the elasticity of substitution between varieties produced in the same country. $C_{F,t}^i$ is domestic consumption of imported varieties from the other members of the currency area

$$C_{F,t}^i = \exp \int_0^1 \log C_{f,t}^i df \quad (3.4)$$

and it is in turn a function of an aggregator combining all varieties j produced in each foreign country f

$$C_{f,t}^i = \left[\int_0^1 c_{f,t}^i(j)^{\frac{\epsilon_p-1}{\epsilon_p}} dj \right]^{\frac{\epsilon_p}{\epsilon_p-1}} \quad (3.5)$$

The parameter α can be interpreted as a measure either of home bias in private consumption or of openness towards the rest of country members.²

Defining for each country i the aggregate price index of domestically produced goods (i.e. the producer price index) as

$$P_t^i = \left[\int_0^1 P_t^i(j)^{1-\epsilon_p} dj \right]^{\frac{1}{1-\epsilon_p}} \quad (3.6)$$

the union wide price index as

$$P_t^* = \exp \int_0^1 \log P_t^f df \quad (3.7)$$

and the consumer price index for each country i

$$P_{c,t}^i = (P_t^i)^{1-\alpha} (P_t^*)^\alpha \quad (3.8)$$

²As long as $\alpha < 1$, because of the home bias, countries are consuming different consumption bundles. As a consequence, CPI inflation differentials may arise even if the law of one price is assumed to hold. Were absent the home bias, one would observe producer price inflation differentials only.

optimal intra-temporal allocation among varieties implies the following equations³

$$C_{i,t}^i(j) = \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\epsilon_p} C_{i,t}^i \quad (3.9)$$

$$P_t^f C_{f,t}^i = P_t^* C_{F,t}^i \quad (3.10)$$

$$P_t^i C_{i,t}^i = (1 - \alpha) P_{c,t}^i C_t^i \quad P_t^* C_{F,t}^i = \alpha P_{c,t}^i C_t^i \quad (3.11)$$

Given optimal allocation of expenditure, the period budget constraint can be written as

$$P_{c,t}^i C_t^i + E_t \{ Q_{t,t+1} D_{t+1}^i \} \leq D_t^i + (1 + \tau^w) W_t^i N_t^i + T_t^i \quad (3.12)$$

$W_t^i N_t^i$ is nominal labor income, τ_w^i is a proportional subsidy to labor income and T_t^i are lump-sum taxes. In addition, households hold a portfolio that is including state contingent assets and shares in foreign and domestic firms. D_{t+1}^i denotes the nominal payoff of the portfolio in $t + 1$, $Q_{t,t+1}$ is the one-period ahead stochastic discount factor and it is such that $E_t \{ Q_{t,t+1} \} R_t^* = 1$, where R_t^* is the risk-free nominal interest rate factor of the currency area.

Labor services offered by households are regarded by firms as imperfect substitutes, where the elasticity of substitution is equal to $\epsilon_w > 1$. As in the standard monopolistic competition set up, total labor demand faced by each household is given by

$$N_t^i(h) = \left[\frac{W_t^i(h)}{W_t^i} \right]^{-\epsilon_w} N_t^i \quad (3.13)$$

where

$$N_t^i = \left[\int_0^1 N_t^i(h)^{\frac{\epsilon_w-1}{\epsilon_w}} dh \right]^{\frac{\epsilon_w}{\epsilon_w-1}} \quad (3.14)$$

³Price indexes P_t^i , P_t^* and $P_{c,t}^i$ are defined so that the minimum cost of consumption bundles $C_{i,t}^i$, $C_{F,t}^i$ and C_t^i are respectively $P_t^i C_{i,t}^i$, $P_t^* C_{F,t}^i$, and $P_{c,t}^i C_t^i$. Moreover, $P_t^i C_{i,t}^i + P_t^* C_{F,t}^i = P_{c,t}^i C_t^i$.

is the aggregate labor index combining in the Dixit-Stiglitz from the total quantity sold of each variety and

$$W_t^i = \left[\int_0^1 (W_t^i(h))^{1-\epsilon_w} dh \right]^{\frac{1}{1-\epsilon_w}} \quad (3.15)$$

can be interpreted as the aggregate wage, defined so that the minimum cost of the aggregate labor index $\int_0^1 N_t^i(h)W_t^i(h)dh$ is $W_t^i N_t^i$.

Utility maximization subject to the period budget constraints and labor demand yields the standard optimality conditions⁴

$$C_t^i (N_t^i)^\varphi = (1 - \chi) \frac{W_t^i}{P_{c,t}^i} \quad (3.16)$$

$$\beta \left(\frac{C_t^i}{C_{t+1}^i} \right) \left(\frac{P_{c,t}^i}{P_{c,t+1}^i} \right) = Q_{t,t+1} \quad (3.17)$$

In order to introduce a tension between inflation and output gap stabilization, it is assumed from now on that the wage mark-up fluctuates exogenously around its mean value⁵. Hence, equation (3.16) is modified accordingly to include a random shock

$$\exp\{\mu_t^w\} C_t^i (N_t^i)^\varphi = (1 - \chi) \frac{W_t^i}{P_{c,t}^i} \quad (3.18)$$

$\mu_t^{w,i}$ follows an autoregressive process represented by

$$\mu_{t+1}^{w,i} = \rho_u \mu_t^{w,i} + \varepsilon_{t+1,u}^i \quad (3.19)$$

where $\varepsilon_{t,u}^i$ is white noise with standard deviation denoted by $\sigma_{\varepsilon,u}$. $\varepsilon_{t,u}^i$ and $\varepsilon_{t,u}^j$ are assumed to be uncorrelated for all t and for all $i \neq j$.

⁴The wage equation already takes into account that the subsidy to labor income is set so as to offset market power

⁵The assumption could be rationalized by any real or nominal friction in the wage contracting process. See also Clarida et al. (1999), Galí (2003) and Woodford (2003)

After rewriting (3.17) as a conventional Euler equation

$$\beta R_t^* E_t \left\{ \left(\frac{C_t^i}{C_{t+1}^i} \right) \left(\frac{P_{c,t}^i}{P_{c,t+1}^i} \right) \right\} = 1 \quad (3.20)$$

complete financial markets imply the following international risk sharing condition⁶

$$C_t^i = C_t^f (S_{f,t}^i)^{1-\alpha} \quad (3.21)$$

where $S_{f,t}^i$ stands for the bilateral terms of trade between any country i and f and it is defined as

$$S_{f,t}^i = \frac{P_t^f}{P_t^i}$$

so that the effective terms of trade of any country i against the rest of the currency area are

$$\begin{aligned} S_t^i &= \frac{P_t^*}{P_t^i} & (3.22) \\ &= \exp \int_0^1 (\log P_t^f - \log P_t^i) df \\ &= \exp \int_0^1 \log S_{f,t}^i df \end{aligned}$$

Note finally that the terms of trade can be related to CPI by

$$P_{c,t}^i = P_t^i (S_t^i)^\alpha \quad (3.23)$$

this implying the following relation between CPI and domestic inflation

$$\pi_{c,t}^i = \pi_t^i + \alpha \Delta S_t^i \quad (3.24)$$

⁶(3.21) holds under the assumption of symmetric initial conditions and initial zero net foreign asset holdings.

3.3.2 Firms

Each country is populated by a continuum of firms indexed by j on the unit interval $[0, 1]$, each producing a variety with a constant return to scale technology

$$Y_t^i(j) = A_t^i N_t^i(j) \quad (3.25)$$

Country-specific productivity is denoted by A_t^i and follows an autoregressive process represented by

$$\log A_{t+1}^i = \rho_a \log A_t^i + \varepsilon_{t+1,a}^i \quad (3.26)$$

where ε_t^i is white noise with standard deviation $\sigma_{\varepsilon,a}$. ε_t^i and ε_t^j are assumed to be uncorrelated for all t and all $i \neq j$.

Prices are staggered à la Calvo, then in every period firms face a constant probability θ of changing the price. The optimal (log) price charged by firms that are allowed to re-optimize in period t is⁷

$$\bar{p}_t^i = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ mc_{t+k}^i + p_{t+k}^i \} \quad (3.27)$$

where $\mu = \log \frac{\epsilon_p}{\epsilon_p - 1}$ is the log of the optimal mark-up. mc_t stands for the log of the marginal cost and is equal to

$$mc_t^i = -\log(1 - \tau_p^i) + w_t^i - p_t^i - a_t^i + \mu_t^{w,i} \quad (3.28)$$

and τ_p^i is a proportional production subsidy. Finally, it can be easily shown that the aggregate production function is given by

$$Y_t^i Z_t^i = A_t^i N_t^i \quad (3.29)$$

where Z_t^i is defined as

$$Z_t^i = \int_0^1 \frac{Y_t^i(j)}{Y_t^i} dj \quad (3.30)$$

⁷To a first order approximation.

and represents a measure of relative price dispersion⁸.

3.3.3 Government Expenditure

Define aggregate government expenditure as

$$G_t^i = \left[\int_0^1 G_t^i(j)^{\frac{\epsilon_p-1}{\epsilon_p}} dj \right]^{\frac{\epsilon_p}{\epsilon_p-1}} \quad (3.31)$$

where $G_t^i(j)$ is the quantity of public consumption of variety j . Note that, differently from households, government is assumed to consume only domestically produced goods.⁹ Given G_t^i , the government chooses $G_t^i(j)$ so as to minimize expenditure. Hence the following condition has to be satisfied

$$G_t^i(j) = \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\epsilon_p} G_t^i \quad (3.32)$$

3.3.4 Market Clearing

After defining aggregate output as

$$Y_t^i = \left[\int_0^1 Y_t^i(j)^{\frac{\epsilon_p-1}{\epsilon_p}} dj \right]^{\frac{\epsilon_p}{\epsilon_p-1}} \quad (3.33)$$

one can show that the clearing of all goods markets, along with conditions for optimal intra-temporal allocation among varieties¹⁰, implies that

$$Y_t^i = A_t^i N_t^i = C_t^i (S_t^i)^\alpha + G_t^i \quad (3.34)$$

⁸It can be proved that $\log(Z)$ is a function of the cross sectional variance of relative prices and it is of second order.

⁹The assumption that the government consumes domestically produced goods only is not as strong as it may look like: the empirical evidence in fact is in favor of a considerably higher home bias in public consumption than private consumption.

¹⁰For further details see Galí and Monacelli (2005).

3.3.5 The Pareto Optimum

The Pareto efficient equilibrium is determined by solving the problem of a planner who wishes to maximize utility of the union as a whole

$$\int_0^1 U(C_t^i, N_t^i, G_t^i) di \quad (3.35)$$

subject to technology and resource constraints

$$Y_t^i = A_t^i N_t^i \quad (3.36)$$

$$Y_t^i = C_{i,t}^i + \int_0^1 C_{i,t}^f df + G_t^i \quad (3.37)$$

for all $i \in [0, 1]$ The corresponding first order conditions determine the following efficient outcome for country i

$$\bar{N}_t^i = 1; \quad \bar{Y}_t^i = A_t^i; \quad \bar{C}_t^i = (1 - \chi)(1 - \alpha)(A_t^i)^{1-\alpha}(A_t^*)^\alpha; \quad \bar{G}_t^i = \chi A_t^i \quad (3.38)$$

Aggregating over i yields the union-wide Pareto optimum

$$\bar{N}_t^* = 1; \quad \bar{Y}_t^* = A_t^*; \quad \bar{C}_t^* = (1 - \chi)A_t^*; \quad \bar{G}_t^* = \chi A_t^* \quad (3.39)$$

The evolution of the terms of trade at an efficient equilibrium has to be

$$\bar{S}_t^i = \left(\frac{\bar{C}_t^i}{\bar{C}_t^*} \right)^{\frac{1}{1-\alpha}} = \frac{A_t^i}{A_t^*} \quad (3.40)$$

3.3.6 Equilibrium Dynamics

As efficiency will constitute the benchmark for welfare analysis, it is convenient to describe the equilibrium dynamics in terms of deviation from first best outcomes. Let output, government expenditure and fiscal gaps be respectively defined as

$$\tilde{y}_t = y_t - \bar{y}_t; \quad \tilde{g}_t = g_t - \bar{g}_t; \quad \tilde{f}_t = \tilde{g}_t - \tilde{y}_t; \quad (3.41)$$

\tilde{f}_t can be interpreted as the percentage deviation from efficiency of government expenditure, as a fraction of GDP. The steady state of the model coincides with the first best steady state because of two reasons. First, fiscal policy is assumed to subsidize production to undo the static distortion induced by monopolistic competition in the goods markets. The absence of distortionary taxation allows to restore static efficiency. Moreover, the choice of the subsidy is not influenced by the desire of manipulating terms of trade in a country's favor. This is because fiscal policy is assumed to be set cooperatively across countries.

One can show that country i 's inflation and output gap are fully described by the following equations (in log deviations from the efficient steady state)

$$\pi_t^i = \beta E_t \{\pi_{t+1}^i\} + \lambda(1 + \varphi)\tilde{y}_t^i - \lambda \frac{\chi}{1 - \chi} \tilde{f}_t^i + \lambda \mu_t^{w,i} \quad (3.42)$$

$$\Delta \tilde{y}_t^i - \Delta \tilde{y}_t^* = \frac{\chi}{1 - \chi} (\Delta \tilde{f}_t^i - \Delta \tilde{f}_t^*) - [(\pi_t^i - \pi_t^*) + (\Delta a_t^i - \Delta a_t^*)] \quad (3.43)$$

as a function of domestic fiscal policy $\{\tilde{f}_t^i\}$, given productivity differentials and the evolution of union-wide inflation and output gap, where the following definitions apply

$$\pi_t^* = \int_0^1 \pi_t^i di \quad \tilde{y}_t^* = \int_0^1 \tilde{y}_t^i di \quad \tilde{f}_t^* = \int_0^1 \tilde{f}_t^i di \quad (3.44)$$

and λ is a convolution of deep parameters

$$\lambda = \frac{(1 - \theta)(1 - \theta\beta)}{\theta}$$

Equation (3.43) is peculiar to the case of a currency area. It relates the evolution of output gap differentials to fiscal gap, inflation and productivity differentials. In particular, note that $\Delta a_t^i - \Delta a_t^*$ is the efficient change in the terms of trade. As in a monetary union the nominal exchange rate cannot adjust so as to keep the terms of trade at their efficient level, price stickiness implies that each country can increase its own output gap relatively to the average, by creating deflation and then pushing the

terms of trade above their efficient level. Hence, other things equal, devaluations of the real exchange rate increase domestic output gap through a beggar thy neighbor policy.

Finally, after specifying a monetary policy rule, the equilibrium of the currency area as a whole can be determined using union-wide versions of the standard closed-economy Phillips and IS curves

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \lambda(1 + \varphi)\tilde{y}_t^* - \lambda\frac{\chi}{1 - \chi}\tilde{f}_t^* + \lambda\mu_t^{w,*} \quad (3.45)$$

$$\tilde{y}_t^* = E_t\tilde{y}_{t+1}^* + \frac{\chi}{1 - \chi}\tilde{f}_t^* - \frac{\chi}{1 - \chi}E_t\tilde{f}_{t+1}^* - (r_t^* - E_t\{\pi_{t+1}^*\} - rr_t^*) \quad (3.46)$$

where rr_t^* is a function of TFP shocks

$$rr_t^* = \rho + E_t\{\Delta a_{t+1}^*\} \quad (3.47)$$

3.4 The Policy Problem

A second order approximation to the sum of utilities of union households around the efficient steady-state yields

$$W = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \int_0^1 \left(\frac{\epsilon_p}{\lambda} (\pi_t^i)^2 + (1 + \varphi) (\tilde{y}_t^i)^2 + \frac{\chi}{1 - \chi} (\tilde{f}_t^i)^2 \right) di \quad (3.48)$$

Nominal rigidities, cost push disturbances and the asymmetry of shocks make it impossible to attain the Pareto efficient allocation. Therefore, the question of how to design monetary and fiscal policy rules is a non-trivial issue.

Following Beetsma and Jensen (2004, 2005), we solve the model by applying Aoki (1981) factorization of the variables into averages and differences from the average. As countries are ex-ante symmetric and of equal size, the factorization allows to split the full optimization programs of both authorities into a currency area part and a relative part, completely independent from each other. Defining country i inflation, output gap and fiscal gap differentials

$$\pi_t^{di} = \pi_t^i - \pi_t^* \quad \tilde{f}_t^{di} = \tilde{f}_t^i - \tilde{f}_t^* \quad \tilde{y}_t^{di} = \tilde{y}_t^i - \tilde{y}_t^* \quad (3.49)$$

the welfare function (3.48) and the constraints (3.42), (3.43), (3.44) and (3.46) can be rewritten as

$$W = W^* + W^d \quad (3.50)$$

$$\pi_t^* = \beta E_t \{\pi_{t+1}^*\} + \lambda(1 + \varphi) \tilde{y}_t^* - \lambda \frac{\chi}{1 - \chi} \tilde{f}_t^* + \lambda \mu_t^{w,*} \quad (3.51)$$

$$\tilde{y}_t^* = E_t \tilde{y}_{t+1}^* + \frac{\chi}{1 - \chi} \tilde{f}_t^* - \frac{\chi}{1 - \chi} E_t \tilde{f}_{t+1}^* - (r_t^* - E_t \{\pi_{t+1}^*\} - r r_t^*) \quad (3.52)$$

$$\pi_t^{di} = \beta E_t \{\pi_{t+1}^{di}\} + \lambda(1 + \varphi) \tilde{y}_t^{di} - \lambda \frac{\chi}{1 - \chi} \tilde{f}_t^{di} + \lambda (\mu_t^{w,i} - \mu_t^{w,*}) \quad (3.53)$$

$$\Delta \tilde{y}_t^{di} = \frac{\chi}{1 - \chi} \Delta \tilde{f}_t^{di} - [\pi_t^{di} + (\Delta a_t^i - \Delta a_t^*)] \quad (3.54)$$

$$\int_0^1 \pi_t^{di} di = 0 \quad \int_0^1 \tilde{y}_t^{di} di = 0 \quad \int_0^1 \tilde{f}_t^{di} di = 0 \quad (3.55)$$

where

$$W^* = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\epsilon_p}{\lambda} (\pi_t^*)^2 + (1 + \varphi) (\tilde{y}_t^*)^2 + \frac{\chi}{1 - \chi} (\tilde{f}_t^*)^2 \right) \quad (3.56)$$

$$W^d = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \left(\frac{\epsilon_p}{\lambda} (\pi_t^{di})^2 + (1 + \varphi) (\tilde{y}_t^{di})^2 + \frac{\chi}{1 - \chi} (\tilde{f}_t^{di})^2 \right) di \quad (3.57)$$

To retrieve country- i variables, it is sufficient to apply (3.49).

3.5 Perfect Coordination

Before studying monetary and fiscal policy interaction, it is useful to look as a benchmark at the case of perfect coordination, where the two authorities share the same objectives and operate under the same regime. We first recall the full commitment solution for the currency area as a whole, derived by Galí and Monacelli (2005). Then, we derive the policy mix under discretion. It is interesting to note that, under both regimes, it is completely indifferent whether monetary and fiscal policy are chosen by

a single authority or simultaneously chosen by two independent authorities that are taking as given the policy instrument of the other. These findings are reminiscent of the ones by Dixit and Lambertini (2003b) and Adam and Billi (2007), even though in the context of a different model. The crucial assumption driving the result is that there is not any disagreement about the targets and about the costs and benefits associated to policy actions.

The optimal policy under commitment for the currency area is defined by a rule for the fiscal gap $\{\tilde{f}_t^*\}$ and the union-wide nominal interest rate $\{r_t^*\}$ that maximize (3.56) subject to (3.51)¹¹. The optimal policy mix implies

$$\lambda^{-1}\epsilon_p\pi_t^* + \lambda^{-1}\Delta\tilde{y}_t^* = 0 \quad (3.58)$$

$$\tilde{f}_t^* = -\tilde{y}_t^* \quad (3.59)$$

(3.58) and (3.59) define the second best, or equivalently the constrained efficient allocation in terms of union-wide variables.

Under discretion policy makers do not choose once and for all the state-contingent path of policy instruments, they are rather allowed to re-optimize in every period. As a consequence, they do not take into account the impact of current choices on past variables through the expectation channel. The resulting policy mix is

$$\lambda^{-1}\epsilon_p\pi_t^* + \lambda^{-1}\tilde{y}_t^* = 0 \quad (3.60)$$

$$\tilde{f}_t^* = -\tilde{y}_t^* \quad (3.61)$$

Equations (3.58), (3.59), (3.60) and (3.61) correspond exactly to the standard rules that would characterize a closed economy sharing preferences and technology of the union's member countries. The features of optimal monetary policy and its advantages over a discretionary regime are well known facts in the literature. However, it may be useful

¹¹The IS equation, (3.52), can be implemented ex-post, by choosing the interest rate consistently with optimal inflation, output and fiscal gaps

to recall that (3.58) and (3.60) differ because the latter overlooks the marginal gain of committing to future deflations in terms of current output gap, $\lambda^{-1}\tilde{y}_t^*$, which is in fact appearing lagged one period in (3.58) but not in (3.60). In the event of an adverse cost-push shock, committed policy makers can contain inflationary pressures through a lower interest rate increase (a lower output contraction), simply by announcing future higher rates (lower future inflation). Through this mechanism, it is possible to smooth the impact of the shock over time. Such a policy is not time consistent and then it cannot be implemented under discretion, as it would not be credible. It follows that discretionary policy makers would evaluate the policy tightening implemented by a committed authority in the face of an adverse cost-push shock as too recessionary.

In addition, some interesting conclusions about fiscal policy can be drawn. First, (3.59) implies

$$\tilde{g}_t^* = 0 \tag{3.62}$$

Hence, in the optimal policy mix, government expenditure is set to its first best level, or equivalently, fiscal policy is not used as a stabilization tool. Therefore, the central bank is the only responsible for addressing aggregate fluctuations. This is due to the asymmetry of costs associated to the use of the policy instruments. The absence of transaction frictions allows to vary the nominal interest rate, without generating welfare costs. On the contrary, fluctuations in government expenditure are costly, as they imply a departure from efficient public goods provision.

In addition, under perfect coordination, irrespectively of the policy regime, (3.62) still holds. Hence, discretion per se does not produce inefficiency losses in public goods provision. This ceases to be true when monetary policy optimally reacts to governments' lack of commitment, as it will be clear in the following sections.

3.6 Optimal Monetary Policy under Fiscal Discretion

We turn now to the case where monetary and fiscal policy are conducted by two independent authorities, sharing the same objectives. Only the latter is able to credibly commit to future policies, while the fiscal policy maker chooses the fiscal gap sequentially, i.e. she solves the policy problem in each period, in order to determine the current instrument only. Because of the lack of commitment, the government cannot directly control future fiscal gaps. As a consequence, the impact of current actions on past expectations is not internalized. Being private sector forward looking, this is costly as long as policy choices are subject to time inconsistency problems. As in Dixit and Lambertini (2003a), we model strategic interaction as a Stackelberg game. The committed authority, the central bank in our case, is assumed to be the leader, while fiscal policy is the follower. As such, the latter takes the union-wide nominal interest rate as given in each period and the IS equation is perceived to be a constraint imposed by monetary policy. The model is solved by backward induction: after solving for the fiscal rule of the government, the central bank determines at time zero the optimal state contingent path of output, inflation and fiscal gaps, taking into account the fiscal policy reaction function. In the remainder of the section, we first define the policy game. Then, we characterize the equilibrium of the currency area as a whole and of the representative country. We refer to the appendix for derivations.

Definition 3.6.1 *Discretionary fiscal policy is defined as the solution to the following problems. The currency area problem consists in selecting a fiscal policy rule for the union-wide fiscal gap $\{\tilde{f}_t^*\}_{t=0}^\infty$ maximizing (3.56) subject to (3.51) and (3.52), given the union-wide nominal interest rate and the exogenous stochastic processes. Finally, optimization of the welfare function (3.57) subject to (3.53), (3.54) and (3.55) determines the state-contingent path of fiscal gap differentials $\{\tilde{f}_t^{di}\}_{t=0}^\infty$, for all $i \in [0, 1]$*

Before defining the monetary policy problem, observe that fiscal policy fully determines inflation, output and fiscal gap differentials. The result reflects the fact that monetary policy does not have enough instruments to stabilize fluctuations of single country variables. Two are the main implications. First, given the constraints imposed by national governments, the central bank has one degree of freedom to choose the union-wide policy rule, but she does not have any leverage on differentials. Equivalently, the monetary authority has to solve the union-wide part of the optimization program, while the relative part is determined by the constraints. Second, there is no strategic interaction, and then no policy game, as far as single country stabilization issues are concerned. The solution to the relative part of the fiscal optimization problem is completely irrelevant for the policy game, which in turns determines currency area equilibrium only.

Definition 3.6.2 *Optimal monetary policy under fiscal discretion is defined as the state contingent path for the common interest rate $\{r_t^*\}$, together with the associated union-wide policy outcomes, $\{\pi_t^*\}$, $\{\tilde{y}_t^*\}$ and $\{\tilde{f}_t^*\}$ maximizing welfare (3.56) subject to (3.51) **and** the union-wide fiscal rule.*

3.6.1 Union-wide Equilibrium

The union-wide fiscal policy rule is

$$\tilde{f}_t^* = -\tilde{y}_t^* - \varphi(\tilde{y}_t^* + \epsilon_p \pi_t^*) \quad (3.63)$$

while the targeting rule of the central bank is

$$\epsilon_p \pi_t^* + \Delta \tilde{y}_t^* = \chi(1 + \varphi \epsilon_p \lambda)(\tilde{f}_t^* + \tilde{y}_t^*) - \chi(\tilde{f}_{t-1}^* + \tilde{y}_{t-1}^*) \quad (3.64)$$

The equilibrium of the currency area as a whole is fully described by the rules (3.63) and (3.64), together with the union-wide Phillips curve (3.51). A comparison of (3.63) and (3.64) with the rules characterizing the case of perfect coordination allows to gain some important economic insights:

- As it has been stressed in section 3.5, governments evaluate policy tightening as more recessionary than a central bank who is able to manipulate expectations. The disagreement about the costs and benefits associated to monetary policy actions generates inefficient public spending over-expansion in case of negative output gaps. Therefore, fiscal policy exacerbates the trade-off faced by the monetary authority as long as she is also concerned about fiscal gap stabilization. Note that if the central bank behaves in a discretionary fashion, the deviation of government expenditure from its commitment level vanishes¹². A committed central bank could still set the nominal interest rate so as to eliminate completely government over-reaction. However, she should accept the inflation and output gap variability associated to full discretion. This would not be optimal and a combination of positive inflation, output and fiscal gap variability is preferred.
- Optimal monetary policy involves the targeting of fiscal gap deviations from the full commitment (second best) level. In particular, coherently with the reaction function of the government, higher deviations call for higher inflation or higher output gap. This allows the central bank to reduce government over-reaction. Moreover, if the government does not deviate from the full commitment solution, the monetary policy rule (3.64) converges to (3.58).
- As in the standard case, optimal monetary policy under fiscal discretion is inertial: the lagged fiscal gap appears in the targeting rule. Then, for given future fiscal gaps, the central bank commits to tighten future monetary policy in the event of an increase of the current fiscal gap above its second best level. This improves the current trade-off between inflation and fiscal gap stabilization by reducing expected future inflation.

¹²It can also be proved that in such a case the Lagrange multiplier attached to the IS equation in the fiscal policy problem is equal to zero. This is because, despite the lack of coordination there is no disagreement between the two authorities so that monetary policy does not impose any constraint on fiscal policy

- In the absence of cost-push shocks, the full commitment solution can be implemented even in the case of fiscal discretion. Keeping inflation and output at their natural level eliminates any incentive of over-expansion on the part of the government. In fact, absent any short-run stabilization trade-off, time inconsistency is not an issue as the efficient allocation is feasible.

Finally, note that the equilibrium evolution of the currency area as a whole is exactly the same that would be observed in a closed economy sharing preferences and technology of the union's member countries. Hence, from now on, all starred variables can be interpreted either as union-wide or closed economy variables.

3.6.2 Equilibrium in The Representative Country

The representative country part of the problem is more involved than the case of the currency area, as the lagged values of fiscal and output gap appear in equation (3.54). This means that expectations of future variables cannot be taken as given. In fact, even restricting to Markov strategies, one has to take into account that in any stationary equilibrium expectations of future states will depend on their own lags. To solve the model we use the same method as Clarida et al. (1999) and Beetsma and Jensen (2004, 2005). What has to be taken as given is how private sector expectations react to current policy, rather than expectations. Hence, we conjecture that expectations are a linear function of current states for some arbitrary coefficients. Those coefficients are defined to be such to coincide with the parameters entering the state space representation of the rational expectation equilibrium¹³. We refer to the appendix for all technical details and we report below the fiscal policy rule for country i

$$\begin{aligned} \varphi\epsilon_p\lambda\pi_t^{di} + (1 + \varphi)(d_1 - \lambda\varphi)\tilde{y}_t^{di} + (1 + \varphi)d_1\tilde{f}_t^{di} &= \\ \beta E_t \left[\varphi\epsilon_p\lambda\pi_{t+1}^{di} + (1 + \varphi)\tilde{y}_{t+1}^{di} + \tilde{f}_{t+1} \right] & \end{aligned} \quad (3.65)$$

¹³This is a conventional fixed point problem that can be easily solved either via undetermined coefficients or through some recursive numerical method.

where

$$d_1 = 1 + \beta(1 - c_1) + \lambda(1 + \varphi) \quad (3.66)$$

and c_1 is a state space coefficient defined in the appendix. It is immediate to see that the fiscal policy rule is entirely forward looking. This is because, due to the lack of commitment, the government fails to internalize the effect of policy on past expectations.

3.7 Impulse Responses and Second Moments

3.7.1 The Currency Area

Let each country be subject to cost-push shocks following the process (3.19). Aggregating across countries and applying the definition $\mu_t^{w,*} = \int_0^1 \mu_t^{w,i} di$ yields

$$\mu_{t+1}^{w,*} = \rho_u \mu_t^{w,*} + \varepsilon_{t+1,u}^* \quad (3.67)$$

where $\varepsilon_{t,u}^* = \int_0^1 \varepsilon_{t,u}^i di$ is white noise with standard deviation denoted by $\sigma_{\varepsilon,u}$. Given the stochastic process (3.67), equations (3.63), (3.64) and the union-wide Phillips curve allow to compute the impulse response functions of starred variables. They can be interpreted either as the response of a closed economy or as the response of the currency area to a shock hitting every member country i . Structural parameters are the same as in Galí and Monacelli (2005) and they are reported in Table 3.1. φ is set equal to 3, implying a labor supply elasticity of 1/3. The elasticity of substitution among goods and labor types, ϵ_p and ϵ_w are equal to 6, which is consistent with average mark-ups of 20 percent. θ and β are respectively set to 0.75 and 0.99. The steady-state share of government spending in output, $\gamma = \chi$, is parameterized to 0.25, the average of final government consumption for the euro zone. TFP standard deviation is calibrated to the conventional value 0.0071. Two alternative calibrations for serial correlation have been chosen: $\rho_u = 0.95$ and $\rho_u = 0$. Figures 3.1 and 3.2 display the response

of output gap, fiscal gap and inflation to a cost-push shock, under the two regimes of full commitment and discretionary fiscal policy. Table 3.2 and 3.3 report percentage standard deviations of inflation, output, fiscal and government expenditure gaps. We normalize to one the relative standard deviation of the cost-push shock with respect to TFP. Sensitivity analysis of welfare to changes in the standard deviation and the serial correlation are postponed to section 3.8. Some features are worth to be stressed:

- Under fiscal discretion, the fiscal gap response to cost-push shocks is significantly stronger. The volatility of public spending translates through aggregate demand into higher than optimal volatility either of aggregate inflation or aggregate output gap. Fluctuations in the fiscal gap can only be dampened either by tolerating more volatile inflation or by under-stabilizing the output gap. Which option is preferred by the central bank depends on the persistence of the shock. As suggested by Table 3.2 and 3.3, the first one is preferred when the serial correlation of the shock is high.
- Although the fiscal rule targets contemporaneous variables, monetary policy can induce inertia by suitably choosing her policy instrument. In fact, the central bank has a first mover advantage over the government, who takes monetary policy as given. This is evident from Figure 3.2, showing the case of serially uncorrelated cost-push shocks.
- Comparing Figures 3.1 and 3.2, it is immediate to see that higher serial correlation magnifies fiscal policy over-reaction to a negative cost push shock, implying persistently higher inflation.

3.7.2 The Representative Country

As it has been previously stressed, there is no strategic interaction at the country level: fiscal policy is "alone" in the task of addressing asymmetric shocks. This meaning that

our discretionary fiscal policy is the same as in Beetsma and Jensen (2004, 2005). In fact, our results are in line with theirs, even if the first order conditions are not directly comparable. Figures 3.3 and 3.4 report impulse responses to TFP and cost-push shocks respectively. Serial correlation of both shocks is set to 0.95. Note that the response of country- i variables and of differentials from the union-average coincide in the case of a country-specific shock with zero mass, as union-wide variables are unaffected. This is not the case when all countries are simultaneously hit by shocks.

In the absence of nominal rigidities, asymmetric shocks to productivity would require the terms of trade to adjust in order to keep output and public spending at their first best level. However, when prices are sticky inflation fluctuations are costly and it is optimal to smooth price changes over time, by allowing a temporary departure of output from efficiency. In the transition to the steady state, an expansionary fiscal policy reduces the cost in terms of output, both under discretion and commitment. However, under discretion the effect of the shock on prices is persistently stronger, since the government cannot control expectations to improve current stabilization trade-offs. On the other hand, output and fiscal gaps are less volatile than optimal.

In the case of a cost-push shock, fiscal stance is tightened. Monetary policy cannot stabilize national business cycles and government expenditure is the only available instrument to address the distortions induced by sticky prices and inefficient fluctuations in the wage mark-up. Therefore, it is not possible to close all the gaps: national governments have to choose a combination of positive inflation and negative output gap. As a consequence, the fiscal gap has to fall in order to counteract inflationary pressures, by contracting aggregate demand. Again, under commitment the fiscal authority manipulates private sector expectations in order to improve the trade-off at the time the shock hits the economy. On the contrary, a discretionary government generates on impact more volatile responses to shocks. Moreover, prices, output gap and fiscal gap are higher than optimal during the transition to the steady state.

Tables 3.4 and 3.5 report percentage standard deviations of differentials: the lack of commitment worsens inflation stabilization so much that the fiscal authority is induced to stabilize output and fiscal gap more than optimally. Hence, discretionary fiscal policy is less active than it should. The result does not contradict the fact that the fiscal stance is inefficiently loose at the currency area level. As all governments are over-reacting to shocks, the union-wide fiscal gap is fluctuating too much, while fiscal gap differentials are fluctuating too little.

3.8 Welfare Analysis

Recent literature claims that, in a currency area, a committed fiscal policy enhances welfare through the stabilization of asymmetric shocks. After evaluating the costs generated by fiscal discretion, we ask whether the result survives when governments act in a discretionary fashion, without coordinating with the central bank. We compute welfare as a function of serial correlation and of relative standard deviation of the cost-push shock. All welfare differences across regimes are measured in consumption equivalents, i.e. the percentage variation of steady state consumption under the benchmark policy that is making agents indifferent to the alternative policy regime.

Figure 3.5 plots the contour sets of the cost generated by discretion, with respect to full commitment. Not surprisingly, discretion entails welfare costs. This is due to two reasons: on one hand, the union-wide fiscal gap is too volatile, making harder the job of the central bank in stabilizing inflation and output gap. On the other hand, discretion leads to sub-optimal fluctuations of inflation, output and fiscal gap differentials. The relative importance of the two components is assessed in Figure 3.6, displaying the fraction of the total cost due to inefficient union-wide fluctuations. Note that this is the least important part of the cost (always less than a half). The intuition is that, while monetary policy can at least partially cope with fiscal misbehavior at the union level, there is no possibility to influence single country variables and then the behavior

of differentials.

Given the cost stemming from fiscal discretion, it is interesting to ask whether it is sensible to use public spending as an instrument to stabilize national business cycles, rather than confining governments to the role of efficiently providing public goods. To answer this question, we compare welfare under full commitment and under fiscal discretion against the case of inactive fiscal policy, meaning expenditure set at its efficient level at all times. Figure 3.7 plots the welfare gain of the full commitment solution. The stabilizing role of fiscal policy always generates welfare gains, the magnitude depending on the stochastic properties of the shocks. Figure 3.8 displays welfare differences between the fiscal discretion regime and the case of inactive fiscal policy. Such differences can be decomposed into two parts. The first is always negative and captures the cost arising from the fact that the fiscal stance is inefficiently loose at the union level. The second component measures differences due to fluctuations in inflation, output and fiscal gap differentials: it can be positive or negative, depending whether the welfare improving role of fiscal policy survives to discretion. Figures 3.9 and 3.10 show the two components. For most of parameter combinations, the only cost imposed by fiscal discretion is represented by excessive fiscal gap variability at the union level. But, interestingly, for some parameter combinations, even the positive role of fiscal policy in stabilizing asymmetric shocks is compromised by the inability of steering inflation expectations due to lack of commitment.

Overall, welfare analysis casts some doubt, at least for some plausible calibrations of parameters, on the desirability of using fiscal policy to address asymmetric shocks.

3.9 Conclusion

This Chapter studies the optimal monetary and fiscal policy mix in a currency area, where only the central bank is able to commit to future policies. The contribution is twofold. First, we show that the optimal reaction on the part of monetary policy to

fiscal discretion involves the targeting of union-wide fiscal stance, on top of inflation and output gap stabilization. Moreover, we perform welfare analysis and we find that the costs generated by discretion may offset the benefits of using fiscal policy for stabilization purposes. In those cases, it is welfare enhancing to confine governments to the role of efficiently providing public goods. The result opens the question of designing a suitable institutional framework coping with the problem of fiscal discretion.

The issue deserves further theoretical and empirical investigation. In particular, some relevant distortions we are abstracting from could push welfare results in opposite directions, either strengthening or weakening our argument. In fact, on one hand the introduction of distortionary taxation and debt may worsen the effects of discretionary fiscal policy as emphasized by Leith and Wren-Lewis (2006). In this perspective, our analysis would just provide a lower bound of the costs generated by the lack of commitment on the fiscal side. On the other hand, transaction frictions would reduce the cost of using public spending as a stabilization instrument, relatively to the nominal interest rate. This provides a motive for the use of government expenditure as a union-wide stabilization tool, even under full commitment.

Table 3.1: **Baseline Calibration**

Parameter	Value
φ	3
$\epsilon_p = \epsilon_w$	6
θ	0.75
β	0.99
$\gamma = \chi$	0.25
ρ_a	0.95
$\sigma_{\varepsilon,a}$	0.0071

Table 3.2: **Percentage standard deviations of union-wide variables in the case of serially correlated cost-push shocks.** Cost-push shock standard deviation is set equal to TFP standard deviation.

$\rho = 0.95$	Inflation	Output Gap	Fiscal Gap	Gov. Exp. Gap
Discretionary Fiscal	0.0451	0.4630	1.0845	0.6351
Full Commitment	0.0209	0.5093	0.5093	0

Table 3.3: **Percentage standard deviations of union-wide variables in the case of serially uncorrelated cost-push shocks.** Cost-push shock standard deviation is set equal to TFP standard deviation.

$\rho = 0$	Inflation	Output Gap	Fiscal Gap	Gov. Exp. Gap
Discretionary Fiscal	0.0580	0.3301	0.4906	0.3548
Full Commitment	0.0621	0.3046	0.3046	0

Table 3.4: **Percentage standard deviations of differentials.** Cost-push shock standard deviation is set equal to TFP standard deviation.

$\rho = 0.95$	Inflation	Output Gap	Fiscal Gap	Gov. Exp. Gap
Discretionary Fiscal	0.3748	0.6475	3.0429	2.7601
Full Commitment	0.3264	0.7384	3.4430	3.4526

Table 3.5: **Percentage standard deviations of differentials.** Cost-push shock standard deviation is set equal to TFP standard deviation.

$\rho = 0$	Inflation	Output Gap	Fiscal Gap	Gov. Exp. Gap
Discretionary Fiscal	0.3801	0.3719	2.8466	2.8205
Full Commitment	0.3399	0.3690	3.4473	3.4847

Figure 3.1: **Impulse responses to a union-wide cost-push shock.** Serial correlation has been set to 0.95. Parameters are calibrated as in Table 1.

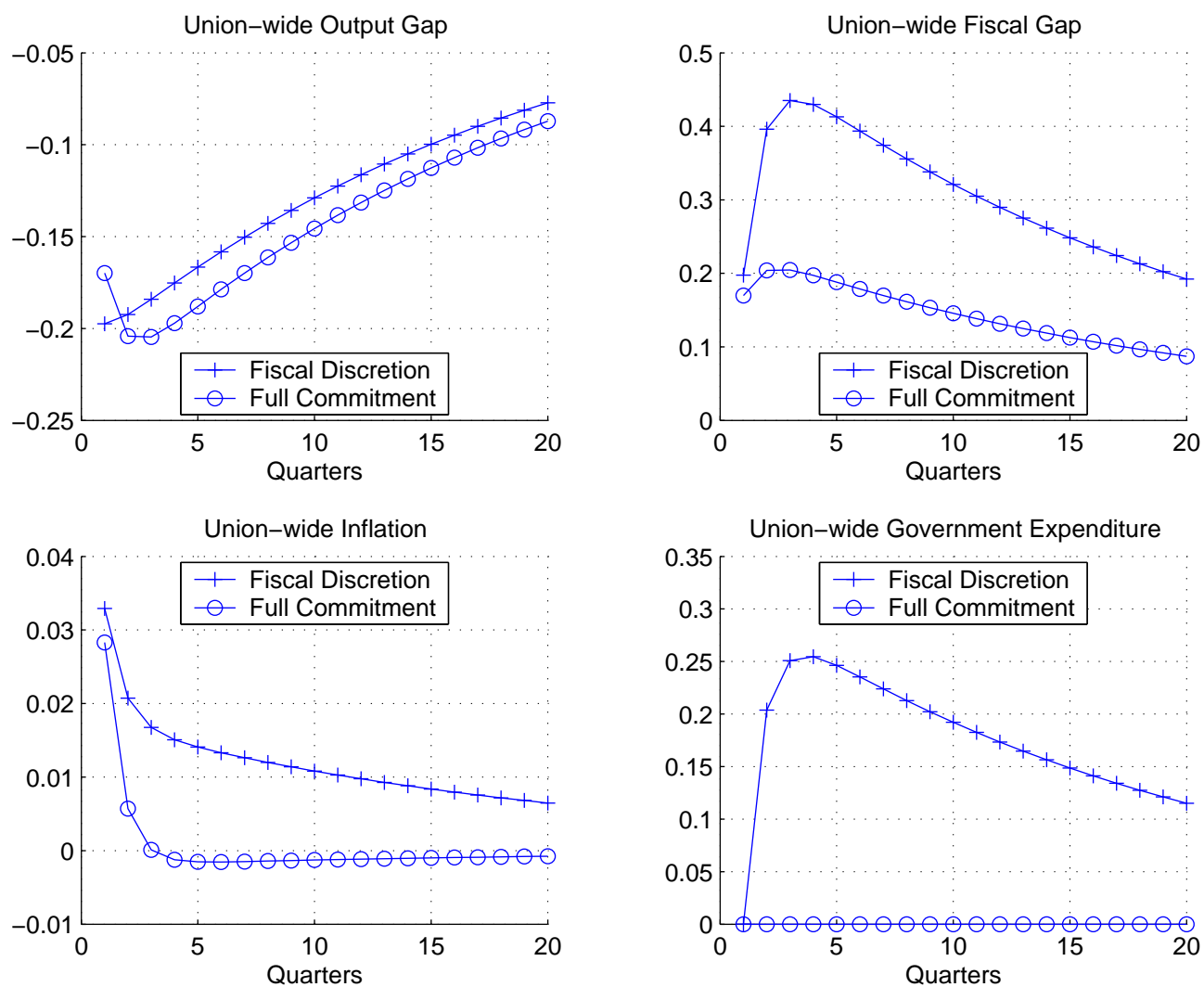


Figure 3.2: **Impulse responses to a union-wide cost-push.** Serial correlation has been set to 0. Parameters are calibrated as in Table 4.1.

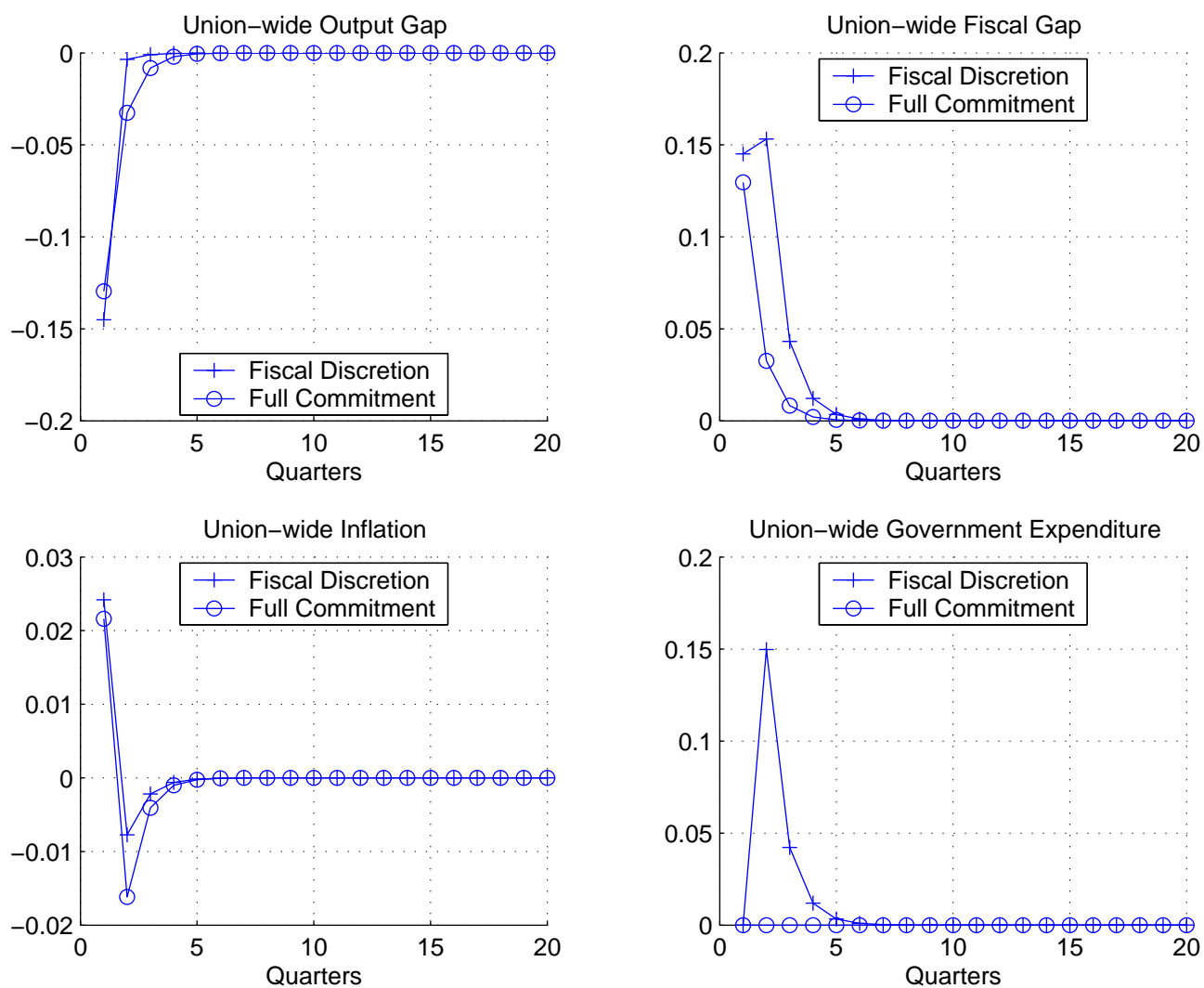


Figure 3.3: **Impulse responses to a single-country TFP shock.** Serial correlation has been set to 0.95. Parameters are calibrated as in Table 4.1.

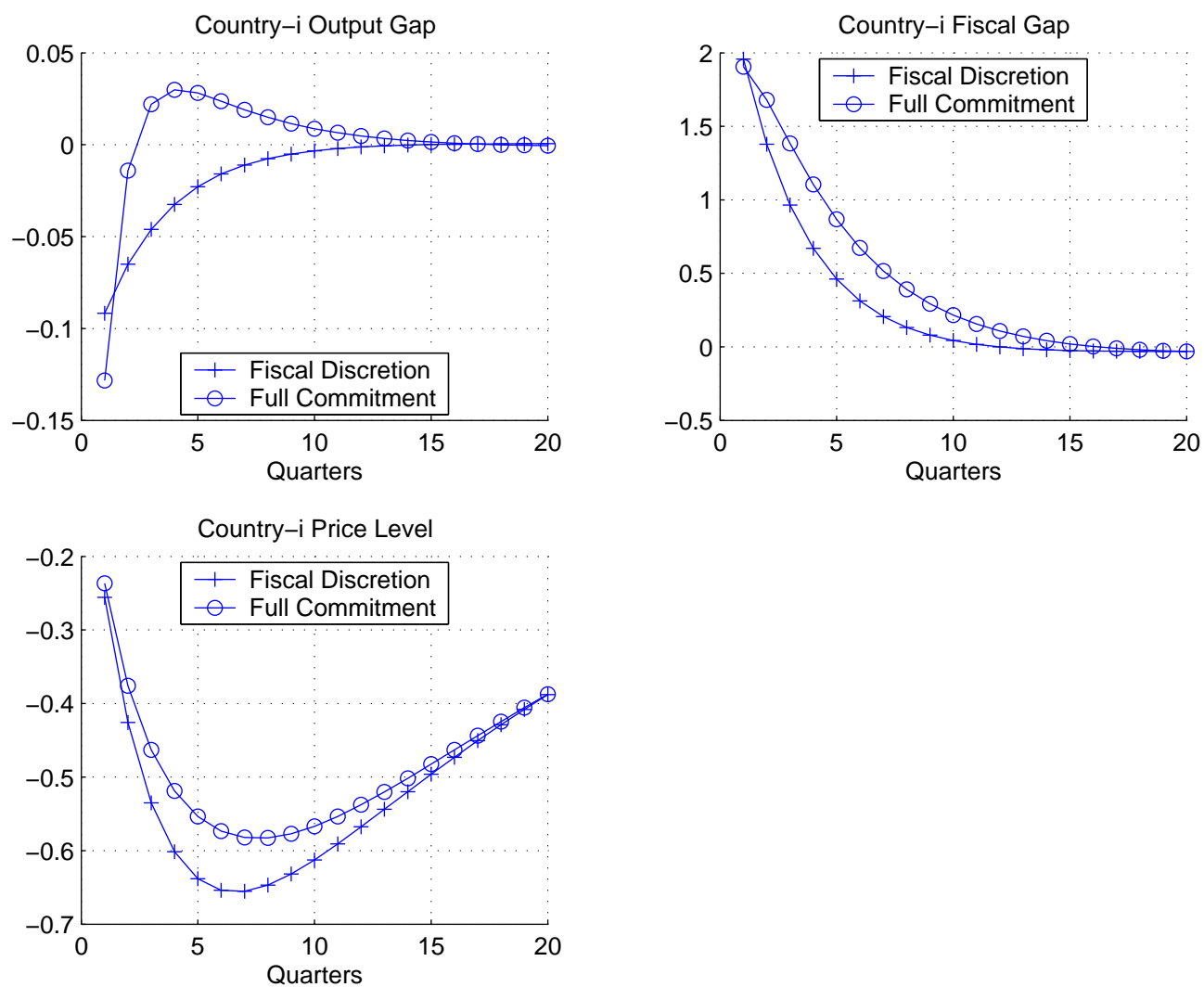


Figure 3.4: **Impulse responses to a single-country cost-push shock.** Serial correlation has been set to 0.95. Parameters are calibrated as in Table 4.1.

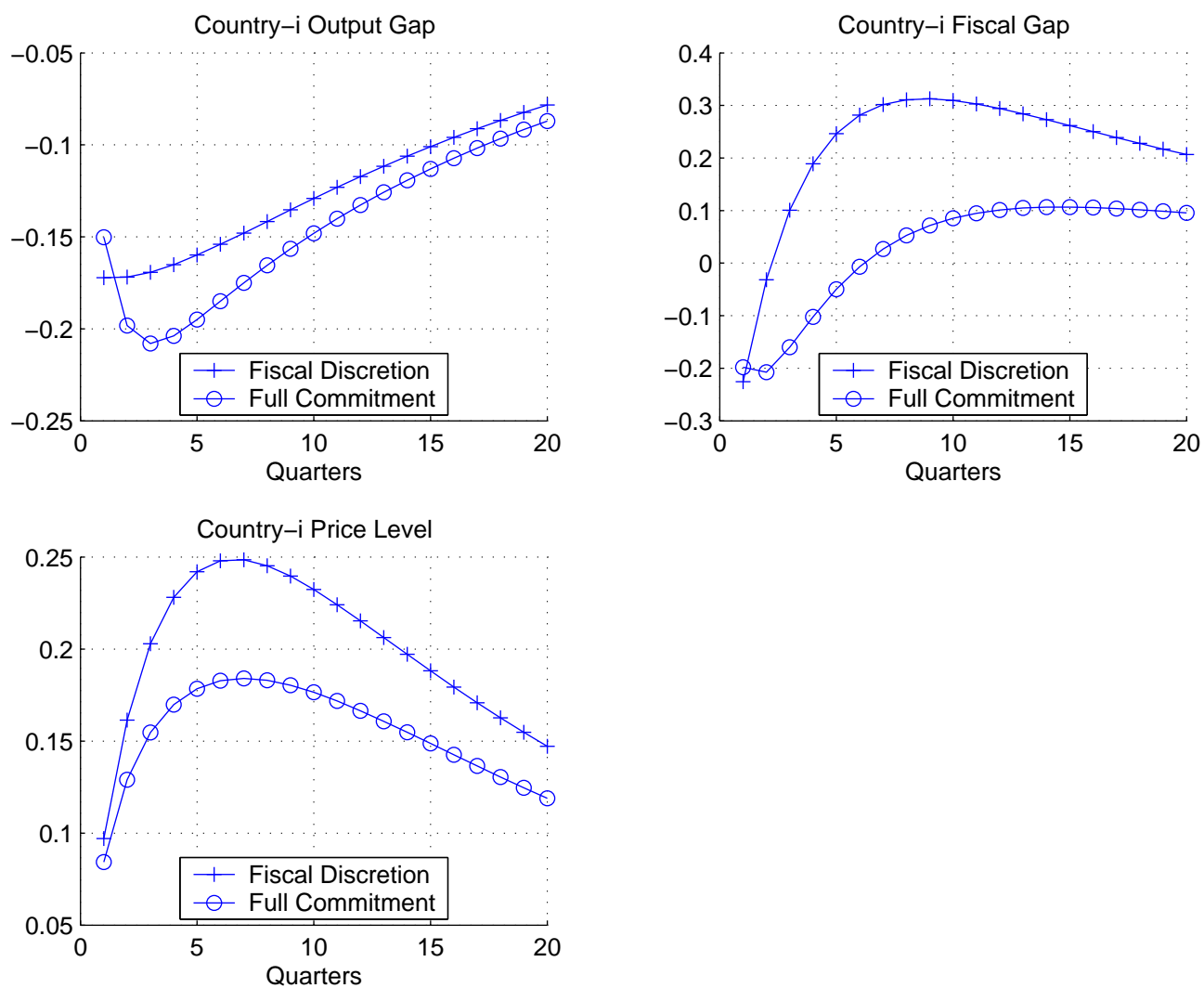


Figure 3.5: **Welfare cost of discretion.** Contour sets of the welfare cost of discretion as a function of cost-push shock serial correlation and relative standard deviation. Welfare cost is measured in consumption equivalents, i.e. as the percentage decrease of steady state consumption under full commitment in order to be indifferent to the fiscal discretion regime.

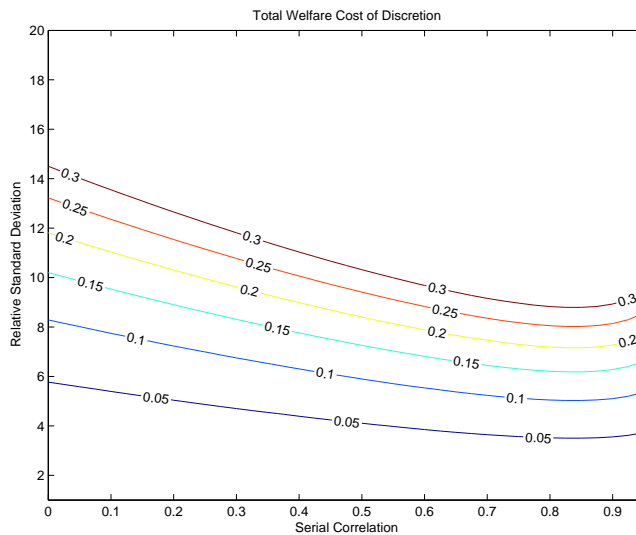


Figure 3.6: **Welfare cost of discretion: union-wide component.** The graph displays contour sets of the union-wide component as a fraction of the total cost of discretion. The cost is measured in consumption equivalents.

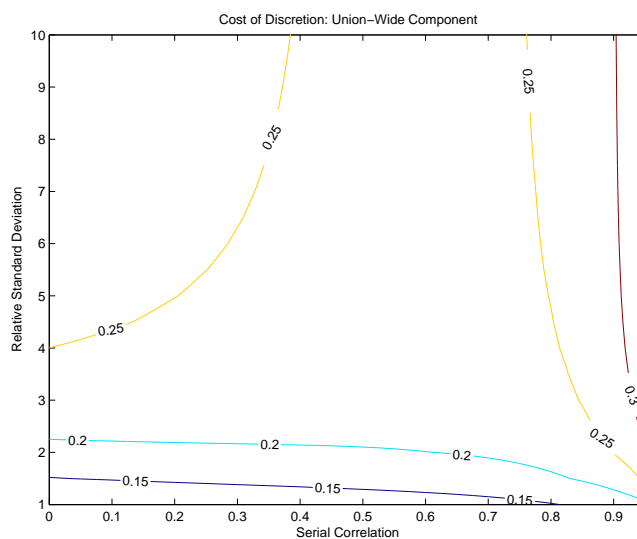


Figure 3.7: **Welfare gain from committed fiscal policy.** The gain is computed with respect to inactive fiscal policy, i.e. a regime where fiscal policy is constrained to efficient provision of public goods.

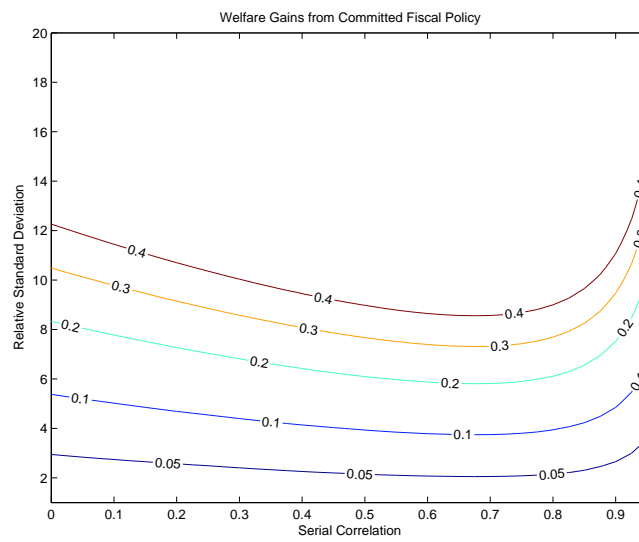


Figure 3.8: **Welfare gain from discretionary fiscal policy.** The gain is computed with respect to inactive fiscal policy.

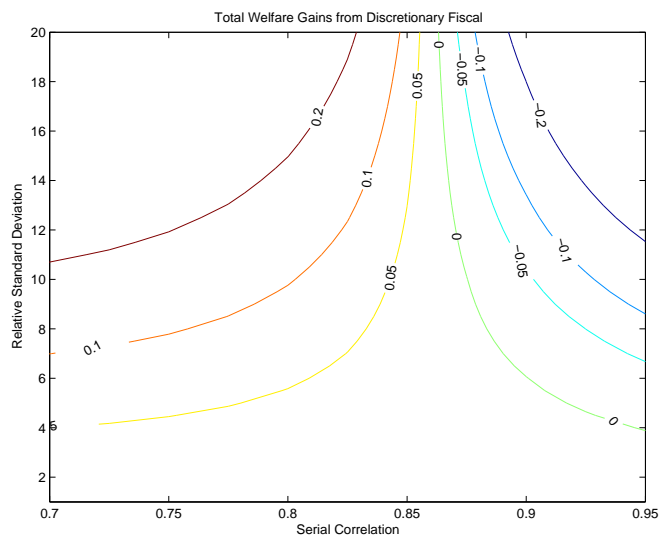


Figure 3.9: **Welfare gain from discretionary fiscal policy: union-wide component.**

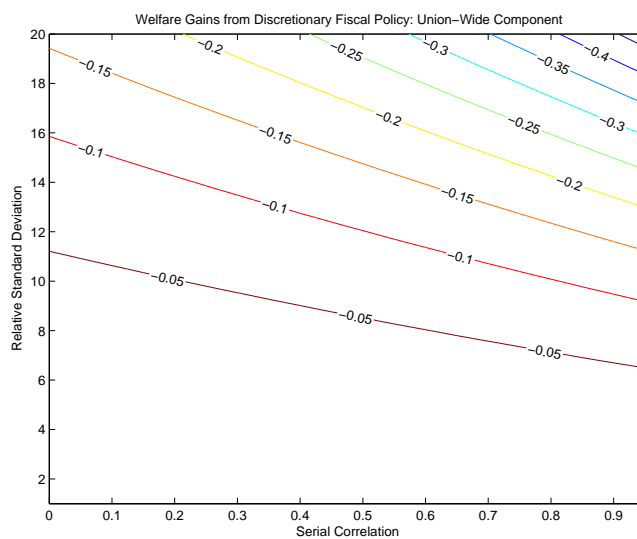
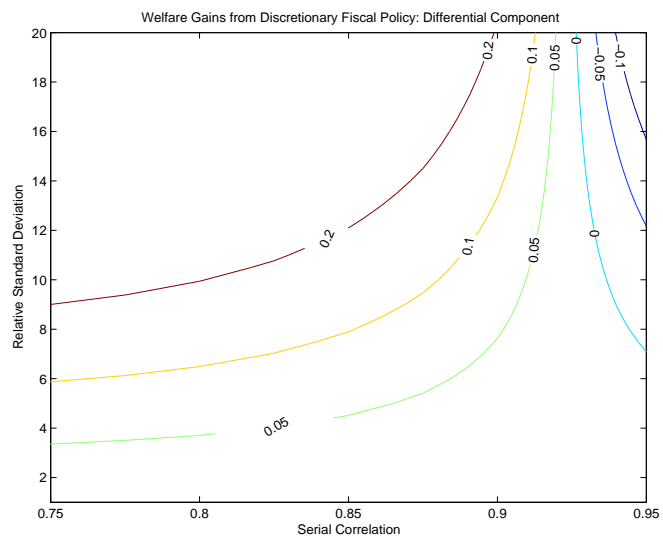


Figure 3.10: Welfare gain from discretionary fiscal policy: differential component.



Appendix A

Addendum to Chapter 1

This appendix first derives the elasticity of labor demand as a function of CWS. Then, section A.2 shows that fluctuations of labor demand elasticity generate non significant variations of endogenous variables out of the steady state.

A.1 Labor Demand Elasticity

Let the real wage be

$$w_t = \frac{W_t}{P_t} \quad (\text{A.1})$$

hence

$$w_t = \left[\int_0^1 w_t(i)^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}} \quad (\text{A.2})$$

Considering that the representative union takes as given the wage of the workers of other unions and that the wage is the same for the workers of union j

$$\begin{aligned} \frac{\partial w_t}{\partial w_{t,j}} &= \frac{\partial}{\partial w_{t,j}} \left[\int_0^1 w_t(i)^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}} \\ &= \frac{\partial}{\partial w_{t,j}} \left[\int_{i \in j} w_t(i)^{1-\theta_w} di + \int_{i \notin j} w_t(i)^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}} \\ &= \frac{1}{n} \left[\frac{w_{t,j}}{w_t} \right]^{-\theta_w} = \frac{1}{n} \end{aligned} \quad (\text{A.3})$$

the result follows immediately from the definition of the real aggregate wage index. The last equality holds because of symmetry at equilibrium. Note that, because of symmetry, it is also true that

$$\frac{\partial w_t}{\partial w_{t,j}} \frac{w_{t,j}}{w_t} = \frac{\partial w_t}{\partial w_{t,j}} = \frac{1}{n} \quad (\text{A.4})$$

The elasticity of labor demand perceived by the j -th union can be derived in three steps

Step 1: The elasticity of inflation to the aggregate real wage

From equations (1.21), (1.24) and (1.25) the elasticity of inflation to the aggregate real wage is

$$\Sigma_{\Pi,t} \equiv \frac{\partial \log \Pi}{\partial \log w} = \Pi_t^{1-\theta_p} \left(\frac{K_t}{F_t} \right)^{1-\theta_p} \frac{1-\alpha}{\alpha} \frac{MC_t}{K_t} \quad (\text{A.5})$$

At the zero inflation steady state

$$\Sigma_{\Pi} = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \quad (\text{A.6})$$

Step 2: The elasticity of aggregate labor demand to the aggregate wage index

Aggregate labor demand is a function of aggregate demand faced by firms. The elasticity of aggregate labor to aggregate demand is constant and equal to 1. It follows from aggregate demand (1.12) and the elasticity of inflation to the aggregate real wage index (A.5) that

$$\Sigma_{L,t} \equiv -\frac{\partial \log L}{\partial \log w} = -\frac{\partial \log L}{\partial \log \Pi} \Sigma_{\Pi,t} = \gamma_{\pi} \Sigma_{\Pi,t} \quad (\text{A.7})$$

At the zero inflation steady state

$$\Sigma_L = \gamma_{\pi} \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \quad (\text{A.8})$$

Step 3: The elasticity of type j labor demand to union's j real wage

From firms' optimization problem

$$L_t^*(j) = \left[\frac{w_t(j)}{w_t} \right]^{-\theta_w} L_t \quad (\text{A.9})$$

Equation (A.9) allows the j -th wage setter to compute the perceived elasticity of its own labor demand with respect to the real wage charged (**differently from the standard case**, aggregate labor is **NOT** taken as given, but it is perceived to be a function of the real wage through the strategic interaction with the central bank as it is showed in steps 1 and 2). Hence,

$$\begin{aligned} \eta_t &\equiv - \frac{\partial \log L_{t,j}}{\partial \log w_{t,j}} \\ &= \theta_w - \frac{1}{n} \theta_w + \frac{1}{n} \Sigma_{L,t} \\ &= \theta_w - \frac{1}{n} \theta_w + \frac{1}{n} \gamma_\pi \Pi_t^{1-\theta_p} \frac{1-\alpha \Pi_t^{\theta_p-1}}{1-\alpha} \frac{1-\alpha}{\alpha} \frac{MC_t}{K_t} \end{aligned} \quad (\text{A.10})$$

θ_w is assumed to be such that labor demand is elastic, that is $\eta > 1$. It is immediate to see from (A.10) that labor elasticity is not constant over time. This implies that the wage mark-up fluctuates over time. At the zero inflation steady state

$$\eta = \theta_w - \frac{1}{n} \theta_w + \frac{1}{n} \gamma_\pi \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \quad (\text{A.11})$$

Equations (1.11), (1.12), (1.19), (1.21), (1.24), (1.25), (1.26), (1.28), (1.30), (1.31), and (A.10) together with the specification of exogenous processes and an initial value for price dispersion Δ fully characterize the equilibrium dynamics.

A.2 Simulation and Numerical Results

The impulse responses to a technology shock are computed using the baseline calibration displayed in Table A.1. The same exercise is repeated under the assumption that

labor demand elasticity is constantly equal to its steady state value. Figure A.1 reports impulse responses. It is immediate to see that the time variation in the wage mark-up induced by elasticity fluctuations does not generate quantitatively significant variation out of the steady state.

The result still holds for alternative calibrations of the parameters. Figures A.2 and A.3 report the percentage standard deviation of inflation and output gap, for alternative degrees of wage setting centralization and for alternative values of the steady state wage mark-up. The elasticity of substitution among labor types, θ_w , is chosen to match the values of η , given the other parameters that are calibrated as in the baseline specification. The difference in the standard deviations generated by elasticity fluctuations is always less than a hundredth of a percentage point.

Figure A.1: Impulse Responses to a Technology Shock (percentage deviation from steady state)

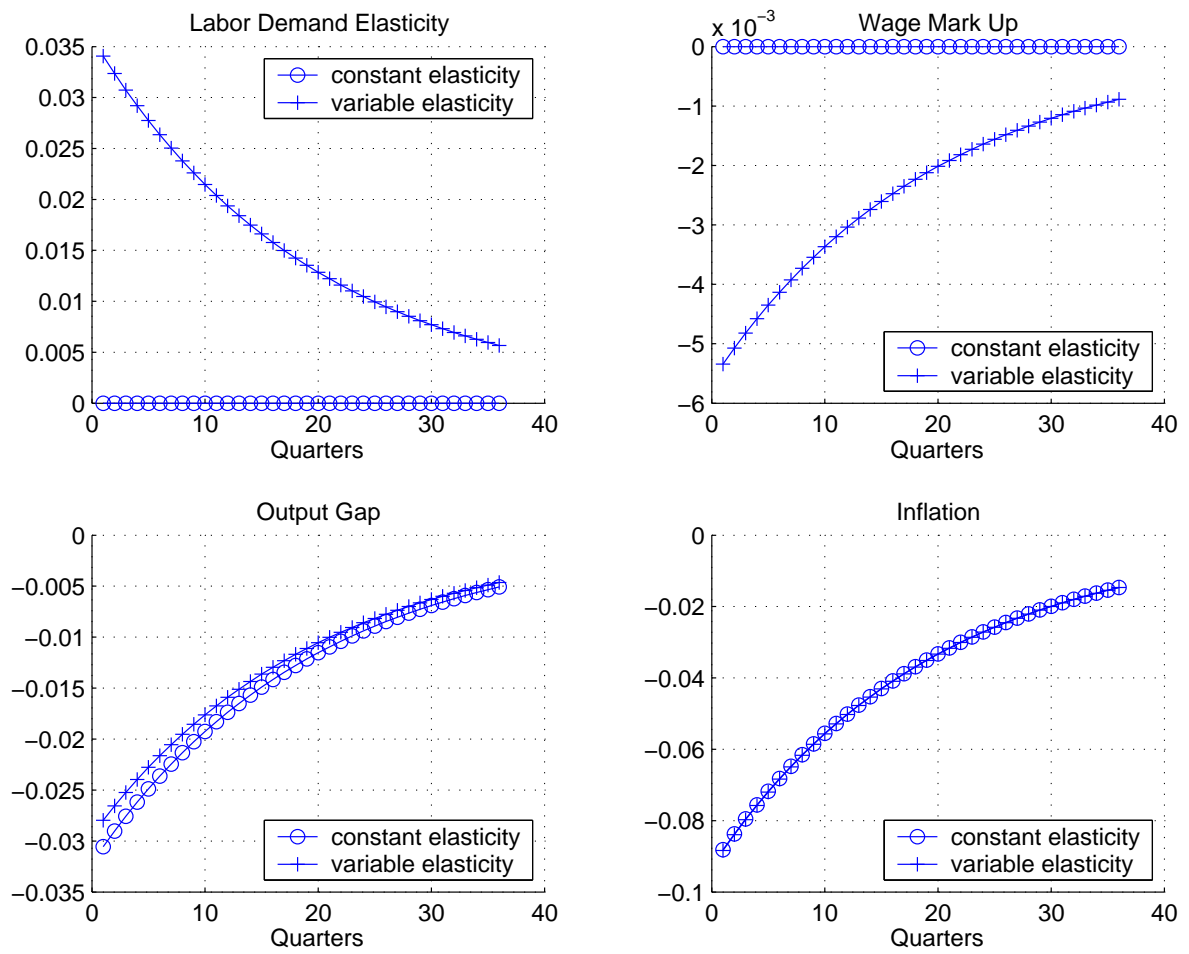


Figure A.2: Percentage Standard Deviations (constant elasticity of labor demand)

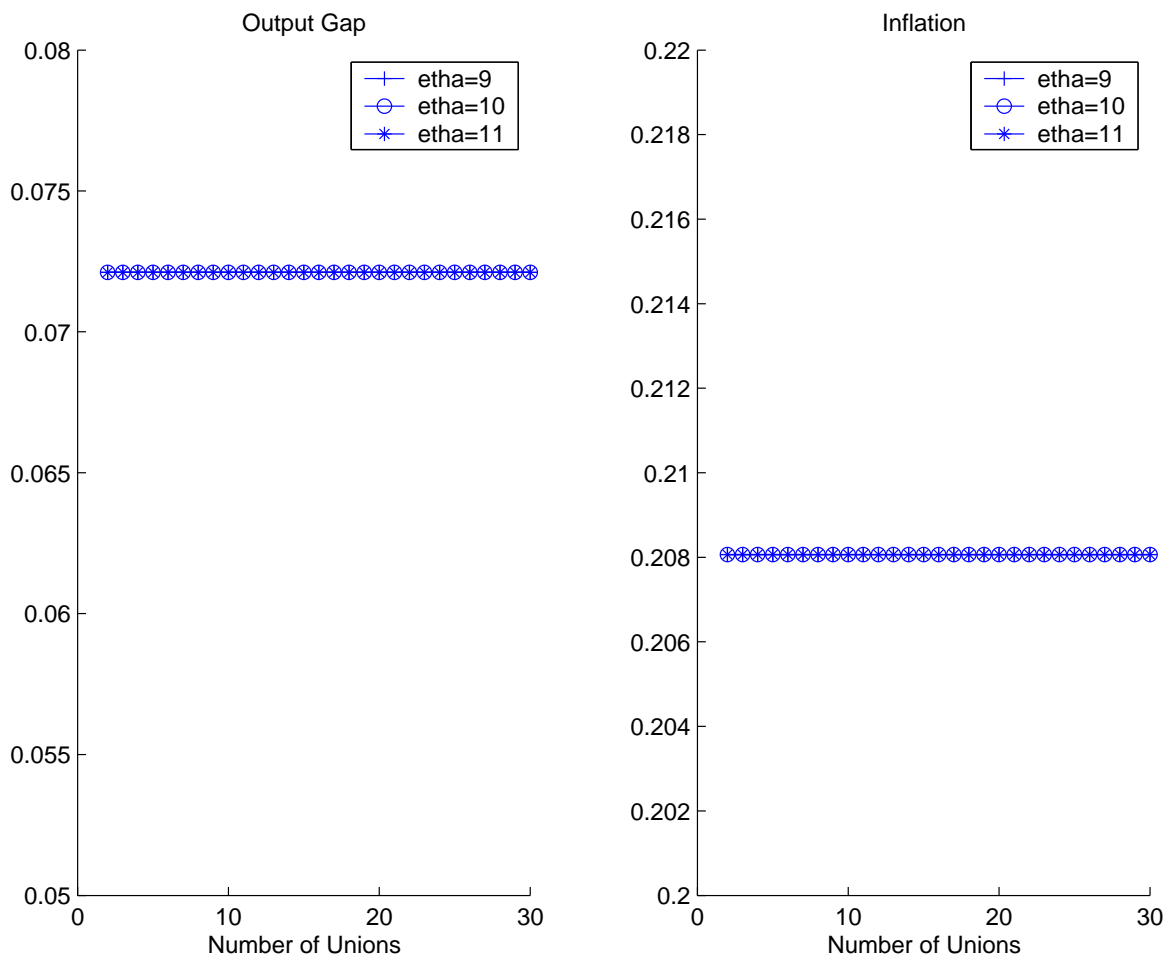


Figure A.3: Percentage Standard Deviations (time varying elasticity of labor demand)

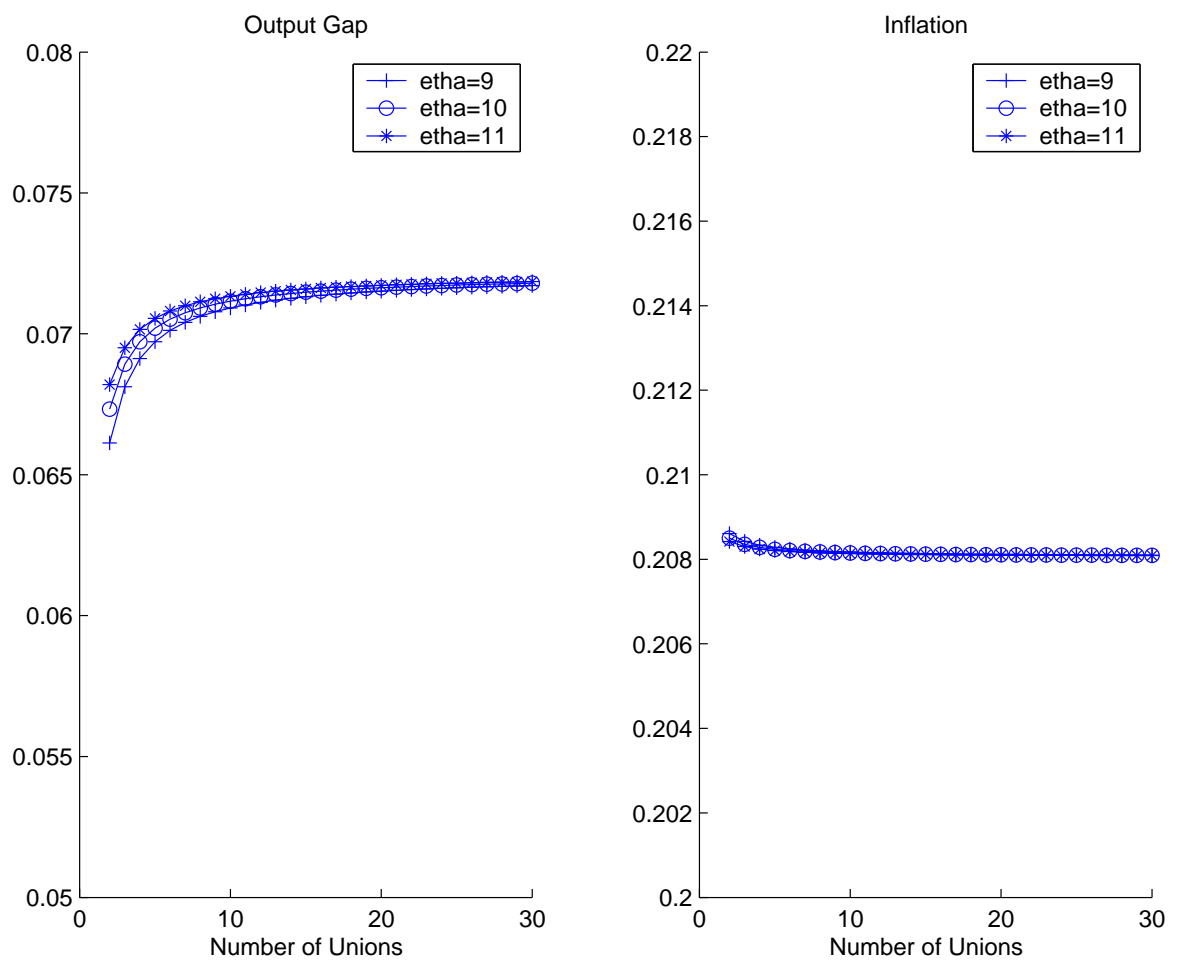


Table A.1: **Baseline Calibration**

Parameter	Value
Price Stickiness	0.75
Discount Factor	0.99
Monetary Policy Stance	1.5
Elast. Subst. Goods	11
Elast. Subst. Labor Types	11
Elast. Marginal Disutility Labor	1
Number of Unions	3
TFP Autocorrelation	0.95
TFP Std. Dev. Innovation	0.0071

Appendix B

Addendum to Chapter 2

B.1 Appendix: Derivation of equation (2.4)

The welfare criterion (2.4) is derived using the method proposed by Benigno and Woodford (2005) for the evaluation of suboptimal policy rules. First, it is characterized the timelessly optimal policy, i.e. an optimal policy that validates private sector's expectations at time zero. Then it is computed an approximation to the value of deviating from the timelessly optimal policy. That value is finally subtracted from the second order approximation of households' lifetime utility.

In the case of non-atomistic wage setters the procedure differs with respect to the one treated in Benigno and Woodford (2005) in that average distortion is not independent of policy. However it can be shown that the timelessly optimal problem can be suitably redefined and solved in two steps: the determination of the timelessly optimal allocation as a function of average distortion and then the choice of the average distortion that maximizes households' utility subject to the constraint of implementing a timelessly optimal allocation.

This further complication makes convenient to introduce the notion of timelessly optimal fluctuations (or timelessly optimal stabilization policy). Recall that $x_t = (Y_t, \Pi_t, \Delta_t)$ and $X_t = (F_t, K_t)$.

Definition 1: Let $\{x_t^*(\Phi), X_t^*(\Phi)\}_{t=0}^\infty$ be the solution to the following problem

$$\text{Max } U_0 \text{ s.t.}$$

$$\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} = \left(\frac{K_t}{F_t} \right)^{1 - \theta_p} \quad (\text{B.1})$$

$$K_t = [1 - \Phi]^{-1} \exp\{\mu_t^w\} \left(\frac{Y_t}{A_t} \right)^{1 + \phi} \Delta_t^\phi + \alpha \beta E_t \{ (\Pi_{t+1})^{\theta_p} K_{t+1} \} \quad (\text{B.2})$$

$$F_t = 1 + \alpha \beta E_t \{ (\Pi_{t+1})^{\theta_p - 1} F_{t+1} \} \quad (\text{B.3})$$

$$\Delta_t = (1 - \alpha) \left(\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} \right)^{\frac{\theta_p}{\theta_p - 1}} + \alpha \Pi_t^{\theta_p} \Delta_{t-1} \quad (\text{B.4})$$

$$X_0 = X_0^* \quad (\text{B.5})$$

given Δ_{-1} , $\{A_t, \mu_t^w\}_{t=0}^\infty$, X_0^* and a value for average distortion Φ . If X_0^* is chosen in such a way that $\{x_t^*(\Phi), X_t^*(\Phi)\}_{t=0}^\infty$ is a time invariant function of exogenous states¹, then $\{x_t^*(\Phi), X_t^*(\Phi)\}_{t=0}^\infty$ is defined to be the timelessly optimal stabilization policy.

Note that the timelessly optimal stabilization policy is **conditional** on Φ , it is in other terms the best response to shocks, given a certain degree of average distortion. In line with the timeless perspective, the initial value of forward looking variables is constrained in the stabilization policy problem. Technically, these constraints allow

¹see Woodford (2003) and Giannoni and Woodford (2002)

to make recursive a problem that naturally is not. Economically, imposing those constraints is equivalent to ask the policy maker not to take advantage of expectations already in place at the time of choosing the commitment.

If the central bank were not constrained by a simple rule, she could choose whatever degree of average distortion she liked, Φ^* , and then implement the timelessly optimal stabilization policy consistent with that degree of average distortion by selecting an appropriate policy rule. $\{x_t^*(\Phi^*), X_t^*(\Phi^*), \Phi^*\}_{t=0}^\infty$ would then be a full characterization of the timelessly optimal policy. Formally the follow definition applies.

Definition 2: Let $\{x_t^*, X_t^*, \Phi^*\}_{t=0}^\infty$ be the solution to the following problem

$$\text{Max } U_0 \text{ s.t.}$$

$$\{x_t^*\}_{t=0}^\infty = \{x_t^*(\Phi)\}_{t=0}^\infty$$

$$\{X_t^*\}_{t=0}^\infty = \{X_t^*(\Phi)\}_{t=0}^\infty$$

given Δ_{-1} and $\{A_t, \mu_t^w\}_{t=0}^\infty$. Then $\{x_t^*, X_t^*, \Phi^*\}_{t=0}^\infty$ is defined to be the timelessly optimal policy.

Hence, the timelessly optimal policy problem can be broken in two steps: first the choice of optimal fluctuations compatible with any degree of average distortion and then the choice of average distortion or, equivalently, the choice of the non-stochastic steady state.

Concerning the second step, it is assumed that whenever the bank has the chance to choose monetary policy without restricting to a simple rule, the best average distortion is zero. This amounts to assume that the marginal benefits of reducing average distortion are greater than the marginal costs. It has been checked numerically that this is always the case for all calibrations considered here.

The rest of the section develops as follows: section 1 characterizes and approximates to first order the timelessly optimal stabilization policy; section 2 derives the welfare criterion for the evaluation of simple policy rules.

B.2 Timelessly Optimal Fluctuations

The problem associated to Definition 1 has no closed form solution. However, using a linear-quadratic approach allows to obtain an approximate characterization of the timelessly optimal stabilization policy at a first order accuracy. Before resorting to local approximation techniques it is shown the existence of a non-stochastic steady state.

The constraints implied by the initial commitments $X_0 = X_0^*$ can be equivalently rewritten as

$$\Pi_0^{\theta_p-1} F_0 = \Pi_0^{*\theta_p-1} F_0^* \quad (\text{B.6})$$

$$\Pi_0^{\theta_p} K_0 = \Pi_0^{*\theta_p} K_0^* \quad (\text{B.7})$$

where Π_0^* is the inflation rate consistent with X_0^* according to equation (B.1). Let $\psi_{1,t}$ through $\psi_{4,t}$ denote the Lagrange multipliers corresponding to constraints (B.1) through (B.4) and let $-\alpha\psi_{2,-1}^*$ $-\alpha\psi_{3,-1}^*$ denote the Lagrange multipliers corresponding to constraints (B.6) and (B.7). Hence, the problem associated to Definition 1 can be restated using the following Lagrangian function

$$\begin{aligned} \Lambda_t &= E_0 \sum_{t=0}^{\infty} \beta^t \{h(\psi_t, \psi_{t-1}; x_t, x_{t-1}, X_t)\} \\ &\quad - \psi_{2,-1}^* \alpha \left[\Pi_0^{\theta_p-1} F_0 - \Pi_0^{*\theta_p-1} F_0^* \right] \\ &\quad - \psi_{3,-1}^* \alpha \left[\Pi_0^{\theta_p} K_0 - \Pi_0^{*\theta_p} K_0^* \right] \end{aligned}$$

where ψ is the vector of Lagrange multipliers and $h(\cdot)$ is defined as

$$\begin{aligned}
h(\psi_t, \psi_{t-1}; x_t, x_{t-1}, X_t) = & u_t + \psi_{1,t} \left[K_t \left(\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} \right)^{\frac{1}{\theta_p - 1}} - F_t \right] \\
& + \psi_{2,t} [F_t - 1 - \alpha \beta (\Pi_{t+1})^{\theta_p - 1} F_{t+1}] \\
& + \psi_{3,t} [K_t - MC_t - \alpha \beta (\Pi_{t+1})^{\theta_p} K_{t+1}] \\
& + \psi_{4,t} \left[\Delta_t - (1 - \alpha) \left(\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} \right)^{\frac{\theta_p}{\theta_p - 1}} - \alpha \Pi_t^{\theta_p} \Delta_{t-1} \right]
\end{aligned}$$

For convenience the following definitions have been used

$$MC_t = [1 - \Phi]^{-1} \exp\{\mu_t^w\} \left(\frac{Y_t}{A_t} \right)^{1+\phi} \Delta_t^\phi$$

$$u_t = \log Y_t - \frac{\left(\frac{Y_t \Delta_t}{A_t} \right)^{(1+\phi)}}{1 + \phi}$$

The marginal benefit of relaxing constraints (B.6) and (B.7) is equal to the value of the corresponding Lagrange multipliers and it can be interpreted as the marginal gain of fooling agents at time zero.

Rearranging terms, the Lagrangian can be rewritten (up to a constant) in the following discounted stationary form so that a time invariant system of first order conditions can be trivially obtained

$$\Lambda_t = E_0 \sum_{t=0}^{\infty} \beta^t g(\psi_t, \psi_{t-1}; x_t, x_{t-1}, X_t) \quad (\text{B.8})$$

where $g(\cdot)$ is now defined as

$$\begin{aligned}
g(\psi_t, \psi_{t-1}; x_t, x_{t-1}, X_t) = & u_t + \psi_{1,t} \left[K_t \left(\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} \right)^{\frac{1}{\theta_p - 1}} - F_t \right] \\
& + \psi_{2,t} [F_t - 1] - \alpha \psi_{2,t-1} [(\Pi_t)^{\theta_p - 1} F_t] \\
& + \psi_{3,t} [K_t - MC_t] - \alpha \psi_{3,t-1} [(\Pi_t)^{\theta_p} K_t] \\
& + \psi_{4,t} \left[\Delta_t - (1 - \alpha) \left(\frac{1 - \alpha \Pi_t^{\theta_p - 1}}{1 - \alpha} \right)^{\frac{\theta_p}{\theta_p - 1}} - \alpha \Pi_t^{\theta_p} \Delta_{t-1} \right]
\end{aligned}$$

(B.8) has the same form as the one used by Benigno and Woodford (2005)² and it can be immediately seen that their results apply to the case with non-atomistic wage setters. Hence I refer to their paper in stating the following results.

Proposition 1: *The non-stochastic steady state of the problem associated to Definition 1 exists and is such that*

$$K = F = (1 - \alpha\beta)^{-1}$$

$$\Pi = \Delta = 1$$

$$Y = (1 - \Phi)^{\frac{1}{1+\phi}}$$

Proposition 2: *A second order approximation to lifetime utility (2.3) yields*

$$\frac{\bar{U}}{1 - \beta} + E_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi \hat{y}_t - \frac{1}{2} u_{yy} \hat{y}_t^2 - \frac{1}{2} u_{\pi\pi} \hat{\pi}_t^2 + u_{ya} \hat{y}_t a_t - \frac{1}{2} u_{aa} a_t^2 + u_a a_t \right]$$

where \hat{y}_t measures deviations of aggregate output from its steady state level and the coefficients entering equation (B.9) are

²see their Appendix B1

$$\begin{aligned}
u_{yy} = u_{ya} = u_{aa} &= (1 - \Phi)(1 + \phi) \\
u_{\pi} &= (1 - \Phi) \frac{\theta_p}{\lambda} \\
u_a &= (1 - \Phi) \\
\lambda &= \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha}
\end{aligned}$$

Note that when the steady state is distorted, a non-zero linear term appears in (B.9), implying that you cannot evaluate utility to the second order using an approximate solution for output that is accurate to first order only. However, the linear term can be substituted out using a second order approximation to the aggregate supply (B.1)

Proposition 3: *The second order approximation to lifetime utility (B.9) can be rewritten in the following purely quadratic form*

$$\begin{aligned}
W_0 &= \frac{\bar{U}(\Phi)}{1 - \beta} - \frac{1}{2} \frac{\theta_p}{\lambda} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + (1 + \phi) \frac{\lambda}{\theta_p} \hat{x}_t^2 \right] + \\
&\quad - \frac{1}{2} \frac{\Phi(1 - \Phi)}{1 + \phi} E_0 \sum_{t=0}^{\infty} \beta^t (\mu_t^w)^2 + E_0 \sum_{t=0}^{\infty} \beta^t \hat{y}_t^* + T_0 \tag{B.9}
\end{aligned}$$

Proof: *A second order approximation to the aggregate supply (B.1) yields*

$$\begin{aligned}
V_0 &= \lambda E_0 \sum_{t=0}^{\infty} \beta^t \left[v_y \hat{y}_t + \frac{1}{2} v_{yy} \hat{y}_t^2 + \frac{1}{2} v_{\pi} \hat{\pi}_t^2 - v_{ya} \hat{y}_t a_t + \frac{1}{2} v_{aa} a_t^2 - v_a a_t + \right. \\
&\quad \left. + \mu_t^w + \frac{1}{2} (\mu_t^w)^2 + (1 + \phi) \mu_t^w (\hat{y}_t - a_t) \right] \tag{B.10}
\end{aligned}$$

where

$$\begin{aligned}
v_{yy} = v_{ya} = v_{aa} &= (1 + \phi)^2 \\
v_{\pi} &= (1 + \phi) \frac{\theta_p}{\lambda} \\
v_a &= (1 + \phi) \\
v_y &= (1 + \phi) \\
\hat{x}_t &= \hat{y}_t - \hat{y}_t^* \\
\hat{y}_t^* &= a_t - \frac{\Phi}{1 + \phi} \mu_t^w
\end{aligned}$$

Subtracting $\frac{\Phi}{\lambda(1+\phi)}V_0$ from U_0 one can obtain (B.9) where T_0

$$T_0 = \frac{\Phi}{\lambda(1 + \phi)}V_0$$

is a deterministic component that depends only on the initial commitments on the forward looking variables and that is predetermined at the time of the policy choice.

These results can be used to derive a first order approximation to the timelessly optimal stabilization policy. Within a linear-quadratic framework the problem associated to Definition 1 can be reformulated as follows

$$\begin{aligned}
&\text{Min } \frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \tilde{\lambda} \hat{x}_t^2 \right] \text{ s.t.} \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa \hat{x}_t + (1 - \Phi) \lambda \mu_t^w \\
\pi_0 &= \pi_0^*
\end{aligned}$$

where

$$\begin{aligned}
\kappa &= \lambda(1 + \phi) \\
\tilde{\lambda} &= \kappa/\theta_p
\end{aligned}$$

Defining φ as the Lagrange multiplier associated to the log-linear version of the aggregate supply, the first order conditions are

$$\begin{aligned}\pi_t + \varphi_t - \varphi_{t-1} &= 0 \\ \tilde{\lambda}\hat{x}_t - \kappa\varphi_t &= 0\end{aligned}$$

These conditions can be rearranged in order to have either a targeting rule

$$\pi_t + \frac{\tilde{\lambda}}{\kappa}(\hat{x}_t - \hat{x}_{t-1}) = 0 \quad (\text{B.11})$$

or an equation describing the evolution of the Lagrange multiplier

$$E_t\varphi_{t+1} - \frac{1}{\beta}\left(1 + \beta + \frac{\kappa^2}{\tilde{\lambda}}\right)\varphi_t + \frac{1}{\beta}\varphi_{t-1} = \frac{1}{\beta}(1 - \Phi)\lambda\mu_t^w \quad (\text{B.12})$$

It can be shown that the characteristic equation

$$\mu^2 - \frac{1}{\beta}\left(1 + \beta + \frac{\kappa^2}{\tilde{\lambda}}\right)\mu + \frac{1}{\beta} = 0$$

has two roots μ_1 and μ_2 such that $0 < \mu_1 < 1 < \mu_2$. Hence, equation (B.12) has a unique bounded solution and

$$\varphi_t = \mu\varphi_{t-1} - \mu(1 - \Phi)\lambda E_0 \sum_{t=0}^{\infty} (\beta\mu)^t \mu_t^w \quad (\text{B.13})$$

where $\mu \equiv \mu_1$. If a process for the mark-up shock of the form (2.2) is assumed, (B.13) becomes

$$\varphi_t = \mu\varphi_{t-1} - \frac{\mu(1 - \Phi)\lambda}{1 - \beta\rho_u\mu} \mu_t^w \quad (\text{B.14})$$

The Lagrange multiplier can be solved as a function of the history of wage mark-up shocks

$$\varphi_t = -\frac{\mu(1-\Phi)\lambda}{1-\beta\rho_u\mu} \sum_{j=0}^{\infty} \mu^j \mu_{t-j}^w \quad (\text{B.15})$$

Finally (B.15), together with the log-linear version of the Phillips curve and the first order conditions, determines inflation and output gap as a function of the history of shocks and average distortion. In the timelessly optimal policy average distortion is zero, hence it follows that

$$\varphi_t^* = -\frac{\mu\lambda}{1-\beta\rho_u\mu} \sum_{j=t}^{\infty} \mu^j \mu_{t-j}^w \quad (\text{B.16})$$

(B.16) can be interpreted as a first order approximation to the marginal value of deviating from the timelessly optimal policy.

B.3 Evaluation of suboptimal rules

Although expected lifetime utility as of time zero has been used in determining the timelessly optimal policy, W_0 cannot serve the purpose of evaluating policy rules. This is because of the time inconsistency issue.

In a timeless perspective, initial commitments guarantee that policy confirms past expectations about current outcomes. However, it may be the case that the optimal initial commitments are not feasible within the class of rules under consideration. In turn the violation of initial commitments may give an advantage to those rules, because of the usual time inconsistency that naturally arises in any Ramsey problem.

Notwithstanding, it is undesirable to prefer rules that are improving the stabilization trade-off by fooling the agents at the time of policy selection. Therefore, Benigno and Woodford (2005) propose to use a welfare criterion that is still based on expected lifetime utility but that penalizes deviations from the timelessly optimal commitments. In particular the criterion is modified in such a way that if the class is flexible enough

to contain the timelessly optimal policy, then the rule implementing the timelessly optimal policy is selected as the best one. Hence the new criterion becomes

$$\hat{U}_0 = U_0 - \psi_{2,-1}^* \alpha \left[\Pi_0^{\theta_p-1} F_0 - \Pi_0^{*\theta_p-1} F_0^* \right] - \psi_{3,-1}^* \alpha \left[\Pi_0^{\theta_p} K_0 - \Pi_0^{*\theta_p} K_0^* \right] \quad (\text{B.17})$$

Note that any rational expectation equilibrium that is maximizing (B.17) and is satisfying the timelessly optimal commitments is by definition the timelessly optimal allocation. It is in fact the solution to the problems associated to Definition 1 and Definition 2. In addition the following result holds

Proposition 4: *A second order approximation to the modified welfare criterion (B.17) can be written in the following purely quadratic form*

$$\begin{aligned} W_0 - \varphi_{-1}^* (\pi_0 - \pi_0^*) &= \frac{\bar{U}(\Phi)}{1-\beta} - \frac{1}{2} \frac{\Phi(1-\Phi)}{1+\phi} E_0 \sum_{t=0}^{\infty} \beta^t (\mu_t^w)^2 + E_0 \sum_{t=0}^{\infty} \beta^t \hat{y}_t^* \quad (\text{B.18}) \\ &\quad - \frac{1}{2} \frac{\theta_p}{\lambda} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + (1+\phi) \frac{\lambda}{\theta_p} \hat{x}_t^2 \right] - \varphi_{-1}^* (\pi_0 - \pi_0^*) \end{aligned}$$

where φ_{-1}^* is the Lagrange multiplier associated to the timelessly optimal policy problem in its linear-quadratic version and π_0^* is a first order approximation to the timelessly optimal initial commitment

Since the Lagrange multiplier depends on the history of shocks prior to the policy choice, in the spirit of the timeless it is computed the unconditional expectation of (B.18) integrating over all possible histories of the shocks

$$\begin{aligned}
\hat{W}_0 &= E \{ W_0 - \varphi_{-1}^*(\pi_0 - \pi_0^*) \} \\
&= \frac{\bar{U}(\Phi)}{1 - \beta} - \frac{1}{2} \frac{\Phi(1 - \Phi)}{1 + \phi} E \sum_{t=0}^{\infty} \beta^t (\mu_t^w)^2 + \\
&\quad - \frac{1}{2} \frac{\theta_p}{\lambda} E \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + (1 + \phi) \frac{\lambda}{\theta_p} \hat{x}_t^2 \right] - E \{ \varphi_{-1}^*(\pi_0 - \pi_0^*) \} \quad (\text{B.19})
\end{aligned}$$

Defining $E\Psi(\mu^{w,0})$ as $E \{ \varphi_{-1}^*(\pi_0 - \pi_0^*) \}$, (B.19) becomes (2.4).

B.4 Appendix: Derivation of coefficients $f_{\pi,a}$, $f_{\pi,u}$, $f_{x,a}$ and $f_{x,u}$

The system of stochastic difference equation (2.12) has a unique solution of the form

$$\begin{aligned}
\begin{bmatrix} \hat{x}_t \\ \pi_t \end{bmatrix} &= -(1 - \rho_a) [I - \rho_a A]^{-1} B a_t + \\
&\quad [I - \rho_u A]^{-1} \left[\frac{(1 - \rho_u) \Phi B}{1 + \phi} + \lambda(1 - \Phi) C \right] \mu_t^w
\end{aligned}$$

Defining

$$TFP = -(1 - \rho_a) [I - \rho_a A]^{-1} B$$

and

$$CP = [I - \rho_u A]^{-1} \left[\frac{(1 - \rho_u) \Phi B}{1 + \phi} + \lambda(1 - \Phi) C \right]$$

it follows that $f_{\pi,a} = TFP(2,1)$, $f_{x,a} = TFP(1,1)$, $f_{\pi,u} = CP(2,1)$ and $f_{x,u} = CP(1,1)$.

B.5 Appendix: Derivation of equation (2.15)

Define

$$\sigma_u^2 = \frac{\sigma_{\varepsilon,u}^2}{1 - \rho_u^2}$$

$$\sigma_a^2 = \frac{\sigma_{\varepsilon,a}^2}{1 - \rho_a^2}$$

Using (2.13) and (2.14), the third term of (B.19) becomes

$$E \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + (1 + \phi) \frac{\lambda}{\theta_p} \hat{x}_t^2 \right] \right\} = \frac{\sigma_a^2}{1 - \beta} (f_{\pi,a}^2 + \tilde{\lambda} f_{x,a}^2) + \frac{\sigma_u^2}{1 - \beta} (f_{\pi,u}^2 + \tilde{\lambda} f_{x,u}^2) \quad (\text{B.20})$$

From the solution of the Lagrange multiplier (B.15)

$$E \{ \varphi_{t-1} \mu_t^w \} = - \frac{\mu \lambda \rho_u \sigma_u^2}{(1 - \beta \rho_u \mu)(1 - \rho_u \mu)}$$

Substituting the previous equation and (2.13) in the fourth term of (B.19) yields

$$\begin{aligned} E \{ \varphi_{-1}^* (\pi_0 - \pi_0^*) \} &= f_{\pi,u} E \{ \varphi_{-1}^* \pi_0 \} + t.i.p. \\ &= -f_{\pi,u} \frac{\mu \lambda \rho_u \sigma_u^2}{(1 - \beta \rho_u \mu)(1 - \rho_u \mu)} + t.i.p. \\ &= -f_{\pi,u} \lambda \Gamma + t.i.p. \end{aligned} \quad (\text{B.21})$$

where

$$\Gamma = \frac{\mu \rho_u \sigma_u^2}{(1 - \beta \rho_u \mu)(1 - \rho_u \mu)}$$

Finally, given the stochastic properties of the wage mark-up shock,

$$\frac{1}{2} \frac{\Phi(1-\Phi)}{1+\phi} E \sum_{t=0}^{\infty} \beta^t (\mu_t^w)^2 = \frac{1}{2} \frac{\Phi(1-\Phi)}{1+\phi} \frac{\sigma_u^2}{1-\beta} \quad (\text{B.22})$$

Using (B.20), (B.21) and (B.22) in (B.19), (2.15) can be immediately obtained.

Appendix C

Addendum to Chapter 3

C.1 Perfect Coordination

Defining country i inflation, output gap and fiscal gap differentials

$$\pi_t^{di} = \pi_t^i - \pi_t^* \quad \tilde{f}_t^{di} = \tilde{f}_t^i - \tilde{f}_t^* \quad \tilde{y}_t^{di} = \tilde{y}_t^i - \tilde{y}_t^* \quad (\text{C.1})$$

the welfare function (3.48) and the constraints (3.42), (3.43), (3.44) and (3.46) can be rewritten as

$$W = W^* + W^d \quad (\text{C.2})$$

$$\pi_t^* = \beta E_t \{ \pi_{t+1}^* \} + \lambda(1 + \varphi) \tilde{y}_t^* - \lambda \frac{\chi}{1 - \chi} \tilde{f}_t^* + \lambda \mu_t^{w,*} \quad (\text{C.3})$$

$$\tilde{y}_t^* = E_t \tilde{y}_{t+1}^* + \frac{\chi}{1 - \chi} \tilde{f}_t^* - \frac{\chi}{1 - \chi} E_t \tilde{f}_{t+1}^* - (r_t^* - E_t \{ \pi_{t+1}^* \} - r r_t^*) \quad (\text{C.4})$$

$$\pi_t^{di} = \beta E_t \{ \pi_{t+1}^{di} \} + \lambda(1 + \varphi) \tilde{y}_t^{di} - \lambda \frac{\chi}{1 - \chi} \tilde{f}_t^{di} + \lambda (\mu_t^{w,i} - \mu_t^{w,*}) \quad (\text{C.5})$$

$$\Delta \tilde{y}_t^{di} = \frac{\chi}{1 - \chi} \Delta \tilde{f}_t^{di} - [\pi_t^{di} + (\Delta a_t^i - \Delta a_t^*)] \quad (\text{C.6})$$

$$\int_0^1 \pi_t^{di} di = 0 \quad \int_0^1 \tilde{y}_t^{di} di = 0 \quad \int_0^1 \tilde{f}_t^{di} di = 0 \quad (\text{C.7})$$

where

$$W^* = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left(\frac{\epsilon_p}{\lambda} (\pi_t^*)^2 + (1 + \varphi) (\tilde{y}_t^*)^2 + \frac{\chi}{1 - \chi} (\tilde{f}_t^*)^2 \right) + tips \quad (\text{C.8})$$

$$W^d = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \int_0^1 \left(\frac{\epsilon_p}{\lambda} (\pi_t^{di})^2 + (1 + \varphi) (\tilde{y}_t^{di})^2 + \frac{\chi}{1 - \chi} (\tilde{f}_t^{di})^2 \right) di + tips \quad (\text{C.9})$$

Concerning the optimal policy mix under commitment for the currency area as a whole we simply refer to Galí and Monacelli (2005).

We solve the currency area optimization problem under discretion by restricting to Markov perfect equilibria. Since the problem does not involve endogenous state variables, future variables are functions of future exogenous states only. As a consequence, a discretionary government that cannot manipulate private beliefs has to take expectations as given. Therefore, the currency area problem reduces to a sequence of static problems. Maximizing (C.8) subject to (C.3) with respect to inflation, output and fiscal gaps yields

$$\frac{\epsilon_p}{\lambda} \pi_t^* + \nu_{\pi,t}^* = 0 \quad (\text{C.10})$$

$$(1 + \varphi) \tilde{y}_t^* - \lambda(1 + \varphi) \nu_{\pi,t}^* = 0 \quad (\text{C.11})$$

$$\frac{\chi}{1 - \chi} \tilde{f}_t^* + \lambda \frac{\chi}{1 - \chi} \nu_{\pi,t}^* = 0 \quad (\text{C.12})$$

Combining (C.10)-(C.12) the policy rules (3.60) and (3.61) can be easily obtained.

C.2 The Discretionary Fiscal Policy Problem

The fiscal policy problem can be split in two independent parts. The currency area problem consists in selecting a fiscal policy rule for the union-wide fiscal gap $\{\tilde{f}_t^*\}_{t=0}^{\infty}$ maximizing (C.8) subject to (C.3) and (C.4), given the union-wide nominal interest rate and the exogenous stochastic processes. Finally, optimization of the welfare function (C.9) subject to (C.5), (C.6) and (C.7) determines the state-contingent path of fiscal gap differentials $\{\tilde{f}_t^{di}\}_{t=0}^{\infty}$, for all $i \in [0, 1]$.

C.2.1 The Currency Area Problem

First order conditions are the following

$$\frac{\epsilon_p}{\lambda} \pi_t^* + \psi_{\pi,t}^* = 0 \quad (\text{C.13})$$

$$(1 + \varphi) \tilde{y}_t^* - \lambda(1 + \varphi) \psi_{\pi,t}^* + \psi_{r,t}^* = 0 \quad (\text{C.14})$$

$$\frac{\chi}{1 - \chi} \tilde{f}_t^* + \lambda \frac{\chi}{1 - \chi} \psi_{\pi,t}^* - \frac{\chi}{1 - \chi} \psi_{r,t}^* = 0 \quad (\text{C.15})$$

together with the constraints (C.3) and (C.4), where $\psi_{\pi,t}^*$ and $\psi_{r,t}^*$ are the lagrange multipliers respectively associated to (C.3) and (C.4). The system can be equivalently rewritten as

$$\frac{\epsilon_p}{\lambda} \pi_t^* + \psi_{\pi,t}^* = 0 \quad (\text{C.16})$$

$$(1 + \varphi) \tilde{y}_t^* - \lambda(1 + \varphi) \psi_{\pi,t}^* + \psi_{r,t}^* = 0 \quad (\text{C.17})$$

$$\tilde{f}_t^* = -\tilde{y}_t^* - \varphi(\tilde{y}_t^* + \epsilon_p \pi_t^*) \quad (\text{C.18})$$

where (C.18) is the fiscal policy rule reported in the text, (3.63), and the first two equations, given the solution that the central bank wants to implement, serve the only purpose to determine lagrange multipliers.

C.2.2 The Representative Country Problem

The differential part of the fiscal optimization program is more involved than the currency area problem, as, even restricting to Markov strategies, in any stationary equilibrium endogenous variables depend on their own lags. This is because the lagged values of fiscal and output gaps enter equation (C.6). As an implication, expectations cannot be taken as given. Therefore, we conjecture that the private sector forecasts future variables as linear functions of current states for some arbitrary coefficients. At the rational expectation equilibrium, those coefficients are defined to be such to

coincide with the true fundamental parameters of the state space representation. To keep the problem tractable, we substitute out inflation using its definition

$$\pi_t^{di} = p_t^{di} - p_{t-1}^{di} \quad (\text{C.19})$$

and the fact that

$$y_t^{di} = \frac{\chi}{1-\chi} f_t^{di} - [p_t^{di} + (a_t^i - a_t^*)] \quad (\text{C.20})$$

This allows to reduce the number of endogenous states, by replacing (C.6) with (C.20). The equivalent optimization program features two controls, y_t^{di} and f_t^{di} , and an endogenous state, p_t^{di} . It is guessed that

$$p_t^{di} = c_1 p_{t-1}^{di} + c_2 (a_t^i - a_t^*) + c_3 (\mu_t^i - \mu_t^*) \quad (\text{C.21})$$

equation (C.20) can be used in the Phillips curve to write f_t^{di} in terms of current and past states only

$$\begin{aligned} \tilde{f}_t^{di} &= \frac{1-\chi}{\chi\lambda\varphi} [1 + \beta(1-c_1) + \lambda(1+\varphi)] p_t^{di} - \frac{1-\chi}{\chi\lambda\varphi} p_{t-1}^{di} \\ &+ \frac{1-\chi}{\chi\lambda\varphi} [\lambda(1+\varphi) - \beta\rho c_2] (a_t^i - a_t^*) \\ &- \frac{1-\chi}{\chi\lambda\varphi} (\lambda + \beta\rho c_3) (\mu_t^i - \mu_t^*) \end{aligned} \quad (\text{C.22})$$

Plugging (C.22) back into (C.20) yields

$$\begin{aligned} \tilde{y}_t^{di} &= \left\{ \frac{1}{\lambda\varphi} [1 + \beta(1-c_1) + \lambda(1+\varphi)] - 1 \right\} p_t^{di} - \frac{1}{\lambda\varphi} p_{t-1}^{di} \\ &+ \left\{ \frac{1}{\lambda\varphi} [\lambda(1+\varphi) - \beta\rho c_2] - 1 \right\} (a_t^i - a_t^*) \\ &- \frac{1}{\lambda\varphi} (\lambda + \beta\rho c_3) (\mu_t^i - \mu_t^*) \end{aligned} \quad (\text{C.23})$$

The problem consists in minimizing the value function

$$V_t = \min \left\{ \frac{\varepsilon}{\lambda} (p_t^{di} - p_{t-1}^{di})^2 + (1+\varphi) (\tilde{y}_t^{di})^2 + \frac{\chi}{1-\chi} (\tilde{f}_t^{di})^2 + \beta E_t V_{t+1} \right\} \quad (\text{C.24})$$

subject to (C.22) and (C.23). The corresponding first order condition is

$$\begin{aligned} \frac{2\varepsilon}{\lambda}(p_t^{di} - p_{t-1}^{di}) + 2(1 + \varphi) \left\{ \frac{1}{\lambda\varphi} [1 + \beta(1 - c_1) + \lambda(1 + \varphi)] - 1 \right\} \tilde{y}_t^{di} \\ + \frac{2}{\lambda\varphi} [1 + \beta(1 - c_1) + \lambda(1 + \varphi)] \tilde{f}_t^{di} + \beta E_t \frac{\partial V_{t+1}}{\partial p_t} = 0 \end{aligned} \quad (\text{C.25})$$

Updating one period ahead the envelope condition

$$\frac{\partial V_t}{\partial p_{t-1}} = -\frac{2\varepsilon}{\lambda}(p_t^{di} - p_{t-1}^{di}) - \frac{2(1 + \varphi)}{\lambda\varphi} \tilde{y}_t^{di} - \frac{2}{\lambda\varphi} \tilde{f}_t^{di} \quad (\text{C.26})$$

and substituting it in (C.25) yields equation (3.65) in the text.

C.3 The Monetary Policy Problem

The central bank has to choose a state contingent path for the union-wide policy outcomes $\left\{ \pi_t^*, \tilde{y}_t^*, \tilde{f}_t^* \right\}_{t=0}^{\infty}$ in order to maximize W^* subject to (3.51) and (C.18). The nominal interest rate is chosen ex-post, consistently with the union-wide IS equations. The associated first order conditions are

$$\frac{\epsilon_p}{\lambda} \pi_t^i + \Delta \xi_{\pi,t}^* + \varphi \epsilon_p \xi_{f,t}^* = 0 \quad (\text{C.27})$$

$$(1 + \varphi) \tilde{y}_t^* - \lambda(1 + \varphi) \xi_{\pi,t}^* + (1 + \varphi) \xi_{f,t}^* = 0 \quad (\text{C.28})$$

$$\frac{\chi}{1 - \chi} \tilde{f}_t^* + \lambda \frac{\chi}{1 - \chi} \xi_{\pi,t}^* + \xi_{f,t}^* = 0 \quad (\text{C.29})$$

where $\xi_{\pi,t}^*$, $\xi_{f,t}^*$ are the lagrange multipliers respectively associated to (3.51) and (C.18). (C.28) and (C.29) allow to express lagrange multipliers as functions of output and fiscal gaps

$$\xi_{\pi,t}^* = \frac{1 - \chi}{\lambda} \tilde{y}_t^* - \frac{\chi}{\lambda} \tilde{f}_t^* \quad (\text{C.30})$$

$$\xi_{f,t}^* = -\chi(\tilde{y}_t^* + \tilde{f}_t^*) \quad (\text{C.31})$$

Substituting back into (C.27) yields the monetary policy rule (3.64) in the text.

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