Essays on Monetary Fiscal and Trade Policy in Open Economies

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Abstract

English: In this thesis I study different kinds of monetary and fiscal policy issues by using fully microfounded general equilibrium models. The first chapter addresses the question of how monetary and fiscal policy should be conducted in a monetary union where there is a single central bank that sets the common interest rate while governments still retain full independence in fiscal policy decisions. The second chapter is devoted to study whether it is possible to rationalize, within a fully microfounded New Keynesian framework, the existence of a monetary union. The last chapter investigates to what extent the incentive of open economy policy makers to improve the terms of trade in their favour can be outweighed by the production relocation externality (the so called home market effect).

Resumen

Español: En esta tesis estudio varias cuestiones de política monetaria y fiscal usando modelos de equilibrio generales completamente micro-fundados. El primer capítulo trata la cuestión de cómo las políticas monetarias y fiscales se deben conducir en una unión monetaria donde hay un solo banco central que fija el tipo de interés común mientras que los gobiernos todavía conservan independencia completa en las decisiones de polticas fiscales. El segundo capítulo se dedica a estudiar si es posible racionalizar en un modelo keynesiano completamente micro-fundado la existencia de una unión monetaria. El ultimo capítulo investiga en qué medida el incentivo de las autoridades de política económica en una economía abierta de mejorar los términos de intercambio en su favor, se puede compensar por la externalidad de relocalización de la producción (home market effect).

Preface

Both the trade and the *New Open Economic Macroeconomic* literatures have emphasized how policy choices of open economy authorities may be biased by the incentive of improving the terms of trade in their favour. This incentive causes a negative externality on the welfare of the other country consumers and may give reason of why the policy prescriptions valid in a closed economy do not hold once the economy under consideration is open.

In this thesis, I first explore the implications of this terms of trade externality for two different monetary policy issues. In particular, in chapter one I address the question of how monetary (and fiscal) policy should be conducted in a monetary union where, as in the EMU, there is one single central bank that sets the common interest rate, but fiscal policies are chosen by autonomous authorities. In this kind of setting, governments are concerned exclusively about the welfare of the households living in their own country and have an incentive to affect their terms of trade in their favour. Indeed by improving their terms of trade they can reduce domestic production and induce an increase in leisure that, thanks to consumption risk sharing, more than compensates the corresponding fall in consumption. This incentive is the cause of beggar-thy-neighbor policies and leads to an inefficiently high size of governments and inefficiently low level of output. These distortions are taken into account by the common central bank that looks at the welfare of all the households living in the monetary union. In fact according to my findings, but in contrast with the previous literature, when governments are autonomous (i.e. not coordinated) the monetary authority does not stabilize the average union inflation as if it were in a closed economy. More specifically, in response to technology shocks it is not optimal to pursue stability: given the inefficiently high size of the public sector, that kind of policy does not allow to close the output gap. In response to mark up shocks instead the common central bank weighs more inflation than output stabilization because in this way the inefficiently low level of average output may increase.

In chapter two I investigate within a fully microfounded New Keynesian framework whether the incentive to improve the terms of trade can justify the existence of a monetary union. To this end I used a DSGE model where the world economy is constituted by small open countries that are split in two areas. Then I compare, in terms of welfare, two policy regimes. Under the first policy regime, in one area there is a common currency, while in the other area countries still retain their autonomous monetary policy. Under the second policy regime, there are two monetary unions. In the first regime where there is monetary independence, monetary policy can stabilize optimally the effects of country-specific shocks. However, in that case, single countries policy makers overlook the spillover effects produced on other countries welfare due to their attempt to improve their terms of trade. Moreover they do not realize how their joint action affects the world economy performance and the terms of trade across areas. This explains why the second regime may be preferable to the first one: being in a currency area entails not only a cost for not tailoring monetary policy to single country economic conditions, but even some gains associated with the improvement upon the conduct of national monetary policies. My results show that under markup shocks and plausible calibrations, there may be welfare gains from adopting a common currency.

Conversely the last chapter of the thesis[1] studies to what extent the incentive to improve the terms of trade can be offset by the production relocation effect (the so called home market effect). Indeed, the trade literature has emphasized how in the presence of trade costs and increasing returns open economy policy makers may have an incentive to impose a tariff on imports or to subsidize production in order to induce firms to relocate to the domestic economy. We consider optimal trade policy in a Krugman type model of trade. We conduct a general analysis allowing for two different instruments: tariffs on imports and a production subsidy. For each instrument we examine the optimal policy under cooperation and no-cooperation. Contrary to the existing literature on trade policy in the Krugman model, we find that optimal trade policy is not driven by a production relocation externality. Instead, we show that when

[1]This chapter is based on a joint paper with Alessia Campolmi and Harald Fadinger.

properly modeling general equilibrium effects of taxes/tariffs, policy makers' behaviour is determined by a standard terms of trade effect and the desire to eliminate distortions arising from monopolistic competition.

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1 Optimal monetary and fiscal policy in the EMU: Does fiscal policy coordination matter?

1.1 Introduction

The birth of the European Monetary Union (EMU) has sparkled interest in the question of how to conduct monetary and fiscal policy for a group of countries that share the same currency. There is a growing body of research that has tried to assess this issue within a fully micro-founded dynamic general equilibrium framework. However literature relies on the existence of a supra-national authority to which all monetary and fiscal policy decisions have been delegated. Yet, as matter of fact, in the EMU only the monetary policy is under the control of a common authority, the European Central Bank (ECB), whereas, even if bound by the Stability and Growth Pact (SGP), fiscal policies are still decided at national level. Consequently the following questions arise: How should monetary and fiscal policies be conducted in a monetary union where there is a common central bank but autonomous fiscal policies? Does this institutional arrangement lead to different normative prescriptions with respect to those highlighted by the previous literature?

In order to answer such questions, in this chapter I uses a generalized version of the DSGE model laid out by Galí and Monacelli (2009)[1] and compares two different policy regimes: the regime of fiscal policy coordination considered as a benchmark, already analyzed by Galí and Monacelli (2009) themselves, Beetsma and Jensen (2004) and (2005)[2] and the regime of fiscal policy no-coordination.

[1]See also Galí and Monacelli (2005). Differently from Galí and Monacelli (2009), not only final private goods but even public goods and intermediate inputs are traded, while the elasticity of substitution between home and foreign goods is not restricted to be equal to one. Moreover in the preference specification the intertemporal elasticities of substitution of public and private consumption are not necessarily equal. As it will be clarified below, the first two generalizations strengthen the incentive of uncoordinated fiscal policymakers to generate aggregate distortions. Conversely the last assumption is crucial to explain the results in the case of shocks to technology.

[2]Even Ferrero (2009) contributed to this debate. He analyzed the case of coordination in which, however, the exogenous government expenditure is financed through distortionary taxes and riskless

In our basic setup, the world is framed as a continuum of small open economies. Each country government chooses the optimal provision of a public consumption good and sets a time-invariant labour subsidy. The presence of lump sum taxes ensures compliance with SGP limits and rules out the additional problem of choosing how to finance optimally the public expenditure. Within this framework, under fiscal policy coordination, monetary and fiscal policies are chosen by a common policymaker in order to maximize the average union welfare. Conversely, under fiscal policy no-coordination[3], there is a multiplicity of policy authorities each of them taking as given other policymakers' decisions: governments that are concerned only about the welfare of their own country and the central bank of the Monetary Union that has the maximization of the average union welfare as objective.

According to my results, the no-coordination among fiscal authorities matters for the design of both optimal monetary and fiscal policies. The driving force of this finding stems from countries monopoly power on their terms of trade. Indeed, given the imperfect substitutability between bundles produced in different countries, uncoordinated policymakers have an incentive to try to influence the terms of trade in their favour. This incentive works both at the steady state and over the business cycle[4]. At the steady state, independent fiscal authorities act as a monopolist. They try to increase the demand of the home produced goods and to decrease their supply by over-expanding government expenditure and reducing the labour subsidy. In this way they seek to render domestic goods relatively more expensive in order to reduce their production. In fact given that there is consumption risk sharing across countries, the increase in leisure associated with a terms of trade improvement more than compensates the corresponding fall in consumption. In other words through a terms of trade improvement

bonds.

[3]There are some old contributions that consider the case of no-coordination (for instance Lambertini and Dixit (2003)). However, in general these papers do not assume fully-micro-founded welfare criteria. An exception in this respect is the work by Lombardo and Sutherland (2004). Yet they treat only marginally the case of a monetary union and reach results opposite to those of this paper by assuming an efficient steady state and considering only the case of optimal simple rules.

[4] ...as pointed out by the previous literature: see, among others, Corsetti and Pesenti (2001), Benigno and Benigno (2003) and Epifani and Gancia (2009).

governments seek to externalize labour effort to other countries consumers.

Over the business cycle instead they use government expenditure to restrain the terms of trade volatility and hence reduce the cost of the volatility of output or private consumption at other countries' expense.

This mechanism explains the differences in policy prescriptions under coordination and no-coordination. Under the benchmark case of fiscal policy coordination, Galí and Monacelli (2009), Beetsma and Jensen (2004) and (2005) have pointed out two main findings. Firstly, under the optimal policy mix, the common monetary policy should seek to stabilize the average union inflation following the same normative prescriptions valid in a closed economy. Therefore, under technology shocks, it should pursue the stability of the average union price level; under mark up shocks, it has to trade off between stabilizing the average inflation and the average output gap. Secondly, in a monetary union fiscal policy is a useful tool for macroeconomic stabilization of single country economies. Indeed, at single country level fiscal policy should be employed to stabilize the effects of idiosyncratic shocks given that, because of the adoption of the common currency, the central bank is able only to stabilize the aggregate economy. However, at the aggregate level fiscal policy should only ensure on average the efficient provision of the public goods.

Under fiscal policy no-coordination, the previous results no longer hold. With regard to monetary policy, the common central bank should cope with the aggregate distortions generated both at the steady state and over the business cycle by independent governments and not stabilize the average union economy as if it were a closed economy. Therefore in the presence of productivity shocks strict inflation targeting is in general not optimal. In fact, under flexible prices output volatility is inefficiently high for the at least two reasons. On the one hand national authorities have an incentive to manipulate the terms of trade to their own advantage even over the business cycle. On the other hand the steady state government expenditure share in output is inefficiently high and thus amplifies the effects of government expenditure shocks on output fluctuations[5]. Moreover, in the response to mark up shocks, the monetary authority should be much more aggressive in fighting inflation under no-coordination than under coordination. This finding is explained by the inefficiently low steady state level of output. Given that distortion, an increase in output volatility in response to mark up shocks has some beneficial effects because it makes consumers willing to work more on average driving the economy, by so doing, towards the efficient allocation.

With regard to fiscal policies independent governments do not ensure on average, given their incentives, the efficient provision of the public goods. And, in the case of mark up shocks they use government expenditure for stabilization purposes even if shocks are symmetric. Indeed, by taking as given what other policymakers are doing, they do not realize that the common central bank is already stabilizing the aggregate economy and they go on seeking to stabilize, on their own, the undesirable effects of mark up shocks.

The chapter is organized as follows. Section 2 describes the basic framework. Section 3 introduces the equilibrium conditions. Section 4 examines the case of full coordination. Section 5 the case of no-coordination. Section 6 concludes.

1.2 The model

The currency union consists of a continuum of small open economies[6]. In each country there are two sectors: a competitive sector that produces one final good by using both home and foreign country intermediate inputs; a monopolistic competitive sector that produces a continuum of intermediate differentiated goods by using as input labour which is assumed immobile across countries.

[5]...at least under the baseline calibration. Given the inefficiently high steady state government expenditure share in output, one percentage increase in the government expenditure expands more output under no-coordination than under coordination. Galí (1994) has already emphasized that the government's size may have an effect on output volatility.

[6]The general framework draws on Galí and Monacelli (2005) and Galí and Monacelli (2009).

1.2.1 Preferences

Preferences of a generic country representative household are defined over a private consumption bundle, C_t, a public consumption bundle, G_t and hours of labour $N_t(h)$[7]:

$$\sum_{t=0}^{\infty}\beta^t E_0\left[\frac{C_t^{1-\sigma}}{1-\sigma}+\chi\frac{G_t^{1-\gamma}}{1-\gamma}-\frac{N_t(h)^{\varphi+1}}{\varphi+1}\right] \quad 0<\beta<1 \tag{1.1}$$

where, as usual, β stands for the intertemporal preferences discount factor and χ is the weight attached to public consumption. Agents consume all the goods produced in the world economy. However preferences exhibit home bias. The private consumption index is, in fact, a CES aggregation of the following type:

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \quad \eta>0 \tag{1.2}$$

with $1-\alpha$ being the degree of home bias in the private consumption and η denoting the elasticity of substitution between $C_{H,t}$, and $C_{F,t}$. $C_{H,t}$ represents the home household's consumption of the single home final good while $C_{F,t}$ is a CES aggregation of the goods produced in foreign countries namely:

$$C_{F,t} \equiv \left[\int_0^1 C_{j,t}^{\frac{\eta-1}{\eta}}\,dj\right]^{\frac{\eta}{\eta-1}} \tag{1.3}$$

η then represents even the elasticity of substitution between different foreign goods.

The public bundle is defined similarly to the private bundle, that is:

$$G_t \equiv \left[(1-\nu)^{\frac{1}{\eta}}G_{H,t}^{\frac{\eta-1}{\eta}}+\nu^{\frac{1}{\eta}}G_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \quad \eta>0 \tag{1.4}$$

with

$$G_{F,t} \equiv \left[\int_0^1 G_{j,t}^{\frac{\eta-1}{\eta}}\,dj\right]^{\frac{\eta}{\eta-1}} \tag{1.5}$$

where $1-\nu$ indicates the degree of home bias in the public consumption which, in general, is allowed to be different from $1-\alpha$[8].

[7] In this and in the following subsections we abstract from indexing the small open economy of reference.

[8] In fact Brülhart and Trionfetti (2004) point out that the home bias of public goods is higher than home bias of private goods.

Public and private consumption index definitions (1.2), (1.4), (1.3) and (1.5) allow to determine consistent definitions of price indexes[9]. In particular, $P_{C,t}$ and $P_{G,t}$, the private and the public consumers' price indexes[10] are given by:

$$P_{C,t} \equiv \left[(1-\alpha)P_t^{1-\eta} + \alpha P_t^{*1-\eta}\right]^{\frac{1}{1-\eta}} \tag{1.6}$$

$$P_{G,t} \equiv \left[(1-\nu)P_t^{1-\eta} + \nu P_t^{*1-\eta}\right]^{\frac{1}{1-\eta}} \tag{1.7}$$

with P_t^* being specified as:

$$P_t^* \equiv \left[\int_0^1 P_t^{j\,1-\eta} dj\right]^{\frac{1}{1-\eta}} \tag{1.8}$$

Thus P_t and P_t^j are producers' price indexes [11]. There are no trading frictions being the law of one price assumed to hold in all single good markets. However, given the home biased preferences, the purchasing power parity does not hold for indexes $P_{C,t}$ and $P_{G,t}$.

1.2.2 Consumption demand, portfolio choices and labour supply

The consumption and price index definitions allow to solve the consumer problem in three stages. In the first two stages, agents decide how much real net income to allocate to buy goods produced at home and abroad. According to the set of optimal conditions, it is possible to determine agent demands for $C_{H,t}$, $C_{F,t}$ and $C_{j,t}$, as:

$$C_{H,t} = (1-\alpha)\left(\frac{P_t}{P_{C,t}}\right)^{-\eta} C_t \qquad C_{F,t} = \alpha\left(\frac{P_t^*}{P_{C,t}}\right)^{-\eta} C_t \qquad C_{j,t} = \left(\frac{P_t^j}{P_t^*}\right)^{-\eta} C_{F,t} \tag{1.9}$$

for all j. The third stage coincides with the standard consumer problem. Agents are monopolistic competitive labour suppliers. Thus they maximize (1.1) with respect to C_t, D_{t+1} and $N_t(h)$ subject to the following sequence of constraints:

$$E_t\{Q_{t,t+1}D_{t+1}\} = D_t + W_t(h)N_t(h) - P_{C,t}C_t + T_t \tag{1.10}$$

[9] Namely price and consumption indexes are such that at the optimum expenditures for total consumption of both private and public goods, $P_tC_{H,t} + \int_0^1 P_t^j C_{j,t}dj$ and $P_tG_{H,t} + \int_0^1 P_t^j G_{j,t}dj$ are equal respectively to $P_{C,t}C_t$ and $P_{G,t}G_t$.

[10] In what follows, CPI stands for consumers' price index.

[11] Again in what follows, PPI stands for producers' price index.

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\upsilon_t} N_t \tag{1.11}$$

where:

$$W_t \equiv \left[\int_0^1 W_t(h)^{1-\upsilon_t} dh\right]^{\frac{1}{1-\upsilon_t}} \tag{1.12}$$

Constraint (1.10) is the budget constraints which states that nominal saving, net of lump sum transfers, has to equalize the nominal value of a state contingent portfolio. In fact $W_t(h)$ stands for the per hour nominal wage, $Q_{t,t+1}$ denotes what is usually called the stochastic discount factor and D_{t+1} is the payoff of one maturity portfolio that includes firm shares.

Constraint (1.11) is a consequence of a CES aggregation of labour inputs which will be specified in the next sub-section and implicitly assumes that the elasticity of demand of labour, υ_t, is time-varying but equal across agents as in Clarida, Galí and Gertler (2002). Finally (1.12) is simply the aggregate wage index. Domestic and international markets are assumed to be complete.

By the optimality conditions of the household problem:

$$(1 + \mu_t) N_t(h)^{\varphi} C_t^{\sigma} = \frac{W_t}{P_{C,t}} \tag{1.13}$$

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_{C,t}}{P_{C,t+1}}\right) = Q_{t,t+1} \tag{1.14}$$

which hold in all states of nature and at all periods and where $\mu_t \equiv \frac{1}{\upsilon_t - 1}$.

According to (1.13), workers set the real wage as mark up over the marginal rate of substitution between consumption and leisure, while the value of the intertemporal marginal rate of substitution of consumption should equalize the stochastic discount factor. Notice that since wages are perfectly flexible $N_t(h){=}N_t$ and $W_t(h){=}W_t$ for all h and t.

1.2.3 Final good aggregate demand

In each country the demand for the final good is the sum of four components: the demands of domestic and foreign households and governments namely:

$$Y_t = C_{H,t} + \int_0^1 C^j_{H,t} dj + G_{H,t} + \int_0^1 G^j_{H,t} dj \tag{1.15}$$

Condition (1.15) can be rewritten as:

$$Y_t = (1-\alpha)\left(\frac{P_t}{P_{C,t}}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_t}{P^j_{C,t}}\right)^{-\eta} C^j_t dj + (1-\nu)\left(\frac{P_t}{P_{G,t}}\right)^{-\eta} G_t + \nu \int_0^1 \left(\frac{P_t}{P^j_{G,t}}\right)^{-\eta} G^j_t dj \tag{1.16}$$

which follows from equation (1.3)[12] and the fact that:

$$G_{H,t} = (1-\nu)\left(\frac{P_t}{P_{G,t}}\right)^{-\eta} G_t \qquad G_{F,t} = \nu \left(\frac{P^*_t}{P_{G,t}}\right)^{-\eta} G_t \qquad G_{j,t} = \left(\frac{P^j_t}{P^*_t}\right)^{-\eta} G_{F,t} \tag{1.17}$$

for all j. According to (1.17) independently of the aggregate level of G_t, governments choose good demands by minimizing the total expenditure $P_t G_{H,t} + \int_0^1 P^j_t G_{j,t} dj$.

1.2.4 Firms and technology in the final good sector

Each final good is produced by using both home and foreign inputs according to the following CES technology:

$$Y_t = \left[(1-\psi)^{\frac{1}{\eta}} \left(Y^I_{H,t}\right)^{\frac{\eta-1}{\eta}} + \psi^{\frac{1}{\eta}} \left(Y^I_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \qquad \eta > 0 \tag{1.18}$$

where $1-\psi$ is the degree of home bias in intermediate goods. Y^I_H and Y^I_F are defined as:

$$Y^I_{H,t} \equiv \left[\int_0^1 \left(y^I_{H,t}(k)\right)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad Y^I_{F,t} \equiv \left[\int_0^1 \left(Y^I_{j,t}\right)^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \tag{1.19}$$

with $Y^I_{j,t} \equiv \left[\int_0^1 \left(y^I_{j,t}(k)\right)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}}$ for all j and $y^I_{H,t}(k)$ and $y^I_{j,t}(k)$ being the demands for the k type of intermediate good produced in the home country and in country j respectively.

[12]... with the symmetric equations for foreign countries.

The final sector is perfectly competitive. Therefore firms maximize profits taking P_t, the price of the final good, as given. The optimality conditions of this problem lead to the following single and aggregate input demands:

$$y^I_{H,t}(k) = \left(\frac{p_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon} Y^I_{H,t} \qquad y^I_{j,t}(k) = \left(\frac{p_{j,t}(k)}{P_{j,t}}\right)^{-\varepsilon} Y^I_{j,t} \tag{1.20}$$

$$Y^I_{H,t} = (1-\psi)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} Y_t \qquad Y^I_{F,t} = \psi\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} Y_t \qquad Y^I_{j,t} = \left(\frac{P_{j,t}}{P_{F,t}}\right)^{-\eta} Y^I_{F,t} \tag{1.21}$$

which allow to determine consistently the price indexes for final and intermediate goods as:

$$P_t = \left[(1-\psi)\left(P_{H,t}\right)^{1-\eta} + \psi\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{1.22}$$

$$P_{H,t} = \left[\int_0^1 p_{H,t}(k)^{1-\varepsilon}dk\right]^{\frac{1}{1-\varepsilon}} \qquad P_{F,t} = \left[\int_0^1 \left(P_{j,t}\right)^{1-\eta}dj\right]^{\frac{1}{1-\eta}} \qquad P_{j,t} = \left[\int_0^1 p_{j,t}(k)^{1-\varepsilon}dk\right]^{\frac{1}{1-\varepsilon}} \tag{1.23}$$

where $p_{j,t}(k)$ is the price of intermediate input k produced in country j.

1.2.5 Intermediate good aggregate demand

The demand for home intermediate goods is generated by the demands of both home and foreign final good producers, namely:

$$y_{H,t}(k) \equiv y^I_{H,t}(k) + \int_0^1 y^{I,j}_{H,t}(k)dj \tag{1.24}$$

Condition (1.24) can be rewritten as:

$$y_{H,t}(k) = \left(\frac{p_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon}\left[(1-\psi)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} Y_t + \psi\int_0^1 \left(\frac{P_{H,t}}{P^j_t}\right)^{-\eta} Y^j_t dj\right] \tag{1.25}$$

which follows from equations (1.20) and (1.21)[13]. Given (1.25) it is possible to recover the aggregate demand $Y_{H,t} \equiv \left(\int_0^1 (y_{H,t}(k))^{\frac{\varepsilon-1}{\varepsilon}}dk\right)^{\frac{\varepsilon}{\varepsilon-1}}$. In fact by properly integrating (1.25) we obtain:

$$Y_{H,t} = \left[(1-\psi)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} Y_t + \psi\int_0^1 \left(\frac{P_{H,t}}{P^j_t}\right)^{-\eta} Y^j_t dj\right] \tag{1.26}$$

[13]... with the symmetric equations for foreign countries.

1.2.6 Firm technology and price setting in the intermediate good sector

In the intermediate sector each firm produces a single differentiated good with a constant return to scale technology of the type:

$$y_{H,t}(k) = A_t N_t(k) \tag{1.27}$$

with $N_t(k) = \left[\int_0^1 N_t(h)^{\frac{\upsilon_t - 1}{\upsilon_t}} dh\right]^{\frac{\upsilon_t}{\upsilon_t - 1}}$ and being the labour input and A_t the specific country technology shock. Given (1.27) and the fact that $N_t = N_t(h)$ for all h, the aggregate relationship between output and labour can be read as:

$$N_t = \frac{Y_{H,t}}{A_t} Z_t \tag{1.28}$$

where $Z_t \equiv \int_0^1 \frac{y_{H,t}(k)}{Y_{H,t}} dk$, and $N_t \equiv \int_0^1 N_t(k) dk$. Given (1.24) and (1.26) then $Z_t \equiv \int_0^1 \left(\frac{p_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon} dk$; thus Z_t can be interpreted as an index of the relative price dispersion across firms. We assume that good prices adjust according to a staggered mechanism *à la* Calvo. Therefore in each period a given firm can reoptimize its price only with probability $1 - \theta$. As a result the fraction of firms that set a new price is fixed and the aggregate producer price index of the intermediate goods evolves accordingly to:

$$P_{H,t}^{(1-\varepsilon)} = \theta P_{H,t-1}^{(1-\varepsilon)} + (1-\theta) \tilde{p}_{H,t}(k)^{(1-\varepsilon)} \tag{1.29}$$

with $\tilde{p}_{H,t}(k)$ being the optimal price. Firms maximize the discounted expected sum of the future profits that would be collected if the optimal price could not be changed namely:

$$\sum_{s=0}^{\infty} (\theta)^s E_t \left\{ Q_{t,t+s} y_{H,t+s}(k) \left[\tilde{p}_{H,t}(k) - MC_{t+s}^n\right] \right\} \tag{1.30}$$

where $y_{H,t}(k)$ is given by (1.25) and $MC_t^n = \frac{(1-\tau)W_t}{A_t}$ is the nominal marginal cost with τ indicating a labour subsidy distributed to firms by the fiscal authority which is not supposed to vary over the business cycle. Taking into account (1.14) and that $MC_t \equiv \frac{MC_t^n}{P_{H,t}}$, the optimality condition of the firm problem can be written as:

$$\sum_{s=0}^{\infty} (\beta\theta)^s E_t \left\{ C_{t+s}^{-\sigma} \left(\frac{\tilde{p}_{H,t}(k)}{P_{H,t+s}}\right)^{-\varepsilon} Y_{H,t+s} \frac{P_{H,t}}{P_{C,t+s}} \left[\frac{\tilde{p}_{H,t}(k)}{P_{H,t}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_{H,t+s}}{P_{H,t}} MC_{t+s}\right] \right\} = 0 \tag{1.31}$$

Condition (1.31) states implicitly that firms reset their prices as a mark up over a weighted average of the current and expected marginal costs, where the weight of the expected marginal cost at some date $t+s$ depends on the probability that the price is still effective at that date.

1.3 Equilibrium

The purpose of this section is twofold: on the one hand to recover the full set of conditions necessary and sufficient to determine the equilibrium of the monetary union; on the other hand to rewrite the single country equilibrium conditions in terms only of single country and average union variables. Indeed in this way, it is possible to simplify the fiscal policy problem under no-coordination. Being infinitesimally small, single country behaviour does not affect the average union performance. As a consequence, under no-coordination, the fiscal policy problem can be formulated just considering single country (and not the full set of the monetary union) equilibrium conditions.

1.3.1 International risk sharing

Under complete markets[14], condition (1.14) and the corresponding conditions for other countries imply:

$$\frac{P^j_{C,t}}{P_{C,t}} = \left(\frac{C^j_t}{C_t}\right)^{-\sigma} \tag{1.32}$$

for all t and j.

Notice that $P^*_t = \left[\int_0^1 (P^j_{C,t})^{1-\eta} dj\right]^{\frac{1}{1-\eta}}$ and let:

$$C^*_t \equiv \left[\int_0^1 (C^j_t)^{\sigma(\eta-1)} dj\right]^{\frac{1}{\sigma(\eta-1)}} \tag{1.33}$$

Hence by properly integrating (1.32) we obtain:

$$\frac{P^*_t}{P_{C,t}} = \left(\frac{C^*_t}{C_t}\right)^{-\sigma} \tag{1.34}$$

[14]...and the assumption that the state contingent wealth at time zero is such that the lifetime discounted budget constraints are identical across agents.

According to equation (1.32) and its aggregate version (1.34), when financial markets are complete, the marginal rate of substitution between home and other country consumption (or the average union consumption) has to be equal to the corresponding relative price. Thus when there is increase in the home relative to foreign CPI, domestic households decrease consumption relative to foreigners. Actually the terms of trade improvement in the home country[15] - being associated with the relative increase in the CPI - induces private agents to reallocate the consumption between home and foreign goods. Then, because of the home bias, the home country consumers would decrease the *total* private consumption more than foreigners [16].

By combining (1.34) with (1.6), (1.7) and (1.22) and considering that $P_t^* = P_{F,t}$, it follows that:

$$\frac{P_t}{P_{C,t}} = \left[\frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\left(\frac{C_t^*}{C_t}\right)^{-\sigma(1-\eta)}\right]^{\frac{1}{1-\eta}} \tag{1.35}$$

$$\frac{P_{G,t}}{P_{C,t}} = \left[(1-\nu)\frac{P_t}{P_{C,t}}^{1-\eta} + \nu\left(\frac{C_t^*}{C_t}\right)^{-\sigma(1-\eta)}\right]^{\frac{1}{1-\eta}} \tag{1.36}$$

$$\frac{P_{H,t}}{P_t} = \left[\frac{1}{1-\psi} - \frac{\psi}{1-\psi}\left(\frac{C_t^*}{C_t}\right)^{-\sigma(1-\eta)}\left(\frac{P_t}{P_{C,t}}\right)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{1.37}$$

which say that all the single country relative prices $P_t/P_{C,t}$, $P_{G,t}/P_{C,t}$ and $P_{H,t}/P_t$ and the terms of trade P_t^*/P_t and $P_t^*/P_{H,t}$ [17] are function exclusively of the difference between single country and average union private consumption.

[15]Namely the prices of the foreign goods in terms of home goods, that is P_t^*/P_t and $P_t^*/P_{H,t}$.

[16]In fact because of the home bias, even if there are complete markets, private agents consumption is not equal across countries.

[17]In fact by (1.6) and (1.22), $P_{C,t}/P_t = \left[(1-\alpha) + \alpha\,(P_t^*/P_t)^{(1-\eta)}\right]^{\frac{1}{1-\eta}}$ and $P_t/P_{H,t} = \left[(1-\psi) + \psi\,(P_t^*/P_{H,t})^{(1-\eta)}\right]^{\frac{1}{1-\eta}}$. P_t^*/P_t and $P_t^*/P_{H,t}$ are the so called *effective* terms of trade. In what follows, unless specified differently, we will refer only to the effective terms of trade.

In addition given (1.14) and (1.34):

$$\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\Pi^{*-1}_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\Pi^{-1}_{C,t+1} \tag{1.38}$$

$$= \left(\frac{P_t}{P_{C,t}}\right)\left(\frac{P_{Ht}}{P_{C,t}}\right)\Pi^{-1}_{H,t+1}\left(\frac{P_t}{P_{C,t}}\right)^{-1}\left(\frac{P_{H,t}}{P_{C,t}}\right)^{-1} \tag{1.39}$$

with $\Pi^*_t \equiv P^*_t/P^*_{t-1}$ and $\Pi_{C,t} \equiv P_{C,t}/P_{C,t-1}$.

Thus in equilibrium the value of intertemporal marginal rate of substitution of private consumption should be equal across countries. This last condition combined with (1.35) and (1.37) can be log-linearized as:

$$\pi_{H,t} - \pi^*_t = -\omega_4\left(\Delta\hat{c}_t - \Delta\hat{c}^*_t\right) \tag{1.40}$$

where $\omega_4 \equiv \frac{\sigma}{(1-\alpha)(1-\psi)}$[18]. (1.38) and (1.40) relate consumption variations differential from the union average to the corresponding *domestic* inflation differential. Moreover under complete markets, from conditions (1.14) and (1.34) it follows:

$$\frac{1}{1+i^*_t} = E_t\{Q_{t,t+1}\} = \beta E_t\left\{\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\left(\Pi^{*-1}_{t+1}\right)\right\} \tag{1.41}$$

When markets are complete, the price of a riskless portfolio should be equal to the price of the one-period riskless bond, being i^*_t its gross yield.

The log-linear approximation of (1.41) leads to:

$$\hat{c}^*_t = E_t\{\hat{c}^*_{t+1}\} - \frac{1}{\sigma}(i^*_t - E_t\{\pi^*_{t+1}\} - \varrho) \tag{1.42}$$

where $\varrho \equiv -log\beta$. Condition (1.42) is the so called IS curve that relates the average union intertemporal marginal rate of substitution of private consumption with the real interest rate.

By(1.38), (1.41) can be read as:

$$\frac{1}{1+i^*_t} = \beta E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\Pi^{-1}_{C,t}\right)\right\} \tag{1.43}$$

[18] Henceforth the following conventions are used: $\hat{x}_t$ stands for the log deviation of X_t from the symmetric zero inflation steady state while $\Delta\hat{x}_t \equiv \hat{x}_t - \hat{x}_{t-1}$ and $\hat{x}^*_t \equiv \int_0^1 \hat{x}^i_t di$.

In other words by (1.38) condition (1.41) is satisfied even at the single country level[19]. Notice that outside a monetary union, where exchange rates are floating, (1.38) and (1.41) do not necessarily hold because the fluctuations of nominal exchange rates themselves assure the equality between the values of intertemporal marginal rate of substitution of private consumption across countries and give reason of differences in the nominal interest rates. For this reason we can interpret (1.38) as a constraint imposed by the adoption of a common currency. In fact according to this condition in response to asymmetric shocks the terms of trade cannot adjust instantaneously because of the sluggish prices adjustment and the fixed exchange rates.

1.3.2 Good market clearing conditions

To rewrite the resource constraints as function of only aggregate variables, note that $P_t/P^j_{C,t} = (P_t/P_{C,t})(P_{C,t}/P^j_{C,t})$. Similarly $P_t/P^j_{G,t} = (P_t/P_{C,t})\,(P_{C,t}/P^j_{C,t})(P^j_{C,t}/P^j_{G,t})$ and $P_{H,t}P^j_t = (P_{H,t}/P_t)(P_t/P_{C,t})(P_{C,t}/P^j_{C,t})(P^j_{C,t}/P_{j,t})$. Then by substituting (1.32) in (1.16) and (1.26) we can express the good market clearing conditions as:

$$Y_t = \left(\frac{P_t}{P_{C,t}}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha C_t^{\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}}{P_{G,t}}\right)^{-\eta} G_t + \nu C_t^{\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta}\right] \tag{1.44}$$

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\psi)Y_t + \psi\left(\frac{P_t}{P_{C,t}}\right)^{-\eta} C_t^{\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta}\right] \tag{1.45}$$

where:

$$\Upsilon_{C,t} \equiv \left[\int_0^1 {C_t^j}^{1-\sigma\eta} dj\right]^{\frac{1}{1-\sigma\eta}}$$

$$\Upsilon_{G,t} \equiv \left[\int_0^1 {C_t^j}^{-\sigma\eta}\left(\frac{P^j_{C,t}}{P^j_{G,t}}\right)^{-\eta} G^j_t dj\right]^{\frac{1}{1-\sigma\eta}} \qquad \Upsilon_{Y,t} \equiv \left[\int_0^1 {C_t^j}^{-\sigma\eta}\left(\frac{P^j_{C,t}}{P_{j,t}}\right)^{-\eta} Y^j_t dj\right]^{\frac{1}{1-\sigma\eta}} \tag{1.46}$$

[19] However (1.38) is not only a sufficient but also a necessary condition for (1.43) to be satisfied given (1.41).

Rewriting the good market clearing conditions in this way leads to the following consideration: any improvement in the domestic terms of trade makes private agents willing to switch expenditure from home to foreign goods[20]; however being isolated, this same improvement does not increase the demands for final and intermediate foreign goods because countries are assumed to be small.

The log-linear approximations of resource constraints (1.44) and (1.45) and of conditions (1.35), (1.36) and (1.37) allow to retrieve the following condition:

$$\hat{y}_{H,t} + \frac{\psi}{1-\psi}(\hat{y}_{H,t} - \hat{y}_t^*) = \rho\hat{c}_t + \rho(\delta-1)(\hat{c}_t - \hat{c}_t^*) + (1-\rho)\hat{g}_t - (1-\rho)\nu(\hat{g}_t - \hat{g}_t^*) \quad (1.47)$$

where

$$\delta \equiv \delta_1 + \delta_2\frac{(1-\rho)}{\rho} + \delta_3\frac{1}{\rho}$$

$$\delta_1 \equiv (1-\alpha) + \xi\alpha(2-\alpha) \qquad \delta_2 \equiv \xi\nu(2-\nu) \qquad \delta_3 \equiv \frac{\xi\psi(2-\psi)}{(1-\psi)} \quad (1.48)$$

with $\rho \equiv \frac{C}{Y}$ and $\xi \equiv \eta\sigma/(1-\alpha)$.

1.3.3 The Phillips curve

Given condition (1.31) the optimal price is determined as:

$$\frac{\tilde{p}_{H,t}(k)}{P_{H,t}} = \frac{K_t}{F_t} \quad (1.49)$$

with:

$$K_t \equiv \sum_{s=0}^{\infty} (\beta\theta)^s E_t \left[C_{t+s}^{-\sigma} Y_{H,t+s} \left(\frac{P_{H,t+s}}{P_{H,t}}\right)^{\varepsilon} \frac{P_{H,t+s}}{P_{C,t+s}} \frac{\varepsilon}{\varepsilon-1} MC_{t+s} \right] \quad (1.50)$$

$$F_t \equiv \sum_{s=0}^{\infty} (\beta\theta)^s E_t \left[C_{t+s}^{-\sigma} Y_{H,t+s} \left(\frac{P_{H,t+s}}{P_{H,t}}\right)^{\varepsilon-1} \frac{P_{H,t+s}}{P_{C,t+s}} \right] \quad (1.51)$$

which can be read as:

$$K_t = C_t^{-\sigma} Y_{H,t} \frac{P_{H,t}}{P_{C,t}} \frac{\varepsilon}{\varepsilon-1} MC_t + \beta\theta E_t \left\{ \Pi_{H,t+s}^{\varepsilon} K_{t+1} \right\} \quad (1.52)$$

$$F_t = C_t^{-\sigma} Y_{H,t} \frac{P_{H,t}}{P_{C,t}} + \beta\theta E_t \left\{ \Pi_{H,t+1}^{\varepsilon-1} F_{t+1} \right\} \quad (1.53)$$

[20] what in the literature is called the *switching expenditure* effect.

where:

$$MC_t = \frac{W_t}{P_{C,t}} \left(\frac{P_t}{P_{C,t}} \right)^{-1} \frac{1}{A_t} \tag{1.54}$$

Following Benigno and Woodford (2005), from (1.49) and (1.29) we can retrieve the next conditions:

$$\frac{1-\theta \Pi_{H,t}^{\varepsilon-1}}{1-\theta} = \left(\frac{F_t}{K_t} \right)^{\varepsilon-1} \tag{1.55}$$

$$Z_t = \theta Z_{t-1} \Pi_{H,t}^{\varepsilon} + (1-\theta) \left(\frac{1-\theta \Pi_{H,t}^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.56}$$

which determines the law of motion of firms price dispersion. From the log linear approximation of (1.13) (1.50) (1.51) and (1.56):

$$\pi_{H,t} = \lambda \widehat{mc}_t + \beta E_t \{\pi_{H,t+1}\} \tag{1.57}$$

where:

$$\begin{aligned} \widehat{mc}_t &= (\widehat{w}_t - \hat{p}_{c,t}) - (\widehat{p}_t - \hat{p}_{c,t}) - (\widehat{p}_{H,t} - \hat{p}_t) - \hat{a}_t \\ &= \varphi \hat{y}_{H,t} + \sigma \hat{c}_t + \omega_4((1-\psi)\alpha + \psi)(\hat{c}_t - \hat{c}_t^*) - (1+\varphi)\hat{a}_t + \hat{\mu}_t \end{aligned} \tag{1.58}$$

Condition (1.57) is the New Keynesian Phillips Curve direct consequence of the Calvo mechanism. As usual, current *domestic* inflation depends on the expectation on future *domestic* inflation and the current real marginal cost of producing intermediate goods. Being the economy open, in equilibrium that cost is determined by the real wage (which is equal to the marginal rate of substitution between consumption and leisure), the labour productivity and the relative prices of intermediate and final goods. These prices are determined as made clear by (1.35) and (1.37) by the differences of private consumption from the average.

The rational expectation stochastic equilibrium of the monetary union is then defined as the sequence of $\{C_t^i, Y_t^i, Y_{H,t}^i, \Pi_{H,t}^i, Z_t^i, F_t^i, K_t^i, \Pi_t^* \}_{t=0}^{\infty}$ for all i which, given $\{G_t^i$, $i_t^* \}_{t=0}^{\infty}$ for all i, τ and the initial condition Z_{-1}, satisfies (1.39), (1.41), (1.44), (1.45), (1.52), (1.53), (1.55) and (1.56) for all i where $W_t^i/P_{C,t}^i$ $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$ and MC_t^i are determined according to (1.13), (1.35), (1.36), (1.37) and (1.54).

What it is still missing is to determine the optimal monetary and fiscal policies. This is the purpose of the next paragraphs.

1.4 The optimal policies

As mentioned in the introduction, the optimal monetary and fiscal policy mix is analysed under two different policy regimes: coordination and no-coordination. Under coordination there is a common authority responsible for both monetary and fiscal policies which has the maximization of the average union welfare as objective. Under no-coordination there is a plurality of independent policy makers each of them taking other policy authorities' decisions as given. The central bank on the one hand seeks to minimize the average losses of union households; governments on the other hand are instead concerned about the average losses of their country households. The solutions to the optimal policy problems under both regimes are derived by using the linear quadratic approach proposed by Benigno and Woodford (2005). This method requires to assume that policies are optimal from *timeless* perspective[21] and can be implemented as follows. First the zero-inflation deterministic steady state is retrieved; then a purely quadratic approximation to the single country and monetary union welfare around the deterministic steady state is obtained. Since in the case of no-coordination the deterministic steady state is distorted, these approximations are derived by using the second order approximations to the structural equations. Finally, given the purely quadratic approximations of policy makers' objectives, the optimal policies[22] are recovered by using as constraints the equilibrium conditions approximated up to the first order.

[21]See also Benigno and Benigno (2006), Benigno and De Paoli (2009) and De Paoli (2009).

[22]In the case of no-coordination, the Nash equilibrium policies are determined by the solutions to both the monetary and fiscal policy problems.

1.4.1 The case of coordination

Under coordination the optimal policy problem of the common authority can be formulated as the maximization of:

$$\sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \left[\frac{C_t^{i\,1-\sigma}}{1-\sigma} + \chi\frac{G_t^{i\,1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}^i}{A_t^i}\right)^{\varphi+1}\right] di \tag{1.59}$$

with respect to C_t^i, G_t^i, Y_t^i, $Y_{H,t}^i$, $\Pi_{H,t}^i$, Z_t^i, F_t^i and K_t^i for all i, subject to the equilibrium conditions (1.39), (1.44), (1.45), (1.52), (1.53), (1.55), and (1.56) for all i where $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$ are determined according to (1.35), (1.36), (1.37). It is easy to show that the symmetric zero inflation deterministic steady state[23] is reduced to the following system of equations:

$$C^{-\sigma} = Y^{\varphi} \tag{1.60}$$

$$\chi G^{-\gamma} = Y^{\varphi} \tag{1.61}$$

$$Y = C + G \tag{1.62}$$

where $A = 1$. Conditions (1.60) and (1.61) equate the marginal rates of substitution (MRS) between private consumption and leisure and between public consumption and leisure to their marginal rates of transformation (MRT) while condition (1.62) is the resource constraint, that ensures the equilibrium between the demand for final goods and its relative supply. Thus, under coordination the steady state allocation is Pareto efficient. Notice that in order to implement this allocation, the common policy maker needs two instruments: government expenditure to provide an efficient level of public goods consistently with (1.60) , (1.61) and (1.62), and a labour subsidy to completely offset the monopolistic distortions of both labour and good markets.

The welfare approximation. As shown in the appendix, under coordination the average

[23]See the appendix.

welfare of union households can be approximated as follows:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \left[\frac{\varepsilon}{\lambda}(\pi^i_{H,t})^2 + \varphi(\tilde{y}^{c,i}_{H,t})^2 + \gamma(1-\rho)(\tilde{g}^{c,i}_t)^2 + \sigma\rho(\tilde{c}^{c,i}_t)^2 \right.$$
$$\left. +2\varpi^c_1(\tilde{g}^{c,i}_t - \tilde{g}^{c,*}_t)(\tilde{c}^{c,i}_t - \tilde{c}^{c,*}_t) + \varpi^c_2(\tilde{c}^{c,i}_t - \tilde{c}^{c,*}_t)^2\right] di + s.o.t.i.p. \tag{1.63}$$

with:

$\varpi^c_1 \equiv (1-\rho)\xi(1-\nu)(\nu+\psi)$

$\varpi^c_2 \equiv \rho\omega_1 + (1-\rho)\omega_2 + \omega_3 + 2\xi\psi(\rho\delta_1 + (1-\rho)\delta_2)$

$\xi \equiv \frac{\eta\sigma}{1-\alpha}$ [24]

and where $\tilde{x}_t^{c,i} \equiv \hat{x}^i_t - \hat{x}^{c,i}_t$ and $\hat{x}^{c,i}_t$ indicates the target of the common authority under coordination.

According to (1.63), under coordination, welfare losses are increasing in inflation, output, private consumption and public expenditure gaps. At the same time, these losses are affected by the gaps of terms of trade[25] and the consequent mis-allocation in private consumption, public expenditure and production which crucially depends on the different degrees of openness and the elasticity of substitution among bundles produced in different countries.

The target. The target of the coordinated policy maker is the Pareto efficient allocation and corresponds to:

$$\varphi\hat{y}^{c,i}_{H,t} + \gamma\hat{g}^{c,i}_t = (\varphi+1)a^i_t - \frac{\varpi^c_1}{1-\rho}(\hat{c}^{c,i}_t - \hat{c}^{c,*}_t) \tag{1.64}$$

$$\sigma\hat{c}^{c,i}_t - \gamma\hat{g}^{c,i}_t = \frac{1}{1-\rho}\varpi^c_1(\hat{c}^{c,i}_t - \hat{c}^{c,*}_t) - \frac{1}{\rho}\left[\varpi^c_1(\hat{g}^{c,i}_t - \hat{g}^{c,*}_t) + \varpi^c_2(\hat{c}^{c,i}_t - \hat{c}^{c,*}_t)\right] \tag{1.65}$$

$$\hat{y}^{c,i}_{H,t} + \frac{\psi}{1-\psi}(\hat{y}^{c,i}_{H,t} - \hat{y}^{c,*}_t) =$$
$$\rho\hat{c}^{c,i}_t + (\delta-1)\rho(\hat{c}^{c,i}_t - \hat{c}^{c,*}_t) + (1-\rho)\hat{g}^{c,i}_t - \nu(1-\rho)(\hat{g}^{c,i}_t - \hat{g}^{c,*}_t) \tag{1.66}$$

[24] For the definition of ω_1, ω_2 and ω_3 see the appendix.

[25] Notice in fact that by (1.35), (1.36) and (1.37), $\hat{c}_t{}^i - \hat{c}_t{}^*$ is perfectly negative correlated with the terms of trade.

The difference across countries embodied in (1.64) - (1.66) are explained as efficient responses to asymmetric shocks to producitvity. If, for instance, its technological shock is above the union average, then a single country economy experiences a terms of trade worsening and efficiently increases the demand for domestic goods relative to those for foreign goods.

However, once the system of equations (1.64) - (1.66) is integrated:

$$\varphi \hat{y}_t^{c,*} + \sigma \hat{c}_t^{c,*} = (1+\varphi) a_t^* \quad \gamma \hat{g}_t^{c,*} = \sigma \hat{c}_t^{c,*} \quad \hat{y}_t^{c,*} = \rho \hat{c}_t^{c,*} + (1-\rho) \hat{g}_t^{c,*} \tag{1.67}$$

Thus, under coordination, the target of the common authority on average corresponds exactly to the target of the policy maker of a closed economy where the only existing distortion is due to price stickiness.

The optimal policy mix. Given (1.64) - (1.66), the set of constraints relevant the optimal policy problem - the resource constraint, the Phillips Curve and condition (1.40) - can be rewritten in terms of gaps as:

$$\begin{aligned} &\tilde{y}_{H,t}^{c,i} + \frac{\psi}{1-\psi}(\tilde{y}_{H,t}^{c,i} - \tilde{y}_t^{c,*}) = \\ &\rho \tilde{c}_t^{c,i} + (\delta - 1)\rho(\tilde{c}_t^{c,i} - \tilde{c}_t^{c,*}) + (1-\rho)\tilde{g}_t^{i,c} - \nu(1-\rho)(\tilde{g}_t^{c,i} - \tilde{g}_t^{c,*}) \end{aligned} \tag{1.68}$$

$$\pi_{H,t}^i = \lambda \left[\varphi \tilde{y}_{H,t}^{c,i} + \sigma \tilde{c}_t^{c,i} + \omega_4((1-\psi)\alpha + \psi)(\tilde{c}_t^{c,i} - \tilde{c}_t^{c,*}) \right] + \beta E_t\{\pi_{H,t+1}^i\} + \lambda \hat{\mu}_t^i \tag{1.69}$$

$$\pi_{H,t}^i - \pi_t^* = -\omega_4(\Delta \tilde{c}_t^{c,i} - \Delta \tilde{c}_t^{c,*}) - \omega_4(\Delta \upsilon_{1,t}^i - \Delta \upsilon_{1,t}^*) \tag{1.70}$$

for all t and i and where $\upsilon_{1,t}^i \equiv \hat{c}_t^{c,i}$.

This system of equations clarifies the tradeoffs of the common policy maker under coordination. From the union wide perspective, there is only a tradeoff due to the presence of mark up shocks. As in a closed economy, a mark up shock affects inefficiently firms marginal costs making it impossible to fully stabilize both inflation and output gap. Nevertheless, when shocks are just to technology, the optimal policy mix - namely the efficient provision of public goods and (average) strict inflation targeting - allows to close all the gaps and reach on average the efficient allocation.

This is possible only at the average union level. At the single country level, independently of which type of shocks hits the economy, the adoption a common currency implies always a tradeoff between inflation and output stabilization: if the nominal exchange rates are fixed and prices are sticky, the terms of trade cannot adjust instantaneously in response to asymmetric shocks and the flexible price allocation (in the case the first best allocation) cannot be implemented. Therefore, as long as shocks are asymmetric:

$$\pi^i_{H,t} \neq 0 \tag{1.71}$$

for all i for some t. This explains why, as highlighted by Galí and Monacelli (2009), under coordination there is room to use the single country government expenditure as a stabilization tool.

1.5 The case of no-coordination

Under no-coordination, fiscal authorities are coordinated neither among each other nor with the common central bank. The monetary and fiscal policy problems are then formulated as follows[26].

Single countries' governments maximize the welfare of the small open economy representative agent:

$$\sum_{t=0}^{\infty}\beta^t E_0\left[\frac{C_t^{1-\sigma}}{1-\sigma}+\chi\frac{G_t^{1-\gamma}}{1-\gamma}-\frac{1}{\varphi+1}\left(\frac{Y_{H,t}}{A_t}\right)^{\varphi+1}\right] \tag{1.72}$$

with respect to C_t, Y_t, $Y_{H,t}$, $\Pi_{H,t}$, Z_t, F_t and K_t, subject to the single country equilibrium conditions (1.39), (1.44), (1.45), (1.52), (1.53), (1.55), and (1.56), where $P_t/P_{C,t}$, $P_{G,t}/P_{C,t}$, $P_{H,t}/P_t$ are determined according to (1.35), (1.36), (1.37) and taking as given the union average variables including the nominal interest rate chosen by the common central bank.

[26]For more details, see the appendix.

Conversely, the monetary authority maximizes the average union welfare:

$$\sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \left[\frac{C_t^{i\,1-\sigma}}{1-\sigma} + \chi \frac{G_t^{i\,1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}^i}{A_t^i}\right)^{\varphi+1}\right] di \tag{1.73}$$

with respect to C_t^i, Y_t^i, $Y_{H,t}^i$, $\Pi_{H,t}^i$, Z_t^i, F_t^i and K_t^i for all i, subject to equilibrium conditions (1.39), (1.44), (1.45), (1.52), (1.53), (1.55), and (1.56) for all i, where $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$ are determined according to (1.35), (1.36), (1.37) and taking fiscal policies as given[27].

Given the monetary and fiscal policy problems, it can be shown that at the symmetric deterministic steady state, zero inflation is a Nash equilibrium policy[28]. In particular, the optimality conditions evaluated at the zero inflation steady state lead to:

$$C^{-\sigma} = (1-\psi)\left[\delta_1 + \delta_2 \frac{G}{C} + \delta_3 \frac{Y}{C}\right] Y^{\varphi} \tag{1.74}$$

$$\chi G^{-\gamma} = (1-\psi)(1-\nu) Y^{\varphi} \tag{1.75}$$

$$Y = C + G \tag{1.76}$$

where $A = 1$. The comparison between the systems of equations (1.74) - (1.76) and (1.60) - (1.62) makes clear that, when uncoordinated, fiscal policy makers generate static distortions. Indeed at the symmetric steady state, under coordination, the MRS between both leisure and private consumption and leisure and public expenditure are set equal to the correspondent MRT; instead under no-coordination they are respectively determined by $(1-\psi)\left[\delta_1 + \delta_2 \frac{G}{C} + \delta_3 \frac{Y}{C}\right] > 1$ and $(1-\psi)(1-\nu) < 1$. In other words, uncoordinated fiscal authorities have an incentive to expand government spending and to tax labour beyond the efficient level. The interpretation of these findings is the following. By boosting the demand for domestic goods and reducing its supply,

[27]Namely G_t^i and τ^i for all countries and in all periods.
[28]See the appendix.

autonomous governments seek to improve their terms of trade. They realize that rendering home produced goods more expensive than foreign goods raises profits revenue of households and makes up for the reduction in labour income and the increase in lump-sum taxes. Then, households can consume more public goods and work less than under coordination. The decrease in private consumption due to the terms of trade improvement is more than compensated by the higher public good provision and the lower labour effort. In this way countries seek to externalize both the costs of production and taxation to foreign consumers[29]. Obviously given that at the symmetric steady state everybody is doing the same, in equilibrium the prices of all goods are equal and everybody is worse off.

To get a sense of the magnitude of the inefficiency generated at the steady state by uncoordinated policies it is sufficient to look at the following table:

	Coordination	No-Coordination
$\frac{C}{Y}$	0.97	0.73

Under the baseline calibration, according to which $\alpha = 0.4$, $\nu = 0.2$ and $\psi = 0.4$[30], if fiscal policies are not coordinated the steady state consumption output ratio is equal to the 73% (as in the European Monetary Union) whereas, if they are coordinated, it reaches 97%. In other words at the steady state governments' size is highly inefficient. And as it will be made clear in the next subsections, this static distortion will be key even in determining what effects lack of coordination produces over the business cycle.

1.5.1 Fiscal policy

The welfare approximation. For the fiscal policy maker the single country welfare has been approximated as:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\left[\frac{\varepsilon}{\lambda}(\pi_{H,t})^2+\varphi(\tilde{y}^f_{H,t})^2+\varpi^f_1(\tilde{g_t}^f)^2+\varpi^f_2(\tilde{c_t}^f)^2+2\varpi^f_3\tilde{g_t}^f\tilde{c_t}^f\right]$$
$$+t.i.f.p. \tag{1.77}$$

[29]In fact at the steady state the incentive to over-expand the government expenditure is present even when the labour supply is completely inelastic.

[30]and that will be discussed in details below.

with

$\varpi_1^f \equiv (1-\rho)(1-\psi)(1-\nu)\gamma$

$\varpi_2^f \equiv \rho(1-\psi)\delta\sigma + \varsigma_2$

$\varpi_3^f \equiv (1-\rho)(1-\nu)(\varsigma_1 - \xi\nu\psi)$

where ς_1 and ς_2 are properly defined in the appendix, $t.i.f.p$ stands for terms independent of fiscal policy, $\tilde{x}_t^f \equiv \hat{x}_t - \hat{x}_t^f$ and $\hat{x}_t^f$ indicates the target of the fiscal authority. Variables, target and weights that enter in this loss are different than those under coordination. Fiscal authorities take as given the average union allocation and weigh more fluctuations in the private consumption gap and less those in the public expenditure gap.

The target. The target of the fiscal policy maker is determined by the following conditions:

$$\varphi\hat{y}_{H,t}^f + \gamma\hat{g}_t^f = (\varphi+1)a_t - ((1-\rho)(1-\psi)(1-\nu))^{-1}\varpi_3^f(\hat{c}_t^f - \hat{c}_t^*) \tag{1.78}$$

$$\begin{aligned}\varphi\hat{y}_{H,t}^f + \sigma\hat{c}_t^f = (\varphi+1)a_t - ((1-\psi)\rho\delta)^{-1}\Big[\varpi_3^f(\hat{g}_t^f - \hat{g}_t^*) + \varsigma_2(\hat{c}_t^f - \hat{c}_t^*) \\ +(1-\psi)\delta_3(\hat{y}_t^* - \hat{c}_t^*) + (1-\psi)(1-\rho)\delta_2(\hat{g}_t^* - \hat{c}_t^*)\Big]\end{aligned} \tag{1.79}$$

$$\begin{aligned}&\hat{y}_{H,t}^f + \frac{\psi}{1-\psi}(\hat{y}_{H,t}^f - \hat{y}_t^*) = \\ &\rho\hat{c}_t^f + (\delta-1)\rho(\hat{c}_t^f - \hat{c}_t^*) + (1-\rho)\hat{g}_t^f - \nu(1-\rho)(\hat{g}_t^f - \hat{g}_t^*)\end{aligned} \tag{1.80}$$

That target is the efficient allocation from the small open economy point of view[31]. It corresponds to the flexible price allocation in the hypotheses that independent governments are not coordinated. To grasp some insights on the incentives of these authorities and the dynamic union wide effects produced by their policy choices, assume that shocks are symmetric. Then, if *implemented*, the target determined by (1.78), (1.79)

[31]It is recovered by maximizing the purely quadratic approximation of (1.72) recovered in the appendix, subject to condition (1.32) and taking the average union variables as given.

and (1.80) satisfies the following conditions:

$$\varphi\hat{y}_t^f + \gamma\hat{g}_t^f = (1+\varphi)a_t^* \tag{1.81}$$

$$\delta\rho(\gamma\hat{g}_t^f - \sigma\hat{c}_t^f) = \delta_3(\hat{y}_t^f - \hat{c}_t^f) + (1-\rho)\delta_2(\hat{g}_t^f - \hat{c}_t^f) \tag{1.82}$$

$$\hat{y}_t^f = \rho\hat{c}_t^f + (1-\rho)\hat{g}_t^f \tag{1.83}$$

This set of conditions (that is nothing more than the log linear approximation of (1.74) - (1.76)) differs from the target of the coordinated authority in two main respects.

Firstly, as already pointed out under no-coordination the government expenditure share in output is inefficiently high because at the steady state both government expenditure is over-expanded and output under-produced. Under the baseline calibration, this static distortion has a clear consequence: it inefficiently amplifies the impact of the government expenditure shocks over output fluctuations. Indeed, according to the baseline calibration, the intertemporal elasticity of substitution of the government expenditure, γ^{-1}, is higher than that of private consumption, σ^{-1}. Hence over the cycles policy makers want to substitute private consumption with public expenditure in order to smooth the path of the more inelastic bundle. As a consequence public expenditure fluctuates more than private consumption. Then, being steady state public expenditure share in output inefficiently high, one percent increase in government spending would expand more output under no-coordination than under coordination.

Secondly according to (1.82), $\widehat{mrs}_t$ between private consumption and public expenditure is not equal on average to the corresponding $\widehat{mrt}_t$ as it would be under coordination. This is because uncoordinated fiscal authorities try to influence the terms of trade to their own advantage even over the business cycle. In fact, as long as $\gamma \neq \sigma$ and intermediate or public goods are traded [32], they have an incentive to generate procyclical response of the average public spending beyond the efficient provision of the public goods. In other words they seek to inefficiently boost the volatility of public consumption and dampen that of labour in order to reduce the volatility of terms of trade,

[32] i.e. even $\psi > 0$ or $\nu > 0$ Note that if $\psi > 0$ or $\nu > 0$ then $\delta_3 > 0$ or $\delta_2 > 0$.

output and private consumption. The underlying reason that explains this behaviour is the attempt to reduce the cost of uncertainty for domestic consumers - that are risk adverse - by externalizing business cycle fluctuations to other countries' households.

The optimal policy. Fiscal policy makers maximize (1.77) subject to (1.47), (1.57) and (1.40) properly rewritten in terms of gaps:

$$\tilde{y}^f_{H,t} = (1-\psi)\delta\rho\tilde{c}_t{}^f + (1-\psi)(1-\nu)(1-\rho)\tilde{g}^f_t \tag{1.84}$$

$$\pi_{H,t} = \lambda\left[\varphi\tilde{y}^f_{H,t} + \omega_4\tilde{c}^f_t\right] + \beta E_t\{\pi_{H,t+1}\} + \lambda(\hat{\mu}_t + \upsilon_{2,t}) \tag{1.85}$$

$$\pi_{H,t} = -\omega_4\Delta\tilde{c}^f_t + \upsilon_{3,t} \tag{1.86}$$

where

$\upsilon_{2,t} \equiv \varpi^f_4(\hat{c}^f_t - \hat{c}^*_t) - ((1-\psi)\rho\delta)^{-1}\Big[\varpi^f_3(\hat{g}^f_t - \hat{g}^*_t) + (1-\psi)\delta_3(\hat{y}^*_t - \hat{c}^*_t) + (1-\psi)(1-\rho)\delta_2(\hat{g}^*_t - \hat{c}^*_t),$

$\upsilon_{3,t} \equiv \pi^*_t - \omega_4(\Delta\hat{c}^f_t - \Delta\hat{c}^*_t)$

$\varpi^f_4 \equiv (\omega_4((1-\psi)\alpha + \psi) - ((1-\psi)\rho\delta)^{-1}\varsigma_2).$

According to first order conditions of this problem with respect to $\tilde{y}^f_{H,t}$, $\tilde{g}^f_t$, $\tilde{c}^f_t$ and $\pi_{H,t}$:

$$\begin{aligned}\pi_{H,t} &= -\frac{1}{\varphi\varepsilon}A(L)\left[\varphi\tilde{y}^f_t + \gamma\tilde{g}^f_t + ((1-\rho)(1-\psi)(1-\nu))^{-1}\varpi^f_3\tilde{c}^f_t\right] \\ &+ \frac{\lambda}{\omega_4\varepsilon}B(L)\left[(1-\psi)\delta\rho(\varphi\tilde{y}^f_t + \sigma\tilde{c}^f_t) + \varpi^f_3\tilde{g}^f_t + \varsigma_2\tilde{c}^f_t\right]\end{aligned} \tag{1.87}$$

where $A(L) \equiv \left[(1-L) + \lambda\left[\frac{(1-\psi)\delta\rho\varphi}{\omega_4} + 1\right]B(L)\right]$ and $B(L) \equiv (1 - E_t L^{-1})^{-1}$.

This system of equations (1.84)-(1.87) determines the gaps of the small open economy under uncoordinated fiscal policies for a given path of the aggregate variables [33]. According to these conditions in general, uncoordinated governments always face a tradeoff between stabilizing inflation and output gap. Indeed, differently from what

[33] To recover the average union allocation one has to find the optimal average level of provision of public expenditure and then determine the average union private consumption and output by using the other equilibrium conditions: the average union resource constraint, the average union Phillips curve and the IS curve.

happens under coordination they are not able to achieve their target even in the absence of idiosyncratic and mark up shocks. This is made made clear by condition (1.85): unless special parametric restrictions are met, it is not possible to fully stabilize the $\widehat{mrs}_t$ between private consumption and leisure at the desired level despite the complete stabilization of the home inflation. The key reason of this outcome is the incentive of independent fiscal authorities to affect the terms of trade in their favour even over the business cycles. As already emphasized, these policy makers, in fact, want the $\widehat{mrs}_t$ between private consumption and leisure to fluctuate less than its $\widehat{mrt}_t$ to restrain private consumption and output volatility at foreign consumers' expense.

However there are specific restrictions under which this result may be reversed and the optimal fiscal policy is consistent with home price stability. Specifically if intermediate and public goods are not traded - i .e. $\psi = 0$ and $\nu = 0$ - or the intertemporal elasticities of substitution of public and private consumption are equal - $\gamma = \sigma$ - $\pi_{H,t} = 0$ for all t is optimal as long shocks are symmetric and to technology. To see why consider: 1) If shocks are symmetric there is no additional trade off generated by the adoption of a common currency (i.e. $\upsilon_{3,t} = 0$) 2) If $\psi = 0$ and $\nu = 0$ or $\gamma = \sigma$ then stabilizing the $\widehat{mrs}_t$ between both private and public consumption and the $\widehat{mrt}_t$ is the target of the uncoordinated fiscal authorities (then i.e. $\upsilon_{2,t} = 0$). And in fact it can be shown that under these restrictions all the conditions (1.84)-(1.87) are simultaneously satisfied.

Nevertheless notice that this last finding is conditional on the willingness of the monetary policy maker to completely stabilize inflation. Whether she finds it optimal or not it will be clarified in the next paragraph.

1.5.2 Monetary policy

The welfare approximation. Under fiscal policy no-coordination the objective of the common central bank can be approximated as:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[\zeta_3\frac{\varepsilon}{\lambda}(\pi_t^*)^2+\zeta_3\varphi(\tilde{y}_t^{m,*})^2+\varpi_1^m(\tilde{c}_t^{m,*})^2+2\zeta_2\sigma\tilde{c}_t^{m,*}\tilde{y}_t^{m,*}\Big]$$

$$+s.o.t.i.m.p. \tag{1.88}$$

with

$\varpi_1^m \equiv \rho\delta(\sigma-1)+\zeta_1\rho-\zeta_2\sigma^2$

$\zeta_1 \equiv \frac{(1-\psi)\delta\varphi\rho+\sigma}{\varphi\rho+\sigma}$

$\zeta_2 \equiv \frac{((1-\psi)\delta-1)\rho}{\varphi\rho+\sigma}$

$\zeta_3 \equiv \frac{(\varphi+1)(1-\psi)\delta\rho+\sigma-\rho}{\varphi\rho+\sigma}$

where $\tilde{x}_t^{m,i} \equiv \hat{x}_t^i - \hat{x}_t^{m,i}$ and $\hat{x}_t^{m,i}$ is the target for $\hat{x}_t^i$ chosen by the central bank.

The approximation to the objective of the central bank under no-coordination diverges from those of the uncoordinated fiscal authority and of the common policy maker under coordination. And this is true not only because the central bank does not choose the optimal provision of public goods. Indeed even abstracting from this consideration, there are striking differences in target, weights and variables that enter in the approximation. The key determinant of these divergences is the steady state distortion as shown by the dependence of the weights and of the average target from ρ, ζ_1, ζ_2 and ζ_3.

The target. The target of the central bank can be retrieved from[34]:

$$\varphi\hat{y}_t^{m,*}+\sigma\hat{c}_t^{m,*}=(1+\varphi)a_t^*-\frac{\zeta_2}{\zeta_3}\left[\frac{\sigma}{\rho}(\hat{y}_t^{m,*}-\hat{c}_t^{m,*})+(1+\varphi)\hat{\mu}_t^*\right] \tag{1.89}$$

$$\hat{y}_t^{m,*}=\rho\hat{c}_t^{m,*}+(1-\rho)\hat{g}_t^* \tag{1.90}$$

According to (1.89) and (1.90) in general, the target of a common central bank is not the first best and thus does not coincide with the target of the coordinated common authority under either technological or mark up shocks. This is due to various reasons.

[34]This target is determined by maximizing the purely quadratic approximation of (1.73) shown in the appendix subject to (1.32) and taking as given $\hat{g}_t^*$.

Some of these reasons have been already underlined in the previous paragraph. First, under the baseline calibration fiscal shocks expand sub-optimally output fluctuations because at the steady state the governments' size is inefficiently high; secondly, autonomous governments try to manipulate the terms of trade even over the business cycles. Hence, in general uncoordinated fiscal policies produce average dynamic distortions that need to be internalized in the target and in the policy decisions of the common central bank. It is worth noticing that, as made explicit by (1.89) and (1.90), the impact of these spillovers on monetary policy choices depends crucially on the difference between $\hat{y}_t^{m,*}$ and $\hat{c}_t^{m,*}$. In fact, the closer are $\hat{y}_t^{m,*}$ and $\hat{c}_t^{m,*}$, the less is the intratemporal substitution between private consumption and output - and the business cycle distortions generated by this substitution -, the closer is target of the common central bank to the average flexible price allocation. Actually when $\hat{y}_t^{m,*} = \hat{c}_t^{m,*}$ the effects of these dynamic inefficiencies disappear on average.

The other reason that explains the influence of independent governments' decisions on the target of the common central bank is related to the state distortion and the long run effects of monetary policy. According to (1.89) and differently from the case of coordination the target of the common central bank does react to mark up shocks. Why? To answer this question, consider the special case in which shocks are symmetric and restricted to the mark up. Then, under flexible prices, from condition (1.13), it follows:

$$E\left\{\frac{W_t}{P_{C,t}}\right\} = E\left\{\Gamma(Y_t)\right\} E\left\{(1+\mu_t)\right\} + Cov\left\{(1+\mu_t)\Gamma(Y_t)\right\} \tag{1.91}$$

where $\Gamma(Y_t) \equiv Y_t^{\varphi}(Y_t - G_t)^{\sigma}$ and $\Gamma_Y(Y_t) > 0$.

According to (1.91), the lower is the covariance between mark up shocks and output, the higher is the average per-period output for a given level of the average per-period real wage. In other words, if there is an increase in output fluctuations in response to mark up shocks - which corresponds to a decrease of the covariance between the mark up and output given that a positive mark up shock tends to reduce output - domestic consumers have to increase their average labour effort if they want to maintain the same average real wage. As a consequence, the common central bank recognizes that by becoming more aggressive in fighting inflation and allowing for an increase in output fluctuations monetary policy can have beneficial effects: it can shift upward the average labour supply curve generating an efficient increase average level of output.

The constraints to the monetary policy problem can then be rewritten in terms of gaps as:

$$\tilde{y}_t^{m,*} = \rho \tilde{c}_t^{m,*} \tag{1.92}$$

$$\pi_t^* = \lambda \left[\varphi \tilde{y}_t^{m,*} + \sigma \tilde{c}_t^{m,*} \right] + \beta E_t \{ \pi_{t+1}^* \} + \lambda (\frac{1}{\zeta_3} \hat{\mu}_t^* + \upsilon_{5,t}^*) \tag{1.93}$$

with: $\upsilon_{5,t}^* \equiv -\frac{\zeta_2}{\zeta_3} \frac{\sigma}{\rho} (\hat{y}_t^{m,*} - \hat{c}_t^{m,*})$

Moreover, the system of optimality conditions of the monetary policy maker can be rewritten as:

$$\pi_t^* = -\frac{\rho(1-L)}{\varepsilon(\sigma + \varphi\rho)} \left[\varphi \tilde{y}_t^{m,*} + \sigma \tilde{c}_t^{m,*} + \frac{\zeta_2}{\zeta_3} \frac{\sigma}{\rho} (\tilde{y}_t^{m,*} - \tilde{c}_t^{m,*}) \right] \tag{1.94}$$

Thanks to this set of equations it is possible to recover the average union allocation determined by the optimal reaction of the common central bank to given fiscal policies. Clearly given the changes in the target and as stressed by (1.93), the common central bank faces different tradeoffs than those of the policy authority under coordination. On the one hand, in general, even if shocks are only to technology, strict inflation targeting is not optimal ($\upsilon_{5,t}^* \neq 0$). That result contrasts with the findings of Galí and Monacelli (2009) and Beetsma and Jensen (2005). Fully stabilizing the average union inflation is not optimal because it does not allow to close the average output gap. Indeed the flexible price allocation is not efficient and the monetary authority wants to correct the dynamic distortions due to the lack of coordination among fiscal policy makers. On the other hand if shocks are to mark up, the central bank tries to stabilize more inflation than output compared to the case of coordination. It realizes that an increase in output fluctuations in response to mark up shocks can boost the inefficiently low level of per-period output output.

Now it is possible to answer the question raised at the end of the last the paragraph. In presence of productivity shocks under which conditions does the central bank find it optimal to completely stabilize the average union inflation?

Suppose that according to a policy rule $\hat{g}_t^* = \hat{c}_t^{m,*}$ for all t and there are only technological shocks. In that case $\pi_t^* = 0$ for all t satisfies conditions (1.92), (1.93) and (1.94). However when $\hat{g}_t^* \neq \hat{c}_t^{m,*}$ for some t, then $\pi_t^* = 0$ for all t cannot be optimal. Thus even when there is no trade in public and intermediate goods, namely $\nu = \psi = 0$ the monetary policy maker would not stabilize the average union inflation even under symmetric productivity shocks, while in that case fiscal authorities would be willing to do that.

1.5.3 The case for average price stability

The analysis of the previous sections allows to formulate the following proposition:

Proposition 1. *Under fiscal policy no-coordination, $\pi_t^* = 0$ for all t is a Nash equilibrium outcome of the monetary and fiscal policy game if and only if $\sigma = \gamma$ with shocks to technology only.*

Proof: See the appendix.

Proposition 1 can be interpreted as follows: when $\sigma = \gamma$ and shocks are to technology, the lack of coordination among fiscal policy makers yields on *average* only *static* distortions[35] namely the steady state distortions. In fact under this parametric restriction and if the average union inflation is completely stabilized, two conditions are simultaneously satisfied. On the one side the average marginal rates of substitution between private and public consumption and private consumption and leisure fluctuate as the marginal rates of transformation between the same variables, i.e. $\sigma\hat{c}_t^* - \gamma\hat{g}_t^* = 0$ and $\varphi(\hat{y}_t^* - \hat{a}_t^*) + \sigma\hat{c}_t^* = \hat{a}_t^*$; on the other side the average union output co-moves with private and public consumption, i.e. $\hat{y}_t^* = \hat{g}_t^* = \hat{c}_t^*$. These two conditions ensure that,

[35]...at least up to a first order approximation of the optimal policies.

even if fiscal policies are uncoordinated, under flexible prices the average union fluctuations of output, and public and private consumption, replicate the fluctuations that would be achieved if the fiscal policies were coordinated. As a consequence the monetary authority seeks to remove the only remaining distortion that can be corrected: the average price stickiness. Stabilizing completely the average inflation is then optimal: it allows at the same time to eliminate on average the inefficiencies produced by price rigidities and to keep the average allocation at the constrained-efficient level.

1.5.4 The general case

This section analyzes the general case allowing for different intertemporal elasticities of substitution of private and public consumption and different kinds of shocks. These differences generate an incentive for the fiscal authorities to seek to substitute intratemporally the public and private consumption: in the case of different elasticities, in order to smooth intertemporally the path of more inelastic goods; in the case of mark up shocks, in order to reduce the home country private consumption and output gap. As a result because of this intratemporal substitution, it is no more true that, under technological shocks, the symmetric allocation is proportional to the efficient one. And both monetary and fiscal policies at the average union level do not correspond to the ones that are optimal under coordination. In fact neither under technological shocks the common central bank should seek to pursue price stability nor fiscal policies ensure on average the efficient provision of public goods.

1.5.5 Calibration

Impulse responses to a one percent rise in technology and mark up under optimal policies are recovered using the calibration indicated in the appendix which is close to those of Galí and Monacelli (2009) and Galí and Monacelli (2005). In particular γ^{-1} and φ^{-1} the intertemporal elasticities of substitution of public consumption and labour, α the degree of openness in private consumption, ε, the elasticity of substitution among goods produced in the same country, β the preferences discount factor, θ the parameter that governs the level of price stickiness in the economy and *ac* the first order autocorrelation of shocks[36] are set according to their calibration. Conversely σ^{-1} the intertemporal elasticity of substitution of private consumption and η the elasticity of substitution between bundles produced in different countries are set according to Benigno and Benigno (2006), ψ the degree of openness in the intermediate goods is equal to α and $\nu = 0.2$ as partially suggested by Brülhart and Trionfetti (2004). Finally χ the parameter that regulates the relative weight of the public good in the preferences is calibrated to match the average consumption output ratio of European Monetary Union.

1.5.6 Dynamic simulations

The appendix shows the impulse responses to a one percent increase in aggregate technology and mark up under the optimal policies. They may be interpreted as follows.

Technological shock. When shocks are to technology and fiscal policies are coordinated, the optimal policy mix embodies two clear prescriptions for the average union economy: the nominal interest rate should be set to fully stabilize the average inflation, while the government expenditure should ensure, on average, the efficient provision of public goods. These policies allow to close all the gaps at the union level and reach the efficient fluctuations. However under fiscal policy no-coordination none of these prescriptions remain valid. The first is not valid because of the dynamic effects produced by uncoordinated fiscal policies. In particular, given that $\gamma < \sigma$ and the incentive of independent fiscal authorities to manipulate the terms of trade in their favour, a technology shock increases the provision of public goods more than private consumption. As a consequence, because of the inefficiently high share of government expenditure in output, there is an overexpansion of output. Thus, the common central bank has to trade off between stabilizing the average union inflation and reducing the output gap. This explains why monetary policy allows for a certain degree of average union deflation,

[36] Both mark up and productivity shocks are suppose to be AR(1).

being more restrictive under no-coordination that under coordination (as emphasized by the different path of the nominal interest rates). Obviously in these circumstances not even the average public good provision is efficient. Fiscal policy makers seek to implement a beggar-my-neighbour policy even over the business cycle, disregarding the aggregate distortions resulting from their joint action. By over-expanding the provision of the public good beyond the efficient level, they want to reduce the terms of trade volatility in order to externalize the cost of private consumption and output fluctuations to other country consumers. And in fact according to the impulse responses the government expenditure expansion is σ/γ greater than that of private consumption.

Mark up shock. When shocks are to the mark up, the policy prescriptions under coordination are twofold. Fiscal policy is not a useful tool to stabilize the average effects of the mark up shocks: for this purpose it is more efficient to use the nominal interest rate which is a costless instrument. Therefore under mark up shocks, the average union government expenditure should be kept at the steady state level. At the same time, the monetary authority should trade off between stabilizing inflation and closing the output gap given the consequences of an inefficient shock to the mark up. The policy prescriptions under no-coordination are quite different.

Firstly, in response to a positive mark up shock optimal monetary policy becomes more aggressive in reducing inflation than under coordination. Indeeed as made clear by (1.91), the common central bank wants output to fluctuate more in response to mark up shocks because in this way it induces domestic consumers to raise their per-period labour supply. As a result there is a beneficial increase in the inefficiently low level of the per-period output. According to the impulse responses, the nominal interest rate is then higher, while the average inflation and output are lower, under no-coordination than under coordination.

Secondly, autonomous governments lower the provision of public goods. This is the result of the balance between different objectives. On the one hand the aggregate mark up shock induce a fall in the average union private consumption and output contracting foreigners' demands for home produced goods. In response to these external shocks she perceives as efficient, the non-coordinated policy maker would like to decrease domestic private and public consumption increasing the leisure[37]. However she has to trade off between this purpose and stabilizing the undesired effect of the domestic mark up shock: the boost in the home inflation and output gap. Thus, the provision of public goods falls, but not much more than the private consumption in order to alleviate the reduction of the private consumption itself that actually after the first periods is higher than under coordination. Hence, while under coordination, the common authority recognizes that only monetary policy should be used to stabilize the average effects of mark up shocks, under no-coordination the single country government takes the actions of other policy makers as given and tries on its own to stabilize the effects of the domestic mark up shock in its country.

1.6 Conclusions

According to this chapter within a monetary union the lack of coordination among fiscal policy makers has relevant implications for both optimal monetary and fiscal policies. In fact, only under a special parametric restriction and when shocks are to technology, fiscal policy no-coordination does not matter for the optimal monetary policy design. However, in general, this result is not verified and, as opposed to the case of coordination, under no-coordination it is possible to reach the following conclusions: first when shocks are to technology, stabilizing the average union prices is not optimal; second under mark up shocks, the monetary authority is mainly focused on the stabilization of the average union inflation. Finally even if shocks are symmetric, fiscal policies are used as stabilization tool.

The analysis of the interactions between monetary and fiscal policies with a monetary union - for both cases of coordination and no-coordination - may be extended in

[37]This is made clear by the (1.78)-(1.80).

several directions. On the one hand there is scope for relaxing some of the key assumptions of the model used in this paper: for instance, by introducing sticky wages and allowing for incomplete financial markets. On the other hand future research should investigate the implications both at the steady state and over the business cycles of different monetary and fiscal policy games, as the Stackelberg one in which the monetary authority chooses its policy before the uncoordinated fiscal policy makers.

Baseline calibration

$\sigma^{-1} = 1/3$	Intertemporal elasticity of substitution of the private goods;
$\gamma^{-1} = 1$	Intertemporal elasticity of substitution of the public goods;
$\eta = 4.5$	Elasticity of substitution between home and foreign private goods;
$\varphi^{-1} = 1$	Intertemporal elasticity of substitution of labor;
$1 - \alpha = 0.6$	Degree of home bias in the private bundle;
$1 - \nu = 0.8$	Degree of home bias in the public bundle;
$1 - \psi = 0.6$	Degree of home bias in the intermediate input;
$\chi = 0.03$	Weight of the public bundle in the preferences.
	It implies a steady state consumption output ratio under no-coordination of 0.73;
$\varepsilon = 6$	Elasticity of substitution among goods produced in the same country;
$\beta = 0.99$	Preferences discount factor.
$ac = 0.95$	Autocorrelation of shocks;

2 On the benefits of a monetary union: Does it pay to be bigger?

2.1 Introduction

What are the costs and the benefits of a monetary union? Should independent countries abandon their own currency to delegate monetary policy to a common central bank? These questions are far from new[1] but have been revitalized by the debate on the creation of the European Monetary Union (EMU). On theoretical grounds the costs of losing monetary autonomy are well known: in presence of nominal rigidities countries that share the same currency cannot properly stabilize asymmetric shocks. By contrast, the sources of welfare benefits that can rationalize the existence of a currency area have not been identified[2], at least if we restrict ourselves to the new open economy macroeconomic literature in which the objectives of the policy makers are fully microfounded[3].

However there is a key aspect seemingly overlooked so far which can explain why a monetary union can be beneficial for its members. Especially, if we refer to the EMU experience, it is clear that the European Central Bank (ECB) sets the nominal interest rate for an economic area which is much *bigger* than each national economy. The difference in size may induce an improvement upon the conduct of the single country monetary policy given that, as stressed also by the recent literature[4], open economy policy makers seek to affect their terms of trade to other countries consumers' expense. Indeed on the one hand, by being concerned about the welfare of all consumers living in the area, the central bank of the monetary union internalizes the spillover effects that

[1] See Mundell (1961).

[2] As emphasized by the so called Delors report (1989), there are microeconomic benefits from adopting a common currency like, for instance, saving in transaction costs. However it would be difficult to incorporate this kind of costs in a macroeconomics model.

[3] ....namely derived directly from the welfare of the representative household. See Rotemberg and Woodford (1997) and Benigno and Woodford (2005). There is a recent contribution of Corsetti (2008) on this issue who, in a model with heterogeneous countries, identifies the conditions under which monetary policy in a currency union is as efficient as under monetary autonomy.

[4] See for instance Corsetti and Pesenti (2001) and Epifani and Gancia (2009).

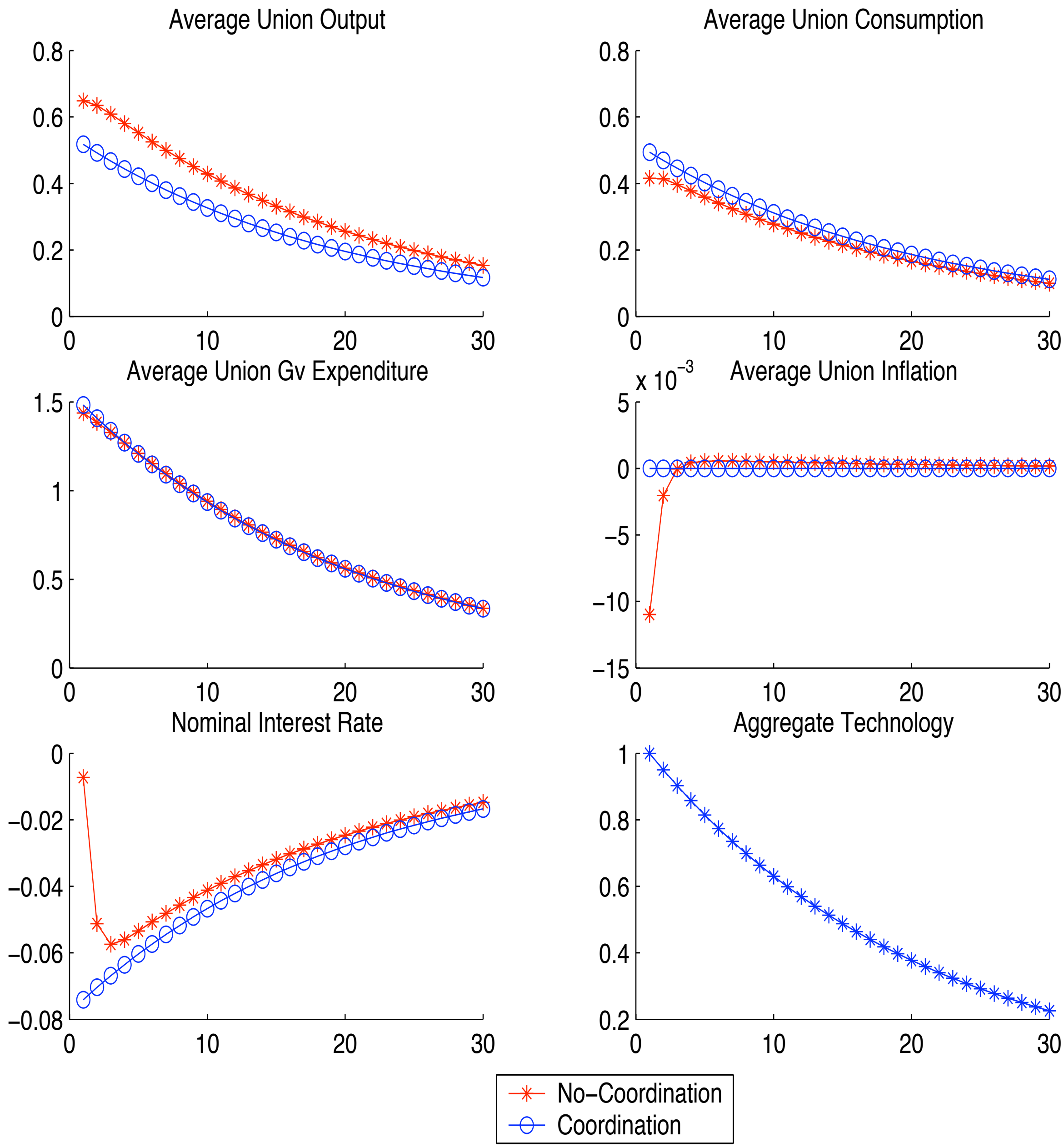

Figure 1.1: Impulse responses to a one percent rise in the technology. Variables are expressed as log-deviations from the steady state

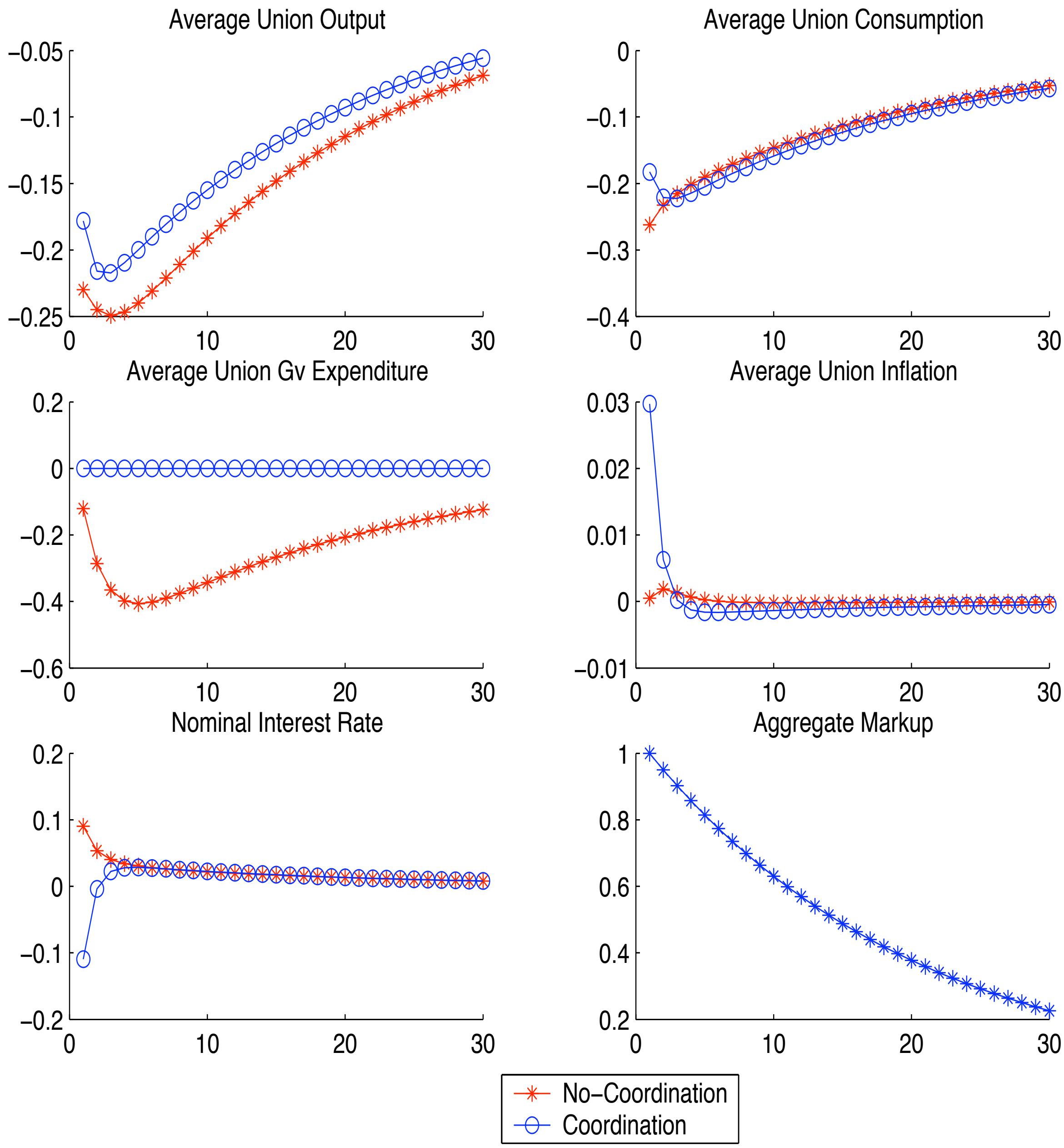

Figure 1.2: Impulse responses to a one percent rise in the markup. Variables are expressed as log-deviations from the steady state

single country's policy makers would produce *inside* the area if there were monetary autonomy. On the other hand, by setting the monetary policy for the union as whole, the common authority better realizes the impact of its policy decisions on the *outside* world and the feedback effects on welfare in its own economy.

The contribution of this chapter is to verify whether, once these channels are taken into account, the adoption of a common currency generates gains in terms of welfare that outweigh the costs of renouncing monetary policy independence. To this end, I develop a dynamic stochastic general equilibrium open economy model in which the world is constituted by a continuum of small open regions as in Galí and Monacelli (2005). Each region produces a bundle of differentiated goods. Preferences exhibit home bias and the elasticity of substitution between home and foreign bundles is different from one. Prices are staggered implying a cost for the adoption of a common currency due to the impossibility to properly stabilize asymmetric shocks.

The regions are split in two areas, H and F. In area F all regions belong to a single country (as in the U.S.). Conversely area H is formed by a collection of sovereign small open economies (as in Europe). In this setup two different policy regimes (called A and B) are considered. Under regime A, in area H there are flexible exchange rates and each small open economy has its own autonomous central bank; under regime B in area H there is a single currency and monetary policy is under the control of a common central bank (ECB). Instead in area F, independently of the policy regimes, all regions share a common currency and monetary policy is delegated to a single authority (FED). Moreover, in both regimes A and B monetary policies are chosen under commitment and are optimal from the *timeless* perspective[5].

In this kind of setting optimal policy decisions of open economy authorities are biased by a free riding problem. Under the assumption of complete financial markets, consumption is highly correlated both across area and across regions. Because of this

[5]See Benigno and Woodford (2005) and Woodford (2003).

consumption sharing, single countries' policy makers have an incentive to seek to improve their terms of trade in their favor[6]. Through a terms of trade improvement, they try to squeeze the domestic/foreign output ratio to outsource labour effort. However their optimal monetary policies are affected in different ways by this incentive depending on the dimension of their own economy.

Under the baseline calibration small countries' policy makers perceive per-period domestic output as inefficiently high. They would rather prefer to lower home production and substitute consumption with leisure. Indeed since the economy is small, one unit decrease in domestic output - which does not affect world output - brings about a marginal decrease in labour that more than outweighs the consumption drop. This has a clear consequence for optimal monetary policy. In fact, by increasing the covariance between output and mark up shocks (which is typically negative), the authorities of a small open economy can induce domestic workers to enjoy more leisure contracting, by so doing, the per-period domestic production. In other words given that they regard home output as too high, these policy makers have a motive to focus more on output than inflation stabilization in response to a global mark up shock.

By contrast, under the baseline calibration the central bank of the monetary union considers per-period domestic production as too low. This is because its incentive to manipulate the terms of trade is much weaker than that of policy makers of the small open economy. Indeed, the authority of the currency union internalizes the feedback effects of its policy decisions stemming from the other area. Then it realizes that a terms of trade improvement is dampening the domestic/ foreign output ratio not only by squeezing the demand for domestic goods (reducing domestic production) but even by boosting that for foreign ones (thus increasing foreign output). As a result, this policy maker is willing to adopt a policy that weights more inflation than output stabilization allowing for a rise in the per-period domestic production.

[6]The effects of this externality are amplified by the hypothesis of home bias for both the bundles produced within the region and within the area as well as by the assumption that the elasticity of substitution between home and foreign bundles is different from one. For a discussion see Obstfeld and Rogoff (2002) , Benigno and Benigno (2003) and Pappa (2004). Notice that other policy instruments to affect terms of trade, such as tariffs, cannot be used in the WTO.

These differences in incentives explain the differences in outcomes across policy regimes. In regime B policy makers are exactly symmetric; thus under global mark up shocks, they choose the same optimal monetary policy, thereby ensuring the same economic performance in the two areas. Conversely, in regime A, the monetary authorities of the two areas have opposite goals. By seeking to reduce per-period domestic output, the central banks of the small open countries weighs more output than inflation stabilization. On the other hand given that from their perspective domestic output is on average too low, the policy maker of the monetary union focuses more on output than on inflation stabilization. Then in response, for instance, to a negative symmetric mark up shock, small open countries' authorities adopt a more restrictive policy than the policy maker of the monetary union. Therefore there is more deflation in area H and output in area F expands more than under regime B.

These differences across regimes explain why, despite the presence of idiosyncratic shocks, households of area H can be better off by sharing a common currency. Indeed, I show that, in the presence of mark up shocks, adopting the same currency may generate welfare benefits under reasonable calibrations. T his finding is quite robust: even for relatively low level of the intertemporal elasticity of substitution between home and foreign bundles and high levels of the variance of the idiosyncratic shocks, welfare gains may be significant.

This chapter is organized as follows. Section 2 describes the basic setup, section 3 determines the equilibrium conditions, section 4 formulates the optimal policy problems, section 5 describes the dynamic simulation and section 6 reports the results about the welfare evaluation.

2.2 The basic framework

The world consists of a continuum of small open regions indexed by $i \in [0, 1]$[7]. The regions are subdivided in two areas, H and F. In area H, there is a continuum of

[7] This model is a general version of the basic framework layout by Galí and Monacelli (2005) and Galí and Monacelli (2009).

regions indexed by $i \in [0, \frac{1}{2})$, which are independent countries. Area F consists of regions indexed by $i \in [\frac{1}{2}, 1]$, which belong to a single country. Each region produces a continuum of imperfect substitutable goods. Labour is immobile across both regions and areas.

2.2.1 Preferences

Agents are infinitely lived and maximize the expected value of the discounted sum of the period utility. Preferences of a generic region i representative household are defined over a private consumption bundle, C_t^i and labor $N_t^i(s)$:

$$\sum_{t=0}^{\infty} \beta^t E_0 \left[\frac{C_t^{i\,1-\sigma}}{1-\sigma} - \frac{N_t^i(s)^{\varphi+1}}{\varphi+1} \right] \quad 0 < \beta < 1 \tag{2.1}$$

where β stands for the intertemporal preferences discount factor. Agents consume all the goods produced in the world economy. However, preferences exhibit home bias. The private consumption index is a CES aggregation of the following type:

$$C_t^i \equiv \left[\alpha_s^{\frac{1}{\eta}} C_{i,t}^{i\,\frac{\eta-1}{\eta}} + (\alpha_b - \alpha_s)^{\frac{1}{\eta}} C_{H,t}^{i\,\frac{\eta-1}{\eta}} + (1-\alpha_b)^{\frac{1}{\eta}} C_{F,t}^{i\,\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \qquad i \in \left[0, \frac{1}{2}\right) \tag{2.2}$$

$$C_t^i \equiv \left[\alpha_s^{\frac{1}{\eta}} C_{i,t}^{i\,\frac{\eta-1}{\eta}} + (\alpha_b - \alpha_s)^{\frac{1}{\eta}} C_{F,t}^{i\,\frac{\eta-1}{\eta}} + (1-\alpha_b)^{\frac{1}{\eta}} C_{H,t}^{i\,\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \qquad i \in \left[\frac{1}{2}, 1\right] \tag{2.3}$$

$\eta > 0$, $0 < \alpha_s < \alpha_b$ and $\frac{1}{2} < \alpha_b < 1$. α_s and α_b are the degrees of home bias for the goods produced within region i and the area to which region i belongs. Moreover, η denotes the elasticity of substitution among $C_{H,t}^i$ $C_{F,t}^i$, $C_{i,t}^i$ which are defined as:

$$C_{H,t}^i \equiv \left[2^{\frac{1}{\eta}} \int_0^{\frac{1}{2}} C_{j,t}^{i\,\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}} \quad C_{F,t}^i \equiv \left[2^{\frac{1}{\eta}} \int_{\frac{1}{2}}^1 C_{j,t}^{i\,\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}} \tag{2.4}$$

$$C_{j,t}^i \equiv \left(\int_0^1 c_t^i(h^j)^{\frac{\varepsilon-1}{\varepsilon}} dh^j \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad j \in \left[0, \frac{1}{2}\right) \quad C_{j,t}^i \equiv \left(\int_0^1 c_t^i(f^j)^{\frac{\varepsilon-1}{\varepsilon}} df^j \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad i \in \left[\frac{1}{2}, 1\right] \tag{2.5}$$

where ε is the elasticity of substitution among goods produced in the same region. The definitions of the private consumption indexes (2.2), (2.4) and (2.5) enable us to determine consistent definitions of price indexes. In particular, $P_{C^i,t}$, the consumers' price index of region i, is:

$$P_{C^i,t} \equiv \left[\alpha_s {P_{i,t}}^{1-\eta} + (\alpha_b - \alpha_s){P_{H,t}}^{1-\eta} + (1-\alpha_b){P_{F,t}}^{1-\eta}\right]^{\frac{1}{1-\eta}} \qquad i \in \left[0, \frac{1}{2}\right) \quad (2.6)$$

$$P_{C^i,t} \equiv \left[\alpha_s {P_{i,t}}^{1-\eta} + (\alpha_b - \alpha_s){P_{F,t}}^{1-\eta} + (1-\alpha_b){P_{H,t}}^{1-\eta}\right]^{\frac{1}{1-\eta}} \qquad i \in \left[\frac{1}{2}, 1\right] \quad (2.7)$$

$$P_{H,t} \equiv \left[2\int_0^{\frac{1}{2}} {P_{j,t}}^{1-\eta} dj\right]^{\frac{1}{1-\eta}} \qquad P_{F,t} \equiv \left[2\int_{\frac{1}{2}}^1 {P_{j,t}}^{\frac{1}{1-\eta}} dj\right]^{\frac{1}{1-\eta}}$$

$$P_{j,t} \equiv \left(\int_0^1 p_t(h^j)^{1-\varepsilon} dh^j\right)^{\frac{1}{1-\varepsilon}} \quad j \in \left[0, \frac{1}{2}\right) \quad P_{j,t} \equiv \left(\int_0^1 p_t(f^j)^{1-\varepsilon} df^j\right)^{\frac{1}{1-\varepsilon}} \quad j \in \left[\frac{1}{2}, 1\right]$$

where all prices are denominated in the currency of the home country. Thus $P_{i,t}$, $P_{H,t}$ and $P_{F,t}$ are producers' price indexes. The law of one price is assumed to hold in all single good markets. However, given the home biased preferences, in general the purchasing power parity does not hold for indexes $P_{C^i,t}$.

2.2.2 Consumption demand, portfolio choices and labor supply

The consumption and price index definitions allow to solve the consumer problem in two stages. In a first stage, agents decide how much real net income to allocate to buy goods produced at home and abroad. According to the set of optimality conditions, it is possible to determine agents' demands as:

$$C^i_{i,t} = \alpha_s \left(\frac{P_{i,t}}{P_{C^i,t}}\right)^{-\eta} C^i_t \quad C^i_{H,t} = (\alpha_b - \alpha_s)\left(\frac{P_{H,t}}{P_{C^i,t}}\right)^{-\eta} C^i_t \quad C^i_{F,t} = (1-\alpha_b)\left(\frac{P_{F,t}}{P_{C^i,t}}\right)^{-\eta} C^i_t \quad i \in \left[0, \frac{1}{2}\right) \quad (2.8)$$

$$C^i_{i,t} = \alpha_s \left(\frac{P_{i,t}}{P_{C^i,t}}\right)^{-\eta} C^i_t \quad C^i_{F,t} = (\alpha_b - \alpha_s)\left(\frac{P_{F,t}}{P_{C^i,t}}\right)^{-\eta} C^i_t \quad C^i_{H,t} = (1-\alpha_b)\left(\frac{P_{H,t}}{P_{C^i,t}}\right)^{-\eta} C^i_t \quad i \in \left(\frac{1}{2}, 1\right] \quad (2.9)$$

and for $i \in \left[0, \frac{1}{2}\right)$:

$$C^i_{j,t} = 2\left(\frac{P_{j,t}}{P_{H,t}}\right)^{-\eta} C^i_{H,t} \quad j \in \left[0, \frac{1}{2}\right) \quad C^i_{j,t} = 2\left(\frac{P_{j,t}}{P_{F,t}}\right)^{-\eta} C^i_{F,t} \quad j \in \left(\frac{1}{2}, 1\right] \quad (2.10)$$

$$c^i_t(h^j) = \left(\frac{p_t(h^j)}{P_{j,t}}\right)^{-\varepsilon} C^i_{j,t} \quad j \in \left[0, \frac{1}{2}\right) \quad c^i_t(f^j) = \left(\frac{p_t(f^j)}{P_{j,t}}\right)^{-\varepsilon} C^i_{j,t} \quad j \in \left(\frac{1}{2}, 1\right] \quad (2.11)$$

while for $i \in \left(\frac{1}{2}, 1\right]$:

$$C^i_{j,t} = 2\left(\frac{P_{j,t}}{P_{F,t}}\right)^{-\eta} C^i_{F,t} \quad j \in \left[0, \frac{1}{2}\right) \quad C^i_{j,t} = 2\left(\frac{P_{j,t}}{P_{H,t}}\right)^{-\eta} C^i_{H,t} \quad j \in \left(\frac{1}{2}, 1\right] \tag{2.12}$$

$$c^i_t(f^j) = \left(\frac{p_t(f^j)}{P_{j,t}}\right)^{-\varepsilon} C^i_{j,t} \quad j \in \left[0, \frac{1}{2}\right) \quad c^i_t(h^j) = \left(\frac{p_t(h^j)}{P_{j,t}}\right)^{-\varepsilon} C^i_{j,t} \quad j \in \left(\frac{1}{2}, 1\right] \tag{2.13}$$

The second stage coincides with the standard consumer problem. Agents maximize (2.1) with respect to C^i_t, D^i_{t+1} and $N^i_t(s)$ subject to the following sequence of budget constraints:

$$E_t\{Q^i_{t,t+1}D^i_{t+1}\} = D^i_t + W_{i,t}(s)N^i_t(s) - P_{C^i,t}C^i_t + T^i_t \tag{2.14}$$

$$N^i_t(s) = \left(\frac{W_{i,t}(s)}{W_{i,t}}\right)^{-\upsilon^i_t} N^i_t \tag{2.15}$$

where:

$$W_{i,t} \equiv \left[\int_0^1 W_{i,t}(s)^{1-\upsilon^i_t} ds\right]^{\frac{1}{1-\upsilon^i_t}} \tag{2.16}$$

Condition (2.14) is the budget constraint which states that nominal saving, net of lump sum transfers, has to equalize the nominal value of a state contingent portfolio. In fact, $W_{i,t}(s)$ stands for the per hour nominal wage, $Q^i_{t,t+1}$ denotes what is usually called the stochastic discount factor and D^i_{t+1} is the payoff of one period maturity portfolio of firm shares.

Constraint (2.15) is a consequence of a CES aggregation of labor inputs which will be specified below and states that the labour market is monopolistically competitive. Indeed each agent offers a different kind of labour service. Thus υ^i_t stands for the elasticity of demand of labor which is time-varying and region-specific as in Clarida et al. (2002). Finally, (2.16) is simply the aggregate wage index. Domestic and international markets are assumed to be complete.

By the optimality conditions of the household problem:

$$(1+\mu^i_t)N^i_t(s)^{\varphi}C^{i\sigma}_t = \frac{W_{i,t}}{P_{C^i,t}} \tag{2.17}$$

$$\beta\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\sigma}\left(\frac{P_{C^i,t}}{P_{C^i,t+1}}\right) = Q^i_{t,t+1} \tag{2.18}$$

which hold in all states of nature and at all periods and where $\mu_t^i \equiv \frac{1}{\upsilon_t^i - 1}$. According to (2.17), workers set the real wage as mark up over the marginal rate of substitution between consumption and leisure, while the value of the intertemporal marginal rate of substitution of consumption should equalize the stochastic discount factor expressed in terms of the currency of region i. Notice that since wages are perfectly flexible, $N_t^i(s){=}N_t^i$ and $W_{i,t}(s){=}W_{i,t}$ for all s and t.

2.2.3 Firms, technology and price setting

In each region i there is a continuum of firms. Each of them produces a single differentiated good with a constant return to scale technology of the type:

$$y_t(h^i) = A_t^i N_t(h^i) \tag{2.19}$$

with $N_t(h^i) = \left[\int_0^1 N_t^i(s)^{\frac{\upsilon_t^i - 1}{\upsilon_t^i}} ds\right]^{\frac{\upsilon_t^i}{\upsilon_t^i - 1}}$ being the labor input and A_t^i the region-specific technology shock. Given (2.19) and the fact that $N_t^i = N_t^i(s)$ for all h^i, the aggregate relationship between output and labor can be read as:

$$N_t^i = \frac{Y_t^i}{A_t^i} Z_t^i \tag{2.20}$$

where $Y_t^i \equiv \left[\int_0^1 y_t^i(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $Z_t^i \equiv \int_0^1 \frac{y_t(h^i)}{Y_t^i} dh^i$, and $N_t^i \equiv \int_0^1 N_t(h^i) dh^i$. Using (2.10) and (2.11) I will show below that $Z_t \equiv \int_0^1 \left(\frac{p_t(h^i)}{P_{i,t}}\right)^{-\varepsilon} dh^i$; thus Z_t^i can be interpreted as an index of the relative price dispersion across firms. We assume that good prices adjust according to a staggered mechanism *à la* Calvo. Therefore, in each period a given firm can reoptimize its price only with probability $1-\theta$. As a result, the fraction of firms that set a new price is fixed and the aggregate producer price index of the intermediate goods evolves accordingly to:

$$P_{i,t}^{(1-\varepsilon)} = \theta P_{i,t-1}^{(1-\varepsilon)} + (1-\theta)\tilde{p}_{i,t}(h^i)^{(1-\varepsilon)} \tag{2.21}$$

with $\tilde{p}_t(h^i)$ being the optimal price. Firms maximize the discounted expected sum of the future profits that would be collected if the optimal price could not be changed.

$$\sum_{s=0}^{\infty} (\theta)^s E_t \left\{ Q_{t,t+s}^i y_{t+s}(h^i) \left[\tilde{p}_t(h^i) - MC_{i,t+s}^n\right]\right\} \tag{2.22}$$

where $y_t(h^i) = (\frac{p_t(h^i)}{P_{i,t}})^{-\varepsilon} Y_t^i$ and $MC_{i,t}^n = \frac{(1-\tau^i)W_{i,t}}{A_t^i}$ is the nominal marginal cost with τ^i denoting a constant labor subsidy. Taking into account (2.18) and that $MC_{i,t} \equiv \frac{MC_{i,t}^n}{P_{i,,t}}$, the optimality condition of the firm problem can be written as:

$$\sum_{s=0}^{\infty} (\beta\theta)^s E_t \left\{ {C_{t+s}^i}^{-\sigma} \left(\frac{\tilde{p}_t(h^i)}{P_{i,t+s}} \right)^{-\varepsilon} Y_{t+s}^i \frac{P_{i,t}}{P_{C^i,t+s}} \left[\frac{\tilde{p}_t(h^i)}{P_{i,t}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_{i,t+s}}{P_{i,t}} MC_{i,t+s} \right] \right\} = 0 \tag{2.23}$$

Condition (2.23) states implicitly that firms reset their prices as a mark up over a weighted average of the current and expected marginal costs, where the weight of the expected marginal cost at some date $t+s$ depends on the probability that the price is still effective at that date.

2.3 Equilibrium

International risk sharing

The assumption of complete markets implies:

$$\frac{{C_t^i}^{-\sigma}}{P_{C^i,t}} = \frac{{C_t^j}^{-\sigma}}{\mathcal{E}_{ij,t} P_{C^j,t}} \tag{2.24}$$

for all t, $i \in \left[0, \frac{1}{2}\right)$ and $j \in \left(\frac{1}{2}, 1\right]$. According to (2.24), the value of marginal utility of consumption is equalized across regions. However, given the home bias in consumption, even if the law of one price holds, the purchasing power parity does not. As a consequence, consumption can be different across both regions and areas.

By properly integrating this equation we obtain:

$$\frac{{C_t^i}^{-\sigma}}{P_{C^i,t}} = \frac{{C_{H,t}^*}^{-\sigma}}{\mathcal{E}_{iH,t} P_{H,t}^*} \quad i \in \left[0, \frac{1}{2}\right) \quad \frac{{C_t^i}^{-\sigma}}{P_{C^i,t}} = \frac{{C_{F,t}^*}^{-\sigma}}{\mathcal{E}_{iF,t} P_{F,t}^*} \quad i \in \left(\frac{1}{2}, 1\right] \quad \frac{{C_{H,t}^*}^{-\sigma}}{P_{H,t}^*} = \frac{{C_{F,t}^*}^{-\sigma}}{\mathcal{E}_{HF,t} P_{F,t}^*} \tag{2.25}$$

for all i, where $\mathcal{E}_{ij,t}$ stands for the nominal exchange rate of region j currency to region i currency[8]. Here $C_{H,t}^* \equiv \left[2 \int_0^{\frac{1}{2}} {C_t^i}^{-\sigma(1-\eta)} di\right]^{\frac{-1}{\sigma(1-\eta)}}$, $C_{F,t}^* \equiv \left[2 \int_{\frac{1}{2}}^1 {C_t^j}^{-\sigma(1-\eta)} dj\right]^{\frac{-1}{\sigma(1-\eta)}}$, $P_{H,t}^* \equiv \left[2 \int_0^{\frac{1}{2}} \left(\mathcal{E}_{Hj,t} P_{C^j,t}\right)^{(1-\eta)} dj\right]^{\frac{1}{(1-\eta)}}$ and $P_{F,t}^* \equiv \left[2 \int_{\frac{1}{2}}^1 {P_{C^j,t}}^{(1-\eta)} dj\right]^{\frac{1}{(1-\eta)}}$.

[8]... and $\mathcal{E}_{Hj,t}$ stands for the nominal exchange rate of region j currency to a common unit of account of area H.

Regarding conditions (2.25), notice the following. Within area F, there is always a common currency, independently of the policy regime. Thus, $\mathcal{E}_{Fi,t} = 1$ for all $i \in [\frac{1}{2}, 1]$. Conversely within area H, $\mathcal{E}_{Hi,t} = 1$ for all $i \in [0, \frac{1}{2})$ only under regime B when there is a common currency and the exchange rates are fixed. Finally, in general, $\mathcal{E}_{HF,t}$ is floating under both regimes A and B. As shown in the appendix, it follows from to (2.25) and (2.24) that:

$$\frac{P_{i,t}}{P_{C^i,t}} = \left[\gamma_s + (\gamma_b - \gamma_s)\left(\frac{C^*_{H,t}}{C^i_t}\right)^{-\sigma(1-\eta)} + (1-\gamma_b)\left(\frac{C^*_{F,t}}{C^i_t}\right)^{-\sigma(1-\eta)}\right]^{\frac{1}{1-\eta}} \tag{2.26}$$

for $i \in \left[0, \frac{1}{2}\right)$ and where $\gamma_s \equiv \frac{1}{\alpha_s}$ and $\gamma_b \equiv \frac{-\alpha_b}{1-2\alpha_b}$. A corresponding condition can be retrieved for area F:

$$\frac{P_{i,t}}{P_{C^i,t}} = \left[\gamma_s + (\gamma_b - \gamma_s)\left(\frac{C^*_{F,t}}{C^i_t}\right)^{-\sigma(1-\eta)} + (1-\gamma_b)\left(\frac{C^*_{H,t}}{C^i_t}\right)^{-\sigma(1-\eta)}\right]^{\frac{1}{1-\eta}} \tag{2.27}$$

for all $i \in \left(\frac{1}{2}, 1\right]$.

At the same time, (2.6) and (2.7) can be log-linearized as:

$$\hat{p}_{i,t} - \hat{p}^i_{c,t} = -(\alpha_b - \alpha_s)\,\hat{s}_{iH,t} - (1-\alpha_b)\,\hat{s}_{iF,t} \qquad i \in \left[0, \frac{1}{2}\right) \tag{2.28}$$

$$\hat{p}_{i,t} - \hat{p}^i_{c,t} = -(\alpha_b - \alpha_s)\,\hat{s}_{iF,t} - (1-\alpha_b)\,\hat{s}_{iH,t} \qquad i \in \left[\frac{1}{2}, 1\right] \tag{2.29}$$

where $\hat{s}_{iH,t} \equiv e_{iH,t} + \hat{p}_{H,t} - \hat{p}_{i,t}$ and $\hat{s}_{iF,t} \equiv e_{iF,t} + \hat{p}_{F,t} - \hat{p}_{i,t}$ denote the terms of trade of a small open economy i and areas H and F respectively[9] and where $\hat{c}_{H,t} \equiv 2\int_0^{\frac{1}{2}} \hat{c}^j_t dj$ and $\hat{c}_{F,t} \equiv 2\int_{\frac{1}{2}}^1 \hat{c}^j_t dj$[10]. By combining (2.6) and (2.7) with (2.28) and (2.29) and using (2.25) :

$$\hat{s}_{iH,t} = -\frac{\sigma}{\alpha_s}(\hat{c}_{H,t} - \hat{c}^i_t) \quad i \in \left[0, \frac{1}{2}\right) \qquad \hat{s}_{iF,t} = -\frac{\sigma}{\alpha_s}(\hat{c}_{F,t} - \hat{c}^i_t) \quad i \in \left[\frac{1}{2}, 1\right] \tag{2.30}$$

[9]...namely the average price of the goods produced in the small open economy i relative to the average price of the goods produced in areas H and F. With a notational abuse $\hat{p}_{F,t}$ indicates the log-deviation of the average price in area F expressed in terms of the common currency of that area. Similar interpretation applies to $\hat{p}_{H,t}$.

[10]We will use this as a general notation. For a given variable $\hat{x}_t$, $\hat{x}_{H,t} \equiv 2\int_0^{\frac{1}{2}} \hat{x}^j_t dj$ and $\hat{x}_{F,t} \equiv 2\int_{\frac{1}{2}}^1 \hat{x}^j_t dj$.

Moreover, by properly integrating the log-linear approximation of (2.26) and (2.28), it is easy to show that:

$$\hat{s}_{HF,t} = -\sigma \left(\frac{1}{2\alpha_b - 1} \right) (\hat{c}_{F,t} - \hat{c}_{H,t}) \tag{2.31}$$

where $\hat{s}_{HF,t} \equiv \hat{e}_{HF,t} + \hat{p}_{F,t} - \hat{p}_{H,t}$ stands for the terms of trade between area F and area H. According to (2.31), in equilibrium a rise in the terms trade of the two areas reduces their relative consumption ratio as long as $\alpha_b > 1 - \alpha_b$[11]. A terms of trade worsening[12] makes home consumers substitute the goods produced in area F with the goods produced in area H and increase their overall consumption because they relatively prefer the bundle produced in their own area. Notice that the impact of an improvement on the terms of trade on consumption differentials depends critically on the household relative risk adversion (or the inverse of the intertemporal elasticity of substitution of consumption) σ. The higher is σ, the lower is the difference in average consumption across areas associated with a movement in the terms of trade. More risk adverse households are more willing to share risk across different states of the world (or less willing to shift consumption across periods). Finally, by taking (2.30) in differences, it follows:

$$\Delta e_{iH,t} + \pi_{H,t} - \pi_{i,t} = -\sigma\gamma_s(\Delta\hat{c}_{H,t} - \Delta\hat{c}_t^i) \qquad i \in \left[0, \frac{1}{2}\right) \tag{2.32}$$

$$\pi_{F,t} - \pi_{i,t} = -\sigma\gamma_s(\Delta\hat{c}_{F,t} - \Delta\hat{c}_t^i) \qquad i \in \left[\frac{1}{2}, 1\right] \tag{2.33}$$

Equation (2.33), and in regime B also equation (2.32), can be interpreted as a constraint imposed by the adoption of a common currency according to which, in response to asymmetric shocks, the terms of trade cannot adjust instantaneously because of the sluggish price adjustment and the fixed exchange rates. Conversely under regime A in area H, when there is monetary autonomy, the fluctuations of the nominal exchange rates assure that condition (2.32) is always satisfied.

[11] That is $\alpha_b > \frac{1}{2}$ as previously assumed.

[12] ...namely an increase of $\hat{s}_{HF,t}$.

2.3.1 IS curve

Given (2.18) and (2.25), we can recover the following condition for area F:

$$\frac{1}{1+r_{F,t}} = \beta E_t \left\{ \left(\frac{C^*_{F,t+1}}{C^*_{F,t}} \right)^{-\sigma} \Pi^{*\ -1}_{F,t+1} \right\} \tag{2.34}$$

where $\frac{1}{1+r_{F,t}} = E_t\{Q^i_{t,t+1}\}$. When markets are complete, the expected value of the intertemporal marginal rate of substitution of private consumption, namely the price of a riskless portfolio, equalizes the price of the riskless bond, being $r_{F,t}$ the nominal interest rate. The analogue of (2.34) for area H is

$$\frac{1}{1+r^i_t} = \beta E_t \left\{ \left(\frac{C^i_{t+1}}{C^i_t} \right)^{-\sigma} \Pi^{-1}_{C^i,t+1} \right\} \tag{2.35}$$

under regime A and:

$$\frac{1}{1+r_{H,t}} = \beta E_t \left\{ \left(\frac{C^*_{H,t+1}}{C^*_{H,t}} \right)^{-\sigma} \Pi^{*\ -1}_{H,t+1} \right\} \tag{2.36}$$

otherwise. The log-linear approximation of conditions (2.34), (2.35) and (2.36) leads to:

$$r_{F,t} - \rho = E_t\{\pi_{F,t+1}\} - \sigma E_t\{\Delta\hat{c}_{F,t+1} + (1-\gamma_b)(\Delta\hat{c}_{H,t+1} - \Delta\hat{c}_{F,t+1})\} \tag{2.37}$$

$$\begin{aligned} r^i_t - \rho &= E_t\{\pi_{i,t+1}\} - \sigma E_t\{\Delta\hat{c}^i_{t+1} + (\gamma_b - \gamma_s)(\Delta\hat{c}_{H,t+1} - \Delta\hat{c}^i_{t+1}) \\ &\quad +(1-\gamma_b)(\Delta\hat{c}_{F,t+1} - \Delta\hat{c}^i_{t+1})\} \end{aligned} \tag{2.38}$$

$$r_{H,t} - \rho = E_t\{\pi_{H,t+1}\} - \sigma E_t\{\Delta\hat{c}_{H,t+1} + (1-\gamma_b)(\Delta\hat{c}_{F,t+1} - \Delta\hat{c}_{H,t+1})\} \tag{2.39}$$

where $\rho \equiv -log(\beta)$. Conditions (2.37), (2.38) and (2.39) are the so called IS curves. Notice that under regime A, r^i_t can be different across the regions in area H being national central banks independent in their policy decisions. Conversely under regime B, $r^i_t = r_{H,t}$ for all i, being the nominal interest of area H set by the common central bank of the monetary union.

2.3.2 Aggregate demand

In each region i of area H the demand for a specific good, $y_t(h^i)$, is determined by the demand of home and foreign consumers namely:

$$y_t^i(h) = c_{i,t}^i(h) + \int_0^{\frac{1}{2}} c_{i,t}^j(h)dj + \int_{\frac{1}{2}}^1 c_{i,t}^j(h)dj \tag{2.40}$$

for all $i \in \left[0, \frac{1}{2}\right)$. Given (2.8), condition (2.40) can be read as:

$$Y_t^i = \alpha_s \left(\frac{P_{i,t}}{P_{C^i,t}}\right)^{-\eta} C_t + 2(\alpha_b - \alpha_s) \int_0^{\frac{1}{2}} \left(\frac{P_{i,t}}{P_{C^j,t}}\right)^{-\eta} C_t^j dj + 2(1-\alpha_b) \int_{\frac{1}{2}}^1 \left(\frac{P_{i,t}}{P_{C^j,t}}\right)^{-\eta} C_t^j dj \tag{2.41}$$

with $Y_t^i \equiv \left[\int_0^1 y_t^i(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$ Because of (2.24), the aggregate demand for region i can be written as:

$$Y_t^i = \left(\frac{P_{i,t}}{P_{C^i,t}}\right)^{-\eta} \left[\alpha_s C_t^i + (\alpha_b - \alpha_s) C_t^{i\sigma\eta} \mathcal{C}_{H,t} + (1-\alpha_b) C_t^{i\sigma\eta} \mathcal{C}_{F,t}\right] \tag{2.42}$$

with:

$$\mathcal{C}_{H,t} \equiv 2\int_0^{\frac{1}{2}} C_t^{j\,1-\sigma\eta} dj \qquad \mathcal{C}_{F,t} \equiv 2\int_{\frac{1}{2}}^1 C_t^{j\,1-\sigma\eta} dj \tag{2.43}$$

for all $i \in \left[0, \frac{1}{2}\right)$. A symmetric condition can be stated for all $i \in \left(\frac{1}{2}, 1\right]$, namely:

$$Y_t^i = \left(\frac{P_{i,t}}{P_{C^i,t}}\right)^{-\eta} \left[\alpha_s C_t^i + (\alpha_b - \alpha_s) C_t^{i\sigma\eta} \mathcal{C}_{F,t} + (1-\alpha_b) C_t^{i\sigma\eta} \mathcal{C}_{H,t}\right] \tag{2.44}$$

It is easy to show that the first order approximation of (2.42) and (2.44) corresponds to:

$$\hat{y}_t^i = \hat{c}_t^i + (\delta_b - \delta_s)(\hat{c}_{H,t} - \hat{c}_t^i) + (1-\delta_b)(\hat{c}_{F,t} - \hat{c}_t^i) \qquad i \in \left[0, \frac{1}{2}\right) \tag{2.45}$$

$$\hat{y}_t^i = \hat{c}_t^i + (\delta_b - \delta_s)(\hat{c}_{F,t} - \hat{c}_t^i) + (1-\delta_b)(\hat{c}_{H,t} - \hat{c}_t^i) \qquad i \in \left[\frac{1}{2}, 1\right] \tag{2.46}$$

where $\delta_s \equiv \gamma_s\eta\sigma + \alpha_s(1-\eta\sigma)$ and $\delta_b \equiv \gamma_b\eta\sigma + \alpha_b(1-\eta\sigma)$. According to (2.45), the aggregate demand of goods produced in region i depends directly on the terms of trade (through (2.30)). Any terms of trade improvement[13] between region i and areas H or

[13] namely a decrease of $\hat{s}_{iH,t}$ or $\hat{s}_{iF,t}$.

F switches the expenditure of both home and foreign households toward foreign goods. Aggregating (2.45) and (2.46), we obtain:

$$\hat{y}_{H,t} = \hat{c}_{H,t} + (1-\delta_b)(\hat{c}_{F,t} - \hat{c}_{H,t}) \quad i \in \left[0, \frac{1}{2}\right) \tag{2.47}$$

$$\hat{y}_{F,t} = \hat{c}_{F,t} + (1-\delta_b)(\hat{c}_{H,t} - \hat{c}_{F,t}) \quad i \in \left[\frac{1}{2}, 1\right] \tag{2.48}$$

2.3.3 Aggregate supply

Given condition (2.23), the optimal price is determined as:

$$\frac{\tilde{p}_t(h^i)}{P_{i,t}} = \frac{K_t^i}{F_t^i} \tag{2.49}$$

with:

$$K_t^i \equiv \sum_{s=0}^{\infty} (\beta\theta)^s E_t \left[{C_{t+s}^i}^{-\sigma} Y_{t+s}^i \left(\frac{P_{i,t+s}}{P_{i,t}}\right)^{\varepsilon} \frac{P_{i,t+s}}{P_{C^i,t+s}} \frac{\varepsilon}{\varepsilon-1} MC_{i,t+s} \right] \tag{2.50}$$

$$F_t^i \equiv \sum_{s=0}^{\infty} (\beta\theta)^s E_t \left[{C_{t+s}^i}^{-\sigma} Y_{t+s}^i \left(\frac{P_{i,t+s}^i}{P_t^i}\right)^{\varepsilon-1} \frac{P_{i,t+s}}{P_{C^i,t+s}} \right] \tag{2.51}$$

which can be read as:

$$K_t^i = C_t^{i-\sigma} Y_{i,t}^i \frac{P_{i,t}}{P_{C^i,t}} \frac{\varepsilon}{\varepsilon-1} MC_{i,t} + \beta\theta E_t \left\{ \Pi_{i,t+s}^{\varepsilon} K_{t+1}^i \right\} \tag{2.52}$$

$$F_t^i = C_t^{i-\sigma} Y_t^i \frac{P_{i,t}}{P_{C^i,t}} + \beta\theta E_t \left\{ \Pi_{i,t+1}^{\varepsilon-1} F_{t+1}^i \right\} \tag{2.53}$$

where $\Pi_{i,t} \equiv \frac{P_{i,t}}{P_{i,t-1}}$. Following Benigno and Woodford (2005), from (2.49) and (2.21) we can retrieve the next conditions:

$$\frac{1-\theta\Pi_{i,t}^{\varepsilon-1}}{1-\theta} = \left(\frac{F_t^i}{K_t^i}\right)^{\varepsilon-1} \tag{2.54}$$

$$Z_t^i = \theta Z_{t-1}^i \Pi_{i,t}^{\varepsilon} + (1-\theta) \left(\frac{1-\theta\Pi_{i,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.55}$$

By the log-linear approximation of (2.17) (2.50) (2.51) and (2.55):

$$\pi_{i,t} = \lambda \widehat{mc}_t^i + \beta E_t\{\pi_{i,t+1}\} \tag{2.56}$$

with $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$ and where:

$$\widehat{mc}_t^i = \left(\hat{w}_t^i - \hat{p}_{c,t}^i\right) - \left(\hat{p}_{i,t} - \hat{p}_{c,t}^i\right) - \hat{a}_t^i \tag{2.57}$$

for all t and i. Condition (2.56) is the New Keynesian Phillips Curve which results from the Calvo mechanism. As usual, current *domestic* inflation depends on the expectation on future *domestic* inflation and the current real marginal cost of producing goods. Being the economy open in equilibrium this cost is determined by the real wage, which is equal to the marginal rate of substitution between consumption and leisure, the labour productivity and the product price index relative to the consumption price index (2.26) and (2.27). By substituting(2.28) and log-linear approximation of (2.26) we obtain:

$$\begin{aligned}\widehat{mc}_t^i &= \varphi\hat{y}_t^i + \sigma\hat{c}_t^i + (\alpha_b - \alpha_s)\hat{s}_{iH,t} + (1-\alpha_b)\hat{s}_{iF,t} - (1+\varphi)\hat{a}_t^i + \hat{\mu}_t^i \\ &= \varphi\hat{y}_t^i + \sigma\hat{c}_t^i + \sigma\left[(\gamma_b - \gamma_s)\left(\hat{c}_{H,t} - \hat{c}_t^i\right) + (1-\gamma_b)\left(\hat{c}_{F,t} - \hat{c}_t^i\right)\right] - (1+\varphi)\hat{a}_t^i + \hat{\mu}_t^i\end{aligned} \tag{2.58}$$

for all $i \in \left[0, \frac{1}{2}\right)$. According to (2.58), an improvement of the terms of trade of region i[14] lowers firms' real marginal costs. Given (2.58), we can rewrite condition (2.56) for $i \in \left[0, \frac{1}{2}\right)$ and its symmetric condition for $i \in \left[\frac{1}{2}, 1\right]$ as:

$$\pi_{i,t} = \lambda\left[\varphi\hat{y}_t^i + \sigma\left(\gamma_s\hat{c}_t^i + (\gamma_b - \gamma_s)\,\hat{c}_{H,t} + (1-\gamma_b)\,\hat{c}_{F,t}\right) - (1+\varphi)\hat{a}_t^i + \hat{\mu}_t^i\right] + \beta E_t\{\pi_{i,t+1}\} \tag{2.59}$$

$$\pi_{i,t} = \lambda\left[\varphi\hat{y}_t^i + \sigma\left(\gamma_s\hat{c}_t^i + (\gamma_b - \gamma_s)\,\hat{c}_{F,t} + (1-\gamma_b)\,\hat{c}_{H,t}\right) - (1+\varphi)\hat{a}_t^i + \hat{\mu}_t^i\right] + \beta E_t\{\pi_{i,t+1}\} \tag{2.60}$$

Under regime A the rational expectation stochastic equilibrium is characterized by (2.38), (2.45) and (2.59) for all $i \in \left[0, \frac{1}{2}\right)$ and by (2.33), (2.37), (2.46) and (2.60) for all $i \in \left[\frac{1}{2}, 1\right]$, while under regime B by (2.32), (2.39), (2.45) and (2.59) for all $i \in \left[0, \frac{1}{2}\right)$ and by (2.33), (2.37), (2.46) and (2.60) for all $i \in \left[\frac{1}{2}, 1\right]$.

It remains to determine to determine the optimal monetary policy.

[14] namely a decrease of $\hat{s}_{iH,t}$ or $\hat{s}_{iF,t}$.

2.4 Optimal monetary policy problems

As anticipated in the introduction, the main objective of this chapter is to compare in terms of welfare costs and benefits of a monetary union in a fully new-keynesian micro-founded model. For this purpose we consider two policy regimes. Under regime A, while there is a common currency in area F, countries in area H retain their own central banks; by contrast under regime B, there are two monetary unions, one in the area F and the other in the area H. Independently of the policy regimes we assume that all monetary authorities (the central banks of the monetary unions and those of the small open economies) are benevolent, take as given other policy makers' choices and can commit credibly to past and future promises[15]. These hypotheses allow to find the Nash equilibrium policies by using the linear quadratic approach pioneered by Benigno and Woodford (2005) and Benigno and Woodford (2006). Thanks to the optimal policies, it is possible to quantify the difference in welfare for the households of area H across the two policy regimes and to identify which regime is preferable depending on the parameters of the model.

The linear quadratic approach is implemented as follows[16]. First the non-linear optimal policy problems are specified. Second, the zero inflation deterministic steady state of these problems is determined. Then, a purely quadratic approximation to the objectives for both the small open economy and the monetary unions authorities employing the second order approximation of the structural equations are retrieved. Finally the optimal policies can be found by maximizing these quadratic approximations subject to the equilibrium conditions approximated to the first order.

2.4.1 The deterministic steady state

The steady state level of output is determined by a constant and generic labour subsidy τ. We assume τ equal across countries and across regimes. As shown in the appendix,

[15] In other words policies are supposed optimal from the timeless perspective.

[16] For more specific details on the non-linear optimal policy problem, on the zero inflation steady state and on the quadratic approximation, see the appendix.

under these assumptions, for any τ there exists a symmetric deterministic steady state at which zero inflation is a Nash equilibrium policy for all policy makers in areas H and F under both regimes A and B. Thus, the equilibrium equations and the objectives of the policy makers are approximated around the following steady state:

$$Y = (1-\tilde{\tau})^{-\frac{1}{\sigma+\varphi}} \tag{2.61}$$

$$C = Y \tag{2.62}$$

$$F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}(1-\tilde{\tau})}{1-\theta} \tag{2.63}$$

$$\Pi_H = \Pi_F = 1 \qquad Z = 1 \tag{2.64}$$

where

$$\tilde{\tau} \equiv 1-(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1}$$

Allowing for different level of labor subsidies enable us to emphasize two special cases of interest. In particular, as clarified below it is possible to identify the steady state levels of domestic output considered efficient from the viewpoint of both the small open economy authorities and the central banks of the monetary union. Given their different incentives, at the steady state these policy makers consider efficient two different levels of domestic output. As a consequence, they have different "perceptions" of the steady state distortion. As it will be made clear in the next sections, this same divergence is crucial in explaining the different monetary policies over the business cycle.

In the case of the small open economy i, the efficient level of steady state output can be retrieved by maximizing:

$$\sum_{t=0}^{\infty}\beta^t E_0\left[\frac{{C_t^i}^{1-\sigma}}{1-\sigma}-\frac{1}{\varphi+1}\left(\frac{Y_t^i}{A_t^i}\right)^{\varphi+1}\right]$$

with respect to C_t^i and Y_t^i, subject to (2.42) where $P_{i,t}/P_{C^i,t}$ are determined consistently with (2.26), while $C^*_{H,t}$, $C^*_{F,t}$, $\mathcal{C}_{H,t}$ and $\mathcal{C}_{F,t}$ are taken as given. According to the first order conditions at the symmetric deterministic steady state:

$$Y_s = \delta_s^{\frac{-1}{\sigma+\varphi}} \tag{2.65}$$

where, as above, $\delta_s \equiv \gamma_s \eta\sigma + \alpha_s(1 - \eta\sigma)$ which is always greater than 1 as long as $\sigma\eta > 1$. The optimal labour subsidy that allows to implement this allocation is given by:

$$\tilde{\tau}_s = 1 - \delta_s \tag{2.66}$$

Thus, the small open policy makers would not like to reach the Pareto efficient steady state at which the monopolistic distortions are exactly eliminated[17]. They would rather prefer a lower level of steady state production. This is because financial markets are complete and consumption is highly correlated across regions. So domestic utility rises if domestic production falls (relative to other countries' production) and the terms of trade improve[18]. Indeed even if a terms of trade improvement causes consumption to drop, its contraction is more than compensated in terms of welfare by the corresponding increase in leisure. In other words, by manipulating the terms of trade in their favor small open economy policy makers (as those of the monetary union) attempt to externalize labour effort to other countries' workers.

Notice that in the case of the small open economy the incentive to outsource production is stronger the higher is δ_s which depends positively on η, the elasticity of substitution between home and foreign goods, σ, the inverse of the intertemporal elasticity of substitution of consumption and $1 - \alpha_s$ the degree of openness of the small country. Indeed the higher are η and σ, the more home households are inclined to substitute consumption of the domestic goods with that of foreign goods (i.e. the higher is the switching effect), and then the less the overall consumption falls because of the reduction in the domestic production.

In the case of the policy maker of the monetary union, the desired steady state

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[17] The Pareto efficient allocation corresponds to $Y = 1$ which can be achieved by setting $\tilde{\tau} = 0$.

[18] And in fact the optimal labour subsidy is set equal to $1 - \delta_s$, a parameter related with the average elasticity of the domestic goods demand with respect to the terms of trade of the small open economy. As made clear by (2.45) and (2.59), two are the relevant terms of trade from the small open economy point of view: those of the small open economy and areas H and F.

output can be determined by maximizing:

$$\sum_{t=0}^{\infty}\beta^t E_0\left[\int_0^{\frac{1}{2}}\left(\frac{C_t^{i\,1-\sigma}}{1-\sigma}-\frac{1}{\varphi+1}\left(\frac{Y_t^i}{A_t^i}\right)^{\varphi+1}\right)di\right] \tag{2.67}$$

with respect to C_t^i and Y_t^i for all $i \in [0,1]$ subject to:

$$\frac{P_{i,t}}{P_{C^i,t}}=\frac{(1-\tilde{\tau})}{A_t^{i\,\varphi+1}}\frac{Y_t^{i\varphi}}{C_t^{i\,-\sigma}} \tag{2.68}$$

for all $i \in [\frac{1}{2},1]$, (2.42) and (2.44) and where $P_{i,t}/P_{C^i,t}$, $\mathcal{C}_{H,t}$ and $\mathcal{C}_{F,t}$ are determined according to (2.26), (2.27) and (2.43)[19]. F rom the first order conditions of this problem it follows that at the symmetric steady state:

$$Y_b=\left[1-\frac{(1-\delta_b)(\sigma+\varphi)}{(\delta_b\varphi+\gamma_b\sigma)}\right]^{\frac{-1}{\sigma+\varphi}} \tag{2.69}$$

This allocation can be achieved by setting the labour subsidy:

$$\tilde{\tau}_b=\frac{(1-\delta_b)(\sigma+\varphi)}{(\delta_b\varphi+\gamma_b\sigma)} \tag{2.70}$$

where $\delta_b \equiv \gamma_b\eta\sigma+\alpha_b(1-\eta\sigma)$ which is always greater than 1 as long as $\sigma\eta>1$. According to these conditions, even in the case of the big economy, the policy makers seek to improve the terms of trade by reducing domestic production with respect to what would be Pareto efficient. However by the comparing (2.69) and (2.65), it can be shown that under reasonable calibrations:

$$1>Y_b>Y_s \tag{2.71}$$

Thus, at the symmetric steady state, the policy maker of the monetary union would choose a level of domestic output higher than that considered efficient by the small open economy authorities. The reasons for this outcome are threefold. First of all, bigger countries are less open. Then the incentive of their policy makers to improve the terms of trade is weaker. Secondly, big economy authorities realize to hold monopoly

[19] Implicitly (2.68) states that the policy maker of area H takes as given the strategy $\tilde{\tau}$ chosen by a symmetric policy maker in area F.

power only on the terms of trade across areas[20] and they internalize the external effects produced within the monetary union. Finally they take into account the impact of their policies on the foreign economy. In particular they are aware that a terms of trade improvement causes an increase in foreign production thanks to the boost in foreign good demand[21]. So they recognize that a lower labor tax rate (lower than that set by the small open economy policy maker which take as given foreign output) allows to reach the same desired level of domestic/foreign output ratio. All these motives contribute to weaken the desire of influencing their terms of trade.

Summing up, the difference in size between small and big countries affects the incentives of their policy makers and thus the desired steady state level of domestic output. Specifically in the case of the monetary union this level is closer to Pareto efficiency than in the case of the small open economy. As explained in the next sections, these different "perceptions" in the steady state distortion are key even for optimal policy decisions over the business cycle.

2.4.2 The case of a closed economy

In this section we step behind doing a small digression to explain how and why the steady state distortion influences optimal monetary policy decisions in a closed economy[22]. To this end consider the approximation of objective of the small open economy policy maker[23] in the limiting case of $\alpha_s = \alpha_b = 1$:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\left[\varpi_{c,1}\pi_t^2 + \varpi_{c,2}(\tilde{y}_t^c)^2 - 2\varpi_{c,3}\tilde{c}_t^c\tilde{y}_t^c + \varpi_{c,4}(\tilde{c}_t^c)^2\right] + t.i.p. \tag{2.72}$$

[20]This explains the dependence of $\tilde{\tau}_b$ on $1-\delta_b$, a parameter that governs the elasticity of aggregate demand for the domestic goods to the terms of trade across areas.

[21]...at least under flexible prices which is the relevant case for the steady state analysis.

[22]On this topic the seminal contributions have been those of Benigno and Woodford (2005) and Benigno and Woodford (2006).

[23]See the appendix.

with:

$$
\begin{aligned}
\varpi_{c,1} &\equiv [1-\zeta_c(\varphi+1)]\,\frac{\varepsilon}{\lambda} \\
\varpi_{c,2} &\equiv [1-\zeta_c(\varphi+1)]\,\varphi \\
\varpi_{c,3} &\equiv \zeta_c\sigma \\
\varpi_{c,4} &\equiv \left[(1-\tilde{\tau})(\sigma-1)+\zeta_c\sigma^2+(1-\zeta_c\varphi)\right] \\
\zeta_c &\equiv \frac{\tilde{\tau}}{\varphi+\sigma}
\end{aligned}
$$

and where t.i.p. stands for terms independent of policy. (2.72) expresses the utility losses (approximated up to second order) as function of inflation and the welfare-relevant consumption and output gap. In fact $\tilde{x}_t^c \equiv \hat{x}_t - \hat{x}_t^c$ denotes the deviations of x_t from the target of monetary authority when the economy is closed. This target can be retrieved by equations (B.40) -(B.43) under the assumption that $\alpha_s = \alpha_b = 1$ and satisfies the next conditions:

$$[1-\zeta_c(\varphi+1)]\,(\widehat{mrs}_t^c - \widehat{mrt}_t^c) = \zeta_c(\varphi+1)\hat{\mu}_t \tag{2.73}$$

$$\hat{y}_t^c = \hat{c}_t^c \tag{2.74}$$

with

$$\widehat{mrs}_t^c - \widehat{mrt}_t^c = \varphi\hat{y}_t^c + \sigma\hat{c}_t^c - (\varphi+1)\hat{a}_t$$

$\widehat{mrs}_t^c$ and $\widehat{mrt}_t^c$ represent the log-deviation of the marginal rate of substitution and of the marginal rate of transformation between consumption and output.

Thanks to conditions (2.73) and (2.74) we can achieve the following conclusions about the goals of the monetary authority:

1. When the steady state is efficient (i.e. $\tilde{\tau} = 0$ and $\zeta_c = 0$), we go back to the standard result of closed economy literature[24] for which the central bank would like to close the gap between $\widehat{mrs}_t^c$ and $\widehat{mrt}_t^c$ because in this way it reaches the first best allocation.

[24] See among others Galí (2008) and Woodford (2003).

2. Under technological shocks, the monetary authority still wishes to close that gap even if the steady state is inefficient (i.e. $\zeta_c \neq 0$). Indeed in that case she cannot influence the distortions due to the steady state inefficiency. Therefore she seeks to replicate the fluctuations of the first best allocation. In other words, under technological shocks, the target of the monetary authority is not affected by the steady state inefficiency because the flexible price allocation is constrain efficient.

3. Conversely under mark up shocks the monetary authority is willing to bear a difference between $\widehat{mrs}_t^c$ and $\widehat{mrt}_t^c$, where this difference depends on the mark up shocks themselves and on the size of the steady state distortion. In particular if the steady state output is inefficiently high (i.e. $\zeta_c > 0$), the monetary authority wants output to negatively comove with these shocks. As a result, the central bank would focus more on inflation than output stabilization (more than what would do if the steady state were efficient) given that a positive mark up shock tends to reduce output. Viceversa[25] an inefficient low level of steady state output (i.e. $\zeta_c < 0$) would imply a monetary policy that weighs more output than inflation stabilization.

At a first glance the third result is quite puzzling: mark up shocks generate inefficient fluctuations in consumption and output. Intuitively we could expect that then the central bank would like to completely stabilize output and consumption (as in fact it is willing to do when the steady state is efficient). Instead, it wants output and consumption to react to these shocks. Why? The underling reason can be understood by considering condition (2.17) (in its closed economy case counterpart), when prices are flexible and there are no shocks to technology:

$$E\left\{\frac{W_t}{P_t}\right\} = E\left\{Y_t^{\varphi+\sigma}\right\} E\left\{(1+\mu_t)\right\} + Cov\left\{Y_t^{\varphi+\sigma}(1+\mu_t)\right\} \tag{2.75}$$

According to (2.75), the lower is the covariance between mark up shocks and output, the higher is the average per-period output for a given level of per-period real wage.

[25]under the parametric restriction: $\tilde{\tau} < \frac{\sigma+\varphi}{\varphi+1}$.

Indeed, if output fluctuates more in response to mark up shocks - which corresponds to a decrease in the covariance between output and the mark up shocks themselves - consumers have to rise on average their labour effort in order to get the same real wage. Then if output is on average inefficiently low because of the steady state distortion, allowing for negative comovements between output and mark up shocks have beneficial effects because it shifts downward the average supply curve engendering an efficient increase in the average level of per-period output.

2.4.3 The case of the small open economy

As shown in the appendix, the objective of the small open economy policy maker of country i in area H can be approximated up to the second order as:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\left[\varpi_{s,1}(\pi_{i,t})^2+\varpi_{s,2}(\tilde{y}_t^{i,s})^2-2\varpi_{s,3}(\tilde{c}_t^{i,s}\tilde{y}_t^{i,s})+\varpi_{s,4}(\tilde{c}_t^{i,s})^2\right]+t.i.p. \tag{2.76}$$

where

$$\varpi_{s,1}\equiv[1-\zeta_s(\varphi+1)]\frac{\varepsilon}{\lambda}$$
$$\varpi_{s,2}\equiv[1-\zeta_s(\varphi+1)]\varphi$$
$$\varpi_{s,3}\equiv\zeta_s\gamma_s\sigma$$
$$\varpi_{s,4}\equiv(1-\tilde{\tau})(\sigma-1)+\zeta_s\gamma_s^2\sigma^2+(1-\zeta_s\varphi)(\delta_s+\omega_1)$$
$$\zeta_s\equiv\frac{\delta_s-(1-\tilde{\tau})}{\delta_s\varphi+\gamma_s\sigma}$$

ω_1 is properly defined in the appendix and $\tilde{x}_t^{i,s}\equiv\hat{x}_t^i-\hat{x}_t^{i,s}$. $\hat{x}_t^{i,s}$ indicates the target of the small open economy monetary authority which is determined by (B.40) -(B.43)[26]. Notice that in the case of the small open economy the *t.i.p.*, the terms independent of monetary policy, include the aggregate variables of both areas H and F.

[26]This target can be interpreted as the constrained efficient allocation from the small open economy viewpoint, namely the allocation that would be chosen by a small open economy policy maker that has as objective the maximization of (2.76) subject *exclusively* to constraint (2.45).

The welfare approximation in (2.76) contains the output gap, the consumption gap and inflation like in a closed economy. What is crucially different are the weights attached to these variables and the target that the authority would like to implement. This divergence with respect to the closed economy case is rationalized again by the desire of open economy policy makers to manipulate the terms of trade in their favour. In fact, on the one hand, this incentive works even over the business cycle and gives reason, for instance, for the higher weight attributed to consumption gap volatility: policy makers realize that fluctuations in consumption are associated with fluctuations in the terms of trade. On the other hand, this same incentive explains why from the small open economy policy makers viewpoint, the steady state is efficient as long as $Y = Y_s$ - which implies $\tilde{\tau} = \tilde{\tau}_s$ and thus $\zeta_s = 0$ - and not when $Y = Y_c = 1$ as in a closed economy. This has clear consequences for the weights in (2.76) (given that ζ_s depends critically on the difference between $\tilde{\tau}$ and $\tilde{\tau}_s$) and for channels through which openness modifies the conduct of small open economy central banks.

To better investigate these channels consider the subsequent conditions:

$$[1 - \zeta_s(\varphi + 1)](\widehat{mrs}^s_{H,t} - \widehat{mrt}^s_{H,t}) = \zeta_s(\varphi + 1)\hat{\mu}_{H,t} + \kappa_s \hat{s}^s_{HF,t} \tag{2.77}$$

$$\hat{y}^s_{H,t} = \hat{c}^s_{H,t} + (1 - \delta_b)\frac{(2\alpha_b - 1)}{\sigma}\hat{s}^s_{HF,t} \tag{2.78}$$

where

$$\widehat{mrs}^s_{H,t} - \widehat{mrt}^s_{H,t} = \varphi\hat{y}^s_{H,t} + \sigma\hat{c}^s_{H,t} - (\varphi + 1)\hat{a}_{H,t} + \sigma(1 - \gamma_b)(\hat{c}_{F,t} - \hat{c}^s_{H,t})$$

$$\hat{s}^s_{HF,t} = -\frac{\sigma}{2\alpha_b - 1}(\hat{c}_{F,t} - \hat{c}^s_{H,t})$$

$$\kappa_s \equiv (2\alpha_b - 1)\sigma^{-1}\delta_s^{-1}\left[(1 - \zeta_s\varphi)(\sigma(1 - \gamma_b)\delta_b - \omega_2) + \zeta_s\gamma_s((1 - \delta_b) - \sigma(1 - \gamma_b))\right]$$

Conditions (2.77) and (2.78) are recovered by properly rearranging and integrating (B.40) -(B.43), the equations determining the target[27] of the small open economy authority. The comparison with their akin of the closed economy, namely (2.73) and (2.74), allows to stress the following findings:

[27]under the assumption that the target is *implemented* which ensures that $\int_0^{\frac{1}{2}} \hat{x}^{i,s}_t di = \hat{x}_{H,t}$.

1 As indicated by the terms $\kappa_s \hat{s}_{HF,t}$, even when the steady state is efficient from the small open economy viewpoint (i.e. $\zeta_s = 0$), in general, the target does not coincide with the flexible price allocation. This is because small country policy makers try to manipulate their terms of trade even over the business cycle[28].

2 The target reacts to domestic mark up shocks if and only if there is a steady state inefficiency from the small open economy perspective (i.e. $\zeta_s \neq 0$).

This second result confirms our intuition that from the small country viewpoint the welfare relevant distortion is determined by the difference between $\tilde{\tau}$, the actual steady state labour subsidy, and $\tilde{\tau}_s = 1 - \delta_s$, its desired level (which in turn governs the value of ζ_s). As long as $\tilde{\tau} \neq \tilde{\tau}_s$, the small open policy maker considers inefficient the average per-period wedge between the marginal rate of substitution between consumption and labour and its marginal rate of transformation. In particular, under the baseline calibration the per-period output is regarded as inefficiently high (i.e $\tilde{\tau} > 1 - \delta_s$ and $\zeta_s > 0$). Indeed, once the aggregate world variables are taken as given, at the margin an increase in leisure rises utility by more than an increase in consumption. This generates a motive for the central banks of the small open economies to seek to squeeze the average per-period output and to modify their inflation output trade-off. In fact, by focusing more on output than on inflation stabilization[29] in response to mark up shocks, these authorities can induce domestic households to work more. In this way per-period domestic output, which is perceived as too high, can fall.

The *timelessly* optimal monetary policy can be retrieved by maximizing (2.76) with respect to $\tilde{y}_t^{i,s}$, $\tilde{c}_t^{i,s}$ and $\pi_{i,t}$ subject to the following sequence of constraints:

$$\tilde{y}_t^i = \delta_s \tilde{c}_t^i \tag{2.79}$$

$$\pi_{i,t} = \lambda \left[\varphi \tilde{y}_t^{i,s} + \sigma \gamma_s \tilde{c}_t^{i,s} \right] + \lambda \upsilon_t^{i,s} + \beta E_t \{ \pi_{i,t+1} \} \tag{2.80}$$

for all t where:

[28]Notice that not surprisingly if $\alpha_b = 1$ then $\kappa_b = 0$. In fact if the area is closed, then on average the effects due to the terms of trade externality disappear.

[29] as long as $\frac{\delta_s \varphi + \gamma_s \sigma}{\varphi + 1} > \delta_s - (1 - \tilde{\tau})$.

$$\upsilon_t^{i,s} = \varphi\hat{y}_t^{i,s} + \sigma\gamma_s\hat{c}_t^{i,s} + \sigma\left(\gamma_b - \gamma_s\right)\hat{c}_{H,t} + \sigma\left(1 - \gamma_b\right)\hat{c}_{F,t} - (1+\varphi)\hat{a}_t^i + \hat{\mu}_t^i.$$

2.4.4 The case of the monetary union

As shown in the appendix, if there is a monetary union in area H, the objective of the monetary policy maker can be approximated in a purely quadratic way as:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[\varpi_{b,1}(\pi_{H,t})^2 + \varpi_{b,2}(\tilde{y}_{H,t}^b)^2 + 2\varpi_{b,3}\tilde{c}_{H,t}^b\tilde{y}_{H,t}^b + 2\varpi_{b,4}\tilde{c}_{F,t}^b\tilde{y}_{H,t}^b + \varpi_{b,5}(\tilde{y}_{F,t}^b)^2$$
$$+2\varpi_{b,6}\tilde{c}_{F,t}^b\tilde{y}_{F,t}^b + 2\varpi_{b,7}\tilde{c}_{H,t}^b\tilde{y}_{F,t}^b + \varpi_{b,8}(\tilde{c}_{F,t}^b)^2 + \varpi_{b,9}(\tilde{c}_{H,t}^b)^2 + 2\varpi_{b,10}\tilde{c}_{H,t}^b\tilde{c}_{F,t}^b\Big] + t.i.p. \tag{2.81}$$

where

$$\varpi_{b,1} \equiv [1 - \zeta_b(\varphi+1)]\frac{\varepsilon}{\lambda}$$
$$\varpi_{b,2} \equiv [1 - \zeta_b(\varphi+1)]\varphi$$
$$\varpi_{b,3} \equiv -\zeta_b\sigma\gamma_b$$
$$\varpi_{b,4} \equiv -\zeta_b\sigma\left(1-\gamma_b\right)$$
$$\varpi_{b,5} \equiv -(\xi - \zeta_b)(\varphi+1)\varphi$$
$$\varpi_{b,6} \equiv -(\xi - \zeta_b)\sigma\gamma_b$$
$$\varpi_{b,7} \equiv (\xi - \zeta_b)\sigma\left(1-\gamma_b\right)$$
$$\varpi_{b,8} \equiv -(1-\zeta_b(\varphi+1))(1-\delta_b) - (\xi-\zeta_b)\varphi\delta_b + \zeta_b\sigma^2(1-\gamma_b)^2 + (1-\zeta_b\varphi)\eta\sigma^2\gamma_b(1-\gamma_b)$$
$$\varpi_{b,9} \equiv (\sigma-1)(1-\tilde{\tau}) + (1-\zeta_b(\varphi+1))\delta_b - (\xi-\zeta_b)\varphi(1-\delta_b) - (1-\zeta_b\varphi)\eta\sigma^2\gamma_b(1-\gamma_b)$$
$$+\zeta_b\sigma^2\gamma_b^2 + (\xi-\zeta_b)\sigma^2\left(1-\gamma_b\right)^2$$
$$\varpi_{b,10} \equiv (1-\zeta_b\varphi)\eta\sigma^2\gamma_b(1-\gamma_b) + \zeta_b\sigma^2\gamma_b(1-\gamma_b) + (\xi-\zeta_b)\sigma^2\left(1-\gamma_b\right)\gamma_b$$
$$\zeta_b \equiv \frac{1}{2}\frac{\tilde{\tau}}{\sigma+\varphi} - \frac{\delta_b - 1 + (1/2)\tilde{\tau}}{(1-2\gamma_b)\sigma + (1-2\delta_b)\varphi}$$
$$\xi \equiv \frac{\tilde{\tau}}{\sigma+\varphi}$$

and $\tilde{x}_t^b \equiv \hat{x}_t - \hat{x}_t^b$. $\hat{x}_t^b$ denotes the target of the monetary union central bank which can be determined from (2.47)-(2.48) and (B.59)-(B.63)[30]. In addition *t.i.p.*, the terms independent of policy, include the state contingent path of $\pi_{F,t}$ decided by the policy maker of the monetary union in area F and the differentials between country specific and average union variables[31].

To grasp some insights the incentives driving the optimal monetary policy of the monetary union consider the following "targeting" condition:

$$[1-\zeta_b(\varphi+1)](\widehat{mrs}^b_{H,t}-\widehat{mrt}^b_{H,t})-(\xi-\zeta_b)(\varphi+1)(\widehat{mrs}^b_{F,t}-\widehat{mrt}^b_{F,t})=$$
$$\zeta_b(\varphi+1)\hat{\mu}_{H,t}+(\xi-\zeta_b)(\varphi+1)\hat{\mu}_{F,t}+\kappa_b\hat{s}^b_{HF,t} \tag{2.82}$$

where:

$$\widehat{mrs}^b_{H,t}-\widehat{mrt}^b_{H,t}=\varphi\hat{y}^b_{H,t}+\sigma\hat{c}^b_{H,t}-(\varphi+1)\hat{a}_{H,t}+\sigma(1-\gamma_b)(\hat{c}^b_{F,t}-\hat{c}^b_{H,t})$$

$$\widehat{mrs}^b_{F,t}-\widehat{mrt}^b_{F,t}=\varphi\hat{y}^b_{F,t}+\sigma\hat{c}^b_{F,t}-(\varphi+1)\hat{a}_{F,t}+\sigma(1-\gamma_b)(\hat{c}^b_{F,t}-\hat{c}^b_{F,t})$$

$$\kappa_b\equiv(2\alpha_b-1)\sigma^{-1}(1-\zeta_b\left[(\varphi+\sigma))((1-\delta_b)-\sigma(1-\gamma_b))-(\xi-\zeta_b)(\varphi+\sigma)(\delta_b-\sigma\gamma_b)\right]$$

Thus $\widehat{mrs}^b_{H,t}$ and $\widehat{mrt}^b_{H,t}$ (as $\widehat{mrs}^b_{F,t}$ and $\widehat{mrt}^b_{F,t}$) stand for the average marginal rate of substitution and transformation between consumption and output in area H (in area F). Like its analogue (2.77), condition (2.82) is derived from the equations that determine the target of the monetary union policy makers, namely (2.47)-(2.48) and (B.59)-(B.63).

Condition (2.82) leads to the next conclusions:

1 Differently from the case of the small open economy, the common central bank in area H wants to stabilize a weighted average between the gap between $\widehat{mrs}^b_{H,t}$ and $\widehat{mrt}^b_{H,t}$ in area H and this same gap in area F. In fact, the monetary authority of the currency area takes into account how its decisions affect the demand and the supply of foreign goods and the related feedback effects on its own economy.

[30] namely the constraint efficient allocation from the perspective of the policy maker of area H. This allocation corresponds to the allocation chosen by a policy maker that maximizes (2.81) subject *exclusively* to constraints (2.47) and (2.48). See the appendix.

[31] Indeed, by choosing the average union inflation, the common central bank can influence only the average union performance. However, these terms have to be taken into account for the welfare evaluation.

2 However the central bank of the monetary union attaches different weights to home and foreign variables.

3 Moreover that authority balances the need to stabilize the gaps between the marginal rates of substitution and transformation with a twofold desire: on the one hand, as indicated by the term κ_b, it wants to manipulate the terms of trade in its favor over the business cycles; on the other hand, in the presence of domestic mark up shocks, it seeks to influence the average per-period levels of both domestic and foreign output.

These features are direct consequence of the desire to improve the terms of trade already emphasized above. This incentive stems from a free riding problem. Under the assumption of complete financial markets, consumption is highly correlated across countries. Given that labour effort lowers utility, this risk sharing in consumption generates a conflict on where to produce output. Indeed, the higher is the substitutability between home and foreign bundles, the more countries wish to outsource production and squeeze domestic output *relatively* to foreign output. In this manner they can reduce labour effort without decreasing too much consumption.

This mechanism gives reason of why the size of the economy shapes optimal monetary policy decisions. In the limiting case in which the economy is small, the only way monetary policy can lessen domestic/foreign relative output ratio is through a contraction of domestic production. In fact, the economic performance of a single small open economy is irrelevant for aggregate output behavior. Conversely when the economy is big, policy makers realize that their decisions can diminish the domestic/foreign per-period output ratio through either a reduction of the domestic output or an increase of the foreign one. As consequence, and as highlighted by condition (2.82), the target of the monetary union central bank depends on foreign mark up shocks and attaches asymmetric weights to domestic and foreign economic performances. In particular under the baseline calibration, per-period foreign production is considered inefficiently low even more than domestic one[32]. In other words, policy makers of the big economies want households abroad to work more than consumers at home.

The optimal monetary policy problem of the common central bank in area H can be formulated as maximizing (2.81) with respect to $\tilde{y}^b_{H,t}$, $\tilde{y}^b_{F,t}$, $\tilde{c}^b_{H,t}$, $\tilde{c}^b_{F,t}$ and $\pi_{H,t}$ subject to the following sequence of constraints:

$$\tilde{y}^b_{H,t} = \tilde{c}^b_{H,t} + (1-\delta_b)(\tilde{c}^b_{F,t} - \tilde{c}^b_{H,t})$$

$$\tilde{y}^b_{F,t} = \tilde{c}^b_{F,t} + (1-\delta_b)(\tilde{c}^b_{H,t} - \tilde{c}^b_{F,t})$$

$$\pi_{H,t} = \lambda\left[\varphi\tilde{y}^b_{H,t} + \sigma\left(\tilde{c}^b_{H,t} + (1-\gamma_b)\left(\tilde{c}^b_{F,t} - \tilde{c}^b_{H,t}\right)\right)\right] + \lambda\upsilon^b_{H,t} + \beta E_t\{\pi_{H,t+1}\}$$

$$\pi_{F,t} = \lambda\left[\varphi\tilde{y}^b_{F,t} + \sigma\left(\tilde{c}^b_{F,t} + (1-\gamma_b)\left(\tilde{c}^b_{H,t} - \tilde{c}^b_{F,t}\right)\right)\right] + \lambda\upsilon^b_{F,t} + \beta E_t\{\pi_{F,t+1}\}$$

for all t where

$$\tilde{\upsilon}^b_t = \varphi\hat{y}^b_{H,t} + \sigma\hat{c}^b_{H,t} + \sigma(1-\gamma_b)\left(\hat{c}^b_{F,t} - \hat{c}^b_{H,t}\right) - (1+\varphi)\hat{a}_{H,t} + \hat{\mu}_{H,t}$$
$$\tilde{\upsilon}^b_{F,t} = \varphi\hat{y}^b_{F,t} + \sigma\hat{c}^b_{F,t} + \sigma(1-\gamma_b)\left(\hat{c}^b_{H,t} - \hat{c}^b_{F,t}\right) - (1+\varphi)\hat{a}_{F,t} + \hat{\mu}_{F,t}$$

The solution to this problem allows to determine the average inflation in area H and all the other area variables, given a state contingent path of the average inflation in area F. A symmetric problem can be stated for the foreign area. Notice that once the average union variables are determined, the region specific variables can be recovered directly from the equilibrium conditions namely (2.32), (2.33), (2.45), (2.46), (2.59) and (2.60). Moreover, under this formulation, the optimal monetary policy problem is independent of whether there is either monetary autonomy among countries or a monetary union in the other area.

2.5 Optimal monetary policies

The solution to the optimal policy problems of both the small open policy maker and the central bank of the monetary union enable us to simulate the impulse responses to

[32]Notice that in order to reach the efficient level of foreign output, the labour subsidy should be set equal to $\tilde{\tau} = -\frac{(1-\delta_b)(\sigma+\varphi)}{((1-\delta_b)\varphi+(1-\gamma_b)\sigma)}$, a level such that foreign labour is over-subsidized!

a one percent decrease in home and foreign mark ups under regimes A and B. These impulses responses are plotted in figures 2.1-2.2. The baseline calibration is listed in the appendix and is in line with the literature[33].

2.5.1 Dynamic Simulation

The impulse responses to a global negative mark up shock can be interpreted as follows.

As shown in figures 2.1 and 2.2, under optimal policies, given the fall in their marginal costs, both home and foreign firms cut prices and expand output supply. Workers increase consumption and reduce leisure. Monetary policies have then to trade off between output and inflation stabilization. These patterns are common to both areas and regimes. However, under regime A, consumption and output in area H increase by less than in regime B, while deflation in area H and output in area F increase by more. These differences are explained *exclusively* by the diverging conduct of policy makers under the two policy regimes.

Regime B. Under regime B, when there are two currency unions, impulse responses are symmetric across areas. Under the baseline calibration both domestic and foreign per-period output are perceived as too low. However, the distortion in the foreign production is considered relatively stronger (i.e. the desired steady state output ratio $\frac{Y_H}{Y_F} < 1.$). As consequence under global mark up shocks, monetary union policy makers would like foreign output to fluctuate relatively more. In other words they attempt to generate a positive covariance between mark up shocks and their terms of trade in order to induce foreign consumers to rise their production by more than domestic households. Obviously, in equilibrium none of the policy makers in area H and F reaches her goal. Indeed given symmetry, home and foreign output perfectly commove in such a way that their relative average per-period ratio is always equal to one.

Regime A. Under regime A, the conduct of the monetary policy makers in area

[33] See in particular Galí and Monacelli (2009), Galí and Monacelli (2005) and Pappa (2004).

H is dissimilar from that in regime B in two respects. On the one hand, under the baseline calibration, the per-period domestic output is too high from the small open economy perspective. As a result, when there is monetary independence, they are more focused on output stabilization than the central bank of the monetary union. Indeed, in response to a negative mark up shock, they seek to restrain output expansion and allow for a higher deflation by increasing on average the nominal interest rate by more than what the single central bank of the monetary union does in regime B. In this way they push the economy in the direction of an improvement of the terms of trade in their area. In fact being their economies small, these monetary authorities consider what happens in the world economy as exogenous. Thus, they do not take into account (as the monetary authority of a currency area does) how their joint action affects per-period foreign production. Therefore they do not realize that boosting the negative covariance between foreign production and mark up shocks can be beneficial: it induces foreign workers to produce additional output that can be consumed even by domestic households thanks to the consumption risk sharing.

Given the restrictive monetary policy in area H, the monetary authority of area F restrains monetary policy as well, but not as much as the central banks of area H, allowing for a terms of trade worsening. By doing so, she wants to oppose the restrictive policies of the other area, because she finds an expansion of foreign output beneficial. Nevertheless, she also wants to stabilize domestic price dynamics. Deflation response in area F is similar across regimes, whereas output and consumption are influenced by the restrictive policy of the policy makers in area H.

There is a crucial question that is still left open. When are the consumers of area H better off? In regime A or in regime B? This question is addressed in the next section.

2.6 Welfare evaluation

The analysis of the previous section reveals that, in the presence of mark up shocks, there are potential welfare benefits from the adoption of a common currency. Moreover, it makes clear which are the sources of these benefits: on the one hand the internalization of the spillover effects generated within area H; on the other hand the gains in monopoly power in controlling the terms of trade across areas. The household welfare based criterion derived in (2.81) allows to quantify the welfare gains of being in a currency area as average per-period losses expressed as a fraction of the steady state consumption. The results are quite robust: under mark up shocks, even for relatively low levels of the elasticity of substitution between home and foreign bundles, there are welfare benefits of forming a monetary union. In the next sections we analyze how these benefits vary according to the key parameters of the model.

The intertemporal and the intratemporal elasticities of substitution. Both the intratemporal elasticity of substitution between home and foreign bundles, η, and the relative risk adversion coefficient (the inverse of intertemporal elasticity of substitution of consumption), σ, are crucial to determine the size of the welfare gains (or losses) of abandoning monetary autonomy[34]. Indeed they influence directly the effects that movements in the terms of trade produce on the demand of foreign goods. The higher are η and σ[35], the larger is the switching effect from domestic towards foreign goods, the stronger is the increase in foreign production due to a terms of trade improvement and the more domestic production (and leisure) decreases allowing home households to reach a higher level of utility. Summing up, these parameters govern the real effects of the beggar-thy-neighbour policies and therefore the benefits of policy coordination that

[34]This finding is actually consistent with the literature. See in particular Benigno and Benigno (2003), Benigno and Benigno (2006) and Pappa (2004).

[35]The lower is the intertemporal elasticity of consumption, the higher is the incentive to smooth consumption across periods. Thus, when there is a terms of trade improvement, consumers are more inclined to keep the same level of overall consumption, buying more foreign goods or working more to substitute between the present and future consumption.

arise from being in a monetary union. Figure 2.3 plots how welfare benefits increase in area H relatively to an increase of η and σ. η varies from 1 to 3, while σ varies from 1 to 2.5. Within this range, these gains reach a maximum of 0.3 percentage of the steady state consumption. However, for low levels η the adoption of a common currency brings about welfare losses up to 0.1 percentage of steady state consumption.

The degree of home bias. The welfare benefits of a monetary union are due to two main channels: the internalization of all the external effects produced within the monetary union by the national authorities; the gains of monopoly power (due to the bigger size of the area) on the terms of trade (and thus average output differentials) across areas. A relevant question is which of these channels contributes more to explain the welfare benefits themselves. For this reason, we investigate to what extent these gains depend on the degree of home bias of area H, α_b.

Figure 2.4 plots the welfare gains of being in the regime B for the consumers of area H relatively to different degree of η (from 1 to 3) and α_b (from 0.6 to 1) and shows the following result. For low degree of η the welfare gains - which are actually losses - are lower in a closed economy (i.e. $\alpha_b = 1$), whereas for high degree of η the converse is true. This finding can be explained as follows. If η and α_b are high, the main sources of welfare gains is due to the elimination of the spillover effects within the union in area H. Indeed if the area is very closed, the welfare benefits due to an increase in control on the terms of trade across area and on average area output differential are not important. However if η and α_b are low the main gain in adopting the same currency is due to the internalization of both the impact of its actions on the foreign area and related feedback effects on the same area H.

In order to better disentangle these two sources of welfare gains, it would be useful to allow for different elasticities of substitution between bundles produced in different regions and in different areas. In this way, in fact, it would be possible to understand how the welfare gains of forming a monetary union vary in response to a variation of a parameter, the elasticity of substitution between bundles produced in different areas, that affects *exclusively* exeternalities generated by the big economy on the terms of trade across areas.

The correlation between region specific shocks. For the purpose of this chapter, it is important to check how welfare gains depend on the correlation between region specific shocks. Indeed, this correlation is the key determinant of the costs due to the loss of an independent instrument of policy that can suit specific country economic conditions.

Figure 2.5 plots the welfare gains of the consumers in area H relative to the elasticity of substitution between home and foreign bundles η and to that ς_1. Not surprisingly according to that figure the lower is the correlation between regional shocks the lower are the welfare benefits of adopting a common currency. In fact for small levels of η and ς_1 there are significant welfare losses across policy regimes up to 0.15 of the steady state consumption. However for high level of η independently of the degree of correlation between region specific shocks, the welfare gains of having the same currency are always greater than 0.1 percent of the steady state consumption.

2.7 Conclusion

This chapter has shown that, in the presence of mark up shocks, under plausible calibration there are welfare gains due to the adoption of a common currency. This finding is obtained in a New Keynesian open economy framework in which forming a monetary union entails a meaningful trade-off: on the one hand, because of nominal rigidities, losing monetary independence implies the welfare costs of renouncing to a policy instrument that can stabilize country-specific shocks; on the other hand, delegating the monetary policy to the monetary union's central bank generates welfare gains by improving the conduct of the national authorities. In a world constituted by two economic areas as the one laid out in our basic setup, two are the main sources of this improvement. The first is due to the internalization of the spillover effects produced by autonomous authorities within the monetary union. The second is due by the gain in monopoly power in controlling the terms of trade across areas and the feedback effects of the policy maker decision.

Baseline Calibration

$\sigma^{-1} = 1/2$	Intertemporal elasticity of substitution of the private goods;
$\eta = 2$	Elasticity of substitution between home and foreign private goods;
$\varphi^{-1} = 1/3$	Intertemporal elasticity of substitution of labor;
$\alpha_s = 0.6$	Degree of home bias for the bundle of the region;
$\alpha_b = 0.8$	Degree of home bias for the bundle of the area;
$\varepsilon = 6$	Elasticity of substitution among goods produced in the same region;
$\beta = 0.99$	Preferences discount factor;
$SDva = 0.0071$	Standard deviation of the white noise of the aggregate technological shocks;
$SDv\mu = 0.03$	Standard deviation of the white noise of the aggregate markup shocks;
$ac = 0.9$	Autocorrelation of shocks;
$\tilde{\tau} = -1$	Steady state labour subsidy;
$\varsigma_1 = 0.33$	Correlation between region specific shocks.

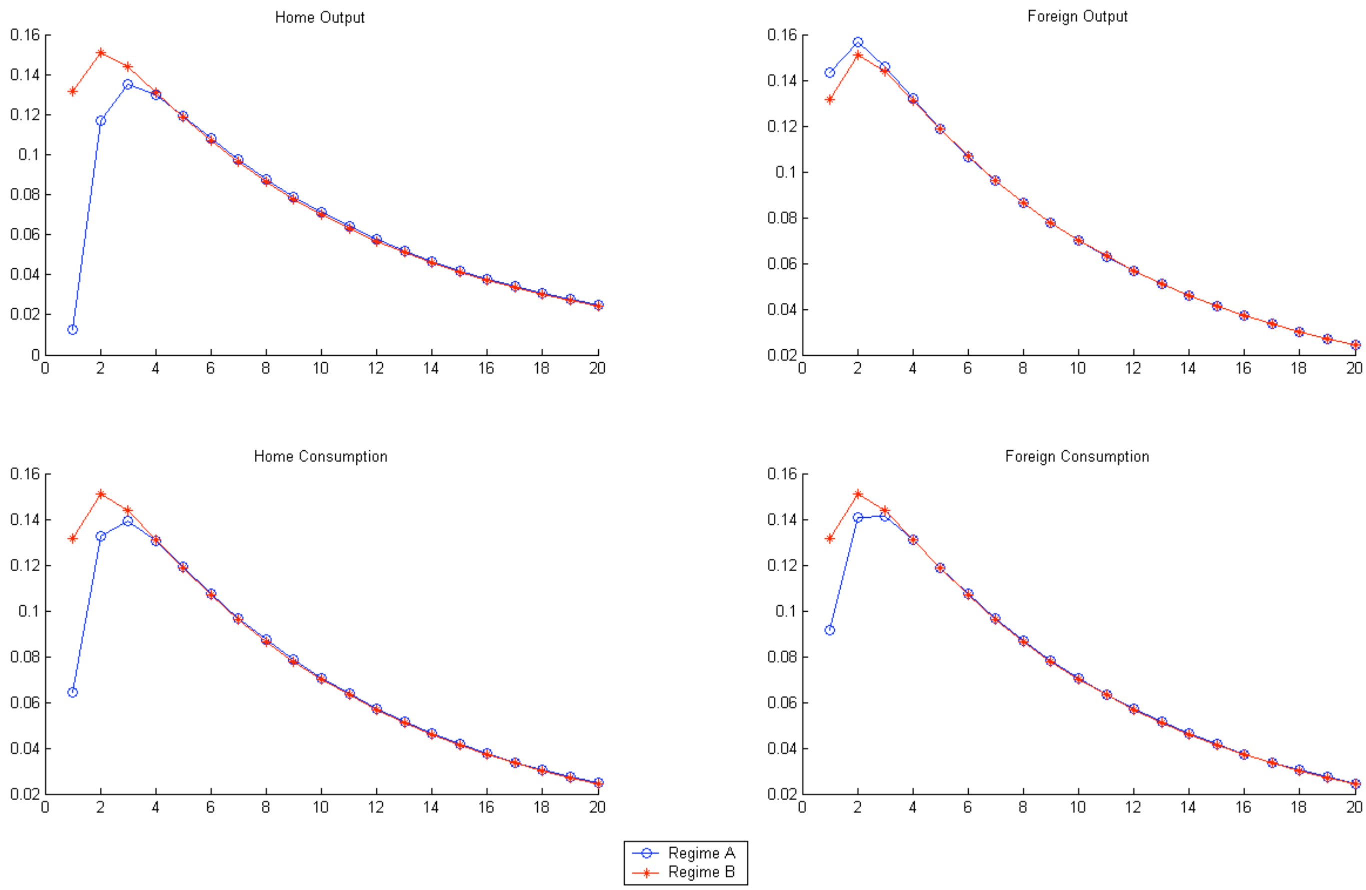

Figure 2.1: Impulse responses to a negative aggregate markup shock.

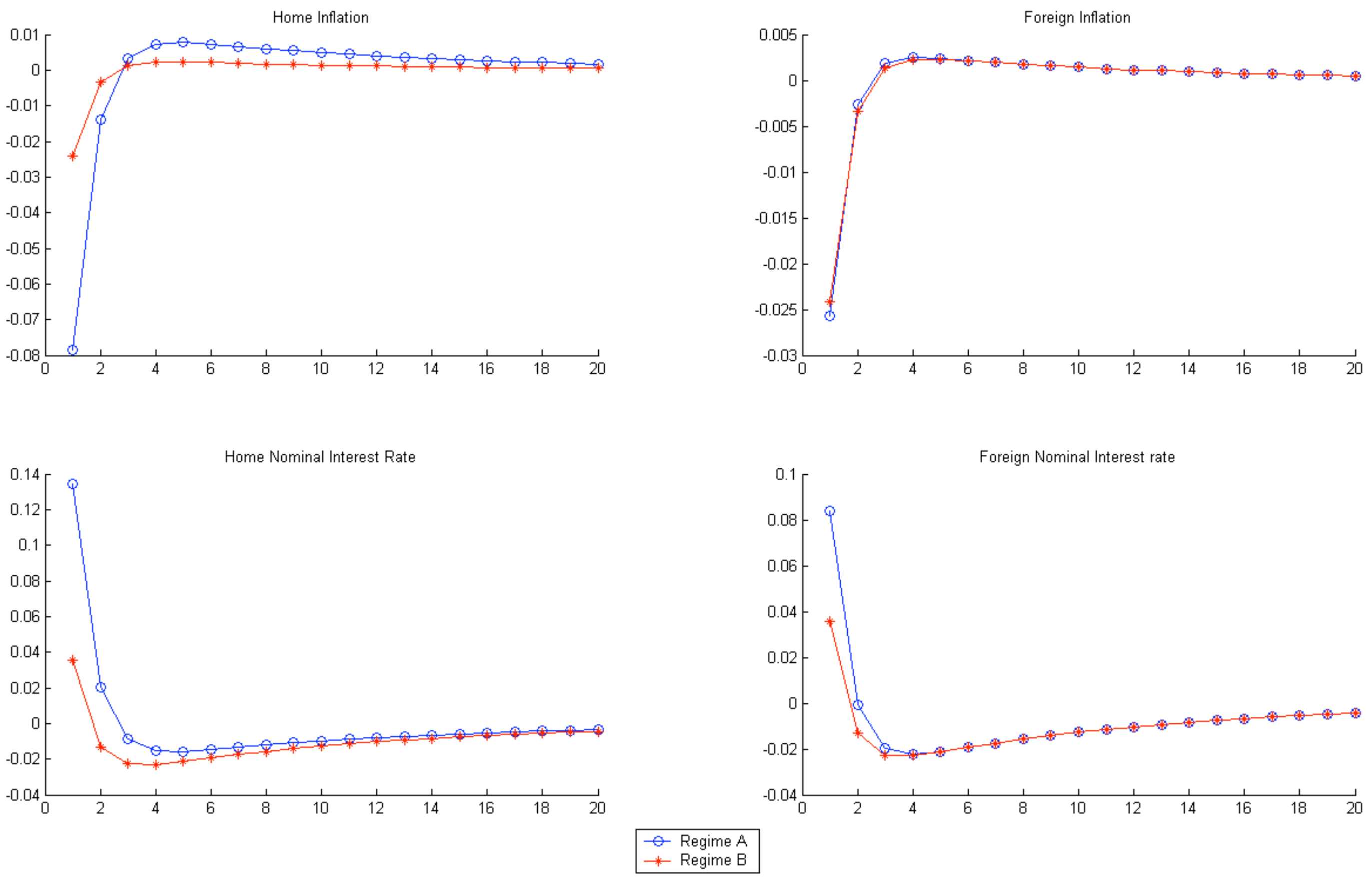

Figure 2.2: Impulse responses to a negative aggregate markup shock.

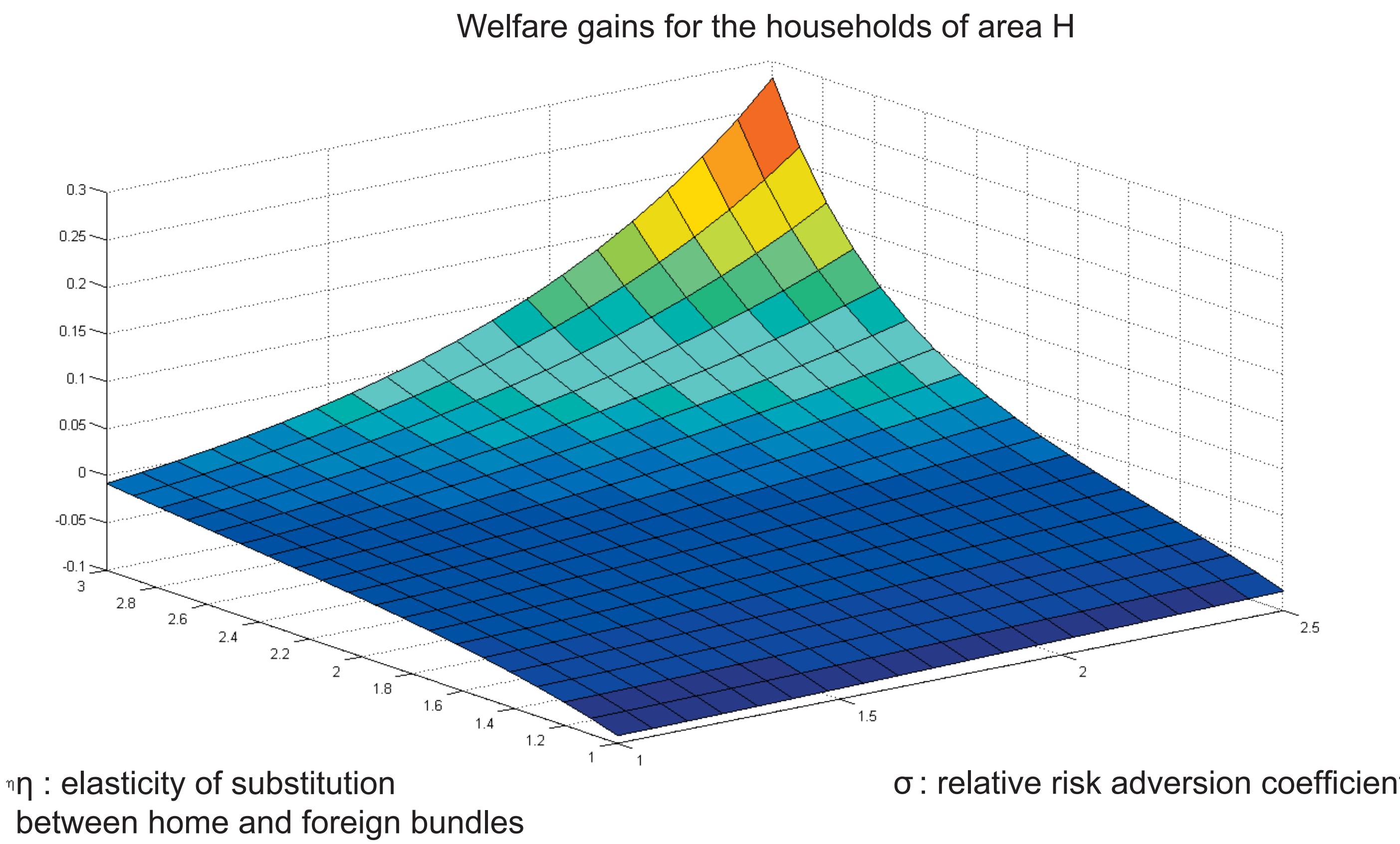

Figure 2.3: Welfare gains for area H expressed as percentage of the steady state consumption.

Figure 2.4: Welfare gains for area H expressed as percentage of the steady state consumption.

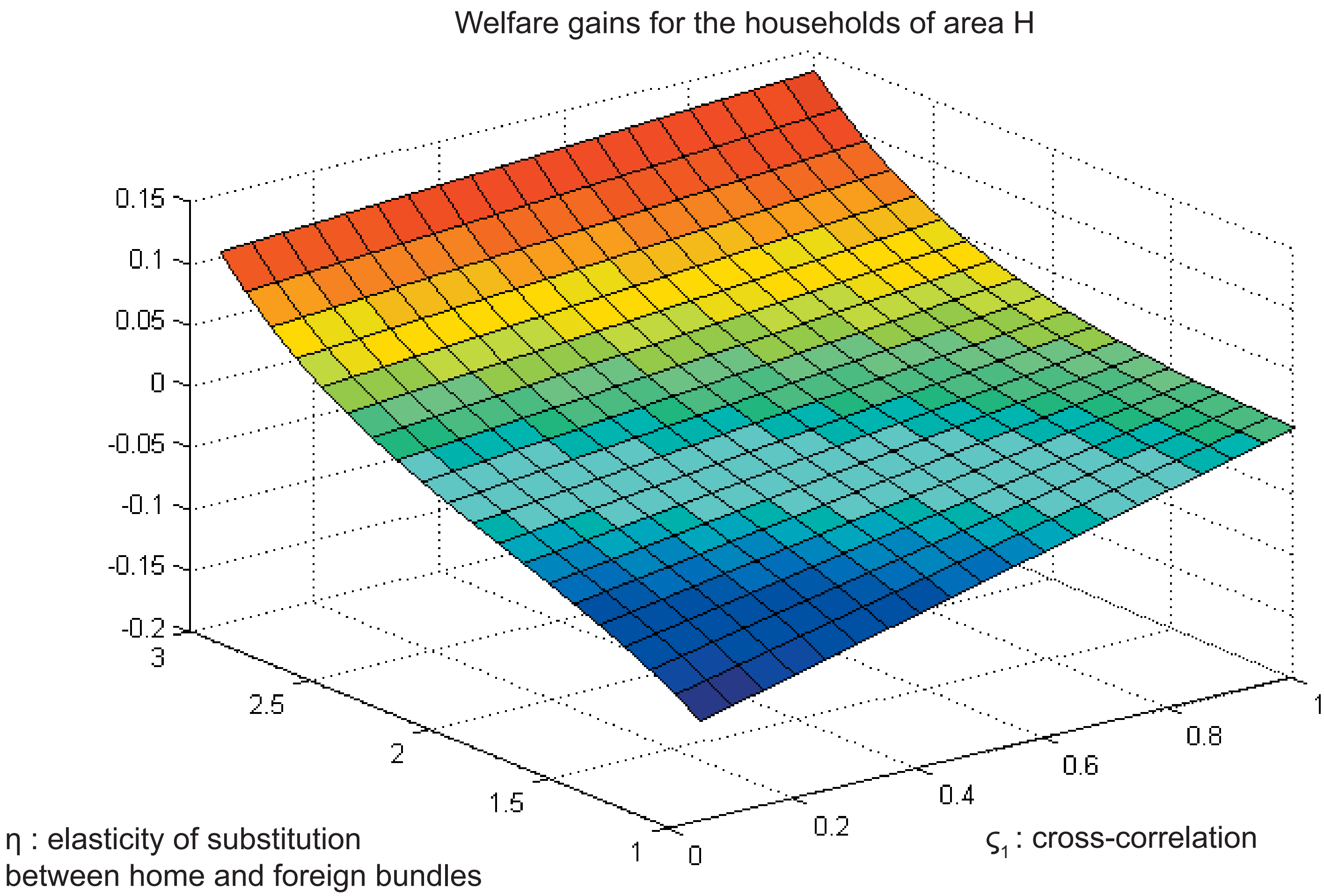

Figure 2.5: Welfare gains for area H expressed as percentage of the steady state consumption.

3 Optimal trade policy: Home market effect vs. terms of trade externality

3.1 Introduction

The aim of this chapter[1] is to study optimal trade policy in a version of the Krugman (1980) model of intra-industry trade due to monopolistic competition and increasing returns. We consider a generalized version of the Krugman model with two countries and two sectors - one with monopolistic competition, increasing returns and iceberg trade costs and one that features perfect competition and constant or decreasing returns. Within this framework we study the optimal non-cooperative and cooperative determination of import tariffs and production subsidies.

While the standard result for optimal tariffs [2] implies that countries have incentives to impose a tariff on imports and/or exports in order to improve their terms of trade[3], a strand of literature from the 1980's[4], as well as some recent contributions[5] emphasize a second channel - a production relocation effect (home market effect).

The idea behind the production relocation effect is that in the presence of trade costs and increasing returns countries have an incentive to impose a tariff on imports or to subsidize production in order to induce firms to relocate to the domestic economy. In this setup firms locate in the country where demand for their goods is relatively high in order to minimize shipping costs that cut into their profits. A tariff on imports shifts demand towards domestically produced goods, thereby increases the size of the home market relative to the foreign one and causes firms to relocate to the domestic economy. A production subsidy has a similar effect. This benefits domestic consumers since they

[1]This chapter is based on a joint paper with Alessia Campolmi and Harald Fadinger.

[2]See, for example, Helpman and Krugman (1989), Feenstra (2004).

[3]Through all the chapter we define the terms of trade as the price of imports relative to the price of exports. Countries have an incentive to improve their terms of trade by making domestically produced goods more expensive in order to obtain more foreign produced goods in exchange for the same amount of exports.

[4]Venables (1987) and Helpman and Krugman (1989), Chapter 7.

[5]Melitz and Ottaviano (2008), Ossa (2008).

pay lower prices on more varieties - as they need not pay transport costs on locally produced varieties. Foreign consumers however are worst off because less varieties are produced in their country.

We show that optimal trade policy is not driven by a production relocation externality even in a setting where the potential for the home market effect to work is maximized. Instead, optimal trade policy is always driven by the standard terms of trade externality.

The explanation of the difference between our findings and those of the existing literature is twofold. On the one hand most of the previous results have been derived supposing that tariffs are pure waste[6]. As we clarify in more detail in section 3.5.4, while assuming that tariffs revenues are not rebated to consumers has been claimed to make the case for the home market effect even stronger, what it does is actually to make the model partial equilibrium. In the present chapter we show how allowing for tariff redistribution generates wealth effects that are crucial for optimal trade policy. On the other hand the inefficiency due to monopolistic competition has always been overlooked. Indeed we demonstrate that the market equilibrium without any policy intervention implies an inefficiently low number of varieties due to the presence of monopolistic competition. As a consequence of that uncoordinated policy makers try to increase the amount of domestic varieties by imposing, for instance, a tariff on imports. This result has been misinterpreted as a home market effect. Once the monopolistic distortion is eliminated by a lump sum financed subsidy to production of differentiated goods, optimal trade policy under non-coordination is driven exclusively by the terms of trade externality and domestic policy makers seek to externalize production of differentiated varieties.

The following conclusions can be drawn from our analysis. First, in the two-sector model analyzed, the market equilibrium allocation is inefficient due to monopolistic competition. If production subsidies are available, the first best allocation can be reached under coordination and there is no role for import tariffs. Contrary to the

[6]Venables (1987), Helpman and Krugman (1989), Ossa (2008).

result of Venables (1987) we find that it does not pay to try to induce firms to locate in the domestic economy by over-subsidizing production. Conversely countries have an incentive to deviate from the cooperative solution lowering their subsidy in order to improve their terms of trade. Second, if import tariffs/subsidies are the only instrument, the optimal policy under coordination is to subsidize imports in both countries but the first best allocation is not implementable anymore. Third, if the number of differentiated varieties is Pareto-optimal due to a production subsidies and there are constant returns to scale in the homogenous sector, the optimal non-cooperative trade policy consists of a positive import subsidy that improves domestic terms of trade, instead of a tariff (subsidy). Instead if there are strong decreasing returns for the homogenous good, the same term trade improvement can be achieved by taxing imports[7].

Finally, if the number of differentiated varieties is inefficiently low, the optimal non-cooperative trade policy is given by a positive import tariff. Such a trade policy is not motivated by a production relocation externality but rather by the attempt to correct for the distortion arising from monopolistic competition.

The chapter proceeds as follows: section 3.2 presents the related literature, section 3.3 the model, section 3.4 the equilibrium, section 3.5 the optimal trade policy problem and section 3.6 concludes.

3.2 Literature Review

This chapter[8] builds on Krugman (1980)'s model of intra-industry trade. Krugman shows that in a two-sector model with increasing returns and transport costs, given the incentives to concentrate industries closer to the biggest market, each country is a net exporter of the goods produced in the sector for which it has a relatively larger domestic demand. This is what is usually referred to as the *home market effect*. As shown first in

[7]This is because a reduction of the homogenous good production increases marginal productivity in that sector and thus the relative wages. This effect which is absent in the case of constant returns to scale, more than compensates the terms of trade trade worsening due to the relative decrease in the number of varieties in the differentiated sector.

[8]This chapter builds on a joint work with Alessia Campolmi and Harald Fadinger.

Krugman (1980) and discussed more in detail by Davis (1998), in a special case where one sector is characterized by monopolistic competition and transport costs while the other operates in a regime of perfect competition and no transport costs, if the two countries differ only in size, the bigger country is a net exporter of differentiated goods, while the smaller country exports the homogeneous good. As a consequence, consumers in the bigger country experience higher welfare because they save on transport costs. In such a context, Venables (1987) studies import tariffs and export and production subsidies. He concentrates on non-strategic interactions (i.e., unilateral changes in the policy instrument of one country with no retaliation) and shows that: first, a country's welfare is raised by a unilateral increase in its import tariffs when tariff revenues are not redistributed; second, a production subsidy also increases welfare. He interprets those results in the light of a home market effect. In his analysis Venables never corrects for the presence of inefficiency in the economy due to monopolistic competition. We extend Venables (1987) in several dimensions, which we believe to be crucial for a better understanding of the incentives that shape trade policies. The first extension is to the case when such inefficiency is taken care of in order to disentangle the different incentives behind trade policies. Second, we allow for redistribution of all tariff revenues. Finally, we consider not only the non-strategic interaction but also the cooperative solution and the Nash equilibrium for each policy instrument. We show that when tariff revenues are redistributed and allocations are optimal without tariffs, policy makers choose an import subsidy (they pay for increasing foreign demand) in order to reduce the domestic number of firms and to improve the welfare relevant terms of trade. We also show that for Cobb-Douglas utility the subsidy to domestic production that is optimal from the domestic policy maker's viewpoint is always smaller than the one chosen by the world planner. This implies that domestic policy makers try to improve their terms of trade rather than to increase the number of domestic firms above the efficient level. Still, they choose a positive level of subsidy because the number of firms in the decentralized equilibrium without policy intervention is too low.

In their standard work on trade policy under imperfect competition Helpman and

Krugman (1989) confirm Venables' results. They also discuss the "production efficiency effect" in a somewhat special model with a specific factor and factor price equalization for the mobile factor[9]. The difference between the price and the marginal cost of domestically produced varieties caused by monopolistic competition induces domestic consumers to consume too little of domestic goods. A tariff on imports can correct for this, even when relative factor prices and hence the relative price of individual varieties are not affected by a tariff. We find that this "production efficiency effect" is in fact also the explanation for the result that there is an optimal tariff in the two sector Krugman model, when monopolistic distortions are not eliminated by a subsidy.

Gros (1987) analyzes optimal (strategic and non-strategic) import tariffs in the one sector Krugman model without trade costs. He shows that the competitive equilibrium (with or without trade) is Pareto-optimal in this set-up. Since there are no trade costs, factor prices are equalized as long as there are no tariffs. In the presence of tariffs factor prices may differ, so there is room for a terms of trade effect, even though the number of firms is pinned down by labor market clearing. He computes the optimal tariff on imports and shows that it is positive even for a small open economy. In the small open economy the optimal import tariff equates the social marginal cost of domestic goods - given by the marginal cost of production - to the social marginal cost of foreign goods - given by their price. To make consumers aware of the monopolistic markup they have to pay to foreign firms on imports, foreign goods must be more expensive than domestic ones by the markup factor (since the markup of domestic firms is just a transfer from domestic firms to domestic consumers), so the optimal tariff on imports equals the markup. For large economies the optimal import tariff is larger because it internalizes the effect that lower aggregate home demand for foreign varieties improves aggregate terms of trade.

Our model collapses to the model analyzed by Gros (1987) when there is only one sector and becomes qualitatively very similar when there are strong decreasing returns in the production of the homogeneous good. In this case relative wages are pinned

[9]Their analysis is based on Flam and Helpman (1987).

down in the homogeneous sector and dominate the effect of variety on aggregate terms of trade. Uncoordinated policy makers now have an incentive to set a positive import tariff (instead of a subsidy), which increases production in the differentiated sector and therefore the domestic relative wage in order to improve the terms of trade.

This chapter is also related to a recent contribution by Ossa (2008). Using a version of Venables' model with a Cobb-Douglas utility function, he studies optimal import tariffs in a non-cooperative game. He finds that governments have incentives to unilaterally impose (infinite) import tariffs and interprets this as a production relocation externality. While the main results are derived under the assumption that tariff revenues are wasted, in the appendix he also considers a redistribution of tariff revenues and still finds an optimal finite tariff on imports. His interpretation (which coincides with Venables (1987) and Helpman and Krugman (1989)) is that the optimal tariff is now finite because a too large tariff implies lower tariff revenues. However, he does not eliminate the allocational distortion between the sectors due to monopolistic competition, which is in fact driving this result.

Finally, Melitz and Ottaviano (2008) study unilateral trade liberalization in a model with heterogeneous firms, variable markups and a freely traded outside good. They demonstrate that in the long run (when entry and exit of firms is allowed for) a unilateral reduction of import tariffs (that are again assumed to be wasted) reduces domestic welfare. In their model, a reduction in tariffs causes some firms to leave the domestic market, which may reduce domestic welfare if the number of firms in the differentiated sector is too low because of monopoly distortions, which they do not correct. In addition, in their model a reduction in the number of firms in a market induces producers to charge higher markups thereby increasing monopoly distortions. Hence, it is in principle possible that optimal uncoordinated trade policy in their model involves a production relocation externality even when the number of differentiated firms is efficient with zero tariffs because there is the additional benefit of having more firms - increased competition leads to lower markups, which benefits domestic consumers. Whether this is in fact the case would require a formal investigation.

3.3 The Model

The world economy is composed by two countries: Home and Foreign. Each country produces a homogenous good and a continuum of differentiated goods. All goods are tradable but only the differentiated goods are subject to transport costs. The differentiated goods sector is characterized by monopolistic competition while perfect competition is assumed in the homogenous good sector. The two countries are identical in terms of preferences, production technology and market structure. The model is solved under the assumption of financial autarchy. In what follows foreign variables will be denoted by a (*).

3.3.1 Households

Household's utility function in the Home country is given by:

$$U(C,Z) \equiv \; C^{\alpha} Z^{1-\alpha} \tag{3.1}$$

where C aggregates over the differentiated goods, Z represents the homogeneous good and α is the share of the differentiated goods in the aggregate consumption basket. While the homogenous good is identical across countries, each country produces a different subset of differentiated goods. In particular, N varieties are produced in the Home country (C_H) while N^* are produced by Foreign (C_F). We allow for a general specification of the consumption aggregators with two different elasticity of substitutions, one between home and foreign goods (η) and one between goods produced in the same country (ε):

$$C = \left[C_H^{\frac{\eta-1}{\eta}} + C_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \qquad \eta > 0 \tag{3.2}$$

$$C_H = \left[\int_0^N c(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad C_F = \left[\int_0^{N^*} c(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad \varepsilon > 1 \tag{3.3}$$

Foreign consumers face an analogous utility function. Let $p(h)$ ($p^*(h)$) be the price payed by home (foreign) consumers on domestically produced goods while $p(f)$ ($p^*(f)$) is the price payed by home (foreign) consumers on imported goods. In general, $p(h) \neq p^*(f)$ and $p^*(h) \neq p(f)$ because of transport costs and tariffs/subsidies on imports. Households inelastically supply L units of labor. The budget constraint of Home consumers reads as follows:

$$PC + p_Z Z = WL + T + \Pi, \tag{3.4}$$

where W is the wage, p_Z is the price payed for the homogeneous good, Π are firm profits redistributed to consumers and T is a lump sum tax/transfer which depends on the tariff/subsidy scheme adopted by the domestic government and which will be defined later. Then the solution to the consumer problem gives the following demand functions and price indices:

- Home's and Foreign's demand for differentiated varieties produced by Home:

$$c(h) = \left[\frac{p(h)}{P_H}\right]^{-\varepsilon} C_H \qquad c^*(f) = \left[\frac{p^*(f)}{P_F^*}\right]^{-\varepsilon} C_F^* \tag{3.5}$$

$$C_H = \left[\frac{P_H}{P}\right]^{-\eta} C \qquad C_F^* = \left[\frac{P_F^*}{P^*}\right]^{-\eta} C^* \tag{3.6}$$

- Home's and Foreign's demand for differentiated varieties produced by Foreign:

$$c(f) = \left[\frac{p(f)}{P_F}\right]^{-\varepsilon} C_F \qquad c^*(h) = \left[\frac{p^*(h)}{P_H^*}\right]^{-\varepsilon} C_H^* \tag{3.7}$$

$$C_F = \left[\frac{P_F}{P}\right]^{-\eta} C \qquad C_H^* = \left[\frac{P_H^*}{P^*}\right]^{-\eta} C^* \tag{3.8}$$

- Demand for the homogeneous good in Home and Foreign:

$$Z = \frac{1-\alpha}{\alpha}\frac{P}{p_Z}C \qquad\qquad Z^* = \frac{1-\alpha}{\alpha}\frac{P^*}{p_Z^*}C^* \tag{3.9}$$

- Domestic price indexes:

$$P = \left[P_H^{1-\eta} + P_F^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{3.10}$$

$$P_H = \left[\int_0^N p(h)^{1-\varepsilon}dh\right]^{\frac{1}{1-\varepsilon}} \qquad\qquad P_F = \left[\int_0^{N^*} p(f)^{1-\varepsilon}df\right]^{\frac{1}{1-\varepsilon}} \tag{3.11}$$

- Foreign price indexes:

$$P^* = \left[P_H^{*\,1-\eta} + P_F^{*\,1-\eta}\right]^{\frac{1}{1-\eta}} \tag{3.12}$$

$$P_H^* = \left[\int_0^{N^*} p^*(h)^{1-\varepsilon}dh\right]^{\frac{1}{1-\varepsilon}} \qquad\qquad P_F^* = \left[\int_0^{N} p^*(f)^{1-\varepsilon}df\right]^{\frac{1}{1-\varepsilon}} \tag{3.13}$$

3.3.2 Firms in the Differentiated Sector

Firms in the differentiated sector operate in a regime of monopolistic competition. They pay a per period fixed cost in terms of labor f and then produce with a constant returns to scale technology:

$$Y(h) = L_C(h) - F, \tag{3.14}$$

where $L_C(h)$ is the amount of labor allocated to the production of the differentiated good h. Goods sold in the foreign market are subject to an iceberg transport cost $\tau \geq 1$. Governments in both countries can use two policy instruments: a production subsidy on fixed and marginal costs (τ_C) and tariffs/subsidies on imports (τ_I). A (*) indicates the Foreign policy instruments. We assume that those subsidies (taxes) are received (payed) directly by the firms. Equivalently, we could have consumers receiving (paying) them to the government. Solving the profit maximization problem, given the constant price elasticity of demand, optimal prices charged by Home firms in the domestic market are a fixed markup over their perceived marginal cost $(1-\tau_C)W$ and optimal prices payed by foreign consumers equal domestic prices augmented by transport costs and tariffs:

$$p(h) = (1-\tau_C)\frac{\varepsilon}{\varepsilon-1}W \qquad\qquad p^*(f) = \tau_I^*\tau p(h) \tag{3.15}$$

In the same way, Foreign firms' optimal pricing decisions lead to:

$$p^*(h) = (1-\tau_C^*)\frac{\varepsilon}{\varepsilon-1}W^* \qquad\qquad p(f) = \tau_I\tau p^*(h) \tag{3.16}$$

Given that all firms use the same production technology, in equilibrium all firms in the same country will charge the same price and we have perfect symmetry within firms in the differentiated sector of each country.

3.3.3 Homogeneous good sector

Both countries produce a homogenous good which can be traded with no transport costs. The two countries share the same production technology:

$$Q_Z = L_Z^{\gamma} \qquad \gamma \leq 1, \tag{3.17}$$

where L_Z is the amount of labor allocated to producing the homogeneous good. The good is sold in a perfectly competitive market without trade costs. Consequently, the price equals marginal cost and is the same across the two countries:

$$p_Z = \frac{1}{\gamma}L_Z^{1-\gamma}W \qquad\qquad p_Z = p_Z^* \tag{3.18}$$

If $\gamma = 1$ (constant returns to scale) and as long as the homogeneous good is produced in both countries in equilibrium, there is factor price equalization:

$$p_Z = p_Z^* = W = W^* \tag{3.19}$$

3.3.4 Government

The government in each country disposes of 3 fiscal instruments. A production tax/subsidy (τ_C) and tariffs/subsidies on imports (τ_I). All government revenues are redistributed to consumers through a lump sum transfer T. The government is assumed to run a balanced budget. Hence, the government's budget constraint is:

$$(\tau_I - 1)\tau P_H^* C_F + -\tau_C W \int_0^N (Y(h) + F)dh = T \tag{3.20}$$

3.4 Equilibrium

Given that firms share the same production technology, the equilibrium is symmetric - firms in the differentiated sector of one country charge the same price and produce the same quantity. This implies that in equilibrium price indices can be written as:

$$\frac{p(h)}{P_H} = N^{\frac{1}{\varepsilon-1}} \qquad \frac{p^*(h)}{P_H^*} = N^{*\frac{1}{\varepsilon-1}} \tag{3.21}$$

$$P_F = \tau_I \tau P_H^* \qquad P_F^* = \tau_I^* \tau P_H \tag{3.22}$$

3.4.1 Free Entry in the Differentiated Sector

The assumption of free entry in the differentiated sector implies that monopolistic producers in the differentiated sector make zero profits in equilibrium:[10]

$$\Pi(h) = c(h)\left[p(h) - (1-\tau_C)W\right] + c^*(f)\left[\tau p(h) - \tau(1-\tau_C)W\right] - FW(1-\tau_C) = 0 \tag{3.23}$$

[10] Remember that firms pay (receive) taxes (subsidies) to (from) the government. Taking this into account, firms' revenues from exporting are given by $c^*(f)\frac{p^*(f)}{\tau_I^*} = c^*(f)\tau p(h)$.

Using the optimal pricing rule into equation (3.23), we obtain:

$$c(h) + \tau c^*(f) = (\varepsilon - 1)F \tag{3.24}$$

Substituting the demand functions in (3.24) and using (3.21) and (3.22), the zero profit condition for firms in the domestic differentiated sector can be rewritten as:

$$(\varepsilon - 1)F = N^{\frac{\varepsilon}{1-\varepsilon}} \left(\frac{P_H}{p_z}\right)^{-\eta} \left[\left(\frac{P}{p_z}\right)^{\eta} C + \tau^{1-\eta}(\tau_I^*)^{-\eta} \left(\frac{P^*}{p_z}\right)^{\eta} C^*\right] \tag{3.25}$$

An analogous condition can be derived for firms located in the foreign country:

$$(\varepsilon - 1)F = N^{*\frac{\varepsilon}{1-\varepsilon}} \left(\frac{P_H^*}{p_z}\right)^{-\eta} \left[\left(\frac{P^*}{p_z}\right)^{\eta} C^* + \tau_I{}^{-\eta}\tau^{1-\eta} \left(\frac{P}{p_z}\right)^{\eta} C\right] \tag{3.26}$$

3.4.2 Goods and Labor Markets Clearing Conditions

For each differentiated variety produced by Home the following market clearing condition must be verified:

$$y(h) = c(h) + \tau c^*(f) \tag{3.27}$$

Therefore, the zero profit condition (3.24) and market clearing (3.27) imply that the production of each variety is fixed and the same is true for the varieties produced by Foreign:

$$y(h) = (\varepsilon - 1)F \qquad\qquad y^*(h) = (\varepsilon - 1)F \tag{3.28}$$

The market clearing condition for the homogeneous good is given by:

$$Q_Z + Q_Z^* = Z + Z^* \tag{3.29}$$

which, using the demand functions, can be written as:

$$Q_Z + Q_Z^* = \frac{(1-\alpha)}{\alpha} \left[\frac{P}{p_z} C + \frac{P^*}{p_z} C^*\right] \tag{3.30}$$

Finally, equilibrium in the labor market implies that $L = L_C + L_Z$ with $L_C = NL_C(h)$ in the symmetric equilibrium. Making use of (3.14) and (3.28), we have:

$$L_C = N\varepsilon F \qquad\qquad Q_Z = [L - N\varepsilon F]^{\gamma} \tag{3.31}$$

and for Foreign:

$$Q_Z^* = [L^* - N^*\varepsilon F]^{\gamma} \tag{3.32}$$

3.4.3 Balanced Trade Condition

The model is solved under the assumption of financial autarky, so trade is balanced. The net-export of the homogenous good by Home is defined as:

$$Z^X - Z^M \equiv Q_Z - \frac{1-\alpha}{\alpha}\frac{P}{p_Z}C \tag{3.33}$$

Hence, the balanced trade condition reads as follows[11]:

$$\tau P_H C_F^* + p_Z\left(Z^X - Z^M\right) = \tau P_H^* C_F \tag{3.34}$$

Combining (3.33) with (3.34), (3.22) and the demand functions, we can rewrite the balanced trade condition as follows:

$$Q_Z = \frac{(1-\alpha)}{\alpha}\frac{P}{p_z}C + \tau_I^{-\eta}(\tau)^{1-\eta}\left(\frac{P_H^*}{p_z}\right)^{1-\eta}\left(\frac{P}{p_z}\right)^{\eta}C - \tau_I^{*-\eta}\tau^{1-\eta}\left(\frac{P_H}{p_z}\right)^{1-\eta}\left(\frac{P^*}{p_z}\right)^{\eta}C^* \tag{3.35}$$

3.4.4 Price Indices

Using the optimal pricing rules (3.15) and (3.18) together with equations (3.17) and (3.21) (and the corresponding one for Foreign), relative prices can be written as follows:

[11] Import tariffs/subsidies are collected directly by the governments at the border so they do not enter into this condition.

$$\frac{P_H}{p_z} = \frac{\varepsilon}{\varepsilon - 1}\gamma(1-\tau_C)N^{\frac{1}{1-\varepsilon}}Q_Z^{\frac{\gamma-1}{\gamma}} \qquad \frac{P_H^*}{p_z} = \frac{\varepsilon}{\varepsilon - 1}\gamma(1-\tau_C^*)N^{*\frac{1}{1-\varepsilon}}Q_Z^{*\frac{\gamma-1}{\gamma}} \tag{3.36}$$

$$\frac{P}{p_z} = \left[\left(\frac{P_H}{p_z}\right)^{1-\eta} + (\tau_I\tau)^{1-\eta}\left(\frac{P_H^*}{p_z}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \quad \frac{P^*}{p_z} = \left[\left(\frac{P_H^*}{p_z}\right)^{1-\eta} + (\tau_I^*\tau)^{1-\eta}\left(\frac{P_H}{p_z}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{3.37}$$

The free entry conditions for the two countries (3.25) and (3.26), the market clearing for the homogeneous good (3.30) and the balanced trade condition (3.35) together with the expressions for price indices just derived and (3.31) and (3.32) fully characterize the equilibrium of the economy.

3.4.5 Terms of Trade

A crucial aspect in this model is the relevant definition of the terms of trade. In our model there are two relative world market prices that are of interest for domestic policy makers: $(\tau P_H^*)/(P_H)$ and $(\tau P_H^*)/p_z$ if Home is an exporter of the homogeneous good and $p_z/(\tau P_H)$ if Home is an importer of the homogeneous good. Using the definition of the price indices, we can write $\left(\tau\frac{P_H^*}{P_H}\right) = \left(\frac{N}{N^*}\right)^{\frac{1}{\varepsilon-1}} \frac{W^*\tau(1-\tau_C^*)}{W(1-\tau_C)}$. Hence, this relative international price of imports of differentiated goods in terms of exports depends positively on the relative number of varieties produced domestically and negatively on the relative domestic wages.

To gain intuition consider two extreme cases: constant returns to scale in the production of the homogeneous good ($\gamma = 1$) and a one sector economy ($\alpha = 1$).

If $\gamma = 1$ relative wages are one, so that $\left(\frac{\tau P_H^*}{P_H}\right) = \left(\frac{N}{N^*}\right)^{\frac{1}{\varepsilon-1}} \frac{\tau(1-\tau_C^*)}{(1-\tau_C)}$. Hence, this measure of the terms of trade depends on production subsidies and on the relative number of domestic varieties. An increase in the relative number of varieties produced at home increases the relative price of imports of differentiated varieties, since a larger number of domestic varieties has to be exchanged for the same number of foreign varieties. The other relevant relative prices become $p_z/(\tau P_H) = \frac{N^{1/(\varepsilon-1)}}{\tau\frac{\varepsilon}{\varepsilon-1}(1-\tau_C)}$ and

$P_F/p_z = \tau(N^*)^{1/(1-\varepsilon)}\frac{\varepsilon}{\varepsilon-1}(1-\tau_C^*)$. Hence, the relative price of imports of homogeneous goods is increasing in domestic varieties, while the relative price of exports of homogeneous goods is increasing in the number of Foreign varieties.

In the second extreme, if $\alpha = 1$, there is only one sector, so that the number of domestic and foreign varieties is fixed by equilibrium firm size and labor supply and with equal country size we have $N = N^* = \frac{L}{\varepsilon F}$. In this case $(\tau P_H^*)/(P_H) = \frac{\tau(1-\tau_C^*)W^*}{(1-\tau_C)W}$ is the only relevant relative import price and it is affected only by changes in relative wages.

In general, with $\gamma < 1$ and $\alpha \in (0,1)$ both margins of adjustment - change in the relative numbers of varieties and changes in relative wages matter for the movements in the terms of trade.

3.5 Optimal Trade Policy

In this section we study optimal trade policy both from the perspective of single country policy makers[12] and from the perspective of a cooperative authority that maximizes average welfare of the world economy. We consider two possible trade policy instruments in turn: taxes/subsidies on the production of differentiated goods (τ_C, τ_C^*) and import tariffs (τ_I, τ_I^*). The general set up of the two problems is specified in the next two sub-sections. We then analyze both the cooperative solution and the non-cooperative Nash solution for each policy instrument.

Since we want to compare our results with the existing literature, the main set of results is derived under the assumptions of $\gamma = 1$ (constant returns in the homogeneous sector, which - together with costless trade in this sector - guarantees factor price equalization) and $\eta = \varepsilon$ (elasticity of substitution between the domestic and the foreign bundle equal to the elasticity of substitution between varieties). For completeness we compare those outcomes with the ones that would arise under the assumption of decreasing returns in the homogeneous sector ($\gamma < 1$).

[12] In this case we study the Nash equilibrium of the game.

3.5.1 Cooperative Policy Problem

Policy makers in the two countries choose fiscal instruments in order to maximize joined utility[13], taking the equilibrium conditions as a constraint:

$$\max_{C,C^*,N,N^*,\tau_i,\tau_i^*} \quad \left(\frac{P}{p_z}\right)^{1-\alpha} C + \left(\frac{P^*}{p_z}\right)^{1-\alpha} C^*$$

subject to (3.25), (3.26), (3.30)[14] and (3.35) and where $i \in \{C, I\}$ and Q_Z, Q_Z^* are defined according to (3.31) and (3.32) and the price indices are defined as in section 3.4.4.

For the complete derivation of the cooperative solution for the two policy instruments we referee to the Appendix.

3.5.2 Non-Cooperative Policy Problem

The policy maker of Home solves the following problem:

$$\max_{C,C^*,N,N^*,\tau_i} \quad \left(\frac{P}{p_z}\right)^{1-\alpha} C$$

subject to (3.25), (3.26), (3.30) and (3.35) taking τ_i^* as given and where $i \in \{C, I\}$ and Q_Z, Q_Z^* are defined according to (3.31) and (3.32) and the price indices are defined as in section 3.4.4. The policy maker of Foreign solves a symmetric problem. The solution of the game is the Nash equilibrium. In order to better understand the intuition behind the Nash equilibrium, we will also study the non-strategic behavior of the single country policy maker i.e., we will underline the mechanisms and the incentives which induce the policy maker to deviate from the cooperative solution.

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[13]Using (3.9) we can rewrite (3.1) as $U(C) = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{P}{p_z}\right)^{1-\alpha} C$. We neglect the constant in the maximization problem.

[14]Using equations (3.33) and (3.34) it is possible to rewrite the market clearing for the homogeneous good (3.30) in the following way: $Q_Z^* = \frac{(1-\alpha)}{\alpha}\frac{P^*}{p_z}C^* + \tau^{1-\eta}\tau_I^{*-\eta}\left(\frac{P_H}{p_z}\right)^{1-\eta}\left(\frac{P^*}{p_z}\right)^{\eta} C^* - \tau^{1-\eta}(\tau_I)^{-\eta}\left(\frac{P_H^*}{p_z}\right)^{1-\eta}\left(\frac{P}{p_z}\right)^{\eta} C$. Using this expression instead of the original formulation of the homogeneous good market clearing condition makes the solution of the optimal problem simpler. We will do the same for the single country optimal policy problem.

3.5.3 Production Subsidies

The case of cooperation

For the time being, we restrict the available policy instruments to a subsidy/tax on the production of the differentiated sector. Since we have two sectors - one with monopolistic competition and a competitive one - the monopolistic pricing decision distorts the competitive allocation towards having too few firms in the differentiated sector. Therefore, a the Pareto-efficient allocation chosen by a hypothetical world planner can be implemented by using a lump sum tax financed subsidy to production of differentiated goods.

If τ_C and τ_C^* are the only instruments, the optimal cooperative solution is to set[15] $\tau_C = \tau_C^* = \frac{1}{\varepsilon}$, i.e. to completely offset the distortion coming from the presence of monopolistic competition in the differentiated sector. Indeed, even if in a one-sector model the presence of monopolistic competition does not introduce any inefficiency in a model with endogenous number of varieties and fixed labor supply[16], this is no longer true in a two-sector model. In particular, if not corrected by the production subsidy, the price markup charged by firms in the differentiated sector leads to an equilibrium with an inefficiently low number of varieties and an inefficiently high level of production of the homogeneous good because the marginal rate of substitution between the two sectors does not equal the marginal rate of transformation.[17]

This result remains unaffected by the assumption of decreasing returns.

[15]See appendix for the analytical derivation of the results discussed here and in the following sections.

[16]This result is proved in Gros (1987). The basic intuition is that firm size is optimal in the market solution of any model with Dixit-Stiglitz utility (see Dixit and Stiglitz (1977)) and with one sector and homogeneous firms markups do not distort any decision of consumers because all relative goods prices equal one, so that also the number of varieties is chosen optimally.

[17]The result that the market equilibrium leads to too little variety in the two sector Dixit-Stiglitz model has been proved by Dixit and Stiglitz (1977). For the case of $\gamma = 1$ our economy coincides with the example in section 6 of Benassy (1996). He shows that the unconstrained Pareto-optimal allocation involves an equilibrium number of varieties equal to $N = \alpha L/((\varepsilon + \alpha - 1)F)$, while the market solution is $N = \alpha L/(\varepsilon F)$, so the market provides too little variety. Note that a lump-sum financed production subsidy that corrects for the markup implements the first best solution.

The case of non-cooperation

Non-strategic production subsidies To gain intuition we look at changes in domestic policy, while holding foreign policy constant. We first study a unilateral deviation from a Pareto-efficient situation, where both countries set an optimal subsidy and Home unilaterally deviates by lowering the subsidy on production.

For our numerical example we consider the following calibration. We set $\varepsilon = 4$, a standard value in the literature. For convenience we choose transport costs $\tau = 1.7$ and an expenditure share on the differentiated sector equal to $\alpha = 0.4$[18].

Looking at figure 3.6, which plots domestic and foreign variables against the level of the domestic production subsidy, while holding the foreign subsidy at the optimal level $\tau_C^* = 1/\varepsilon$, we see that a unilateral decrease in the domestic subsidy from the efficient level $1/\varepsilon = 0.25$ initially increases domestic utility, even though it lowers domestic consumption of differentiated goods. Hence, the optimal strategy given that the other country chooses an efficient subsidy, is to deviate to a smaller subsidy, that causes exit of firms in Home and entry in Foreign, reduces the domestic subsidy bill and improves domestic terms of trade (defined here as the relative price of imports of differentiated goods in terms of exports), while lowering the aggregate level of efficiency. There is no overexpansion of subsidies above the efficient level and consequently no home market effect - policy makers do not find it optimal to try to attract more firms to their economy if the aggregate amount of firms is efficient. Instead, they improve domestic terms of trade by reducing the number of varieties produced domestically. This implies that given fixed relative factor prices, a smaller amount of domestic varieties can be exchanged against the larger bundle of foreign varieties which also frees resources to produce more of the homogeneous good - which Home now exports at improved conditions, as the relative price of imports of the differentiated bundle in terms of homogeneous good also falls.

When there are decreasing returns in the homogeneous sector, relative wages are no

[18]This calibration just exemplifies our results, which are robust for any $\eta = \varepsilon > 1$, $\tau > 1$, and $\alpha \in (0, 1)$.

longer fixed but determined by the relative marginal product in the homogeneous sector. This implies that the terms of trade, defined as the world market price of imports of differentiated goods in terms of their exports or in terms of exports of the homogeneous good, now not only depend negatively on the (relative) number of domestic differentiated varieties but also positively on the domestic relative wage. Under this scenario a reduction of the domestic subsidy reduces domestic relative wage which works in the direction of worsening the terms of trade. Still, the direct effects on prices through the lower number of domestic varieties and the lower subsidy predominate leading to the same conclusions as with constant return.

In a second scenario we consider a domestic deviation from an equilibrium where both countries initially set zero production subsidies, to a positive domestic subsidy. Again, figure 3.6 plots a number of domestic and foreign variables as functions of the domestic production subsidy. An increase in the domestic subsidy causes firms in the differentiated sector to enter the domestic market and leave the foreign one - so the subsidy to production causes agglomeration. The production in the homogeneous sector is reduced domestically to free resources for production in the differentiated sector. Consumption of differentiated goods at Home increases, even though the terms of trade move against Home. Note that domestic utility is initially increasing in the subsidy and then starts to decrease and that the level of subsidy that maximizes domestic utility, is strictly smaller than the efficient subsidy, $1/\varepsilon = 0.25$. This implies that the presence of monopolistic distortions gives an incentive to the domestic policy maker to subsidize production of differentiated goods in order to bring consumption of differentiated goods closer to the first best level. Since this subsidy comes at the cost of worsened terms of trade the domestic policy maker chooses a suboptimally low subsidy level. As a consequence, there is no home market effect in the sense that country policy makers do not have the incentive to over-subsidize production in order to expand the domestic number of firms by too much. Instead, policy makers trade off increased efficiency against worsened terms of trade. This conclusion holds also for the case of decreasing returns.

Optimal production subsidy under non-cooperation

Having discussed the incentives of individual policy makers, we now take a look at the strategic interaction of policy makers. For simplicity, we study exclusively symmetric Nash equilibria of the policy game, where domestic and foreign policy makers simultaneously choose the optimal production subsidy. Since no analytical solutions of the equilibrium strategies can be obtained, we rely on numerical simulations. Again, we set $\alpha = 0.4$[19] and we plot the Nash-equilibrium subsidy and the subsidy of the cooperative solution against ε for various values of the transport cost τ. It is apparent from figure 3.3 that the subsidy in the noncooperative equilibrium is always strictly lower than the efficient subsidy. Domestic policy makers are willing to set some positive subsidy in order to get closer to efficiency, while on the other hand they try to obtain better terms of trade, inducing them to reduce the subsidy on production of differentiated goods. The equilibrium outcome is a positive, but inefficiently low subsidy of production. Thus, the equilibrium of the subsidy game does not feature any home market effect, since policy makers do not over-subsidize production of differentiated goods, corroborating the intuition from the non-strategic analysis. Instead, the standard terms of trade effect prevails.[20]

3.5.4 Tariff on Imports

In this section we assume that the only strategic policy instrument available to trade authorities is a tariff on imports. We discuss our findings for the case of cooperation and non-cooperation under two alternative assumptions. Under the first hypothesis the monopolistic distortion of the differentiated sector is offset by an appropriate production subsidy (i.e. $\tau_C = \tau_C^* = 1/\varepsilon$); under the second, the monopolistic distortion is not corrected (i.e. $\tau_C = \tau_C^* = 0$). This distinction is key to clarify what drives the incentives of non-coordinated governments.

[19]The solutions are not sensitive to the choice of α.

[20]Note that our interpretation differs from Venables (1987), who shows in a non-strategic setting that a deviation from a zero to a small positive subsidy on production is always profitable from the viewpoint of domestic policy makers. The true reason why this is the case is the inefficiency of the market solution and not a Home Market Effect.

The case of cooperation

The solution of the cooperative problem with respect to the choice of the optimal $\tau_I = \tau_I^*$ is given by[21]:

$$\frac{1+\tau_I^{1-\varepsilon}\tau^{1-\varepsilon}}{1+\tau_I^{-\varepsilon}\tau^{1-\varepsilon}}\left[1+\tau_I^{-\varepsilon}\tau^{1-\varepsilon}-\frac{\tau_I-1}{\tau_I}\right]=\frac{1}{1-\tau_C}\frac{\varepsilon-1}{\varepsilon}\left[1+\tau_I^{-\varepsilon}\tau^{1-\varepsilon}-\frac{\varepsilon}{1-\alpha}\frac{\tau_I-1}{\tau_I}\right] \quad (3.38)$$

When $\tau_C = \tau_C^* = 1/\varepsilon$ the cooperative solution is $\tau_I = \tau_I^* = 1$. This can be seen also by looking at figure 3.6[22] which plots the endogenous responses of welfare and equilibrium variables to a simultaneous shift in home and foreign tariffs. The intuition behind this result is pretty straightforward: when the distortions due to monopolistic competition are removed by means of production subsidies the economy is already at the first best allocation therefore, utility of Home and Foreign is maximized in absence of tariffs. These results continue to hold if there are decreasing returns.

When $\tau_C = \tau_C^* = 0$ we have already seen in section 3.5.3 that the number of varieties produced in the economy is inefficiently low. Hence, the cooperative authority seeks to correct this distortion by subsidizing imports. Figure 3.6 plots the endogenous responses of welfare and equilibrium variables to a simultaneous shift in home and foreign tariffs for this case. Subsidizing imports in both countries reduces the relative price of imported differentiated varieties (both relatively to the domestic varieties and the homogeneous good). This will increase N and N^* while reducing the demand for the homogeneous good Z (whose production is above efficiency when $\tau_I = \tau_I^* = 1$). A positive tariff on the contrary would bring the economy further away from the efficient allocation. This is why household's utility is maximized for $\tau_I = \tau_I^* < 1$. Notice however that not surprisingly, the inefficiency due to monopolistic competition cannot be completely offset through tariffs: welfare is lower in figure 3.6 when $\tau_C = 0$ than in figure 3.6 when $\tau_C = 1/\varepsilon$. In order to show that the optimality of an import subsidy is

[21] See appendix for the derivation.

[22] As for the production subsidy, all the exercises in this section are carried out under the baseline calibration $\varepsilon = 4$, $\alpha = 0.4$ and $\tau = 1.7$ but results are robust for any $\varepsilon > 1$, $\tau > 1$, and $\alpha \in (0, 1)$.

not limited to the specific calibration used for figure 3.6, figure 3.4 reports the optimal import subsidy for the cooperative solution against ε for different values of the transport cost τ.

Note that the optimal cooperative subsidy becomes smaller and smaller the larger decreasing returns are. The reason is that decreasing returns bring the market allocation closer to the efficient one, because producing additional units of the homogeneous good has every time larger opportunity costs in terms of foregone production of differentiated goods.

The case of non-cooperation

As for production subsidies, before discussing the Nash equilibrium, we study the effects of a unilateral change. The exercise makes clear why single country policy makers may want to deviate from the cooperative policy. As clarified below, uncoordinated policies aim to improve the terms of trade and not to render domestic good cheaper by agglomerating firms in the domestic economy.

Non-strategic tariffs

Figure 3.6 plots some endogenous domestic and foreign variables as function of the home tariff under the assumption that the distortion due to monopolistic competition is removed. Suppose that domestic authorities decides unilaterally to provide a small subsidy to imports[23]. Such a policy improves domestic welfare at the expense of the foreign country. A subsidy to imports renders local differentiated goods relatively more expensive and makes households increase their demand for foreign goods. As a result firms agglomerate in the foreign economy and domestic varieties are reduced while foreign ones are boosted. If the number of firms diminishes, governments can cut production subsidies and tax bills. This causes a positive wealth effect which more than compensates that due to the subsidy to imports. Indeed, the demand of the homogenous good rises as does its consumption. At the same time the price of the differentiated goods augments and the terms of trade improve allowing to import more

[23]In a neighborhood of $\tau_I = \tau_I^* = 1$.

goods for each unit of exports. As a consequence, domestic households can increase also the overall consumption of the differentiated goods C, even if some of the domestic varieties have been destroyed.

The figure also clarifies why we can interpret our results in the light of the standard terms of trade externality. The incentives of uncoordinated authorities to deviate from the efficient allocation are explained as the attempt to exert the monopoly power on the production of domestic varieties. As a monopolist, these policy makers seek to reduce home output to render local goods relatively more expensive. Hence contrary to Venables (1987) and Ossa (2008), raising import tariffs would decrease domestic welfare.

Figure 3.8 depicts a domestic deviation in import tariffs for strongly decreasing returns ($\gamma = 0.3$). It is apparent that now domestic policy makers have an incentive to set a positive tariff on imports instead of a subsidy (like with constant returns). Nevertheless, the interpretation is still a domestic terms of trade improvement. An increase in the production of differentiated goods at home induced by the import tariff, reduces production of the homogeneous good and therefore increases domestic relative wages, thus improving the terms of trade. For strongly decreasing returns this effect dominates the negative effect on the terms of trade that works through the larger number of domestic varieties. Overall, there is a positive wealth effect that allows to increase the consumption of homogeneous goods which have become relative cheaper.

Figure 3.6 plots the effects of a unilateral change of the domestic tariff when the number of varieties is inefficiently low (i.e. $\tau_C = \tau_C^* = 0$). In this scenario a positive tariff improves domestic welfare. This outcome is in line with the analysis when the instrument available to policy makers is a production subsidy. These results are also consistent with Venables (1987) and Ossa (2008) but the analysis just conducted highlights a quite different interpretation. Uncoordinated authorities face a trade off between correcting the monopolistic distortion and improving the terms of trade. In equilibrium however the first incentive prevails. Indeed a tax on imports renders domestic differentiated goods cheaper and pushes firms to relocate in the domestic economy.

But differently from the case when $\tau_C = \tau_C^* = 1/\varepsilon$ the consumption of differentiated goods at Home is boosted. In fact, the increase in consumption of domestic differentiated varieties that have no transport cost brings prices closer to marginal cost, allows for a more efficient use of resources and reduces the domestic dead weight loss due to firm monopoly power. This is exactly why single country policy makers accepts the cost of terms of trade worsening. Notice that also the consumption of the homogenous good can augment thanks to the increase in the foreign production and the positive wealth effect due to tariff revenue redistribution.

When there are strongly decreasing returns $\gamma = 0.3$ (see figure 3.10) there is still an incentive to tax imports of differentiated goods. However, this domestic policy deviation is not caused by the incentive to bring the consumption of differentiated goods closer to the efficient level any more but by the fact that a tariff now leads to a strong terms of trade improvement. Our conclusion is then that in this model the incentive to tax imports has to be interpreted either as a "production efficiency effect" or a terms of trade effect and not as the attempt to exploit the home market effect in order to minimize transportation costs. Overall, we do not find any incentive to use import tariffs or production subsidies to push the number of domestically produced varieties above the efficient level. Rather the contrary.

Optimal tariffs under non-cooperation

After having clarified in the previous section the incentives that move policy makers away from the cooperative solution, we now analyze the equilibrium outcomes of the non-cooperative game. As for the case of the production subsidy, we restrict ourselves to the symmetric Nash equilibria of the game. Once again, we have to rely on numerical simulation. We calibrate $\alpha = 0.4$ and then study the Nash solution for $\varepsilon \in [1, 7]$ and $\tau = (1.1; 1.5; 1.9)$. We consider two cases: $\tau_C = \tau_C^* = 1/\varepsilon$ and $\tau_C = \tau_C^* = 0$.

Figure 3.11 reports the optimal import tariff for the case where the monopolistic distortion has been eliminated by means of the production subsidy. The first thing to be noticed is that no Nash equilibrium exists for $\varepsilon < 2$. More importantly, for all other values of ε and for all values of τ, the Nash equilibrium entails a subsidy to imports[24]. When the production subsidies eliminate the monopolistic competition distortions, single country policy makers' choices are driven solely by the incentive to manipulate the terms of trade in their favor. This incentive is stronger the higher their market power i.e., the lower the elasticity of substitution between varieties ε[25]. Coherently, the lower the elasticity the more the two countries are subsidizing their imports (τ_I moves closer and closer to zero).

Figure 3.12 depicts the same case of figure 3.11 when there are strong decreasing returns i.e. when $\gamma = 0.3$. Consistently with the analysis on non-strategic tariffs the policy makers are always willing to tax and not to subsidize imports. This is because with strong decreasing returns boosting differentiated good production improves the terms of trade through the effect on the relative wage. Obviously this incentive to improve the terms of trade is weaker the higher is the elasticity of substitution between home and foreign bundles.

Figure 3.13 shows the Nash solution for the case when the monopolistic distortion has not been eliminated and there is a constant returns to scale technology in the homogenous sector. Because of the assumption $\eta = \varepsilon$, a higher ε means at the same time lower monopolistic distortion (i.e. less inefficiency in the economy to be taken care of) and lower market power of each country (i.e. lower incentive to manipulate the terms of trade). As we saw in the previous analysis, domestic policy makers are willing to accept a worsening in the terms of trade in order to increase domestic production of the differentiated good to move closer to the efficient level. The optimal import tariff approaches one as the elasticity increases. Indeed, the higher the elasticity the lower the inefficiency in the economy but also the lower the incentive to manipulate the terms of trade. For relatively low values of ε instead the trade off faced by the policy maker

[24]For relatively high values of transport costs there are multiple equilibria but all of them imply $\tau_I < 1$.

[25]Recall that for the moment we are following the practice common in the related literature of assuming $\eta = \varepsilon$. The elasticity which is relevant for the term of trade externality is η. Considering different values for the two elasticities would allow us to disentangle the inefficiency due to monopolistic competition from the incentives to manipulate the terms of trade.

becomes stronger. On the one hand, the lower the elasticity the higher the distortion due to monopolistic competition and the stronger the incentive to impose a positive import tariff to increase the number of domestic varieties towards efficiency. On the other hand, lower elasticity gives higher market power to each country therefore making a stronger case for trying to improve the terms of trade in their favor by means of an import subsidy. While the incentive to correct for the monopolistic distortion prevails for values of ε above 1.5, the terms of trade incentive kicks in strongly for low values of the elasticity, thus the hump shaped form of the optimal import tariff. The above findings still hold if there are strong decreasing returns as made clear by the figure 3.14.

The wealth effect

As emphasized in the introduction, the redistribution of tariff revenues is key to explain the difference between our results and those of the previous literature. Neglecting this redistribution means neglecting wealth effects that are crucial for the open policy makers' optimal decisions. To see why consider that in our model[26]:

$$Z = (1 - \alpha)m$$

, namely the expenditure for the homogenous good is a constant share of consumers' wealth m. If, as in Venables (1987) and Ossa (2008), tariffs are pure waste (i.e $m = wL$) in equilibrium $Z = (1 - \alpha)L$. This outcome follows directly from the assumptions that preferences are Cobb-Douglas, there is free trade and the homogenous sector is perfectly competitive with a constant return to scale technology[27]. Intuitively, the substitution and the income effects due to a change in the relative price always compensate exactly. Then the equilibrium level of the homogenous good is exogenously determined and independent of trade policies. In this set up single country policy makers will find it optimal to tax imports as much as possible[28]. A positive tariff shifts local demand

[26]For the sake of simplicity in this paragraph we set $p_z = 1$.
[27]These last assumptions imply $w = p_z = 1$.
[28]In other words an unilateral increase of import tariffs increases always domestic welfare. See Ossa (2008).

towards home produced goods and induces firms of the differentiated sector to agglomerate in the domestic economy. This relocation benefits home consumers. Indeed the higher is the number of varieties locally produced, the lower are transport costs and the price of the differentiated goods and the more households can buy of them. At the same time, even if the homogeneous good becomes relatively more expensive, in equilibrium it is always consumed in the same amount exactly because labor income is fixed and tariffs are a pure waste.

However, once tariff revenues are redistributed, uncoordinated policy maker's optimal behavior is driven by the incentive to improve the terms of trade. In order to clarify our argument we analyzed two cases: 1) the lump sum transfers rebated to consumers consist of just the tariffs on imports of differentiated goods (i.e $m = L + T_{\tau_I}$ and $Z = (1-\alpha)(L + T_{\tau_I})$); 2) these transfers include the tariffs on imports and the taxes collected by governments to finance the correction of the monopolistic distortion in the differentiated sector (i.e $m = L + T_{\tau_I} - T_{\tau_C}$ and $Z = (1-\alpha)(L + T_{\tau_I} - T_{\tau_C})$). In both these cases the demand of the homogenous good is not invariant to policy decisions. Still, in response to a change in the relative price (and in tariffs) income and substitution effects cancel out. Nevertheless in equilibrium the demand of the homogenous goods is shifted by the wealth effect due to the transfers. Which impact a tariff produces on the homogenous good then depends on the sign of this wealth effect.

Tariff revenue is hump shaped because of the Laffer curve argument. Hence, when there are no taxes to subsidize production, the optimal tariff is positive but finite[29]. Increasing a tariff always boosts consumption of differentiated goods. But a too high tariff reduces that of the homogenous good because it decreases tariff revenues and thus consumers' income (i.e. the wealth effect becomes negative). Then raising tariffs unilaterally generates a tradeoff for single country trade authorities: in order to consume more differentiated goods, domestic households have to consume less of the homogenous good.

When governments tax consumers to eliminate the monopolistic distortion, the

[29] Ossa (2008) in the appendix.

wealth effect on the demand of the homogenous good associated with a tariff is negative even for low values of τ_I (more precisely in sufficiently small neighborhood of $\tau_I = 1$). Indeed, such kind of policy augments the production of the differentiated goods increasing then the taxes paid by consumers to finance production subsidies. This effect more than compensates that on tariff revenues. As a result, transfers decrease thereby reducing total household income. Thus, in equilibrium domestic demand for the homogenous good is reduced too and the domestic policy maker faces a trade off between increasing C and reducing Z[30].

3.6 Conclusion

In a two sector variant of the Krugman (1980) model of the intra-industry trade we study optimal trade policy of uncoordinated and coordinated authorities for different policy instruments: a production subsidy and a tariff on imports. According to all our findings, in this type of framework uncoordinated policy makers' behavior is explained by two opposite incentives: the willingness to exploit their monopoly power to improve the terms of trade (the standard terms of trade externality) and the need of correcting the distortion due to monopolistic competition (known as production efficiency effect). This result is clearly in contrast with some contributions of the trade literature (see in particular Venables (1987), Ossa (2008) and Helpman and Krugman (1989)) which, within the same type of set up have claimed that single country policies aim at inefficiently agglomerating firms in the domestic economy, in order to minimize transportation costs (the so called home market effect). What explains the difference between our conclusions and those of the existing analysis is the following. On the one

[30]One may wonder if these findings hold even with a more general utility function. Venables (1987) for instance considers the class of weakly separable preferences. Within this class our results would not change as long as the homogenous and the differentiated goods are gross-substitutes. In that case the Walrasian demand of the homogenous good $Z(P_C, m)$ is such that $\frac{\partial Z(P_C,m)}{\partial P_C} > 0$. Hence under this assumption a policy that renders the differentiated goods cheaper and reduces the demand for the homogenous good, would shrink that demand even more than when preferences are Cobb-Douglas. Notice that both quasi-linear and CES utility functions may satisfy the property of gross substitutability. In particular for CES it is sufficient that the elasticity between Z and C is greater than 1.

hand, we relax one simplifying assumption common to the literature, which is to model tariffs as a pure waste. Neglecting the redistribution of tariff revenues implies eliminating wealth effects that are key for optimal policy choices. On the other hand, we show that in this kind of framework the equilibrium allocation is not Pareto-efficient. Indeed, given that there are two sectors and endogenous firm entry, the production of differentiated goods is not fixed by labor supply and a proper production subsidy improves consumers' welfare.

Overlooking these features of the model is what has led to a misinterpretation of the underlying mechanism driving uncoordinated policy decisions. In fact, once the monopolistic distortion is offset by an appropriate subsidy and tariff revenues are redistributed, it is clear that single country policy makers' behavior can be explained only in the light of the standard terms of trade externality: they find it optimal to subsidize imports(!); they want to induce firm exit from the domestic market in order to render domestic goods more expensive than foreign ones. Conversely, if there are no production subsidies, we do find - like Venables (1987) and Ossa (2008) - that a positive tariff is optimal from the uncoordinated authority viewpoint. However, this result should not be interpreted as an attempt to induce firms to relocate in the domestic economy in order to render differentiated goods cheaper, but as a way to push the economy towards a more efficient use of resources.

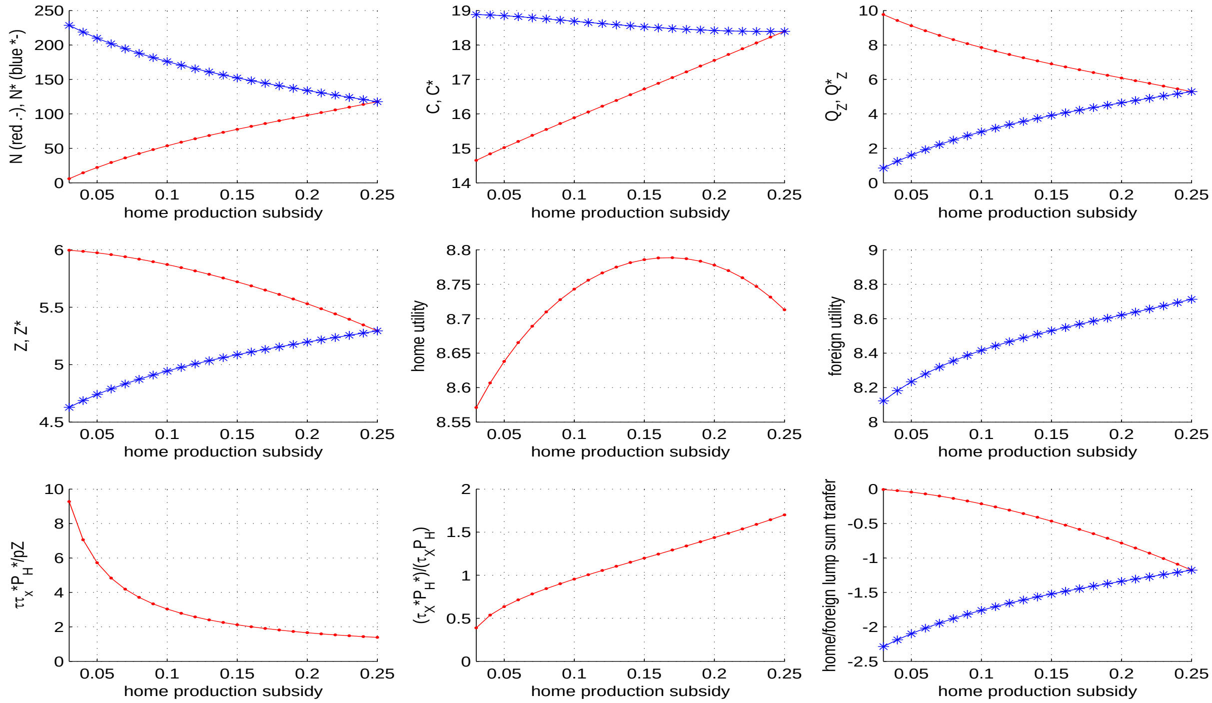

Figure 3.1: Effects of an unilateral shift of the domestic production subsidy when $\tau_C^* = 1/\varepsilon$.

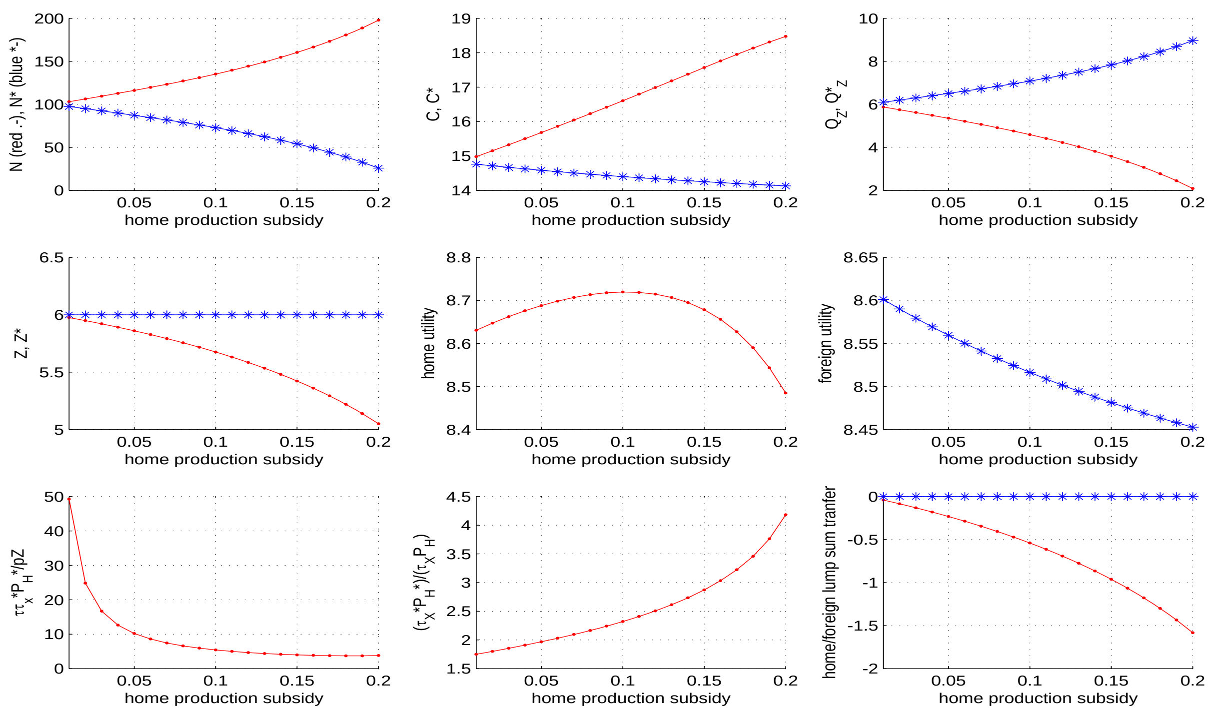

Figure 3.2: Effects of an unilateral shift of the domestic production subsidy when $\tau_C^* = 0$.

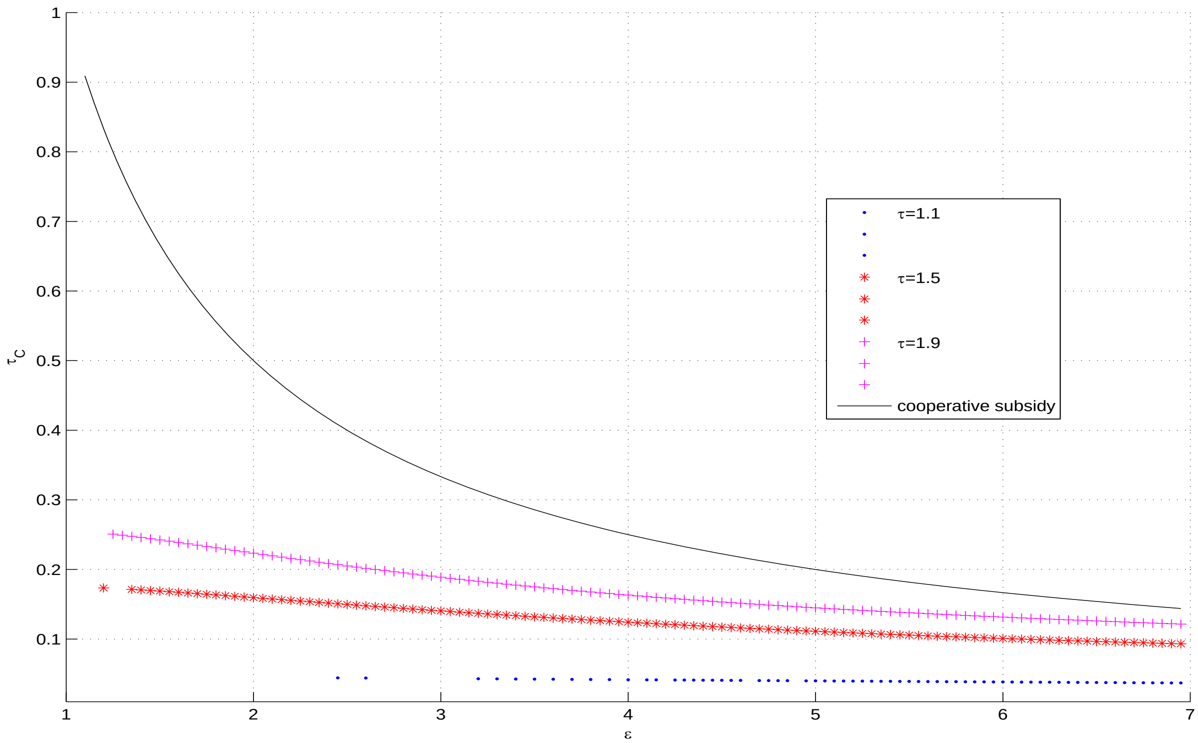

Figure 3.3: Nash solution for a production subsidy.

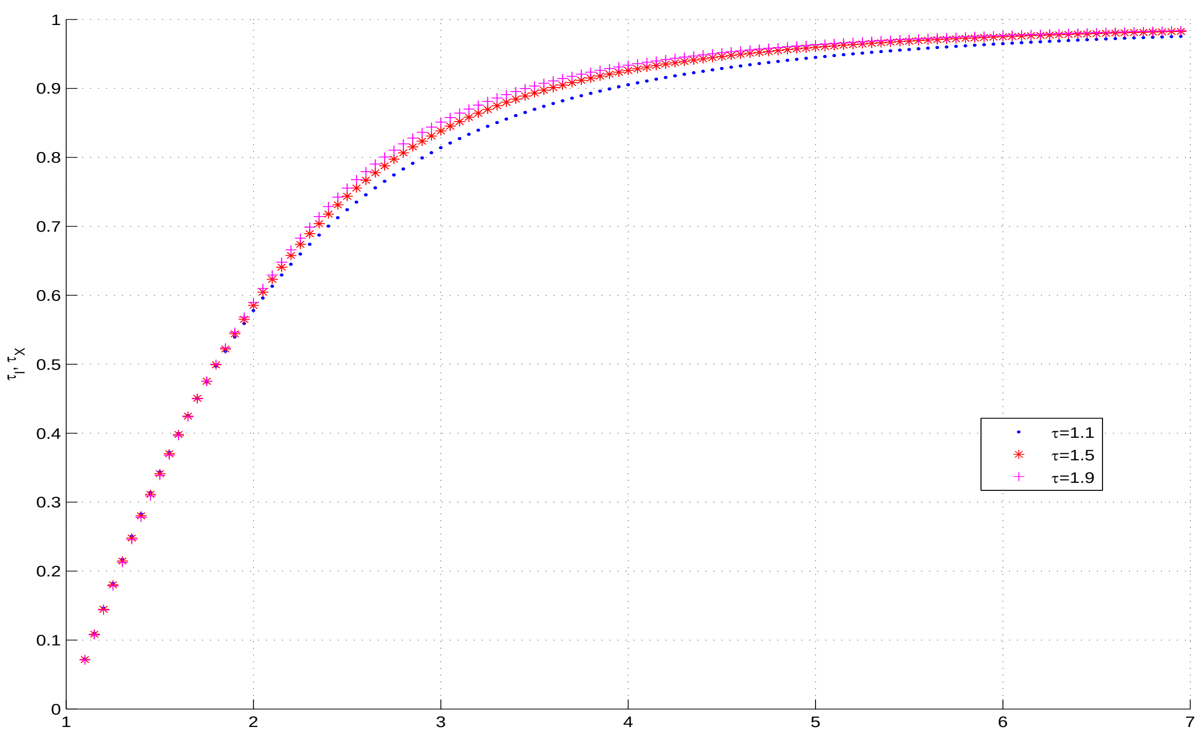

Figure 3.4: Cooperative solution for import tariffs when $\tau_C = \tau_C^* = 0$.

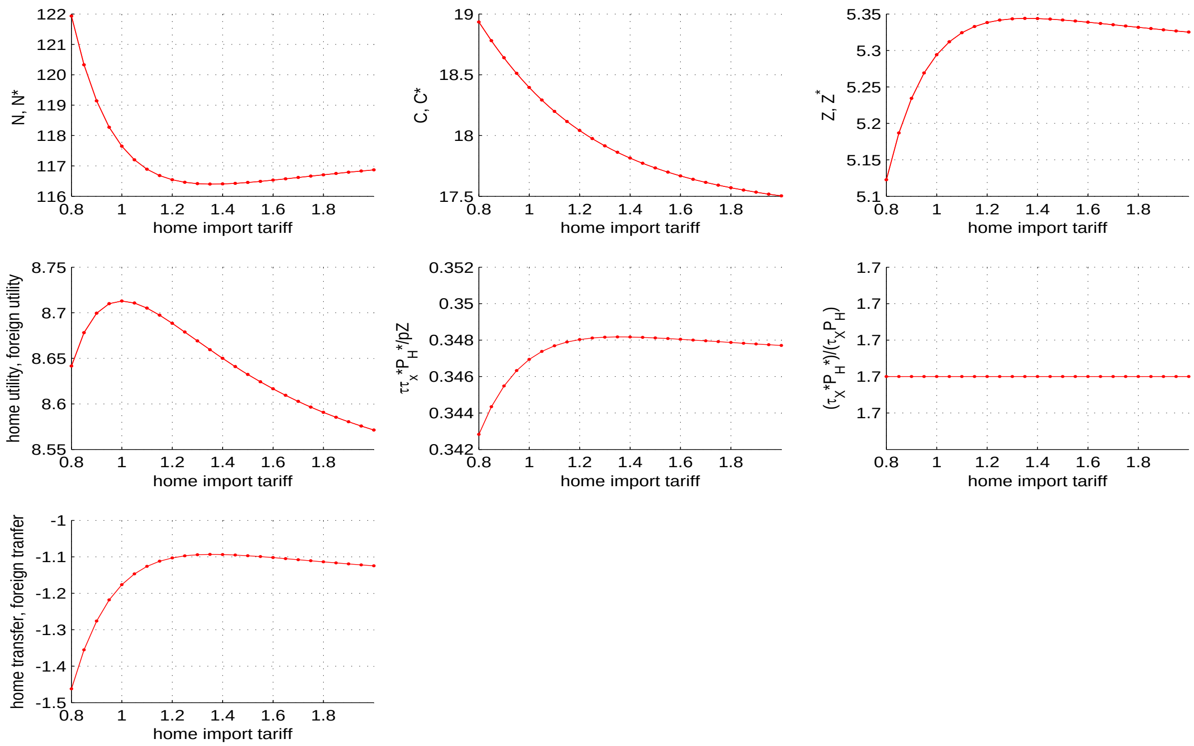

Figure 3.5: Effects of a simultaneous shift in home and foreign tariffs when $\tau_C = \tau_C^* = 1/\varepsilon$.

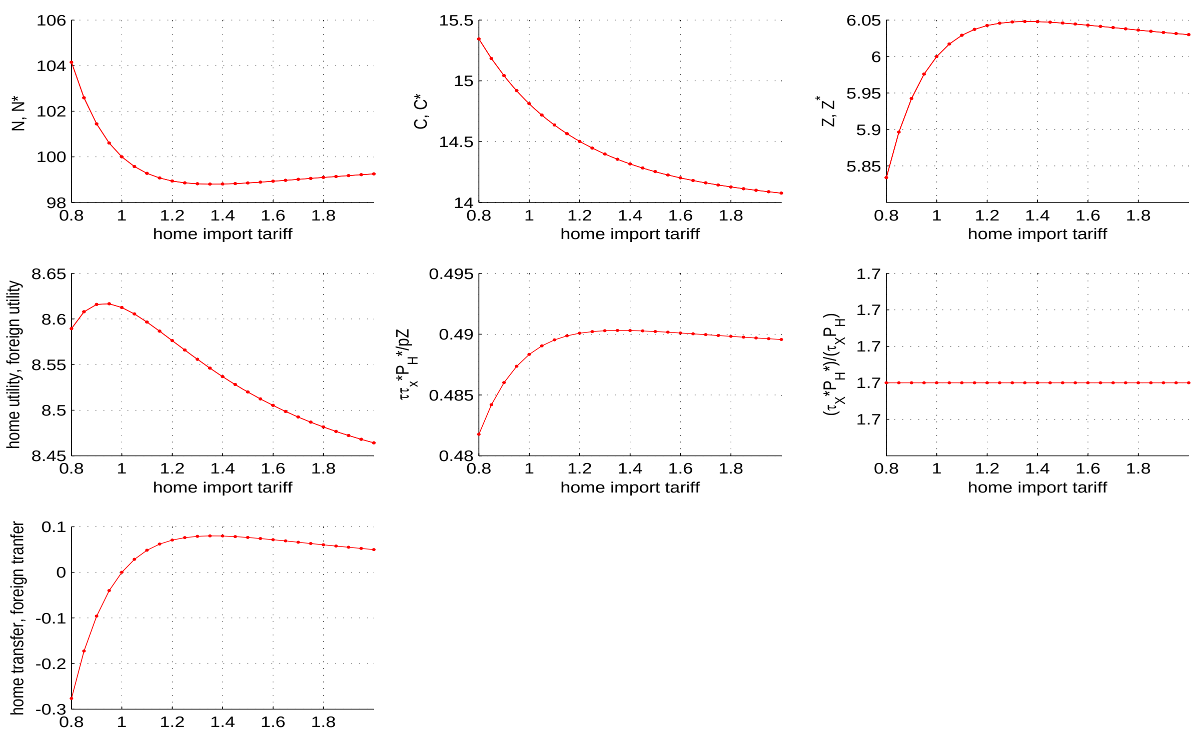

Figure 3.6: Effects of a simultaneous shift in home and foreign tariffs when $\tau_C = \tau_C^* = 0$.

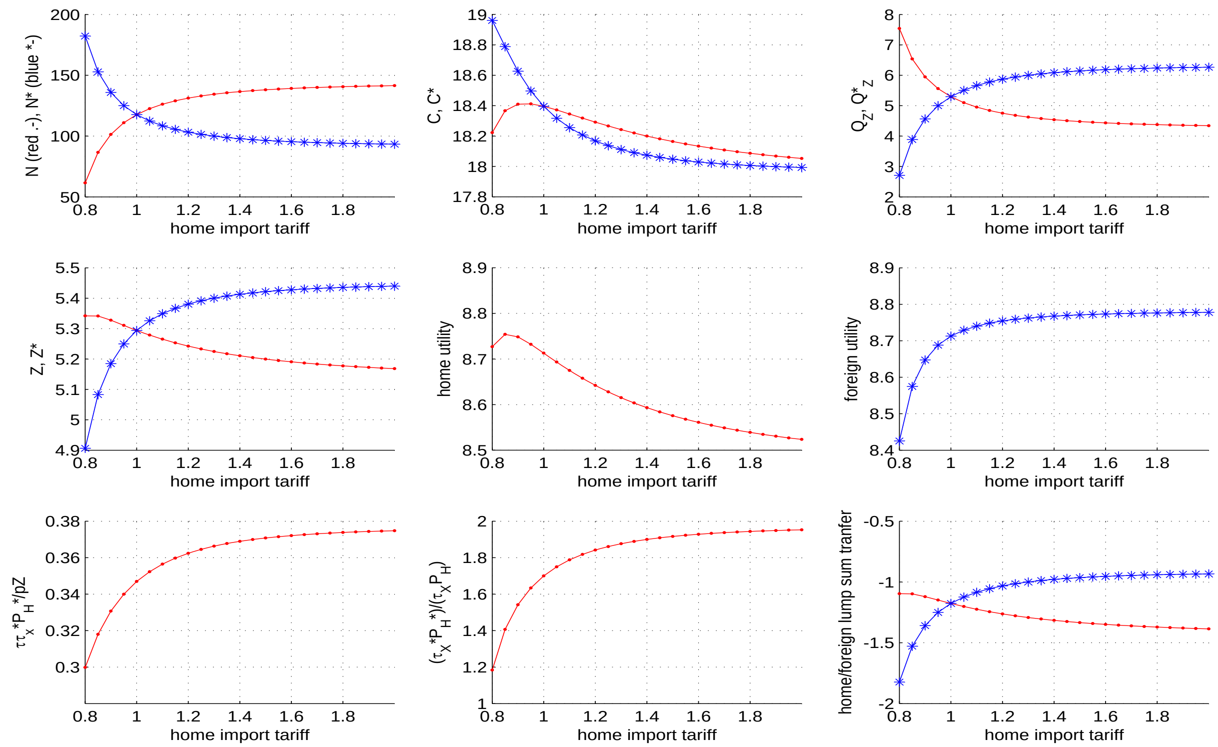

Figure 3.7: Effects of an unilateral shift of the domestic tariff when $\tau_C = \tau_C^* = 1/\varepsilon$.

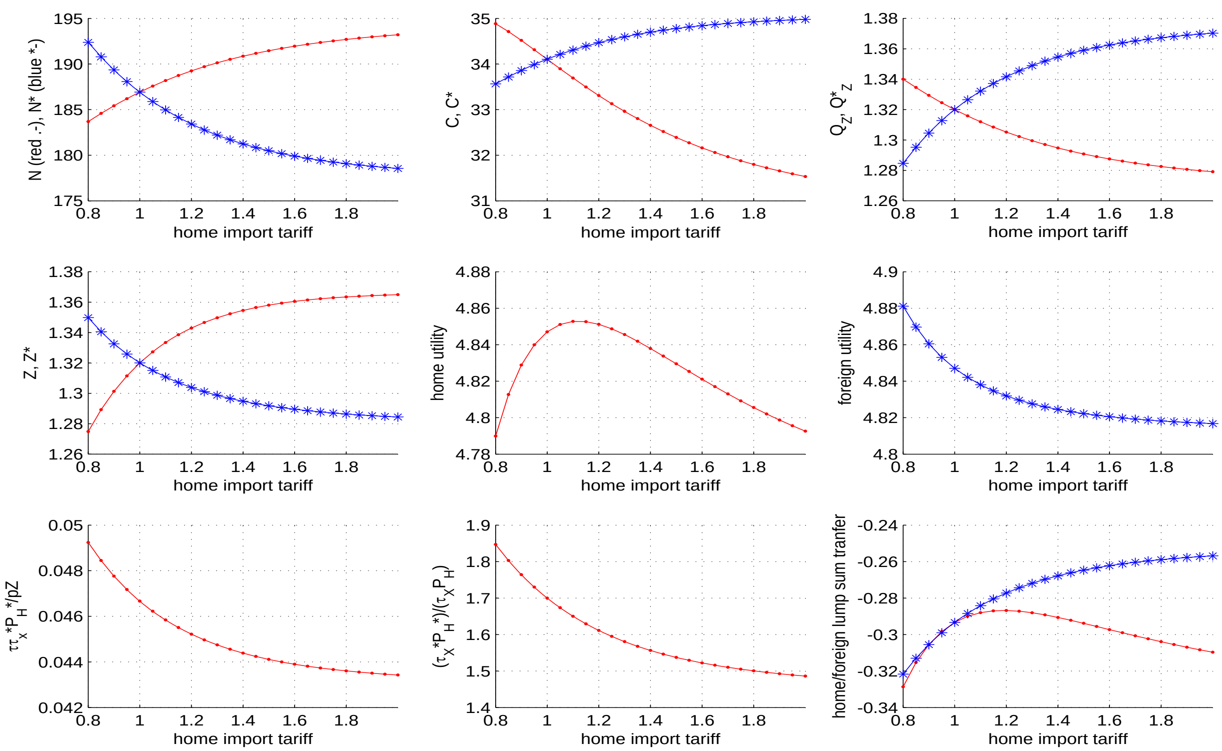

Figure 3.8: Effects of an unilateral shift of the domestic tariff when $\tau_C = \tau_C^* = 1/\varepsilon$ and $\gamma = 0.3$.

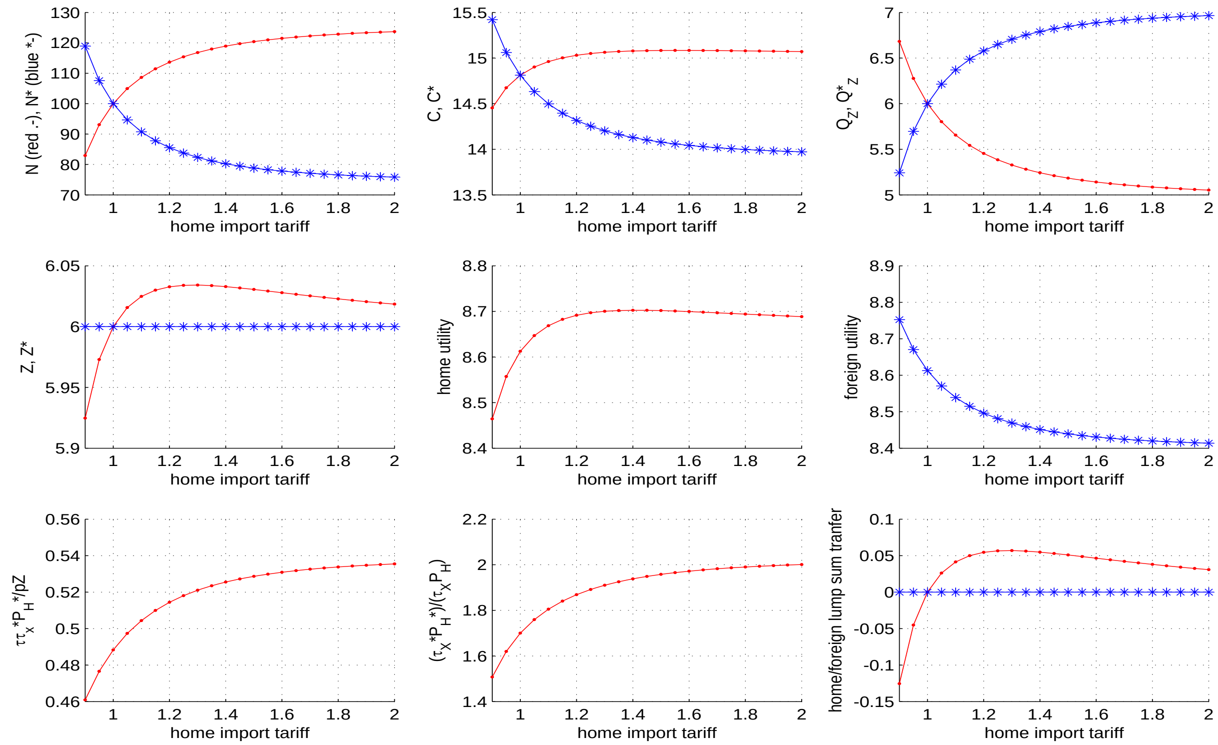

Figure 3.9: Effects of an unilateral shift of the domestic tariff when $\tau_C = \tau_C^* = 0$.

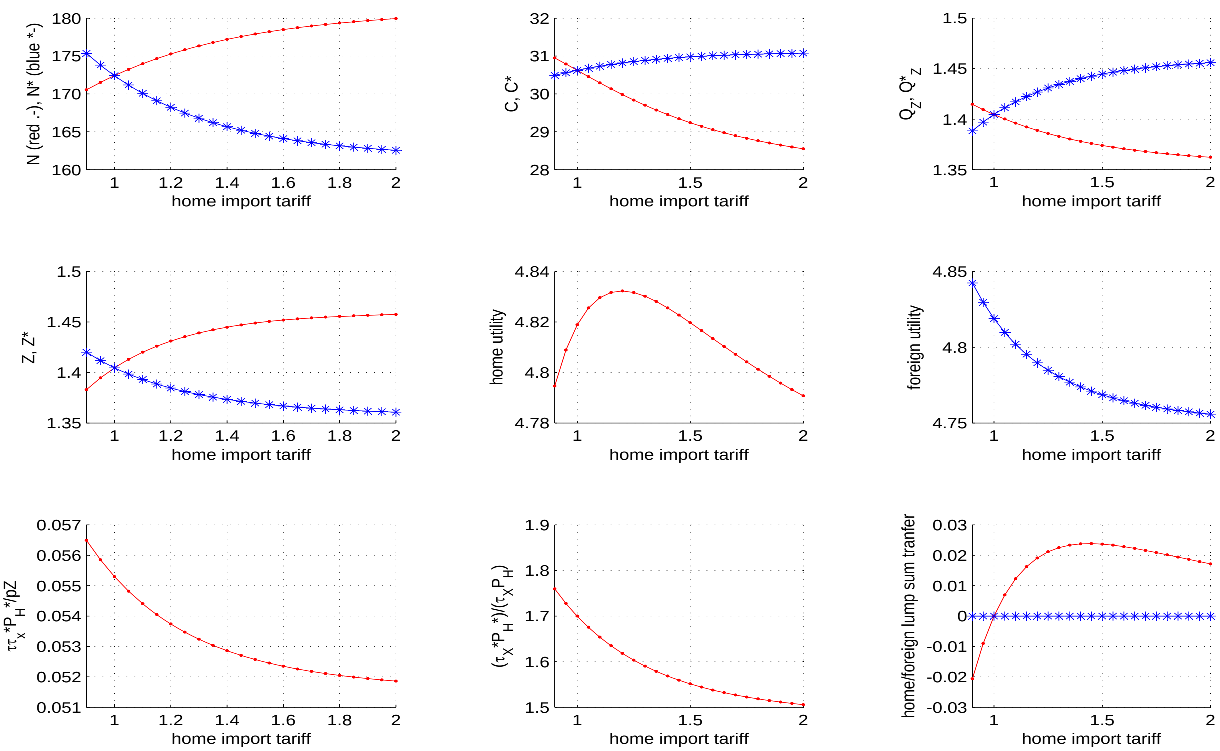

Figure 3.10: Effects of an unilateral shift of the domestic tariff when $\tau_C = \tau_C^* = 0$ and $\gamma = 0.3$.

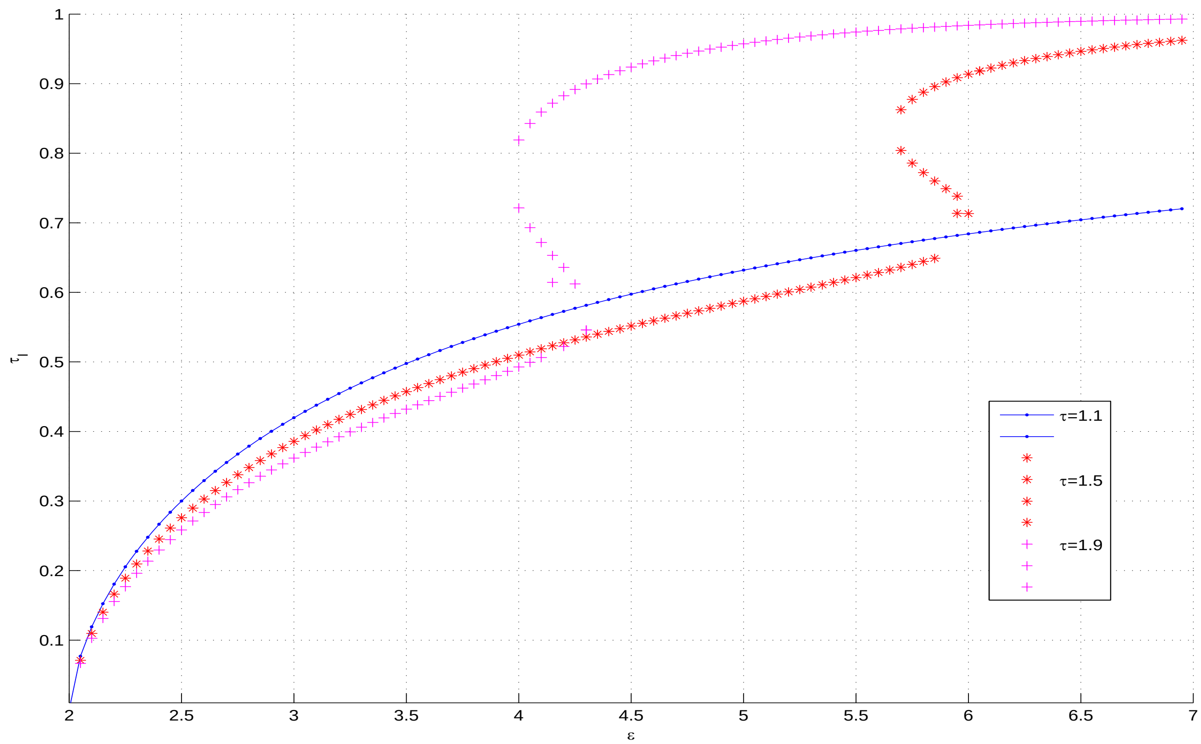

Figure 3.11: Nash solution for import tariff when $\tau_C = \tau_C^* = 1/\varepsilon$.

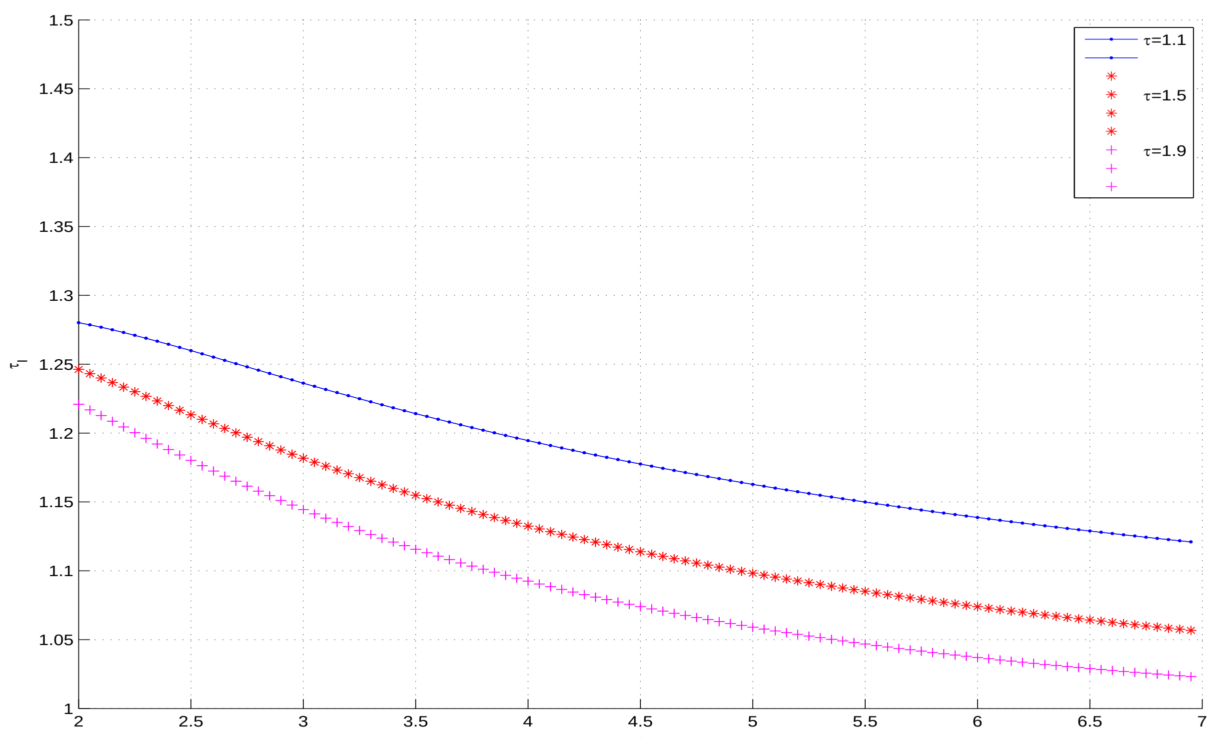

Figure 3.12: Nash solution for import tariff when $\tau_C = \tau_C^* = 1/\varepsilon$ and $\gamma = 0.3$.

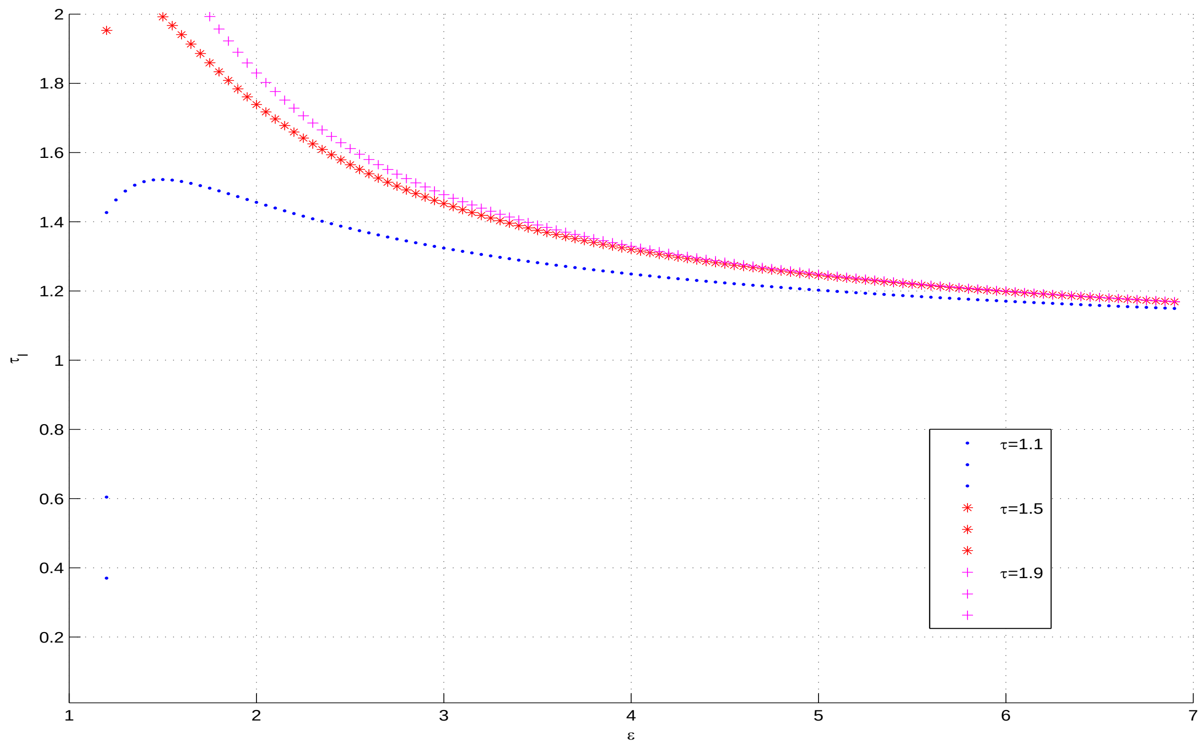

Figure 3.13: Nash solution for import tariff when $\tau_C = \tau_C^* = 0$.

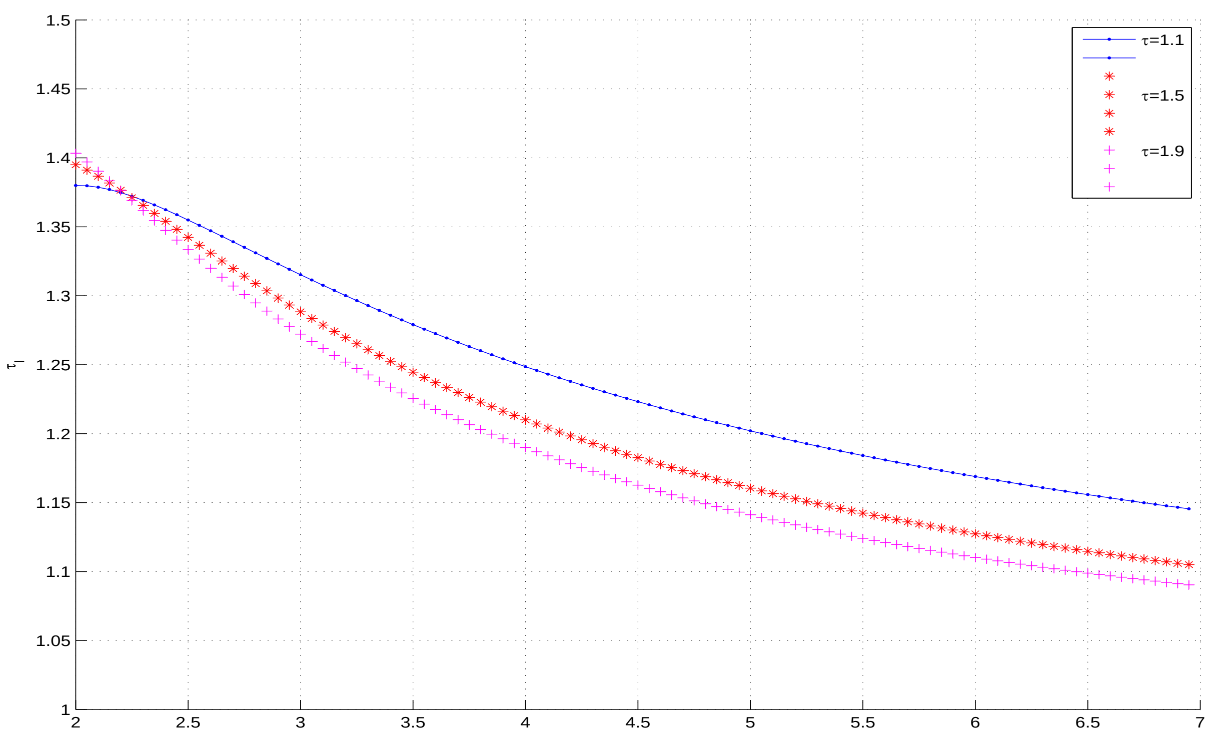

Figure 3.14: Nash solution for import tariff when $\tau_C = \tau_C^* = 0$ and $\gamma = 0.3$.

A Appendix to chapter 1

A.1 Proof of proposition 1

First part of the proof. If $\gamma = \sigma$ for all t then it can be shown that $\tilde{g}_t^{f,*} = \tilde{c}_t^{f,*} = \tilde{y}_t^{f,*}$ $\pi_t^* = 0$ satisfies the average of conditions (1.87)-(1.86). Then $\hat{g}_t^* = \hat{c}_t^* = \hat{y}_t^*$ which implies that $\tilde{g}_t^{m,*} = \tilde{c}_t^{m,*} = \tilde{y}_t^{m,*} = 0$.

The second part of the proof can be obtained by contradiction. If $\pi_t^* = 0$ for all t, then by (1.94) and (1.93) $\hat{c}_t^{m,*} = \hat{y}_t^{m,*}$ which implies that $\hat{c}_t^* = \hat{g}_t^*$. However $\hat{c}_t^* = \hat{g}_t^*$ is consistent with the average of conditions (1.87)-(1.86) only if only if $\gamma = \sigma$ which contradicts our initial hypothesis.

A.2 The zero inflation deterministic steady states

A.2.1 The policy problem under coordination

Under coordination, the policy maker maximizes the following lagragian with respect to C_t^i, G_t^i, Y_t^i, $Y_{H,t}^i$, Z_t^i, K_t^i, F_t^i and $\Pi_{H,t}^i$ for all i and t:

$$
\begin{aligned}
L = \sum_{t=0}^{\infty} \beta^t E_0 \int_0^1 \Bigg\{ & \frac{C_t^{i\,1-\sigma}}{1-\sigma} + \chi \frac{G_t^{i\,1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}^i Z_t^i}{A_t^i}\right)^{\varphi+1} \\
&+\lambda_{1,t}^{i,c}\left[Y_t^i - \left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta}\left((1-\alpha)C_t^i + \alpha C_t^{i\,\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}^i}{P_{G,t}^i}\right)^{-\eta} G_t^i + \nu C_t^{i\,\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta}\right)\right] \\
&+\lambda_{2,t}^{i,c}\left[Y_{H,t}^i - \left(\frac{P_{H,t}^i}{P_t^i}\right)^{-\eta}\left((1-\psi)Y_t^i + \psi\left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta} C_t^{i\,\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta}\right)\right] \\
&+\lambda_{3,t}^{i,c}\left[K_t^i - \left(\frac{Y_{H,t}^i}{A_t^i}\right)^{\varphi+1} Z_t^{i\varphi}(1-\tau)(1+\mu_t^i)\frac{\varepsilon}{\varepsilon-1}\right] - \lambda_{3,t-1}^{i,c}\theta \Pi_{H,t}^{i\ \varepsilon} K_t^i \\
&+\lambda_{4,t}^{i,c}\left[F_t^i - Y_{H,t}^i C_t^{i\,-\sigma}\frac{P_t^i}{P_{C,t}^i}\frac{P_{H,t}^i}{P_t^i}\right] - \lambda_{4,t-1}^{i,c}\theta \Pi_{H,t}^{i\ (\varepsilon-1)} F_t^i
\end{aligned}
$$

$$+\lambda^{i,c}_{5,t}\left[F^i_t - K^i_t\left(\frac{1-\theta{\Pi^i_{H,t}}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right]$$

$$+\lambda^{i,c}_{6,t}\left[Z^i_t - \theta Z^i_{t-1}{\Pi^i}^{\varepsilon}_{H,t} - (1-\theta)\left(\frac{1-\theta{\Pi^i_{H,t}}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]$$

$$+\lambda^{i,c}_{7,t}\left[\left(\frac{C^i_t}{C^i_{t-1}}\right)^{-\sigma}\left(\frac{C^*_{t-1}}{C^*_t}\right)^{-\sigma}\Pi^*_t - \frac{P^i_{C,t}}{P^i_t}\frac{P^i_t}{P^i_{H,t}}\Pi^i_{H,t}\frac{P^i_{H,t-1}}{P^i_{t-1}}\frac{P^i_{t-1}}{P^i_{C,t-1}}\right]\Big\}di$$

where $P^i_t/P^i_{C,t}$, $P^i_{G,t}/P^i_{C,t}$, $P^i_{H,t}/P^i_t$, C^*_t, $\Upsilon_{C,t}$, $\Upsilon_{G,t}$ and $\Upsilon_{Y,t}$ are determined according (1.35), (1.36), (1.37), (1.33) and (1.46) and $Z_{-1} = 1$. According to the first order conditions evaluated at the zero inflation symmetric non-stochastic steady state:

$$C^{-\sigma} = \lambda^c_1 - \lambda^c_4\sigma Y C^{-\sigma-1}$$

$$\chi G^{-\gamma} = \lambda^c_1$$

$$\lambda^c_1 = \lambda^c_2$$

$$Y^{\varphi} = \lambda^c_2 - \lambda^c_3(\varphi+1)Y^{\varphi}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1} - \lambda^c_4 C^{-\sigma}$$

$$Y^{\varphi+1} = -\lambda^c_3\varphi Y^{\varphi+1} + \lambda^c_6(1-\theta)$$

$$\lambda^c_3(1-\theta) = \lambda^c_5$$

$$\lambda^c_4(1-\theta) = -\lambda^c_5$$

$$\lambda^c_3\theta\varepsilon K = -\lambda^c_4\theta(\varepsilon-1)F + \lambda^c_5\frac{\theta}{1-\theta}K$$

If $(1-\tau) = (1/(1+\mu))(\varepsilon-1)/\varepsilon$[1], this system of equations jointly with (1.38), (1.44), (1.45), (1.52), (1.53), (1.55), and (1.56) can be satisfied by the following solution:

$$C^{-\sigma} = Y^{\varphi}$$

$$\chi G^{-\gamma} = Y^{\varphi}$$

[1]Namely if even τ is chosen optimally in such a way $\lambda_3 = -\lambda_4 = 0$

$$Y = C + G$$

$$F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}}{1-\theta}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1}$$

$$Y_H = Y \qquad \Pi_H = 1 \qquad Z = 1$$

$$\lambda_1^c = Y^{\varphi} \qquad \lambda_2^c = \lambda_1^c \qquad \lambda_3^c = -\lambda_4^c = \frac{\lambda_5^c}{1-\theta} = 0 \qquad \lambda_6^c = \frac{Y^{\varphi}}{1-\theta} \qquad \lambda_7^c = 0$$

A.2.2 The fiscal policy problem under no-coordination

The fiscal policy makers maximize the following lagrangian with respect to C_t, G_t, Y_t, $Y_{H,t}$, Z_t, K_t, F_t and $\Pi_{H,t}$:

$$\begin{aligned}
L = \sum_{t=0}^{\infty}\beta^t E_0 \Bigg\{ & \frac{C_t^{1-\sigma}}{1-\sigma} + \chi\frac{G_t^{1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}Z_t}{A_t}\right)^{\varphi+1} \\
& +\lambda_{1,t}^f\left[Y_t - \left(\frac{P_t}{P_{C,t}}\right)^{-\eta}\left((1-\alpha)C_t + \alpha C_t^{\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}}{P_{G,t}}\right)^{-\eta} G_t + \nu C_t^{\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta}\right)\right] \\
& +\lambda_{2,t}^f\left[Y_{H,t} - \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left((1-\psi)Y_t + \psi\left(\frac{P_t}{P_{C,t}}\right)^{-\eta} C_t^{\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta}\right)\right] \\
& +\lambda_{3,t}^f\left[K_t - \left(\frac{Y_{H,t}}{A_t}\right)^{\varphi+1} Z_t^{\varphi}(1-\tau)(1+\mu_t)\frac{\varepsilon}{\varepsilon-1}\right] - \lambda_{3,t-1}^f\theta\Pi_{H,t}^{\varepsilon}K_t \\
& +\lambda_{4,t}^f\left[F_t - Y_{H,t}C_t^{-\sigma}\frac{P_t}{P_{C,t}}\frac{P_{H,t}}{P_t}\right] - \lambda_{4,t-1}^f\theta\Pi_{H,t}^{(\varepsilon-1)}F_t \\
& +\lambda_{5,t}^f\left[F_t - K_t\left(\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right] \\
& +\lambda_{6,t}^f\left[Z_t - \theta Z_{t-1}\Pi_{H,t}^{\varepsilon} - (1-\theta)\left(\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right] \\
& +\lambda_{7,t}^f\left[\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\left(\frac{C_{t-1}^*}{C_t^*}\right)^{-\sigma}\Pi_t^* - \frac{P_{C,t}}{P_t}\frac{P_t}{P_{H,t}}\Pi_{H,t}\frac{P_{H,t-1}}{P_{t-1}}\frac{P_{t-1}}{P_{C,t-1}}\right]\Bigg\}
\end{aligned}$$

where $P_t/P_{C,t}$, $P_{G,t}/P_{C,t}$ and $P_{H,t}/P_t$ are determined according (1.35), (1.36) and (1.37) and C_t^*, $\Upsilon_{C,t}$, $\Upsilon_{G,t}$ and $\Upsilon_{Y,t}$ are taken as given. According to first order conditions evaluated at the zero inflation symmetric non-stochastic steady state:

$$C^{-\sigma} = \lambda_1^f(\delta_1 + \delta_2 \frac{\rho}{1-\rho}) + \lambda_2^f(1-\psi)\delta_3 \frac{1}{\rho} - \lambda_4^f Y C^{-\sigma}[C^{-1}\sigma + (\omega_4 - 1)] + \lambda_7^f \frac{(1-\beta)}{C}[\sigma - (\omega_4 - 1)]$$

$$\chi G^{-\gamma} = \lambda_1^f$$

$$\lambda_1^f = \lambda_2^f(1-\psi)$$

$$Y^{\varphi} = \lambda_2^f - \lambda_3^f(\varphi+1)Y^{\varphi}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1} - \lambda_4^f C^{-\sigma}$$

$$Y^{\varphi+1} = -\lambda_3^f \varphi Y^{\varphi+1} + \lambda_6^f(1-\theta)$$

$$\lambda_3^f(1-\theta) = \lambda_5^f$$

$$\lambda_4^f(1-\theta) = -\lambda_5^f$$

$$\lambda_3^f \theta \varepsilon K = -\lambda_4^f \theta(\varepsilon-1)F + \lambda_5^f \frac{\theta}{1-\theta}K - \lambda_7^f$$

If $(1-\tau) = ((1/(1+\mu))(\varepsilon-1)/\varepsilon)(1-\psi)\left[\delta_1 + \delta_2 \frac{G}{C} + \delta_3 \frac{Y}{C}\right]$[2] this system of equations jointly with (1.38), (1.44), (1.45), (1.52), (1.53), (1.55), and (1.56) can be satisfied by the following solution:

$$C^{-\sigma} = (1-\psi)\left[\delta_1 + \delta_2 \frac{G}{C} + \delta_3 \frac{Y}{C}\right] Y^{\varphi}$$

$$\chi G^{-\gamma} = (1-\psi)(1-\nu)Y^{\varphi}$$

$$Y = C + G$$

$$F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}}{1-\theta}(1+\mu)(1-\tau)\frac{\varepsilon}{\varepsilon-1}$$

$$Y_H = Y \qquad \Pi_H = 1 \qquad Z = 1$$

$$\lambda_1^f = (1-\psi)Y^{\varphi} \qquad \lambda_2^f = Y^{\varphi} \qquad \lambda_3^f = -\lambda_4^f = \frac{\lambda_5^f}{1-\theta} = 0 \qquad \lambda_6^f = \frac{Y^{\varphi+1}}{1-\theta} \qquad \lambda_7^f = 0$$

A.2.3 The monetary policy problem under no-coordination

The monetary policy maker maximizes with respect to C_t^i, Y_t^i, $Y_{H,t}^i$, Z_t^i, K_t^i, F_t^i and $\Pi_{H,t}^i$ for all i and t the following lagragian:

[2]Namely if even τ is chosen to maximize the objective of the fiscal policy maker ensuring $\lambda_3^f = 0$.

$$L = \sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \left\{ \frac{C_t^{i\,1-\sigma}}{1-\sigma} + \chi \frac{G_t^{i\,1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}^i Z_t^i}{A_t^i}\right)^{\varphi+1} \right.$$

$$+\lambda_{1,t}^{i,m}\left[Y_t^i - \left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta}\left((1-\alpha)C_t^i + \alpha C_t^{i\,\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}^i}{P_{G,t}^i}\right)^{-\eta} G_t^i + \nu C_t^{i\,\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta}\right)\right]$$

$$+\lambda_{2,t}^{i,m}\left[Y_{H,t}^i - \left(\frac{P_{H,t}^i}{P_t^i}\right)^{-\eta}\left((1-\psi)Y_t^i + \psi\left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta} C_t^{i\,\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta}\right)\right]$$

$$+\lambda_{3,t}^{i,m}\left[K_t^i - \left(\frac{Y_{H,t}^i}{A_t^i}\right)^{\varphi+1} Z_t^{i\,\varphi}(1-\tau)(1+\mu_t^i)\frac{\varepsilon}{\varepsilon-1}\right] - \lambda_{3,t-1}^{i,m}\theta \Pi_{H,t}^{i\;\varepsilon} K_t^i$$

$$+\lambda_{4,t}^{i,m}\left[F_t^i - Y_{H,t}^i C_t^{i\,-\sigma}\frac{P_t^i}{P_{C,t}^i}\frac{P_{H,t}^i}{P_t^i}\right] - \lambda_{4,t-1}^{i,m}\theta\Pi_{H,t}^{i\;(\varepsilon-1)}F_t^i$$

$$+\lambda_{5,t}^{i,m}\left[F_t^i - K_t^i\left(\frac{1-\theta\Pi_{H,t}^{i\;\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right]$$

$$+\lambda_{6,t}^{i,m}\left[Z_t^i - \theta Z_{t-1}^i \Pi_{H,t}^{i\,\varepsilon} - (1-\theta)\left(\frac{1-\theta\Pi_{H,t}^{i\;\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]$$

$$\left. +\lambda_{7,t}^{i,m}\left[\left(\frac{C_t^i}{C_{t-1}^i}\right)^{-\sigma}\left(\frac{C_{t-1}^*}{C_t^*}\right)^{-\sigma}\Pi_t^* - \frac{P_{C,t}^i}{P_t^i}\frac{P_t^i}{P_{H,t}^i}\Pi_{H,t}^i\frac{P_{H,t-1}^i}{P_{t-1}^i}\frac{P_{t-1}^i}{P_{C,t-1}^i}\right]\right\} di$$

where $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$, C_t^*, $\Upsilon_{C,t}$, $\Upsilon_{G,t}$ and $\Upsilon_{Y,t}$ are determined according (1.35), (1.36), (1.37), (1.33) and (1.46) G_t^i is taken as given for all i and t and $Z_{-1} = 1$. According to the first order conditions evaluated at the zero inflation symmetric non-stochastic steady state:

$$C^{-\sigma} = \lambda_1^m - \lambda_4^m \sigma Y C^{-\sigma-1}$$

$$\lambda_1^m = \lambda_2^m$$

$$Y^{\varphi} = \lambda_2^m - \lambda_3^m(\varphi+1)Y^{\varphi}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1} - \lambda_4^m C^{-\sigma}$$

$$Y^{\varphi+1} = -\lambda_3^m \varphi Y^{\varphi+1} + \lambda_6^m(1-\theta)$$

$$\lambda_3^m(1-\theta) = \lambda_5^m$$

$$\lambda_4^m(1-\theta) = -\lambda_5^m$$

$$\lambda_3^m\theta\varepsilon K = -\lambda_4^m\theta(\varepsilon-1)F + \lambda_5^m\frac{\theta}{1-\theta}K$$

It easy to show that if $(1-\tau) = ((1/(1+\mu))(\varepsilon-1)/\varepsilon)(1-\psi)\left[\delta_1 + \delta_2\frac{G}{C} + \delta_3\frac{Y}{C}\right]$ this system of equations jointly with (1.38), (1.44), (1.45), (1.52), (1.53), (1.55), and (1.56) can be satisfied by the following solution:

$$C^{-\sigma} = (1-\psi)\left[\delta_1 + \delta_2\frac{G}{C} + \delta_3\frac{Y}{C}\right]Y^{\varphi}$$

$$Y = C + G$$

$$F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}}{1-\theta}(1+\mu)(1-\tau)\frac{\varepsilon}{\varepsilon-1}$$

$$Y_H = Y \qquad \Pi_H = 1 \qquad Z = 1$$

$$\lambda_1^m = Y^{\varphi}\left[\frac{C}{Y}\delta\varphi + \sigma\right]\Big/\left[\frac{C}{Y}\varphi + \sigma\right] \qquad \lambda_2^m = \lambda_1^m$$

$$\lambda_3^m = -\lambda_4^m = \frac{\lambda_5^m}{1-\theta} = \frac{C}{Y}\frac{(\delta-1)}{\delta}\Big/\left[\frac{C}{Y}\varphi + \sigma\right] \qquad \lambda_6^m = \frac{Y^{\varphi+1}(1-\varphi\lambda_4^m)}{1-\theta} \qquad \lambda_7^m = 0$$

A.3 A purely quadratic approximation to policy makers' objectives

In order to recover the optimal policies we need to approximate up to the second order single country representative agent utility given by (1.1) in the following way.

First we can approximate the utility derived from private consumption as:

$$\frac{C_t^{1-\sigma}}{1-\sigma} \simeq \frac{C^{1-\sigma}}{1-\sigma} + C^{1-\sigma}(\hat{c}_t + \frac{1}{2}\hat{c}_t^2) - \frac{\sigma}{2}C^{1-\sigma}\hat{c}_t^2 + t.i.p. \tag{A.1}$$

where $\hat{c}_t$ stands for the log-deviations of private consumption from the steady state[3].

[3] From now this convention will be used: $\hat{x}_t$ represents the log-deviation of X_t from the steady state.

Similarly the utility derived from the consumption of public goods can be approximated:

$$\frac{G_t^{1-\gamma}}{1-\gamma} \simeq \frac{G^{1-\gamma}}{1-\gamma} + G^{1-\gamma}(\hat{g}_t + \frac{1}{2}\hat{g}_t^2) - \frac{\gamma}{2}G^{1-\gamma}\hat{g}_t^2 + t.i.p. \tag{A.2}$$

The labor disutility can be approximated by taking into account that $N_t = \frac{Y_{H,t}Z_t}{A_t}$ and, as showed by Galí and Monacelli (2009), being $Z_t = \int_0^1 \left(\frac{p_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon} dk$:

$$\hat{z}_t \simeq \frac{\varepsilon}{2} Var_k(p_{H,t}(k)) \tag{A.3}$$

In words the approximation of Z_t around the symmetric steady state is purely quadratic. Moreover following Woodford (2001, NBER WP8071) it is possible to show that $\sum_{t=0}^{\infty}\beta^t Var_k(p_{H,t}(k)) = \frac{1}{\lambda}\sum_{t=0}^{\infty}\beta^t \pi_{H,t}^2$ with $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Thus:

$$\frac{1}{\varphi+1}\left(\frac{Y_t Z_t}{A_t}\right)^{\varphi+1} \simeq \frac{1}{\varphi+1}Y^{\varphi+1} + Y^{\varphi+1}(\hat{y}_{H,t} + \frac{1}{2}\hat{y}_{H,t}^2) + Y^{\varphi+1}\frac{\varepsilon}{2\lambda}(\pi_{H,t})^2 + \frac{\varphi}{2}Y^{\varphi+1}\hat{y}_{H,t}^2$$
$$-(\varphi+1)Y^{\varphi+1}\hat{y}_{H,t}a_t + t.i.p. \tag{A.4}$$

A.3.1 The welfare approximation under coordination

Under coordination, at the steady state, the fiscal authority chooses to produce the efficient level of public goods. Therefore $C^{-\sigma} = \chi G^{-\gamma} = Y^{\varphi}$ which implies that the second order approximation of the average union welfare can be rewritten as:

$$\sum_{t=0}^{\infty}\beta^t Y^{\varphi+1} E_0 \int_0^1 \left[\hat{s}_t^{i'} z_s - \frac{1}{2}\hat{s}_t^{i'} Z_{s,s}\hat{s}_t^i + \hat{s}_t^{i'} Z_{s,a}\hat{a}_t^i\right] + t.i.p. \tag{A.5}$$

where

$$\hat{s}_t' \equiv \left[\hat{y}_{H,t}^i,\ \hat{g}_t^i,\ \hat{c}_t^i,\ \pi_{H,t}^i\right] \qquad z_s' \equiv [-1,\ \rho,\ (1-\rho),\ 0]$$

$$Z_{s,s} \equiv \begin{bmatrix} (\varphi+1) & 0 & 0 & 0 \\ 0 & (\gamma-1)(1-\rho) & 0 & 0 \\ 0 & 0 & (\sigma-1)\rho & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix}$$

$$Z_{s,a} \equiv \begin{bmatrix} (1+\varphi) \\ 0 \\ 0 \end{bmatrix}$$

Again it is possible to substitute the linear quadratic terms of (A.1) by using the second order approximation of the resource constraints namely:

$$0 \simeq -\int_0^1 \hat{y}_t^i di - \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \int_0^1 \hat{s}_t^i di' \; h_s + \frac{1}{2}\int_0^1 \hat{s}_t^{i'} H_{s,s} \hat{s}_t^i di + \frac{1}{2}\int_0^1 \hat{s}_t^i di' H_{S,S} \int_0^1 \hat{s}_t^i di + t.i.p. \tag{A.6}$$

$$0 \simeq \int_0^1 \hat{s}_t^i di' \; p_s + \int_0^1 \hat{y}_t^i di + \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \frac{1}{2}\int_0^1 \hat{s}_t^{i'} P_{s,s} \hat{s}_t^i di + \frac{1}{2}\int_0^1 \hat{s}_t^i di P_{S,S} \int_0^1 \hat{s}_t^i di + \int_0^1 \hat{y}_t^i P_{y,s} \hat{s}_t^i di$$
$$+ \int_0^1 \hat{y}_t^i di P_{Y,S} \int_0^1 \hat{s}_t^i di + t.i.p. \tag{A.7}$$

where

$$h_s' \equiv [0, \; (1-\rho), \; \rho, \; 0]$$

$$H_{s,s} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (1-\rho) & \xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & \xi\nu(1-\nu)(1-\rho) & \rho + \omega_1\rho + \omega_2(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H_{S,S} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & -\xi\nu(1-\nu)(1-\rho) & -\omega_1\rho - \omega_2(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$p_s' \equiv [-1, \; 0, \; 0, \; 0] \qquad P_{y,s} \equiv \begin{bmatrix} 0 & 0 & \xi\psi & 0 \end{bmatrix} \qquad P_{Y,S} \equiv \begin{bmatrix} 0 & 0 & -\xi\psi & 0 \end{bmatrix}$$

$$P_{s,s} \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad P_{S,S} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$\xi \equiv \frac{\eta\sigma}{1-\alpha}$$

$$\omega_1 \equiv \frac{\alpha\,\eta\,\sigma\;(\sigma - (1-\alpha)\;\alpha\;(1-\eta\,\sigma))}{(1-\alpha)^2}$$

$$\omega_2 \equiv \frac{\eta\,\nu\,\sigma\;((\nu-1) + (\sigma-1) - (1-2\,\eta)\;(1-\nu)\;\nu\,\sigma - \alpha\;(\nu-2)\;(1+(1-\eta)\;\sigma) - (1-\eta)\;\sigma\nu)}{(1-\alpha)^2}$$

$$\omega_3 \equiv -\left(\frac{\eta\,\sigma\,\psi\;((1-\alpha-\alpha\;(1-\eta)\;\sigma)\;(1-\psi)\;(2-\psi) - \sigma\;(1+\eta\;(1-\psi)\;\psi))}{(1-\alpha)^2\,(1-\psi)^2}\right)$$

Given (A.22) and (A.23) it is easy to show that:

$$0 \simeq \int_0^1 \hat{s}_t^i di' \ r_s + \frac{1}{2}\int_0^1 \hat{s}_t^{i'} R_{s,s} \hat{s}_t^i di + \frac{1}{2}\int_0^1 \hat{s}_t^i di' R_{S,S} \int_0^1 \hat{s}_t^i di + t.i.p. \tag{A.8}$$

where

$$r_s \equiv p_s + h_s \qquad R_{s,s} \equiv P_{s,s} + H_{s,s} + h_y P_{y,s} + P'_{y,s} h'_y$$

$$R_{S,S} \equiv \ P_{S,S} + H_{S,S} + h_Y P_{y,s} + P'_{y,s} h'_Y + h_s P_{Y,S} + P'_{Y,S} h'_s \tag{A.9}$$

and

$$h_y^{'} \equiv [0,\ (1-\nu)(1-\rho),\ \delta_1\rho + \delta_2(1-\rho),\ 0] \qquad h_Y^{'} \equiv [0,\ \nu(1-\rho), \rho - \delta_1\rho - \delta_2(1-\rho),\ 0]$$

where $\varsigma_1 \equiv \xi(\psi + \nu)$ $\varsigma_3 \equiv \rho\omega_1 + (1-\rho)\omega_2 + \omega_3 + 2\xi\psi(\rho\delta_1 + (1-\rho)\delta_2)$

Given that

$$z_s = r_s \tag{A.10}$$

under coordination, the second order approximation to the average union welfare can be rewritten as:

$$Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[-\frac{1}{2}\int_0^1 \hat{s}_t^{i'}\Omega_{s,s}\hat{s}_t^i di - \frac{1}{2}\int_0^1 \hat{s}_t^i di'\Omega_{S,S}\int_0^1 \hat{s}_t^i di + \int_0^1 \hat{s}_t^{i'}\Omega_{s,a}\hat{a}_t^i di + \int_0^1 \hat{s}_t^i di'\Omega_{S,A}\int_0^1 \hat{a}_t^i di\Big]$$
$$+t.i.p. \tag{A.11}$$

where

$$\Omega_{s,s} \equiv Z_{s,s} + R_{s,s} \qquad \Omega_{S,S} \equiv R_{S,S}$$

$$\Omega_{s,a} \equiv Z_{s,a}$$

are equal to:

$$\Omega_{s,s} = \begin{bmatrix} \varphi & 0 & 0 & 0 \\ 0 & \gamma(1-\rho) & (1-\rho)(1-\nu)\varsigma_1 & 0 \\ 0 & (1-\rho)(1-\nu)\varsigma_1 & \sigma\rho + \varsigma_3 & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix}$$

$$\Omega_{S,S} = \begin{bmatrix} 0 & -(1-\rho)(1-\nu)\varsigma_1 & 0 \\ -(1-\rho)(1-\nu)\varsigma_1 & -\varsigma_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A.3.2 The welfare approximation to the objective of the fiscal authority

By combining (A.1),(A.2) and (A.4) and considering that at the steady state $C^{-\sigma} = (1-\psi)\delta Y^{\varphi}$and $\chi G^{-\gamma} = (1-\psi)(1-\nu)Y^{\varphi}$ the second order approximation of single country representative agent welfare can be written in matrix notation as:

$$\sum_{t=0}^{\infty} \beta^t Y^{\varphi+1} E_0 \left[\hat{s}_t' w_s - \frac{1}{2} \hat{s}_t' W_{s,s} \hat{s}_t + \hat{s}_t' W_{s,e} \hat{e}_t \right] + t.i.f.p. \tag{A.12}$$

where

$$\hat{s}_t' \equiv [\hat{y}_{H,t},\ \hat{g}_t,\ \hat{c}_t,\ \pi_{H,t}] \qquad w_s^{'} \equiv [-1,\ (1-\psi)(1-\nu)(1-\rho),\ (1-\psi)\delta\rho,\ 0] \qquad \hat{e}_t^{'} \equiv [\hat{y}_t^*,\ \hat{g}_t^*,\ \hat{c}_t^*,\ a_t]$$

$$W_{s,s} \equiv \begin{bmatrix} (\varphi+1) & 0 & 0 & 0 \\ 0 & (\gamma-1)(1-\psi)(1-\nu)(1-\rho) & 0 & 0 \\ 0 & 0 & (\sigma-1)(1-\psi)\delta\rho & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix}$$

$$W_{s,e} \equiv \begin{bmatrix} 0 & 0 & 0 & (1+\varphi) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and with $\hat{y}_t^* \equiv \int_0^1 \hat{y}_t^j dj$, $\hat{g}_t^* \equiv \int_0^1 \hat{g}_t^j dj$ and $\hat{c}_t^* \equiv \int_0^1 \hat{c}_t^j dj$. This approximation can be written in purely quadratic way by using the second order approximation of the single country market clearing conditions (1.44) and (1.45). In particular notice that the second order approximation of these constraints can be read as:

$$0 \simeq \left[-\hat{y}_t - \tfrac{1}{2}\hat{y}_t^2 + \hat{s}_t' f_s - \hat{e}_t' f_e + \tfrac{1}{2}\hat{s}_t' F_{s,s}\hat{s}_t - \hat{s}_t' F_{s,e} e_t \right] + s.o.t.i.f.p. \tag{A.13}$$

$$0 \simeq \left[\hat{s}_t'\iota_s - \hat{e}_t'\iota_e + \hat{y}_t\iota_y + \tfrac{1}{2}\hat{y}_t^2\iota_y + \tfrac{1}{2}\hat{s}_t^{'} I_{s,s}\hat{s}_t - \hat{s}_t' I_{s,e}\hat{e}_t + \hat{y}_t I_{y,s}\hat{s}_t - \hat{y}_t I_{y,e}\hat{e}_t \right] + s.o.t.i.f.p. \tag{A.14}$$

where

$$f_s^{'} \equiv [0,\ (1-\nu)(1-\rho),\ \delta_1\rho+\delta_2(1-\rho),\ 0] \qquad f_e^{'} \equiv [0,\ -\nu(1-\rho),\ -\rho+(\delta_1\rho+\delta_2(1-\rho)),\ 0]$$

$$F_{s,s} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (1-\nu)(1-\rho) & \xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & \xi\nu(1-\nu)(1-\rho) & \delta_1\rho+\delta_2(1-\rho)+\omega_1\rho+\omega_2(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F_{s,e} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & -\xi\nu(1-\rho) & \omega_1\rho+\omega_2(1-\rho)+\xi\nu(2-\nu)(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\iota_s^{'} \equiv [-1,\ 0,\ (1-\psi)\delta_3,\ 0] \qquad \iota_e^{'} \equiv [-\psi,\ 0,\ (1-\psi)\delta_3,\ 0] \qquad \iota_y \equiv [(1-\psi)]$$

$$I_{s,s} \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\psi)\delta_3+\omega_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad I_{s,e} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\xi\psi}{(1-\psi)} & 0 & \omega_3+\frac{\xi\psi(2-\psi)}{(1-\psi)} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$I_{y,s} \equiv \begin{bmatrix} 0 & 0 & \xi\psi & 0 \end{bmatrix} \qquad I_{y,e} \equiv \begin{bmatrix} 0 & 0 & \xi\psi & 0 \end{bmatrix}$$

Given (A.13), (A.14) an be rewritten as:

$$0 \simeq \hat{s}_t^{'}(\iota_s+(1-\psi)f_s)-\hat{e}_t^{'}(\iota_e+(1-\psi)f_e)+\frac{1}{2}\hat{s}_t^{'}\left(I_{s,s}+(1-\psi)F_{s,s}+f_sI_{y,s}+I_{y,s}^{'}f_s^{'}\right)\hat{s}_t$$
$$-\hat{s}_t^{'}\left[I_{s,e}+(1-\psi)F_{s,e}+f_sI_{y,e}+f_eI_{y,s}\right]\hat{e}_t+s.o.t.i.f.p. \tag{A.15}$$

Again thanks to conditions (1.74), (1.75) and (1.76) it follows that:

$$w_s = \iota_s+(1-\psi)f_s \tag{A.16}$$

Therefore by using (A.15), (A.12) can be approximated as:

$$Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[-\frac{1}{2}s_t^{'}\Omega_{s,s}s_t+s_t^{'}\Omega_{s,e}e_t\Big]+t.i.f.p. \tag{A.17}$$

which is purely quadratic and where $\Omega_{s,s} \equiv W_{s,s}+I_{s,s}+(1-\psi)F_{s,s}+f_sI_{y,s}+I_{y,s}^{\prime}f_s^{\prime}$ and $\Omega_{s,e} \equiv W_{s,e}+I_{s,e}+(1-\psi)F_{s,e}+f_sI_{y,e}+f_eI_{y,s}$ are respectively equal to:

$$\begin{bmatrix} \varphi & 0 & 0 & 0 \\ 0 & \gamma(1-\rho)(1-\nu)(1-\psi) & (1-\rho)(1-\nu)(\varsigma_1-\xi\nu\psi) & 0 \\ 0 & (1-\rho)(1-\nu)(\varsigma_1-\xi\nu\psi) & (1-\psi)\rho\sigma\delta+\varsigma_2 & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix} \tag{A.18}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1+\varphi \\ 0 & 0 & (1-\rho)(1-\nu)(\varsigma_1-\xi\nu\psi) & 0 \\ -(1-\psi)\delta_3 & -(1-\rho)(1-\psi)\delta_2+(1-\rho)(1-\nu)(\varsigma_1-\xi\nu\psi) & (1-\psi)((1-\rho)\delta_2+\delta_3)+\varsigma_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(A.19)}$$

with $\delta \equiv \delta_1 + \frac{(1-\rho)}{\rho}\delta_2 + \frac{1}{\rho}\delta_3$, $\varsigma_1 \equiv \xi(\nu+\psi)$ $\varsigma_2 \equiv (1-\psi)(\omega_1\rho + \omega_2(1-\rho)) + \omega_3 + 2\xi\psi(\rho\delta_1 + (1-\rho)\delta_2)$.

A.3.3 The welfare approximation to the objective of the monetary authority

The central bank of the monetary union maximizes:

$$\sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \left[\frac{C_t^{i\,1-\sigma}}{1-\sigma} + \chi\frac{G_t^{i\,1-\gamma}}{1-\gamma} - \frac{N_t^{i\,\varphi+1}}{\varphi+1}\right] di \quad 0<\beta<1 \tag{A.20}$$

By combining (A.1) and (A.4) and given that $C^{-\sigma} = (1-\psi)\delta Y^{\varphi}$, the second order approximation of (A.20) can be written as:

$$\sum_{t=0}^{\infty}\beta^t Y^{\varphi+1} E_0 \int_0^1 \left[\hat{l}_t^{i\prime} w_l - \frac{1}{2}\hat{l}_t^{i\prime} W_{l,l}\hat{l}_t^i + \hat{l}_t^{i\prime} W_{l,u}\hat{u}_t^i\right] di + t.i.m.p. \tag{A.21}$$

where

$$\hat{l}_t^{i\prime} \equiv \left[\hat{y}_{H,t}^i,\ \hat{c}_t^i,\ \hat{\pi}_{H,t}^i\right] \qquad \hat{u}_t^{i\prime} \equiv \left[\hat{g}_t^i,\ a_t^i, \mu_t^i\right] \qquad w_l' \equiv [-1,\ (1-\psi)\delta\rho,\ 0]$$

$$W_{l,l} \equiv \begin{bmatrix} (\varphi+1) & 0 & 0 \\ 0 & (\sigma-1)(1-\psi)\delta\rho & 0 \\ 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix} \qquad W_{l,u} \equiv \begin{bmatrix} 0 & (\varphi+1) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $t.i.m.p.$ stands for terms independent of monetary policy inclusive of the government expenditure. In order to express that approximation in a purely quadratic way, it is necessary to recover the second order approximations of (1.44), (1.45), (1.55), (1.50) and (1.51). By integrating the first two approximation we obtain:

$$0 \simeq -\int_0^1 \hat{y}_t^i di - \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \int_0^1 \hat{l}_t^i di' \ f_l - \int_0^1 \hat{u}_t^i di' \ f_u + \frac{1}{2}\int_0^1 \hat{l}_t^{i'} F_{l,l}\hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i dj' F_{L,L}\int_0^1 \hat{l}_t^i di$$
$$- \int_0^1 \hat{l}_t^{i'} F_{l,u}\hat{u}_t^i di - \int_0^1 \hat{l}_t^{i'} di F_{L,U}\int_0^1 \hat{u}_t^i di + s.o.t.i.m.p. \tag{A.22}$$

$$0 \simeq \int_0^1 \hat{l}_t^i di' \ \iota_l + \int_0^1 \hat{y}_t^i di + \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \frac{1}{2}\int_0^1 \hat{l}_t^{i'} I_{l,l}\hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i di' I_{L,L}\int_0^1 \hat{l}_t^i di + \int_0^1 \hat{y}_t^i I_{y,l}\hat{l}_t^i di$$
$$+ \int_0^1 \hat{y}_t^i di I_{Y,L}\int_0^1 \hat{l}_t^i di + s.o.t.i.m.p. \tag{A.23}$$

where

$$f_l' \equiv [0,\ \rho,\ 0] \qquad f_u' \equiv [-(1-\rho),\ 0, 0]$$

$$F_{l,l} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho + \omega_1\rho + \omega_2(1-\rho) & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad F_{l,u} \equiv \begin{bmatrix} 0 & 0 & 0 \\ -\xi\nu(1-\nu)(1-\rho) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$F_{L,L} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\omega_1\rho - \omega_2(1-\rho) & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad F_{L,U} \equiv \begin{bmatrix} 0 & 0 & 0 \\ \xi\nu(1-\nu)(1-\rho) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\iota_l' \equiv [-1,\ 0,\ 0] \qquad I_{y,l} \equiv \begin{bmatrix} 0 & \xi\psi & 0 \end{bmatrix} \qquad I_{Y,L} \equiv \begin{bmatrix} 0 & -\xi\psi & 0 \end{bmatrix}$$
$$I_{l,l} \equiv \begin{bmatrix} -1 & 0 & 0 \\ 0 & \omega_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad I_{L,L} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\omega_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Given (A.22) and (A.23) it is easy to show that:

$$0 \simeq \int_0^1 \hat{l}_t^i di' \ r_l - \int_0^1 \hat{u}_t^i di' \ r_u + \frac{1}{2}\int_0^1 \hat{l}_t^{i'} R_{l,l}\hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i di' R_{L,L}\int_0^1 \hat{l}_t^i di$$
$$- \int_0^1 \hat{l}_t^{i'} R_{l,u} u_t^i di - \int_0^1 \hat{l}_t^{i'} di R_{L,U}\int_0^1 u_t^i di + s.o.t.i.m.p. \tag{A.24}$$

where

$$r_l \equiv \iota_l + f_l \qquad r_u \equiv \ f_u$$

$$R_{l,l} \equiv I_{l,l} + F_{l,l} + f_y I_{y,l} + I_{y,l}' f_y' \qquad R_{L,L} \equiv \ I_{L,L} + F_{L,L} + f_Y I_{y,l} + I_{y,l}' f_Y + f_l I_{Y,L} + I_{Y,L}' f_l'$$

$$R_{l,u} \equiv F_{l,u} + I_{y,l}' f_g' \qquad R_{L,U} \equiv F_{L,U} + f_G I_{y,l} + I_{Y,L}' f_u' \tag{A.25}$$

and

$$f_y' \equiv [0,\ \delta_1\rho + \delta_2(1-\rho),\ 0] \qquad f_Y' \equiv [0,\ \rho - \delta_1\rho - \delta_2(1-\rho),\ 0]$$

$$f_g' \equiv [-(1-\nu)(1-\rho),\ 0] \qquad f_G' \equiv [-\nu(1-\rho),\ 0]$$

By combining the second order approximation of the (1.55), (1.52) and (1.53) as in Benigno and Woodford (2005), we obtain the following condition:

$$\begin{aligned} V_0 \ &= \ \frac{1-\theta}{\theta}(1-\beta\theta)\sum_{t=0}^{\infty}\beta^t E_0\Big[\int_0^1 \hat{l}_t^i di' v_l - \int_0^1 \hat{u}_t^{i'} v_u + \frac{1}{2}\int_0^1 \hat{l}_t^{i'} V_{l,l}\hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i di' V_{L,L}\int_0^1 \hat{l}_t^i di \\ &\quad - \int_0^1 \hat{l}_t^{i'} V_{l,u}\hat{u}_t^i di\Big] + s.o.t.i.m.p. \end{aligned} \tag{A.26}$$

where

$$v_l' \equiv [\varphi,\ \sigma,\ 0] \qquad v_u' \equiv [0,\ (\varphi+1),\ -1]$$

$$V_{l,l} \equiv \begin{bmatrix} \varphi(\varphi+2) & \omega_4 & 0 \\ \omega_4 & -\omega_4^2+\omega_5 & 0 \\ 0 & 0 & \frac{\varepsilon(\varphi+1)}{\lambda} \end{bmatrix} \qquad V_{l,u} \equiv \begin{bmatrix} 0 & (\varphi+1)^2 & -(\varphi+1) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V_{L,L} \equiv \begin{bmatrix} 0 & \sigma-\omega_4 & 0 \\ \sigma-\omega_4 & -\sigma^2+\omega_4^2-\omega_5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with

$$\begin{aligned} \omega_5 &\equiv -\frac{\sigma\,\psi\ (-1+(1-\eta)\ \sigma\ (1+\alpha\ (1-\psi))+\alpha\ (1-\psi)+(1-\sigma)\ \psi)}{(1-\alpha)^2\,(1-\psi)^2} \\ &+\frac{\alpha\,\sigma\ (1-\alpha\ (1-\sigma)-(1-\eta)\ \sigma)}{(1-\alpha)^2} \end{aligned}$$

$$\omega_4 \equiv \frac{\sigma}{(1-\alpha)\ (1-\psi)} \qquad \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

Conditions (A.24) and (A.26) allow to substitute the linear term of the union welfare approximation with purely quadratic terms. In fact given these conditions:

$$\begin{aligned} 0 \ &\simeq \ Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[\int_0^1 \hat{l}_t^i di'(\zeta_1 r_l + \zeta_2 v_l) + \frac{1}{2}\int_0^1 \hat{l}_t^{i'}(\zeta_1 R_{l,l}+\zeta_2 V_{l,l})\hat{l}_t^i di - \int_0^1 \hat{l}_t^{i'}(\zeta_1 R_{l,u}+\zeta_2 V_{l,u})\hat{u}_t^i di \\ &+\frac{1}{2}\int_0^1 \hat{l}_t^i di'(\zeta_1 R_{L,L}+\zeta_2 V_{L,L})\int_0^1 \hat{l}_t^i di - \int_0^1 \hat{l}_t^i di'(\zeta_1 R_{L,U})\int_0^1 \hat{u}_t^i di\Big] + t.i.m.p. \end{aligned} \tag{A.27}$$

where $\zeta_1 \equiv \frac{(1-\psi)\delta\varphi\rho+\sigma}{\varphi\rho+\sigma}$ and $\zeta_2 \equiv \frac{((1-\psi)\delta-1)\rho}{\varphi\rho+\sigma}$. It is easy to show that:

$$w_l = \zeta_1 r_l + \zeta_2 v_l \tag{A.28}$$

Hence we can write the second order approximation of union welfare as:

$$Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[-\frac{1}{2}\int_0^1 \hat{l}_t^{i\prime}\Omega_{l,l}\hat{l}_t^i di - \frac{1}{2}\int_0^1 \hat{l}_t^i di'\Omega_{L,L}\int_0^1 \hat{l}_t^i di + \int_0^1 \hat{l}_t^{i\prime}\Omega_{l,u}\hat{u}_t^i di + \int_0^1 \hat{l}_t^i di'\Omega_{L,U}\int_0^1 \hat{u}_t^i di\Big]$$
$$+t.i.m.p. \tag{A.29}$$

where

$$\Omega_{l,l} \equiv W_{l,l} + \zeta_1 R_{l,l} + \zeta_2 V_{l,l} \qquad \Omega_{L,L} \equiv \zeta_1 R_{L,L} + \zeta_2 V_{L,L}$$
$$\Omega_{l,u} \equiv W_{l,u} + \zeta_1 R_{l,u} + \zeta_2 V_{l,u} \qquad \Omega_{L,U} \equiv \zeta_1 R_{L,U} \tag{A.30}$$

are equal to:

$$\Omega_{l,l} = \begin{bmatrix} \varphi\zeta_3 & \zeta_2\omega_4 & 0 \\ \zeta_2\omega_4 & \delta(\sigma-1)(1-\psi)\rho + \zeta_1(\rho+\varsigma_3) + \zeta_2(\omega_5-\omega_4^2) & 0 \\ 0 & 0 & \frac{\varepsilon\zeta_3}{\lambda} \end{bmatrix}$$
$$\Omega_{L,L} = \begin{bmatrix} 0 & \zeta_2(\sigma-\omega_4) & 0 \\ \zeta_2(\sigma-\omega_4) & -\zeta_1\varsigma_3 - \zeta_2(\sigma^2+\omega_5-\omega_4^2) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$\Omega_{l,u} = \begin{bmatrix} 0 & (\varphi+1)\zeta_3 & -(\varphi+1)\zeta_2 \\ -\zeta_1(1-\nu)(1-\rho)\varsigma_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \Omega_{L,U} = \begin{bmatrix} 0 & 0 & 0 \\ \zeta_1(1-\nu)(1-\rho)\varsigma_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with $\zeta_3 \equiv 1 + (\varphi+1)\zeta_2$ and $\varsigma_3 \equiv \rho\omega_1 + (1-\rho)\omega_2 + \omega_3 + 2\xi\psi(\rho\delta_1 + (1-\rho)\delta_2)$.

B Appendix to chapter 2

B.1 Retrieving condition (2.26)

Given the definitions of $P^*_{H,t}$ and $P^*_{F,t}$ it is easy to show that:

$$\mathcal{E}_{iH,t}P^*_{H,t} = [\alpha_b {P_{H,t}}^{1-\eta} + (1-\alpha_b){P_{F,t}}^{1-\eta}]^{\frac{1}{1-\eta}} \qquad \mathcal{E}_{iF,t}P^*_{F,t} = [\alpha_b {P_{F,t}}^{1-\eta} + (1-\alpha_b){P_{H,t}}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{B.1}$$

By (B.1):

$$\frac{\mathcal{E}_{iH,t}P^*_{H,t}}{P_{H,t}} = \left[\alpha_b + (1-\alpha_b)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \qquad \frac{\mathcal{E}_{iF,t}P^*_{F,t}}{P_{F,t}} = \left[\alpha_b + (1-\alpha_b)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{B.2}$$

which jointly with (2.25) leads to:

$$\left(\frac{C^*_{F,t}}{C^*_{H,t}}\right) = \left[\frac{\alpha_b\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\eta} + (1-\alpha_b)}{(1-\alpha_b)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\eta} + \alpha_b}\right]^{-\frac{1}{\sigma(1-\eta)}} \tag{B.3}$$

Moreover thanks to (2.6):

$$\frac{P_{i,t}}{P_{C^i,t}} = \left[\frac{1}{\alpha_s} - \frac{\alpha_b - \alpha_s}{\alpha_s}\left(\frac{P_{H,t}}{P_{C^i,t}}\right)^{1-\eta} - \frac{(1-\alpha_b)}{\alpha_s}\left(\frac{P_{F,t}}{P_{C^i,t}}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \qquad i \in \left[0, \frac{1}{2}\right) \tag{B.4}$$

which can be read as:

$$\frac{P_{i,t}}{P_{C^i,t}} = \left[\frac{1}{\alpha_s} - \frac{\alpha_b - \alpha_s}{\alpha_s}\left(\frac{P_{H,t}}{\mathcal{E}_{i,H}P^*_{H,t}}\right)^{(1-\eta)}\left(\frac{C^*_{H,t}}{C^i_t}\right)^{-\sigma(1-\eta)} - \frac{(1-\alpha_b)}{\alpha_s}\left(\frac{P_{F,t}}{\mathcal{E}_{i,F}P^*_{F,t}}\right)^{(1-\eta)}\left(\frac{C^*_{F,t}}{C^i_t}\right)^{-\sigma(1-\eta)}\right]^{\frac{1}{1-\eta}} \tag{B.5}$$

Finally by using (B.2) and (B.3) we can rewrite (B.4) as (2.26)

B.2 Zero Inflation Deterministic Steady State

In this section we show that, given appropriate initial conditions, under both regimes, A and B, at the deterministic steady state, zero inflation is a Nash equilibrium policy.

In the regime A the *timelessly* optimal policy problem of a monetary authority of country i in the area H can be formulated as the maximization of the following Lagragian:

$$\begin{aligned}
L^i = \sum_{t=0}^{\infty} \beta^t E_0 \Big\{ & \frac{C_t^{i\,1-\sigma}}{1-\sigma} - \frac{1}{\varphi+1}\left(\frac{Y_t^i Z_t^i}{A_t^i}\right)^{\varphi+1} \\
& + \zeta_{1,t}^{s,i}\left[Y_t^i - \left(\frac{P_{i,t}}{P_{C^i,t}}\right)^{-\eta}\left(\alpha_s C_t^i + (\alpha_b-\alpha_s) C_t^{i\sigma\eta}\mathcal{C}_{H,t} + (1-\alpha_b) C_t^{i\sigma\eta}\mathcal{C}_{F,t}\right)\right] \\
& + \zeta_{2,t}^{s,i}\left[K_t^i - \left(\frac{Y_t^i}{A_t^i}\right)^{\varphi+1} Z_t^{i\varphi}(1+\mu_t^i)(1-\tau^i)\frac{\varepsilon}{\varepsilon-1}\right] - \zeta_{2,t-1}^{s,i}\theta\Pi_{i,t}^{\varepsilon}K_t^i \\
& + \zeta_{3,t}^{s,i}\left[F_t^i - Y_t^i C_t^{i-\sigma}\frac{P_t^i}{P_{C^i,t}}\right] - \zeta_{3,t-1}^{s,i}\theta\Pi_{i,t}^{(\varepsilon-1)}F_t^i \\
& + \zeta_{4,t}^{s,i}\left[F_t^i - K_t^i\left(\frac{1-\theta\Pi_{i,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right] \\
& + \zeta_{5,t}^{s,i}\left[Z_t^i - \theta Z_{t-1}^i\Pi_{i,t}^{\varepsilon} - (1-\theta)\left(\frac{1-\theta\Pi_{i,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]\Big\}
\end{aligned}$$

with respect to C_t^i, Y_t^i, Z_t^i, K_t^i, F_t^i and $\Pi_{i,t}$ and where $P_{i,t}/P_{C^i,t}$ are determined consistently with (2.26), while $C_{H,t}^*$, $C_{F,t}^*$, $\mathcal{C}_{H,t}$ and $\mathcal{C}_{F,t}$ are taken as given. Assume that $\mu_t^i = \mu$, $A_t^j = A$, $\tau^j = \tau$ and $Z_t^j = \Pi_{j,t} = 1$ for all $j \in [0,1]$ and t. Assume in addition that $Z_{-1}^i = 1$. Recalling that $\tilde{\tau} = 1-(1-\tau)(1+\mu)\frac{(\varepsilon)}{\varepsilon-1}$ it can be shown that according to the first order conditions at the symmetric deterministic steady state:

$$C^{-\sigma} = \zeta_1^s\delta_s - \zeta_3^s\sigma\gamma_s Y C^{-\sigma-1} \tag{B.6}$$

$$Y^{\varphi} = \zeta_1^s - \zeta_2^s(\varphi+1)Y^{\varphi}(1-\tilde{\tau}) - \zeta_3^s C^{-\sigma} \tag{B.7}$$

$$Y^{\varphi+1} = -\zeta_2^s\varphi Y^{\varphi+1} + \zeta_5^s(1-\theta) \tag{B.8}$$

$$\zeta_2^s(1-\theta) = \zeta_4^s \tag{B.9}$$

$$\zeta_3^s(1-\theta) = -\zeta_4^s \tag{B.10}$$

$$\zeta_2^s\theta\varepsilon K = -\zeta_3^s\theta(\varepsilon-1)F + \zeta_4^s\frac{\theta}{1-\theta}K \tag{B.11}$$

with $\delta_s = \alpha_s(1-\sigma\eta) + \gamma_s\eta\sigma$. Then

$$Y = (1-\tilde{\tau})^{-\frac{1}{\sigma+\varphi}}$$

$$C = Y$$

$$F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}(1-\tilde{\tau})}{1-\theta}$$

$$\Pi_H = \Pi_F = 1 \qquad Z = 1$$

$$\zeta_1^s = \frac{\gamma_s\sigma + (1-\tilde{\tau}\varphi)}{\delta_s\varphi + \gamma_s\sigma}$$

$$\zeta_2^s = \frac{\zeta_4^s}{1-\theta} = -\zeta_3^s = -\frac{\delta_s - (1-\tilde{\tau})\varphi}{(\delta_s\varphi + \gamma_s\sigma)(1-\tilde{\tau})} \qquad \zeta_5^s = \frac{Y^{\varphi+1}(1-\varphi\zeta_3^s)}{1-\theta}$$

is a steady state symmetric solution of the optimal policy problem just stated[1].

Consider now the monetary union in the area F[2]. Suppose that for all $i \in [0\frac{1}{2})$ $\Pi_t^i = 1$ at all times[3]. Then we want to show that given other policymakers strategy, $\Pi_t^i = 1$ for all $i \in [\frac{1}{2}, 1]$ and t is optimal.

If for all $i \in [0\frac{1}{2})$ $\Pi_t^i = 1$ at all times, the optimal policy problem of the monetary authority in the area F can be written as maximizing:

$$L = \sum_{t=0}^{\infty}\beta^t E_0 \Bigg\{ \int_{\frac{1}{2}}^{1} \left[\frac{C_t^{i\,1-\sigma}}{1-\sigma} - \frac{1}{\varphi+1}\left(\frac{Y_t^i Z_t^i}{A_t^i}\right)^{\varphi+1} \right]$$
$$+\zeta_{1,t}^{b,i}\left[Y_t^i - \left(\frac{P_{i,t}}{P_{C^i,t}}\right)^{-\eta} \left(\alpha_s C_t^i + 2(\alpha_b - \alpha_s) C_t^{i\,\sigma\eta} \int_{\frac{1}{2}}^{1} C_t^{j\,1-\sigma\eta} dj + 2(1-\alpha_b) C_t^{i\,\sigma\eta} \int_0^{\frac{1}{2}} C_t^{j\,1-\sigma\eta} dj \right) \right]$$
$$+\zeta_{2,t}^{b,i}\left[K_t^i - \left(\frac{Y_t^i}{A_t^i}\right)^{\varphi+1} Z_t^{i\varphi}(1+\mu_t^i)(1-\tau^i)\frac{\varepsilon}{\varepsilon-1} \right] - \zeta_{2,t-1}^{b,i}\theta\Pi_{i,t}^{\varepsilon}K_t^i$$
$$+\zeta_{3,t}^{b,i}\left[F_t^i - Y_t^i C_t^{i\,-\sigma}\frac{P_t^i}{P_{C^i,t}} \right] - \zeta_{3,t-1}^{s}\theta\Pi_{i,t}^{(\varepsilon-1)}F_t^i$$
$$+\zeta_{4,t}^{b,i}\left[F_t^i - K_t^i \left(\frac{1-\theta\Pi_{i,t}^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{\varepsilon-1}} \right]$$

[1] In other words, given a zero inflation policy of the other central banks, zero inflation is a best response of the central bank of the country i.

[2] We follow closely Benigno and Benigno (2006).

[3] ..which implies that $F_t^i = F$, $K_t^i = K$ and $\frac{F_t^i}{K_t^i} = 1$ for all i and t

$$+\zeta_{5,t}^{b,i}\left[Z_t^i-\theta Z_{t-1}^i\Pi_{i,t}^{\varepsilon}-(1-\theta)\left(\frac{1-\theta\Pi_{i,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]$$
$$+\zeta_{6,t}^{b,i}\left[\left(\frac{C_{F,t}^*}{C_{F,t-1}^*}\right)^{-\sigma}\frac{P_{F,t}}{P_{F,t}^*}\frac{P_{F,t-1}^*}{P_{F,t-1}}\Pi_{F,t}^{-1}-\left(\frac{C_t^i}{C_{t-1}^i}\right)^{-\sigma}\frac{P_{i,t}}{P_{C^i,t}}\frac{P_{C^i,t-1}}{P_{i,t-1}}\Pi_{i,t}^{-1}\right]\Bigg\}di$$
$$+\int_0^{\frac{1}{2}}\Bigg\{\zeta_{7,t}^{b,i}\left[Y_t^i-\left(\frac{P_{i,t}}{P_{C^i,t}}\right)^{-\eta}\left(\alpha_s C_t^i+2(\alpha_b-\alpha_s)C_t^{i\sigma\eta}\int_0^{\frac{1}{2}}C_t^{j\,1-\sigma\eta}dj+2(1-\alpha_b)C_t^{i\sigma\eta}\int_{\frac{1}{2}}^1 C_t^{j\,1-\sigma\eta}dj\right)\right]$$
$$+\zeta_{8,t}^{b,i}\left[\left((1+\mu_t^i)(1-\tau)\frac{\varepsilon}{\varepsilon-1}\left(\frac{Y_t^i}{A_t^i}\right)^{\varphi+1}\right)-\frac{P_{i,t}}{P_{C^i,t}}Y_t^iC_t^{i-\sigma}\right]di\Bigg\}$$

with respect to C_t^i, Y_t^i for all i and Z_t^i, K_t^i, F_t^i and $\Pi_{i,t}$ all $i\in[\frac{1}{2},1]$ and where $P_{i,t}/P_{C^i,t}$ and $P_{F,t}^*/P_{F,t}$ are determined consistently with (2.26) , (2.27), (B.2) and (B.3). Assume that $\mu_t^i=\mu$, $A_t^j=A$, $\tau^j=\tau$ and $Z_t^j=\Pi_{j,t}=1$ for all $j\in[0,1]$ and t. Moreover assume that $Z_{-1}^i=1$ for all $i\in\left[\frac{1}{2},1\right]$ Given that $\tilde{\tau}=1-(1-\tau)(1+\mu)\frac{(\varepsilon)}{\varepsilon-1}$. Then according to the first order conditions at the symmetric deterministic steady state:

$$C^{-\sigma}=\zeta_1^b\delta_b+\zeta_7^b(1-\delta_b)-\zeta_3^b\sigma\gamma_bYC^{-\sigma-1}-\zeta_8^b\sigma(1-\gamma_b)YC^{-\sigma-1} \tag{B.12}$$

$$Y^{\varphi}=\zeta_1^b-\zeta_2^b(\varphi+1)Y^{\varphi}(1-\tilde{\tau})-\zeta_3^bC^{-\sigma} \tag{B.13}$$

$$Y^{\varphi+1}=-\zeta_2^b\varphi Y^{\varphi+1}+\zeta_5^b(1-\theta) \tag{B.14}$$

$$\zeta_2^b(1-\theta)=\zeta_4^b \tag{B.15}$$

$$\zeta_3^b(1-\theta)=-\zeta_4^b \tag{B.16}$$

$$\zeta_2^b\theta\varepsilon K=-\zeta_3^b\theta(\varepsilon-1)F+\zeta_4^b\frac{\theta}{1-\theta}K \tag{B.17}$$

$$0=\zeta_1^b(1-\delta_b)+\zeta_7^b\delta_b-\zeta_3^b\sigma(1-\gamma_b)YC^{-\sigma-1}-\zeta_8^b\sigma\gamma_bYC^{-\sigma-1} \tag{B.18}$$

$$0=\zeta_7^b+\zeta_8^b\left[(\varphi+1)Y^{\varphi}(1-\tilde{\tau})-C^{-\sigma}\right] \tag{B.19}$$

where $\delta_b \equiv (1-\sigma\eta)\alpha_b + \eta\sigma\gamma_b$. Then it is easy to show:

$$Y = (1-\tilde{\tau})^{-\frac{1}{\sigma+\varphi}} \tag{B.20}$$

$$C = Y \tag{B.21}$$

$$F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}(1-\tilde{\tau})}{1-\theta} \tag{B.22}$$

$$\Pi_H = \Pi_F = 1 \qquad Z = 1 \tag{B.23}$$

$$\zeta_1^b = Y^{\varphi} \qquad \zeta_2^b = \frac{\zeta_4^b}{1-\theta} = -\zeta_3^b = 0 \qquad \zeta_5^b = \frac{Y^{\varphi+1}}{1-\theta} \tag{B.24}$$

$$\zeta_6^b = 0 \qquad \zeta_7^b = \frac{\tilde{\tau}}{(1-\tilde{\tau})(\sigma+\varphi)} \qquad \zeta_8^b = \frac{-Y^{\varphi}\tilde{\tau}}{(\sigma+\varphi)} \tag{B.25}$$

Hence being the best response of both monetary union and the small open economy policymakers, zero inflation is a Nash equilibrium solution in regime A. Consider now the case of regime B and suppose that the central bank of area H set $\Pi_{H,t}^{-1} = 1$ for all t. The central bank of the monetary union in area F maximizes:

$$\begin{aligned}
L = \sum_{t=0}^{\infty}\beta^t E_0\Bigg\{ \int_{\frac{1}{2}}^{1} & \left[\frac{C_t^{i\,1-\sigma}}{1-\sigma} - \frac{1}{\varphi+1}\left(\frac{Y_t^i Z_t^i}{A_t^i}\right)^{\varphi+1}\right] \\
&+\zeta_{1,t}^{b,i}\left[Y_t^i - \left(\frac{P_{i,t}}{P_{C^i,t}}\right)^{-\eta}\left(\alpha_s C_t^i + 2(\alpha_b-\alpha_s)C_t^{i\sigma\eta}\int_{\frac{1}{2}}^{1} C_t^{j\,1-\sigma\eta}dj + 2(1-\alpha_b)C_t^{i\sigma\eta}\int_0^{\frac{1}{2}} C_t^{j\,1-\sigma\eta}dj\right)\right] \\
&+\zeta_{2,t}^{b,i}\left[K_t^i - \left(\frac{Y_t^i}{A_t^i}\right)^{\varphi+1} Z_t^{i\varphi}(1+\mu_t^i)(1-\tau^i)\frac{\varepsilon}{\varepsilon-1}\right] - \zeta_{2,t-1}^{b,i}\theta\Pi_{i,t}^{\varepsilon}K_t^i \\
&+\zeta_{3,t}^{b,i}\left[F_t^i - Y_t^i C_t^{i-\sigma}\frac{P_t^i}{P_{C^i,t}}\right] - \zeta_{3,t-1}^{s}\theta\Pi_{i,t}^{(\varepsilon-1)}F_t^i \\
&+\zeta_{4,t}^{b,i}\left[F_t^i - K_t^i\left(\frac{1-\theta\Pi_{i,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right] \\
&+\zeta_{5,t}^{b,i}\left[Z_t^i - \theta Z_{t-1}^i\Pi_{i,t}^{\varepsilon} - (1-\theta)\left(\frac{1-\theta\Pi_{i,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right] \\
&+\zeta_{6,t}^{b,i}\left[\left(\frac{C_{F,t}^*}{C_{F,t-1}^*}\right)^{-\sigma}\frac{P_{F,t}}{P_{F,t}^*}\frac{P_{F,t-1}^*}{P_{F,t-1}}\Pi_{F,t}^{-1} - \left(\frac{C_t^i}{C_{t-1}^i}\right)^{-\sigma}\frac{P_{i,t}}{P_{C^i,t}}\frac{P_{C^i,t-1}}{P_{i,t-1}}\Pi_{i,t}^{-1}\right]\Bigg\}di
\end{aligned}$$

$$
\begin{aligned}
&+\int_0^{\frac{1}{2}}\Bigg\{\zeta_{7,t}^{b,i}\left[Y_t^i-\left(\frac{P_{i,t}}{P_{C^i,t}}\right)^{-\eta}\left(\alpha_s C_t^i+2(\alpha_b-\alpha_s)C_t^{i\sigma\eta}\int_0^{\frac{1}{2}}C_t^{j\,1-\sigma\eta}dj+2(1-\alpha_b)C_t^{i\sigma\eta}\int_{\frac{1}{2}}^{1}C_t^{j\,1-\sigma\eta}dj\right)\right]\\
&+\zeta_{8,t}^{b,i}\left[K_t^i-\left(\frac{Y_t^i}{A_t^i}\right)^{\varphi+1}Z_t^{i\varphi}(1+\mu_t^i)(1-\tau^i)\frac{\varepsilon}{\varepsilon-1}\right]-\zeta_{8,t-1}^{b,i}\theta\Pi_{i,t}^{\varepsilon}K_t^i\\
&+\zeta_{9,t}^{b,i}\left[F_t^i-Y_t^iC^{i-\sigma}_t\frac{P_t^i}{P_{C^i,t}}\right]-\zeta_{9,t-1}^{s}\theta\Pi_{i,t}^{(\varepsilon-1)}F_t^i\\
&+\zeta_{10,t}^{b,i}\left[F_t^i-K_t^i\left(\frac{1-\theta\Pi_{i,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right]\\
&+\zeta_{11,t}^{b,i}\left[Z_t^i-\theta Z_{t-1}^i\Pi_{i,t}^{\varepsilon}-(1-\theta)\left(\frac{1-\theta\Pi_{i,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]\\
&+\zeta_{12,t}^{b,i}\left[\left(\frac{C_{H,t}^*}{C_{H,t-1}^*}\right)^{-\sigma}\frac{P_{H,t}}{P_{H,t}^*}\frac{P_{H,t-1}^*}{P_{H,t-1}}-\left(\frac{C_t^i}{C_{t-1}^i}\right)^{-\sigma}\frac{P_{i,t}}{P_{C^i,t}}\frac{P_{C^i,t-1}}{P_{i,t-1}}\Pi_{i,t}^{-1}\right]\Bigg\}di
\end{aligned}
$$

with respect to C_t^i, Y_t^i, Z_t^i, K_t^i, F_t^i and $\Pi_{i,t}$ all i and where$P_{i,t}/P_{C^i,t}$, $P_{F,t}^*/P_{F,t}$ and $P_{H,t}^*/P_{H,t}$ are determined consistently with (2.26) , (2.27), (B.2) and (B.3). Assume that $\mu_t^i=\mu$, $A_t^j=A$, $\tau^j=\tau$ and $Z_t^j=\Pi_{j,t}=1$ for all $j\in[0,1]$ and t. Moreover assume that $Z_{-1}^i=1$ for all i Given that $\tilde{\tau}=1-(1-\tau)(1+\mu)\frac{(\varepsilon)}{\varepsilon-1}$. Then according to the first order conditions at the symmetric deterministic steady state it can be shown:

$$Y=(1-\tilde{\tau})^{-\frac{1}{\sigma+\varphi}} \tag{B.26}$$

$$C=Y \tag{B.27}$$

$$F=K=\frac{YC^{-\sigma}}{1-\theta}=\frac{Y^{\varphi+1}(1-\tilde{\tau})}{1-\theta} \tag{B.28}$$

$$\Pi_H=\Pi_F=1 \qquad Z=1 \tag{B.29}$$

Therefore for the policymaker of the area F zero inflation is a best response to a zero inflation policy of the policymaker in the area H. A symmetric problem can be stated for the policymaker of the monetary union of the area H Thus zero inflation is a Nash equilibrium policy.

B.3 The purely quadratic approximation of the welfare

In order to recover the optimal policies we need to approximate up to the second order single country representative agent utility given by (2.1) in the following way.

First we can approximate the utility derived from private consumption for generic region i as:

$$\frac{C_t^{i\,1-\sigma}}{1-\sigma} \simeq \frac{C^{1-\sigma}}{1-\sigma} + C^{1-\sigma}(\hat{c}_t^i + \frac{1}{2}(\hat{c}_t^i)^2) - \frac{\sigma}{2}C^{1-\sigma}(\hat{c}_t^i)^2 + t.i.p. \quad \text{(B.30)}$$

where $\hat{c}_t^i$ stands for the log-deviations of private consumption from the non-stochastic symmetric steady state[4].

Similarly the labor disutility can be approximated by taking into account that $N_t^i = \frac{Y_t^i Z_t^i}{A_t^i}$ and, as showed by Galí and Monacelli (2005), being $Z_t^i = \int_0^1 \left(\frac{p_t(h^i)}{P_{i,t}}\right)^{-\varepsilon} dh^i$:

$$\hat{z}_t^i \simeq \frac{\varepsilon}{2} Var_{h^i}(p_t(h^i)) \quad \text{(B.31)}$$

In words the approximation of Z_t^i around the symmetric steady state is purely quadratic. Moreover following Woodford (2001, NBER WP8071) it is possible to show that $\sum_{t=0}^{\infty}\beta^t Var_{h^i}(p_t(h^i)) = \frac{1}{\lambda}\sum_{t=0}^{\infty}\beta^t \pi_{i,t}^2$ with $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Thus:

$$\frac{1}{\varphi+1}\left(\frac{Y_t^i Z_t^i}{A_t^i}\right)^{\varphi+1} \simeq \frac{1}{\varphi+1}Y^{\varphi+1} + Y^{\varphi+1}(\hat{y}_t^i + \frac{1}{2}(\hat{y}_t^i)^2) + Y^{\varphi+1}\frac{\varepsilon}{2\lambda}(\pi_{i,t})^2 + \frac{\varphi}{2}Y^{\varphi+1}(\hat{y}_t^i)^2$$
$$-(\varphi+1)Y^{\varphi+1}\hat{y}_t^i a_t^i + t.i.p. \quad \text{(B.32)}$$

B.3.1 The case of the small open economy

By combining (B.30) and (B.32) and taking into account that at the steady state $C^{-\sigma} = (1-\tilde{\tau})Y^{\varphi}$, the second order approximation of welfare of the region i households can be written as:

$$\sum_{t=0}^{\infty}\beta^t Y^{\varphi+1} E_0\left[\hat{s}_t^{i\prime} w_s - \frac{1}{2}\hat{s}_t^{i\prime} W_{s,s}\hat{s}_t^i + \hat{s}_t^{i\prime} W_{s,e}\hat{e}_t^i\right] + t.i.p. \quad \text{(B.33)}$$

[4]From now this convention will be used: $\hat{x}_t$ represents the log-deviation of X_t from the steady state.

where

$$\hat{s}_t^{i'} \equiv \left[\hat{y}_t^i,\ \hat{c}_t^i,\ \pi_{i,t}\right] \qquad w_s^{'} \equiv [-1,\ (1-\tilde{\tau}),\ \ 0] \qquad \hat{e}_t^{i'} \equiv \left[\hat{c}_{H,t},\ \hat{c}_{F,t},\ a_t^i,\ \ \mu_t^i\right]$$

$$W_{s,s} \equiv \begin{bmatrix} (\varphi+1) & 0 & 0 \\ 0 & (1-\tilde{\tau})(\sigma-1) & 0 \\ 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix} \qquad W_{s,e} \equiv \begin{bmatrix} 0 & 0 & (\varphi+1) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and with $i \in [0, \frac{1}{2})$ $\hat{c}_{H,t} \equiv 2\int_0^{\frac{1}{2}} \hat{c}_t^j dj$ and $\hat{c}_{F,t} \equiv 2\int_{\frac{1}{2}}^1 \hat{c}_t^j dj$. In order to recover a purely quadratic approximation to the welfare for the central bank of the small open economy, we have to use both the second order approximation to the demand and to the Phillips curves.

The second order approximation to the demand curve can be written as:

$$0 \simeq \left[\hat{s}_t^{i'} g_s - \hat{e}_t^{i'} g_e + \tfrac{1}{2}\hat{s}_t^{i'} G_{s,s}\hat{s}_t - \hat{s}_t^{i'} G_{s,e} e_t\right] + s.o.t.i.p. \tag{B.34}$$

where

$$g_s^{'} \equiv [-1,\ \delta_s,\ 0] \qquad g_e^{'} \equiv [-(\delta_b-\delta_s),\ -(1-\delta_b),\ 0,\ 0]$$

$$G_{s,s} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & \delta_s+\omega_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad G_{s,e} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ \omega_1+\omega_2 & -\omega_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where $\delta_s \equiv \alpha_s(1-\eta\sigma) + \eta\sigma/\alpha_s$, $\delta_b \equiv \alpha_b(1-\eta\sigma) - \alpha_b\eta\sigma/(1-2\alpha_b)$ and:

$$\omega_1 \equiv \frac{(1-\alpha_s)\eta\sigma(\sigma-(1-\alpha_s)\alpha_s(1-\eta\sigma))}{\alpha_s^2} \qquad \omega_2 \equiv \frac{(1-\alpha_b)\eta\sigma\left(\sigma+\left(\alpha_s^2+(1-2\alpha_b)\right)(1-\eta\sigma)\right)}{\alpha_s(1-2\alpha_b)}$$

As in Benigno and Woodford (2005) the second order approximation to the (2.54) and be combined with (2.52) and (2.53) to obtain:

$$V_0 = \frac{1-\theta}{\theta}(1-\beta\theta)\sum_{t=0}^{\infty}\beta^t E_0\left[\hat{s}_t^{i'} v_s - \hat{e}_t^{i'} v_e + \frac{1}{2}\hat{s}_t^{i'} V_{s,s}\hat{s}_t^i - \hat{s}_t^{i'} V_{s,e}\hat{e}_t^i\right] + s.o.t.i.p. \tag{B.35}$$

where

$$v_s' \equiv [\varphi,\ \sigma\gamma_s,\ 0] \qquad v_e' \equiv [\sigma(\gamma_s - \gamma_b),\ -\sigma(1-\gamma_b),\ -(\varphi+1),\ 1]$$

$$V_{s,s} \equiv \begin{bmatrix} \varphi(\varphi+2) & \sigma\gamma_s & 0 \\ \sigma\gamma_s & -\sigma^2\gamma_s^2 & 0 \\ 0 & 0 & \frac{\varepsilon(\varphi+1)}{\lambda} \end{bmatrix}$$

$$V_{s,e} \equiv \begin{bmatrix} \sigma(\gamma_s-\gamma_b) & -\sigma(1-\gamma_b) & (\varphi+1)^2 & -(\varphi+1) \\ \sigma^2\gamma_s(\gamma_b-\gamma_s) & \sigma^2\gamma_s(1-\gamma_b) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Given (B.34) and (B.35), it is possible to rewrite (B.33) in a purely quadratic way. Indeed thanks to these conditions:

$$0 \simeq Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[\hat{l}_t^{i'}(1-\varphi\zeta_s)f_s - \zeta_s v_s) + \frac{1}{2}\hat{s}_t^{i'}((1-\varphi\zeta_s)F_{s,s} - \zeta_s V_{s,s})\hat{s}_t^i - \hat{s}_t^{i'}((1-\varphi\zeta_s)F_{s,e}$$
$$-\hat{s}_t^{i'}((1-\varphi\zeta_s)F_{s,e} - \zeta_s V_{s,e})\hat{e}_t^i\Big] + t.i.p. \qquad \text{(B.36)}$$

where $\zeta_s = (\delta_s - (1-\tilde{\tau}))/(\delta_s\varphi + \gamma_s\sigma)$. Notice that $\zeta_3^s = \zeta_s(1-\tilde{\tau})$ and $\zeta_1^s = (1-\varphi\zeta_s)$ with ζ_1^s and ζ_3^s being the lagrange multipliers previously recovered for the optimal policy problem of the small economy policymaker[5]. It is easy to show that:

$$w_s = (1-\varphi\zeta_s)f_s - \zeta_s v_s \qquad \text{(B.37)}$$

Hence we can write the second order approximation of union welfare as:

$$Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\left[-\frac{1}{2}\hat{s}_t^{i'}\Omega_{s,s}\hat{s}_t^i + \hat{s}_t^{i'}\Omega_{s,e}\hat{e}_t^i\right] + t.i.p. \qquad \text{(B.38)}$$

where

$$\Omega_{s,s} \equiv W_{s,s} + (1-\varphi\zeta_s)G_{s,s} - \zeta_s V_{s,s} \qquad \Omega_{s,e} \equiv W_{s,e} + (1-\varphi\zeta_s)G_{s,e} - \zeta_s V_{s,e} \qquad \text{(B.39)}$$

and $\Omega_{s,s}$ and $\Omega'_{s,e}$ are respectively equal to:

[5]See Benigno and Woodford (2005)

$$\begin{bmatrix} (1-\zeta_s(\varphi+1))\varphi & -\zeta_s\gamma_s\sigma & 0 \\ -\zeta_s\gamma_s\sigma & (1-\tilde{\tau})(\sigma-1)+\zeta_s\gamma_s^2\sigma^2+(1-\zeta_s\varphi)(\delta_s+\omega_1)) & 0 \\ 0 & 0 & (1-\zeta_s(\varphi+1))\varepsilon/\lambda \end{bmatrix}$$

$$\begin{bmatrix} -\zeta_s\sigma(\gamma_s-\gamma_b) & \zeta_s\sigma^2\gamma_s(\gamma_s-\gamma_b)+(1-\zeta_s\varphi)\,(\omega_1+\omega_2) & 0 \\ \zeta_s\sigma(1-\gamma_b) & -\zeta_s\sigma^2\gamma_s(1-\gamma_b)-(1-\zeta_s\varphi)\,\omega_2 & 0 \\ (1-\zeta_s(\varphi+1))(\varphi+1) & 0 & 0 \\ \zeta_s(\varphi+1) & 0 & 0 \end{bmatrix}$$

Now we would like to rewrite this approximation in terms of deviations from the target of the small open economy policymaker. It can be shown this target can be determined by maximizing (B.36) with respect to $\hat{y}_t^i$, $\hat{c}_t^i$ and $\pi_{i,t}$ taking as given the aggregate variables in the area H and F and subject to (2.45). According to the first order conditions of this problem:

$$(1-\zeta_s(\varphi+1))\varphi\hat{y}_t^{i,s}-\zeta_s\gamma_s\sigma\hat{c}_t^{i,s}+\zeta_s\sigma(\gamma_s-\gamma_b)\hat{c}_{H,t}-\zeta_s\sigma(1-\gamma_b)\hat{c}_{F,t}-(1-\zeta_s(\varphi+1))(\varphi+1)\hat{a}_t^i$$

$$-\zeta_s(\varphi+1)\hat{\mu}_t^i=\phi_{1,t}^i \tag{B.40}$$

$$(1-\tilde{\tau})(\sigma-1)+\zeta_s\gamma_s^2\sigma^2+(1-\zeta_s\varphi)(\delta_s+\omega_1))\hat{c}_t^{i,s}-\zeta_s\gamma_s\sigma\hat{y}_t^{i,s}-(\zeta_s\sigma^2\gamma_s(1-\gamma_b)-(1-\zeta_s\varphi)\,\omega_2)\hat{c}_{F,t}$$

$$+(\zeta_s\sigma^2\gamma_s(\gamma_s-\gamma_b)+(1-\zeta_s\varphi)\,(\omega_1+\omega_2))\hat{c}_{H,t}=-\delta_s\phi_{1,t}^i \tag{B.41}$$

$$(1-\zeta_s(\varphi+1))\frac{\varepsilon}{\lambda}\pi_{i,t}=0 \tag{B.42}$$

$$\hat{y}_t^{i,s}=\delta_s\hat{c}_t^{i,s}+(\delta_b-\delta_s)\hat{c}_{H,t}+(1-\delta_b)\hat{c}_{F,t} \tag{B.43}$$

for all $i\in[0,\frac{1}{2})$ and where ϕ_t^i is the lagrange multiplier of (2.45). Notice that in the perspective of the small open monetary authority $\hat{c}_{H,t}$ and $\hat{c}_{F,t}$ are taken as exogenous.

Then it is easy to show that (B.38) can be rewritten as (2.76). Indeed it is sufficient to add and subtract the corresponding target in each terms of (B.38) and then use the fist order conditions just listed.

B.3.2 The case of the Monetary Union

If in the area H there is a Monetary Union, then the second order approximation of average welfare of the union household can be read as:

$$\sum_{t=0}^{\infty}\beta^t Y^{\varphi+1}\int_0^{\frac{1}{2}} E_0\left[\hat{s}_t^{i\prime} w_s - \frac{1}{2}\hat{l}_t^{i\prime} W_{s,s}\hat{s}_t^i + \hat{l}_t^{i\prime} W_{s,u}\hat{u}_t^i\right] di + t.i.p. \qquad \text{(B.44)}$$

$$\hat{s}_t^{i\prime} \equiv \left[\hat{y}_t^i,\ \hat{c}_t^i,\ \pi_{i,t}\right] \qquad w_s^{\prime} \equiv [-1,\ (1-\tilde{\tau}),\ \ 0] \qquad \hat{u}_t^{i\prime} \equiv \left[a_t^i,\ \mu_t^i\right]$$

$$W_{s,s} \equiv \begin{bmatrix} (\varphi+1) & 0 & 0 \\ 0 & (1-\tilde{\tau})(\sigma-1) & 0 \\ 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix} \qquad W_{s,e} \equiv \begin{bmatrix} (\varphi+1) & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

A purely quadratic approximation to the welfare of the union households can be retrieved thanks to the second order approximations of the demand and of the supply curves.

The second order approximation to the demand curve of a generic region i in the area H can be read as:

$$0 \simeq \hat{s}_t^{i\prime} f_s + \int_0^{\frac{1}{2}} \hat{s}_t^i di' g_{S_H} + \int_{\frac{1}{2}}^1 \hat{s}_t^i di' g_{S_F} + \frac{1}{2}\hat{s}_t^{i\prime} G_{s,s}\hat{s}_t^i + \frac{1}{2}\int_0^{\frac{1}{2}} \hat{s}_t^{i\prime} G_{s_H,s_H}\hat{s}_t^i di + \frac{1}{2}\int_{\frac{1}{2}}^1 \hat{s}_t^{i\prime} G_{s_F,s_F}\hat{s}_t^i di$$
$$+\frac{1}{2}\int_0^{\frac{1}{2}} \hat{s}_t^i di' G_{S_H,S_H}\int_0^{\frac{1}{2}} \hat{s}_t^i di + \frac{1}{2}\int_{\frac{1}{2}}^1 \hat{s}_t^i di' G_{S_F,S_F}\int_{\frac{1}{2}}^1 \hat{s}_t^i di + \hat{s}_t^{i\prime} G_{s,S_H}\int_0^{\frac{1}{2}} \hat{s}_t^i di + \hat{s}_t^{i\prime} G_{s,S_F}\int_{\frac{1}{2}}^1 \hat{s}_t^i di$$
$$+\int_0^{\frac{1}{2}} \hat{s}_t^i di' G_{S_H,S_F}\int_{\frac{1}{2}}^1 \hat{s}_t^i di + s.o.t.i.p. \qquad \text{(B.45)}$$

where

$$g_s^{\prime} \equiv [-1,\ \delta_s,\ 0] \qquad g_{S_H}^{\prime} \equiv [0,\ 2(\delta_b-\delta_s),\ 0] \qquad g_{S_F}^{\prime} \equiv [0,\ 2(1-\delta_b),\ 0,]$$

$$G_{s,s} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & \delta_s + \omega_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad G_{s_F,s_F} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & (1-\delta_b) + \omega_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{s_H,s_H} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\eta\sigma^2(1-\gamma_s^2) + (\delta_b - \delta_s) - (\omega_1 + \omega_3) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{S_H,S_H} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & (\eta\sigma^2(1-\gamma_s^2) - \eta\sigma^2\gamma_b(1-\gamma_b) + 2\omega_1 + 2\omega_2 + \omega_3) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{S_F,S_F} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -(\eta\sigma^2\gamma_b(1-\gamma_b) + \omega_3) & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad G_{s,S_H} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -(\omega_1 + \omega_2) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{s,S_F} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & \omega_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad G_{S_H,S_F} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & (\eta\sigma^2\gamma_b(1-\gamma_b) - \omega_2) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and where

$$\omega_3 \equiv \frac{(1-\alpha_b)\eta\sigma(\sigma + 2(1-\alpha_b)(1-\eta\sigma))}{1-2\alpha_b}$$

By integrating (B.45):

$$0 \simeq + \int_0^{\frac{1}{2}} \hat{s}_t^i di' h_{S_H} + \int_{\frac{1}{2}}^1 \hat{s}_t^i di' h_{S_F} + \frac{1}{2}\int_0^{\frac{1}{2}} \hat{s}_t^{i'} H_{s_H,s_H} \hat{s}_t^i di + \frac{1}{2}\int_{\frac{1}{2}}^1 \hat{s}_t^{i'} H_{s_F,s_F} \hat{s}_t^i di$$

$$+\frac{1}{2}\int_0^{\frac{1}{2}} \hat{s}_t^i di' H_{S_H,S_H} \int_0^{\frac{1}{2}} \hat{s}_t^i di + \frac{1}{2}\int_{\frac{1}{2}}^1 \hat{s}_t^i di' H_{S_F,S_F} \int_{\frac{1}{2}}^1 \hat{s}_t^i di + \int_0^{\frac{1}{2}} \hat{s}_t^i di' H_{S_H,S_F} \int_{\frac{1}{2}}^1 \hat{s}_t^i di + s.o.t.i.p.$$

with

$$h_{S_H}^{'} \equiv [-1,\ \delta_b,\ 0] \qquad h_{S_F}^{'} \equiv [0,\ (1-\delta_b),\ 0,]$$

$$H_{s_H,s_H} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\eta\sigma^2(1-\gamma_s^2) + \delta_b - \omega_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad H_{s_F,s_F} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & (1-\delta_b) + \omega_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$H_{S_H,S_H} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & \eta\sigma^2(1-\gamma_s^2) - \eta\sigma^2\gamma_b(1-\gamma_b) + \omega_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$H_{S_F,S_F} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -(\eta\sigma^2(1-\gamma_b)\gamma_b + \omega_3) & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad H_{S_H,S_F} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & \eta\sigma^2\gamma_b(1-\gamma_b) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A symmetric approximation can be stated for the resource constraints of the regions in the area F namely:

$$0 \simeq + \int_{\frac{1}{2}}^{1} \hat{s}_t^i di' f_{S_F} + \int_0^{\frac{1}{2}} \hat{s}_t^i di' f_{S_H} + \frac{1}{2}\int_{\frac{1}{2}}^{1} \hat{s}_t^{i'} F_{s_F,s_F} \hat{s}_t^i di + \frac{1}{2}\int_0^{\frac{1}{2}} \hat{s}_t^{i'} F_{s_H,s_H} \hat{s}_t^i di$$

$$+\frac{1}{2}\int_{\frac{1}{2}}^{1} \hat{s}_t^i di' F_{S_F,S_F} \int_{\frac{1}{2}}^{1} \hat{s}_t^i di + \frac{1}{2}\int_0^{\frac{1}{2}} \hat{s}_t^i di' F_{S_H,S_H} \int_0^{\frac{1}{2}} \hat{s}_t^i di + \int_0^{\frac{1}{2}} \hat{s}_t^i di' F_{S_H,S_F} \int_{\frac{1}{2}}^{1} \hat{s}_t^i di + s.o.t.i.p.$$

with $f_{S_F} \equiv h_{S_H}$, $f_{S_H} \equiv h_{S_F}$, $F_{s_F,s_F} \equiv H_{s_H,s_H}$, $F_{s_H,s_H} \equiv H_{s_F,s_F}$, $F_{S_F,S_F} \equiv H_{S_H,S_H}$, $F_{S_H,S_H} \equiv H_{S_F,S_F}$ and $F_{S_F,S_H} \equiv H_{S_H,S_F}$.

Conversely the second order approximation of the (2.54) for the area F can be obtained by combining (2.52) and (2.53):

$$\begin{aligned} V_0 \;=\; & \frac{1-\theta}{\theta}(1-\beta\theta)\sum_{t=0}^{\infty}\beta^t E_0\Big[\hat{s}_t^{i'} v_s + \int_{\frac{1}{2}}^{1} \hat{s}_t^i di' v_{S_F} + \int_0^{\frac{1}{2}} \hat{s}_t^i di' v_{S_H} - \hat{u}_t^{i'} v_u + \frac{1}{2}\hat{s}_t^i V_{s,s}\hat{s}_t^i \\ & +\frac{1}{2}\int_{\frac{1}{2}}^{1} \hat{s}_t^i di' V_{S_F,S_F} \int_{\frac{1}{2}}^{1} \hat{s}_t^i di + \frac{1}{2}\int_0^{\frac{1}{2}} \hat{s}_t^i di' V_{S_H,S_H} \int_0^{\frac{1}{2}} \hat{s}_t^i di + \hat{s}_t^{i'} V_{s,S_F} \int_{\frac{1}{2}}^{1} \hat{s}_t^i di \\ & +\hat{s}_t^{i'} V_{s,S_H} \int_0^{\frac{1}{2}} \hat{s}_t^i di + \int_0^{\frac{1}{2}} \hat{s}_t^i di' V_{S_H,S_F} \int_{\frac{1}{2}}^{1} \hat{s}_t^i di - \hat{s}_t^{i'} V_{s,u}\hat{u}_t^i\Big] + s.o.t.i.p. \end{aligned} \qquad \text{(B.46)}$$

where

$$v_s' \equiv [\varphi,\ \sigma\gamma_s,\ 0] \qquad v_{S_F}' \equiv [0,\ 2\sigma(\gamma_b-\gamma_s),\ 0] \qquad v_{S_H}' \equiv [0,\ 2\sigma(1-\gamma_b),\ 0] \qquad v_u' \equiv [\ (\varphi+1),\ -1]$$

$$V_{s,s} \equiv \begin{bmatrix} \varphi(\varphi+2) & \sigma\gamma_s & 0 \\ \sigma\gamma_s & -\sigma^2\gamma_s^2 & 0 \\ 0 & 0 & \frac{\varepsilon(\varphi+1)}{\lambda} \end{bmatrix} \qquad V_{S_F,S_F} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma^2(\gamma_b-\gamma_s)^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V_{S_H,S_H} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sigma^2(1-\gamma_b)^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad V_{s,S_F} \equiv \begin{bmatrix} 0 & \sigma(\gamma_b-\gamma_s) & 0 \\ 0 & -\sigma^2\gamma_s(\gamma_b-\gamma_s) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V_{s,S_H} \equiv \begin{bmatrix} 0 & \sigma(1-\gamma_b) & 0 \\ 0 & -\sigma^2\gamma_s(1-\gamma_b) & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad V_{S_F,S_H} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sigma^2(1-\gamma_b)(\gamma_b-\gamma_s) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V_{s,u} \equiv \begin{bmatrix} (\varphi+1)^2 & -(\varphi+1) \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By integrating (B.48) over $\left[\frac{1}{2}, 1\right]$

$$\frac{1}{2}V_0 = \frac{1-\theta}{\theta}(1-\beta\theta)\sum_{t=0}^{\infty}\beta^t E_0\Big[\int_{\frac{1}{2}}^{1}\hat{s}_t^i di' r_{S_F} + \int_0^{\frac{1}{2}}\hat{s}_t^i di' r_{S_H} - \int_{\frac{1}{2}}^{1}\hat{u}_t^i di' r_U + \frac{1}{2}\int_{\frac{1}{2}}^{1}\hat{s}_t^{i'} R_{s_F,s_F}\hat{s}_t^i di$$
$$+\frac{1}{2}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di' R_{S_F,S_F}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di + \frac{1}{2}\int_0^{\frac{1}{2}}\hat{s}_t^i di' R_{S_H,S_H}\int_0^{\frac{1}{2}}\hat{s}_t^i di + \int_0^{\frac{1}{2}}\hat{s}_t^i di' R_{S_F,S_H}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di$$
$$-\int_{\frac{1}{2}}^{1}\hat{s}_t^{i'} R_{s_F,u}\hat{u}_t^i di\Big] + s.o.t.i.p. \tag{B.47}$$

where:

$$r'_{S_F} \equiv [\varphi,\ \sigma\gamma_b,\ 0] \qquad r'_{S_H} \equiv [0,\ \sigma(1-\gamma_b),\ 0] \qquad r'_u \equiv [(\varphi+1),\ -1]$$

$$R_{s_F,s_F} \equiv \begin{bmatrix} \varphi(\varphi+2) & \sigma\gamma_s & 0 \\ \sigma\gamma_s & -\sigma^2\gamma_s^2 & 0 \\ 0 & 0 & \frac{\varepsilon(\varphi+1)}{\lambda} \end{bmatrix} \qquad R_{S_F,S_F} \equiv \begin{bmatrix} 0 & \sigma(\gamma_b-\gamma_s) & 0 \\ \sigma(\gamma_b-\gamma_s) & -\sigma^2(\gamma_b^2-\gamma_s^2) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_{S_H,S_H} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sigma^2(1-\gamma_b)^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad R_{S_F,S_H} \equiv \begin{bmatrix} 0 & \sigma(1-\gamma_b) & 0 \\ 0 & -\sigma^2\gamma_b(1-\gamma_b) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_{s_F,u} \equiv \begin{bmatrix} (\varphi+1)^2 & -(\varphi+1) \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Again a symmetric condition can be stated for the regions of the area H namely:

$$\frac{1}{2}V_0 = \frac{1-\theta}{\theta}(1-\beta\theta)\sum_{t=0}^{\infty}\beta^t E_0\Big[\int_{\frac{1}{2}}^{1}\hat{s}_t^i di' k_{S_H} + \int_0^{\frac{1}{2}}\hat{s}_t^i di' k_{S_F} - \int_{\frac{1}{2}}^{1}\hat{u}_t^i di' k_U + \frac{1}{2}\int_{\frac{1}{2}}^{1}\hat{s}_t^i K_{s_H,s_H}\hat{s}_t^i di$$
$$+\frac{1}{2}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di' K_{S_H,S_H}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di + \frac{1}{2}\int_0^{\frac{1}{2}}\hat{s}_t^i di' K_{S_F,S_F}\int_0^{\frac{1}{2}}\hat{s}_t^i di + \int_0^{\frac{1}{2}}\hat{s}_t^i di' K_{S_H,S_F}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di$$
$$-\int_{\frac{1}{2}}^{1}\hat{s}_t^{i'} K_{s_H,u}\hat{u}_t^i di\Big] + s.o.t.i.p. \tag{B.48}$$

where $k_{S_H} = r_{S_F}$, $k_{S_F} = r_{S_H}$, $k_U = r_U$, $K_{s_H,s_H} = R_{s_F,s_F}$, $K_{S_H,S_H} = R_{S_F,S_F}$, $K_{S_F,S_F} = R_{S_H,S_H}$, $K_{S_H,S_F} = R_{S_F,S_H}$ and $K_{s_H,u} = R_{s_F,u}$.

Then it can be shown that:

$$w_s = (1-\varphi\zeta_b)h_{S_H} - (\xi-\zeta_b)\varphi f_{S_H} - \zeta_b k_{S_H} - (\xi-\zeta_b)r_{S_H}$$
$$0 = (1-\varphi\zeta_b)h_{S_F} - (\xi-\zeta_b)\varphi f_{S_F} - \zeta_b k_{S_F} - (\xi-\zeta_b)r_{S_F} \qquad \text{(B.49)}$$

where $\zeta_b = \frac{1}{2}\frac{\tilde{\tau}}{\sigma+\varphi} - \frac{\delta_b - 1 + (1/2)\tilde{\tau}}{(1-2\gamma_b)\sigma + (1-2\delta_b)\varphi}$ and $\xi = \frac{\tilde{\tau}}{\sigma+\varphi}$ Hence we can write the second order approximation of union welfare as:

$$Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[\frac{1}{2}\int_0^{\frac{1}{2}} \hat{s}_t^{i\prime}\Omega_{s_H,s_H}\hat{s}_t^i di + \frac{1}{2}\int_{\frac{1}{2}}^{1} \hat{s}_t^{i\prime}\Omega_{s_F,s_F}\hat{s}_t^i di + \frac{1}{2}\int_0^{\frac{1}{2}} \hat{s}_t^i di'\Omega_{S_H,S_H}\int_0^{\frac{1}{2}} \hat{s}_t^i di$$
$$+\frac{1}{2}\int_{\frac{1}{2}}^{1} \hat{s}_t^i di'\Omega_{S_F,S_F}\int_{\frac{1}{2}}^{1} \hat{s}_t^i di + \int_0^{\frac{1}{2}} \hat{s}_t^i di'\Omega_{S_H,S_F}\int_{\frac{1}{2}}^{1} \hat{s}_t^i di - \int_0^{\frac{1}{2}} \hat{s}_t^{i\prime}\Omega_{s_H,u}\hat{u}_t^i di - \int_{\frac{1}{2}}^{1} \hat{s}_t^{i\prime}\Omega_{s_F,u}\hat{u}_t^i di\Big]$$
$$+t.i.p. \qquad \text{(B.50)}$$

where

$$\Omega_{s_H,s_H} \equiv W_{s,s} + (1-\varphi\zeta_b)H_{s_H,s_H} - (\xi-\zeta_b)\varphi F_{s_H,s_H} - \zeta_b K_{s_H,s_H}$$
$$\Omega_{s_F,s_F} \equiv (1-\zeta_b\varphi)H_{s_F,s_F} - (\xi-\zeta_b)\varphi F_{s_F,s_F} - (\xi-\zeta_b)R_{s_F,s_F}$$
$$\Omega_{S_H,S_H} \equiv (1-\zeta_b\varphi)H_{S_H,S_H} - (\xi-\zeta_b)\varphi F_{S_H,S_H} - \zeta_b K_{S_H,S_H} - (\xi-\zeta_b)R_{S_H,S_H}$$
$$\Omega_{S_F,S_F} \equiv (1-\zeta_b\varphi)H_{S_F,S_F} - (\xi-\zeta_b)\varphi F_{S_F,S_F} - \zeta_b K_{S_F,S_F} - (\xi-\zeta_b)R_{S_F,S_F}$$
$$\Omega_{S_H,S_F} \equiv (1-\zeta_b\varphi)H_{S_H,S_F} - (\xi-\zeta_b)\varphi F_{S_H,S_F} - \zeta_b K_{S_H,S_F} - (\xi-\zeta_b)R'_{S_F,S_H}$$
$$\Omega_{s_H,u} \equiv W_{s,u} - \zeta_b K_{s_H,u} \qquad \Omega_{s_F,u} \equiv -(\xi-\zeta_b)R_{s_F,u} \qquad \text{(B.51)}$$

and Ω_{s_H,s_H}, Ω_{s_F,s_F}, Ω_{S_H,S_H}, Ω_{S_F,S_F}, Ω_{S_H,S_F}, $\Omega_{s_H,u}$ and $\Omega_{s_F,u}$ are respectively equal to:

$$\begin{bmatrix} (1-\zeta_b(\varphi+1))\varphi & -\zeta_b\sigma\gamma_s & 0 \\ -\zeta_b\sigma\gamma_s & \omega_{sHsH} & 0 \\ 0 & 0 & \frac{(1-\zeta_b(\varphi+1))\epsilon}{\lambda} \end{bmatrix}$$

$$\begin{bmatrix} -(\xi-\zeta_b)(\varphi+1)\varphi & -(\xi-\zeta_b)\sigma\gamma_s & 0 \\ -(\xi-\zeta_b)\sigma\gamma_s & \omega_{sFsF} & 0 \\ 0 & 0 & -\frac{((\xi-\zeta_b)(\varphi+1))\varepsilon}{\lambda} \end{bmatrix}$$

$$\begin{bmatrix} 0 & -\zeta_b\sigma\left(\gamma_b-\gamma_s\right) & 0 \\ -\zeta_b\sigma\left(\gamma_b-\gamma_s\right) & \omega_{SHSH} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -(\xi-\zeta_b)\sigma\left(\gamma_b-\gamma_s\right) & 0 \\ -(\xi-\zeta_b)\sigma\left(\gamma_b-\gamma_s\right) & \omega_{SFSF} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -\zeta_b\sigma\left(1-\gamma_b\right) & 0 \\ -(\xi-\zeta_b)\sigma\left(1-\gamma_b\right) & \omega_{SHSF} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} (1-\zeta_b(\varphi+1))(\varphi+1) & \zeta_b(\varphi+1) \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} -(\xi-\zeta_b)(\varphi+1)^2 & (\xi-\zeta_b)(\varphi+1) \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

with

$$\begin{aligned} \omega_{sHsH} \equiv \quad & (\sigma-1)\left(1-\tilde{\tau}\right) \\ & +(1-\zeta_b\varphi)\left(-\eta\sigma^2\left(1-\gamma_s^2\right)+\delta_b-\omega_3\right) \\ & -(\xi-\zeta_b)\varphi((1-\delta_b)+\omega_3) \\ & +\zeta_b\sigma^2\gamma_s^2 \end{aligned} \tag{B.52}$$

$$\begin{aligned} \omega_{sFsF} \equiv \quad & (1-\zeta_b\varphi)((1-\delta_b)+\omega_3) \\ & -(\xi-\zeta_b)\varphi(-\eta\sigma^2\left(1-\gamma_s^2\right)+\delta_b-\omega_3) \\ & +(\xi-\zeta_b)\sigma^2\gamma_s^2 \end{aligned} \tag{B.53}$$

$$\begin{aligned} \omega_{SHSH} \equiv \quad & (1-\zeta_b\varphi)(\eta\sigma^2\left(1-\gamma_s^2-\gamma_b\left(1-\gamma_b\right)\right)+\omega_3) \\ & +(\xi-\zeta_b)\varphi(\eta\sigma^2\gamma_b\left(1-\gamma_b\right)+\omega_3) \\ & +\zeta_b\sigma^2\left(\gamma_b^2-\gamma_s^2\right) \\ & +(\xi-\zeta_b)\sigma^2\left(1-\gamma_b\right)^2 \end{aligned} \tag{B.54}$$

$$\begin{aligned} \omega_{SFSF} \equiv \quad & -(1-\zeta_b\varphi)(\eta\sigma^2\gamma_b\left(1-\gamma_b\right)+\omega_3) \\ & -(\xi-\zeta_b)\varphi(\eta\sigma^2\left(1-\gamma_s^2-\gamma_b\left(1-\gamma_b\right)\right)+\omega_3) \\ & +\zeta_b\sigma^2\left(1-\gamma_b\right)^2 \\ & +(\xi-\zeta_b)\sigma^2\left(\gamma_b^2-\gamma_s^2\right) \end{aligned} \tag{B.55}$$

$$\begin{aligned}\omega_{SHSF} \equiv \quad & (1-\zeta_b\varphi)\eta\sigma^2\gamma_b\,(1-\gamma_b) \\ & -(\xi-\zeta_b)\varphi\eta\sigma^2\gamma_b\,(1-\gamma_b) \\ & +\zeta_b\sigma^2\gamma_b\,(1-\gamma_b) \\ & +(\xi-\zeta_b)\sigma^2\gamma_b\,(1-\gamma_b)\end{aligned} \tag{B.56}$$

Now we would like to split this welfare approximation in (B.50) in two components namely:

$$\begin{aligned} & Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[\frac{1}{2}\int_0^{\frac{1}{2}}\left(\hat{s}_t^i-\frac{1}{2}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di\right)'\Omega_{s_H,s_H}\left(\hat{s}_t^i-\frac{1}{2}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di\right)di \\ & +\frac{1}{2}\int_{\frac{1}{2}}^{1}\left(\hat{s}_t^i-\frac{1}{2}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di\right)'\Omega_{s_F,s_F}\left(\hat{s}_t^i-\frac{1}{2}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di\right)'di \\ & -\int_0^{\frac{1}{2}}\left(\hat{s}_t^i-\int_0^{\frac{1}{2}}\hat{s}_t^i di\right)'\Omega_{s_H,u}\left(\hat{u}_t^i-\int_0^{\frac{1}{2}}\hat{u}_t^i di\right)di \\ & -\int_{\frac{1}{2}}^{1}\left(\hat{s}_t^i-\int_{\frac{1}{2}}^{1}\hat{s}_t^i di\right)'\Omega_{s_F,u}\left(\hat{u}_t^i-\int_{\frac{1}{2}}^{1}\hat{u}_t^i di\right)di \\ & +\frac{1}{2}\int_0^{\frac{1}{2}}\hat{s}_t^i di'(\Omega_{s_H,s_H}+\Omega_{S_H,S_H})\int_0^{\frac{1}{2}}\hat{s}_t^i di \\ & +\frac{1}{2}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di'(\Omega_{s_F,s_F}+\Omega_{S_F,S_F})\int_{\frac{1}{2}}^{1}\hat{s}_t^i di \\ & +\int_0^{\frac{1}{2}}\hat{s}_t^i di'\Omega_{S_H,S_F}\int_{\frac{1}{2}}^{1}\hat{s}_t^i di \\ & -\int_0^{\frac{1}{2}}\hat{s}_t^{i'} di\Omega_{s_H,u}\int_0^{\frac{1}{2}}\hat{u}_t^i di \\ & -\int_{\frac{1}{2}}^{1}\hat{s}_t^{i'} di\Omega_{s_F,u}\int_{\frac{1}{2}}^{1}\hat{u}_t^i di\Big]+t.i.p. \end{aligned} \tag{B.57}$$

The first component depends only on the average union variables whereas the second depends only on the differences between specific country and average union variables. However this second component can be considered as terms independent of policy (even if they should be taken into account for welfare evaluation) because having as a monetary policy instrument of the average union interest rate, the policy decisions of the Monetary Union Central Bank can just influence the average union economic performance. Thus (B.57) can be read as:

$$Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[\frac{1}{2}\int_0^{\frac{1}{2}}\hat{s}_t^i di'(\Omega_{s_H,s_H}+\Omega_{S_H,S_H})\int_0^{\frac{1}{2}}\hat{s}_t^i di+\frac{1}{2}\int_{\frac{1}{2}}^1\hat{s}_t^i di'(\Omega_{s_F,s_F}+\Omega_{S_F,S_F})\int_{\frac{1}{2}}^1\hat{s}_t^i di$$
$$+\int_0^{\frac{1}{2}}\hat{s}_t^i di'\Omega_{S_H,S_F}\int_{\frac{1}{2}}^1\hat{s}_t^i di-\int_0^{\frac{1}{2}}\hat{s}_t^{i'} di\Omega_{s_H,u}\int_0^{\frac{1}{2}}\hat{u}_t^i di-\int_{\frac{1}{2}}^1\hat{s}_t^{i'} di\Omega_{s_F,u}\int_{\frac{1}{2}}^1\hat{u}_t^i di\Big]+t.i.p.$$

The last step consists in rewriting (B.58) in terms of gaps with respect to the target of the policymaker of the monetary union. It is easy to show that target is determined by maximizing (B.58) with respect $\hat{y}_{H,t}$, $\hat{y}_{F,t}$, $\hat{c}_{H,t}$, $\hat{c}_{F,t}$ and $\pi_{H,t}$ subject to:

$$\hat{y}_{H,t}=\delta_b\hat{c}_{H,t}+(1-\delta_b)\hat{c}_{F,t} \qquad i\in\left[0,\frac{1}{2}\right)$$
$$\hat{y}_{F,t}=\delta_b\hat{c}_{F,t}+(1-\delta_b)\hat{c}_{H,t} \qquad i\in\left[\frac{1}{2},0\right] \tag{B.58}$$

In other words the target of the benevolent central bank of the monetary union coincides with the constrained efficient allocation (namely the allocation that a planner would choose having as objective (B.58)). According to the first order conditions with respect to $\hat{y}_{H,t}^b$, $\hat{y}_{F,t}^b$, $\hat{c}_{H,t}^b$, $\hat{c}_{F,t}^b$ and $\pi_{H,t}$:

$$(1-\zeta_b(\varphi+1))\varphi\hat{y}_{H,t}^b-\zeta_b\sigma\left(\gamma_b\hat{c}_{H,t}^b+(1-\gamma_b)\,\hat{c}_{F,t}^b\right)-(1-\zeta_b(\varphi+1))(\varphi+1)\hat{a}_{H,t}$$
$$-\zeta_b(\varphi+1)\hat{\mu}_{H,t}=\phi_{1,t}^H \tag{B.59}$$

$$-(\xi-\zeta_b)(\varphi+1)\varphi\hat{y}_{F,t}^b-(\xi-\zeta_b)\sigma\left(\gamma_b\hat{c}_{F,t}^b+(1-\gamma_b)\,\hat{c}_{H,t}^b\right)+(\xi-\zeta_b)(\varphi+1)^2\hat{a}_{F,t}$$
$$-(\xi-\zeta_b)(\varphi+1)\hat{\mu}_{F,t}=\phi_{1,t}^F \tag{B.60}$$

$$[(\sigma-1)(1-\tilde{\tau})+(1-\zeta_b\varphi)\delta_b-(\xi-\zeta_b)\varphi(1-\delta_b)]\,\hat{c}^b_{H,t}-\zeta_b\sigma\gamma_b\hat{y}^b_{H,t}-(\xi-\zeta_b)\sigma\left(1-\gamma_b\right)\hat{y}^b_{F,t}$$
$$+\zeta_b\sigma^2\gamma_b\left(\gamma_b\hat{c}^b_{H,t}+\left(1-\gamma_b\right)\hat{c}^b_{F,t}\right)-(\xi-\zeta_b)\sigma^2\left(1-\gamma_b\right)\left(\gamma_b\hat{c}^b_{F,t}+\left(1-\gamma_b\right)\hat{c}^b_{H,t}\right)$$
$$+(1-\xi\varphi)\eta\sigma^2\gamma_b(1-\gamma_b)(\hat{c}^b_{H,t}-\hat{c}^b_{F,t})=-(\delta_b\phi^H_{1,t}+(1-\delta_b)\phi^F_{1,t}) \qquad \text{(B.61)}$$

$$[(1-\zeta_b\varphi)(1-\delta_b)-(\xi-\zeta_b)\varphi\delta_b]\,\hat{c}^b_{F,t}-(\xi-\zeta_b)\sigma\gamma_b\hat{y}^b_{F,t}-\zeta_b\sigma(1-\gamma_b)\hat{y}^b_{H,t}$$
$$+(\xi-\zeta_b)\sigma^2\gamma_b\left(\gamma_b\hat{c}^b_{F,t}+\left(1-\gamma_b\right)\hat{c}^b_{H,t}\right)+\zeta_b\sigma^2(1-\gamma_b)\left(\gamma_b\hat{c}^b_{H,t}+\left(1-\gamma_b\right)\hat{c}^b_{F,t}\right)$$
$$-(1-\xi\varphi)\eta\sigma^2\gamma_b(1-\gamma_b)(\hat{c}^b_{F,t}-\hat{c}^b_{H,t})=-(\delta_b\phi^F_{1,t}+(1-\delta_b)\phi^H_{1,t}) \qquad \text{(B.62)}$$

$$(1-\zeta_b(\varphi+1))\frac{\varepsilon}{\lambda}\pi_{H,t}=0 \qquad \text{(B.63)}$$

where $\phi^H_{1,t}$ and $\phi^F_{1,t}$ are the lagrange multipliers of constraints (2.47) and (2.48). Then it can be shown that (B.50) corresponds to (2.81) (again by adding and subtracting the target in each term of (B.50) and then using the conditions just listed) where:

$$\varrho_H\equiv\Big[(\sigma-1)(1-\tilde{\tau})+(1-\zeta_b(\varphi+1))\delta_b-(\xi-\zeta_b)\varphi(1-\delta_b)-(1-\zeta_b\varphi)\eta\sigma^2\gamma_b(1-\gamma_b)$$
$$+\zeta_b\sigma^2\gamma_b^2+(\xi-\zeta_b)\sigma^2\left(1-\gamma_b\right)^2\Big]$$
$$\varrho_F\equiv\Big[(1-\zeta_b(\varphi+1))(1-\delta_b)-(\xi-\zeta_b)\varphi\delta_b+\zeta_b\sigma^2(1-\gamma_b)^2+(1-\zeta_b\varphi)\eta\sigma^2\gamma_b(1-\gamma_b)$$
$$+(\xi-\zeta_b)\sigma^2\gamma_b^2\Big]$$
$$\varrho_{H,F}\equiv(1-\zeta_b\varphi)\eta\sigma^2\gamma_b(1-\gamma_b)+\zeta_b\sigma^2\gamma_b(1-\gamma_b)+(\xi-\zeta_b)\sigma^2\left(1-\gamma_b\right)\gamma_b$$

C Appendix to chapter 3

C.1 Some useful derivatives

Before writing down the optimal policy problems it is useful to compute some derivatives. For the shake of brevity we report them after having imposed symmetry.

- Derivatives w.r.t. N and N^*:

$$\frac{\partial P/p_Z}{\partial N} = \frac{\partial P^*/p_Z}{\partial N^*} = -\phi^{1-\eta}\frac{P}{p_Z}\left[\frac{N^{-1}}{\varepsilon - 1} - \varepsilon F(1-\gamma)Q_Z^{\frac{-1}{\gamma}}\right] \tag{C.1}$$

$$\frac{\partial P/p_Z}{\partial N^*} = \frac{\partial P^*/p_Z}{\partial N} = (\tau_I \tau)^{1-\eta}\frac{\partial P/p_Z}{\partial N} \tag{C.2}$$

$$\frac{\partial P_H/p_Z}{\partial N} = \frac{\partial P_H^*/p_Z}{\partial N^*} = \phi^{\eta}\frac{\partial P/p_Z}{\partial N} \qquad \frac{\partial Q_Z}{\partial N} = \frac{\partial Q_Z^*}{\partial N^*} = -\gamma\varepsilon F Q_Z^{\frac{\gamma-1}{\gamma}} \tag{C.3}$$

- Derivatives w.r.t. τ_C and τ_C^*:

$$\frac{\partial P/p_Z}{\partial \tau_C} = \frac{\partial P^*/p_Z}{\partial \tau_C^*} = -\phi^{1-\eta}\frac{P}{p_Z}\frac{1}{1-\tau_C} \qquad \frac{\partial P^*/p_Z}{\partial \tau_C} = \frac{\partial P/p_Z}{\partial \tau_C^*} = (\tau\tau_I)^{1-\eta}\frac{\partial P/p_Z}{\partial \tau_C} \tag{C.4}$$

$$\frac{\partial P_H/p_Z}{\partial \tau_C} = \frac{\partial P_H^*/p_Z}{\partial \tau_C^*} = \phi^{\eta}\frac{\partial P/p_Z}{\partial \tau_C} \tag{C.5}$$

- Derivatives w.r.t. τ_I and τ_I^*:

$$\frac{\partial P/p_Z}{\partial \tau_I} = \frac{\partial P^*/p_Z}{\partial \tau_I^*} = \phi^{1-\eta}\tau_I^{-\eta}(\tau)^{1-\eta}\frac{P}{p_Z} \tag{C.6}$$

where $\phi \equiv [1-(\tau\tau_I)^{1-\eta}]^{\frac{1}{\eta-1}}$

C.2 Cooperative optimal policy problem

As already specified in the paper, even thought we consider three sets of policy instruments, (τ_C, τ_C^*), (τ_I and τ_I^*), we allow policy makers to choose optimally only one instrument at a time. Therefore, the general set up of the cooperative problem is a follow:

$$\max_{C,C^*,N,N^*,\tau_i,\tau_i^*} \quad \left(\frac{P}{p_Z}\right)^{1-\alpha} C + \left(\frac{P^*}{p_Z}\right)^{1-\alpha} C^*$$

subject to:

$$(\varepsilon-1) f N^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{P_H}{p_Z}\right)^{\eta} = \left[\left(\frac{P}{p_Z}\right)^{\eta} C + \tau^{1-\eta}(\tau_I^*)^{-\eta}\left(\frac{P^*}{p_Z}\right)^{\eta} C^*\right] \tag{C.7}$$

$$(\varepsilon-1) f N^{*\frac{\varepsilon}{\varepsilon-1}} \left(\frac{P_H^*}{p_Z}\right)^{\eta} = \left[\left(\frac{P^*}{p_Z}\right)^{\eta} C^* + \tau^{1-\eta}(\tau_I)^{-\eta}\left(\frac{P}{p_Z}\right)^{\eta} C\right] \tag{C.8}$$

$$Q_Z = \frac{(1-\alpha)}{\alpha}\frac{P}{p_Z}C + \tau_I^{-\eta}(\tau)^{1-\eta}\left(\frac{P_H^*}{p_Z}\right)^{1-\eta}\left(\frac{P}{p_Z}\right)^{\eta} C - \tau_I^{*-\eta}(\tau)^{1-\eta}\left(\frac{P_H}{p_Z}\right)^{1-\eta}\left(\frac{P^*}{p_Z}\right)^{\eta} C^* \tag{C.9}$$

$$Q_Z^* = \frac{(1-\alpha)}{\alpha}\frac{P^*}{p_Z}C^* + \tau_I^{*-\eta}(\tau)^{1-\eta}\left(\frac{P_H}{p_Z}\right)^{1-\eta}\left(\frac{P^*}{p_Z}\right)^{\eta} C^* - \tau_I^{-\eta}\tau^{1-\eta}\left(\frac{P_H^*}{p_Z}\right)^{1-\eta}\left(\frac{P}{p_Z}\right)^{\eta} C. \tag{C.10}$$

where $i \in \{C, I, X\}$ and Q_Z, Q_Z^* are defined according to (3.31) and (3.32) and the price indices are defined as in section 3.4.4. Let λ_i be the lagrange multiplier associated with the i-th constraint. Note that the first order conditions w.r.t. C, C^*, N and N^* remain the same for all the three policy problems. After imposing symmetry, we can write those conditions as follows:

- C :

$$\left(\frac{P}{p_Z}\right)^{1-\alpha} = \lambda_1\left(\frac{P}{p_Z}\right)^{\eta}\left[1+\tau^{1-\eta}(\tau_I)^{-\eta}\right] + \lambda_3\frac{1-\alpha}{\alpha}\frac{P}{p_Z} \tag{C.11}$$

- C_* :

$$\lambda_1 = \lambda_2 \tag{C.12}$$

- N :

$$(1-\alpha)\left(\frac{P}{p_Z}\right)^{-\alpha} C\phi^{\eta-1}\frac{\partial P/p_Z}{\partial N} + \lambda_1\left(\frac{P}{p_Z}\right)^{\eta} f\varepsilon N^{\frac{1}{\varepsilon-1}}\phi^{\eta} = \lambda_3\left[\frac{1-\alpha}{\alpha}C\frac{\partial P/p_Z}{\partial N}\phi^{\eta-1} - \frac{\partial Q_Z}{\partial N}\right] \tag{C.13}$$

- N^* :

$$\lambda_3 = \lambda_4 \tag{C.14}$$

To obtain those expressions (and in what follows) we also make use of the constraints which, after imposing symmetry, collapse to two:

$$(\varepsilon - 1) f N^{\frac{\varepsilon}{\varepsilon-1}} \phi^{\eta} = C \left[1 + \tau^{1-\eta} (\tau_I)^{-\eta}\right] \tag{C.15}$$

$$Q_Z = \frac{1-\alpha}{\alpha} \frac{P}{p_Z} C \tag{C.16}$$

C.2.1 Cooperative Production Subsidies

When solving for the optimal cooperative production subsidies, we set $\tau_I = \tau_I^* = 1$. After imposing symmetry the first order conditions with respect to τ_C and τ_C^* implies, respectively:

- τ_C :

$$(1-\alpha) \left(\frac{P}{p_Z}\right)^{-\alpha} \phi^{\eta-1} = \lambda_1 \left(\frac{P}{p_Z}\right)^{\eta-1} \eta \left(1 + \tau^{2(1-\eta)} + 2\tau^{1-\eta} - \frac{\varepsilon - 1}{C} f N^{\frac{\varepsilon-1}{\varepsilon}} \phi^{2\eta-1}\right) + \lambda_3 \frac{1-\alpha}{\alpha} \phi^{\eta-1} \tag{C.17}$$

- τ_C^* :

$$\lambda_3 = \lambda_4 \tag{C.18}$$

Using (C.15) equation (C.17) simplifies to:

$$(1-\alpha) \left(\frac{P}{p_Z}\right)^{-\alpha} \phi^{\eta-1} = \lambda_3 \frac{1-\alpha}{\alpha} \phi^{\eta-1} \tag{C.19}$$

Multiplying this last equation by $C\frac{\partial P/p_Z}{\partial N}$ and subtracting it from (C.13) we obtain:

$$\lambda_1 \left(\frac{P}{p_Z}\right)^{\eta} N^{\frac{1}{\varepsilon-1}} \phi^{\eta} = \lambda_3 \gamma Q_z^{\frac{\gamma-1}{\gamma}} \tag{C.20}$$

From (C.19) we have $\lambda_3 = \alpha \left(\frac{P}{p_Z}\right)^{-\alpha}$. Combining (C.11) with the expression for λ_3, with (C.20) and the expression for $\frac{P}{p_Z} = \phi^{-1} \gamma \frac{\varepsilon}{\varepsilon-1} (1 - \tau_C) N^{\frac{1}{1-\varepsilon}} Q_Z^{\frac{\gamma-1}{\gamma}}$, we obtain:

$$1 - \tau_C = \frac{\varepsilon - 1}{\varepsilon} \tag{C.21}$$

The optimal cooperative solution implies that each country should set the production subsidy in order to exactly offset the distortion coming from the presence of monopolistic competition.

C.2.2 Cooperative Import Tariffs

For the moment we do not make any assumption on τ_C and τ_C^* other than that of symmetry. After imposing symmetry the first order conditions with respect to τ_I and τ_I^* implies, respectively:

- τ_I :

$$(1-\alpha)\left(\frac{P}{p_Z}\right)^{-\alpha} = \lambda_3 \frac{1-\alpha}{\alpha} + \lambda_1 \left(\frac{P}{p_Z}\right)^{\eta-1} \eta \frac{\tau_I - 1}{\tau_I} \tag{C.22}$$

- τ_I^* :

$$\lambda_3 = \lambda_4 \tag{C.23}$$

We can use (C.11) to eliminate λ_3 from the other equations. In particular, combining (C.11) with (C.22) we can write the F.O.C. w.r.t. τ_I as:

$$\alpha \left(\frac{P}{p_Z}\right)^{-\alpha} = \lambda_1 \left(\frac{P}{p_Z}\right)^{\eta-1} \left[1 + \tau^{1-\eta}\tau_I^{-\eta} - \eta \frac{\tau_I - 1}{\tau_I}\right] \tag{C.24}$$

Using again (C.11) to eliminate λ_3 from the F.O.C. w.r.t. N, (C.13), and combining this with (C.24) it is possible to derive the following expression:

$$\varepsilon F N^{\frac{1}{\varepsilon-1}} \phi^{\eta} = -\phi^{\eta-1}\eta C \left(\frac{P}{p_Z}\right)^{-1} \frac{\partial P/p_Z}{\partial N} \frac{\tau_I - 1}{\tau_I} - \frac{\partial Q_Z}{\partial N}\left(\frac{P}{p_Z}\right)^{-1} l \left[1 + \tau^{1-\eta}\tau_I^{-\eta} - \frac{\eta}{1-\alpha}\frac{\tau_I - 1}{\tau_I}\right] \tag{C.25}$$

Using (3.36) together with (C.3) we have:

$$\frac{\partial Q_Z}{\partial N}\left(\frac{P}{p_Z}\right)^{-1} = -F\phi \frac{\varepsilon - 1}{1 - \tau_C} N^{-\frac{1}{1-\varepsilon}} \tag{C.26}$$

Substituting this last expression and the one for $\frac{\partial P/p_Z}{\partial N}$ we can rewrite (C.25) as:

$$\varepsilon F\phi^{\eta-1} = -N^{\frac{1}{1-\varepsilon}}\eta\phi^{-1}\left[C\varepsilon F(1-\gamma)Q_Z^{-\frac{1}{\gamma}} - N^{-1}\frac{C}{\varepsilon-1}\right]\frac{\tau_I-1}{\tau_I} + F\frac{\varepsilon-1}{1-\tau_C}B \qquad \text{(C.27)}$$

where $B \equiv 1+\tau^{1-\eta}\tau_I^{-\eta} - \frac{\eta}{1-\alpha}\frac{\tau_I-1}{\tau_I}$. Finally note that, combining (3.36) with (C.16) we have:

$$CQ_Z^{-\frac{1}{\gamma}}\varepsilon\phi^{-1}N^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon-1}{\gamma}\frac{1}{1-\tau_C}\frac{\alpha}{1-\alpha} \qquad \text{(C.28)}$$

while from (C.15):

$$N^{\frac{\varepsilon}{1-\varepsilon}}\frac{C}{\varepsilon-1} = F\phi^{\eta}\left[1+\tau^{1-\eta}\tau_I^{-\eta}\right]^{-1} \qquad \text{(C.29)}$$

Using those last two equations into (C.27) we obtain:

$$\phi^{\eta-1}\left[\varepsilon - \eta\frac{\tau_I-1}{\tau_I}\left(1+\tau^{1-\eta}\tau_I^{-\eta}\right)^{-1}\right] = \frac{\varepsilon-1}{1-\tau_C}\left[B - \eta\frac{1-\gamma}{\gamma}\frac{\alpha}{1-\alpha}\frac{\tau_I-1}{\tau_I}\right] \qquad \text{(C.30)}$$

which, $\eta = \varepsilon$ and $\gamma = 1$, when simplifies to (3.38).

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