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B

List of publications

This thesis has summarized the author's research experience in the field of fuzzy modeling for the last seven years. Obviously, during its development we have published several articles related to this work. Here we enumerate them and we also include their abstracts. They are sorted by its publication date in descending order.

1. An Intelligible Approach for the Synthesis of Intelligible Fuzzy Models

Submitted to Fuzzy Sets and Systems (but not accepted yet) International Fuzzy Systems Association & Elsevier Elsevier, 2005

We present an heuristic methodology devised to address the problems encountered in designing an intelligible fuzzy model to fit a set of inputoutput data. In particular, we are able to determine the number of fuzzy sets, place them in the universe of scope and propose a set of linguistic rules that relate them.

The resulting method is very simple and also intelligible. Therefore, it performs the final models with a low computational cost but furthermore, if helps the tuning of its different options based on the nature of the problem and the nature of the users. Thus, observe that we will focus this work not only on the intelligibility of the model but also on the intelligibility of the method itself.

We do not seek to conclude that our method is better than others but to obtain an acceptable error in comparison while keeping the linguistic capacities of the fuzzy model. In fact with this methodology we will be able to choose the precision of the model and consequently its degree of intelligibility.

2. A Fast Approach to Synthesize Intelligible Fuzzy Systems from Input-Output Data

Proceedings of the IFSA'05 Word Congress International Fuzzy Systems Association Beijing, 2005

We present a set of heuristic criteria devised to address the problems encountered in designing a fuzzy controller to fit a set of input-output data. The objective is to obtain in a fast and simple manner an intelligible model able to undergo further refinements. We detail the method, compare it with other alternatives and finally some examples are given.

3. Intelligible Fuzzy Models Applied to Time Series Prediction and Control

Poster for the 7-th CCIA Catalan Association for Artificial Intelligence Barcelona, 2004

A simple and fast method to build fuzzy systems from input-output data consists in computing optimal fuzzy curves whose linearization can define the necessary fuzzy sets. In this paper we review this method and show its capacities to predict the popular Box and Jenkins' time series and to control the ball and beam system.

4. Building Controllers from Optimal Fuzzy-Curves: A Simple and Intelligible Approach

Proceedings of the 2004-EUROFUSE Workshop on Data and Knowledge Engineering

European Chapter of the International Fuzzy Systems Association Warsaw, 2004

In this paper we present a set of heuristic criteria devised to address the problems encountered in designing a fuzzy controller to fit a set of input-output data. The objective is to obtain an intelligible starting control able to undergo further refinements. The method is exemplified by means of two popular cases: the truck backer-upper and the ball and beam problem.

5. An Approach of Fuzzy Modeling towards Intelligible Modeling

Proceedings of the 5th WSEAS Int.Conf. on fuzzy sets and fuzzy systems

World Scientific and Engineering Academy and Society Udine, 2004

In this paper we present a set of heuristic criteria devised to address the

problems encountered in designing a fuzzy system to fit a set of inputoutput data. The objective is to obtain in a simple and fast manner a good starting model to undergo further refinements. The result is a simple algorithm with a similar performance than other techniques but with a low computational cost.

6. Min-square Fitting of Fuzzy Curves

Proceedings of the IFSA'03 World Congress International Fuzzy Systems Association Istanbul, 2003

Fuzzy curves proposed by Lin *et al.* deliver a smooth representation of the relation between two variables from the weighted average of near samples. This average is taken from samples inside a window of adjustable size by means of a parameter defined in the fuzzy curves, the b or β parameter, which is adjusted empirically until the moment. This paper proposes a method to fit this parameter so that the fuzzy curve presents the minimum square error between its result and the samples.

7. Extracting Relevant Information from Input-Output Data Poster for the 4-th CCIA

Catalan Association for Artificial Intelligence Barcelona, 2001

In this paper a set of heuristic criteria devised to address the problems encountered in designing a fuzzy system from input-output data is presented. In particular, we show how to discriminate unsignificant linguistic variables, determine the number of fuzzy sets, place them in the universe of scope, and propose a set of linguistic rules. The objective is to obtain in a simple and fast manner an algorithm simpler than the standard ones with minimum computational cost but still similar performance and more intelligibility in most cases.

8. Automatic Process for the Synthesis of Fuzzy Systems from Input-Output Data

Proceedings of the 1999 EUSFLAT-ESTYLF Joint Conference European Society for Fuzzy Logic and Technology Palma de Mallorca, 1999

In this paper we present a set heuristic criteria devised to address the problems encountered in designing a fuzzy system to fit a set of inputoutput data. In particular, we discriminate unsignificant linguistic variables, determine the number of fuzzy sets, place them in the universe of scope, propose a set of linguistic rules and give the necessary number of bits to represent each variable. The objective is to obtain in a simple and fast manner a good starting model to undergo further refinements.

C

Algorithms

Here we provide the algorithm of the methodology that we have proposed in this work with the Wang&Mendel's option and the Chiu's clustering option. We detail all the steps except for some few computations that we have considered in the real program in order to diminish the elapsed time.

In order to facilitate its understanding we have considered for every variable the same name in all procedures. Every variable is treated as a global variable and thus, the whole algorithm can be obtained by joining directly all the procedures. Furthermore we have included some comments where the methodology could be difficult to understand.

Every variable is identified with an intelligible name and in most cases they have the same name that we have used when we have explained the methodology.

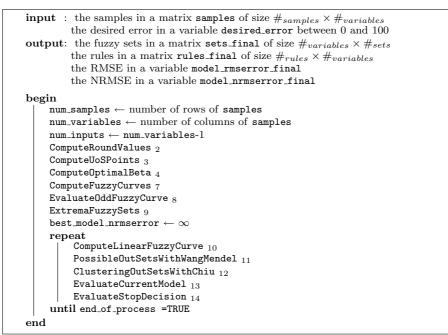
Anyway, among all the variables there is one of them, $fuzzy_curve$, probably one of the most significant, which should be introduced here because it has not been explained before. This variable is a 3-dimensional array where the first number indicates the input variable, the second number indicates the fuzzy set of this variable and the third number indicates the type of information which may be a 1 if it indicates the value of the universe of scope (UoS), a 2 if it indicates the value of the fuzzy curve in this point, a 3 if it indicates the value of the linearized fuzzy curve in this point, a 4 if it indicates the error between the real value and the linearized value in this point or a 5 if it indicates the rounded value of this error.

Furthermore in every call to a function we include with a subscript the number of the algorithm in order to locate it quickly. For example the call ComputeOptimalBeta4 means that this procedure can be found in the algorithm number 4.

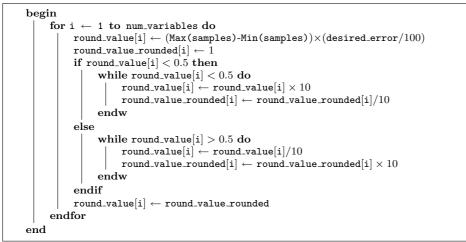
Nevertheless, some generic functions are not detailed here. These are the

following:

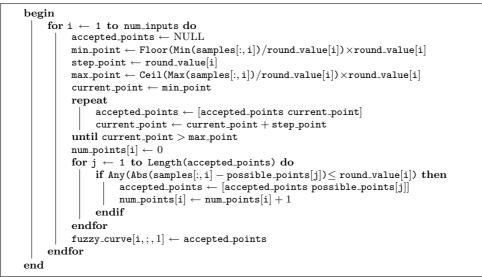
- Abs \rightarrow Absolute value.
- All \rightarrow True if all elements of a vector are nonzero.
- Any \rightarrow True if any element of a vector is nonzero.
- Bisection \rightarrow Bisection method.
- Ceil \rightarrow Round towards plus infinity.
- $Exp \rightarrow Exponential.$
- Find \rightarrow Find indices of nonzero elements.
- Floor \rightarrow Round towards minus infinity.
- FuzzySystem \rightarrow Perform fuzzy inference calculations.
- Length \rightarrow Length of a vector or matrix.
- $Log \rightarrow Natural logarithm.$
- LogCommon \rightarrow Common base 10 logarithm.
- LogDivision \rightarrow Logarithmically spaced vector.
- $Max \rightarrow Largest$ value.
- Mean \rightarrow Mean value.
- $Min \rightarrow Smallest$ value.
- $\operatorname{Prod} \to \operatorname{Product}$ of values.
- Rand \rightarrow Uniformly distributed random numbers.
- Round \rightarrow Round towards nearest integer.
- Sort \rightarrow Sort in ascending order.
- Sqrt \rightarrow Square root.
- $Std \rightarrow Standard deviation.$
- Sum \rightarrow Sum of values.
- $TStudentInv \rightarrow$ Inverse of Student's T cumulative distribution function.



Algorithm 1: Main procedure



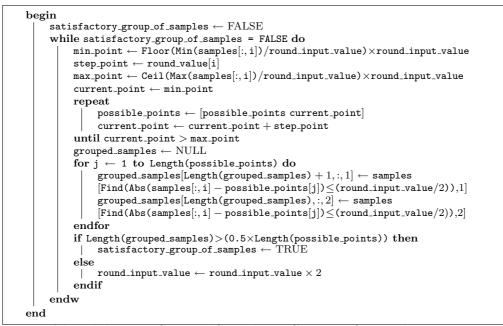
Algorithm 2: ComputeRoundValues



Algorithm 3: ComputeUoSPoints

begin
for $i \leftarrow 1$ to num_inputs do
$\texttt{round_input_value} \leftarrow \texttt{round_value[i]}$
${\tt GroupedSamplesToComputeOptimalBeta}{}_5$
$optimal_beta_mean \leftarrow NULL$
$optimal_beta_std \leftarrow NULL$
current_error $\leftarrow \infty$
$\texttt{current_iteration} \leftarrow 1$
while (current_error > desired_error)OR(current_iteration < 21) do
TestTrainPointsToComputeOptimalBeta 6
$possible_min \leftarrow NULL$
for $j \leftarrow 2$ to Length(derivative_square_error) do
if (derivative_square_error[j,2] \geq
0) AND (derivative_square_error $[i-1,2] < 0$) then
possible_min[:, 1] \leftarrow
$[possible_min[:, 1] derivative_square_error[j - 1, 1]]$
possible_min[:,2] \leftarrow [possible_min[:,2] derivative_square_error[j,1]]
endif
endfor
local_min ← Bisection(possible_min)
if Length(local_min)=1 then
optimal_beta[current_iteration] ← local_min
else
Comment: Choosing the global minimum.
square_error \leftarrow NULL
for $k \leftarrow 1$ to Length(local_min) do
$ square_error[k] \leftarrow 0$
for $q \leftarrow 1$ to Length(grouped_samples) do
$ $ numerator \leftarrow Sum(Exp(-((train_points):
$(1] - \text{test_points}[q, 1])/(\text{local_min}[k]))) \times \text{train_points}[;, 2])$
denominator \leftarrow Sum(Exp(-((train_points[:
$(1] - \text{test_points}[q, 1])/(\text{local_min}[k])))$
square_error[k] \leftarrow square_error[k] + 0.5 × (test_points[q, 2] -
(numerator/denominator)) ²
endfor
endfor
optimal_beta[current_iteration] ←
Mean(local_min[Find(square_error = Min(square_error))])
endif
$ptimal_beta_mean[current_iteration] \leftarrow Mean(optimal_beta)$
$optimal_beta_std[current_iteration] \leftarrow$
Std(optimal_beta/Sqrt(current_iteration))
current_error ←
100 × TStudentInv((200-desired_error)/200,current_iteration-1) ×
optimal_beta_std[current_iteration]/optimal_beta_mean[current_iteration]
$current_iteration \leftarrow current_iteration + 1$
\leftarrow current_iteration \leftarrow current_iteration $+1$ endw
$optimal_beta[i] \leftarrow optimal_beta_mean[current_iteration - 1]$
$[$ optimal_beta[1] \leftarrow optimal_beta_mean[current_iteration - 1] endfor
end

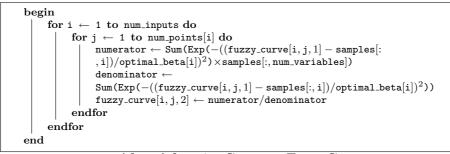
Algorithm 4: ComputeOptimalBeta



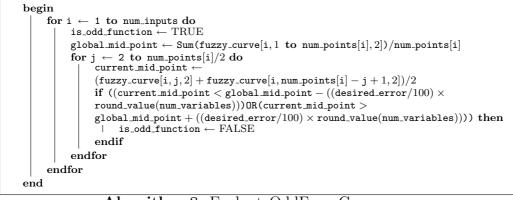
Algorithm 5: GroupedSamplesToComputeOptimalBeta

```
begin
      satisfactory\_partition \leftarrow FALSE
      while satisfactory_partition = FALSE do
             \texttt{test\_points} \leftarrow \texttt{NULL}
             \texttt{train\_points} \gets \text{NULL}
             for j \leftarrow 1 to Length(grouped_samples) do
                    k \leftarrow Rand(1 to Length(grouped_samples))
                    \texttt{test\_points}[:, 1] \leftarrow [\texttt{test\_points}[:, 1] \texttt{ grouped\_samples}[\texttt{j}, \texttt{k}[1], 1]]
                    \texttt{test_points}[:, 2] \leftarrow [\texttt{test_points}[:, 2] \texttt{ grouped_samples}[\texttt{j}, \texttt{k}[1], 2]]
                    \begin{array}{l} \texttt{train_points}[:,1] \leftarrow [\texttt{train_points}[:,1] \texttt{ grouped\_samples}[j,k[2],1]] \\ \texttt{train_points}[:,2] \leftarrow [\texttt{train_points}[:,2] \texttt{ grouped\_samples}[j,k[2],2]] \end{array}
             endfor
             \texttt{beta\_min} \gets \infty
             beta_max \leftarrow 0
             for j \leftarrow 1 to Length(grouped_samples) do
                    current_dist \leftarrow Sort(Abs(test_points[i, 1] - train_points[:, 1]))
                    \texttt{min\_dist} \gets \texttt{current\_dist}
                    [Min(Find(((current_dist-Min(current_dist))>0)))]^2 - Min(current_dist)^2
                    \begin{split} & \texttt{beta_min} \gets \texttt{Min([beta_min \ \texttt{Sqrt}(-(\texttt{min_dist}/\texttt{Log}(0.01 \times \texttt{desired\_error})))])} \\ & \texttt{max_dist} \gets \texttt{Max}(\texttt{current_dist})^2 - \texttt{Min}(\texttt{current\_dist})^2 \end{split}
                    /Log(0.01×(100-desired_error))))])
             endfor
             num\_decades \leftarrow LogCommon(beta\_max)-LogCommon(beta\_min)
             \texttt{points\_per\_decade} \gets 3
             square\_error[:,1] \leftarrow LogDivision(beta\_min to
             beta_max,num_decades,points_per_decade)
             \texttt{derivative\_square\_error}[:,1] \leftarrow \texttt{square\_error}[:,1]
             for j \leftarrow 1 to Length(derivative_square_error) do
                    derivative_square_error[j,2] \leftarrow 0
                    for k \leftarrow 1 to Length(grouped_samples) do
                           \begin{array}{l} \textbf{A} \leftarrow \texttt{Sum}(\texttt{Exp}(-((\texttt{test_points}[k,1]-\texttt{train_points}[:,1])/\texttt{derivative\_square\_error}[j,1])^2) \times (\texttt{test_points}[k,2]-\texttt{train_points}[:,1]) \\ \end{array}
                           ,2]))
                           B \leftarrow Sum(Exp(-((test_points[k, 1] - train_points[:
                           , 1])/\texttt{derivative\_square\_error[j, 1])^2}) \times ((\texttt{test\_points[k, 1]} - \texttt{train\_points[:}
                           (,1])^2) \times \texttt{train_points}[:,2])
                           C \leftarrow Sum(Exp(-((test_points[k, 1] - train_points[:
                           , 1]) / \texttt{derivative\_square\_error[j, 1])^2)) \\
                           D \leftarrow Sum(Exp(-((test_points[k, 1] - train_points[:
                           , 1]) / \texttt{derivative\_square\_error[j, 1])^2}) \times ((\texttt{test\_points[k, 1]} - \texttt{train\_points[:}
                            (,1])^{(2)})
                           E \leftarrow Sum(Exp(-((test_points[k, 1] - train_points[:
                           ,1])/derivative_square_error[j,1])^2) \times train_points[:,2])
                           derivative_square_error[j,2] \leftarrow
                           derivative_square_error[j, 2] + (A \times (-B \times C + D \times E))/(C^3)
                    endfor
                    \texttt{derivative\_square\_error[j,2]} \leftarrow
                    \texttt{derivative\_square\_error[j,2]/(\texttt{derivative\_square\_error[j,1]^2})}
             endfor
             for j \leftarrow 2 to Length(derivative_square_error) do
                    if (derivative_square_error[j, 2 \ge 0) AND(derivative_square_error[j - 1, 2] \le 0)
                    then
                           \texttt{satisfactory\_partition} \gets TRUE
                    endif
             endfor
      endw
end
```

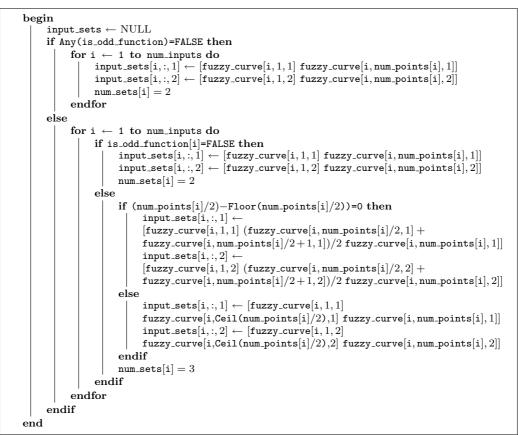
Algorithm 6: TestTrainPointsToComputeOptimalBeta



Algorithm 7: ComputeFuzzyCurve



Algorithm 8: EvaluateOddFuzzyCurve



Algorithm 9: ExtremaFuzzySets



Algorithm 10: ComputeLinearFuzzyCurve



Algorithm 11: PossibleOutSetsWithWangMendel

```
begin
         sets\_to\_cluster \leftarrow Sort(rules[:,num\_variables])
         for i \leftarrow 1 to num_inputs do
                  \texttt{possible\_points} \leftarrow \texttt{NULL}
                   min_point +
                   Round(Min(sets_to_cluster)/round_value[num_variables])×round_value[num_variables]
                   step_point \leftarrow round_value[num_variables]
                   max_point +
                   Round(Max(sets_to_cluster)/round_value[num_variables])×round_value[num_variables]
                   \texttt{current\_point} \gets \texttt{min\_point}
                   repeat
                            \texttt{possible\_points} \gets [\texttt{possible\_points current\_point}]
                             \texttt{current\_point} \gets \texttt{current\_point} + \texttt{step\_point}
                   until current_point > max_point
         endfor
         for i \leftarrow 1 to Length(possible_points) do
                   if Any(Abs(points - possible_points[i]) \le (round_value/2)) then
                     points ← [points possible_points[i]]
                   endif
         endfor
         n \leftarrow Length(points)
         \texttt{radius\_a} \leftarrow (\texttt{desired\_error}/100) \times (\texttt{Max(sets\_to\_cluster)} - \texttt{Min(sets\_to\_cluster)})
         \texttt{radius\_b} \gets \texttt{radius\_a}
         for i \leftarrow 1 to n do
                \texttt{p\_factor[i]} \gets \texttt{Sum(Exp(-alpha\_parameter \times (points[i] - points)^2))}
         endfor
         last_max_p_factor \leftarrow Max(p_factor)
         \texttt{max\_p\_factor} \gets \texttt{last\_max\_p\_factor}
         Comment: Initial cluster.
         possible_cluster \leftarrow sets_to_cluster[Find(p_factor = max_p_factor)]
         cluster \leftarrow possible_cluster[1]
         added_cluster \leftarrow possible_cluster[1]
         for i \leftarrow 2 to Length(possible_cluster) do
                   if All(cluster \neq possible_cluster[i]) then
                            cluster \leftarrow [cluster possible_cluster[i]]
                            added_cluster \leftarrow [added_cluster possible_cluster[i]]
                   endif
         endfor
         Comment: Next clusters
         \texttt{no\_more\_clusters} \leftarrow FALSE
         while no_more_clusters = FALSE do
                   for i \leftarrow 1 to Length(added_cluster) do
                          p_factor \leftarrow p_factor - last_max_p_factor \times Exp(-beta_parameter \times (points - p_factor + p
                            added_cluster[i])^2)
                   endfor
                   \textbf{if last_max_p_factor} > ((\texttt{desired\_error}/100) \times \texttt{max_p_factor}) \textbf{ then }
                            possible\_cluster \leftarrow points[Find(p\_factor = last\_max\_p\_factor)]
                             \texttt{added\_cluster} \gets \text{NULL}
                            for i \leftarrow 1 to Length(possible_cluster) do
                                      if All(cluster \neq possible_cluster[i]) then
                                                cluster \leftarrow [cluster possible_cluster[i]]
                                                added_cluster \leftarrow [added_cluster possible_cluster[i]]
                                      \mathbf{endif}
                             endfor
                   else
                           no\_more\_clusters \leftarrow TRUE
                   endif
         endw
         end
```

Algorithm 12: ClusteringOutSetsWithChiu



Algorithm 13: EvaluateCurrentModel

then	
	Comment: Successful end of process.
	$\texttt{end_of_process} \leftarrow \text{TRUE}$
else	if Sum(Sum(fuzzy_curve[:,:,5]=0)=num_points)=num_inputs then
	Comment: No more sets are considered due to a high desired error.
	$\texttt{end_of_process} \leftarrow \text{TRUE}$
else	Comment. Second the variable with highest spren in order to diminish it
	Comment: Search the variable with highest error in order to diminish it.
	old_sets ← input_sets
	nighest_error ← Max(fuzzy_curve[:,:,5])
	input_with_highest_error ~ Find(highest_error = Max(highest_error))
	or i ← 1 to Length(input_with_highest_error) do new_set ← NULL
	possible_new_set \leftarrow NULL
	$max_error_found \leftarrow -\infty$
	for $j \leftarrow 1$ to num_points[input_with_highest_error[i]] do
	if fuzzy_curve[input_with_highest_error[i], j, 5] =
	highest_error[input_with_highest_error[i] then
	if fuzzy_curve[input_with_highest_error[i], j, 4] > max_error_fou
	then
	possible_new_set ←
	[fuzzy_curve[input_with_highest_error[i], j, 1]
	fuzzy_curve[input_with_highest_error[i], j, 2]]
	max_error_found
	else if
	fuzzy_curve[input_with_highest_error[i], j, 4] = max_error_found
	possible_new_set[1] \leftarrow
	[possible_new_set[1] fuzzy_curve[input_with_highest_error]
	possible_new_set[2] \leftarrow
	[possible_new_set[2] fuzzy_curve[input_with_highest_error]
	endif
	endif
	if (fuzzy_curve[input_with_highest_error[i], j, 5] <
	highest_error[input_with_highest_error[i]])AND(Length(possible_new
	0) then
	$ \texttt{new_set}[1] \leftarrow [\texttt{new_set}[1] \texttt{Mean}(\texttt{possible_new_set}[1])]$
	$\texttt{new_set}[2] \leftarrow [\texttt{new_set}[2] \texttt{Mean}(\texttt{possible_new_set}[2])]$
	possible_new_set ~ NULL
	$max_error_found \leftarrow -\infty$
	endif
	endfor
	if Length(possible_new_set) then
	$ new_set[1] \leftarrow [new_set[1] Mean(possible_new_set[1])]$
	$new_set[2] \leftarrow [new_set[2] Mean(possible_new_set[2])]$
	endif
	sets_current_variable[:,1] ← [input_sets[input_with_highest_error[i],1 t
	num_sets[input_with_highest_error[i], 1] new_set[1]]
	sets_current_variable[:,2] ← [input_sets[input_with_highest_error[i],1 t
	num_sets[input_with_highest_error[i], 2] new_set[2]]
	sets_current_variable ← Sort(sets_current_variable)
	input_sets[input_with_highest_error[i],:,1] ← sets_current_variable[:,1]
	input_sets[input_with_highest_error[i],:,2]
	num_sets[input_with_highest_error[i]] ~
	$ Sum(input_sets[input_with_highest_error[i],:,1]] > -\infty)$ endfor

Algorithm 14: EvaluateStopDecision