Limiting Debt in the Optimal Taxation Setup

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To my mother

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Contents

1 Introduction: Optimal Fiscal Policy in Models of Real Business Cycles

How should fiscal policy be set in the short run, over the long run, and over the business cycle? Unfortunately for the policy makers, different economic models generate very different policy predictions. The questions I consider important for any macroeconomic model of fiscal policy include: Which tax instruments should a benevolent government use to finance its expenditures? Should there be tax cuts during recessions to cushion the economy in a downturn? How should the tax rates react when unanticipated shocks hit the government budget constraint? I will show how the answers to these three policy questions change if I introduce limits on debt into the standard neoclassical model of real business cycles.

In the present thesis, I consider the setup in which a benevolent government raises all revenues through time-varying flat rate taxes on capital income, net of depreciation allowances, and labor income. This approach has a long tradition in public economics starting with Frank Ramsey's seminal contribution in 1927. The problem of a government that chooses an optimal taxation structure when only distorting taxes are available is called Ramsey optimal taxation problem. The framework used in the modern literature on optimal taxation combines the public finance approach and the general equilibrium tradition in macroeconomics. The models of optimal taxation build on the primal approach of Atkinson and Stiglitz (1980). The basic idea is to characterize the set of allocations that can be implemented as a competitive equilibrium with distorting taxes by the resource constraint and the implementability constraint. This primal approach, in essence, involves finding optimal wedges between marginal rates of substitution and marginal rates of transformation. Another strand of the literature that uses the primal approach studies business cycle properties of economies with exogenously given policies. The examples of such "fine tuning" exercises are numerous[1]. The key difference between the Ramsey setup and the economies with exogenous policies is: the Ramsey government understands that, whatever tax system it adopted, consumers and firms in the economy would react in their own interest through a system of competitive markets. Therefore the government must take into consideration the best responses of consumers and firms to the announced tax policies. As a consequence, the solution to the Ramsey optimal taxation problem consists of a tax policy that maximizes the welfare of the consumers, together with allocations and prices that constitute a competitive equilibrium. For the purpose of this work, I choose to ignore the well-known issue of time inconsistency of the Ramsey solution raised by Lucas and Stokey (1983). As is usually done, I assume that the government has a commitment technology that allows it to implement exactly the sequence of policies announced at the start of the economy.[2]

My thesis has the following structure: In the first chapter, I review the extensive literature on optimal taxation, emphasizing the main policy prescription it gives for capital and labor income taxes. Below I discuss at length the famous finding that the capital income taxation should be abolished after a short number of periods. This so-called Chamley-Judd result is very robust to many environments. However, when I introduce asset market incompleteness, it becomes optimal to tax capital income if either the consumers or the government face binding limits on

[1] See Chari, Christiano and Kehoe (1995) and references therein or Kim and Kim (2002).

[2] Recent literature specifies several kinds of such commitment mechanisms, like overaccumulation of capital as in Benhabib and Rustichini (1997), or a specific term structure of debt as in Barro (1997).

the amount of state-contingent bonds they want to buy. In the second chapter, I show that, in a stochastic simple growth model calibrated to mimic some features of the postwar U.S. economy, binding exogenous limits on debt lead to long run optimal capital income tax rates that are different from zero and positive on average. In the third chapter, a collateral requirement for consumers' borrowing may result in positive capital income taxes even in the non-stochastic steady state. As for the labor income taxes, the literature agrees that they should be set in order to smooth the welfare losses from taxation over time and states of nature. The characteristics of the labor taxes implied by this principle are strikingly different: under the assumption of complete asset markets, labor taxes are nearly constant over time and fully inherit the serial correlation properties of the underlying government expenditure shocks. With risk-free debt, labor taxes may end up being random walks regardless of the serial correlations of government expenditures. Both theoretical and numerical results of my Chapter 2 suggest that under incomplete markets (due to borrowing constraints), labor income taxes behave as with risk-free debt and affirm Barro's assertion of random walks that I discuss later in this chapter.

Before I turn to the intuition behind my main findings, let me place my contribution into the context of the vast number of existing works. The second chapter of my thesis relates to three streams within macroeconomic literature: the tax smoothing literature originated from Barro (1979) and Lucas and Stokey (1983), zero long run optimal capital income taxation result of Chamley (1986) and Judd (1985), and business cycle characteristics of optimal capital and labor income taxes by Chari, Christiano and Kehoe (1994). I retain Zhu's (1992) and Chari, Christiano and Kehoe's (1994) environment but add exogenous upper bounds on government debt and asset holdings. Introducing such financial market incompleteness means that the private sector becomes unable to neutralize the effects of a change in financial policy. Barro (1979) pointed out that the presence of some eventual limits on debt may be motivated, on the high side, by the probability of government's default which is measured by the present value of the future taxing capacity. On the low side, as the quantity of government bonds diminishes, public and private debts may become less perfect substitutes in terms of liquidity characteristics. Though nothing rules out the possibility of the government becoming a net creditor for the private sector, "some monopoly power that the government has in the sale of bonds would then prescribe a target lower bound for the debt-income ratio".

The third chapter of my thesis studies the deterministic version of the Model from Chapter 2 to endogenize the borrowing restrictions. I impose credit constraints on the representative consumer in the spirit of Kiyotaki and Moore (1997). To prevent from defaults on borrowing, consumers are required to secure their debts by their capital stock holdings.

1.1 Chamley-Judd Result of Zero Optimal Capital Income Taxation

Ramsey optimal policies smooth distortions over time and states of nature. With complete asset markets and for a fairly general class of the utility functions, smoothing tax distortions over time implies that capital income tax rates should be roughly zero while labor and consumption taxes should be roughly constant, as shown by Zhu (1992) and Chari, Christiano and Kehoe (1994). Ramsey policies also imply that heavily taxing inelastically supplied inputs is optimal. Thus it is optimal to tax capital income at initially high rate and then drop this rate to zero in the long run. The latter result is due to Chamley (1986), who demonstrated that, for a general class of utility functions characterized by Koopmans, the tax rate on capital income is zero in the steady state of a deterministic model. Judd (1985) proved that in a deterministic model with heterogeneous consumers, if the economy converges to the steady state where all agents have a common rate of time preference, no agent will asymptotically choose redistributive capital income taxation. This result is independent of the initial wealth and of the weight the planner puts on different categories of consumers.

The intuition for this very strong result comes from three main principles of public finance, namely taxing necessities more than luxuries, uniform commodity taxation, and no intermediate goods taxation. Auerbach (1979) conjectured that zero optimal capital income tax arises from the infinite long run elasticity of savings in a representative agent models with separable utility. Judd (1985) claimed that this long run elasticity property is not relevant since the same long run zero tax holds even if the long run saving elasticity is finite and differs across individuals. Judd (1999) explains the zero long run capital income tax results by looking at commodity taxation literature. He reviews two basic optimal commodity taxation ideas - the inverse elasticity rule and the non-taxation of intermediate inputs - and shows how these ideas can be used to understand the optimal factor taxation results. Namely, since the Arrow-Debreu model in the dynamic context is closely related to optimal commodity tax theory (Atkinson and Stiglitz), the resulting optimal uniform tax policy creates a uniform wedge between the untaxed good and every other good. The second key argument is the one Diamond and Mirrlees (1971) gave against taxation of intermediate inputs . This is relevant here since capital goods, physical and human, are intermediate goods. In fact, income taxation is equivalent to sales taxation of intermediate goods. Since intermediate good taxation will generally put an economy on the interior of its production possibilities frontier, capital income taxation is likely to produce similar factor distortions, particularly if there are many capital goods.

The Chamley-Judd result generalizes for many other environments. Neither human capital taxation, no consumption taxes are optimal in the long run, as shown by Lucas (1990), Jones, Manuelli and Rossi (1993), Corsetti and Roubini (1996), and Coleman II (2000). If we turn to environments with uncertainty, Zhu's (1992) contribution is that, for general utility functions in the context of a stochastic growth model, the long run capital income taxes may or may not be zero. However, the numerical results of Chari, Christiano and Kehoe (1994) demonstrate that the ex ante expected capital income taxes are statistically very close to zero for the stochastic setup.[3]

The Chamley-Judd result may be broken if we impose extra constraints on the Ramsey problem implied by restrictions on the tax system. The known examples that lead to non-zero

[3]For reasonable specifications of consumers' risk aversion. For very risk-averse consumers, it is optimal to have a very volatile long run capital income taxes, positive on average.

limiting tax rates (in the non-stochastic steady state) are: Aiyagari (1995)with incomplete markets due to uninsurable idiosyncratic risks. The model generates precautionary savings that lead to overaccumulation of capital which can be offset by a positive tax. Judd (1997) shows that when monopolistic competition results in underproduction of goods, a negative tax (meaning a capital income subsidy) is optimal. Jones, Manuelli and Rossi (1993, 1997) construct two examples with a positive steady state tax. In the first one, inelastic labor supply gives rise to profits which the planner cannot fully tax. In the second, the planner is forced to tax both skilled and unskilled labor at the same rate, so he faces an additional restriction that depends on the ratio of marginal productivities of skilled and unskilled labor. In the latter example, it is optimal to tax capital income at 7% in the long run.

For both models considered in the present thesis, I choose the specifications for production and preferences for which under complete markets, the capital income taxes are identically zero after the first two periods. In the second chapter, I show that imposing tight enough exogenous limits on debt and assets of the planner, gives rise to ex ante expected capital income taxes with a positive mean and high variability. In the third chapter, the capital income taxes are positive even in the non-stochastic steady state due to binding collateral constraints that lead to overaccumulation of capital. The main conclusion from these two exercises is the following: if markets are incomplete because of borrowing restrictions then abolishing the capital income taxation after a small number of periods is not an optimal policy anymore.

1.2 Labor Income Tax Smoothing

In a well-known paper, Barro (1979) analyzed a reduced-form model of optimal taxation. In his work, there is no uncertainty and the government chooses a sequence of tax rates on (labor) income to minimize tax collection costs. Tax distortions in his model constitute a single most important deviation from the reference point of the Ricardian equivalence and this second order deviation defines an optimal, smooth path of tax rates over time. This "tax rate smoothing" result has been generalized to the "tax smoothing" prescription, according to which the government should equalize the deadweight burdens of taxation over time and different states of nature.

It turns out that "tax smoothing" may yield very different predictions about the behavior of labor taxes. Barro's (1979) conjecture was that, by analogy with a permanent income model of consumption, labor tax rates should be a martingale, regardless of the stochastic process for government expenditures. The key characteristic is that the serial correlation properties of tax collections are independent of the serial correlation properties of government expenditures. Thus a random walk with small innovation appears to be smooth in the sense of "tax smoothing" that emerges from Barro's analysis. This outcome depends on the debt being risk-free. Barro's model has tax collections adjust permanently by a small amount in response to surprise shocks to government expenditures, and has the next period risk-free debt make the rest of adjustment to enforce the government budget constraint to hold period by period.

The adjustments are very different in Lucas and Stockey's model. Lucas and Stokey (1983) re-examined the optimal taxation problem in a model without capital accumulation where the government issues state-contingent debt. In their analysis, tax smoothing in the form emphasized by Barro does not emerge. Taxes are not a random walk but rather have serial correlation properties that mirror those of government expenditures. This latter characteristic of labor taxes reemerges in the model with capital and labor income taxes considered by Zhu (1992) and Chari, Christiano and Kehoe (1994).

Marcet, Sargent and Seppäla (2000) tried to recover a version of Barro's random walk for optimal labor taxes in the context of Lucas and Stokey (1983) economy but with risk-free debt only, as in Barro, and limits on debt. They stress the important role of borrowing restrictions and show for which type of constraints Barro's random walk result prevails. I follow their strategy of putting "time-invariant ad hoc debt limits" on the planner that I call exogenous limits on debt.

Chari and Kehoe (1998) conjectured that if asset markets were incomplete, then the analysis would depend on the precise details of incompleteness. Scott (1997) shows that introducing risk-free debt into the model of Chari, Christiano and Kehoe (1994) brings a unit root component to the labor income taxes. However, optimal labor tax rates still depend positively on employment as under complete markets.

Chapter 2 studies the model of Chari, Christiano and Kehoe (1994) with fully state-contingent debt but still incomplete assets markets and with both capital and labor income taxes. I find that labor income taxes possess a unit root component, positively depend on employment, and are functions of the expected future solvency of the Ramsey planner. For most of the specifications of exogenous limits on debt, a unit root like behavior dominates the effect of employment on labor taxes so their path is independent of the government expenditure process. Therefore, any unanticipated shock to the government budget has a permanent effect on labor taxes.

One of the important factors when choosing which model to use as an approximation of the real world is how they fit the patterns of the data. Comparing the predictions for persistence and volatility of labor taxes that come out of my model with incomplete markets to the data on G7 countries from Mendoza, Razin and Tesar (1994), I find that the three models with binding limits fit perfectly both the autocorrelation and volatility properties of the labor taxes in the data. The corresponding estimates that come from the model of Chari, Christiano and Kehoe with complete markets are much lower. Therefore, my second chapter contributes to the evidence about market incompleteness that other authors, in particular Marcet and Scott (2001), have found when looking at the behavior of other variables.

2 Optimal Capital-Labor Taxes under Uncertainty and Exogenous Limits on Debt

2.1 Introduction

The on-going discussion in the economic literature, described in Chapter 1, favors abolishing capital income taxation. Theoretical judgement is easily made comparing the short period of severe capital income taxation to the long run benefits of undistorted capital accumulation. If we look closely at the numerical characterization of Chari, Christiano and Kehoe (1994), their model calibrated to the U.S. data predicts just one period of such a high capital income taxation. The price of not taxing capital income ever after consists of a levy on all the capital income and about a half of the existing capital stock. Such a levy is announced for the period following the start of the economy. Using the proceeds from this confiscating capital tax, the government builds up a stock of assets generating sufficient interest income to finance future deficits. After the levy is implemented, it is optimal not to tax capital, set nearly constant labor income tax rates and use the state-contingent government debt to absorb all the fluctuations. Thus the complete markets model of Chari, Christiano and Kehoe predicts that the government should become a net creditor to the private sector[4]. The last time the federal government in the United States had this chance was as "recently" as 1835! Besides, announced capital levies proved unsustainable[5] unless they were effectively replaced by a moderate capital income taxation spread over many years, as shown in a historical study by Eichengreen (1990).

This chapter introduces incomplete markets to study the problem of optimal capital and labor income taxation in a stochastic growth model. I consider the optimal fiscal policy of a government acting as a benevolent Ramsey planner in presence of exogenous limits on each period debt issue. My basic model retains the Chari, Christiano and Kehoe environment while adding limited short sales requirement in the market for government bonds with state-contingent returns.

The motivation for limiting debt is two-fold: on the one hand, I want to see how the presence of market incompleteness of a very "soft" type[6] changes the answers to the three policy questions that started Chapter 1. On the other hand, a recent empirical study of Marcet and Scott (2001) provides evidence in favor of market incompleteness in the U.S. by investigating the behavior of fiscal deficit and government debt in the data. They conclude that risk-free debt assumption seems too strong, so I employ a less restrictive form of market incompleteness.

I solve the model numerically using short run Monte Carlo simulations inside the Parameterized Expectations Algorithm (PEA) by Marcet (1998). Both the expected capital income tax and the labor income tax are negatively correlated with output. Expected capital taxes have a positive mean in the range from 2 to 8% and a high standard deviation, sharing the shock absorber role with state-contingent debt. Capital taxes are quantitatively more important for the purpose of financing unexpected shocks than the labor income taxes. The latter fluctuate much more than under complete markets, are more persistent than the underlying shocks and

[4] Quantitatively, the model suggests that, every period, the government should lend to the consumers an amount close to the period's GDP.

[5] In the sense of creating political lobbying, causing capital flight etc.

[6] Keeping the full vector of state-contingent returns but restricting the amount of bonds that can be issued allows for an equivalent formulation in terms of portfolios of securities - see Bohn (1994).

affirm Barro's (1979) assertion that the optimal taxes should be smooth in the sense of being random walks. My results about stochastic properties of the two taxes are comparable to those of Judd (1989) who examined the nature of optimal taxation of labor and capital income in various stochastic models of quadratic loss minimization. He found that the random walk test for optimality, popularized by Barro, applies only to labor taxation, whereas optimal capital income tax rates are more volatile, sometimes being white noise.

My model is very successful in improving the testable predictions for both capital and labor income taxes. I compare the volatility and correlations properties of both taxes to G7 data on aggregate effective tax rates from Mendoza, Razin and Tesar (1994). My model predicts autocorrelation of labor income taxes to be in the range between 0.888 and 0.912 while the G7 average is 0.91; the standard deviation of labor taxes in the data is 0.010 while my specifications of debt limits suggest a number between 0.012 and 0.016! This result clearly favors the performance of my model relative to the one with complete markets: for the same parameters' specifications, autocorrelation of labor taxes never exceeds 0.71 and the volatility is too low (between 0.001 and 0.002). As for the capital income taxes, my model predicts a positive mean and a relatively high persistence. Yet I don't get as high mean and autocorrelation as in the data, and the volatility of the model tax rates is too high. However, this is still an improvement relative to the complete markets case since, for the baseline specification of Chari, Christiano and Kehoe, the expected capital income tax rates are identically zero, so that neither volatility nor autocorrelation properties can be discussed.

One of the key results of this chapter is that setting limits on current borrowing and saving of the planner is enough for the expected capital income taxes to be different from zero even for the class of utility functions for which the Chamley-Judd result holds under complete markets. Therefore, as an answer to the first fiscal policy question from Chapter 1, the optimal policy implies using both capital and labor tax instruments. The intuition for non-zero expected capital income taxes is the following. The government issues securities contingent on the outcome of shocks to the government budget constraint. Thus, the shocks are absorbed by these securities and not by labor income tax rates. When binding debt limits prevent consumers from buying the required insurance in bonds, stochastic ex post capital income tax rates essentially create a state-contingent return for owners of capital. The substitution between the two contingent smoothing devices is propagated by a simple policy rule followed by the planner: binding upper (lower) limit leads to expected next period capital income subsidy (tax). That is, if the government today is not allowed to sell as much of the state-contingent bond as the consumers want, the capital income subsidy is announced to foster capital investment and thus increase the state-contingent payoff on capital. The reverse happens in the case of the lower limit binding.

My model gives a positive answer to the second policy question: labor income taxes should be cut in recessions to boost employment. This Keynesian kind of result comes from a general equilibrium setup based on microfoundations.

As for the third policy question, the answer to it is closely related to the discussion of Barro's random walk assertion. Random walk of labor taxes originally comes from risk-free debt models (see Marcet et al., 2000). Getting the same outcome with a different specification of market incompleteness gives this result an additional strength.

Last but not the least, debt limits prove to be powerful enough to significantly reduce or completely eliminate such undesirable transitional features of the complete markets model as the announced capital levy and a labor income subsidy criticized in the respective papers of

Eichengreen (1990) and Coleman II (2000). Borrowing restrictions help explain why we do not observe benevolent governments using capital levies or paying subsidies to the workers.

This chapter has the following structure: Section 2 describes the model with debt limits, Section 3 applies the primal approach and describes Ramsey optimal allocations and policies, Section 4 discusses the theoretical implications of market incompleteness for the optimal fiscal policy of the planner. Choices of debt limits and parameters of the model are described in Section 5, Section 6 presents findings from solving the model numerically, and Section 7 concludes the chapter.

2.2 The Economy

The Ramsey planner maximizes the representative consumer's utility over the set of competitive equilibria in the economy. This set is determined by the first-order and transversality conditions from the consumer's problem, the first-order conditions from profits maximization of competitive firms, the government budget constraints, and the market clearing conditions. In addition, I impose restrictions on each period new debt issue of the planner, that I refer to as limits on debt. The upper limit on the amount of new debt constrains the ability of the government to borrow from the public. It reflects legal requirements in most developed countries, as more debt today means passing the tax burden on to future generations. The lower limit (on government assets) is motivated by the planner's concern about consumers accumulating too much debt. Recall the discussion of Section 1 about government being a net creditor to the private sector in the complete markets model.

The economy is decentralized with three perfectly competitive markets: the labor market, the capital market and the market for government bonds with one period maturity and state-contingent returns. Both capital and bond markets open after the technology and government spending shocks are realized. I use the convention that variables dated t are measurable with respect to the history of shocks up to t.

A **competitive equilibrium** for this economy consists of a policy $\pi = (\tau_t, \theta_t)_{t=0}^{\infty}$, an allocation $x = (k_t, l_t, c_t, b_t)_{t=0}^{\infty}$, and a price system $(w_t, r_t, R_{b,t})_{t=0}^{\infty}$ that satisfy

1. the first-order conditions of the representative consumer's problem determining the household's consumption-leisure and consumption-investment choices for capital and government bonds[7]:

$$
\begin{aligned}
1 - \tau_t &= -\frac{u_{l,t}}{u_{c,t} w_t} && (1)\\
u_{c,t} &= \beta E_t u_{c,t+1}(1 + (1 - \theta_{t+1})(r_{t+1} - \delta)) && (2)\\
u_{c,t} &= \beta E_t u_{c,t+1} R_{b,t+1} && (3)
\end{aligned}
$$

2. the budget constraint of the consumer

$$
c_t + k_t + b_t = (1 - \tau_t) w_t l_t + (1 + (1 - \theta_t)(r_t - \delta)) k_{t-1} + R_{b,t} b_{t-1} \quad (4)
$$

3. the factor prices equal to the corresponding marginal productivities

$$
\begin{aligned}
r_t &= F_{k,t}(k_{t-1}, l_t, z_t) && (5)\\
w_t &= F_{l,t}(k_{t-1}, l_t, z_t) && (6)
\end{aligned}
$$

4. the government budget constraint and the limits for the amount of bonds that the planner can issue each period

$$
g_t + R_{b,t} b_{t-1}^{gov} = \tau_t w_t l_t + \theta_t r_t k_{t-1} + b_t^{gov} \quad (7)
$$

[7]A representative consumer solves for the allocations taking as given the government policy (taxes and vectors of state-contingent returns on debt) and the factor prices:
$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$ subject to his budget constraint.

$$\underline{M} \leq b_t^{gov} \leq \overline{M} \tag{8}$$

Following Marcet, Sargent and Seppäla (2000), I assume that the consumer also faces debt limits but less stringent than the planner: $\underline{M}^{cons} \leq \underline{M}$ and $\overline{M} \leq \overline{M}^{cons}$. Therefore, in equilibrium the consumer's problem always has an interior solution.

5. the market clearing conditions for the goods market are satisfied because the consumer's budget constraint and the planner's budget constraint together imply the resource constraint

$$c_t + k_t + g_t = F(k_{t-1,} l_t, z_t) + (1 - \delta)k_{t-1} \tag{9}$$

and the bond market clears by the Walras' law:

$$b_t^{gov} = b_t \tag{10}$$

6. the transversality conditions

$$\lim_{t \to \infty} \beta^t E_t u_{c,t+1} R_{k,t+1} k_t = \lim_{t \to \infty} \beta^t E_t u_{c,t+1} R_{b,t+1} b_t = 0 \tag{11}$$

where $R_{k,t+1} = 1 + (1 - \theta_{t+1})(r_{t+1} - \delta)$ is the gross after-tax rate of return on capital.

2.3 Ramsey Allocations and Policies

The Ramsey planner plays a two-stage Stackelberg game with the public: in period zero, the government announces the policy π and lets the consumers and the firms choose their allocations $x(\pi)$ and factor prices $w(\pi)$ and $r(\pi)$ as the best response to π. In equilibrium, the planner must satisfy his budget constraint and the limits on debt taking as given the reaction function of the agents, i.e. the allocation rule, and the pricing rules. These requirements impose the restrictions on the set of allocations that the government can achieve by varying its policies.

A **Ramsey equilibrium** for this economy is a policy π, an allocation rule $x(\pi)$, and price rules $w(\pi)$ and $r(\pi)$ such that the policy π maximizes the consumer's utility over the set of competitive equilibria in the economy.

I use a standard strategy of recasting the Ramsey problem in terms of a constrained choice of allocations substituting τ_t, θ_{t+1}, $R_{b,t}$, r_t, w_t from the conditions (1),(2),(4)-(6).

Proposition 1 *Under exogenous limits on debt, the competitive equilibrium allocations are characterized by the same resource constraint and period zero implementability constraint as in Chari, Christiano and Kehoe (Proposition 1, page 622) plus a sequence of period-by-period Euler-type of constraints of the form*

$$\underline{M} \leq E_t \sum_{j=1}^{\infty} \beta^{t+j} \frac{u_{c,t+j}c_{t+j} + u_{l,t+j}l_{t+j}}{u_{c,t}} - k_t \leq \overline{M}, \text{ for all } t \geq 0 \tag{12}$$

The proof is given in the Appendix 2.

2.3.1 Ramsey Allocations Problem

The benevolent government maximizes the representative consumer's utility with respect to $(c_t, l_t, k_t, b_t)_{t=0}^{\infty}$

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t, l_t)\} \tag{13}$$

- subject to the economy's resource constraint

  $$c_t + k_t + g_t = F(k_{t-1}, l_t, z_t) + (1-\delta)k_{t-1} \tag{14}$$

  to which I attach the Lagrange multiplier $\beta^t \eta_t$,

- the implementability constraint (the consumer's budget constraint) at $t = 0$[8]

  $$c_0 + k_0 + b_0 = -\frac{u_{l,0}l_0}{u_{c,0}} + ((1-\overline{\theta}_0)(F_{k,0} - \delta) + 1)k_{-1} + \overline{R_{b,0}b_{-1}} \tag{15}$$

  with the Lagrange multiplier λ_0 which is often called in the literature the cost of distortionary taxation,

[8]The presence of period zero implementability constraint comes from the fact that we use the period t budget constraint of the consumer to express $R_{b,t}b_{t-1}$ in terms of allocations. But $R_{b,0}b_{-1}$ is given, so the period zero budget constraint remains an additional restriction on the set of CE allocations.

- the sequence of Euler-type constraints, summarizing the consumer's budget constraints and the Euler equations from the consumer's problem, of the form

$$u_{c,t}(b_t + k_t) = \beta E_t(u_{c,t+1}c_{t+1} + u_{l,t+1}l_{t+1}) + \beta E_t u_{c,t+1}(b_{t+1} + k_{t+1}) \tag{16}$$

  to which I attach $\beta^t \psi_t$,

- and the limits on the planner's borrowing and saving

$$\underline{M} \leq b_t \leq \overline{M} \tag{17}$$

  with $\beta^t \nu_{1,t}$ and $\beta^t \nu_{2,t}$, respectively,

- for given $\overline{R_{b,0}b_{-1}}$, $\overline{\theta}_0$, and k_{-1}.

This Ramsey allocations problem is not recursive since future control variables appear in the Euler-type of constraints facing the planner each period. Thus, the optimal choice at period t is not an invariant function of the natural state variables.

2.3.2 Recursive Formulation

Following the recursive contracts approach of Marcet and Marimon (1998), this problem can be made recursive by enlarging the state space: ψ_{t-1} becomes another state variable, I refer to it as to the costate Lagrange multiplier.

The Lagrangian of the new saddle point minimax problem can be rewritten as

$$\begin{aligned} W_0 = \quad & \min\max E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t, l_t) + (\psi_{t-1} - \psi_t)u_{c,t}(b_t + k_t) + \psi_{t-1}(u_{c,t}c_t + u_{l,t}l_t) + \\ & (\nu_{1,t} - \nu_{2,t})u_{c,t}b_t + (\nu_{2,t}\overline{M} - \nu_{1,t}\underline{M})u_{c,t} + \\ & \eta_t(F(k_{t-1}, l_t, z_t) + (1-\delta)k_{t-1} - c_t - k_t - g_t)\} + \\ & \lambda_0[(u_{c,0}c_0 + u_{l,0}l_0 + u_{c,0}(b_0 + k_0) - u_{c,0}(\varkappa_{initial} + (1 - \overline{\theta}_0)F_{k,0}k_{-1})] \end{aligned} \tag{18}$$

with the maximization variables being $(c_t, l_t, k_t, b_t, \eta_t, \nu_{1,t}, \nu_{2,t})_{t=0}^{\infty}, \lambda_0$ and minimizing with respect to $(\psi_t)_{t=0}^{\infty}$.

The Kuhn-Tucker multipliers $\nu_{1,t}, \nu_{2,t}$ are non-negative for any t.

By notation,

$$\psi_{-1} = 0 \tag{19}$$

$$\varkappa_{initial} = \overline{R_{b,0}b_{-1}} + (1 - \delta(1 - \overline{\theta}_0))k_{-1} \tag{20}$$

where $\overline{R_{b,0}b_{-1}}$, $\overline{\theta}_0$, and k_{-1} are given.

See Appendix 1 for the analysis of the first-order conditions.

2.4 Theoretical Predictions for Ramsey Taxes

I link the condition of one of the debt limits binding today or tomorrow to non-zero expected capital income tax proceeds. Following Zhu (1992), I derive analytical expressions for the optimal tax rates on capital and labor income. I compare the predictions for serial correlation and volatility of labor income taxes to those of Aiyagari, Marcet, Sargent and Seppäla (2001). Both specifications of incomplete markets (debt limits and risk-free debt) add more volatility to labor taxes and there is a unit root component. However, we need simulations to verify whether optimal labor taxes are smooth in the sense of following a random walk.

Issues of Indeterminacy of Capital Income Tax Rates Tax distortions across states of nature can be smoothed by state-contingent taxes on capital as well as state-contingent returns on debt. These contingent devices prove to be quantitatively important. Several authors (see Bohn, 1994) pointed out that in the complete markets stochastic environment, the capital income tax rates and the state-contingent returns on government bonds cannot be uniquely determined by the first-order conditions of the planner. This happens because the planner has access to "too many" state contingent instruments. With the full set of state-contingent returns on debt in hand and allowing the capital income taxes to vary with the states of the economy, the planner can insure against all relevant shocks to the budget in many different ways. For the purpose of financing government spending, a low return on bond in some state S_t can serve as a substitute for a high tax rate on capital in this state. Notice that both instruments are distortionary: higher capital taxes reduce capital accumulation, while higher debt will generate higher interest payments in the future that lead to higher tax rates. Individual investment decisions are based on weighted averages of state-contingent returns. The capital tax rates in the different states can be altered without affecting investment decisions as long as the relevant expectations are left unchanged.

In the model with debt limits, the degree of indeterminacy can be significantly reduced if I specify exogenous limits on debt tight enough to be often binding. Formally, however, just one restriction for a large number of states of the economy is not enough to solve the problem of indeterminacy. I use the *ex ante (expected) capital income tax rate* defined as

$$\theta_t^e = \frac{E_t \theta_{t+1} u_{c,t+1}(F_{k,t+1} - \delta)}{E_t u_{c,t+1}(F_{k,t} - \delta)} \tag{21}$$

The expected capital income tax rate θ_t^e can be interpreted as the ratio of present market value of tax revenue from capital income over the present market value of capital income. Thus, this is a kind of certainty equivalent capital income tax rate.

2.4.1 Taxing Capital Income?

The ex ante expected capital income tax rate defined above pins down the average tax rate on the next period income from capital that the rational representative consumer should expect given the current state of the economy. This rate is uniquely determined by the first-order conditions of the Ramsey planner. The ex post capital income taxes remain indeterminate and may well differ from the ex ante rate. In what follows I solve for the ex ante rate. Substituting the definition (21) into the Euler equation for capital-consumption choice of the consumer (2) gives the following expression for the ex ante capital tax rate

$$\theta_t^e = \frac{\beta E_t u_{c,t+1}(F_{k,t+1} + 1 - \delta) - u_{c,t}}{\beta E_t u_{c,t+1}(F_{k,t} - \delta)} \tag{22}$$

The difference in the numerator of (22) reminds the Euler equation of the planner without distortionary taxation:

$$u_{c,t} = \beta E_t u_{c,t+1}(F_{k,t+1} + 1 - \delta) \tag{23}$$

Remark 1 *The ex ante expected capital income tax rate is zero if and only if and only if* (23) *holds for the planner.*

In my model, the Euler equation of the consumption-capital choice of the planner is of the form:

$$(\psi_t - \psi_{t-1})u_{c,t} + \eta_t = \beta E_t \eta_{t+1}(F_{k,t+1} + 1 - \delta) \tag{24}$$

where the *shadow* price of an additional unit of resources η_t can be expressed as

$$\eta_t = u_{c,t}(1 + \psi_{t-1}(1 - \sigma_c)) + u_{c,t}(\psi_t - \psi_{t-1})\sigma_c \frac{k_t + b_t}{c_t} \tag{25}$$

where I used $\nu_{2,t}(k_t + \overline{M}) - \nu_{1,t}(k_t + \underline{M}) = (\psi_{t-1} - \psi_t)(k_t + b_t)$

Plugging (25) into (24) and rearranging terms leads to a following proposition:

Proposition 2 *For any $t > 0$, $\psi_t \neq \psi_{t-1}$ is enough to give rise to $\theta_t^e \neq 0$.*

(Proof in the Appendix 1)

Corrolary 1 *If for periods t and $t+1$*[9] *both limits on debt are slack, then $\theta_t^e = 0$.*

Looking at the sign of the difference $\psi_t - \psi_{t-1}$ in case of each of the debt limits binding, I get the following result:

Proposition 3 *Lower (upper) limit on debt binding today induces positive (negative) average capital tax collections tomorrow.*

(Proof in the Appendix 2)

The above result has the following intuition: when the upper limit on debt is binding today, this means that the consumers would like to buy more state-contingent bonds than the planner is allowed to issue in the current period. Thus it is optimal for the planner to announce a capital income subsidy for the next period[10] Such a policy instrument would stimulate an additional capital accumulation and enforce the substitution between the state-contingent returns on government debt and the ex post state-contingent returns on capital. The reverse happens in case of the lower limit binding. My numerical results presented in Section 6 show that the upper limit is more often binding when the overall state of the economy is relatively good while the lower limit is hit in relatively bad states. The conclusion from both theoretical and numerical parts of this exercise is that a benevolent policy maker subject to binding debt limits should tax capital more when bad times are coming and compensate by a subsidy when the economy booms.

[9]The formal condition for $t + 1$ is that b_{t+1} falls within the limits with probability equal to one.

[10]On average across the states of the economy.

2.4.2 Random Walk of Labor Income Taxes

For the baseline model of Chari, Christiano and Kehoe, optimal labor income tax rates are given by

$$\tau_t = \frac{\lambda_0}{1 - l_t + \lambda_0} \tag{26}$$

The tax rate is a positive function of employment and thus fully inherits its serial correlation and volatility properties. As proved in Zhu (Proposition 4, p.264), smooth leisure leads to almost constant optimal labor taxes and zero expected capital taxes.

With limits on debt, the consumer's consumption-leisure choice (1) implies

$$(1 - \tau_t)\frac{1-\gamma}{c_t}F_{l,t} = \frac{\gamma}{1 - l_t} \tag{27}$$

while the planner's choice leads to

$$\eta_t F_{l,t} = \frac{\gamma(1 - l_t + \psi_{t-1})}{(1 - l_t)^2} \tag{28}$$

Combining the two gives the following expression for the tax rate on labor income

$$\tau_t = \frac{\psi_{t-1}}{1 - l_t + \psi_{t-1}} + (\psi_t - \psi_{t-1})\mu_t \frac{(1 - l_t)}{1 - l_t + \psi_{t-1}} \tag{29}$$

where I defined $\mu_t = \frac{1}{1-\gamma}\frac{k_t + b_t}{c_t}$ which has a representation in terms of the right-hand side of the Euler-type constraint

$$\mu_t = \frac{1}{1-\gamma}E_t \sum_{j=1}^{\infty} \beta^{t+j}(u_{c,t+j}c_{t+j} + u_{l,t+j}l_{t+j}) \tag{30}$$

The parameter μ_t is equal to the period t present discounted value of all future (primary) government surpluses, $\omega^{gov}_{t+j} = \tau_{t+j}w_{t+j}l_{t+j} + \theta^e_{t+j-1}(r_{t+j} - \delta)k_{t+j-1} - g_{t+j}$, measured in units of current marginal utility[11].

The labor income taxes fully inherit fluctuations in employment if and only if the costate variable ψ_t converges, i.e. if after some period, the state variables converge to a stationary distribution for which none of the debt limits is binding and we are back to a complete markets case.

Recall the discussion from Chapter 1 whether optimal labor taxes should follow a random walk. I take a first-order approximation of τ_{t+1} around ψ_t, l_t and μ_t to get

$$\tau_{t+1} \simeq \tau_t + \frac{\psi_{t-1}(1 - \mu_t \triangle\psi_t)}{(1 - l_t + \psi_{t-1})^2}\triangle l_{t+1} + \frac{1 - l_t}{(1 - l_t + \psi_{t-1})}(\mu_t \triangle\psi_{t+1} + \triangle\psi_t \triangle\mu_{t+1}) \tag{31}$$

[11]It is trivial to show that $u_{c,t}b_t = E_t \sum_{t=0}^{\infty} \beta^j u_{c,t+j}\omega^{gov}_{t+j}$. Combining with the Euler-type constraint, obtains $\mu_t = \frac{1}{1-\gamma}\{\ E_t \beta^j u_{c,t+j}\omega^{gov}_{t+j} + u_{c,t}k_t\}$

The first component makes labor taxes more volatile than employment[12], while the second one is a unit root component which provides additional persistence and volatility. Notice that both terms depend upon μ_t, the solvency of the planner as expected at t. As in case of Scott (1997), there is a tradeoff between the two effects. Section 6 shows that for most of the debt limits specifications, I get a random walk like behavior of the labor income taxes.

[12]Recall that $\psi_t - \psi_{t-1}$ is positive (negative) if the lower (upper) limit is binding and zero otherwise.

2.5 Numerical Aspects

I choose the functional forms and the parameters of the model as close as possible to those of Chari, Christiano and Kehoe (1994). The utility function is separable in consumption and leisure so that the Chamley's result of zero long run expected capital taxes holds for the complete markets. To find a reasonable specification for the exogenous limits on debt, I first solve three models with complete markets and then set the limits as a percent of the average long run GDP of those models.

2.5.1 Functional Forms

The production function is Cobb-Douglas with labor-augmented technological progress

$$F(k_{t-1,}l_t, z_t) = k_{t-1}^{\alpha}(l_t z_t)^{1-\alpha} \tag{32}$$

The instantaneous utility of the consumer is of the form

$$u(c_t, l_t) = (1-\gamma)\frac{c_t^{1-\sigma_c}}{1-\sigma_c} + \gamma\frac{(1-l_t)^{1-\sigma_l}}{1-\sigma_l} \tag{33}$$

I consider the baseline model with $\sigma_c = \sigma_l = 1$ and thus logarithmic preferences.

$$u(c_t, l_t) = (1-\gamma)\ln c_t + \gamma \ln(1-l_t) \tag{34}$$

2.5.2 Processes for Shocks

I assume that both shocks are lognormal and follow an AR(1) process:

$$\ln z_t = \begin{cases} \overline{z} \text{ if } \ln z_t > \overline{z} \\ \underline{z} \text{ if } \ln z_t < \underline{z} \\ \rho_z \ln z_{t-1} + \varepsilon_{z,t} \text{ otherwise} \end{cases} \tag{35}$$

$$\ln \widetilde{g}_t = \begin{cases} \overline{g} \text{ if } \ln g_t > \overline{g} \\ \underline{g} \text{ if } \ln g_t < \underline{g} \\ \rho_g \ln \widetilde{g}_{t-1} + \varepsilon_{g,t} \text{ otherwise} \end{cases} \tag{36}$$

where $\overline{z} = 2\frac{\sigma_z}{\sqrt{1-\rho_z^2}} = -\underline{z}$ and $\overline{g} = 2\frac{\sigma_g}{\sqrt{1-\rho_g^2}} = -\underline{g}$.

Government expenditures follow

$$g_t = G\exp(\ln \widetilde{g}_t) \tag{37}$$

2.5.3 Parameters of the Model

Table 1. Baseline Model Parameter Values

Preferences	$\beta^* = .98$	$\gamma = .75$
Technology	$\alpha = .34$	$\delta^* = .08$
Stochastic process for z_t	$\rho_z = .81$	$\sigma_z = .04$
Stochastic process for g_t ($G = .07$)	$\rho_g = .89$	$\sigma_z = .07$
Initial values	$k_{-1} = 1.0$	$\overline{\theta}_0 = .27$

Source: CCK (1994, p. 632)

Note that the initial model of CCK assumes a balanced growth path of the economy at the rate $\rho = .016$. Therefore, I have to adjust for growth β^* and δ^*[13].

2.5.4 Debt Limits Specifications and the Initial Indebtedness of the Planner

To impose exogenous debt limits, I first solve the model with complete markets for each of the three values of the planner's initial indebtedness($\overline{R_{b,0}b_{-1}}$). The latter positively affects the cost of distortionary taxation λ_0, which is one of the key determinants of the Ramsey allocations and the tax policies. Higher initial indebtedness raises λ_0 increasing debt in period zero and lowering the long run savings of the planner:

Each model with incomplete markets is characterized by a triplet: the two limits on debt and the initial indebtedness of the planner.

Model 1 has an initial indebtedness as in Chari, Christiano and Kehoe (1994) and is amended by loose limits on debt which are of the order of 50% of the average long run output (over 70% of the long run assets of the planner) under complete markets.

Model 2 has half of the initial indebtedness of Model 1 and a pretty tight limit on the consumers' debt (about 2.5% of GDP). The upper limit (on debt of the planner) is set to be equal to $\overline{R_{b,0}b_{-1}}$ of Model 2.

Model 3 has zero initial indebtedness.

Model 3A is characterized by moderate limits on debt (still 50% of the long run output and slightly over 50% of the planner's long run savings under complete markets).

Model 3B has very tight limits on both debt and savings of the planner approximating the case of a balanced budget. The planner is allowed to borrow or save less than 3% of his desired level of assets under complete markets.

[13] The resulting $\beta^* = \beta$ for logarithmic preferences, $\delta = 1 - \frac{1-\delta^*}{e^\rho} = .095$ (see details of the adjustment procedure in Garcia-Milá et al. (1995)).

2.6 Findings

Table 2 below illustrates four of my main findings. First, all the models but Model 1 generate ex ante capital tax rates with a positive mean and a high standard deviation. Second, both ex ante capital income taxes and labor income taxes are negatively correlated with the current period output. Third, labor income taxes fluctuate much more than under complete markets and are more persistent than the underlying shocks. This suggests that the random walk component of (31) dominates the other two effects. The main characteristic of labor taxes under complete markets is their extremely low volatility precluding any serial correlation from having any predictive power. Forth, ex ante capital taxes are negatively correlated to the technological shock and positively to the government spending shock. This result will be further emphasized by conditional distribution histograms for the capital taxes.

Table 2. Cyclical Properties of Taxes for Models 1, 2, 3A, 3B

Table 2.1.

$R_{b,0}b_{-1} = 0.2$	%	θ^e	τ	%	$cor(\theta^e, \cdot)$	$cor(\tau, \cdot)$
CM	*mean*	0	24.1	$cor(\cdot, y)$	NA	48.4
	std	0	0.2	$cor(\cdot, z)$	NA	47.7
	autocor	NA	62.7	$cor(\cdot, g)$	NA	55.0
Model 1	*mean*	0	26.5	$cor(\cdot, y)$	NA	49.6
	std	0	0.2	$cor(\cdot, z)$	NA	49.6
	autocor	NA	69.1	$cor(\cdot, g)$	NA	55.7

Table 2.2.

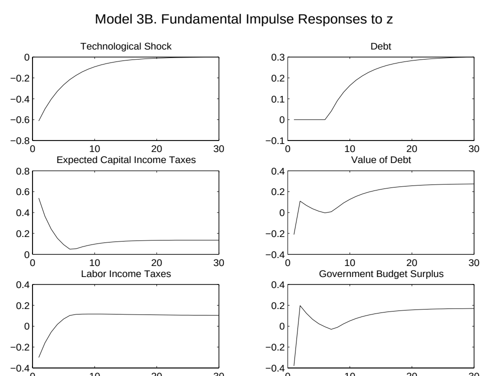
$R_{b,0}b_{-1} = 0.1$	%	θ^e	τ	%	$cor(\theta^e, \cdot)$	$cor(\tau, \cdot)$
CM	*mean*	0	23.5	$cor(\cdot, y)$	NA	48.8
	std	0	0.1	$cor(\cdot, z)$	NA	48.5
	autocor	NA	70.7	$cor(\cdot, g)$	NA	54.7
Model 2	*mean*	8.2	26.4	$cor(\cdot, y)$	-11.4	-21.6
	std	30.7	1.4	$cor(\cdot, z)$	-25.2	7.4
	autocor	48.9	91.2	$cor(\cdot, g)$	29.1	2.0

Table 2.3.

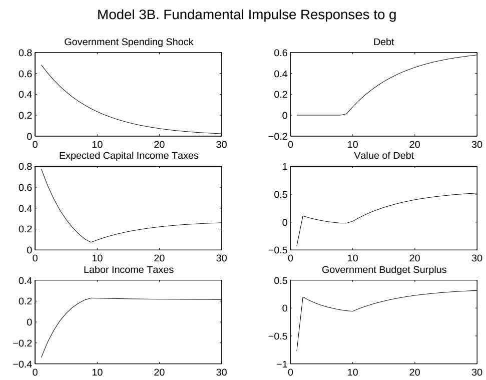
$R_{b,0}b_{-1} = 0.0$	%	θ^e	τ	%	$cor(\theta^e, \cdot)$	$cor(\tau, \cdot)$
CM	*mean*	0	23.2	$cor(\cdot, y)$	NA	49.0
	std	0	0.1	$cor(\cdot, z)$	NA	48.7
	autocor	NA	70.6	$cor(\cdot, g)$	NA	54.7
Model 3A	*mean*	7.5	24.5	$cor(\cdot, y)$	-7.9	-16.5
	std	29.8	1.2	$cor(\cdot, z)$	-19.8	6.7
	autocor	43.5	90.4	$cor(\cdot, g)$	26.9	6.6
Model 3B	*mean*	2.7	26.3	$cor(\cdot, y)$	-31.7	-12.3
	std	33.6	1.6	$cor(\cdot, z)$	-45.0	13.6
	autocor	61.3	88.8	$cor(\cdot, g)$	50.0	-3.8

To get a flavor of the persistence and volatility properties of both taxes I look at the data from Mendoza, Razin and Tesar (1994). The aggregate effective tax rates (averaged over G7 countries) are 31% for the labor income and 36% for the capital income with the standard deviations of 0.01 and 0.027 respectively. The autocorrelation of labor tax rates is 0.91 while that of capital 0.81. The corresponding rows of Tables 2.2 and 2.3 on labor tax rates speak for themselves. My model gives a significantly closer match of the stochastic properties of the data for both taxes.

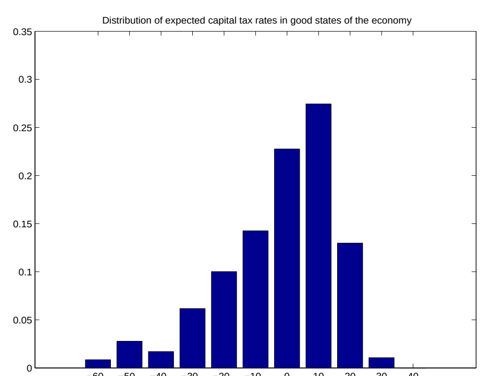
I look at the fundamental impulse responses (measured in units of standard deviations of the variables) of taxes in Model 3B with tight upper and lower limits (see Figures 1 and 2 below). Under debt limits, labor taxes follow a kind of random walk behavior being smooth in Barro's sense. Here the state variables are such that an unexpected innovation to any of the shocks leads to the binding lower limit on debt. The graphs show that both a negative innovation to the technology or a higher government spending are followed by an immediate increase in ex ante capital taxes and a fall in labor tax rates. The response of labor taxes can be interpreted as countercyclical: cut taxes in recessions to boost employment. At the same time capital taxes exhibit procyclical behavior[14]. This finding is similar to a fine tuning exercise of Kim and Kim (2002) who look at the exogenous AR(1) processes for the capital and labor income taxes and consumption taxes in an open economy with incomplete asset markets.



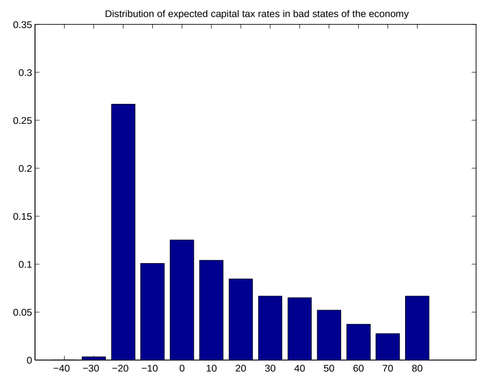
[14]Strictly speaking this result has no Keynesian flavor: a Ramsey government sets tax policy to smooth fluctuations in consumption and leisure directly. Therefore, cushioning the business cycle is not part of its objective.



Histograms of conditional distribution of expected capital taxes in good and bad states of the economy are given in Figures 3 and 4 below. A good state is defined as a state in which the existing capital stock and the technological shock are higher than their long run means and the government spending is relatively low.



The two histograms suggest that the expected capital tax rates should be lower in good and higher in bad times (wars). This is related to the theoretical result of the Proposition 3: the planner should set a positive capital tax when the limit on savings is binding and pay a capital income subsidy when the consumers run short of savings in government bonds.



2.6.1 Lessons from Different Models

Model 1. Model 1 is the closest specification to the complete markets. The upper and lower limits on debt are such that only the upper limit turned out to bind and only at $t = 0$. This model emphasizes the importance of my solution method: without using the short run simulations, the matrix of the states is not invertible.

I find level effects in this model relative to the complete markets case. Under complete markets, higher initial indebtedness of the planner is reflected in the higher cost of distortionary taxation which leads to higher taxes at $t = 0$ (a higher announced expected capital levy and a lower initial labor income subsidy). The model with loose limits on debt converges to a stationary distribution equivalent to a complete markets case with a much higher initial indebtedness of the planner. Therefore, the resulting Ramsey optimal allocation is characterized by lower capital, output and consumption, and higher leisure. Thus there is a level effect on allocations of an even transitionally binding limit on the initial government debt and of never binding (in equilibrium) limit on savings. Model 1 can be used to gain an insight into the cost-benefit analysis of reducing the initial capital levy.

Models 2 and 3B These two models have the same structure of debt limits: the upper one is set equal to the corresponding initial indebtedness of the planner, the lower limit (on government savings) is very close to zero and the same for both models ($\underline{M} = -0.01$). As a result, the two models converge to the same Ramsey allocation except for the equilibrium level of debt which is slightly higher for the Model 2 with a softer upper limit.

As for the Ramsey optimal policies, a tighter and more often binding upper limit of Model 3B implies a lower mean and a higher standard deviation of the expected capital income taxes, as shown in Tables 2.2 and 2.3 above. The intuition is that a binding upper limit leads to expected capital subsidies (Proposition 3). The presence of borrowing and lending limits obliges the government to use capital income taxes as shock absorber.

The labor income taxes become more than ten times more volatile than under complete markets, are both negatively correlated with output and uncorrelated with the government spending shock. The fundamental impulse responses in Figures 1 and 2 suggest that the unit root component of (31) dominates so that the labor income tax rates follow a kind of random walk.

Models 3A and 3B Case of zero initial indebtedness allows for comparisons of the Model 3 to the theoretical conclusions of Zhu for the complete markets case. My simulations confirm Zhu's Proposition 5 (p. 267) that expected capital tax rates are both positive and negative, as under complete markets when the Chamley result of zero expected capital tax rates does not hold.

Model 3A is characterized by much looser upper and lower limits than Model 3B. Less frequently binding debt limits lead to a reduction in absolute value of cross-correlations for θ^e but the signs are preserved. Labor income taxes are smooth in Barro's sense for both specifications of limits.

2.6.2 Initial Period Behavior of Ramsey Taxes

The results given in the Table 3 below show that the capital levy announced at $t = 0$ for the period 1 is reduced or eliminated whenever the upper limit on debt is binding at $t = 0$ (case of all the models except 3A). The same applies to the initial labor income subsidy.

Table 3. Initial Period Behavior of Ramsey Taxes

Table 3.1.

	Model 1	**CM**$_{Model1}$
θ_0^e (%)	373.1	906.6
τ_0 (%)	30.0	-35.2

Table 3.2.

	Model 2	**CM**$_{Model2}$
θ_0^e (%)	205.6	874.9
τ_0 (%)	27.2	-31.0

Table 3.3.

	Models 3A, 3B		**CM**$_{Model3}$
θ_0^e (%)	547.5	65.0	846.6
τ_0 (%)	-22.9	20.7	-26.9

The intuition for a different initial period behavior under incomplete markets is the following. Under complete markets, the government accumulates all its assets through the capital levy announced at $t = 0$ and applied at $t = 1$. In period zero, the deficit is high because of the inherited indebtedness and the labor income subsidy paid to the consumers, it is financed by high initial borrowing.

In my model, a more reasonable period zero behavior comes from three factors. First, a binding upper limit prohibits excessive government borrowing in period zero and thus reduces the cost of future tax increases reflected in the costate ψ_0. Recall from the FOC that $\psi_0 = \lambda_0 - \nu_{2,0}$ replaces λ_0 of the complete markets. Second, restricting the level of the long run assets automatically reduces the amount of levy in period one. Third, future use of capital income tax instrument makes it unnecessary (and impossible) to accumulate the whole amount of budget surplus in period zero. Last but not the least, when markets are incomplete, the sequence of period-by-period Euler-type constraints requires reconsidering the present value of future government surpluses each period.

2.6.3 Debt and Deficit under Incomplete Markets

I also look at the behavior of debt and primary government budget deficit under the exogenous limits on debt and savings of the planner.

In the Table 4 below I compare the signs of impulse responses of my model to those of Marcet and Scott (2001) who described the behavior of debt in the US data and, among others, in a Ramsey model with capital accumulation but without capital income taxes and with risk-free debt only.

Table 4. Models' Predictions for Signs of Impulse Responses

	CM	**IM debt limits**	**IM risk-free debt**	**US data**
$z \uparrow$	$b \uparrow$ $deficit \downarrow$	$b \rightarrow, \downarrow$ $deficit \downarrow$	$b \uparrow$ $deficit \uparrow$	$b \downarrow$ $deficit \downarrow$
$g \uparrow$	$b \downarrow$ $deficit \uparrow$	$b \rightarrow, \uparrow$ $deficit \uparrow$	$b \uparrow$ $deficit \uparrow$	$b \uparrow$ $deficit \uparrow$

The model with debt limits does a better job in reproducing the countercyclical response of debt in the data and is also capable of giving a correct sign of the impulse response of deficit to a positive innovation to the technology.

Finally, as in Marcet and Scott, non-neutrality of debt implies its more volatile behavior. Debt remains negatively correlated with the government spending, though less than under complete markets.

A Frequently Asked Policy Question How to finance a sudden war (or other large financial need)? Unfortunately enough for the policy makers, there are almost as many answers to this crucial question of the public finance as there are different models. The first-best recommendation of taxing the existing, inelastically supplied capital stock is inapplicable (see the discussion about the history of capital levies between the two world wars in Eichengreen

(1990)). Barro's model suggests to raise all tax rates by a small amount so that when held constant at that level, expected value of war is financed. Under complete markets, a temporary small increase in labor income taxes is accompanied by a reduction in outstanding debt in the short run.

The impulse responses show what happens in my model if one day, after the economy has converged to a non-stochastic steady state[15], there is an increase in government spending. The government should recur to the capital income taxation next period, cutting current labor taxes. Before the unexpected positive innovation to the government spending happened, the planner was financing constant spending by constant labor taxes and the return on the (small) assets which were at the lower limit $\underline{M}$. After the shock, the return on assets goes up a lot, as the consumers demand more bonds to insure themselves against future shocks.

Initially high g_t boosts output for the period of fixed capital taxes. Past savings are enough to cut labor taxes, to avoid a big drop in both capital and hours. In the long run, there is the usual incomplete markets effect of a recession caused by a big cut in government expenditure leading to higher labor taxes, less hours and capital and thus less output and consumption, and less government assets. The transitional dynamics suggest that the expected capital tax shares the role of a shock absorber with the debt.

[15] All the variables in the non-stochastic steady state of the Ramsey model are functions of λ_0 evaluated at the Ramsey optimum of the stochastic model.

2.7 Conclusions

Asset market incompleteness may be a possible explanation of why we do observe capital income taxation but we don't see governments that use either capital levies or labor income subsidies. Moreover, stochastic properties of simulated capital and labor income taxes suggest that the U.S. government applies policy schemes close to the ones optimal under Ramsey setting with debt limits. Introducing exogenous limits (possibly binding in equilibrium) on debt and savings of the planner gives rise to non-zero and very volatile expected capital income taxes.

The model considered in this chapter is more appropriate for the presentation of policy implications in the short and medium run than for the very long run. When the economy converges to a very long run, it is most likely to converge to a kind of complete markets, as either the limits on debt stop binding after a while or their importance for the change in the policy function diminishes. I conjecture that there exists a support for the costate Lagrange multiplier ψ_t (depending on both the limits and λ_0) such that once ψ_t is inside, it converges to a constant value meaning that the economy converges to a complete markets solution for some induced level of the initial indebtedness.

I had difficulties with solving the model numerically when I tried to set the upper limit on debt tighter than the initial indebtedness of the government, as the cost of not being able to borrow in period zero gets prohibitively large. Solving this problem may require some special numerical tricks related to the slope of the policy function in period zero.

It is interesting to see what happens if we allow debt limits which distort the first-order conditions of the consumers. One of the most fascinating issues about the capital income taxation, if the government is to make use of it due to market incompleteness, is its redistributive effect. Chari and Kehoe (1998) proved that under complete markets, heterogeneity of consumers is not enough to give rise to non-zero capital taxes even for the case when the planner puts no weight on the capitalists. Hence once more limited short sales can be used to study the redistributive effect of the capital income taxation.

2.8 Appendices

2.8.1 Appendix 1. FOC

The structure of the model is such that the first-order conditions for the period zero are different from the rest of the periods because of the nature of the implementability constraint. For notation simplicity, I assume that the utility function is separable in consumption and leisure.

FOC for $t > 0$

$$\psi_t = \psi_{t-1} + \nu_{1,t} - \nu_{2,t} \quad \text{(FOC with respect to } b_t\text{)}$$

This equations gives us the law of motion of the costate Lagrange multiplier attached to the period t Euler-type constraint. Thus ψ_t can be interpreted as a shadow price of government savings necessary to ensure future solvency. It increases whenever the planner runs short of savings and falls if the upper limit on debt is binding.

$$(\nu_{1,t} - \nu_{2,t})u_{c,t} + \eta_t = \beta E_t \eta_{t+1}(F_{k,t+1} + 1 - \delta) \quad \text{(FOC with respect to } k_t\text{)}$$

The above equation is the consumption-capital investment choice of the planner. Under complete markets, it reduces to the common shape

$$u_{c,t} = \beta E_t u_{c,t+1}(F_{k,t+1} + 1 - \delta) \tag{38}$$

Differentiating with respect to c_t and l_t gives

$$\eta_t = u_{c,t} + \psi_{t-1}(u_{c,t} + u_{cc,t}c_t) + [\nu_{2,t}(k_t + \overline{M}) - \nu_{1,t}(k_t + \underline{M})]u_{cc,t} \quad \text{(FOC with respect to } c_t\text{)}$$

and

$$\eta_t F_{l,t} = -u_{l,t} - \psi_{t-1}(u_{l,t} + u_{ll,t}l_t) \quad \text{(FOC with respect to } l_t\text{)}$$

Again, those are the "good old" $\eta_t = u_{c,t}$ and $u_{c,t}F_{l,t} = -u_{l,t}$ from the first-best, adjusted for distortionary taxation and inequality constraints.

$$u_{c,t}(b_t + k_t) = \beta E_t(u_{c,t+1}c_{t+1} + u_{l,t+1}l_{t+1}) + \beta E_t u_{c,t+1}(b_{t+1} + k_{t+1}) \quad \text{(FOC with respect to } \psi_t\text{)}$$

$$c_t + k_t + g_t = F(k_{t-1}, l_t, z_t) + (1 - \delta)k_{t-1} \quad \text{(resource constraint)}$$

$$\nu_{1,t}(b_t - \underline{M}) = \nu_{2,t}(\overline{M} - b_t) = 0 \quad \text{(Kuhn-Tucker conditions)}$$

$$\underline{M} \le b_t \le \overline{M} \quad \text{(debt limits)}$$

FOC for $t = 0$

$$\psi_0 = \lambda_0 + \nu_{1,0} - \nu_{2,0} \qquad \text{(FOC with respect to } b_0\text{)}$$

$$(\nu_{1,0} - \nu_{2,0})u_{c,0} + \eta_0 = \beta E_0 \eta_1 (F_{k,1} + 1 - \delta) \qquad \text{(FOC with respect to } k_0\text{)}$$

The first-order conditions with respect to c_0 and l_0 are amended by additional terms which contain the initial conditions.

$$\eta_0 = u_{c,0} + \lambda_0(u_{c,0} + u_{cc,0}c_0) + [\nu_{2,0}(k_0 + \overline{M}) - \nu_{1,0}(k_0 + \underline{M})]u_{cc,0} - \lambda_0 u_{cc,0}(\varkappa_{initial} + (1 - \overline{\theta}_0)F_{k,0}k_{-1}) \qquad \text{(FOC with respect to } c_0\text{)}$$

$$\eta_0 F_{l,0} = -u_{l,0} - \lambda_0(u_{l,0} + u_{ll,0}l_0) + \lambda_0 u_{c,0}(1 - \overline{\theta}_0)F_{kl,0}k_{-1} \qquad \text{(FOC with respect to } l_0\text{)}$$

$$u_{c,0}(b_0 + k_0) = \beta E_0(u_{c,1}c_{1+}u_{l,1}l_1) + \beta E_0 u_{c,1}(b_1 + k_1) \qquad \text{(FOC with respect to } \psi_0\text{)}$$

$$c_0 + k_0 + g_0 = F(k_{-1}, l_0, z_0) + (1 - \delta)k_{-1} \qquad \text{(resource constraint)}$$

$$\nu_{1,0}(b_0 - \underline{M}) = \nu_{2,0}(\overline{M} - b_0) = 0 \qquad \text{(Kuhn-Tucker conditions)}$$

$$\underline{M} \leq b_0 \leq \overline{M} \qquad \text{(debt limits)}$$

$$u_{c,0}c_0 + u_{l,0}l_0 + u_{c,0}(b_0 + k_0) = u_{c,0}(\varkappa_{initial} + (1 - \overline{\theta}_0)F_{k,0}k_{-1}) \qquad \text{(implementability at } t = 0\text{)}$$

2.8.2 Appendix 2. Proofs of Propositions

Proof of Proposition 1 My model has an additional constraint on the competitive equilibrium allocations which has the form

$$\underline{M} \leq b_t \leq \overline{M} \tag{39}$$

I derive an expression for debt from the Euler equation for the returns on bonds (3) using the Euler equation for capital (2) and one-period ahead present discounted version of the consumer budget constraint:

$$u_{c,t}(b_t + k_t) = \beta E_t(u_{c,t+1}c_{t+1} + u_{l,t+1}l_{t+1}) + \beta E_t u_{c,t+1}(b_{t+1} + k_{t+1}) \tag{40}$$

Substituting forward $u_{c,t+1}(b_{t+1}+k_{t+1})$ and applying the Law of Iterated Expectations gives us [16]

$$u_{c,t}(b_t + k_t) = E_t \sum_{j=1}^{\infty} \beta^j (u_{c,t+j}c_{t+j} + u_{l,t+j}l_{t+j}) \tag{41}$$

Expressing debt and substituting it into the inequality constraints gives the Euler-type constraints of the form (12).

Proof of Proposition 2 Let $\psi_t = \psi_{t-1}$ for all $t > \widetilde{t}$ and $\psi_{\widetilde{t}} \neq \psi_{\widetilde{t}-1}$ (one of the debt limits is binding for $t = \widetilde{t}$ but never after). Then the Euler equation of the planner takes the form

$$\begin{aligned} & (\psi_{\widetilde{t}} - \psi_{\widetilde{t}-1})(1 + \sigma_c \frac{k_{\widetilde{t}} + b_{\widetilde{t}}}{c_{\widetilde{t}}})u_{c,\widetilde{t}} + (1 + \psi_{\widetilde{t}-1}(1 - \sigma_c))u_{c,\widetilde{t}} \\ = \ & (1 + \psi_{\widetilde{t}}(1 - \sigma_c))\beta E_t u_{c,\widetilde{t}+1}(F_{k,t+1} + 1 - \delta) \end{aligned} \tag{42}$$

Suppose that (38) holds. Then the above equation simplifies to

$$(\psi_{\widetilde{t}} - \psi_{\widetilde{t}-1})\sigma_c \frac{k_{\widetilde{t}} + b_{\widetilde{t}} + c_{\widetilde{t}}}{c_{\widetilde{t}}} u_{c,\widetilde{t}} = 0 \tag{43}$$

or

$$k_{\widetilde{t}} + b_{\widetilde{t}} + c_{\widetilde{t}} = 0 \tag{44}$$

which would require zero wealth of the consumer in period $\widetilde{t}$. Hence, (38) does not hold, and by Proposition 2, we have that the expected value of capital income tax collections is different from zero. By definition of the ex ante expected capital tax rate, θ_t^e is different from zero.

[16] We also use a transversality condition $\lim \beta^T E_t u_{c,t+T} b_{t+T} = 0$.

Proof of Proposition 3 Using the definition of the ex ante expected capital tax rates we can rewrite the expected value of capital tax collections as

$$X_t^e = \theta_t^e E_t u_{c,t+1}(F_{k,t+1} - \delta)k_t \tag{45}$$

It is equal to

$$X_t^e = (\beta E_t u_{c,t+1}(F_{k,t+1} + 1 - \delta) - u_{c,t})k_t \tag{46}$$

Assume that none of the limits is binding at $t+1$. Then $\eta_{t+1} = u_{c,t+1}(1 + \psi_t(1 - \sigma_c))$

$$X_t^e = (\psi_t - \psi_{t-1})\sigma_c \frac{k_t + b_t + c_t}{c_t} u_{c,t} \tag{47}$$

and its sign depends only on the sign of the difference $\psi_t - \psi_{t-1} = \nu_{1,t} - \nu_{2,t}$ which is positive when the lower limit is binding, negative when the upper limit is binding and zero otherwise.

2.8.3 Appendix 3. Algorithm of the Numerical Solution

I use the Parameterized Expectations Algorithm by Marcet (see Marcet and Lorenzoni (1998) for a detailed description of the method) to approximate the conditional expectations at time t by

$$\beta E_t f(S_{t+1}, v_{t+1}) = \Phi(S_t \beta^{PEA_n}) \tag{48}$$

where Φ is a time-invariant smooth function of the current states $S_t = [1\ k_{t-1}\ z_t\ g_t\ \psi_{t-1}]$ and the coefficients of the approximation $\beta^{PEA_n} = (\beta_1^{PEA_n}, ..., \beta_5^{PEA_n})$. See below how to find the initial guess for β^{PEA_n}. I parameterize

- the right-hand side of the Euler equation for consumption-investment choice of the planner,
- the sum of the conditional expectations on the right-hand side of the Euler-type constraint (16) and
- the two conditional expectations that appear in the formula of the ex ante expected capital income tax rates.

The solution strategy consists in the following. Fist, I fix the initial indebtedness of the planner, $R_{b,0}b_{-1}$, and guess a value of $\lambda_0 > 0$, the Lagrange multiplier on the period zero implementability constraint. For a given value of λ_0, I first solve the FOC at $t = 0$ assuming that the debt limits are not binding, compute the solution and check whether debt falls inside the limits. If it doesn't, then set debt equal to its corresponding limit and recompute the solution. Proceed with the FOC for the rest of the periods. With the simulated variables in hand, check the distance between the left-hand side of each approximated equation and $\Phi(S_t \beta^{PEA_n})$. Find β^{PEA_n} which minimize this distance for each $n = 1, ..., 5$. Finally, compute the initial indebtedness of the planner from the implementability constraint right-hand side, and adjust λ_0. I have to iterate on λ_0 until I get the initially fixed $R_{b,0}b_{-1}$.

Short Run Monte Carlo Simulations See Marcet and Marimon (1992) for the description of the method.

I need to recur to the short run Monte Carlo simulations for the following reasons: first, I want to approximate the transition really well. The period zero behavior of the model is totally different from the rest of the periods due to different FOC for c_0 and l_0. Second, I start with a low initial capital stock, very far away from its stationary long run level. I want to look at different specifications for debt limits and to be able to analyze the models with only transitionally binding limits. This is impossible to do with just a one long series of shocks due to degenerate matrix of states in case that there are few observations with binding limits.

Finally, the models with transitionally binding limits, in the very long run converge to the complete markets solution. Therefore, to study the effects of binding debt limits on the optimal taxes, I need to stick to the short and medium run analysis.

I use 200 simulations of the two shocks, each one of the lengths of 100 periods. To compute the statistics, I construct a long vector for each series taking periods between 11 and 90 for each of the 200 Monte Carlo realizations.

PEA and Homotopy The main difficulty with applying the Parameterized Expectations approach is to find the starting values for the vectors of parameters $\beta^{PEA_n} = (\beta_1^{PEA_n}, ..., \beta_5^{PEA_n})$ such that they generate non-explosive stochastic processes for the allocations and bring us to the stationary distribution.

Homotopy is an approach which allows imposing "good" initial conditions for the Parameterized Expectations Algorithm in a systematic way (Marcet and Lorenzoni (1998, p.p. 20-24)). This approach consists in always starting with a simplified version of the model which is easy to solve numerically, and then modifying the parameters slowly, to go to the desired version of the model. As long as the model goes from the known to the desired solution in a smooth way, the initial conditions are good.

To solve the model with debt limits, I proceeded in 5 iterations, i.e. I solved 5 different models, starting from the simplest one and gradually complicating it with additional state variables and constraints. Each previous iteration gave the starting parameterization for the next one.

Iteration 1: Solve a simple stochastic growth model with only technological shock and no labor.

Iteration 2: Introduce labor to the previous model.

Iteration 3: Add the government spending shock.

Iteration 4: Solve the complete markets model.

Iteration 5: Introduce debt limits (hence, enlarge the vector of states by introducing the costate Lagrange multiplier, ψ_{t-1}.

To pass to the next step, I had to iterate on some parameter linking the two models.

3 Optimal Capital Income Taxes with Endogenous Credit Constraints

3.1 Introduction

The famous Chamley-Judd result discussed at length in the first chapter appears to be very robust to different environments. In this chapter, I show that endogenizing borrowing constraints leads to positive limiting capital income taxes even in the non-stochastic steady state. Once again market incompleteness alters policy implications for optimal factor taxation.

The models of optimal taxation that assume a commitment technology for the Ramsey planner are usually silent about what mechanism would prevent the consumers from default on their long run debt. In particular the models with capital accumulation and complete markets predict that the consumers become highly indebted during the short transition and that it is never optimal to repay the debt. One of the characteristics of the capital income taxes that I found solving the model of Chapter 2 numerically is that capital income taxes, on average across the states, are lower in good states and higher in bad ones. This is because in good states, the limit on the high side is binding more often: the representative consumer wants to save. In bad states, the government recurs to capital income taxation and creates a surplus so that the consumer can borrow. The strong (simplifying) assumption of the previous chapter was that the consumer was not fully aware of the tightness of debt limits, so the first-order conditions of the consumer's problem remained undistorted. A naturally interesting question arises: How would the predictions of the model from Chapter 2 change if I set endogenous limits on debt?

I explore the idea of Kiyotaki and Moore (1997) and consider an economy in which lenders cannot force borrowers to repay their debts, unless debts are secured. In such an economy, durable assets such as land, buildings and machinery serve not only as factors of production but also as collateral for loans. In the absence of an asset in fixed supply (like land in their original model), I assume that the physical capital serves as a collateral for loans.

Chari and Kehoe (1998, p. 40) conjectured the following observation: "Zero capital income taxation in the steady state is optimal if the extra constraints do not depend on the capital stock and is not possible if these constraints depend on the capital stock (and, of course, are binding)". The model discussed below is in line with their affirmation. The intuition behind positive optimal taxes on capital income builds upon the following possible explanations. First, binding credit constraints in equilibrium may lead to an overaccumulation of capital, as it happens in the model of Aiyagari (1995) with incomplete markets due to idiosyncratic uninsurable individual shocks. Second, if an initial negative shock leads to a binding credit constraint, the excess demand for borrowing would drive the price of bonds above the kernel. The no arbitrage condition then implies that the resulting decline in returns to state-contingent bonds should be matched by the lower after-tax return on capital. Thus positive capital income taxes become optimal. However positive capital income taxes today increase the likelihood of binding credit constraints tomorrow. Persistence and amplification reinforce each other - similar to the original credit cycles construction of Kiyotaki and Moore.

This chapter starts with the discussion of the competitive equilibria with credit constraints. I then proceed with setting up a Ramsey problem. What follows is the discussion of the mechanism that links binding credit constraints to positive capital income taxes. I give the intuition of why the Chamley-Judd result does not hold anymore in the non-stochastic steady state. Finally, I describe further research perspectives arising from the model of this chapter.

3.2 Decentralized Economy

I consider a deterministic version of the model from the previous chapter. For the moment, I remove the upper limit on government debt to focus on the effects of endogenous credit constraints that distort the consumer's problem.

3.2.1 Consumers

A representative consumer maximizing the present value of utility from consumption and leisure

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to his budget constraint

$$c_t + k_t + q_t b_t = (1 - \tau_t) w_t l_t + (1 + (1 - \theta_t)(r_t - \delta)) k_{t-1} + b_{t-1} \tag{49}$$

The capital owned by agents also serves as a collateral that opens the access to the credit market. In the spirit of Kiyotaki and Moore (1997), I choose to impose the credit constraint on the consumers of the form

$$b_t + \zeta k_t \geq 0 \tag{50}$$

where $0 \leq \zeta < 1$. That is, the value of consumers' debt to be repaid next period, $-b_t$, should not exceed the market value of (a fraction of) the capital stock at $t+1$.[17] The parameter ζ reflects the tightness of the credit constraint: $\zeta = 0$ means no borrowing at all, $\zeta \to 1$ allows borrowing as much as the capital stock next period. Letting $\zeta > 1$ would have taken into account the next period's capital income that I choose not to consider as a collateral here. Nonetheless all the results of this chapter go through for any $\zeta \geq 0$.

The initial conditions are

$$\begin{aligned} k_{-1} &= \bar{k} \\ b_{-1} &= 0 \end{aligned} \tag{51}$$

The consumer's optimal choice should satisfy the following first-order conditions:

$$(1 - \tau_t) w_t = -\frac{u_{l,t}}{u_{c,t}} \tag{52}$$

$$u_{c,t} - \zeta \chi_t = \beta u_{c,t+1}(1 + (1 - \theta_{t+1})(r_{t+1} - \delta)) \tag{53}$$

$$q_t u_{c,t} - \chi_t = \beta u_{c,t+1} \tag{54}$$

$$\chi_t (b_t + \zeta k_t) = 0 \tag{55}$$

$$\chi_t \geq 0 \tag{56}$$

[17] Recall that $b_t > 0$ means government borrowing thus $b_t < 0$ stands for consumer's debt. Both k_t and b_t are chosen at the end of period t. For the extention to the stochastic setup, this means that they are measurable with respect to the history of shocks up to t.

3.2.2 Firms

Competitive firms produce output employing a constant returns to scale technology $y = F(k_{t-1}, l_t)$. They demand capital and labor up to levels that equate the corresponding marginal products and the factor prices

$$r_t = F_k(k_{t-1}, l_t) \tag{57}$$

$$w_t = F_l(k_{t-1}, l_t) \tag{58}$$

3.2.3 Government

The government sets capital income taxes and labor income taxes in order to finance a (deterministic) stream of expenditures that are assumed to be of no use for either the agents or the economy

$$g_t + b_{t-1} = \tau_t w_t l_t + \theta_t (r_t - \delta) k_{t-1} + q_t b_t \tag{59}$$

3.2.4 Competitive Equilibria

A competitive equilibrium for this economy is defined as a sequence of allocations $(c_t, k_t, l_t, b_t)_{t=0}^{\infty}$, tax policies $(\theta_{t+1}, \tau_t)_{t=0}^{\infty}$ and prices $(q_t, r_t, w_t)_{t=0}^{\infty}$ such that agents maximize their utilities to find optimal allocations given prices and policies, firms maximize profits, government sets taxes that satisfy (59), and markets clear. The latter implies the usual resource constraint

$$c_t + k_t + g_t = F(k_{t-1}, l_t) + (1 - \delta) k_{t-1} \tag{60}$$

3.3 Ramsey Planner

A benevolent government, acting as a social planner, maximizes the utility of the consumer over the set of competitive equilibria in the economy. As it is usually done in the literature, I use some of the equations characterizing the competitive equilibria to express prices and policies in terms of allocations and then solve the Ramsey Allocations problem.

Implementability Constraints I try to reduce the number of equations characterizing the set of competitive equilibria feasible for the Ramsey planner. I substitute taxes and prices expressed from the FOC of the consumer and the firms into the sequence of the consumer's budget constraints for $t \geq 0$ that I choose to write in the present discounted value form[18]:

$$\sum_{j=1}^{\infty} \beta^j (u_{c,t+j} c_{t+j} + u_{l,t+j} l_{t+j}) + \sum_{j=1}^{\infty} \beta^j u_{c,t+j} (k_{t+j} + q_{t+j} b_{t+j}) = \sum_{j=1}^{\infty} \beta^j u_{c,t+j} (R_{k,t+j} k_{t+j-1} + b_{t+j-1}) \quad (61)$$

$$u_{c,0} c_0 + u_{l,0} l_0 + u_{c,0}(k_0 + q_0 b_0) = u_{c,0} R_{k,0} \overline{k} \quad (62)$$

where

$$R_{k,t} = 1 + (1 - \theta_t)[F_k(k_{t-1}, l_t) - \delta] \quad (63)$$

$$R_{k,0} = 1 + (1 - \overline{\theta}_0)[F_k(\overline{k}, l_0) - \delta] \quad (64)$$

The present discounted value form allows to simplify all the future values of the assets by using the Euler equations for the consumption-capital and consumption-bonds choice of the consumer.

$$0 = \sum_{j=1}^{\infty} \beta^j (u_{c,t+j} c_{t+j} + u_{l,t+j} l_{t+j}) + \sum_{j=0}^{\infty} \beta^j [u_{c,t+j} - \beta u_{c,t+j+1} R_{k,t+j+1}] k_{t+j} - u_{c,t} k_t \quad (65)$$
$$+ \sum_{j=0}^{\infty} \beta^j [q_{t+j} u_{c,t+j} - \beta u_{c,t+j+1}] b_{t+j} - u_{c,t} q_t b_t$$

The key difference between the complete markets case and credit constraints is due to the distorted Euler equations of the consumer.

$$u_{c,t} [k_t + q_t b_t] = \sum_{j=1}^{\infty} \beta^j (u_{c,t+j} c_{t+j} + u_{l,t+j} l_{t+j}) + \sum_{j=0}^{\infty} \beta^j \chi_{t+j} (b_{t+j} + \zeta k_{t+j}) \quad (66)$$

However, the particular shape of the collateral constraint (50) together with the Kuhn-Tucker condition (55) for every period imply that the second term on the right-hand side is zero and the implementability constraint for every period has exactly the same shape as without the collateral requirement:

[18] To have an equivalent formulation to the sequence of period-by-period budget constraints, we need the present discounted value constraints for all the periods plus the period zero constraint. The latter is of a slightly different shape due to the initial conditions.

$$u_{c,t}\left[k_t + q_t b_t\right] = \sum_{j=1}^{\infty} \beta^j (u_{c,t+j} c_{t+j} + u_{l,t+j} l_{t+j}) \tag{67}$$

The period zero implementability constraint (62), after substituting (67) in, also has the usual shape

$$\sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t + u_{l,t} l_t) = u_{c,0} R_{k,0} \overline{k} \tag{68}$$

It depends on the initial capital stock, $\overline{k}$, and on the period zero capital income tax rate, $\overline{\theta}_0$, that is fixed in order to make the first-best unfeasible.

I use the Euler equation for credit-consumption choice of the consumer (54) to express χ_t as a function of c_t, c_{t+1},and q_t.

3.3.1 Optimal Allocations

The Ramsey planner solves for the optimal allocations $(c_t, k_t, l_t, b_t)_{t=0}^{\infty}$ and a sequence of bond prices $(q_t)_{t=0}^{\infty}$:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{69}$$

subject to

$$\begin{aligned}
c_t + k_t + g_t &= F(k_{t-1}, l_t) + (1-\delta) k_{t-1} && (70)\\
\sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t + u_{l,t} l_t) &= u_{c,0} R_{k,0} \overline{k} && (71)\\
u_{c,t}\left(k_t + q_t b_t\right) &= \sum_{j=1}^{\infty} \beta^j (u_{c,t+j} c_{t+j} + u_{l,t+j} l_{t+j}) && (72)\\
\left(q_t u_{c,t} - \beta u_{c,t+1}\right)\left(b_t + \zeta k_t\right) &= 0 && (73)\\
q_t u_{c,t} - \beta u_{c,t+1} &\geq 0 && (74)\\
b_t + \zeta k_t &\geq 0 && (75)
\end{aligned}$$

The presence of collateral constraints in the consumer's problem imposes new restrictions on the set of Ramsey optimal allocations implementable in a Ramsey equilibrium. In addition to the usual resource constraint (70) and the so-called "period zero implementability constraint" (71), the planner is subject to the sequence of period-by-period implementability restrictions (72), Kuhn-Tucker condition of the consumer (73), and inequality constraints (74) and (75).

As in the previous chapter, I have to convert the maximization problem to a saddle point one enlarging the state space by the appropriate costate Lagrange multipliers on the constraints with conditional expectations. This application of the recursive contracts approach by Marcet and Marimon is less trivial than in the previous chapter due to the presence of non-differentiable Kuhn-Tucker conditions.

The resulting Ramsey allocations problem becomes

$$
\begin{aligned}
W_0 &= \min\max\sum_{t=0}^{\infty}\beta^t\{u(c_t,l_t)+ \\
&\quad +\eta_t\left[F(k_{t-1},l_t)+(1-\delta)k_{t-1}-c_t-k_t-g_t\right]+ \\
&\quad +\left(\Lambda_0+\Phi_{t-1}\right)\left[u_{c,t}c_t+u_{l,t}l_t\right]-\psi_t u_{c,t}\left[k_t+q_t b_t\right]+ \\
&\quad +\omega_t q_t u_{c,t}\left(b_t+\zeta k_t\right)-\omega_{t-1}u_{c,t}\left(b_{t-1}+\zeta k_{t-1}\right)+ \\
&\quad +\mu_t q_t u_{c,t}-\mu_{t-1}u_{c,t}+\nu_t\left(b_t+\zeta k_t-\underline{M}\right)\}- \\
&\quad -\Lambda_0 u_{c,0}\left(1+(1-\theta_0)(F_{k,0}-\delta)\right)\overline{k} \qquad (76)
\end{aligned}
$$

where

$$
\begin{aligned}
\mu_{-1} &= \psi_{-1}=\Phi_{-1}=0 \qquad (77)\\
\Phi_t &= \Phi_{t-1}+\psi_t=\sum_{k=0}^{t}\psi_k \qquad (78)
\end{aligned}
$$

The controls for maximization are $(c_t,k_t,l_t,b_t,q_t,\eta_t,\nu_t)_{t=0}^{\infty}$ and Λ_0, and we minimize with respect to $(\psi_t,\mu_t,\omega_t)_{t=0}^{\infty}$. The Kuhn-Tucker multipliers μ_t and ν_t are non-negative.

The first-order conditions are presented in the Appendix 1 below together with the analysis of sign restrictions for the Kuhn-Tucker multipliers. The implications of the additional restrictions on the set of competitive equilibria implementable by the Ramsey planner are presented below.

Summary of Sign Restrictions As shown in the Appendix 1 of this chapter, all the possible combinations of sings of Kuhn-Tucker multipliers and inequality constraints for any period t require

$$\psi_t = 0 \qquad (79)$$

Hence, we also have

$$\Phi_t = 0$$

There are two possibilities: binding limits:

$$
\begin{aligned}
b_t+\zeta_t k_t &= 0\\
u_{c,t}q_t &> \beta u_{c,t+1}\\
\mu_t &= 0\\
\omega_t &< 0 \text{ if } \nu_t>0\\
\omega_t &= 0 \text{ if } \nu_t=0
\end{aligned}
$$

and non-binding limits:

$$\begin{aligned} b_t + \zeta_t k_t &> 0 \\ u_{c,t} q_t &= \beta u_{c,t+1} \\ \nu_t &= 0 \\ \omega_t &< 0 \text{ if } \mu_t > 0 \\ \omega_t &= 0 \text{ if } \mu_t = 0 \end{aligned}$$

We will see below that the first option gives rise to a positive capital income taxation even in the deterministic steady state while the second one implies zero capital income taxes.

3.3.2 Optimal Capital Income Taxation

From the consumer's capital-consumption choice we have

$$\theta_{t+1} = \frac{\beta u_{c,t+1}(F_{k,t+1} + 1 - \delta) - u_{c,t} + \zeta(q_t u_{c,t} - \beta u_{c,t+1})}{\beta u_{c,t+1}(F_{k,t+1} - \delta)} \tag{80}$$

Combining the First-order conditions of the planner for c_t and k_t (see Appendix 1) with the conclusions of the sign analysis above yields

$$u_{c,t} + \Lambda_0(u_{c,t} + u_{cc,t} c_t) = \beta \left[u_{c,t+1} + \Lambda_0(u_{c,t+1} + u_{cc,t+1} c_{t+1}) \right] (F_{k,t+1} + 1 - \delta)$$

To simplify further analysis, consider logarithmic utility that reduces the above condition to

$$u_{c,t} = \beta u_{c,t+1}(F_{k,t+1} + 1 - \delta)$$

Then for the case of binding limits together with $q_t u_{c,t} > \beta u_{c,t+1}$, optimal capital income taxes should be positive.

Why: when consumers want to borrow more but are not able to due to binding credit constraints, their demand for borrowing pushes up bond prices reducing the next period return on bonds:

$$\frac{1}{q_t} < \frac{u_{c,t}}{\beta u_{c,t+1}} = 1 + (1 - \theta_t)[F_k(k_{t-1}, l_t) - \delta] \tag{81}$$

No arbitrage condition for the two assets requires positive capital income taxes to reduce the net return on capital.

3.4 Steady State

It turns out that the credit constraints may be binding even in the long run. The steady state has the following characteristics: first of all, the planner can achieve the allocation for which the golden rule capital-labor ratio holds.

$$\begin{aligned}
\frac{\overline{k}}{\overline{l}} &= \left(\frac{1-\beta+\beta\delta}{\alpha\beta}\right)^{\frac{1}{\alpha-1}} \\
\overline{c} &= \left[\left(\frac{\overline{k}}{\overline{l}}\right)^{\alpha} - \delta\right]\overline{l} - \overline{g} \\
y &= \left(\frac{1-\beta+\beta\delta}{\alpha\beta}\right)^{\frac{\alpha}{\alpha-1}}\overline{l} \\
\frac{1}{\overline{c}} &= A\frac{1-\overline{l}+\Lambda_0}{\left(1-\overline{l}\right)^2} \\
A &= \left(\frac{1-\beta+\beta\delta}{\alpha\beta}\right)^{\frac{\alpha}{\alpha-1}}\frac{\gamma}{(1-\gamma)(1-\alpha)}
\end{aligned}$$

The labor income taxes are given by

$$\overline{\tau} = \frac{\Lambda_0}{1-\overline{l}+\Lambda_0}$$

The capital income taxes satisfy

$$\overline{\theta} = \frac{\zeta\beta\left(\overline{q}-\beta\right)}{1-\beta}$$

Thus the necessary and sufficient condition for the capital income tax rate to be positive is that bond is priced above the rate of time preferences [19]:

$$\overline{q} > \beta$$

In this latter case, any bond price above the rate of time preferences can be supported as a steady state. If we use the standard procedure (see Chari, Christiano and Kehoe, 1995) of calibrating the steady state level of both capital and labor income taxes to the data, it will pin down the return on bonds as a function of the free parameter of the model - the tightness of the collateral requirement.

The intuition for non-zero limiting capital income taxes is similar to Aiyagari's: incomplete markets due to credit constraints generate precautionary savings that lead to overaccumulation of capital. To achieve the golden rule level, the government sets a positive tax. The transmission mechanism works as follows: the price of bonds above the kernel affects the ability of the consumer to borrow and enhances capital accumulation. If the credit constraint is tight enough to be binding, the excess demand for credit brings down the return on bonds and thus leads to a lower return on capital achieved through positive capital income taxes. The latter, in turn, reinforce the tightness of the credit constraint, as in the original Kiyotaki and Moore's paper.

[19] Notice that $\overline{q} \geq \beta$ by the consumer's FOC

3.5 Conclusion

This work contributes to the existing literature by explicitly defining constraints on the consumer side necessary to avoid default on long run debt. Such constraints distort the first-order conditions of the representative consumer and thus impose additional inequality restrictions on the set of competitive equilibria the Ramsey planner chooses from. As a consequence, a benevolent government should set positive capital income taxes in the long run of a deterministic model. Positive taxes aim at bringing down the net return on holding capital stock to match the lower return on state-contingent bonds due to excess demand for consumer's credit.

It is natural to extend the present model to the stochastic environment and calibrate it to the data, to examine how the shortcomings of the previous chapter in its attempts to fit the data on capital income taxes may be overcome. Another immediate extension is to set the upper limit on the planner's borrowing at a fraction of GDP as in the Growth and Stability Pact. I conjecture that by parameterizing the fraction of the capital stock required as collateral I can significantly improve the predictions for capital income taxes. On the other hand, we saw from the findings of the previous chapter that binding debt limits result in some very sound predictions about the autocorrelation and the volatility of labor taxes.

Finally, a complementary area of research can deal with the introduction of heterogeneous agents into the analyzed model. Heterogeneity would allow a more ample study of the economys wealth distribution. Such a modification of the basic setup may or may not imply significant deviations from the present conclusions, thus representing an important area for further research.

3.6 Appendix 1. First-Order Conditions and Kuhn-Tucker Analysis

The first-order conditions of the planner's saddle point problem are given below.

3.6.1 FOC

$$\psi_t u_{c,t} q_t = \nu_t + \omega_t \left[q_t u_{c,t} - \beta u_{c,t+1} \right] \tag{b_t}$$

$$\psi_t b_t = \mu_t + \omega_t \left[b_t + \zeta k_t \right] \tag{q_t}$$

Kuhn-Tucker conditions:

$$\nu_t \left[b_t + \zeta k_t - \underline{M} \right] = 0 \tag{credit}$$

$$\mu_t \left[q_t u_{c,t} - \beta u_{c,t+1} \right] = 0 \tag{pricing}$$

$$\left(q_t u_{c,t} - \beta u_{c,t+1} \right) \left[b_t + \zeta k_t - \underline{M} \right] = 0 \tag{Kuhn-Tucker}$$

$$\eta_t F_{l,t} = -u_{l,t} - (\Lambda_0 + \Phi_{t-1})(u_{l,t} + u_{ll,t} l_t) \tag{l_t}$$

$$\eta_t = u_{c,t} + (\Lambda_0 + \Phi_{t-1})(u_{c,t} + u_{cc,t} c_t) - [\psi_t k_t + \psi_{t-1} b_{t-1}] u_{cc,t} \tag{c_t}$$

$$\psi_t u_{c,t}(1 - \zeta q_t) + \eta_t = \beta \eta_{t+1}(F_{k,t+1} + 1 - \delta) \tag{k_t}$$

The usual restrictions on the planner's set of feasible allocations:

$$c_t + k_t + g_t = F(k_{t-1}, l_t) + (1 - \delta) k_{t-1} \tag{η_t}$$

$$u_{c,t} \left(k_t + q_t b_t \right) = \sum_{j=1}^{\infty} \beta^j (u_{c,t+j} c_{t+j} + u_{l,t+j} l_{t+j}) \tag{ψ_t}$$

$$\sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t + u_{l,t} l_t) = u_{c,0} \left[1 + (1 - \theta_0)(F_{k,0} - \delta) \right] \overline{k} \tag{Λ_0}$$

Notice that the first-order conditions for consumption and labor and capital in period zero are slightly different because of the right-hand side of the implementability constraint at $t = 0$ and $\Phi_{-1} = \psi_{-1} = 0$:

$$\eta_0 F_{l,0} = -u_{l,0} - \Lambda_0 (u_{l,0} + u_{ll,0} l_0) + \Lambda_0 u_{c,0} (1 - \overline{\theta}_0) F_{kl,0} \overline{k} \tag{l_0}$$

$$\eta_0 = u_{c,0} + \Lambda_0 (u_{c,0} + u_{cc,0} c_0) - [\psi_0 k_0 + \Lambda_0 \left(1 + (1 - \theta_0)(F_{k,0} - \delta) \right) \overline{k}] u_{cc,0} \tag{c_0}$$

$$\psi_0 u_{c,0}(1 - \zeta q_0) + \eta_0 = \beta \eta_1 (F_{k,1} + 1 - \delta) \tag{k_0}$$

3.6.2 Sign Analysis

Let us study the sign restrictions that the first-order conditions (b_t)-(q_t) and the Kuhn-Tucker conditions (credit)-(Kuhn-Tucker) put on the variables. I immediately rule out the case of

$$\mu_t \nu_t > 0 \qquad \text{(impossible 1)}$$

together with the case

$$\nu_t > 0 \Rightarrow b_t + \zeta k_t = 0 \text{ and } u_{c,t} q_t = \beta u_{c,t+1} \qquad \text{(impossible 2)}$$

The remaining four cases are considered below.

Case A: $\nu_t > 0$ implies

$$\begin{aligned} b_t + \zeta k_t &= 0 \\ \mu_t &= 0 \\ u_{c,t} q_t &> \beta u_{c,t+1} \\ \psi_t &= 0 \\ \omega_t &< 0 \end{aligned}$$

Case B: $u_{c,t} q_t > \beta u_{c,t+1}$ implies

$$\begin{aligned} b_t + \zeta k_t &= 0 \\ \mu_t &= 0 \\ \psi_t &= 0 \\ \omega_t &< 0 \text{ if } \nu_t > 0 \\ \omega_t &= 0 \text{ if } \nu_t = 0 \end{aligned}$$

Case C: $b_t + \zeta k_t > 0$ implies

$$\begin{aligned} \nu_t &= 0 \\ u_{c,t} q_t &= \beta u_{c,t+1} \\ \psi_t &= 0 \\ \omega_t &< 0 \text{ if } \mu_t > 0 \\ \omega_t &= 0 \text{ if } \mu_t = 0 \end{aligned}$$

Case D: $\mu_t > 0$ implies

$$\begin{aligned} u_{c,t} q_t &= \beta u_{c,t+1} \\ b_t + \zeta k_t &> 0 \\ \nu_t &= 0 \\ \psi_t &= 0 \\ \omega_t &< 0 \end{aligned}$$

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