Markets with Frictions

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To my father

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Abstract

This thesis consists of three chapters analyzing markets with frictions. In the first two chapters frictions result from consumers not knowing all the prices and searching for them. In the first chapter I study multiproduct price competition in this environment. I find that consumer search induces firms to negatively correlate prices of complements in order to rip-off consumers who do not search enough. In the second chapter I study the effects of consumer search on price competition when firms have different marginal costs. I demonstrate that firms with different costs cannot charge common prices in equilibrium. Due to this, the higher are the costs the higher are the average prices charged by firms. In the third chapter frictions emerge because firms do not have access to all the markets. I analyze quantity competition following a capacity investment stage to show that equilibrium capacity is larger than in a standard Cournot model because of pro-competitive incentives in fragmented markets.

Resumen

Esta tesis consta de tres capítulos en donde analizo mercados con fricciones. En los dos primeros capítulos, estas fricciones surgen debido a la carencia información completa acerca de los precios por parte de los consumidores. Puntualmente, en el primer capítulo, estudio como se desarrolla la competición de precio multiproducto en este tipo de ambiente. Encuentro que la búsqueda por precios bajos por parte de los consumidores conlleva a que las firmas fijen los precios de los productos complementarios de manera correlativamente inversa. De esta manera, las firmas buscan incrementar sus ganancias valiéndose de aquellos consumidores que no investigan lo suficiente. En el siguiente capítulo analizo cuales son los efectos que genera la búsqueda de mejores precios en la determinación de los mismos cuando las firmas tienen diferentes costos marginales. Demuestro que firmas con diferentes estructuras de costos no pueden fijar los mismos precios en equilibrio. Debido a esto, mayores costos conllevan a mayores precios promedios. Finalmente, en el tercer capítulo, las fricciones emergen debido a que las firmas no tienen acceso a todos los mercados. Analizo la competición en cantidades que se desarrolla luego de la etapa de inversión en capacidad productiva. Demuestro que la capacidad productiva es mayor que la generada en un modelo Cournot estándar debido los incentivos pro-competitivos presentes en los mercados fragmentados.

Foreword

This dissertation is a collection of three essays that analyze various aspects of fragmented markets. My efforts on this topic are the result of the conviction that traditional models of competition, which assume that all firms have access to the same market and compete for all the consumers there, is inadequate for many purposes. Market fragmentation can arise for several reasons. Often consumers do not have easy access to all the firms and confine themselves to visiting only a few before making their purchases. Sometimes firms themselves cannot access markets due to lack of licenses, distribution networks etc. Once the market in the traditional sense has been separated into smaller and interconnected markets many conventional rules as fundamental as the "law of one price" can fail.

All three chapters that form this thesis are written with this consideration in mind. In the first two chapters I analyze traditional pricing situations under market fragmentation. These two chapters are closely related to each other due to the shared assumption on the consumer behavior. I assume that consumers do not know all the prices charged by firms in the market and conduct costly search for the best prices.[1] Search for the best price, or rather lack of one, gives firms market power even if the products they sell are assumed to be undifferentiated. The last chapter is somewhat different from the previous two because I assume that it is the firms who do not have access to all the markets. In effect, firms end up competing only with those who serve the markets they serve themselves but still remain in interaction with all others through indirect competition. From here I proceed to a more detailed account of each chapter.

As mentioned above, in the first chapter consumers search for the best prices in the market and in doing so incur search costs. This process inevitably leads to a situation where some consumers visit several shops while others visit only one, giving ground for firms to price above marginal costs. Multiproduct pricing is a textbook matter if consumers have access to only one firm (monopolist) or to all the firms. In the former case pricing depends on the relationship between the two goods, while in the latter firms are forced to price each good at marginal cost regardless of the nature of their interrelation. In the first chapter I study implications

[1] Diamond (1971) was one of the first to point out that even negligible search costs can lead to paradoxical equilibrium outcomes.

for pricing of multiple goods when demand for these goods is dependent. Price competition between firms emerges because of the consumers who are willing to search for the lowest price in the market, thus forcing retailers to undercut each other's prices, while it is softened by consumers who buy from a randomly chosen firm. When firms are induced to frequently change prices (or hold sales) due to price seeking consumers, the interdependency between products becomes an immediate concern if contemplating such changes, providing a link to monopolist pricing.

To study this phenomenon I build a model of price competition between retailers that all sell two interdependent but homogeneous products. I consider the full range of interdependency between the two products from perfect complementarity to independent valuations to perfect substitutability. The informational frictions are introduced by assuming two types of consumers - captives, who visit only one retailer, and shoppers who visit two and buy each good at the lowest price. I show that in equilibrium retailers will use mixed strategies for both prices leading to price dispersion in the market. The dichotomy between complements and substitutes is particularly noteworthy: if the goods are complements their prices within every shop will be negatively correlated, while if they are substitutes or independently valued, the prices will be uncorrelated.

Negative correlation between prices of complements has its roots in the monopoly pricing. When facing consumers who want to buy two complements a monopolist is willing to charge any price combination such that the sum of prices is equal to the highest price consumers are willing to pay for the two goods together. When firms are competing for some consumers but hold a monopoly position with respect to others they offer price combinations with a sum close to the desired monopoly level but vary individual prices to attract the price sensitive consumers. In effect, prices move in opposite directions leading to negative correlation.

In addition to the previous result I demonstrate that product complementarity has an interesting implication for the profitability of retailers. It turns out that when the two goods are complements, the multiproduct offering allows retailers to jointly discriminate between captives and shoppers. When retailers are selling two complements, they can keep the sum of the two prices constant at the joint reservation price of the two goods (ensuring that the profits earned from captives are maximized) while lowering one of the prices and thus engaging aggressively in price competition for shoppers. This practice yields higher profits than selling one composite good because it allows selling one of the complements to shoppers even though the sum of the two prices is kept at the monopoly level. The first chapter provides a number additional insights into pricing of substitutes, product bundling and some other issues that are described in detail there.

In the second chapter I take the same search environment to study price competition in a single good when firms have different marginal costs. Pricing under marginal cost heterogeneity is of great empirical relevance yet is largely understudied. The main contribution of this chapter is to identify a simple equilibrium regularity. I prove that in an equilibrium in which expected demand is equal over a range of prices for two firms with different marginal costs then at most one of these prices can be charged by both. This statement concerns equilibrium demand that generally depends not only on consumer behavior but also on the strategies chosen by other firms. The purpose of this argument is to facilitate finding equilibrium strategies so the fact that this statement involves expected demand that depends on these strategies makes its application nontrivial. Fortunately, several widely used consumer search frameworks allow such application that, in turn, allows to solve the asymmetric generalization relatively easily.

In the second chapter I apply the above mentioned requirement on equilibrium prices to the "clearinghouse" model from Varian (1980) and the "nonsequential search" model from Burdett and Judd (1983). Both models are originally solved for symmetric equilibrium with identical firms. I demonstrate that there is a certain discontinuity in the symmetric equilibrium. Even if firms have slightly different marginal costs in equilibrium they adopt disjoint pricing strategies. By definition, disjoint price distributions cannot converge to the same symmetric mixed strategy even when heterogeneity disappears in the limit.

The empirical relevance of this chapter is rather immediate. My results confirm an obervation by Salop (1979) that most efficient firms will generally have larger market shares. In equilibrium of the two models that I generalize more efficient firms tend to charge lower prices and obtain larger market shares. If consumers who visit one firm can identify firms by their marginal costs (and thus expected price levels) and gradually migrate to the ones that charge lowest prices, the more efficient firms will gain even larger market shares in equilibrium. As expected, migration of loyal consumers towards more efficient firms equates average prices across the firms in equilibrium. This result can help identify marginal cost heterogeneity when product quality across firms is different. Unlike cost heterogeneity, quality differentials translate into long-term price differentials between firms.

The last chapter differs considerably from the previous two in the nature of fragmentation considered there. Unlike before, in the last part of my thesis it is the firms who do not have full access to all the existing markets. This in itself, as long as two or more firms access each market, has no substantial consequence for price or quantity competition. In the former case firms in each market will have to price at marginal cost while in the latter case the actions of firms in each market will be independent of competitive conditions in other markets. What is shown though is that such a market structure will be pivotal for determination of production capacities prior to the competition stage.

Until now, capacity building prior to quantity competition has solely been considered under full market access. Under this scenario simultaneous capacity building is of no interest as it results in a model where costs of production are incurred on two stages but actual outcome only depended on the sum of the two costs and not on their timing. As a result, most of the literature has concentrated on sequential capacity building in the context of entry deterrence. We relax the assumption of full market access and show that even with simultaneous capacity building the model has non-trivial implications.

I model the idea of partial market access by assuming that firms are located on a circle with markets located in between firms. Firms are assumed to have access only to the two neighboring markets. In the first stage firms simultaneously choose how many units of capacity to build at a constant marginal cost. In the second stage, after having observed all others' capacities, each firm decides how many units of the final good to produce (at a constant marginal cost of production) within its capacity and how to allocate this quantity between the two markets that the firm supplies to. The price in each market is then determined by the sum of the two quantities supplied to the market by the two neighboring firms.

The most important result of this chapter is that even though there is no unused capacity in the second stage of competition, the level of capacity built is larger than what would have prevailed if the pairs of firms competed only in one market. If the latter holds then firms have no other destination for their production so observing that one's competitor has built a larger capacity than was expected cannot force the firm to redirect its own production elsewhere. This is not true in the model of this chapter. If a firm builds two additional units of capacity on the first stage, it commits to suppling one more unit to each market it serves and thus will induce its competitors to supply less to the affected markets. In effect, firms have a Stackelberg leader's incentive to overproduce capacity even though they do not have the first-mover advantage. As a result, marginal revenue generated by one more unit of capacity will be larger than marginal revenue from one more unit shipped to a market in the second stage and will induce overcapacity relative to a standard Cournot model. I argue that this result holds for any symmetric market structures including all regular networks provided that all firms have identical marginal costs. Even more importantly, this insight has a potential to be applied to a very general class of fragmented markets and change the state of discourse on the anti-competative nature of irreversible capacity investments.

CONTENTS

1 MULTIPRODUCT PRICING IN OLIGOPOLY

1.1 Introduction

Traditionally, a vast majority of goods are delivered to final consumers through retail stores. While small retailers could carry hundreds of items, the largest ones like Fnac or Tesco offer tens of thousands of goods for sale. Managers of such stores make pricing decisions for many products bearing in mind that the price of nearly any of them has an impact on the demand for its substitutes and complements. Furthermore, these pricing decisions are often taken in the context of competition with other retailers selling similar products. Price competition emerges due to consumers who are willing to search for the lowest price in the market thus forcing retailers to undercut each other's prices while it is softened by those consumers who do not observe all the prices in the market and shop randomly. It has been shown by Varian (1980) and many others in the one-good setting that such consumer behavior leads to price randomization in equilibrium.[1] When firms are induced to frequently change prices due to the consumer search, the interdependency between products becomes an immediate concern when contemplating such changes.[2] It is such environment, characterized by price competition and dependent demands across goods, that appears to be shaping retail pricing decisions. However, most of the previous literature on multiproduct pricing has either analyzed pricing strategy of a monopolist selling interdependent goods (Venkatesh and Kamakura (2003)) or competition with independent goods (Lal and Matutes (1989), McAfee (1995)).

This paper builds a model of price competition between retailers that all sell two interdependent but homogeneous products. We consider the full range of interdependency between the two products from perfect complementarity to independent valuations to perfect substitutability. The information frictions are introduced by assuming two types of consumers - captives, who visit only one retailer and shoppers who visit two and

[1] For one-good models with information frictions see Diamond (1971), Salop and Stiglitz (1977) and Stahl, II (1989) among others.

[2] Hosken and Reiffen (2004) estimate that temporary price discounts account for 20% to 50% of the annual variation in retail prices for most product categories in the United States.

buy each good at the lowest price. We show that, in the static Nash Equilibrium, retailers use mixed strategies for both prices leading to price dispersion in the market. The dichotomy between complements and substitutes is particularly noteworthy: if the goods are complements their prices within every shop will be negatively correlated, while if they are substitutes or independently valued, the prices will be uncorrelated.

The reason for the negative correlation is that if a price of one of the two complements is at its highest equilibrium level then this good is never sold to shoppers because some other retailer is surely setting a lower price. If so, the retailer will want to increase this price further until even captive consumers are about to stop buying the good. While doing so, the retailer has the incentive to combine the highest price for the first good with a relatively low price for the other to be able to increase the former as much as possible and yet sell both goods. Such reasoning leads to negative relationship between prices of complements.

Empirical literature has documented negative relationship between the prices of complementary goods within shops which would be hard to account for with deterministic pricing subject to demand or supply shocks. Van den Poel, Schamphelaereb and Wets (2004) have analyzed consumer decisions based on their baskets of purchases in a large do-it-yourself retailer and found that simultaneous large discounts on two complementary products occur rarely. This is precisely what our model predicts - when one of the complements is at a discount the other one is priced high. Even before disintegrated data on purchases became available, there has been a consensus in the empirical marketing literature that if one of the complementary goods is on sale the other ones are unlikely to be at a discount as well (Mulhern and Leone (1991) and Mulhern and Padgett (1995)). In fact, this view is so well accepted that Van den Poel et al. (2004) proceed to asking what would have happened if, unorthodoxly, prices of complements were reduced simultaneously? Theoretical justification for this negative relationship, until now, has been grounded in the monopoly paradigm: if a store lowers the price for one of the goods the profit-maximizing price for the other rises as the demand for it increases. As a result, having both goods on sale cannot be optimal. It is not obvious why, in the first place, a monopolist would lower one of the prices from its optimal monopoly level. One would imagine that, in equilibrium, all observed prices should give equal expected profits so, unless one explicitly models multiproduct competition, the relationship between prices of complements is a priori ambiguous.

Product complementarity has an interesting implication for the profitability of retailers. It turns out that when the two goods are complements, the multiproduct offering allows retailers to jointly discriminate between captives and shoppers. In the one-good version of this model (Burdett and Judd (1983)) retailers are setting only one price so when lowering it to attract shoppers they also lose the guaranteed profits they could have earned from captives. In contast, if the retailers are selling two complements, they can keep the sum of the two prices constant at the joint reservation price of the two goods (ensuring that the profits earned from captives are maximized) and lower one of the prices, engaging aggressively in price competition for shoppers. This practice yields higher profits than selling one composite good because it allows selling one of the complements to shoppers even if the sum of the two prices is kept at the monopoly level.

The rational behind no correlation between the prices of substitutes is that unless the substitutability is near perfect, in which case retailers abandon the less valued good, retailers still want to sell both substitutes. In order to do so, they have to keep both prices low because the two goods, as substitutes, "compete" with each other. When prices are capped in this way even the highest price ever charged for one of the substitutes can be combined with any price for the other one and, nevertheless, lead to selling both goods. As a result, there is no need to take into account the price of one of the substitutes when pricing the other. The lack of correlation between prices of substitutes does not imply that their prices are chosen as if retailers were selling only one of them. Quite to the contrary, equilibrium price ranges and distributions are determined by the characteristics of the entire product assortment, it is just that in equilibrium there is no correlation between the prices chosen from these distributions. To the best of our knowledge, in the literature, there are no stylized empirical facts on co-pricing of substitutes in a competitive environment. To a certain degree our model rationalizes this absence by predicting no correlation between the prices of substitutes.

We also demonstrate that the kind of joint discrimination that was possible with complements is not feasible if the two goods are substitutes. Substitutes compete with each other so a retailer cannot keep the sum of their prices high and decrease one of them to attract shoppers because by doing so she induces captives to buy only the cheaper good. Not only are retailers incapable of earning additional profits through discriminating between the two groups of consumers, but they also earn lower profits than in the one-good model. When the consumers have to buy a bundled good retailers do not have to take into account the competition between the two substitutes within their store and are able to earn higher profits. This reasoning is unrelated to the nature of competition and is also applicable to a monopolist.

Finally, this paper is the first to point out that bundling can play an important role in the multiproduct competition even if consumers have identical tastes and thus price discrimination through bundling among such consumers is not of interest. We show that price randomization by competitors creates a novel incentive to bundle goods, a consideration that has been overlooked by the previous literature.

Sequential Search Literature

As noted in the introduction, prior literature on multiproduct price competition has concerned itself with independent goods. While several authors worked on models with horizontally differentiated retailers (Lal and Matutes (1989), Matutes and Regibeau (1992)), others have analyzed multiproduct competition in homogeneous goods under some form of sequential price search. Our work is closest to the latter group in that, under product homogeneity, sequential search induces price randomization. Burdett and Malueg (1981) were the first to look at consumer behavior (with a fixed and exogenously given market price distribution) under a sequential skim with multiple products. In a generalization of the concept of reservation price used in one-good sequential search models, they introduce a concept of "acceptance set" that serves a similar purpose. Namely, price vectors in the acceptance set preclude further search. McAfee (1995) introduces competition, and thus market price distribution choice, into the Burdett and Judd (1983) noisy search model.[3] The author shows that, in addition to an equilibrium which is largely a n-good version of a one-good equilibrium, for certain parameter values there is a continuum of equilibria where prices within every shop are highly dependent. McAfee finds that

[3]In the noisy search model Burdett and Judd (1983) assume that consumers incur a cost for sampling a random number of firms where probability of obtaining a sample of size n is exogenously given.

under certain conditions there always exist a continuum of equilibria where the prices are set at the frontier of the acceptance set. In a striking resemblance to our results, McAfee shows that in such equilibria the highest price for one of the goods is always charged along with the lowest prices for all other goods. This result is driven by the sequential search procedure and does not apply to models, including the one presented here, where consumers cannot pay to get additional information. If consumers do not search sequentially then the prices of independent goods are evaluated against willingness to pay and co-pricing motive disappears completely. To show this we demonstrate that in our model with independent goods the unique marginal distribution for the price of each good is that of the one-good model. In our setting incentives to co-price emerge only if we introduce complementarity between the two goods. One could only hypothesize about the effect of sequential search in our model but it is clear outright that negative relationship between the prices of complements can only be strengthened by sequential search assumption.

The rest of the paper is organized as follows: in Section 2 we specify the model and analyze optimal behavior of the consumers and a hypothetical monopolist, in Section 3 we solve the oligopoly model for all the cases, in Section 4 we provide discussion of some extensions of the main model and in Section 5 we conclude.

1.2 The Model

a Assumptions

Consider a market with two retailers selling two homogeneous goods labeled a and b. The marginal cost of production of both goods is assumed to be zero. There is a continuum of consumers who have identical tastes and their mass per retailer is normalized to one. The consumers demand exactly one unit of each good and the triplet $\{v_a, v_b, v_{ab}\}$ describes their reservation prices for one unit of good a, one unit of good b and one unit of each good consumed together. We will assume that consumers get utility of zero if they do not consume anything and can freely dispose the goods they own so $v_{ab} \geq \max(v_a, v_b)$. The triplet of unit valuations describes the entire possible set of demand interrelations between the two goods. If $v_a = v_b = 0$ then good a and good b are perfect complements, if

$v_a + v_b = v_{ab}$ they have independent valuations, and if $v_{ab} = v_a = v_b$ then they are perfect substitutes. We allow the two goods to be asymmetric and without loss of generality we assume that:

Assumption 1.1. $v_b \geq v_a$.

While identical in tastes, consumers differ in their shopping behavior and come in two types: proportion θ of the consumers visits only one retailer at random (we refer to these consumers as captives) while the rest of the consumers (proportion $1 - \theta$) visit both retailers (shoppers).[4] Shoppers can buy each good at the lowest price they observe without paying any extra transportation cost if they choose to buy the goods at different shops.[5] If two retailers charge the same price for a good shoppers are equally likely to purchase from either of them.

Finally, in order to simplify the treatment of border cases we assume that:

Assumption 1.2. *In the event of indifference the consumers respect the following order: buy both goods, buy only good b, buy only good a and do not buy anything.*

Firms compete by setting prices for the two goods simultaneously and we use static Nash Equilibrium as the solution method. We assume that the retailers will not, in addition, set a separate price for a bundle of the two goods. The implications for the model when the retailers are allowed to bundle the two goods are discussed in Section a.

b Consumer Behavior and Monopolist

Since each retailer has monopoly power through captive consumers, the strategies employed in the oligopolistic equilibrium depend on the pricing behavior of a hyphotetical monopolist facing captive consumers. Before proceeding to solving the oligopoly model we will illustrate the optimal behavior of consumers facing any price pair and, subsequently, the profit-maximizing strategy of the monopolist. This section will demonstrate that the pricing by the monopolist is fundamentally different depending on

[4]In this model θ is given exogenously but it can be endogenized as in Burdett and Judd (1983) fixed sample search model.

[5]This amounts to requiring free recall by shoppers, an assumption widely used in the consumer search literature.

whether the two goods are substitutes or complements and so we shall solve the oligopoly model for these two cases separately.

Assume a consumer can buy the goods at a price pair (p_a, p_b). For a captive consumer this pair is the one charged by the only retailer she visits while for a shopper each price is the minimum between the prices of each good from the two retailers. The consumer has a choice of buying both goods, only good a, only good b and none at all, and gets a surplus of $v_{ab} - p_a - p_b$, $v_a - p_a$, $v_b - p_b$ and 0, respectively.

The consumer will buy both goods if and only if:

$$v_{ab} \geq p_a + p_b \tag{1.1}$$
$$v_{ab} \geq p_a + v_b \tag{1.2}$$
$$v_{ab} \geq p_b + v_a. \tag{1.3}$$

She will buy only good i if $v_i \geq p_i$, $v_i - v_j \geq p_i - p_j$ and $v_{ab} - v_i \leq p_j$ hold at the same time (when used along i, subscript j denotes the other good). Figure 1.1 illustrates the consumer choices depending on the prices and the relation between v_{ab} and $v_a + v_b$.

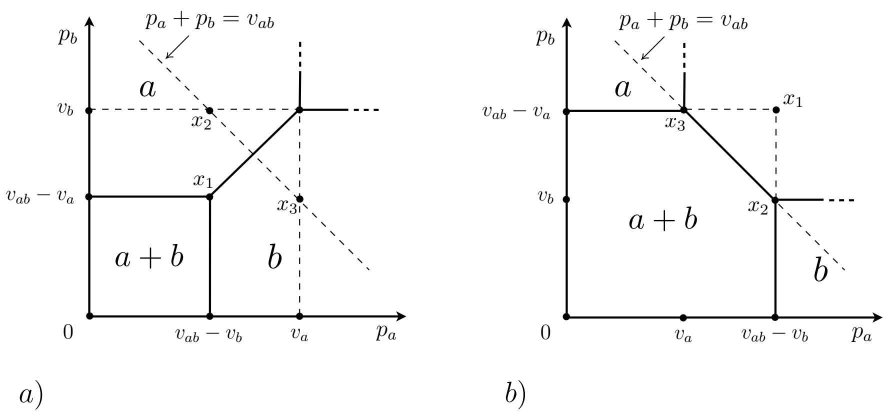

Figure 1.1: Consumer choice when the goods are a) substitutes and b) complements. Labels a, b and $a + b$ indicate the price pairs such that consumers buy only good a, only good b and both goods, respectively. These areas are delimited with solid lines.

If the two goods are complements the most the monopolist can earn when selling both good a and good b is v_{ab}. In this case Inequality 1.1 is binding so she can charge any point on the line connecting x_2 and x_3 in Figure 1.1 b). It is easy to see that this pricing is only feasible when the goods are complements. Figure 1.1 a) shows that if $v_{ab} = p_a + p_b$ then consumers will not buy the two substitutes together, hence the monopolist is unable to earn v_{ab}. When the monopolist aims to sell both substitutes Inequalities 1.2 and 1.3 bind and she earns $2v_{ab} - v_a - v_b < v_{ab}$ by charging the price pair x_1. The inability to earn v_{ab} is the result of substitutability between the goods. When good a and good b are substitutes they effectively compete with each other not allowing the monopolist to extract their joint value from the consumer.[6] It needs to be noted that under unit demands price of one of the substitutes does not affect the profit maximizing price for the other, an artifact that seems to drive the lack of co-pricing in the case of substitutes.

If the monopolist sells only one of the substitutes then she should sell good b (recall Assumption 1.1). She will charge $p_b = v_b$ and any $p_a \geq v_a$ to do so and will earn v_b. Hence, if $2v_{ab} - v_a - v_b < v_b \iff v_{ab} < v_b + \frac{1}{2}v_a$ the monopolist will choose to sell only b, otherwise she will sell both goods.

Having verified the pricing by the monopolist we turn to our oligopolistic model. We will consider complements and substitutes separately as suggested by the analysis of this section.

1.3 Equilibrium

a Complements

In this section we assume that $v_{ab} > v_a + v_b$. Previously, we have demonstrated that in this case the monopolist will charge a pair of prices such that their sum is equal to v_{ab} (e.g. $p_a = v_a$ and $p_b = v_{ab} - v_a$). The competition will force the retailers to undercut each other from the monopoly prices. This pressure on prices is downwards so the retailers will nevertheless sell both goods to captives in equilibrium.

Lemma 1.1. *When the goods are complements, in equilibrium retailers set such prices that captives always buy both goods, that is, Inequalities*

[6] In Section a we discuss the implications for the behavior of the monopolist if she can bundle substitutes. It is shown that the monopolist can sell both goods and still earn v_{ab} if she refuses to sell the goods separately.

1.1-1.3 hold.

Proof. Assume the opposite. For simplicity assume that good a is the one that captives do not buy. It should be clear that shoppers will not buy any good that captives do not buy so the retailer will not sell good a at all. If that is so, the retailer can lower the price of good a to a level such that it is still positive and captives buy both goods, a strategy that increases profits. In terms of Figure 1.1, good b is the only good sold if the price vector is in region b. For any point in this region the retailer can fix the price of good b and lower the price of good a before the price pair is in the region $a+b$. By doing so the retailer will increase her profit because she will be selling good a at a price $v_{ab} - v_b > 0$. More formally good b is the only good sold to captives iff all of the following are true:

$$\begin{aligned} v_{ab} - v_b &< p_a \\ v_b - v_a + p_a &\geq p_b \\ v_b &\geq p_b. \end{aligned}$$

Let the retailer, instead of p_a, charge $\hat{p}_a = v_{ab} - v_b$. One can show that at $(\hat{p}_a, p_b)$ captive consumers will buy both goods and $\hat{p}_a + p_b > p_b$ so the profit earned from captives will increase. Shoppers were not buying good a before and by lowering its price the profit earned from them cannot decrease. □

Once we have verified that firms never charge prices that do not attract consumers we can move to characterizing the equilibrium pricing strategies. First we demonstrate that, in analogy to the one-good models of price dispersion, equilibrium distributions of both prices will be atomless and gapless, defined over a closed and connected support.

Lemma 1.2. *In equilibrium, p_i $(i = a, b)$ will be randomized according to the continuous distribution function $F_i(p_i)$ defined over the interval $[\underline{p}_i, \overline{p}_i]$.*

The reason why there are no atoms in the equilibrium distributions is clear: if some price was charged with a strictly positive probability, there would be a positive probability of a tie at that price and all retailers would have incentive to charge a slightly lower price with the same probability as the old one and serve all shoppers in the case of a tie by others. Moreover, there will be no gaps in the equilibrium price distributions. This is because charging the price at the lower edge of the gap attracts shoppers with the same probability as charging the price at the upper edge of the gap does but the latter price yields higher profits.

Proof. See proofs of Lemmas 3 and 8 from Varian (1980). □

Next we will argue that charging any p_i along with p_j such that both goods are sold to captive will give the same expected profits earned from selling i.

Lemma 1.3. *Expected profits earned from selling good i when charging any $p_i \in [\underline{p}_i, \overline{p}_i]$ are constant for all such p_i and are independent from the price charged for good j.*

Proof. See Appendix. □

Now we proceed to identifying F_i in the equilibrium. We know that due to the fact that F_i is atomless, if any retailer is charging $\overline{p}_i$ for good i then she will not sell i to shoppers because other retailers will charge a lower price for it with probability one. Hence, she should increase $\overline{p}_i$ until captive consumers are indifferent between buying the two goods and either buying only good j or not buying anything at all. Formally,

$$\overline{p}_i = \max\{p_i \mid v_{ab} - v_j \geq p_i \text{ and } v_{ab} - p_j \geq p_i\}. \tag{1.4}$$

Using the last expression and Lemma 1.3 we conclude that the highest price for i will be charged along with the lowest price for good j when $\underline{p}_j \geq v_j$. If $\underline{p}_j < v_j$ then $\overline{p}_i$ will be charged along with some $p_j \leq v_j$ and will be equal to $v_{ab} - v_j$.

Lemma 1.4. *If $\underline{p}_j \geq v_j$ then $\overline{p}_i$ will be charged along with $\underline{p}_j$ and their sum will be equal to v_{ab}.*

Proof. Assume the opposite so that a pair $(\overline{p}_i, \hat{p}_j)$ is charged such that $\hat{p}_j > \underline{p}_j$ when $\underline{p}_j \geq v_j$. The last two inequalities combined imply $\hat{p}_j > v_j$. In the maximization problem in Equation 1.4 the second restriction will bind so $\overline{p}_i = v_{ab} - \hat{p}_j$. But then, the retailer can charge the pair $(v_{ab} - \underline{p}_j, \underline{p}_j)$ and earn higher profits on i without changing the profits earned on j

(Lemma 1.3). From the last argument it follows that if $\underline{p}_j \geq v_j$ then $\overline{p}_i = v_{ab} - \underline{p}_j$. $\square$

If the highest price of good i is restricted by the price of good j (in the sense of Equation 1.4) a retailer should always choose the lowest price for j in order to increase the highest price for i as much as possible.

Now consider the case when $\underline{p}_j \leq v_j$. It should be clear that because the retailer wants to increase $\overline{p}_i$ as much as possible she should always charge $\overline{p}_i$ with some $p_j \leq v_j$ so because of Equation 1.4 we have $\overline{p}_i = v_{ab} - v_j$.

Lemma 1.5. *If $\underline{p}_j \leq v_j$ then $\overline{p}_i$ is always charged along with some $p_j \leq v_j$ and $\overline{p}_i = v_{ab} - v_j$.*

Proof. Assume the opposite so that a retailer charges $\overline{p}_i$ with some $\hat{p}_j > v_j$ in equilibrium. We know that $\overline{p}_i + \hat{p}_j \leq v_{ab}$ so $v_{ab} - v_j > \overline{p}_i$. In this case the retailer can increase her profits by charging a price pair $(v_{ab} - v_j, v_j)$ instead. By doing so she will earn the same profits from selling good j (Lemma 1.3) but will earn strictly higher profits from selling good i, a contradiction. $\square$

Lemmas 1.4 and 1.5 imply that there are four possible cases when good a and good b are complements:

1. $\underline{p}_a \geq v_a$ and $\underline{p}_b \geq v_b \Longrightarrow \overline{p}_a = v_{ab} - \underline{p}_b$ and $\overline{p}_b = v_{ab} - \underline{p}_a$. In this case we refer to good a and good b as Strong Complements.

2. $\underline{p}_a \leq v_a$ and $\underline{p}_b \leq v_b \Longrightarrow \overline{p}_a = v_{ab} - v_b$ and $\overline{p}_b = v_{ab} - v_a$. Weak Complements.

3. $\underline{p}_a > v_a$ and $\underline{p}_b < v_b \Longrightarrow \overline{p}_a = v_{ab} - v_b$ and $\overline{p}_b = v_{ab} - \underline{p}_a$. Intermediate Complements.

4. $\underline{p}_a < v_a$ and $\underline{p}_b > v_b \Longrightarrow \overline{p}_a = v_{ab} - \underline{p}_b$ and $\overline{p}_b = v_{ab} - v_a$. Intermediate Complements II, is impossible due to Assumption 1.1.

Next we will consider each case separately.

a.1 Strong Complements

In this case $\underline{p}_b \geq v_b$ and $\underline{p}_a \geq v_a$. We will demonstrate that these two inequalities hold only if the complementarity is strong enough (i.e. v_{ab}

is large enough with respect to $v_a + v_b$), hence the name for the case. When v_{ab} is large, retailers increase $\overline{p}_i$ up to the point where consumers are indifferent between buying both products or not buying anything at all ($v_{ab} - p_a - p_b = 0$). Retailers never have to be concerned that by increasing $\overline{p}_i$ they may induce consumers to switch to buying only j because prices of both goods are above the individual valuations for the goods.

Using $\underline{p}_i \geq v_i$ $(i = a, b)$ along with Lemma 1.4 gives:

$$\overline{p}_i = v - \underline{p}_j. \tag{1.5}$$

Since $\overline{p}_i$ never attracts shoppers and $\underline{p}_i$ attracts them with probability one, the expected profits from charging either of these two have to be equal so:

$$(2 - \theta)\underline{p}_i = \theta \overline{p}_i. \tag{1.6}$$

Using the last equation along with Equation 1.5 we get:

$$\overline{p}_a = \frac{(2-\theta)}{2} v_{ab} \tag{1.7}$$

$$\overline{p}_b = \frac{(2-\theta)}{2} v_{ab} \tag{1.8}$$

$$\underline{p}_a = \frac{\theta}{2} v_{ab} \tag{1.9}$$

$$\underline{p}_b = \frac{\theta}{2} v_{ab}. \tag{1.10}$$

Recall that $\underline{p}_a \geq v_a$ and $\underline{p}_b \geq v_b$ should hold in this case so:

$$v_{ab} \geq \frac{2}{\theta} v_a$$
$$v_{ab} \geq \frac{2}{\theta} v_b.$$

Using assumption 1.1 we find that if good a and good b are strong Substitutes:

$$v_{ab} \geq \frac{2}{\theta} v_b. \tag{1.11}$$

At this point we need to verify that captive consumers buy both goods at all the price pairs charged in equilibrium or:

$$\overline{p}_a = \frac{2-\theta}{2} v_{ab} \leq v_{ab} - v_b$$
$$\overline{p}_b = \frac{2-\theta}{2} v_{ab} \leq v_{ab} - v_a.$$

These reduce to:

$$v_{ab} \geq \frac{2}{\theta} v_a$$
$$v_{ab} \geq \frac{2}{\theta} v_b,$$

the two conditions we have obtained for Strong Complements.

As mentioned above, we refer to this case as Strong Complements because if v_{ab} is large enough the prices for both goods will always exceed their individual reservation values ($\underline{p}_i \geq v_i$) and the price ranges and equilibrium strategies are independent of v_a and v_b.

The expected profits for a retailer charging a price pair (p_a, p_b) such that $p_a + p_b \leq v_{ab}$ are given by $\pi_{ab} = \pi_a + \pi_b$ where π_i $(i = a, b)$ are:

$$\pi_i = p_i \left[\theta + 2(1-\theta)(1 - F_i(p_i))\right]. \tag{1.12}$$

Given that $p_a + p_b \leq v_{ab}$, the expected profits from selling i is constant for all $p_i \in [\underline{p}_i, \overline{p}_i]$ and is equal to $\pi_i = \theta \overline{p}_i$. As a result, the unique cumulative marginal distribution functions for prices of good a and good b in the equilibrium will be:

$$F_a(p_a) = \frac{(2-\theta)(2p_a - v_{ab}\theta)}{4(1-\theta)p_a} \tag{1.13}$$
$$F_b(p_b) = \frac{(2-\theta)(2p_b - v_{ab}\theta)}{4(1-\theta)p_b}, \tag{1.14}$$

respectively.

What remains to be shown is that there exists a joint distribution function $F(p_a, p_b)$ such that $p_a + p_b \leq v_{ab}$ for all price pairs and the derived marginal distributions are $F_a(p_a)$ and $F_b(p_b)$.

There is no such range of p_a and p_b where the two prices can be randomized independently because for any $p_i \in [\underline{p}_i, \overline{p}_i]$ the restriction $p_i + p_j \leq v_{ab}$ is binding for some p_j. This in particular implies that $F(p_a, p_b) = F_a(p_a) \cdot F_b(p_b)$ cannot be the equilibrium joint distribution.

Next we will solve for a simple randomization rule: randomize the price of good a according to the marginal distribution function in Equation 1.13 and set p_b according to some monotonically decreasing function $b(p_a)$ such that the resulting marginal distribution of the price of good b is exactly as in Equation 1.14. Such function exists and is unique, defined by the equation $F_a(p_a) = 1 - F_b(b(p_a))$. This is because if p_b is a decreasing function of p_a then the probability a price of good a is below p_a should be equal to the probability that a price of good b is above $b(p_a)$. We find:

$$b(p_a) = \frac{\theta(v_{ab} - v_b)p_a}{(1+\theta)p_a - \theta(1-\theta)v_{ab} - \theta^2 v_b}$$

After some algebra one can check that the function $p_a + b(p_a)$ (the sum of prices) is decreasing at $\overline{p}_a$, increasing at $\underline{p}_a$ (at both points it is equal to v_{ab}) and the derivative $\partial(p_a + b(p_a))/\partial p_a$ changes its sign only once on the interval $[\underline{p}_a, \overline{p}_a]$ so $p_a + b(p_a) \leq v_{ab}$ for all $p_a \in [\underline{p}_a, \overline{p}_a]$. It is possible to introduce noise to the function $b(p_a)$ and obtain some other joint distribution function which has the necessary marginals so this could lead to a multiplicity of such functions. Notably, the joint distribution we have derived is unique in the class of monotone single-valued functions (that is when for every p_a there is a unique p_b):

Proposition 1.1. *When $v_{ab} \geq \frac{2}{\theta} v_b$ the equilibrium marginal distributions given in Equations 1.13 and 1.14 are unique. Moreover, the joint distribution function defined by $b(p_a)$ along with $F_a(p_a)$ is unique in the class of single-valued monotone joint distribution functions.*

Proof. Lemmas 1.1 to 1.5 suffice to show that the equilibrium distribution functions given in Equations 1.13 and 1.14 are unique. We have already argued that there exists strictly decreasing function $p_b = b(p_a)$ such that if p_a is randomized according to the marginal distribution function given in Equation 1.13 then the resulting marginal distribution for p_b is exactly that in Equation 1.14 while $p_a + p_b \leq v_{ab}$ holds for all $p_a \in [\underline{p}_a, \overline{p}_a]$. This is sufficient to prove that the joint distribution function described above satisfies all the conditions for the equilibrium. It is also unique in the class of monotone single-valued function becuase $b(\cdot)$ cannot be strictly increasing given that $\overline{p}_a + \overline{p}_b > v_{ab}$. $\square$

Equilibrium marginal densities and the function $b(p_a)$ are illustrated in Figure 1.2. In the equilibrium, only the prices along the curve $b(p_a)$ will be charged, starting from the price pair $(\underline{p}_a, \overline{p}_b)$ and ending with the pair $(\overline{p}_a, \underline{p}_b)$.

It seems natural that any retailer, if possible, would want to have no coordination between the two pricing strategies. If that is not feasible

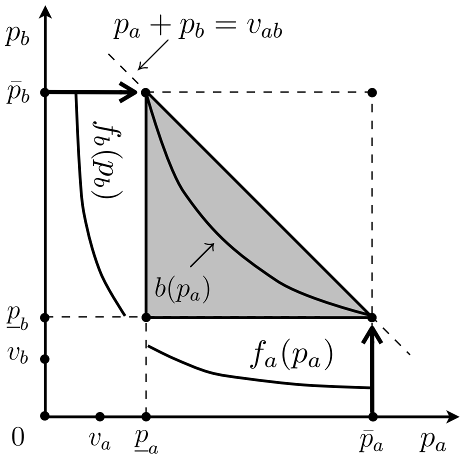

Figure 1.2: Strong Complements. The shaded area indicates price pairs that can be charged in equilibrium ($p_a + p_b \leq v_{ab}$). Short axis are the marginal densities for the price of each good.

then, as in Strong Complements, the retailer would find it easiest to draw one of the prices, say p_a, from the equilibrium price distribution and set p_b according to some monotonic function such that the resulting distribution of p_b is the equilibrium one. In order to pick from potentially multiple equilibria we will introduce intuitive criterion that will be used throughout the paper:

Assumption 1.3. *When indifferent the retailers use the following ordering of the pricing strategies: they charge uncorrelated prices. If the the latter is not possible then they randomize one of the prices and set the other one as a monotonic function of the first, and finally they jointly randomize both prices.*

Given Assumption 1.3 the joint distribution function defined by F_a and $b(\cdot)$ will be unique.

When the goods are Strong Complements retailers earn expected profits of $\pi_{ab} = \theta(2-\theta)v_{ab}$ which are larger than the profits they would have obtained if they sold one good with a valuation v_{ab} ($\pi = \theta v_{ab}$). Intuition for this profit bump is the following: when setting the sum of prices equal to v_{ab} the retailers can surely sell one of the goods to shoppers by setting its price low enough. By doing so, they will earn θv_{ab} from captives and

$\theta(1-\theta)v_{ab}$ from shoppers. If instead, they sold only the composite good, v_{ab} would be the highest price ever charged and at that price only captives would buy the good giving the retailer excepted profits of only θv_{ab}. As this case demonstrates, when the two goods are complements retailers can jointly discriminate between captives and shoppers and earn higher profits than in the one-good model.

a.2 Weak Complements

Here we assume that $\underline{p}_i \leq v_i$ for $i = a, b$. The goods in this section are called Weak Complements because their individual valuations are large enough (relative to v_{ab}) to be higher than at least some prices charged for them. Unlike the case of Strong Complements, here the process of increasing $\overline{p}_i$ stops when consumers are ready to switch to buying only good j and this is a concern because $\underline{p}_j \leq v_j$. We will show that here v_{ab} has to be close enough to $v_a + v_b$ so the case of independent valuations ($v_{ab} = v_a + v_b$) will be approached in the limit.

As in the previous section it should be the case that expected profits at $\underline{p}_i$ and $\overline{p}_i$ are equal so $(2-\theta)\underline{p}_i = \theta\overline{p}_i$. Given this and Lemma 1.5 the boundaries for price distributions will be given by:

$$\overline{p}_b = v_{ab} - v_a \tag{1.15}$$

$$\overline{p}_a = v_{ab} - v_b \tag{1.16}$$

$$\underline{p}_a = \frac{\theta\left(v_{ab} - v_b\right)}{2-\theta} \tag{1.17}$$

$$\underline{p}_b = \frac{\theta\left(v_{ab} - v_a\right)}{2-\theta}. \tag{1.18}$$

We impose $\underline{p}_a \leq v_b$ and $\underline{p}_b \leq v_b$ to get:

$$v_a \geq \frac{\theta\left(v_{ab} - v_b\right)}{2-\theta}$$
$$v_b \geq \frac{\theta\left(v_{ab} - v_a\right)}{2-\theta}.$$

Rewriting in terms of v_{ab} and remembering Assumption 1.1 reduces the last two inequalities to:

$$v_{ab} \leq \frac{(2-\theta)}{\theta}v_b + v_a. \tag{1.19}$$

The marginal price distributions for good a and good b in the equilibrium can be derived as in Section a.1 using Equation 1.12 and we get:

$$F_a(p_a) = \frac{(2-\theta)p_a - \theta(v_{ab} - v_b)}{2(1-\theta)p_a} \tag{1.20}$$

$$F_b(p_b) = \frac{(2-\theta)p_b - \theta(v_{ab} - v_a)}{2(1-\theta)p_b}. \tag{1.21}$$

As in the previous case, the equilibrium joint distribution function should satisfy the following conditions: the derived marginal distributions should coincide with the two we have obtained and for all equilibrium price pairs (p_a, p_b) the sum of prices should be no larger than v_{ab} ($p_a + p_b \leq v_{ab}$).

Note that when $v_{ab} = v_a + v_b$, that is when the goods are independent, $\overline{p}_a = v_a$ and $\overline{p}_b = v_b$ so $p_a + p_b \leq v_{ab}$ for all pairs. In this case there will be no restriction linking the marginal pricing strategies for the two goods so in the equilibrium the strategies can be independent and the joint distribution function can be written as a product of the marginal distributions: $F(p_a, p_b) = F_a(p_a)F_b(p_b)$.[7] Prices of weak complements can be randomized independently, albeit for a subset of the equilibrium range, even when $v_{ab} > v_a + v_b$. To see this note that for any $p_i \leq v_i$ the restriction $p_i + p_j \leq v_{ab}$ is not binding for any $p_j \in [\underline{p}_j, \overline{p}_j]$. Latter implies that if a price of one of the goods is set below its reservation price then the price of the other good can be randomized independently of the first price. When v_{ab} approaches $v_a + v_b$ the probability of $p_i \leq v_i$ approaches one allowing the randomization of the two prices independently as noted above. Figure 1.3 *c*) illustrates the set of possible price pairs in equilibrium for the case of Weak Complements.

We will use Assumption 1.3 to identify single joint distribution function that supports the equilibrium. It is easy to see that both firms will randomize price of good a according to $F_a(p_a)$ and will set price of good b from a monotonically decreasing function $b(p_a)$ which is implicitly define by $F_a(p_a) = 1 - F_b(b(p_a))$.

Proposition 1.2. *When $v_{ab} \leq \frac{(2-\theta)}{\theta} v_b + v_a$ the equilibrium marginal distributions given in Equations 1.20 and 1.21 are unique. Moreover, the joint distribution function defined by $b(p_a)$ along with $F_a(p_a)$ is unique in the class of single-valued monotone joint distribution functions.*

[7]When $v_{ab} = v_a + v_b$ the marginal distribution we derive are identical to those derived by Burdett and Judd (1983).

Proof. See proof of Proposition 1.1. □

For any retailer the expected profits in equilibrium will be equal to $\pi_{ab} = \theta(2v_{ab} - v_a - v_b) \geq \theta v_{ab}$ so they are at least as large as the profits obtained in a one-good model. As expected, the profits converge to the sum of individual goods equilibrium profits when v_{ab} approaches $v_a + v_b$. That is, the additional gain from discriminating between the two groups of consumers disappears as goods become independently valued.

a.3 Intermediate Complements

Here we assume that $\underline{p}_b < v_b$ and $\underline{p}_a > v_a$. The two goods are evidently asymmetric here and we will prove that the case of Intermediate Complements exists if and only if Assumption 1.1 holds with the strict inequality. The latter implies that selling good b yields more profits than selling good a holding the surplus obtained by consumers fixed. This case is a mixture of the previous two in the sense that $\overline{p}_a$ is constrained by $v_{ab} - v_b$ while $\overline{p}_b$ is constrained by $\underline{p}_a$. For this range of v_{ab} the price charged for good a will always exceed its individual reservation price. One could think of a laptop and a laptop bag as an illustrative example. In equilibrium the prices of laptop bags will be always higher than their individual value, pricing that can be frequently observed.

Lemmas 1.4 and 1.5 imply that $\overline{p}_b = v_{ab} - \underline{p}_a$ and $\overline{p}_a = v_{ab} - v_b$. Remembering that $\overline{p}_a$ attracts only captives and $\underline{p}_a$ attracts shoppers with probability one we write:

$$\underline{p}_a = \frac{\theta}{2-\theta}(v_{ab} - v_b). \tag{1.22}$$

From the previous equation we get

$$\overline{p}_b = v_{ab} - \underline{p}_a = \frac{2(1-\theta)v_{ab} + \theta v_b}{2-\theta} \tag{1.23}$$

$$\underline{p}_b = \frac{2(1-\theta)\theta v_{ab} + \theta^2 v_b}{(2-\theta)^2}. \tag{1.24}$$

We have to impose the restrictions for Intermediate Complements to get:

$$v_a < \frac{\theta}{2-\theta}(v_{ab} - v_b)$$
$$v_b > \frac{2(1-\theta)\theta v_{ab} + \theta^2 v_b}{(2-\theta)^2}$$

which simplify to

$$\frac{2}{\theta}v_b > v_{ab} > \frac{(2-\theta)}{\theta}v_b + v_a. \tag{1.25}$$

Note that the goods are Intermediate Complements if and only if $v_b > v_a$. If the two goods are equally valuable then it is impossible that only one of the goods is always sold at a price above its individual reservation price. Possible price pairs for Intermediate Complements are illustrated in Figure 1.3 *b*).

The marginal distribution functions for the prices of good a and good b in the equilibrium will be:

$$F_a(p_a) = \frac{(2-\theta)p_a - \theta(v_{ab} - v_b)}{2(1-\theta)p_a} \tag{1.26}$$
$$F_b(p_b) = \frac{(2-\theta)^2 p_b - 2\theta(1-\theta)v_{ab} - \theta^2 v_b}{(2-\theta)(1-\theta)p_b}. \tag{1.27}$$

The joint distribution function should have derived marginal distributions as in the previous two equations and for all pairs (p_a, p_b) such that $p_a + p_b \leq v_{ab}$. Invoking Assumption 1.3 we conclude that the only equilibrium in this case is when both firms randomize price of good a according to $F_a(p_a)$ and set price of good b from a monotonically decreasing function $b(p_a)$ which is implicitly define by $F_a(p_a) = 1 - F_b(b(p_a))$.

Proposition 1.3. *When $\frac{2}{\theta}v_b > v_{ab} > \frac{(2-\theta)}{\theta}v_b + v_a$ the equilibrium marginal distributions given in Equations 1.26 and 1.27 are unique. Moreover, the joint distribution function defined by $b(p_a)$ along with $F_a(p_a)$ is unique in the class of single-valued monotone joint distribution functions.*

Proof. See proof of Proposition 1.1. □

The equilibrium expected profits are equal to:

$$\pi = \frac{\theta}{2-\theta}\left[(4-3\theta)v_{ab} - (1-\theta)v_b\right] \tag{1.28}$$

and are strictly larger than θv_{ab}. In this case, as well as in the previous two, the retailers are able to increase their profits through joint discrimination of captive consumers.

We have already exhausted all the cases of $v_{ab} > v_a + v_b$. It is trivial to show that the case of Intermediate Complements II cannot occur. To see this one can derive similar conditions for this case and verify that they cannot be satisfied given Assumption 1.1.

b Substitutes

In this section we assume that $v_{ab} \leq v_a + v_b$. In Section b we have demonstrated that the monopolist compares $2v_{ab} - v_a - v_b$ and v_b and prices accordingly. If $v_{ab} \geq \frac{1}{2}v_a + v_b$ the monopolist will charge $p_a = v_{ab} - v_b$ and $p_b = v_{ab} - v_a$, a price pair at which captive consumers buy both goods. Instead, if $v_{ab} < \frac{1}{2}v_a + v_b$ the prices charged will be $p_b = v_b$ and $p_a \geq v_a$ and captive consumers buy only good b. It turns out that these two ranges for v_{ab} are important even when the competition is present. We will call the two goods Weak Substitutes when all the retailers sell both goods which is the case when $v_{ab} \geq \frac{1}{2}v_a + v_b$. When the goods are close enough to being independently valued, all the retailers still choose to sell them both. As the goods become better substitutes the retailers will find it less and less profitable to sell both as this requires lowering both prices and at some point they switch to selling only good b.

When $v_{ab} < \frac{1}{2}v_a + v_b$ the monopolist would sell only good b. We will show that in our model retailers attach a positive probability to selling both goods.

b.1 Weak Substitutes

Here we assume that in the equilibrium retailers sell both goods to captives with probability one. We will prove that this is the case if and only if $v_{ab} \geq \frac{1}{2}v_a + v_b$, that is the two goods are relatively close to being independently valued. One could think of photo and video cameras as an example of weak substitutes. Both devices can perform overlapping tasks but their functions are distinct enough to induce consumers to buy both.

Lemma 1.6. *Both goods are sold to captives with the probability one iff*

$v_{ab} \geq \frac{1}{2}v_a + v_b$.

Proof. Assume retailers sell both goods. If this is true then we can use Lemma 1.2 to argue that the distribution function $F_i(p_i)$ will be atomless and defined over a closed and connected support so the price of i will be randomized over an interval $[\underline{p}_i, \overline{p}_i]$. In order for the retailers to sell both goods it has to be true that $\overline{p}_a \leq v_{ab} - v_b$ and $\overline{p}_b \leq v_{ab} - v_a$. Note that for each price the condition of selling both goods depends only on the price of that good so provided these are true, the expected profits earned on each good will be independent of the price of the other. Since the distribution functions are atomless shoppers will not buy the good priced at $\overline{p}_i$. So any retailer will increase this price up to the maximum possible provided that both goods are sold, that is:

$$\overline{p}_a = v_{ab} - v_b \tag{1.29}$$

$$\overline{p}_b = v_{ab} - v_a. \tag{1.30}$$

The expected profits earned in equilibrium will be $\pi = \theta(2v_{ab} - v_a - v_b)$. We have to make sure no retailer wants to deviate and sell only one of the goods. It is obvious that if i is the only good sold then $p_i > \overline{p}_i$, otherwise the retailer can decrease the price of the other good and sell both which leads to higher profits. If only i is sold to captives when $p_i > \overline{p}_i$ it will never be sold to shoppers because of the competitor so $p_i = v_i$. If this is true the retailer will earn θv_b. For both goods to be sold in equilibrium it has to be the case that $\theta(2v_{ab} - v_a - v_b) \geq \theta v_b \iff v_{ab} \geq \frac{1}{2}v_a + v_b$.

Now assume that $v_{ab} \geq \frac{1}{2}v_a + v_b$. We will argue that in this case all the retailers will choose to sell both goods. Assume the opposite so retailers in the equilibrium sell only good b. As before, we can argue that the highest price of good b is equal to v_b and the equilibrium profits will be θv_b. Charging $p_a = v_{ab} - v_b$ and $p_b = v_{ab} - v_a$ will yield larger profit than $\theta(2v_{ab} - v_a - v_b)$ because in this case retailer will sell good a with probability one and good b with a probability strictly larger than zero. Given $v_{ab} \geq \frac{1}{2}v_a + v_b$ is at least as larger as θv_b. As a result, selling only good b yields strictly less profits than selling both goods at the price pair $(v_{ab} - v_b, v_{ab} - v_a)$. □

The lowest prices ever charged for good a and good b are the ones that attract shoppers with probability one and yield the same profits as charging the highest price for the good and attracting no shoppers so:

$$\underline{p}_a = \frac{\theta}{2-\theta}(v_{ab} - v_b) \tag{1.31}$$

$$\underline{p}_b = \frac{\theta}{2-\theta}(v_{ab} - v_a). \tag{1.32}$$

In equilibrium, price of good i will be randomized in the interval $[\underline{p}_i, \overline{p}_i]$ according to the unique marginal distributions:

$$F_a(p_a) = \frac{(2-\theta)p_a - \theta(v_{ab} - v_b)}{2(1-\theta)p_a} \tag{1.33}$$

$$F_b(p_b) = \frac{(2-\theta)p_b - \theta(v_{ab} - v_a)}{2(1-\theta)p_b}, \tag{1.34}$$

for good a and good b, respectively. The price ranges for Weak Substitutes are illustrated in Figure 1.4 a). Note that the marginal price distributions are identical to those from the case of Weak Complements but the marginal distribution functions in the latter case can never be independent.

Proposition 1.4. *When $v_{ab} \geq \frac{1}{2}v_a + v_b$ the equilibrium marginal distributions given in Equations 1.33 and 1.34 are unique. Given Assumption 1.3 the equilibrium joint distribution function will be the product of marginal distribution functions, that is $F(p_a, p_b) = F_a(p_a) \cdot F_b(p_b)$.*

Proof. The first part of the proposition results from Lemma 1.6 and the proof of Proposition 1.1. The second part follows from Assumption 1.3 and from the fact that $\overline{p}_a + \overline{p}_b \leq v_{ab}$. □

Equilibrium profits in this case ($\pi = \theta(2v_{ab} - v_a - v_b)$) are less than in the one-good model (θv_{ab}). The reason is that the goods are substitutes so there is no opportunity to "discriminate" between captives and shoppers. Monopoly profits from captives obtain only for one price pair ($v_{ab} - v_b, v_{ab} - v_a$) and as a result the retailers do not have the opportunity to keep the monopoly sum constant while lowering one of the prices.

b.2 Intermediate Substitutes

In the previous section we demonstrated that both goods are always sold iff $v_{ab} \geq \frac{1}{2}v_a + v_b$. So if $v_{ab} < \frac{1}{2}v_a + v_b$ with some probability only one good will be bought by captives. In this section we consider the case when probability of selling both goods is still more than zero, albeit less than one. We will argue that in the presence of the price competition good a will never be the only good sold to captives.

Lemma 1.7. *When $v_{ab} < \frac{1}{2}v_a + v_b$ good a will never be the only good sold to captives.*

Proof. Assume the opposite so that for some (p_a, p_b) good a is the only good sold. There are two possibilities: either $v_a \geq p_a > v_{ab} - v_b$ and then it has to be true that $p_b > v_b - v_a + p_a$, or $v_{ab} - v_b \leq p_a$ and then $p_b > v_{ab} - v_a$. Let us consider the latter case first. As before, good b will not be bought by shoppers so the retailer can decrease her price to $v_{ab} - v_a$ and earn strictly higher profits by selling both goods instead of selling only good a. If $v_a \geq p_a > v_{ab} - v_b$ then decreasing the price of good b can only induce captives and shoppers to switch to buying good b but will never lead to selling both goods. Assume that shoppers in this case were buying good a with a probability λ_a. Since $p_a > v_{ab} - v_b$ in the case shoppers buy good a they do not buy anything else from the other retailers and they get overall surplus of $v_a - p_a$. Now consider setting the price of good b at $v_b - v_a + p_a$. Then with the probability λ_a shoppers will buy good b instead of good a. The expected profits will be at least $(v_b - v_a + p_a)(\theta + 2(1-\theta)\lambda_a)$ which is larger than the previous profit of $p_a(\theta + 2(1-\theta)\lambda_a)$, a contradiction. □

We have established that either both goods are bought or only good b is bought by captives. It should be clear that when retailers sell only good b ($p_b > v_{ab} - v_a$ and $v_b - p_b \geq v_a - p_a$) they will randomize p_b in some interval $[\tilde{p}_b, v_b]$ where $\tilde{p}_b > v_{ab} - v_a$. Charging $\tilde{p}_b$ the retailer will sell good b to captives with the same probability they would sell both goods to them if they were to charge $p_a = v_{ab} - v_b$ and $p_b = v_{ab} - v_a$ so $\tilde{p}_b = 2v_{ab} - v_a - v_b$. Because $v_{ab} < v_a + v_B$ the last inequality on $\tilde{p}_b$ will always hold.

If the retailer charges the highest price for good b she will only sell good b and only to captives so $\overline{p}_b = v_b$ and the quilibrium profits have to be θv_b. When $p_b > v_{ab} - v_a$ the distribution function of p_b is:

$$F_b(p_b) = \frac{2p_b - (p_b + v_b)\theta}{2(1-\theta)p_b} \tag{1.35}$$

while p_a can be chosen arbitrarily provided that $p_a \geq v_b - v_a + p_b$. Note that the distribution function for p_b coincides with the one from a one-good model.

Now we will require that:

$$\tilde{p}_b \geq \frac{\theta}{2-\theta} v_b.$$

The latter is necessary because no retailer aiming to sell only good b would ever charge a price below $\frac{\theta}{2-\theta} v_b$. Hence, the condition for Intermediate Substitutes is

$$2v_{ab} - v_a - v_b \geq \frac{\theta}{2-\theta} v_b \Rightarrow v_{ab} \geq \frac{v_a}{2} + \frac{v_b}{2-\theta}. \tag{1.36}$$

Now let us turn to price pairs such that captives buy both goods. This is the case when $p_a \leq v_{ab} - v_b$ and $p_b \leq v_{ab} - v_a$. In this case, if the price of good a is such that it never attracts shoppers then it will be set to the maximum so $\overline{p}_a = v_{ab} - v_b$. Now assume the retailer is charging the highest price for good b of those below $v_{ab} - v_a$. This price will attract shoppers only when other retailers charge p_b above $v_{ab} - v_a$ so the retailer will get the highest profit only when this price is equal to $v_{ab} - v_a$. The expected profits from charging any price pair such that $p_a \leq v_{ab} - v_b$ and $p_b \leq v_{ab} - v_a$ should be equal so for such prices:

$$p_a \left[\theta + 2(1-\theta)(1 - F_a(p_a))\right] + p_b \left[\theta + 2(1-\theta)(1 - F_b(p_b))\right] = \theta v_b. \tag{1.37}$$

The expected profits from charging $(v_{ab} - v_b, v_{ab} - v_a)$ should be such that:

$$\theta v_b = \left[\theta + 2(1-\theta)(1 - F_b(v_{ab} - v_a))\right](2v_{ab} - v_a - v_b), \tag{1.38}$$

which defines $F_b(v_{ab} - v_a)$. We know that $F_b(v_{ab} - v_a) = F_b(\tilde{p}_b)$ and we verify that $\tilde{p}_b = 2v_{ab} - v_a - v_b$ as derived before.

If a retailer charges $p_b = v_{ab} - v_a$ along with some $p_a \leq v_{ab} - v_b$ the profit earned from selling good b is equal to:

$$\frac{\theta(v_{ab} - v_b)v_b}{2v_{ab} - v_a - v_b}. \tag{1.39}$$

The lowest price charged for good b attracts shoppers with probability one and should give the same expected profits so:

$$\underline{p}_b = \frac{\theta(v_{ab} - v_b)v_b}{(2-\theta)(2v_{ab} - v_a - v_b)}. \tag{1.40}$$

The distribution function $F_b(p_b)$ for $p_b \in [\underline{p}_b, v_{ab} - v_a]$ is defined by:

$$p_b \left[\theta + 2(1-\theta)(1 - F_b(p_b))\right] = \frac{\theta(v_{ab} - v_b)v_b}{2v_{ab} - v_a - v_b}. \tag{1.41}$$

If a retailer charges $p_a = v_{ab} - v_b$ along with some $p_b \leq v_{ab} - v_a$ the profits earned from selling good a are equal to:

$$(v_{ab} - v_b)\left[\theta + 2(1-\theta)(1 - F_b(v_{ab} - v_a))\right] = \frac{\theta(v_{ab} - v_a)v_a}{2v_{ab} - v_a - v_b}. \tag{1.42}$$

The lowest price charged for good a attracts shoppers with probability one and earns the same profit so:

$$\underline{p}_a = \frac{\theta(v_{ab} - v_a)v_a}{(2-\theta)(2v_{ab} - v_a - v_b)}. \tag{1.43}$$

The distribution function $F_a(p_a)$ for $p_a \in [\underline{p}_a, v_{ab} - v_b]$ is defined by:

$$p_a \left[\theta + 2(1-\theta)(1 - F_a(p_a))\right] = \frac{\theta(v_{ab} - v_b)v_b}{2v_{ab} - v_a - v_b}. \tag{1.44}$$

To summarize:

Proposition 1.5. *When* $\max[v_b, \frac{1}{2}v_a + \frac{v_b}{2-\theta}] \leq v_{ab} < \frac{1}{2}v_a + v_b$ *the unique equilibrium marginal distribution for* p_a *is defined by Equation 1.44. The unique equilibrium marginal distribution for* $p_b \in [\underline{p}_b, v_{ab} - v_a]$ *is defined by Equation 1.41 and for* $p_b \in [\tilde{p}_b, \overline{p}_b]$ *by Equation 1.38. For* $p_i \leq v_{ab} - v_j$ $(i = a, b)$ *the joint distribution function is given by* $F(p_a, p_b) = F_a(p_a) \cdot F_b(p_b)$.

The price ranges for Intermediate Substitutes are illustrated in Figure 1.4 b). Note that for the range of parameters discussed in this section the oligopolistic industry provides both goods to captives with some probability while the monopolist would only sell good b. In this range the competition leads to a larger variety offered to consumers.

b.3 Strong Substitutes

In this section we consider the case when the only good sold to captives is good b (We have established in the previous section that good a cannot be the only good sold). We have shown so far that if $v_{ab} \geq \frac{1}{2}v_a + \frac{v_b}{2-\theta}$ both goods are sold with nonzero probability. As a result we consider the case when:

$$v_{ab} \in \left[v_b, \max\left(v_b, \frac{1}{2}v_a + \frac{v_b}{2-\theta}\right)\right]. \tag{1.45}$$

We have shown that for such v_{ab} the retailers will choose to sell only good b in the equilibrium. In absence of horizontal differentiation, if the two goods are close enough substitutes, it does not pay off for retailers to charge such pairs of prices that consumers buy both goods. Instead, they will abandon the less valued good. The most retailer can charge for good a if she aims to sell both goods to captives is $v_{ab} - v_b$ which is either less than zero or yields less profits than charging v_b and selling only good b would. Hence, she will sell only good b and earn expected profits of θv_b in the equilibrium. The equilibrium distribution of p_b will be atomless and defined over a closed interval. The highest price ever charged will never attract shoppers so it will be equated to v_b. As a result, the price of good b will be randomized over the interval $[\underline{p}_b, v_b]$ according to the unique equilibrium distribution function:

$$F_b(p_b) = \frac{2p_b - (p_b + v_b)\theta}{2(1-\theta)p_b}, \tag{1.46}$$

and $\underline{p}_b = \frac{\theta}{2-\theta} v_b$.

Proposition 1.6. *If* $v_{ab} \in \left[v_b, \max\left[v_b, \frac{1}{2}v_a + \frac{v_b}{2-\theta}\right]\right]$ *retailers set* p_b *according to the distribution function given in 1.46 over the interval* $[\frac{\theta}{2-\theta}v_b, v_b]$ *and set* $p_a \geq v_b - v_a - p_b$.

Retailers randomize p_b as if good b is the only good available and charge p_a such that consumers never choose to buy good a. The price ranges for Strong Substitutes are illustrated in Figure 1.4 c).

1.4 Extensions

a Bundling

We start with a brief clarification of the terminology used in the bundling literature. As is well understood bundling refers to the practice of selling several goods together at a joint price. Pure bundling (PB) occurs when the goods are not sold separately but only as a bundle while mixed bundling (MB) is, as the name indicates, a pricing strategy where the goods are offered for sale both as separate items and as a bundle. The practice of selling individual goods separately without bundling them is referred to as pure components (PC). In our model we have explicitly solved for the pure components equilibrium while the pure bundling equilibrium is identical to the one-good model solved by Burdett and Judd (1983).

Traditional bundling literature (Adams and Yellen (1976), McAfee, McMillan and Whinston (1989), Lewbel (1985)) deals with consumers that have heterogeneous willingness to pay for goods but observe all the prices. In such environment bundling allows to charge higher overall price while still selling both goods to most of the consumers, especially when the willingness to pay is negatively correlated across goods. It is generally shown that mixed bundling which, by definition, encompasses the other two strategies, is the only one that is deployed in equilibrium (Anderson and Leruth (1993)). Even though one would be tempted to analyze mixed bundling in our framework we will not do so. Importantly, while implementing pure bundling is not necessarily associated with high costs, implementation of mixed bundling is as it requires larger shelve space and more complex logistics than any of the two other strategies. Moreover, whenever expected demand depends on the joint distributions of the two prices and goods are allowed to be complements or substitutes, writing down expected profits in a closed form becomes cumbersome.[8]

Here we will extend our model to allow retailers to use pure bundling in addition to the pure components strategies we have analyzed so far. As such, it is the first such analysis in the price dispersion literature. This omission is probably due to the demand structure associated with the price dispersion. Traditionally, as in our model, consumers are assumed to have identical willingness to pay for the goods and thus the discriminatory ability of bundling cannot be used. Due to this, contrary to what one would initially expect, monopolist in our model has no incentive to bundle complements even if bundling is costless. Monopolist can achieve maximal profits with pure components strategy (any price pair such that $p_a + p_b = v_{ab}$) and would strictly prefer to do so if there were positive costs of bundling. From now on we assume that cost of assembling a bundle is equal to $c \geq 0$.

First important feature to note here is that bundling in our model cannot be a better *type* of strategy *per se*. If one of the firms chooses to bundle the two goods she effectively makes the other firm to bundle the goods as well. Bundling here is a refusal to allow consumers to buy one of the goods from the firm itself and the other one from the competitor so one firm bundling the goods is sufficient to make both bundle.

[8] Venkatesh and Kamakura (2003) use numerical methods even when the distributions are uniform.

If bundling cannot be attractive in itself what additional benefit can it give to a deviating firm? The answer to this question lies in the pricing strategy absent bundling: firm that does not bundle might be randomizing the sum of prices in such a way that allows the other firm to earn additional profits through bundling.

For a simple illustration let us consider the case of independent valuations in our model ($v_{ab} = v_a + v_b$). Assume that one of the retailers randomizes the two prices independently according to the distribution functions given in Equations 1.33 and 1.34. It turns out that charging only one price for a bundle p_{ab} above but close enough to $\underline{p}_a + \underline{p}_b$ outperforms any pure components strategy $(p_a, p_b) : p_i \in [\underline{p}_i, \overline{p}_i]$ where $i = a, b$. This is because the density of $p_a + p_b$ chosen by the complying firm does not give the same expected profits for all $p_{ab} \in [\underline{p}_a + \underline{p}_b, \overline{p}_a + \overline{p}_b]$ chosen by the deviator. Remember that equilibrium profits are given by $\theta v_{ab} = (2-\theta)(\underline{p}_a + \underline{p}_b)$ so for the deviating firm to be indifferent between any $p \in [\underline{p}_a + \underline{p}_b, \overline{p}_a + \overline{p}_b]$ the distribution of $p_a + p_b$ has to be defined by:

$$\theta v_{ab} = p_{ab}(\theta + 2(1-\theta)(1 - F_{ab}(p_{ab}))). \tag{1.47}$$

This distribution is much more concentrated around $\underline{p}_a + \underline{p}_b$ then is the distribution of the sum of p_a and p_b when a firm randomize prices independently. To see this remember that F_{a+b} will be of the order $F_a F_b$ and thus will be going to zero around $\underline{p}_a + \underline{p}_b$ at a larger speed then F_{ab}. This means that charging a bundle price p_{ab} above but sufficiently close to $\underline{p}_a + \underline{p}_b$ will do better than charging bundle price $\underline{p}_a + \underline{p}_b$ or equivalently any component price pair. For independent goods this problem can be removed if firms sell goods separately but positively correlate prices. If p_b is chosen as an increasing function of p_a then the probability distribution of $p_a + p_b$ will be as in Equation 1.47, which eliminates a profitable deviation to bundling.

As has been illustrated with the previous example, attractiveness of bundling is not in using different strategy type (the other firm also effectively bundles) but rather in the fact that competitor does not anticipate it. If he did then he would have chosen his sum of prices accordingly and both firms would play an equilibrium of a single good model with a reservation price equal to v_{ab}. In effect, for complementary goods firms would switch to equilibrium with lower profits.

Anderson and Leruth (1993) and Thisse and Vives (1988) have suggested that firms might choose the type of pricing strategy first (whether to bundle or not) and only having observed each other's choices set actual prices. This is motivated by a premise that bundling decision is harder to change then the pricing one. If we adopt this idea and introduce an additional stage before the pricing game where firms choose what kind of strategy they will use, provided that $v_{ab} \geq v_a + v_b$, the game reduces to:

Firm 1\Firm 2	Pure Bundling	Pure Components
Pure Bundling	$\theta v_{ab} - c, \theta v_{ab} - c$	$\theta v_{ab} - c, \theta v_{ab}$
Pure Components	$\theta v_{ab}, \theta v_{ab} - c$	$\pi^{pc}(v_{ab}), \pi^{pc}(v_{ab})$

Table 1.1: Game of choosing type of pricing for complements. $\pi^{pc}(v_{ab})$ is the equilibrium profit when both firms play pure components strategy.

Proposition 1.7. *Depending on c there are the following equilibria of the two stage game when the goods are complements or independently valued:*

(i) When $c = 0$ there are two equilibria: (PB, PB) and (PC, PC). Out of these only (PC, PC) survives elimination of weakly dominated strategies.

(ii) When $c > 0$ there is a unique equilibrium (PC,PC).

The previous proposition shows that pure components pricing is the most natural solution even if bundling is costless and is the only solution if there are any costs of bundling.

Situation for substitutes is a bit different. Here we will only consider weak substitutes for illustration. We have established in Section b.1 that, unlike complements, the monopolist would bundle substitutes by the fact that $2v_{ab} - v_a - v_b < v_{ab}$. In the oligopolistic competition this consideration is still present. We have shown that both goods are sold in equilibrium and the highest price charged for good i is $v_{ab} - v_j$. Since all the price pairs yield the same profits so does $(v_{ab} - v_b, v_{ab} - v_a)$. At this price vector none of the goods are sold to shoppers. If the retailer, instead, bundles the two goods and charges v_{ab} for the bundle she will sell it to captives but will earn strictly higher profits. This creates a profitable deviation from pure components equilibrium we found.

Next we adopt the two stage game to the weak substitutes. We will not provide proof here but it is fairly straightforward to show that if one of the firms bundles the goods and the other one chooses to sell them separately the equilibrium profits for both firms are equal to θv_{ab}. Remembering that for weak substitutes $\pi^{pc}(v_{ab}) < \theta v_{ab}$ the new game becomes:

Firm 1\Firm 2	Pure Bundling	Pure Components
Pure Bundling	$\theta v_{ab} - c, \theta v_{ab} - c$	$\theta v_{ab} - c, \theta v_{ab}$
Pure Components	$\theta v_{ab}, \theta v_{ab} - c$	$\pi^{pc}(v_{ab}), \pi^{pc}(v_{ab})$

Table 1.2: Game of choosing type of pricing for substitutes. $\pi^{pc}(v_{ab})$ is the equilibrium profit when both firms play pure components strategy.

Proposition 1.8. *Depending on c there are the following equilibria of the two stage game when the goods are weak substitutes:*

(i) When $c = 0$ there are three equilibria: (PB, PB), (PB, PC) and (PC, PB). Only (PB, PB) survives elimination of weakly dominated strategies.

(ii) When $c > 0$ and $\theta v_{ab} - c = \pi^{pc}(v_{ab})$ there are three equilibria: (PB, PC), (PC, PB) and (PC, PC). Only (PC, PC) survives elimination of weakly dominated strategies.

(iii) When $c > 0$ and $\theta v_{ab} - c > \pi^{pc}(v_{ab})$ there are two equilibria: (PB, PC) and (PC, PB).

(iv) When $c > 0$ and $\theta v_{ab} - c < \pi^{pc}(v_{ab})$ there is a unique equilibrium (PC, PC).

As expected, if cost of bundling is sufficiently large there will be no bundling in equilibrium. Even if monopolist would bundle the goods ($\theta v_{ab} - c > \theta(2v_{ab} - v_a - v_b)$) but $c > 0$ we have that only one of the firms bundles. Finally, if there are no costs of bundling then as a monopolist both competitors in equilibrium use bundling.

All the previous analysis, both in the literature and here, assumes that there are two firms in the market. Under such assumption if one of the firms uses pure bundling strategy it effectively forces the other firm to do the same. If there are more than two firms in the market and we are considering a pure components equilibrium, then bundling is weakened as a deviation. This is because now the deviating firm has to undercut not the sum of prices charged by the only remaining firm but rather the sum of minimum prices in the market. Hence, increasing competition further strengthens the pure components equilibrium and we are confident that such study will be an important next step in the analysis of bundling.

b Generalization to n Firms

The analysis so far has be concerned with a duopoly. Clearly one would wish to extend the model to more firms to explore the effect that increase in their number brings. The main ambiguity when generalizing our model to n firms is the behavior of the shoppers. It is easy to see that if they are still visiting two shops as before then the equilibrium we presented is still unique. The reason is that if all the retailers employ identical strategies in the equilibrium and shoppers randomly choose two of them then expected profits of every given retailer from selling good i are still given by Equation 1.12.

Natural question that arises is why would shoppers search only two firms and not more? One way to address this issue is to assume that all the consumers are identical *ex ante* and they engage themselves in the fixed-sample search, that is they decide upfront how many shops to visit and pay a cost t for each visit. Then, as in Burdett and Judd (1983), two symmetric equilibria emerge. In the first equilibrium all the consumers search once and all the retailers charge the same monopoly price pairs. In the second equilibrium some consumers search once (these will be captives) and some search twice (these will be shoppers) and the proportion of captives θ is adjusted in such a way that consumers are indifferent between searching once and searching twice. They will not search three times because the marginal benefit of additional search is decreasing so if the second search decreased average sum of prices by t the third search will not be worth the additional cost of searching.

Another way to address the issue of shoppers is to assume that, as in Varian (1980), shoppers can buy from any of the n retailers while captives are still forced to buy from only one retailer. This slightly changes the equilibria but should not change the main findings of the paper. It proved impossible to show that when the goods are complementary for any n there exists a monotonically decreasing function $b_n(p_a)$ such that $p_a + b_n(p_a) \leq$

v_{ab}. If this function does not exsit for some n then we have to prove that for that n there is some joint distribution function that generates the equilibrium marginal distributions and satisfies the constraint on the sum of the prices. We leave this as an open question for another paper.

1.5 Conclusion

We have presented a two-good price competition model where the goods are complements or substitutes. The model exposed substantial differences between these two kinds of demand dependencies between goods. Namely, it was shown that in the Nash Equilibrium of our model the prices of both goods are randomized in an atomless fashion. This leads to price dispersion and our model is one of the first attempts to study price dispersion in multiproduct setting. When the goods are complements and one of the goods is priced high the other can not be priced high as well, implying a negative correlation between the two prices. The stronger is the complementarity between the goods the more restrictive is the price of one of them for the price of other. This results are supported by the empirical studies on pricing of complementary goods that document the infrequency of simultaneous discounts on complements. We have shown that only when the goods are either independently valued or are substitutes it is feasible that the two prices are randomized independently. This result could suggest why we lack any stylized facts on the co-pricing of substitutes.

It was shown that if the two goods are complements, then retailers are able to discriminate between the more informed consumers (shoppers) and the less informed ones (captives) by enticing the former with one of the goods at a deep discount while taxing the latter by keeping the overall price tag high. This practice requires that the retailers are able to charge different combinations of prices that have a fixed sum and still induce consumers to buy both goods. Through this discrimination the retailers are able to improve their profitability relative to the one-good model. It turns out that when the two goods are substitutable retailers lack this space for manoeuvre and are incapable of discriminating between the two groups of consumers.

Bundling was shown to be a profitable deviation in a pure components equilibrium even if all the consumers have identical willingness to pay.

This is not because bundling is an effective strategy *per se* but rather because competitors price without anticipating such deviation. If, on the other hand, type of pricing is decided before actual prices are chosen then bundling is no longer used in equilibrium when the goods are complements. If the goods are substitutes tendency to bundle in the competitive industry is lower than for monopolist.

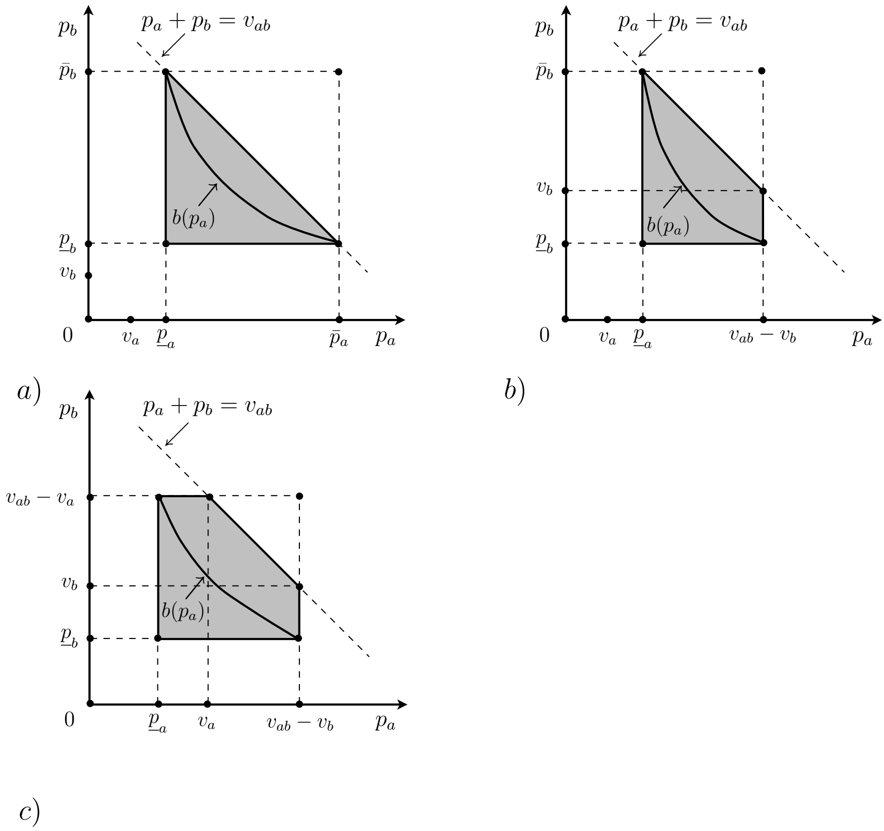

Figure 1.3: Permitted equilibrium price pairs in the case of strong, intermediate and week complements are illustrated with a shaded area in $a)$, $b)$ and $c)$, respectively.

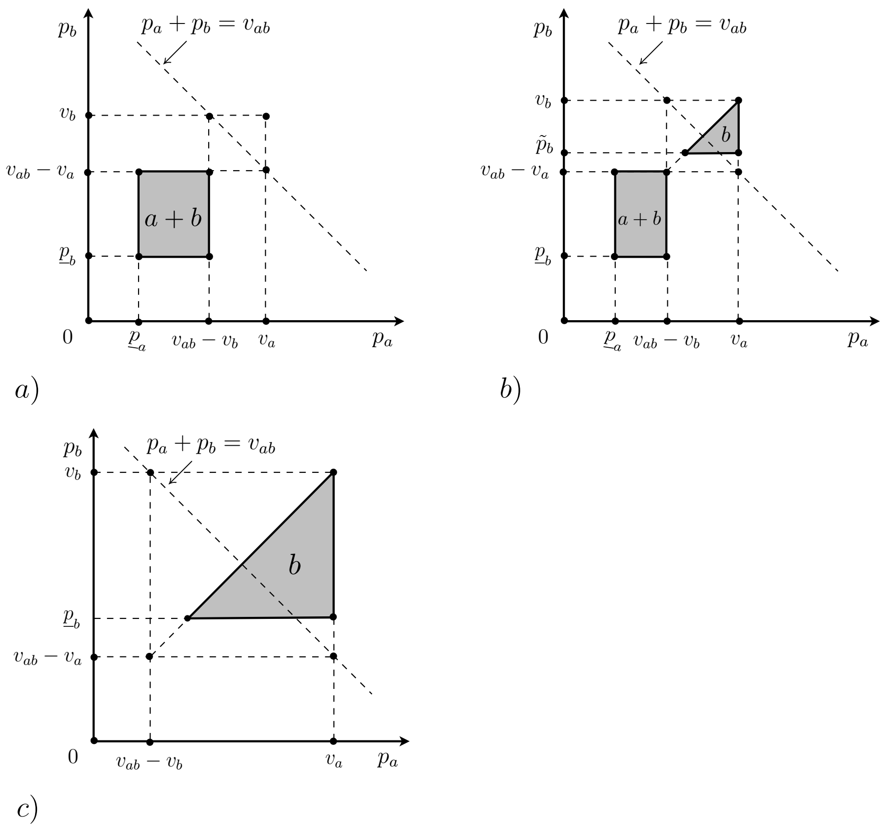

Figure 1.4: Permitted equilibrium price pairs in the case of strong and week substitutes are illustrated with a shaded area in $a)$ and $b)$, respectively. Labels $a+b$ and b indicate areas where both goods are sold and only good b is sold, respectively.

2 HETEROGENEOUS PRICE DISPERSION

2.1 Introduction

The law of one price is so well known to fail that Varian (1980) has famously proclaimed that it is no law at all. All of us have experienced that the same product is frequently sold at different prices in various shops of similar characteristics. There has been no lack of theoretical or empirical studies on the subject but what seems to have been left unexplained is the heterogeneity of its nature. It is clear that some shops vary prices more often than others while some charge higher prices on average than others do. Infrequently you will find discounts in a corner shop or two supermarkets that charge equal average prices. Our paper provides an explanation to both of these phenomena. We argue that (marginal) cost heterogeneity is sufficient to explain why some shops are more expensive than others while different shops have diverse levels of variation in prices. In order to do so we introduce cost asymmetry among firms in two important theoretical models that have been used to explain price dispersion before.

First we generalize Varian (1980) clearinghouse model to allow firms to have different marginal costs and we find that only the two most efficient firms are involved in the competition for informed consumers while all the rest charge monopoly prices. This result can be understood by contrasting the motives of cost-efficient retailers with those of inefficient ones. A retailer has an incentive to lower price of a product if she can rip the benefits of larger demand that this price reduction brings. Since the higher the marginal costs the sharper is the fall in margins from the price reduction, only the most efficient retailers can compete by randomizing prices while the less efficient ones do not change prices at all. Yang (2008) has shown that about 70% of the firms posting prices at *shopping.com* persistently charge monopoly prices and only 30% compete for price-comparing consumers. Our model also justifies anecdotal evidence that small retailers give discounts mostly for logistical purposes and never actually intend to compete with large retailers by doing so.

In the second part of the paper we extend Burdett and Judd (1983) nonsequential search model in a similar fashion by breaking down the continuum of competing firms into groups with different marginal costs. Equilibrium of this model is similar to, but different from, equilibrium of the first part of this paper. As before, firms of lower efficiency cannot share the same price range with more efficient firms, hence each cost type randomizes in its own interval. Unlike the clearinghouse model, in the nonsequential search model firms have the luxury of contesting bits of the informed market exclusively with other firms of their cost type, so less efficient retailers are not forced to charge monopoly prices. That is why, in the price range adequate to their cost level, identical retailers compete by randomize prices and give birth to the second result of our paper: all firms randomize prices, that is give price discounts, but the higher the marginal costs the higher is the average price that the firms charge.

2.2 Literature Review

Starting with the seminal work by Salop and Stiglitz (1977) theoretical economist have provided several explanations of price dispersion in consumer markets. The basic reason for price dispersion in the models that followed are captive or uninformed consumers. In their presence firms are torn between lowering prices to attract shopper and charging monopoly price to rip off captives (see for example Varian (1980), Burdett and Judd (1983), Stahl, II (1989) and Baye and Morgan (2001) among others). These authors analyze price competition among symmetric firms and generally focus on symmetric Nash Equilibrium as the solution method. While plausible in markets where retailers are similar, symmetric models sometimes fail to rationalize the price distributions in markets where retailers are different in their costs, market shares or capacities.

The independent works by Baye, Kovenock and Vries (1992) and Kocas and Kiyak (2006) are remarkable exceptions where the authors consider asymmetric firm equilibria of Varian (1980). Baye et al. (1992) have characterized all the asymmetric solutions of the symmetric Varian (1980) model. Moreover, as Kocas and Kiyak (2006), they consider the clearinghouse model when number of captive consumers per firm is different. They find that, much like in the first part of this paper, firms with larger captive base have less incentives to lower prices and only the two "smallest" firms randomize below the monopoly price. This conclusion (empirically tested by Kocas and Kiyak (2006) for online books market) is counterintuitive in most settings as it suggests that the largest retailers are the ones who charge the highest prices. In contrast, our generalization of the clearinghouse model suggests that the most efficient firms (i.e. large firms who's marginal costs are usually low due to efficiency in operations and bargaining power with suppliers) will charge lowest prices. Unlike the captive base asymmetry, the cost heterogeneity is easily defined and, most importantly, readily modeled in the random search setting, which we illustrate in the second part of this paper.

There is a lot of scientific as well as anecdotal evidence on the price dispersion of homogeneous goods. Lach (2002) has shown that there is persistent asymmetry in the pricing strategies of different retailers in that many of them retain the same relative position in the price distribution in the market from period to period. Also, casual observation suggests that smaller retailers are less prone to giving discounts while large supermarkets do it frequently. Our paper can explain both of these observations. The first part of our model shows that if inefficient (small) retailers directly compete with the efficient (large) ones for consumers then only the most efficient retailers charge price below the monopoly price while other charge monopoly prices only.[1] Our second model shows that if the market is fragmented enough so that bits of it are contested bilaterally then even the least efficient firms compete by randomizing below the monopoly price but they do so at the price level appropriate for their cost type. That is, all the retailers randomize price but different types of retailers do so at different levels.

The rest of the paper is organized as follows: in the next section we state formal conditions on equilibrium demand necessary for our non-overlap argument to hold. We also introduce notation that will be used throughout the two subsequent theoretical sections. In the third section we present a clearinghouse model with a finite number of heterogeneous firms. This section will also lay some groundwork for the fourth section where we present a nonsequential search model with infinite number of firms grouped into finite number of cost types.

2.3 Preliminaries

In the previous section we hinted to the heterogeneous costs as the reason for the pricing strategy asymmetry among dissimilar firms. In this section

[1]Yang (2008) shows that substantial number of firms that advertise price on *shopping.com* do not actually compete for shoppers and set high prices all the time.

we will provide two propositions (the second is a generalization of the first) to formally state our argument. We will also introduce some notation for the cost structure that will be used in the subsequent sections.

Consider a one shot game in a competitive multi-firm setting where firms produce using a constant returns to scale technology and compete by setting prices simultaneously. Let $F_i(p_i)$ denote the equilibrium pricing strategy employed by firm i while $f_i(p_i)$ is the associated density function. For the moment assume that firms face some demand structure that depends on their pricing strategies and demand behavior of consumers. Namely, $D_i(p_i, F_{-i})$ will denote the expected demand firm i faces when charging price p_i against pricing strategies of all opponents F_{-i}. Assume the expected demand is strictly decreasing in p_i.

Proposition 2.1. *If firms i and j have different marginal costs then there does not exist a Nash Equilibrium in which at two different prices p and p' have positive probability measures and at each price firms get equal expected demand, that is $D_i(p) = D_j(p)$ and $D_i(p') = D_j(p')$.*

Proof. Without loss of generality let us assume that $p < p'$ and $c_i < c_j$. Assume that it is possible that in equilibrium $D_i(p) = D_j(p)$ and $D_i(p') = D_j(p')$. Since both firms charge the two prices in equilibrium they should get equal expected profits at these prices. As a result $D_k(p)(p - c_k) = D_k(p')(p' - c_k)$ where $k = i, j$.

$$\begin{aligned} D(p)(p - c_i) &= D(p')(p' - c_i) = D(p')(p' - c_j) + D(p')(c_j - c_i) \\ &= D(p)(p - c_j) + D(p')(c_j - c_i) < D(p)(p - c_j) + D(p)(c_j - c_i) \\ &= D(p)(p - c_i) \end{aligned}$$

where $D(p) = D_i(p) = D_j(p)$ and $D(p') = D_i(p') = D_j(p')$. We arrive at contradiction which proves the proposition. $\square$

Intuitively, demand increases by an equal amount for both firms but the reduction in profit margin per unit is larger for the firm which has higher marginal costs so her profitability has to fall.

This argument can be generalized in two ways. First, expected demand need not be equal at both prices for our argument to hold. It is sufficient that lowering the price brings larger increase in expected demand for the more efficient firm. Second, the two prices need not be charged in equilibrium, but rather, for both firms expected profits at those prices should be equal to the equilibrium profits.

Proposition 2.2. *Let firms i and j produce at the marginal cost c_k where $k = i, j$ and $c_i < c_j$. Assume they earn expected profits π_i and π_j in Nash Equilibrium, respectively. There can be no pair of prices p_1 and p_2 ($p_1 < p_2$) such that the expected profits at those prices for firm k ($k = i, j$) are equal to π_k while $D_i(p_1, F_{-i}) \geq D_j(p_1, F_{-j})$ and $D_i(p_2, F_{-i}) \leq D_j(p_2, F_{-j})$.*

Proof. Let us prove by contradiction. Expected demand from charging p_s for firm k will be denoted by D_{sk} (where $k = i, j$ and $s = 1, 2$) so $\pi_k = D_{1k}(p_1 - c_k) = D_2(p_2 - c_k)$. Using the definitions above we get:

$$\begin{aligned} D_{1i}(p_1 - c_i) &= D_{2i}(p_2 - c_i) = D_{2i}(p_2 - c_j) + D_{2i}(c_j - c_i) \\ &= D_{2j}(p_2 - c_j) + D_{2i}(c_j - c_i) < D_{1j}(p_1 - c_j) + D_{1j}(c_j - c_i) \\ &= D_{1j}(p_1 - c_i) \leq D_{1i}(p_1 - c_i). \end{aligned}$$

We arrive at a contradiction which completes the proof. □

The last proposition will be used to solve both models we present next. It is worth noting that if the number of firms is infinite then expected demand in equilibrium is equal for all the firms (expected demand does not depend on the identity of the firm). If so, Proposition 2.1 can be invoked to prove separation of pricing strategies. It is also clear that with infinite number of firms arbitrary difference in marginal costs will lead to asymmetric pricing which will not disappear even if the cost difference goes to zero in the limit.

When number of firms is finite, unless the two firms use identical pricing strategies, expected demand they face will be different at least for some prices. Even so, in the model we present next, equilibrium demand will have enough regularity to satisfy restrictions imposed by Proposition 2.2.

Throughout the paper we will assume there are K different types of firms, type referring to a production technology employed by firms. There is a large, possibly, infinite number of firms in the market. They employ production technology with associated cost function $C(Q)$ which is characterized by non-increasing average costs. We will assume that:

Assumption 2.1. *All cost functions are continuous for $Q > 0$ and $\forall k' > k$ and $\forall Q' > Q$ we have $C^{k'}(Q') - C^{k'}(Q) > C^k(Q') - C^k(Q)$.*

The second part of Assumption 2.1 implies that types are ranked according to their marginal costs. An illustrative example of cost functions that satisfy the assumption above is when $C^k(Q) = G_k + c_k Q$ where $c_1 < c_2 < \ldots < c_{K-1} < c_K$. We will use this linear example throughout the paper even though all of our results are valid under the more general formulation. In particular, both Proposition 2.1 and 2.2 hold if instead of the linear formulation of cost functions one specifies any cost function that satisfies Assumption 2.1. In the next section we present a clearinghouse model with finite number of firms where we will use Proposition 2.2 to solve for equilibrium in presence of heterogeneous firms.

2.4 Clearinghouse Model

a Consumers and Firms

The model of this section is derived from Varian (1980). There will be two types of consumers - informed and uninformed. The mass of informed consumers is denoted by I while mass of uninformed consumers is denoted by M. Both types of consumers demand at most one unit of a good and are ready to pay up to an amount v, their reservation price. The consumers differ in the information available to them. Informed consumer are able to consult a clearinghouse and obtain prices charged by all the active firms and will buy from the one who charges the lowest price. In case there are several firms charging the lowest price the mass of informed consumers will be divided equally among tying firms. In contrast, uninformed consumers can only observe one price quote and are randomly allocated among all the firms where each firm is equally likely.

There are N firms competing in the market. We will denote mass of uninformed consumers per firm by $U = \frac{M}{N}$. As discussed in the previous section there are K different types of firms where $K \leq N$. We will denote number of firms with a cost function $C^k(Q)$ by N_k where $\sum_{k=1}^{K}(N_k) = N$ and $N_k \leq N$ for all k. When $K = 1$ all the firms are identical (the Varian model) and when $K = N$ all the firms are different from each other. We will also assume (for reasons which will become apparent in the next section) that $vU > C^k(U)$ for all k.

In the next section we will describe all the symmetric equilibria of this model.

b Equilibrium

Two important cases emerge in this model. In the first case there are at least two most efficient firms ($N_1 > 1$). If that is so, we prove that all other firms are driven out of competition for the informed consumers and charge prices equal to v. The second case is when there is only one most efficient firm ($N_1 = 1$). In this case, type one firm and type two firms will randomize over a certain interval while all of the other types will charge v with probability one thus withdrawing from competition for the informed.

We define $\bar{\pi}^k = vU - C^k(U)$. It should be clear that $\bar{\pi}^k$ is the minimum profit that any type k firm can surely get and is positive.[2] The lowest price that will ever be charged by type k firm in equilibrium is a solution to:

$$p(U + I) - C^k(U + I) = vU - C^k(U), \tag{2.1}$$

will be denoted by $\underline{p}^k$ and is equal to:

$$\underline{p}^k = \frac{vU + C^k(U + I) - C^k(U)}{U + I}. \tag{2.2}$$

Note that the left hand side of Equation 2.1 is the profit that type k firm gets if she charges price p and sells to informed consumers with probability one. Charging any price below $\underline{p}^k$ will necessarily bring less profit than charging v does. Charging prices above v yields negative profit so in equilibrium a type k firm can only charge prices in the interval $[\underline{p}^k, v]$. It is important to note that $\underline{p}^k$ is increasing in $C^k(U + I) - C^k(U)$ so $\underline{p}^1 < \underline{p}^2 < ... < \underline{p}^{K-1} < \underline{p}^K$.

Theorem 2.1. *The following are the only symmetric equilibria of the clearinghouse model:*

(i) When $N_1 > 1$, all the type one firms randomize continuously over the interval $[\underline{p}^1, v]$ using the cumulative price distribution $F^1(p) = 1 - \left[\frac{(v-p)U}{(p-c_1)I}\right]^{1/(N_1-1)}$. All other types charge v with probability 1.

[2] Recall that we have assumed $vU > C^k(U)$.

(ii) When $N_1 = 1$ the only type one firm randomizes continuously over the interval $[\underline{p}^2, v)$ using the cumulative price distribution $F^1(p) = 1 - \frac{(v-p)U}{(p-c_2)I(1-F^2(p))^{N_2-1}}$. All the type two firms randomize over the interval $[\underline{p}^2, v)$ using the cumulative price distribution $F^2(p) = 1 - \left[\frac{(v-c_1)U+(c_2-c_1)I}{(p-c_1)I}\right]^{\frac{1}{N_2}}$ and put the probability mass $\left[\frac{c_2-c_1}{v-c_1}\right]^{\frac{1}{N_2}}$ on v. All the remaining types charge v with probability 1.

We will prove Theorem 1 in a sequence of lemmas of the next section. Lemmas 1 through 6 will be necessary to prove both parts (i) and (ii) of the Theorem 1. Lemmas 7 through 9 will conclude the proof of the part (i) while Lemmas 9 completes the proof of part (ii).

Proof of Theorem 2.1

Following Baye et al. (1992) let $\overline{s}_i^k$ and $\underline{s}_i^k$ denote the upper and lower bound for firm i's equilibrium price distribution $F_i^k(p)$, respectively.[3] We will refer to the expected profits of firm i when it charges p against pricing strategies of opponents $F_{-i}(p)$ by $\pi_i(p, F_{-i}(p))$.

A winning tie is defined as situation where two or more firms charge the same price simultaneously and attract informed consumers with a positive probability

Lemma 2.1. *There cannot be a winning tie at any price $p \in [\underline{p}^1, v]$.*

Proof. Suppose not. Any firm that has a point mass at p will find it profitable to transfer that mass to $p-\epsilon$, lose profit of order ϵ and gain a fixed profit by getting all the informed consumers that were being shared them when tying at p. For formalities see proof of Proposition 3 from Varian (1980). Varian proves that in a symmetric equilibrium there are no point masses in equilibrium pricing strategies but the reason his proposition holds is the impossibility of a winning tie at any price. □

Lemma 2.2. *For $\forall i$ the upper bound of the equilibrium price distribution $\overline{s}_i$ is equal to v.*

Proof. Baye et al. (1992) provide a proof of a similar proposition when $K = 1$. Nevertheless, their proof is valid even when number of types is

[3]Superscript k, if present, will indicate that firm i is of type k

larger then one. The proof of Lemma 2.2 follows from Lemmas 1 to 6 from Baye et al. □

The proof of this lemma is technically involved and of little economic interest so we will only provide intuition here. Suppose Lemma 2.2 does not hold so $\exists j$ such that $\overline{s}_j < v$. If that is so, no other firm would charge any price in the open interval $(\overline{s}_j, v)$ as these prices would never attract informed consumers (firm j always charges below) so charging v brings strictly larger profit. There can be no winning tie at $\overline{s}_j$ (Lemma 2.1) which implies that firm j itself prefers charging any price from $(\overline{s}_j, v]$ to charging $\overline{s}_j$.

Lemma 2.3. *Any firm of type k that charges v in equilibrium earns profit equal to $\bar{\pi}^k$.*

Proof. There can be no winning tie at v (Lemma 2.1) , so probability of getting informed consumers at v is equal to zero thus expected profits earned are $\bar{\pi}^k$ □

Lemma 2.4. *Only one firm i can earn more than $\bar{\pi}_i$ in equilibrium.*

Proof. Assume not. Using Lemma 2.3 we conclude that there are at least two firms that do not charge v in equilibrium. Since at least two firms completely randomize (say i and j), for prices close to v, the probability of serving informed consumers approaches zero so their profit cannot be larger than $\bar{\pi}$. More formally: $\lim_{p \to v} \pi_l(p, F_{-l}(p)) = \bar{\pi}_l$ where $l = i, j$ and since $\overline{s}_l = v$ we conclude that both firms can only earn $\bar{\pi}_l$ in equilibrium. □

Lemma 2.5. *In equilibrium all firms of the same type earn equal profit.*

Proof. Assume the opposite. There are two firms i and j of type k which earn different equilibrium profit (say $\pi_j^k > \pi_i^k$). Expected demand for firm i at $\overline{s}_j$ is no smaller than expected demand for firm j at that price $(D_i(\overline{s}_j) \geq D_j(\overline{s}_j))$ so if firm i charges $\overline{s}_j$ with probability one her profit $\pi_i^k(\overline{s}_j) \geq \pi_j^k(\overline{s}_j) = \pi_j^k > \pi_i^k$, a contradiction. □

Lemma 2.6. *No type k firm can earn profit larger than $\bar{\pi}^k$ for $k > 1$.*

Proof. Suppose not. There can only be one type k firm which has profit $\tilde{\pi} > \bar{\pi}^k$ (Lemma 2.4). From Lemma 2.2 we know that the upper limit of an equilibrium price distribution for this firm is v. This implies that all other firms charge v with a positive probability and expected demand for firm j at v is larger than U while for all others it is precisely U. The lower bound of firm $j's$ equilibrium distribution is $\underline{s}_j$. Demand for any firm of type one is at least as large as demand for firm j at $\underline{s}_j$. Using Proposition 2.2 we arrive at contradiction. □

From this point on our discussion will depend on the number of type one firms (N_1). As outlined in the Theorem 1 there are two important cases: when $N_1 = 1$ and $N_1 > 1$.

b.1 Several Type One Firms

Let us first consider the case when $N_1 > 1$. We will argue that all other types will charge v with probability one in equilibrium.

Lemma 2.7. *All type one firms earn profit equal to $\bar{\pi}_1$.*

Proof. Immediate from Lemmas 2.4 and 2.5, and the fact that $N_1 > 1$. □

Let $\underline{s}^{k>1}$denote the lowest price any non-type one firm charges in equilibrium. This price will satisfy $\underline{s}^{k>1} \geq \underline{p}^2$ by definition of $\underline{p}^2$.[4]

Lemma 2.8. *There exist firms i and j of type 1 such that $\underline{s}_i^1 = \underline{s}_j^1 = \underline{p}^1$.*

Proof. Suppose not. As a result there is at most one firm i for which $\underline{s}_i^1 = \underline{p}^1$. Since for all others $\underline{s}_{j\neq i}^1 > p^1$ firm i can earn profit larger than $\bar{\pi}_1$ by charging a price slightly below $min\{\underline{s}_j^1\}_{j\neq i}$ but still above $\underline{p}^1$ while getting all the informed consumers. A contradiction with Lemma 2.7. If $\underline{s}_j^1 > p^1$ for all j then every firm of type 1 can charge some price slightly below $min\{\underline{s}_j^1\}_j$ but still above $\underline{p}^1$ and earn profit larger than $\bar{\pi}_1$, again a contradiction with Lemma 2.7. □

Lemma 2.9. *For all $k > 1$ $\underline{s}^k = v$.*

Proof. Let us prove by contradiction. Assume there is some firm j of type $k > 1$ such that $\underline{s}_j^k < v$. Expected demand for any firm at v is equal to U (implication of Lemmas 2.2 and 2.7). Expected demand for at least one

[4] Recall that $\underline{p}^1 < \underline{p}^2$.

(in fact two) type one firms at $\underline{s}_j^k$ is larger than it is for firm j (Lemma 2.8). Invoking Proposition 2.2 we arrive at a contradiction. $\square$

We have proven that only type one firms can (and two of them certainly will) randomize prices. We are looking for a symmetric equilibrium so all type one firms will randomize over the same interval. One can easily construct an asymmetric solution where only two or more type one firms randomize over the entire interval while other randomize over its proper subsets or charge only v. Baye et al. (1992) provide a detailed guide to constructing such equilibria in the case of only one type but since we have proven that only one type randomizes their solution can be used directly. Let $F^1(p)$ denote the symmetric equilibrium price distribution for the type one firms. As a result, for any price $p \in \left[\underline{p}^1, v\right)$ we have: $(v - c_1)U = (p - c_1)(U + I(1 - F^1(p))^{N_1 - 1}$ which defines $F^1(p)$.

Equilibrium when $N_1 > 1$ is the following:

$\tilde{N}_1(N_1 \geq N_1 \geq 2)$ type one firms randomize using a cumulative distribution function:

$$F^1(p) = 1 - \left[\frac{(v-p)U}{(p-c_1)I}\right]^{1/(N_1-1)}. \tag{2.3}$$

All other types will charge v with probability one. This completes the proof for Theorem 1 part (i). The equilibrium density functions for this case are illustrated in Figure 2.1 *a*).

Note that, given the information structure of the consumers, production allocation is efficient since only type one firms (who produce at the lowest marginal costs) serve the informed consumers

b.2 Single Type One Firm

In this section we consider the case when $N_1 = 1$. Note that this is necessarily the case when $K = N$, that is, all firms are different. In this subsection we show that Propositions 2.1 and 2.2 do not necessarily apply in all the cases. Because there is a single type one firm, she will be forced to randomize in the same interval where other types (namely type two) firms randomize. This will be possible by constructing equilibrium where with lowering price demand for type two firms increases more than for type one firm which means that proposition 2.2 cannot be invoked.

Directly, we can conclude that the only type one firm can always charge a price slightly below $\bar{p}_2$, capture the entire informed market and earn $\tilde{\pi}^1 = vU + C_2(U+I) - C_2(U) - C_1(U+I) = (v-c_1)U + (c_2-c_1)I > \bar{\pi}^1$. We have proven that at most one firm can earn profit larger than $\bar{\pi}$ (Lemma 2.4) so we conclude that the only type one firm is the only one that earns abnormal profit. Next we will argue that the lower bound for its pricing distribution will be equal to $\underline{p}^2$.

Lemma 2.10. *The lower bound of the type one firm's equilibrium price distribution $\underline{s}^1$ will be equal to $\underline{p}^2$.*

Proof. Suppose $\underline{s}^1 \neq \underline{p}^2$.There are two cases: $\underline{s}^1 > \underline{p}^2$ and $\underline{s}^1 < \underline{p}^2$. The former is impossible because no type two firm charges prices below $\underline{p}^2$ so charging any price strictly less than that is not optimal for type one firm. The latter case is more involved. Imagine $\underline{s}^1 > \underline{p}^2$. All type two firms that randomize over $[\underline{p}^2, \underline{s}^1)$ will use price distribution $F^2(p)$ that satisfies: $(v-c_2)U = (p-c_2)(U + I(1-F^2(p))^{N_2-1})$. In such a case profit for type one firm at $\underline{s}^1$ will be smaller than profit at $\underline{p}^2$ which completes the proof. □

As a result of Lemma 2.10 equilibrium profit for type one firm will be precisely $\tilde{\pi}^1$. Because $\overline{s}^1 = v$ we conclude that probability of a tie for all other firms at v should be equal to $\frac{c_2-c_1}{v-c_1}$.[5]

The lower bound for the equilibrium pricing distribution of type two firms will also be $\underline{p}^2$. If all of them charge above $\underline{p}^2$ then the type one firm will have no incentive to randomize near $\underline{p}^2$. Using these two observations we can prove (in the spirit of Lemma 2.11) that all firms of type three and up will charge v with probability one.

Lemma 2.11. *For all $k > 2$ $\underline{s}^k = v$.*

Proof. Identical to the proof of Lemma 2.11. □

We are ready to characterize equilibrium in the case when $N_1 = 1$:

[5]Type one firm should not charge v in equilibrium. It will, of course, charge prices very close to v and at those prices it should earn expected profits equal to $(v-c_1)U+(c_2-c_1)I = (v-c_1)(U + \frac{c_2-c_1}{v-c_1}I)$. As p approaches v, probability of getting informed consumers (tie of all others at v) should approach $\frac{c_2-c_1}{v-c_1}$.

In equilibrium, type one firm will randomize completely on interval $\left[\underline{p}^2, v\right)$. Since the probability of a tie at v has to be equal to $\frac{c_2-c_1}{v-c_1}$ each of the N_2 type two firms will charge v with probability $\left[\frac{c_2-c_1}{v-c_1}\right]^{\frac{1}{N_2}}$.

The pricing strategy of type two firms that randomize will be defined by:

$$(v-c_1)U+(c_2-c_1)I=(p-c_1)(U+I(1-F^2(p))^{N_2}). \tag{2.4}$$

From the Equation 2.4 we derive $F^2(p)$:

$$F^2(p)=1-\left[\frac{(v-p)U+(c_2-c_1)I}{(p-c_1)I}\right]^{\frac{1}{N_2}}. \tag{2.5}$$

The pricing strategy of the only type one firm is given by:

$$(v-c_2)U=(p-c_2)(U+I(1-F^2(p))^{N_2-1}(1-F^1(p))) \tag{2.6}$$

From the Equation 2.6 we derive that:

$$F^1(p)=1-\frac{(v-p)U}{(p-c_2)I(1-F^2(p))^{N_2-1}}. \tag{2.7}$$

The equilibrium cumulative density functions are illustrated in Figure 2.1 *b*). Note that, as expected, $\lim_{p\to v}F^2(p)=1-\left[\frac{c_2-c_1}{v-c_1}\right]^{\frac{1}{N_2}}$ and $\lim_{p\to v}F^1(p)=1$. This completes the proof of Theorem 1 part (ii).

The only case when the equilibrium derived here converges to Varian's equilibrium is when $K=2$, $N_1=1$.

In this case as c_2 approaches c_1 we have that

$$\lim_{c_2\to c_1}F^1(p)=\lim_{c_2\to c_1}F^2(p)=\left[\frac{(v-c_1)U}{(p-c_1)I}\right]^{\frac{1}{N-1}}=F(p). \tag{2.8}$$

Production allocation is inefficient given the information structure. Type two firms will serve informed consumers with a strictly positive probability which leads to inefficiency in production.

c Endogenous Allocation of the Captive Consumers

In the introduction we have stated that the asymmetry in the number of captive consumers per firm has a similar effect on the equilibrium strategies of firms. Specifically, the two smallest firms in that regard will be the only ones randomizing below the monopoly price. It has to be pointed out that if the captive consumers can choose the firm to stick with in the stage prior to the price competition the two effects can cancel each other out.[6] That is, firms with lower marginal costs will charge on average lower prices if the number of captive consumers is equal among all the firms so the captive consumers who chose other firms will have the incentive to switch to the most efficient firms. Increasing the number of captive consumers per efficient firms lowers their incentive to undercut others and the equilibrium will arise where all the firms randomize in the same interval and the average price charged by any of them is equal. The more efficient the firm is the more captive consumers it will have in such equilibrium, reflecting its cost advantage over others. Technicalities of such a solution are not presented here since the model of the next section does not readily allow captive consumers to choose their firm.

In the next section we present a search-theoretic model with an infinite number of firms. Strictly speaking, one can also specify this model when number of firms is finite making it directly comparable to the clearinghouse model. Moreover, in the spirit of Janssen and Moraga-González (2004) we can nest the two into one model. The reason why we abstain from doing so is the loss in elegance and tractability that arises when there is only one firm of some type(s). In this case, solution of the model we present next, becomes obscure. In cases where number of firms of each type is no less than two the solution to the finite firm model is identical to the one presented here subject to a slight reformulation.[7]

2.5 Nonsequential Search Model

a Consumers and Firms

The model of this section is derived from Burdett and Judd (1983). Suppose there is a large number of firms which produce a good that they sell to a fixed number of consumers. Mass of consumers per firm will be denoted

[6]Baye et al. (1992) allow the captives to choose their firm and show that the only equilibrium that survives is the symmetric equilibrium of the Varian (1980) model.

[7]If we require that number of firms of each type N_k is no less than two, solution of the nonsequential search model (presented in the next section) with infinite number of firms can be used if we substitute $\frac{N_k}{N}$ for n_k everywhere. Of course it is necessary to construct a different sequence of lemmas to prove that such equilibrium is the only one possible.

by μ.

There are K types of firms with different marginal costs. Proportion of firms with marginal cost c_k is n_k where $\sum_{k=1}^{K} n_k = 1$ and $n_k \leq 1$. Burdett and Judd model is a special case of ours when $k = 1$. We will assume that $v\mu \geq G_k + c_k\mu$ for all k. The latter assumption guarantees that there is at least one equilibrium where all firms participate in the market.[8] Firms are free to choose a price they charge for the good. Let F denote the cumulative distribution function of prices charged in the market. It is easy to see that $F(v) = 1$ and $F(c_1 - \varepsilon) = 0$ for all $\varepsilon > 0$.

Consumers are assumed to know F or act as if they do. They will not know which firm charges which price though. Consumers will choose to get informed about a subset of firms which will be chosen randomly. Consumers can only buy from a firm who's price quotation they got informed about. Consumer will buy one unit of a good if she decides to buy from a firm who charges no more than v for it, otherwise she buys no units. Expected profits a firm gets from charging a certain price depend on the price she charges, prices that other firms charge and the way consumers obtain price quotations.

Consumers decide how many price quotations to obtain upfront, pay amount t for each quotations and then buy from a firm who quotes the lowest price in their sample provided that the price is not larger than v. Consumers decide how many price quotations to obtain and pay tm if they choose to obtain m price quotes. Value of the parchase is such that it is always optimal for a consumer to obtain at least one quotation. All the prices are drawn randomly from population of firms. Price of purchase for a consumer who searches m firms is $cm + \int_0^\infty mp(1 - F(p))^{m-1} dF(p)$. This function is convex in m and thus has a unique minimum if m is a real number. Since m can only be an integer there can be either one integer m^* or two integers m^* and $m^* + 1$ that minimize the function.

Here we introduce additional notation. Proportion of consumers who decide to obtain m quotations will be denoted by θ_m where $m = 1, 2, 3, \ldots$ and $\sum_{m=1}^{\infty} \theta_m = 1$.

[8] We will need to further restrict $C^k(Q)$ to make sure that all types earn non-negative profit in a dispersed equilibrium. It will be sufficient to assume that $(\underline{p}^k - c_k)D(\underline{p}_k) \geq G_k$ where $D(p)$ and $\underline{p}^k$ will be defined in the next subsection. The former is the expected demand at price p while the latter is the lowest price that type k firms charge in equilibrium.

b Equilibrium

There always is a simple equilibrium in this model. If all firms charge price equal to v then all consumers should search once in equilibrium. Clearly, consumers do not have an incentive to search more than once as the only price they can encounter is v. Firms also do not have incentive to charge price below v as consumers only search one firm and there is no way to attract additional consumers by charging a lower price.

It is easy to see that when $k = 1$ (Burdett and Judd (1983) model) the only equilibrium with some consumers searching more than once is where $1 > \theta_1 > 0$ and $\theta_2 = 1 - \theta_1$. The reason why θ_1 has to be more than zero is intuitive. If all consumers obtain two or more quotations then all firms engage in Bertrand competition and thus single equilibrium price is c_1. If all firms charge the same price it is optimal for consumers to search only once thus $\theta_1 = 1$ which leads to a contradiction. Note that if $k > 1$ this argument is not necessarily true. If $\theta_1 = 0$ then all type K firms will have to charge price equal to c_K. Charging price equal to marginal cost is not an equilibrium strategy for all other types. Some of their consumers will only search them and firms of "higher" types which means that charging price equal to c_k is not an equilibrium for type k ($k < K$) firms. For small enough ε type k firm can always charge $c_{k+1} - \varepsilon$ and surely earn positive profit. Even if all firms charged prices equal to their marginal costs consumers would still have incentive to search more than once because, in equilibrium, different firm types charge different prices. For this reason there can exist equilibria where no consumers search once. In this paper we will only discuss the case when $\theta_1 \neq 0$. Our conjecture is that for a range of search cost parameter t only equilibrium when $\theta_1 > 1$ exists. Furthermore, we also focus on analyzing how Burdett and Judd's symmetric equilibrium can be approached so dropping the model with all consumers searching at least twice is only natural. Analyzing all the equilibria in this framework is a subject of another paper.

Having established that the consumers search either once or twice we turn to characterizing such equilibrium. First, consumers should be indifferent between searching once or twice. Second, firms should only charge prices that earn equal expected profits and all other prices should yield expected profits no larger than the equilibrium one.

We will define:

$$S(F) = \int_0^v pdF(p) - 2\int_0^v p(1-F(p))dF(p) \tag{2.9}$$

as a reduction in expected price of purchase from searching twice insted of searching once. $D(p)$ will denote expected demand from charging p (for any firm) and will be equal to:

$$D(p) = \mu(\theta + 2(1-\theta)(1-F(p))) = \mu\theta + 2\mu(1-\theta)(1-F(p)). \tag{2.10}$$

The first part on the left-hand side of the last expression is demand from those who search only once ($\mu\theta$), the number of consumers per firm times the proportion of consumers who search only once) and the second part is expected demand from those consumers who search twice ($2\mu(1-\theta)$), is the number of queries that end up at any firm so by charging p that firm expects to grab $1-F(p)$ proportion of those who searched it).

Theorem 2.2. *Nonsequential search model has the following symmetric equilibria when $\theta_1 > 0$:*

(i) All firms charge v with probability one and all consumers search once ($\theta_1 = 1$).

(ii) Exists $\tilde{t}$ such that for all $t < \tilde{t}$ type k firms randomize continuously over subsequent intervals $[\underline{p}^k, \underline{p}^{k+1}]$ using the cumulative distribution $F^k(p)$. Proportion $\tilde{\theta}(t)$ of the consumers searches once and the rest search twice.

We have already argued that the equilibrium described in Theorem 2 part (i) always exists. We will prove part (ii) of Theorem 2 in a sequence of lemmas.

Lemma 2.12. *The equilibrium distribution of prices $F(p)$ is continuous with connected support.*

Proof. Suppose there is p' at which $F(p)$ has discontinuity. That is:

$$\lim_{\epsilon \to 0} F(p' - \epsilon) < \lim_{\epsilon \to 0} F(p' + \epsilon).$$

There is a positive probability that consumers search two firms which charge p' which means that those firms have incentive to charge some slightly lower price and get all of those consumers insted of sharing them. While doing so they loose profit of order zero and gain a discrete amount. No assume $F(p)$ is constant on some interval $[p', p'']$ in the convex hull of its support. Charging p'attracts those who search twice exactly with the same probability as $p' + \epsilon < p''$ does. As a result, charging $p' + \epsilon$ brings higher profit which can not be the case in equilibrium. $\square$

Note that, unlike the clearinghouse model, v can not be charged with a positive probability in equilibrium. In the clearinghouse model so as long as two firms never charged v there was no tie at v and undercutting it did not give any obvious advantage. In this model firms compete pairwise meaning that if a mass of firms charges v then there are consumers who search two firms only out of those who charge v and undercutting it slightly becomes profitable.

Expected demand for any firm of any type charging price p is equal because the number of firms is infinite. As a result we can use Proposition 2.1 to conclude that firms of different types will not randomize over any common interval.

Lemma 2.13. *If, in equilibrium, price p is charged by firms of two different types k and k' (assume $k < k'$) then type k firms will not charge any prices above p and type k' firms will not charge any prices below p.*

Proof. If p is charged by the two types we can deduce their equilibrium profits: $\pi^k(p, F) = (p - c_k)D(p)$ and $\pi^{k'}(p, F) = (p - c_{k'})D(p)$ where $D(p) = \mu\theta + 2\mu(1-\theta)(1 - F(p))$ is expected demand for any firm charging p. Take any price $p' > p$. If type k firms charge that price as well we should have that $\pi^k(p, F) = (p' - c_k)D(p')$ but then any type k' firm would want to charge p' and never p because $(p' - c_{k'})\mu(\theta + 2(1-\theta)(1 - F'(p'))) > \pi^{k'}(p, F)$. In the same fashion we can prove that any price $p'' < p$ will not be charged by type k' firms.

$\square$

Lemma 2.14. *If firms of type k charge p' and p'' in equilibrium then they will charge all the prices in the the interval $[p', p'']$ and will be the only type that does so.*

Proof. Assume the opposite. There is type k that charges p' and p'' but not some $p \in (p', p'')$ which is charged by some other type k'. Charging p' and

p'' should give equal profits to type k while p should give at most that profit which gives an inequality: $(p' - c_k)D(p') = (p'' - c_k)D(p'') \geq (p - c_k)D(p)$. For type k' charging p' and p''should not be more profitable than charging p so $(p - c_{k'})D(p) \geq (p' - c_{k'})D(p')$ and $(p - c_{k'})D(p) \geq (p'' - c_{k'})D(p'')$.

There are two cases: $c_k > c_{k'}$ or $c_k < c_{k'}$. Let us consider the latter case first.

$(p' - c_k)D(p') \geq (p - c_k)D(p) \Rightarrow$

$(p' - c_{k'})D(p') + (c_{k'} - c_k)D(p') \geq (p - c_{k'})D(p) + (c_{k'} - c_k)D(p)$.

Since $p' < p$ we know that $D(p') > D(p)$ and we get $(p' - c_{k'})D(p') > (p - c_{k'})D(p)$ which contradicts $(p - c_{k'})D(p) \geq (p' - c_{k'})D(p')$.

Now assume that $c_k < c_{k'}$. $(p'' - c_k)D(p'') \geq (p - c_k)D(p) \Rightarrow (p'' - c_{k'})D(p'') + (c_{k'} - c_k)D(p'') \geq (p - c_{k'})D(p) + (c_{k'} - c_k)D(p)$. Since $p'' > p$ we get that $D(p'') < D(p)$ which along with $c_k < c_{k'}$ implies that $(p'' - c_{k'})D(p'') > (p - c_{k'})D(p)$, a contradiction. □

Lemma 2.15. *The reservation price v will be the upper limit of the support of $F(p)$.*

Proof. Suppose the upper bound is some $x < v$. Firms that charge x never attract those consumers who search more than once so charging v will give them strictly larger profit, a contradiction. □

Lemmas 2.12 and 2.14 imply that types will be charging prices in consecutive intervals. Because those intervals will necessarily touch each other at one point (and no more than that). Lemma 2.13 requires that the sequence of firm types follows their marginal cost levels from the *highest* type charging in the *highest* interval.

We will describe a symmetric equilibrium of this model (symmetric in a sense that all firms of the same type employ identical pricing strategies).[9] All of the previous arguments along with Lemma 2.15 imply that type K firms randomize over the interval $[\underline{p}^K, v]$ where $\underline{p}^K$ is defined by:

$$(\underline{p}^K - c_K)(\theta + 2(1-\theta)n_K) = (v - c_K)\theta. \tag{2.11}$$

[9]Here we will pin down pricing strategy of each type in aggregate. Because number of firms of each type is infinite individual firms of that type can employ arbitrary strategies as long as in aggregate their price distribution is correct. In Appendix we describe how to construct an asymmetric solution where each subgroups of a type randomizes over a separate interval.

From the last expression we obtain:

$$\underline{p}^K = \frac{v\theta + 2c_K n_K(1-\theta)}{\theta + 2n_K(1-\theta)}. \tag{2.12}$$

The equilibrium distribution function $F^K(p)$ for type K firms will be defined by:

$$(v - c_K)\theta = (p - c_K)(\theta + 2(1-\theta)n_K(1 - F^K(p))), \tag{2.13}$$

where $p \in [\underline{p}^K, v]$.

Type $K-j$ $(j = 1, 2, .., (K-1))$ firms will charge prices in the interval $[\underline{p}^{K-j}, \underline{p}^{K-j+1}]$ where $\underline{p}^{K-j}$ is defined by:

$$\begin{aligned}(\underline{p}^{K-j} - c_{K-j})\left[\theta + 2(1-\theta)\sum_{i=1}^{j+1} n_{K-i+1}\right] = \\ = (\underline{p}^{K-j+1} - c_{K-j})\left[\theta + 2(1-\theta)\sum_{i=1}^{j} n_{K-i+1}\right]\end{aligned} \tag{2.14}$$

Since we know $\underline{p}^K$ we can apply this equation recursively to obtain all $\underline{p}^{K-j}$.

The equilibrium price distribution function for type $K-j$ firms $F^{K-j}(p)$ will be defined by:

$$\begin{aligned}(p - c_{K-j})\left[\theta + 2(1-\theta)\left[\sum_{i=1}^{j} n_{K-i+1} + n_{K-j}(1 - F^{K-j}(p))\right]\right] = \\ = \left(\underline{p}^{K-j+1} - c_{K-j}\right)\left[\theta + 2(1-\theta)\sum_{i=1}^{j} n_{K-i+1}\right]\end{aligned} \tag{2.15}$$

where $p \in [\underline{p}^{K-j}, \underline{p}^{K-j+1}]$. One can see that no type k firm has incentive to charge prices outside $[\underline{p}^k, \underline{p}^{k+1}]$. Since $\underline{p}^k$ is charged by type $k-1$ firms Lemma 2.13 implies that type k firm cannot charge prices below $\underline{p}^k$. Using similar reasoning we conclude that they cannot charge prices above $\underline{p}^{k+1}$ either which along with the previous statement proves that they will only charge prices in the interval $[\underline{p}^k, \underline{p}^{k+1}]$. The equilibrium density functions are illustrated in Figure 2.1 c).

When θ approaches 1 all $\underline{p}^k$ will converge to v thus making $S(F(\theta))$ (which is a function of F which in turn is a function of θ) converge to 0. Intuitively, when number of consumers who search twice goes to zero firms start charging prices very close to v to earn the most from consumers who search only once. As a result value of search goes to zero because all firms charge price very close to each other.

Lemma 2.16. *For all* k $\lim_{\theta\to 1}\underline{p}^k = v$ *which implies that* $\lim_{\theta\to 1} S(\theta) = 0$.

Proof. From Equation 2.12 we can see that $\lim_{\theta\to 1}\underline{p}^K = v$. From Equation 2.14 we obtain that $\lim_{\theta\to 1}\underline{p}^{K-j} = \underline{p}^{K-j+1}$. As a result we conclude that $\lim_{\theta\to 1}\underline{p}^k = v$ for all k. In particular, $\lim_{\theta\to 1}\underline{p}^1 = v$. We know that $F(p)$ is defined on an interval $[\underline{p}^1, v]$ so from Equation 2.9 we conclude that $\lim_{\theta\to 1} S(\theta) = 0$. □

For $1 \geq \theta \geq 0 \Rightarrow S(\theta) > 0$ and S is a continuous function of θ so it will reach its maximum for at least one $\theta \in [0, 1]$. Let $\tilde{S} > 0$ denote the corresponding maximum. If cost of obtaining price quotation $t < \tilde{S}$ there will be at least one $\tilde{\theta}(t)$ such that consumers are indifferent between searching once or twice ($S(\tilde{\theta}) = t$). Reduction in expected price from obtaining one more price quote is decreasing in the number of prices quotes obtained (see Section 2.1.6 in Baye, Morgan and Scholten (2006)), as a result no consumer will choose to search three or more times. θ that solves $S(\theta) = t$ and the corresponding price distributions of all types $F^k(p, \theta)$ will constitute equilibrium of this model. This completes the proof of Theorem 2 part (ii).

The aggregate price distribution will be equal to the sum of the price distributions employed by all the types weighted by the proportions of types, that is $F(p) = \sum_{k=1}^{K} n_k F^k(p)$ where $p \in [\underline{p}^1, v]$. When firms become more and more similar ($\forall c_k \to$ c) the aggregate price distribution converges to the one where $K = 1$ and $c_1 = c$ (Burdett and Judd (1983)). This model proves to be more resistant to firm heterogeneity: to the extent that we consider the price distribution observed in the market, as firms become less dissimilar equilibrium converges to the homogeneous firm equilibrium. The symmetric firm equilibrium of Burdett and Judd (1983) can not be reached as the limit of the heterogeneous firm model. Just as before, the argument made in Proposition 2.2 makes symmetric solution unapproachable. Instead, an asymmetric equilibrium of Burdett and Judd model is reached in the limit when costs converge (We describe this asymmetric equilibrium in Appendix).

Given the information structure of the consumers (which is endogenous in this case) production decisions are efficient in the sense that if consumers search two firms with different marginal costs they always buy from the more efficient one.

2.6 Conclusion

In this paper we have analyzed a homogeneous good price dispersion where firms have different marginal costs. We have identified conditions under which the cost heterogeneity leads to asymmetric equilibria that stay such even when in the limit costs converge. We have shown that these conditions are satisfied in two important frameworks of the price dispersion literature. In the generalized costly nonsequential search (based on Burdett and Judd (1983)) and clearinghouse (based on Varian (1980)) models we have shown that equilibrium pricing strategies of heterogeneous firms will have at most one common price in their support. These finding are in line with empirical observation that in markets where prices are dispersed firms have tendency to stay in the same quartile of market price distribution from period to period. This phenomenon cannot be explained by symmetric firm models prominent in the literature (including the ones we have generalized) since they command symmetric equilibrium as the only natural solution. Our nonsequential search model has generated an individual price dispersion, albeit at systematically different levels, that has been documented in many markets and was not explained by heterogeneous firm models to date. We have also justified empirical and anecdotal observations that some retailers persistently charge monopoly prices while others compete among each other by charging prices below the monopoly price .

We are confident that our results can be extended to a more general framework where both of our models are nested into one and sequential search is allowed additionally.[10] Our conjecture is that depending on the search cost parameter the equilibrium would, in its spirit, vary from the one we obtained in the clearinghouse model to the one derived in the non-

[10]Janssen and Moraga-González (2004) nest the two in a symmetric setting.

sequential search model. This would lead to the same stably asymmetric solution that has been presented here in the two different setups.

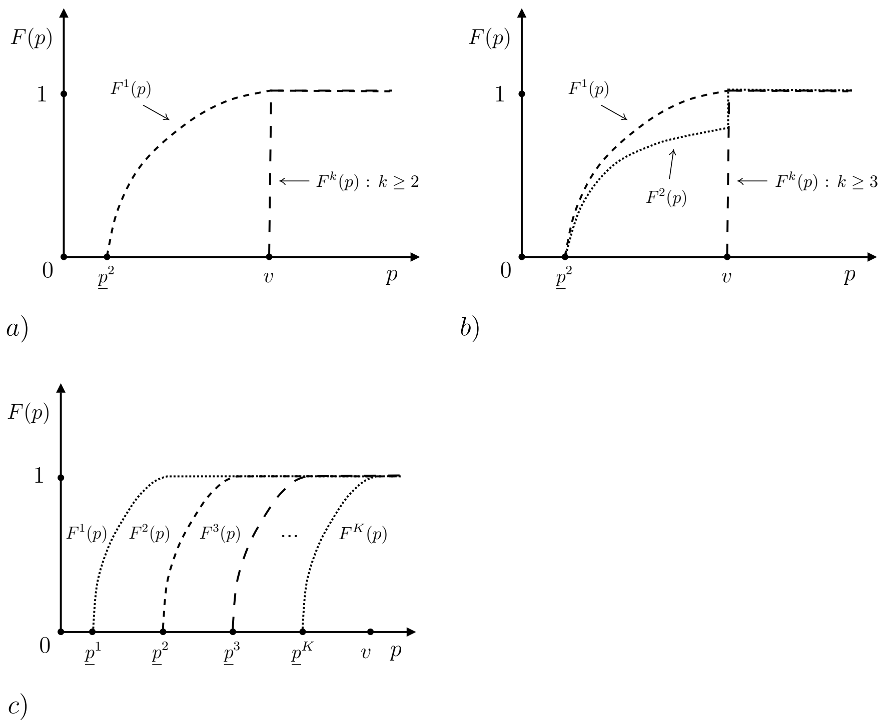

Figure 2.1: Equilibrium cumulative densities: a) for the Clearinghouse model when $N_1 \geq 2$, b) for the Clearinghouse model when $N_1 = 1$, and c) for the Nonsequential Search model.

3 IS A COMPETITOR OF MY COMPETITOR ALSO MY COMPETITOR?

3.1 Introduction

Capacity investment has been long associated with anticompetitive behavior. Its use as an entry barrier in the context of quantity competition is well studied in the literature (Gilbert and Vives (1986), Anderson and Engers (1994), Allen, Deneckere, Faith and Kovenock (2000)). Eaton and Ware (1987) and McLean and Riordan (1989) have shown that in a model with sequential capacity building irreversibility of capacity investment allows early movers to deter some later comers to the market from entering. Eaton and Ware (1987), unlike McLean and Riordan (1989), separate capacity (sunk) investment stage from a simultaneous quantity competition that follows it and nevertheless show that capacity investment plays entry-deterring role.

In order for capacity levels to play any strategic role when on the subsequent production stage quantities are chosen simultaneously it is necessary that capacity acquisition is not simultaneous. If it were then capacity could not play any entry-deterring role and the resulting model would be equivalent to a standard Cournot competition with marginal costs equal to the sum of the cost of acquiring a unit of capacity on the investment stage and the marginal cost of production on the quantity competition stage. In this paper we show that this observation holds true only if all the firms compete in the same market. It is not unusual that after having built their capacity firms ship goods to many markets and do not always face the same competitors in all of them. If this is true then, even if capacities are built simultaneously, each firm will have a tendency to overbuild capacity on the investemnt stage in order to force its competitors to redistribute their production in the second stage towards the markets where the firm itself is not present.

We model this idea by assuming that firms are located on a circle with markets located in between firms. In the first stage firms simultaneously choose how many units of capacity to build at a constant marginal cost. In the second stage, after having observed all others' capacities, each firm decides how many units of the final good to produce (at a constant marginal cost of production) within its capacity and how to allocate this quantity between the two markets that the firm directly borders. Price in each market is then determined by the sum of the two quantities supplied to the market by the two neighboring firms.

We show that in this setting, even though there will be no unused capacity in the second stage, the level of capacity built is larger than would have been if the pairs of firms competed only in one market. To see this imagine that all firms are symmetric and in the first stage they build such a level of capacity that in the second stage each market receives Cournot quantity corresponding to two firms and marginal costs equal to the cost of capacity plus the cost of production. If this is true then marginal revenue from one more unit supplied to each market in the second stage should be equal to the total marginal cost and larger than the marginal cost of the second stage. The latter implies that capacity constraint will be binding in the second stage and thus every unit of additional capacity would have been produced and supplied. In such a case, under standard assumptions on inverse demand function, from the standpoint of the first stage marginal revenues from one unit of additional capacity will be larger than joint marginal costs. If a firm builds two additional units of capacity in the first stage, the firm commits to supply one more unit to each market it serves and thus will induce its competitors to supply less to the affected markets. In effect, firms have Stackelberg leader's incentive to overproduce capacity even though they do not have the first-mover advantage. As a result, marginal revenue generated by one more unit of capacity will be larger than marginal revenue from one more unit shipped to a market in the second stage and will induce overcapacity relative to standard Cournot model. This result holds for any symmetric market structures including all regular networks provided that all firms have identical marginal costs.

Additionally, we consider asymmetry in capacity acquisition costs to find that indirect competition has interesting implication for profitability of firms on the circle. It is shown here that direct competitors of firms own competitors can be perceived as its allies in the following sense: if their marginal costs decrease then equilibrium capacity and profits of the firm itself increase. This result is driven by the strategic substitutability of quantities along with the capacity constraints. When capacity costs of an indirect competitor go down, she increases her capacity in equilibrium. This has two effects on the firm's direct competitor: in the first stage she is forced to build smaller capacity but in the second stage she will supply larger fraction of that capacity to the market contested with the firm itself. Under linear demand specification the first effect is shown to be stronger than the second which leads to smaller quantity supplied to the firm's own market and thus leads to higher capacity and profits for the firm. It remains to be seen whether this effect is sensitive to the linear demand specification but it is clear that generalization of this result to a wide class of demand functions is to be expected.

The rest of the paper is organized in the following way: we present the model in Section 3.2. We characterize identical firm equilibrium under general demand specification in Section 3.3. We solve an asymmetric firm version under linear demand in Section 3.4 and we conclude in Section 3.5.

3.2 The Model

Imagine $n \geq 3$ firms located on the circle. There are n markets located in between the firms. Each firm serves only the two markets which it borders. Competition takes place in two stages. In the first stage all firms

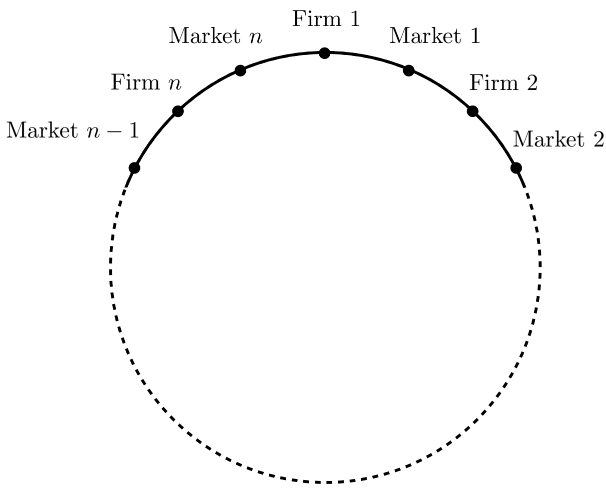

Figure 3.1: Indexing and location of firms and markets on the circle

simultaneously acquire a production capacity k_i at a (potentially seller-specific) constant per-unit cost d_i, which is a common knowledge. The capacity investment in the first stage is assumed to be irreversible and observed by all the firms prior to the second stage. In the second stage all firms simultaneously choose quantities that they ship to the two markets they serve within the capacities determined in the first stage. Marginal cost of production in the second stage is constant and is equal to c_i for firm i. For now we require that $c_i + d_i < p^{-1}(0)$ but later we will impose more conditions on the sum of marginal costs if these are different across firms. Demand is identical in all the markets and the inverse demand function is denoted by $p(q)$ where q is the total quantity supplied to that market. It is assumed that function $p(q)$ is twice continuously differentiable and for all q where the function is positive we have $p'(q) < 0$. Finally, we assume that

Assumption 3.1. $p'(q) + xp''(q) < 0$ *for* $\forall x \in [0, q]$.

The last assumption insures that reaction functions are upward-sloping which is a sufficient condition for the existence of Cournot equilibrium. We assume that firms maximize their profits without discounting amounts between stages. We solve for subgame perfect Nash equilibrium of the model.

Quantity that firm i supplies to the market on its right (left), when facing the circle, will be denote by q_i^l (q_i^r).[1] We will use i rather loosely implying that $1 < i < n$, implicitly assuming that if $i = 1$ then $i - 1$ stands for n and if $i = n$ then $i + 1$ stands for 1.

3.3 Identical Firms

It is assumed in this section that all firms are identical ($c_i = c$ and $d_i = d$ for all i). We characterize only the symmetric equilibrium of the model. It appears to be the only equilibrium when the firms are identical but we do not provide a formal proof.

We solve the model by backward induction. Imagine that in the first stage all firms built certain level of capacity. Let us denote the vector of these capacities by $K = (k_1, k_2, ..., k_n)$. Once capacities are built firms decide how many units to produce within their capacity. Each firm can allocate her capacity between the two markets in any way so in equilibrium of the second stage marginal revenues in both markets have to be equal provided that capacities are not too different.

Lemma 3.1. *If firms are identical marginal revenues in the second stage*

[1] For example, price in the market 1 will be equal to $p(q_1^l + q_2^r)$.

for firm i in both markets will be equal $p(q_i^l + q_{i+1}^r) + p'(q_i^l + q_{i+1}^r)q_i^l = p(q_i^r + q_{i-1}^l) + p'(q_i^r + q_{i-1}^l)q_i^r$.

Proof. Proof follows from a simple observation that if marginal revenues were not equal firm could move one unit of quantity from the market with lower marginal revenue to the one with higher marginal revenue and increase profits. □

As mentioned, previous lemma would not hold if quantities supplied by competitors (q_{i-1}^l, q_{i+1}^r) were so different from each other that a firm would not be able to equate marginal revenues in both markets even after having supplied all of its capacity to the one with higher marginal revenue. In this section firms are identical and we are characterizing a symmetric equilibrium so this consideration is of no great importance. We allow vector K to have different components only insofar as to evaluate profitability of small deviations in the first stage and find equilibrium capacity that will be equal for all firms.

Since the cost of production is c, in the second stage firms will produce up to the point when either the capacity constraint is binding or marginal revenue in both markets is equal to c. Depending on whether common marginal revenue in both markets is larger than c or equal to it the optimal reaction function $q_i^l(q_{i-1}^l, q_{i+1}^r)$ will be a solution to either

$$\begin{aligned} &p(q_i^l + q_{i+1}^r) + p'(q_i^l + q_{i+1}^r)q_i^l = \\ &= p((k_i - q_i^l) + q_{i-1}^l) + p'((k_i - q_i^l) + q_{i-1}^l)q_i^r \end{aligned} \tag{3.1}$$

or

$$p(q_i^l + q_{i+1}^r) + p'(q_i^l + q_{i+1}^r)q_i^l = c, \tag{3.2}$$

respectively. Equilibrium of the second stage will be such pair of quantities produced for the two markets that each firm optimally reacts to all others according to the reaction function specified. From now on let us assume that for any i functions $q_i^l(K)$ and $q_i^r(K)$ are differentiable in K.

Now we turn to the first stage. First observation to be made is that capacities are built simultaneously so in equilibrium there will be no overcapacity.

Lemma 3.2. *There will be no overcapacity in equilibrium. That is, for any firm i the sum of equilibrium quantities supplied in the second stage is equal to the equilibrium capacity built in the first stage* $q_i^r(K^*) + q_i^l(K^*) = k_i^*$.

Proof. This lemma follows from the fact that capacities are built simultaneously and affect outcomes of the second stage only to the extent that they bind quantities supplied at that stage. If for some firm capacity is not binding in the second stage then it can do better by decreasing its capacity to be exactly equal to the overall quantity it supplies in the second stage. By doing so it does not alter the equilibrium of the second stage and strictly increases her overall profits. □

Marginal revenues with respect to quantities produced in the second stage cannot be lower than c. Lemma 3.2 implies that if all firms build an equal capacity in the first stage (as they do in the symmetric equilibrium) then this capacity has to be no larger than k^c defined by:

$$p\left(k^c\right) + \frac{k^c}{2} p'\left(k^c\right) = c. \tag{3.3}$$

This is a total Cournot equilibrium quantity of a model with two firms and marginal costs equal to c.[2] If all the firms build k^c then marginal revenue in all the markets will be equal to c and thus adding one more unit of capacity in the first stage will not result in an increase of quantities produced in the second stage.

Lemma 3.3. *Maximum possible capacity in a symmetric equilibrium is k^c which is total Cournot equilibrium quantity of a model with two firms and marginal costs equal to c.*

If all the firms build equal capacity it will be an equilibrium only if marginal revenue from additional capacity is equal to $c+d$ at that capacity level. To see this observe the maximization problem of a firm i is:

$$\max_{k_i} \quad p\left(q^l_{i-1}(K) + q^r_i(K)\right) q^r_i(K) + p\left(q^r_{i+1}(K) + q^l_i(K)\right) q^l_i(K) - \\ -(c+d)k_i \tag{3.4}$$

By symmetry we know that in equilibrium $q^r_i(K) = q^l_i(K) = \frac{k_i}{2}$ and $q^l_{i-1}(K) = q^r_{i+1}(K)$. Using this the first order condition gives:

$$p(k^*) + k^* p'(k^*) \left[\frac{1}{2} + \left. \frac{\partial q^l_{i-1}(K)}{\partial k_i} \right|_{K=K^*} \right] = c + d. \tag{3.5}$$

From this Equation it is clear that the utility of solving equilibrium of the second stage for an arbitrary vector K is to account for a possible deviation

[2] From this point on k^x will denote total Cournot quantity for marginal costs equal to x.

of one of the firms. This is precisely why we assume that capacities are close enough to allow that marginal revenues in equilibrium of the second stage will always be equal in both markets any firm serves.

By Lemma 3.3 the solution in Equation 3.5 applies only when $k^* \leq k^c$. If it is not, then all the firms will build capacity equal to k^c. Since k^* is increasing in $c+d$ the upper bound on k^* will put a lower bound on d. For any d below a certain d_{min} the equilibrium capacity will be equal to k^c.

Proposition 3.1. *Equilibrium capacity k^* is defined by Equation 3.5 for all $d > d_{min}$ and is equal to k^c for all $d \leq d_{min}$ where d_{min} is the solution of $k^*(c + d_{min}) = k^c$.*

Note that Equation 3.5 is similar to the definition of k^{c+d}, a total Cournot quantity in a competition of two identical firms with marginal costs equal to $c+d$. The only difference between k^{c+d} and k^* is in the second term in the square brackets of Equation 3.5. For a standard Cournot competition this term is zero because the quantities are produced simultaneously. In our model, on the other hand, firms can commit to quantities produced because marginal revenues are larger than marginal costs in the second stage so any capacity built in the first stage will necessarily be used. Subsequently, both firms gain Stackelberg leader incentive to overproduce without having an actual leader advantage. As a result:

Proposition 3.2. *In a symmetric equilibrium all firms build a capacity between Cournot capacity for marginal costs equal to $c + d$ and c, that is $k^{c+d} < k^* \leq k^c$.*

Proof. Proof of the first inequality follows from the assumption on the inverse demand function and the observation that in Equation 3.5 term $\frac{\partial q^l_{i-1}(K)}{\partial k_i}$ is negative. The latter is true due to Assumption 3.1 which implies that optimal reaction to an increased supply by a competitor is to supply less quantity. If all firms start with a capacity equal to k and one of the firms increases capacity then it will supply more to both of its markets and thus in the equilibrium of the second stage both its direct competitors will supply less to the two markets. Having established that $\frac{\partial q^l_{i-1}(K)}{\partial k_i} < 0$ inequality $k^* > k^c$ follows from $p'(q) < 0$ and Assumption 3.1. Inequality $k^c \geq k^*$ is given in Lemma 3.3. $\square$

This proposition has important implication for the intensity of competition. It shows that if costs per unit are payed on two stages then competition is harsher than if they were payed simultaneously. To see that the presence of indirect competition is necessary for this result consider two firms who as in our model build capacity in the first stage, observe each other's capacity and then decide how much to produce and supply to a single market in the second stage. In such a model marginal revenue of the second stage will be equal to $c + d$. Since firms have only one market to supply their capacity to they will never build unused capacity and in the second stage will produce quantity equal to the capacity. As a result, in this model they will build capacities equal to k^{c+d}.

The driving force behind Proposition 3.2 is squarely Assumption 3.1. As such, it can trivially be generalized to any symmetric market structure. Borrowing terminology from the Graph Theory, circle is a particular example of a regular graph with the degree equal to two.[3] Increasing the degree of the graph does not alter the fact that in equilibrium all firms will build equal capacity. Assumption 3.1 ensures that increased capacity of any firm leads to smaller quantities supplied to its markets by competitors in the second stage so:

Corollary 3.1. *In any regular graph with identical markets situated between firms in a symmetric equilibrium all firms build a capacity between Cournot capacity for marginal costs equal to $c + d$ and c, that is $k^{c+d} < k^* \leq k^c$.*

Generally speaking, graphs have a limited ability to describe market structures. So far we have assumed that firms act as nodes while any connection between firms is a market that these two firms contest. This specification does not allow for more than two firms to be present in a single market. Even though we are not able to find a definition that encompasses all the features of the spatial market structures is seems clear that our main result will go through for any " symmetric" structure. By symmetric we mean that firms have identical production technologies and the structure is such that changing labels on firms does not change the market at hand.

Next we proceed to an illustration of the mechanics of the model by solving for the equilibrium capacities for a linear demand function.

[3]Regular graph of a degree m is a graph where all nodes (in our case firms) are connected to m other nodes (in our model contest m markets with m other firms).

a Identical Firms and Linear Demand

For a linear inverse demand function $p(q) = 1 - q$ the equality of marginal revenues in both markets yields

$$q_i^l = \frac{1}{2}k_i + \frac{1}{4}\left[q_{i-1}^l - q_{i+1}^r\right] \text{ and } q_i^r = \frac{1}{2}k_i + \frac{1}{4}\left[q_{i+1}^r - q_{i-1}^l\right]. \tag{3.6}$$

Taking all such conditions for $i = 1...n$ we get the following system of $2n$ equations in $2n$ unknowns:

$$\begin{aligned} q_1^l &= \tfrac{1}{2}k_1 + \tfrac{1}{4}\left[q_n^l - q_2^r\right] \\ q_1^r &= \tfrac{1}{2}k_1 + \tfrac{1}{4}\left[q_n^r - q_2^l\right] \\ &\vdots \\ q_n^l &= \tfrac{1}{2}k_n + \tfrac{1}{4}\left[q_{n-1}^l - q_1^r\right] \\ q_n^l &= \tfrac{1}{2}k_n + \tfrac{1}{4}\left[q_1^r - q_{n-1}^l\right]. \end{aligned}$$

We will solve this system of equations using recursive method. To do so in Equation 3.6 we substitute $k_{i+1} - q_{i+1}^l$ for q_{i+1}^r. We are left with:

$$q_i^l = \frac{1}{2}k_i + \frac{1}{4}\left[q_{i-1}^l - (k_{i+1} - q_{i+1}^l)\right]. \tag{3.7}$$

The last expression can be rewritten as:

$$\frac{1}{\sqrt{3}-2}\left[1 - \frac{L}{2+\sqrt{3}}\right]\left[1 - \frac{2-\sqrt{3}}{L}\right] q_i^l(K) = k_{i-1} - 2k_i. \tag{3.8}$$

where L is a lag operator defined in a usual way ($Lq_i = q_{i-1}$ and $\frac{1}{L}q_i = q_{i+1}$). We will use the fact that for any variable x_i we have $x_i = x_{i+mn}$ where m is any integer. After tedious algebra one arrives at:

$$\begin{aligned} q_i^l(K) = &\frac{k_i}{2} + \left[1 - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{\left(2+\sqrt{3}\right)^n - 1}\right](k_{i-1} - k_{i+1}) + \\ &+ \sum_{t=i+2}^{n} \frac{\left(2+\sqrt{3}\right)^{t-i} - \left(2+\sqrt{3}\right)^{n-t+i}}{2\left(\left(2+\sqrt{3}\right)^n - 1\right)} k_t + \\ &+ \sum_{t=1}^{i-2} \frac{\left(2+\sqrt{3}\right)^{n+t-i} - \left(2+\sqrt{3}\right)^{i-t}}{2\left(\left(2+\sqrt{3}\right)^n - 1\right)} k_t. \end{aligned} \tag{3.9}$$

Here we use Equation 3.5 to determine equilibrium capacity. The idea behind this equation is the following: if k^* is to be an equilibrium capacity then when one of the firms deviates from k^* to k' her profits should not increase. Since all other firms have equal capacity the deviating firm in the second stage will supply half of its capacity to each of the markets. Knowing this and the optimal reaction by other firms to k' from Equation 3.9 we can find optimal k' given k^* through equating marginal revenue from one more additional unit of capacity to $c + d$. Imposing back that in equilibrium $k' = k^*$ results in the solution:

$$k^* = \frac{\left(\sqrt{3}-1\right)\left(1-\left(2-\sqrt{3}\right)^n\right)}{1+\left(2-\sqrt{3}\right)^{1+n}}(1-(c+d)). \tag{3.10}$$

Note first that, as by Proposition 3.2, for any $n > 2$ this capacity is larger than Cournot capacity:

$$k^* = \frac{\left(\sqrt{3}-1\right)\left(1-\left(2-\sqrt{3}\right)^n\right)}{1+\left(2-\sqrt{3}\right)^{1+n}}(1-(c+d)) > \frac{2}{3}(1-(c+d)) = k^{c+d}.$$

It is noteworthy that even though there are always two firms per market equilibrium capacity is increasing in n. This is because the larger is the number of firms on the circle the larger is the number of markets where firms can push out their competitors in the second stage. This leads to even larger buildup of capacity in the first stage. When $n = 2$ the equilibrium capacity is precisely the Cournot one. When there are only two firms on the circle both firms have no choice but to supply their capacity into the two common markets they compete in and thus they build capacity sufficient to supply Cournot quantities to both markets.

From Proposition 3.1 the above solution only applies for $d > d_{min}$. The latter is defined by equation $k^*(c + d_{min}) = k^c$ so in the linear example the equilibrium capacity given in Equation 3.11 applies only for:

$$d > \frac{2-\sqrt{3}-\left(2-\sqrt{3}\right)^{n-1}}{3\left(1-\left(2-\sqrt{3}\right)^n\right)}(1-c) = d_{min}. \tag{3.11}$$

Note that when $d = d_{min}$, as expected, $k^* = \frac{2}{3}(1-c) = k^c$. For any $d \leq d_{min}$ the equilibrium capacity will be equal to $k^* = k^c$.

In the next section we will solve the model when firms have different costs of building capacity in order to study the relationship between these costs and profitability of all other firms.

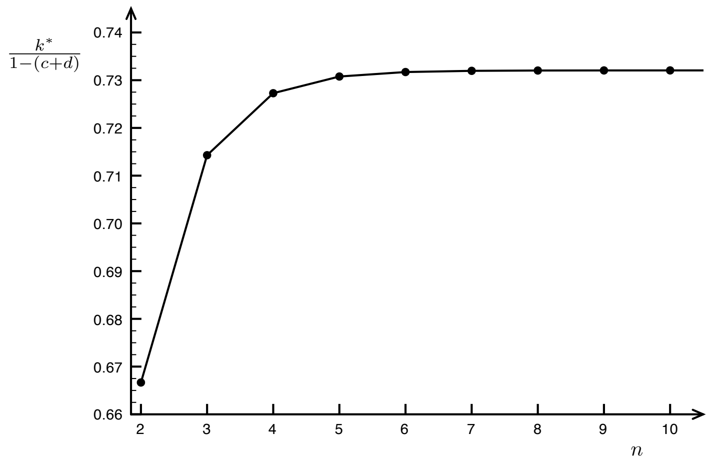

Figure 3.2: Relationship between the equilibrium capacity k^* and the number of firms n.

3.4 Asymmetric Firms and Linear Demand

One immediate problem is that is firms are too asymmetric then we cannot guarantee that production allocation of the second stage equates marginal revenues in both markets. Secondary problem arises because, unlike symmetric firm model, marginal revenues in the second stage can reach the lower threshold of c_i for some firms while not for some others. This is not a difficulty in a symmetric model because this threshold is reached by all the firms simultaneously.

For the above mentioned two reasons we will assume that marginal costs are not diverse enough and that d_i are large enough so that marginal revenues in the second stage are strictly larger than c_i for all firms in equilibrium. We will start solving the model assuming all this and later we put restriction on the primitives of the model to make sure that these condition are satisfied in equilibrium.

Recall that in the second stage quantity supplied by firm j to the market on her left q_j^l is given by Equation 3.9 and is a function of the vector of first period capacities K. What remains to be done is to find optimal capacity for each of the firms in the first stage, taking into account how these capacities affect quantities and prices of the second stage. Using

Equation 3.9 we solve the maximization problem in Equation 3.4. After tedious algebra one finds optimal capacity for firm 1 given capacities of all others:

$$\begin{aligned} k_1 &= \frac{\left(2+\sqrt{3}\right)^n - 1}{\left(1+\sqrt{3}\right)\left(1+\left(2+\sqrt{3}\right)^{n-1}\right)}(1-c_1) - \\ &\quad -\sum_{t=2}^{n} \frac{\left(2+\sqrt{3}\right)^{t-1} + \left(2+\sqrt{3}\right)^{n-t+1}}{2\left(1+\frac{1}{\sqrt{3}}\right)\left(1+\left(2+\sqrt{3}\right)^{n-1}\right)} k_t \end{aligned} \tag{3.12}$$

Analogous conditions apply to all other firms. Vector of equilibrium capacities K^* is the solution of the system of equation that consists of n such conditions.

For illustration purposes here we present the case of six firms with asymmetric capacity building costs and zero marginal costs of production. From the system of equations analogous to Equation 3.12 we derive:

$$k_1 = \tfrac{30}{41} - \tfrac{68760}{41041}c_1 + \tfrac{20415}{41041}(c_2+c_6) - \tfrac{1110}{41041}(c_3+c_5) + \tfrac{120}{41041}c_4 \tag{3.13}$$

After some algebra one finds that price in market 1 will be equal to:

$$p_1 = \tfrac{11}{41} + \tfrac{1439}{3731}c_1 + \tfrac{1439}{3731}c_2 - \tfrac{78}{3731}c_3 - \tfrac{78}{3731}c_6 + \tfrac{4}{3731}c_4 + \tfrac{4}{3731}c_5 \tag{3.14}$$

As one can see, the marginal costs of firms directly involved in competition in market 1 (firms 1 and 2) have positive coefficient in the expression for p_1, while the coefficient for marginal costs of their direct neighbors (firms 3 and 6) is negative though significantly smaller in absolute value. The sign changes for next two firms on the circle (firms 4 and 5) as the relevant coefficients are positive though extremely small. It seems puzzling that increase in marginal costs for 3 and 6 would actually decrease price charged in market 1. The explanation lies in the nature of Cournot competition and in the presence of capacity constraints. The expressions for capacity built by firm 3 (similar logic applies for firm 6) and quantities supplied to markets 2 and 3 are:

$$\begin{aligned} q_3^l &= \tfrac{15}{41} - \tfrac{34380}{41041}c_3 + \tfrac{1864}{41041}c_4 + \tfrac{18551}{41041}c_2 - \tfrac{1006}{41041}c_1 + \tfrac{60}{41041}c_6 - \tfrac{104}{41041}c_5 \\ q_3^r &= \tfrac{15}{41} - \tfrac{34380}{41041}c_3 + \tfrac{1864}{41041}c_2 + \tfrac{18551}{41041}c_4 - \tfrac{1006}{41041}c_5 + \tfrac{60}{41041}c_6 - \tfrac{104}{41041}c_1 \end{aligned}$$

An increase in the cost of capacity for firm 3 decreases overall capacity it builds and also both quantities she supplies. This leads to increased capacities built by her competitors (firms 2 and 4) which in turn decreases capacities built by their competitors (firms 1 and 5). These effects are sharply dieing out so the increase in capacity (and quantities supplied to each market) for firm 2 is larger than the reduction of capacity for firm 1 so joint effect is higher overall quantity supplied to market 1 which leads to reduction of p_1.

It is interesting to see how marginal costs affect profits. In the context of our model it is unclear whether firms are competitors or not if they are not present in the same market.

Definition 3.1. *A firm is a competitor to another firm if a reduction in her capacity acquisition costs leads to a reduction in equilibrium profits of the other firm.*

Using this definition we find that competitors of firms direct competitors are not in fact her own competitors if costs are similar across firms and they are bounded away from one. For example:

$$\begin{aligned}
&\frac{\partial \pi_1}{c_1}(\overline{C}) < 0 \\
&\frac{\partial \pi_1}{c_2}(\overline{C}) = \frac{\partial \pi_1}{c_6}(\overline{C}) > 0 \\
&\frac{\partial \pi_1}{c_3}(\overline{C}) = \frac{\partial \pi_1}{c_5}(\overline{C}) < 0,
\end{aligned}$$

where π_1 is equilibrium profit of firm 1 as a function of the marginal costs of all the firms and $\overline{C}$ is a vector of equal marginal costs.

Now we turn to the numerical solution of the model with asymmetric capacity costs. As we have already emphasized above the effect of capacity costs of firms situated more than 3 markets away from a given market on the price is insignificant. Moreover, after number of firms reaches 10 all relevant (within three markets reach) coefficients remain unchanged up to 5th digit. The following two equations give numeric results for our model with 500 firms:

$$\begin{aligned}
k_i &= 0.73205 - 1.6775c_i + 0.4985(c_{i+1} + c_{i-1}) - \\
&\quad -0.027185(c_{i+2} + c_{i-2}) + 0.0014825(c_{i+3} + c_{i-3}) + \ldots
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
p_i &= 0.26795 + 0.38599(c_i + c_{i+1}) - \\
&\quad -0.021049(c_{i-1} + c_{i+2}) + 0.0011478(c_{i-2} + c_{i+3}) + \ldots
\end{aligned} \tag{3.16}$$

From Equation 3.15 one can see that the direct competitors of firm's own competitors are acting as allies. Effect of their marginal costs on capacity built by the firm are similar (though smaller) to effect of firm's own marginal cost.

3.5 Conclusion

We presented a model of spatial Cournot competition with prior capacity building. We show that, even though there will be no unused capacity in the second stage, due to ability to push competitors into markets where one is not present capacity levels are larger than in a standard Cournot competition. As a result this paper is the first to have documented a pro-competitive effect of capacity building. This result can be generalized beyond circular location model to all symmetric market structures including regular graphs of any degree. Capacity constants and the spacial structure were shown to have paradoxical implications for indirect competition: in our setting indirect competitors of a firm can be perceived as its allies because their efficiency gains translate into the firm's capacity and profit expansion.

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APPENDIX

Proof of Lemma 1.3

Assume a retailer is charging price pair (p_i, p_j). Because of Lemma 1.1 expected profits from charging p_i for i are equal to

$$\pi_i(p_i) = [\theta + 2(1-\theta)(1-F_i(p_i))]p_i.$$

First, we argue that either the profits from selling i are constant or strictly increasing for all $p_i \in [\underline{p}_i, \overline{p}_i]$. The expected profits cannot be decreasing with p_i because if they were then all the retailers would lower prices, a strategy that increases profit earned on i and does not affect the profits earned from j by inducing consumers not to buy both goods.

We have established that $\pi_i(p_i)$ is either constant or strictly increasing at any $p_i \in [\underline{p}_i, \overline{p}_i]$ so if $\partial\pi_i(p_i)/\partial p_i \neq 0$ for some p_i then $\partial\pi_i(p_i)/\partial p_i > 0$. Take any equilibrium price pair $(\tilde{p}_i, \tilde{p}_j)$ such that $\partial\pi_i(\tilde{p}_i)/\partial p_i > 0$. Any retailer would find it optimal to increase $\tilde{p}_i$ so either $\tilde{p}_i \leq v_{ab} - \tilde{p}_j$ or $\tilde{p}_i \leq v_{ab} - v_j$ should bind.

We will consider each case in turn. Assume $\tilde{p}_i \leq v_{ab} - v_j$ binds. This means that $v_j > \tilde{p}_j$. then for any $p_j \in [\tilde{p}_j, v_j]$ it is optimal to charge $\tilde{p}_i$ and thus there is a point mass at $\tilde{p}_i$, a contradiction.

Now assume that $\tilde{p}_i \leq v_{ab} - \tilde{p}_j$ binds so $\tilde{p}_i + \tilde{p}_j = v_{ab}$. It should also be the case that $\partial\pi_j(\tilde{p}_j)/\partial p_j > 0$ for if the profits from selling j were constant at $\tilde{p}_j$ there would be a point mass at $\tilde{p}_i$. The conditions $\tilde{p}_i + \tilde{p}_j = v_{ab}$, $\partial\pi_i(\tilde{p}_i)/\partial p_i > 0$ and $\partial\pi_j(\tilde{p}_j)/\partial p_j > 0$ imply that $F_i(\tilde{p}_i) = 1 - F_j(v_{ab} - \tilde{p}_i)$. This is because any $p_i \leq \tilde{p}_i$ should be charged along with some $p_j \geq \tilde{p}_j = v_{ab} - \tilde{p}_i$.

Let us write the total profits in equilibrium at the price pair $(\tilde{p}_i, v_{ab} - \tilde{p}_i)$:

$$\pi = \theta v_{ab} + 2(1-\theta)(\tilde{p}_i(1-F_i(\tilde{p}_i)) + (v_{ab} - \tilde{p}_i)F_i(\tilde{p}_i)).$$

From this we find that in the neighborhood of $\tilde{p}_i$:

$$F_i(p_i) = \frac{\pi - 2p_i - \theta(v_{ab} - 2p_i)}{2(1-\theta)(v_{ab} - 2p_i)}.$$

First of all note that the cumulative distribution only exists if $\tilde{p}_i \neq \frac{1}{2}v_{ab}$.

The derivative of F_i at $\tilde{p}_i$ has to be strictly positive so:

$$\frac{\pi - v_{ab}}{(1-\theta)(v_{ab} - 2p_i)^2} > 0.$$

This implies that $\pi > v_{ab}$. Because the firms are identical their equilibrium profits are equal so the last inequality implies that joint profits of the two firms are larger than $2v_{ab}$. This is impossible because the most the two firms can earn (when they charge all prices equal to $\frac{1}{2}v_{ab}$) is $2v_{ab}$.

Asymmetric Equilibria of the Symmetric Nonsequential Search Model

Here we describe a subset of the asymmetric equilibria of nonsequential search model when all firms are identical (Burdett and Judd (1983)) that are of a particular interest to us. Model when $K = 1$ has a unique symmetric equilibrium where market price distribution as well as strategy of every firm is given by

$$F(p) = 1 - \frac{(v-p)\theta}{2(1-\theta)(p-c)}$$

, $p \in [\underline{p}, v]$ where $\underline{p} = \frac{v\theta + 2c(1-\theta)}{\theta + 2(1-\theta)}$. Because the number of firms is infinite each individual firm can use an arbitrary pricing strategy so long as the market price distribution is $F(p)$ (Burdett and Judd prove that market distribution has to be precisely $F(p)$). We will only characterize an asymmetric equilibrium where firms are gathered in K groups (or types). We will require that each group randomizes in a separate interval, all the firms in the group use identical pricing strategies and sequence of intervals goes from group K to group 1 descending from v to $\underline{p}$.

Assume there are K different strategies employed by firms. Let n_k denote proportion of firms that employ strategy k. As mentioned above group K will randomize in an interval adjacent to v. The lowest price this group will charge is denoted by $\underline{p}^K$ and has to be equal to:

$$\underline{p}^K = \frac{v\theta + 2cn_K(1-\theta)}{\theta + 2n_K(1-\theta)}.$$

Pricing strategy of the group K will be $F^K(p)$ defined by:

$$(v-c)\theta = (p-c)(\theta + 2(1-\theta)n_K(1 - F^K(p)))$$

Using the same recursive algorithm as in Section 4 we find that type $K-j$ firms will randomize in an interval $[\underline{p}^{K-j}, \underline{p}^{K-j+1}]$ where $\underline{p}^{K-j}$ is defined by:

$$(\underline{p}^{K-j} - c)\left[\theta + 2(1-\theta)\sum_{i=1}^{j+1} n_{K-i+1}\right] =$$
$$= (\underline{p}^{K-j+1} - c)\left[\theta + 2(1-\theta)\sum_{i=1}^{j} n_{K-i+1}\right]$$

and the pricing strategy they use is $F^{K-j}(p)$, derived from:

$$(p-c)\left[\theta + 2(1-\theta)\left[\sum_{i=1}^{j} n_{K-i+1} + n_{K-j}(1 - F^{K-j}(p))\right]\right] =$$
$$= (\underline{p}^{K-j+1} - c)\left[\theta + 2(1-\theta)\sum_{i=1}^{j} n_{K-i+1}\right].$$

It is important to remember that market price distribution will always be equal to $F(p)$ regardless of individual firm pricing strategies so we verify that $F(p) = \sum_{k=1}^{K} n_k F^k(p)$ and $\underline{p}^1 = \underline{p}$ so $p \in [\underline{p}, v]$. As we have argued in Section 4 when the number of types is K the symmetric equilibrium of the heterogeneous firm model will converge to the asymmetric equilibrium described here.